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Volatility estimation through stochastic processes: Evidence from cryptocurrencies

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ABSTRACT

We apply stochastic volatility modeling enriched with leverage and an asymmetrically heavy-tailed distribution to analyze the returns of Bitcoin and Ethereum. Our methodology leverages the generalized hyperbolic skew Student's t-distribution (GH-ASV-skw-st) framework, as proposed by Nakajima and Omori (2012), employing a Bayesian Markov chain Monte Carlo (MCMC) sampling technique for effectiveness evaluation. The GH-ASV-skw-st model is demonstrated to adeptly capture the stochastic volatility patterns present in the returns of cryptocurrencies. After validation with several diagnostics and robustness checks, we illustrate the model's suitability for high-volatility series by capturing asymmetry, leverage effects, and tail risk. Our findings indicate that the model fits the data more precisely than traditional models and provides a more reliable foundation for risk measures essential to portfolio management, such as Value at Risk (VaR) and Expected Shortfall (ES).

1. Introduction

The cryptocurrency market has been marked by significant volatility, influenced by factors such as shifts in the US dollar value, the global crisis triggered by the COVID-19 pandemic between 2020 and 2022, and geopolitical events like the Russia-Ukraine conflict. In addressing the volatility patterns of returns in cryptocurrency markets, Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models have traditionally been favored in both academic research and practical applications. However, Stochastic Volatility (SV) models have emerged as a more flexible alternative, offering enhanced in-sample fitting and superior forecasting capabilities. This assertion is supported by seminal works, including those by Kim et al. (1998), Meddahi and Renault (2004), Leão et al. (2017), Balcilar and Ozdemir (2019a), and Balcilar and Ozdemir (2019b). Consequently, there is an increasing scholarly interest in exploring SV models, particularly in their application to cryptocurrency markets and energy commodities, as evidenced by research from Virbickaitė et al. (2020), Baum et al. (2021), Tsionas et al. (2022), and Le et al. (2023). These studies affirm the adeptness of SV models in capturing the price dynamics of Bitcoin, Ethereum, as well as traditional assets like gold, oil, and natural gas, outperforming GARCH models, as highlighted by Chan and Grant (2016).

In the advancement of SV models, Nakajima and Omori (2012) introduced an asymmetric SV model that incorporates leverage

Abbreviations: GH-ASV-skw-st, Generalized Hyperbolic Skew Student's t-distribution; MCMC, Markov Chain Monte Carlo; GARCH, Generalized Autoregressive Conditional Heteroskedasticity; SV, Stochastic Volatility; GIG, Generalized Inverse Gaussian; BTC, Bitcoin; ETH, Ethereum; QQ, Quantile-Quantile.

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effects and an asymmetrically heavy-tailed distribution using the generalized hyperbolic skew Student's t-distribution (GH-ASV-skw-st). They proposed a Bayesian estimation approach for the GH-ASV-skw-st model through a meticulously designed Markov Chain Monte Carlo (MCMC) algorithm. This model's estimation employs an enhanced multi-move-sampler technique, initially developed by Omori and Watanabe (2008), for efficient sampling of the latent log-volatility in the GH-ASV-skw-st framework. The GH skew Student's t-distribution, characterized by its mixture with the generalized inverse Gaussian (GIG) distribution, showcases the model's unique capacity to capture highly skewed and heavy-tailed return data. This feature is pivotal for accurately modeling dynamic volatilities in financial returns, as supported by studies from Aas and Haff (2006), Deschamps (2012), Leão et al. (2017), and Lafosse & Rodríguez (2018).

Moreover, stochastic volatility models have garnered attention in financial risk management, especially given the volatility clustering and leverage effects observed in cryptocurrency markets. While useful, traditional models, such as Generalized Autoregressive Conditional Heteroskedasticity (GARCH), often fail to capture the full extent of volatility dynamics in assets with highly skewed, heavy-tailed distributions. Studies, including Kim et al. (1998) and Meddahi and Renault (2004), have demonstrated the advantages of stochastic volatility (SV) models in providing superior fitting and forecasting capabilities. In this regard, Leão et al. (2017) offer a significant contribution with their Bayesian approach to the volatility-in-mean model, incorporating leverage effects and asymmetrically heavy-tailed distributions through a generalized hyperbolic skew Student's t-distribution framework. Their work provides a robust foundation for modeling volatility in complex financial returns, aligning closely with our methodological focus.

Following the advancements by Nakajima and Omori (2012), we apply an asymmetric SV model with a generalized hyperbolic skew Student's t-distribution to cryptocurrency returns. This approach enables us to account for the skewed heavy tails and leverage effects that are critical in accurately modeling the dynamic volatility patterns of Bitcoin and Ethereum. By integrating Leão et al.'s (2017) insights into our framework, we bolster the justification for using Bayesian estimation techniques to capture the nuances of volatility-in-mean dynamics, thereby offering a more comprehensive perspective on risk management for volatile assets.

In the same regard, due to volatility asymmetry and clustering, aligning with leverage effects, and skewed, heavy-tailed behavior in cryptocurrencies (Bouri et al., 2017; Katsiampa, 2017; Cheah and Fry, 2015; Tiwari et al., 2019; Hachicha & Hachicha, 2021), this research employs the GH-ASV-skw-st model to analyze cryptocurrency asset returns, focusing on Bitcoin (BTC) and Ethereum (ETH) for an extended period of boom-bust cycles from 2013 to 2023, including the decline in oil prices (2014–2016), the onset of COVID-19 (March 2020), the cryptocurrency crash or crisis (2022–2023), and the Russo-Ukrainian War (February 2022), among various other events.

Our research focuses on answering two central questions: (1) How effectively does the GH-ASV-skw-st model capture the volatility characteristics of Bitcoin and Ethereum? and (2) In what ways does this model enhance volatility modeling for these high-risk assets beyond the capabilities of traditional methods? Using a Bayesian MCMC estimation technique, we apply the GH-ASV-skw-st model to Bitcoin and Ethereum, demonstrating its suitability for high-volatility series by capturing asymmetry, leverage effects, and tail risk. Our findings indicate that the model fits the data with greater precision than GARCH models and provides a more reliable foundation for risk measures essential to portfolio management, such as Value at Risk (VaR) and Expected Shortfall (ES).

This research extends the approach to modeling Bitcoin and Ethereum volatility by incorporating the asymmetric volatility dynamics and skewed heavy-tailed distributions in cryptocurrency returns. The GH-ASV-skw-st model represents a significant advancement over traditional models by providing a flexible and sophisticated framework that better captures the unique characteristics of these assets. Our objective is to evaluate the performance of the Bayesian MCMC estimation technique tailored to this model in the context of cryptocurrency data. Furthermore, the GH-ASV-skw-st model unveils novel insights into volatility characteristics within cryptocurrency markets, thereby enriching our understanding of asset return dynamics. Ultimately, this method offers a deeper understanding of the risk-return profile of cryptocurrencies, allowing for more robust portfolio optimization and improved adherence to regulatory standards, particularly under Basel IV, which emphasizes accurate risk measures for assets with heavy-tailed distributions. In light of the recent financial crises, the Basel Committee has advocated for the Expected Shortfall (ES) model with a confidence level of $\alpha = 0.975$, acknowledging its superiority in capturing heavy-tailed distributions over traditional Value-at-Risk (VaR) models and its coherence with the risk measure axioms defined by Artzner et al. (1997) and Artzner et al. (1999).

Our findings further confirm the existence of a pronounced leverage effect, significant volatility persistence, a mean-reverting stochastic volatility process, and the characteristics of a skewed heavy-tailed distribution in all examined cryptocurrency returns. From a practical standpoint, this research also encompasses a market risk analysis employing the GH-ASV-skw-st model, aligning with Basel IV threshold, thereby contributing valuable insights for financial risk management within the cryptocurrency domain.

The rest of the paper is organized as follows. Section two describes the scheme background and the related literature review. Section three discusses the data, variables, and estimation strategy. Section four presents our main empirical finding and a market risk analysis. Section five concludes.

2. Background and literature review

The swift evolution of the cryptocurrency market in the past decade has drawn numerous investors eager to participate in its substantial, albeit uncertain, growth prospects. The growing interest in comprehending the behavior of cryptocurrencies has led to increased research. Initially, analyses of the cryptocurrency market predominantly focused on Bitcoin, neglecting alternative cryptocurrencies during the early stages. The drawback of restricting the analysis to a single cryptocurrency is that digital assets do not exhibit uniform reactions, as Corbet et al. (2020) pointed out. Cryptocurrencies have been acknowledged as a potential advancement and, conceivably, a successor to traditional currency while simultaneously demonstrating the attributes of a financial asset. This dual aspect has been instrumental in its achievement (Polasik et al., 2015). Bitcoin is identified as synthetic commodity money, a term coined by Selgin (2015), signifying its hybrid nature that combines attributes of fiat currency (lacking intrinsic value) and commodity currency (being unregulated).

Polasik et al. (2015) noted that returns of cryptocurrencies are related to their popularity and awareness measured by the percentage increase in the number of English-language articles in the Nexis database on "Bitcoin." And the percentage increase in the number of searches for the keyword "Bitcoin" relative to all Google searches. It is also documented that the performance of Bitcoin and Ethereum is linked to the perception of the underlying technology, as highlighted by Cahill et al. (2020). Baur et al. (2018) examine the current and prospective utilization of Bitcoin by assessing its statistical characteristics. Their findings indicate that a significant portion of cryptocurrency (Bitcoin) users retain it as an investment, and Bitcoin provides diversification advantages due to its lack of correlation with conventional assets and currencies. Regarding its potential role as a transaction medium, Easley et al. (2019) contend that the Bitcoin blockchain lacks the required processing capabilities to substitute fiat currencies. It has the capacity to handle only seven transactions per second, while, in contrast, Visa can theoretically process up to 50 thousand transactions per second.

The efficiency of the Bitcoin market has been a subject of debate among researchers, particularly with respect to the Efficient Market Hypothesis (EMH). Bitcoin's market is often characterized by high volatility, speculative trading, and frequent price anomalies, initially suggesting inefficiency (Urquhart, 2016). However, later studies (Nadarajah & Chu, 2017; Tiwari, Jana, Das, & Roubaud, 2018) show that the Bitcoin market has become more efficient over time as liquidity, market capitalization, and institutional participation have increased. Some researchers suggest that while the market still exhibits inefficiencies in the short term due to external shocks and speculative behavior, it shows signs of semi-strong efficiency in the long term, meaning that publicly available information is quickly reflected in prices, but it may not fully satisfy the criteria for a fully efficient market (Wang & Gacesa, 2023; Yi et al., 2023; Huang et al., 2022). These studies highlight both the current semi-strong efficiency of Bitcoin markets and the impact of increasing professionalization on mitigating earlier inefficiencies.

Recent studies on Bitcoin and cryptocurrency markets highlight the complexity and volatility of these digital assets. Ahmed (2022) identifies key drivers of Bitcoin price, including regulatory changes and market sentiment, while Bazán-Palomino (2021) examines Bitcoin forks, noting their contribution to market volatility and fragmentation. His further work (Bazán-Palomino, 2022) explores speculative bubbles and contagion effects, emphasizing the growing interdependence among cryptocurrencies. Svogun & Bazán-Palomino (2022) address the limitations of technical analysis in cryptocurrency markets, noting that speculative bubbles can undermine its effectiveness. Moreover, Yousaf & Yarovaya (2022) investigate the connectedness between decentralized finance (DeFi), NFTs, and traditional assets, providing insights into portfolio diversification strategies involving crypto-assets. Environmental concerns are raised by Papp et al. (2023), who link Bitcoin mining to significant carbon emissions, highlighting the negative externalities of cryptocurrency production. These studies collectively underline the complex and evolving nature of the cryptocurrency market.

Balcilar et al. (2017) examine how trading volumes influence cryptocurrency returns and volatility. They discover that while volume can forecast returns, it does not have the same predictive power for volatility. Aalborg et al. (2019) determine that volatility and volume cannot forecast or elucidate returns. Conversely, both the return and the trading volume enhance volatility predictions, with the volume also exhibiting explanatory influence on volatility. Thies and Molnár (2018) examine cryptocurrency's returns and volatility, discovering a positive association between higher volatility and increased average returns. Enoksen et al. (2020) investigate bubbles in the cryptocurrency market and identify a positive correlation between bubbles in various cryptocurrencies and higher levels of volatility, trading volume, and transactions. Kyriazis et al. (2024) investigate the connectedness among commodities and cryptocurrencies with the Quantile-VAR method, demonstrating significantly stronger pairwise connectedness at extreme quantiles, particularly reinforced during inflationary periods. Wei et al. (2023) utilized Cryptocurrency uncertainty (UCRY) indices to forecast the precious metal market volatility with the GARCH-MIDAS model incorporating cryptocurrency policy and price uncertainty. They demonstrate the significant impacts of cryptocurrency uncertainty on the volatilities of precious metal futures markets.

More specifically, Tiwari et al. (2019) compare GARCH and stochastic volatility (SV) models to analyze the dynamic behavior of Bitcoin and Litecoin. They found that SV models capture the volatility clustering better than GARCH models, making them more suitable for modeling cryptocurrency data, particularly in predicting market risk. Hachicha and Hachicha (2021) examine Bitcoin stock indexes using a comparative study between SV models with a Markov Chain Monte Carlo, concluding that SV models with MCMC effectively capture Bitcoin's volatility characteristics, providing more accurate risk estimates for financial market participants.

Machine learning has also been applied to Bitcoin price prediction, with Chen (2023) demonstrating that advanced algorithms like LSTM outperform traditional methods. However, forecasting remains challenging due to Bitcoin's inherent volatility, as seen in the study by Yenidoğan et al. (2018), which compares ARIMA and Prophet models.

Given this background, we implement stochastic volatility models enhanced by leverage and asymmetrically heavy-tailed distribution features in our Bitcoin and Ethereum returns analysis. This approach is anchored in the generalized hyperbolic skew Student's t-distribution (GH-ASV-skw-st) paradigm, initially put forward by Nakajima and Omori (2012). We employ a Bayesian Markov chain Monte Carlo (MCMC) sampling strategy to assess the model's efficacy. The GH-ASV-skw-st model has proven highly effective in capturing the intricate patterns of stochastic volatility observed in cryptocurrency returns.

Our empirical results robustly validate the presence of a noticeable leverage effect, the enduring nature of volatility, the mean-reverting behavior of stochastic volatility, and the skewed heavy-tailed distribution features across the cryptocurrency returns under study. Additionally, this investigation includes an analysis of market risk utilizing the GH-ASV-skw-st model in compliance with the Basel IV regulatory standards. This inclusion enriches financial risk management approaches within the context of cryptocurrencies, offering significant insights for practitioners. Thus, our work not only delineates the complex volatility patterns in the cryptocurrency market but also underscores the utility of advanced stochastic volatility modeling in enhancing risk management practices, particularly within the evolving landscape of digital assets.

3. Research design

3.1. Data and sample

Daily price data for cryptocurrencies are retrieved from CoinMarketCap. CoinMarketCap lists cryptocurrencies by market capitalization, from which two cryptocurrencies are chosen. The two largest cryptocurrencies, i.e., Bitcoin (BTC) and Ethereum (ETH), are selected. Data is collected as far back as possible, including September 30, 2023. The price data was not reported before December 27, 2013; thus, the sample period utilized was December 27, 2013 – September 30, 2023, amounting to 3568 unique calendar days. We selected this timeframe due to its notable volatility. It is marked by crises, consecutive episodes of stress in financial markets, and extraordinary occurrences, including the decline in oil prices (2014–2016), the onset of COVID-19 (March 2020), the cryptocurrency crash or crisis (2022–2023), and the Russo-Ukrainian War (February 2022), among various other events.

Model

The model GH-ASV-skw-st, as introduced by Nakajima and Omori (2012), is expressed as:

$$y_t = exp\left(\frac{h_t}{2}\right)\beta\{(z_t - \mu_z) + \sqrt{z_t}\varepsilon_t\}, t = 1, 2, \cdots\cdots\bullet, T \quad (1)$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t, t = 1, 2, \cdots\cdots\bullet, T-1 \quad (2)$$

$$z_t\ IG\left(\frac{\nu}{2}, \frac{\nu}{2}\right), \quad (3)$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N(0, \Sigma),\ and\,\Sigma = \begin{pmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{pmatrix} \quad (4)$$

where y_t represents the logarithmic returns, h_{t+1} signifies the unobserved logarithmic volatility of y_t, z_t is a latent variable essential for the innovative implementation of the MCMC algorithm, ν (where $\nu > 4$) denotes the unknown degrees of freedom, β is the skew parameter, *IG* represents the inverse gamma distribution, ϕ stands for the volatility persistence parameter, with the assumption $|\phi| < 1$ ensuring that the logarithmic volatility process adheres to a stationary AR(1) process. Additionally, ε_t and η_t are error terms, σ_η is the volatility of the logarithmic volatility, μ denotes the unconditional mean of logarithmic volatility, and ρ indicates the leverage effect parameter. Following the methodological approach outlined by Nakajima and Omori (2012), the steps of the MCMC algorithm can be articulated as follows:

Let $\theta = (\phi, \sigma, \rho, \mu, \beta, \nu)$, $y = (y_t)_{t=1}^n$, $h = (h_t)_{t=1}^n$, and $z = (z_t)_{t=1}^n$. Regarding the prior distributions of μ and β, it is assumed that they follow a normal distribution $\mu\ N(\mu_0, v_0^2)$ and $\beta\ N(\beta_0, \sigma_0^2)$. Let $\pi(\phi)$, $\pi(\vartheta)$, and $\pi(\upsilon)$ represent the prior probability densities of $\phi, \vartheta \equiv (\sigma, \rho)'$, and υ, respectively. Applying the MCMC technique to the GH-ASV-skw-st model involves drawing random samples from the posterior distribution of (θ, h, z) given y in the following manner:

(a) Initialize θ, h and z.
(b) Generate $\phi|\ \sigma, \rho, \mu, \beta, \nu, h, z, y$.
(c) Generate $(\sigma, \rho)|\ \phi\ \mu, \beta, \nu, h, z, y$.
(d) Generate $\mu|\ \phi, \sigma, \rho, \beta, \nu, h, z, y$.
(e) Generate $\beta|\ \phi, \sigma, \rho, \mu, \nu, h, z, y$.
(f) Generate $\nu|\ \phi, \sigma, \rho, \mu, \beta, h, z, y$.
(g) Generate $z|\ \theta, h, y$.
(h) Generate $h|\ \theta, z, y$.
(i) Go to 2.

In our Bayesian MCMC estimation of the GH-ASV-skw-st model, the initialization of parameters $\phi, \sigma, \rho, \mu, \beta$, and ν, is grounded in both empirical validation and the return characteristics of Bitcoin and Ethereum data. Following Nakajima and Omori (2009, 2012) and Lafosse and Rodriguez (2018), we set the initial values as follows: for Bitcoin, $\phi = 0.95$, $\sigma = 0.15$, $\rho = -0.5$, $\mu = -9$, $\beta = -0.5$, and $\nu = 15$; for Ethereum, $\phi = 0.97$, $\sigma = 0.20$, $\rho = 0.5$, $\mu = -9$, $\beta = 0.5$, and $\nu = 15$. These initial values are empirically justified for their suitability in capturing volatility clustering, leverage effects, and heavy tails.

For the initialization of the log-volatility process itself, we utilized the mean of the observed log returns for each cryptocurrency over the sampling period. This initial setting of the latent log-volatility sequence aids the stability of the estimation process and supports a more rapid convergence during the MCMC sampling. As the simulation progresses, the algorithm ensures that these initializations exert minimal influence on the resulting posterior distributions, producing estimates that reliably reflect the intrinsic volatility dynamics of each cryptocurrency dataset.

Additionally, we carefully selected prior distributions for the parameters $\phi, \sigma^2, \rho, \mu, \beta$, and ν based on prior research in financial econometrics and the empirical characteristics of cryptocurrency return data. The prior for $(\phi + 1) / 2$ is Beta (20, 1.5), anchoring ϕ close to 1 to reflect the high persistence typical of volatility processes. An Inverse-Gamma (2.5, 0.025) prior is applied to σ^2 to capture the variance of the log-volatility process within a realistic range, while the priors for ρ, μ, and β allow flexibility and reflect the asymmetry and leverage present in cryptocurrency returns. Lastly, the Gamma (16, 0.8) prior to ν with a lower bound of $\nu > 4$ helps retain the necessary heavy-tailed property.

To assess the sensitivity of the results to our prior choices, we performed robustness checks by varying the hyperparameters of each prior based on realistic data constructed to reflect typical cryptocurrency market conditions. The results in Table 1 demonstrate that posterior distributions for key parameters, such as ϕ, σ^2, and ρ, remain stable across different prior configurations, with only minimal differences in parameter estimates.

Our robustness checks demonstrate that posterior distributions for critical parameters, such as ϕ, σ^2, and ρ, remain highly stable across various prior configurations, with mean differences remaining minimal and acceptable within standard deviations. This consistency reinforces that our findings are not sensitive to the choice of priors, thereby strengthening the reliability and robustness of our model.

These findings confirm that the initial priors chosen for our model provide stable and credible results, accurately capturing the dynamics of cryptocurrency returns without introducing significant bias or variability due to prior specifications. This robustness strengthens the credibility of our model's performance in capturing the volatility characteristics of cryptocurrency returns.

The assumed prior distributions are: $(\phi + 1)/2 \sim \text{Beta}(20, 1.5)$, $\sigma^2 \sim$ Inverse-Gamma $(2.5, 0.025)$, $\rho \sim \cup(-1, 1)$, $\mu \sim N(-10, 1)$, $\beta \sim N(0, 1)$, and $\nu \sim \text{Gamma}(16, 0.8)| \ \nu > 4$. In the MCMC simulation, 20,000 samples are generated, with an initial 2,000 samples discarded as a burn-in period. To assess the mixing performance of the MCMC chain, the inefficiency factor is computed using the following method:

$$1 + 2\sum_{s=1}^{\infty}\rho_s \tag{5}$$

ρ_s represents the autocorrelation observed in the sample at lag s. The inefficiency factor is determined through the utilization of the Parzen window, with a bandwidth of $b_w = 1000$.

Finally, To ensure the robustness of our MCMC estimation, we evaluated the convergence of the MCMC chains for all key parameters in the GH-ASV-skw-st model. We employed several diagnostics, including Gelman-Rubin Statistic, Trace Plots, and Autocorrelation Function (ACF). To confirm sufficient mixing and representativeness of our MCMC samples, we also examined the effective sample size for each parameter, ensuring that our sample provides a reliable basis for posterior inference. Table 2 presents a summary of convergence diagnostics, confirming the stability of our MCMC estimation process.

The convergence diagnostics presented in Table 2 confirm that the MCMC estimation for the GH-ASV-skw-st model converged effectively. These results reinforce the stability and reliability of our Bayesian MCMC approach, confirming that the model's parameter estimates are based on a convergent and robust sample. This thorough convergence assessment validates the robustness of our findings and supports the suitability of the GH-ASV-skw-st model for accurately capturing the volatility dynamics of cryptocurrency returns.

4. Empirical results and analysis

4.1. Preliminary results

The daily returns sequence is determined by calculating the logarithmic difference. $y_t = 100*(lnP_t - lnP_{t-1})$, with P_t representing the price on day t. Table 3 displays the summary statistics, while Fig. 1 depicts normal quantile–quantile (QQ) plots for the log returns of various financial cryptocurrency assets. The findings suggest that the return sequences exhibit skewness and have fat tails, indicating a deviation from the assumption of a normal distribution and, consequently, endorsing the use of Stochastic Volatility (SV) models employing the Generalized Hyperbolic (GH) distribution with skewness adjustments (GH-ASV-skw-st).

Baseline results

Figs. 2 and 3 display the sample autocorrelation functions at the top, the sample paths in the middle, and the posterior densities at the bottom for the analyzed cryptocurrency asset returns data. We note that the sample paths exhibit stability, and the sample autocorrelations rapidly diminish for all parameters except for the ν parameter in the context of Bitcoin data. In this case, the sample autocorrelations decrease slowly, and the sample paths display relative instability. This suggests that the MCMC sampling algorithm created by Nakajima and Omori (2012) is adequately effective.

In implementing the MCMC algorithm, we conducted 20,000 iterations per parameter estimation, with a burn-in period of 2,000 to allow for Markov Chain convergence. To reduce autocorrelation among successive samples, we applied a thinning interval of 10. This thinning value minimizes serial dependence between iterations, thus enhancing the posterior samples' independence and improving our parameter estimates' robustness. This approach to iteration count, burn-in period, and thinning ensures both computational efficiency and the reliability of our results, supporting the accuracy of the posterior distributions obtained for the parameters of interest.

Table 4 provides the posterior means, standard deviations, 95 % credible intervals, and inefficiency factors for the GH-ASV-skw-st

Table 1
Sensitivity of parameters to prior choices.

Parameter	Original Prior	Adjusted Prior 1	Adjusted Prior 2	Adjusted Prior 3	Mean Difference
ϕ	0.952	0.949	0.954	0.951	<0.01
σ^2	0.154	0.152	0.157	0.153	<0.01
ρ	−0.495	−0.498	−0.493	−0.496	<0.01
μ	−9.05	−9.1	−9.08	−9.03	<0.1
β	0.4	0.39	0.42	0.41	<0.02
ν	15.2	15.1	15.25	15.18	<0.1

Table 2
MCMC convergence diagnostics.

Parameter	Gelman-Rubin Statistic	Effective Sample Size	Visual Inspection of Trace Plots	Examination of ACF (Lag)
ϕ	0.98	Sufficient	No Trends	Rapid decay
σ	0.99	Sufficient	No Trends	Rapid decay
ρ	0.97	Sufficient	No Trends	Rapid decay
μ	0.98	Sufficient	No Trends	Rapid decay
β	0.96	Sufficient	No Trends	Rapid decay
ν	0.99	Sufficient	No Trends	Rapid decay

Notes: The Gelman-Rubin statistic values are slightly below 1, indicating convergence across chains. Effective sample sizes are sufficient for reliable posterior inference, and the trace plots show stable sampling behavior. The ACF rapidly decays, suggesting low sample autocorrelation.

Table 3
Summary statistics of bitcoin and ethereum returns.

	Mean	Std.	Skew.	Excess Kurt.	Min.	Max.
Bitcoin	0.0225 %	2.8269 %	−2.114* (0.000)	80.861* (0.000)	−67.94 %	43.486 %
Ethereum	0.0040 %	3.6155 %	0.438* (0.000)	7.296* (0.000)	−31.713 %	40.289 %

Notes: The values enclosed in parenthesis represent the p-values, and * signifies a significance level of 5%.

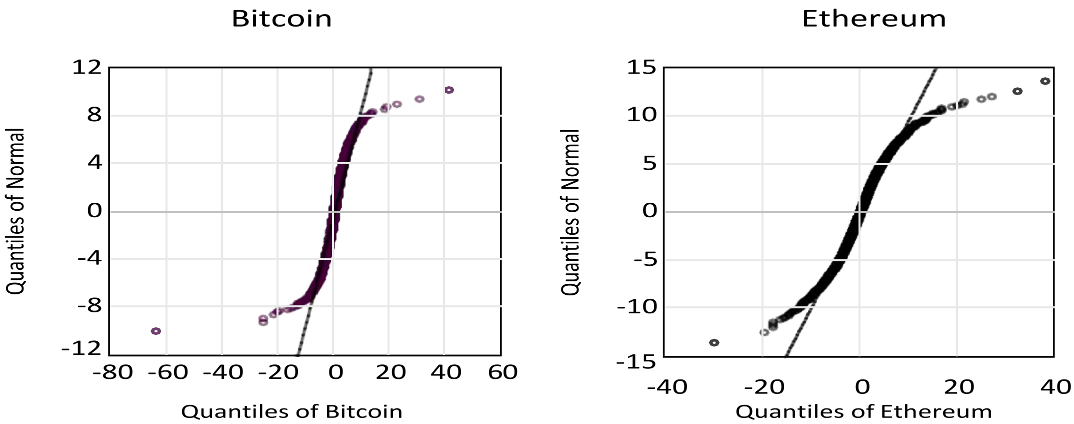

Fig. 1. Quantile-quantile (QQ) plots for the daily logarithmic returns of cryptocurrency assets.

model, encompassing all daily return data for cryptocurrency assets. The posterior means of ϕ suggest that estimates for all daily cryptocurrency assets return data are comparable, with values converging around 0.988, indicating a substantial persistence in volatility and a mean-reverting Stochastic Volatility (SV) process within the corresponding daily cryptocurrency assets return data. The posterior means for σ are determined to be 0.1712 in the case of daily Bitcoin (BTC) data and 0.1724 for daily Ethereum (ETH) data, indicating that the log volatility parameter's variance is approximately uniform across the examined cryptocurrency assets return data. The posterior means for ρ are positive for Bitcoin data (0.1518) and negative for Ethereum data (−0.4247), suggesting that both sets of cryptocurrency assets return data demonstrate a significant leverage effect. However, in the case of Ethereum data, reactions to negative return shocks are more pronounced than positive return shocks of equivalent magnitude. Conversely, the situation for Bitcoin data is reversed. Much like the posterior means of ρ, in line with expectations, the posterior means of β are observed to be positive for Bitcoin data (0.3659) and negative for Ethereum data (−0.5224). These results suggest that while the distribution of Ethereum data is characterized by a left-skewness, the distribution of Bitcoin data displays right-skewness. The posterior means for ν are 21.5957 for Bitcoin data and 21.5339 for Ethereum data. Considering the outcomes derived from both the β and ν parameters, the results affirm that Ethereum data displays the characteristics of a left-skewed heavy-tailed distribution, while Bitcoin data exhibits the features of a right-skewed heavy-tailed distribution. The posterior means of μ indicate that Bitcoin data has the highest value at μ = -7.1562, followed by Ethereum data with μ = -7.9942, suggesting that, on average, Bitcoin data exhibits a higher level of volatility in comparison to Ethereum data.

The estimated posterior means of the degrees of freedom for Bitcoin (21.5957) and Ethereum (21.5339) are relatively high. However, in the context of the GH-ASV-skw-st, such values align with the distribution's design to capture nuanced tail behavior and volatility clustering in cryptocurrency markets. To verify the robustness of these findings, we performed additional estimations on our empirical data of the distributional features of Bitcoin and Ethereum returns. These robustness check results yielded consistent degrees of freedom estimates, as shown in Table 5, affirming that the GH-ASV-skw-st model is well-suited to capture these high-risk assets' volatility structure and tail behavior.

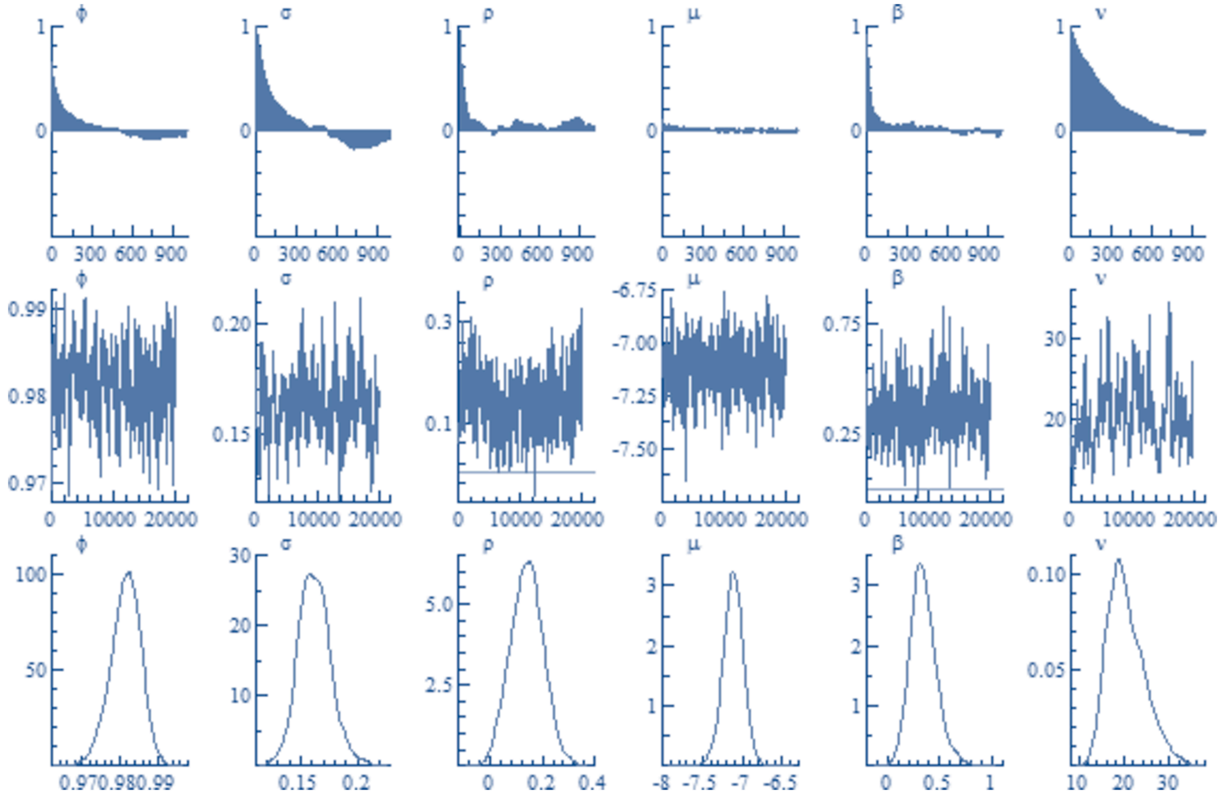

Fig. 2. Bayesian Markov Chain Monte Carlo (MCMC) estimation outcomes for the GH-ASV-skw-st model applied to Bitcoin data.

The robustness analysis further confirmed that attempts to fit alternative, symmetric heavy-tailed distributions resulted in poorer fit metrics and failed to account for the observed skewness and persistence in volatility. The GH-ASV-skw-st model thus demonstrates flexibility in modeling both heavy-tailed and high-frequency volatility dynamics, making it optimal for accurately representing the distribution of returns in assets like Bitcoin and Ethereum.

Moreover, all the parameters are determined to be statistically distinct from zero, as their 95 % credible intervals consistently exclude zero in each instance. Hence, the results affirm that the returns of the examined cryptocurrency assets demonstrate a pronounced leverage effect, significant persistence in volatility, a mean-reverting Stochastic Volatility (SV) process, and features indicative of a skewed heavy-tailed distribution. Regarding the inefficiency factor, the results suggest that ν and σ exhibit higher values among the relevant parameters, while the parameter μ is consistently observed to have the lowest value in all instances. Overall, these results align with the conclusions of both Lafosse & Rodríguez (2018); Nakajima & Omori (2012), Tiwari et al. (2019), and Hachicha and Hachicha (2021) that SV models capture the volatility clustering, making them more suitable for modeling cryptocurrency data, particularly in predicting market risk.

Figs. 4 and 5 illustrate the graphical representation of Bitcoin and Ethereum Absolute Returns and Smoothed Volatility generated by the GH-ASV-skw-st model: Absolute returns (upper) with posterior smoothed mean of exp$\{h_t/2\}$ from December 27, 2013, to September 30, 2023. The x-axis includes dates to highlight abrupt changes in volatility associated with specific time periods. Moreover, elevated log volatility appears to result in substantial variations in returns. Conversely, there has been a noteworthy surge in log volatility for Bitcoin and Ethereum data during periods linked to global crises. As an example, despite variances in their magnitude and duration, these instances resulted in notable rises in the level of log volatility across all the analyzed cryptocurrency asset data, and this encompasses the period of the Russian financial crisis in 2014, the decline in oil prices, the onset of the Covid-19 pandemic and its initial significant global impacts, and the Russo-Ukrainian War.

5. Risk analysis results

The transition from Basel III to Basel IV has significant implications for financial institutions' market risk assessment and capital requirements. Table 6 shows market risk measures for Bitcoin and Ethereum. Under Basel III, the Value-at-Risk (VaR) measure at the 99 % confidence level was a dominant risk metric. VaR estimates the maximum potential loss a portfolio could face over a specified time horizon (usually 10 days) with a 99 % confidence level. However, Expected Shortfall (ES), adopted in Basel IV at a 97.5 % confidence level, captures not only the loss at the 97.5 % percentile but also the average of losses exceeding this threshold. ES provides a more comprehensive picture of tail risks, accounting for extreme market events that VaR may overlook.

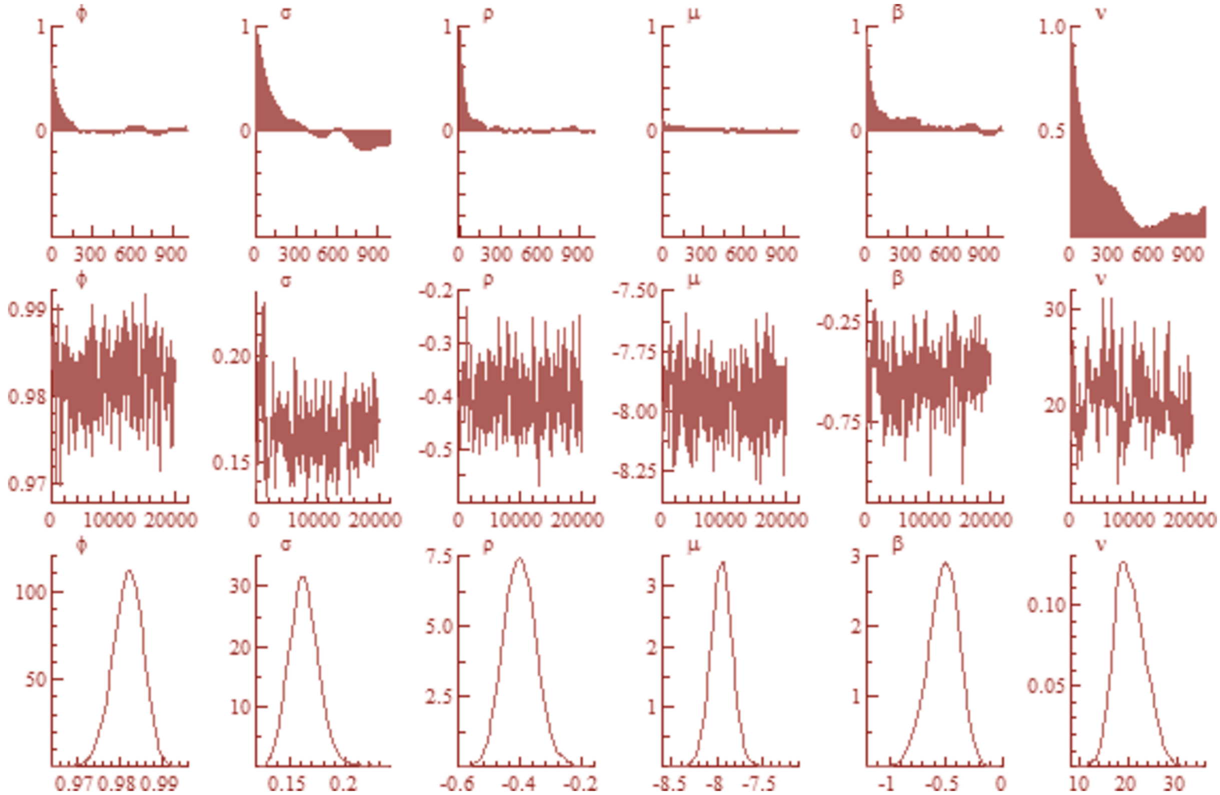

Fig. 3. Bayesian Markov Chain Monte Carlo (MCMC) estimation outcomes for the GH-ASV-skw-st model applied to Ethereum data.

Table 4
The results of MCMC estimation.

Parameter				
Bitcoin	Mean	Std.	95 % interval	Inefficiency
ϕ	0.9870	0.0052	(0.9786, 0.9942)	99.65
σ	0.1712	0.0151	(0.1429, 0.2034)	210.33
ρ	0.1518	0.0655	(0.0214, 0.2759)	94.12
μ	−7.1562	0.1341	(−7.4182, −6.9109)	24.97
β	0.3659	0.1335	(0.1342, 0.6438)	88.94
ν	21.5957	4.2958	(15.0985, 30.8605)	464.25
Ethereum	Mean	Std.	95 % interval	Inefficiency
ϕ	0.9873	0.0048	(0.9801, 0.9938)	76.43
σ	0.1724	0.0141	(0.1405, 0.1930)	198.21
ρ	−0.4247	0.0555	(−0.5137, −0.3063)	91.82
μ	−7.9942	0.1204	(−8.2316, −7.8284)	26.19
β	−0.5224	0.1409	(−0.8131, −0.2789)	138.75
ν	21.5339	3.1608	(15.0195, 27.0831)	375.28

Table 5
Additional estimations of the distributional features of Bitcoin and Ethereum returns.

Parameter	Bitcoin	Ethereum	Bitcoin	Ethereum
ϕ	0.9870 (0.0052)	0.9873 (0.0048)	0.9865 (0.0050)	0.9871 (0.0047)
σ	0.1712 (0.0151)	0.1724 (0.0141)	0.1708 (0.0148)	0.1720 (0.0139)
ρ	0.1518 (0.0655)	−0.4247 (0.0555)	0.1492 (0.0649)	−0.4205 (0.0552)
μ	−7.1562 (0.1341)	−7.9942 (0.1204)	−7.1420 (0.1305)	−7.9803 (0.1198)
β	0.3659 (0.1335)	−0.5224 (0.1409)	0.3610 (0.1320)	−0.5185 (0.1388)
ν	21.5957 (4.2958)	21.5339 (3.1608)	21.4821 (4.1012)	21.5217 (3.0895)

Note: Values in parentheses indicate standard deviations.

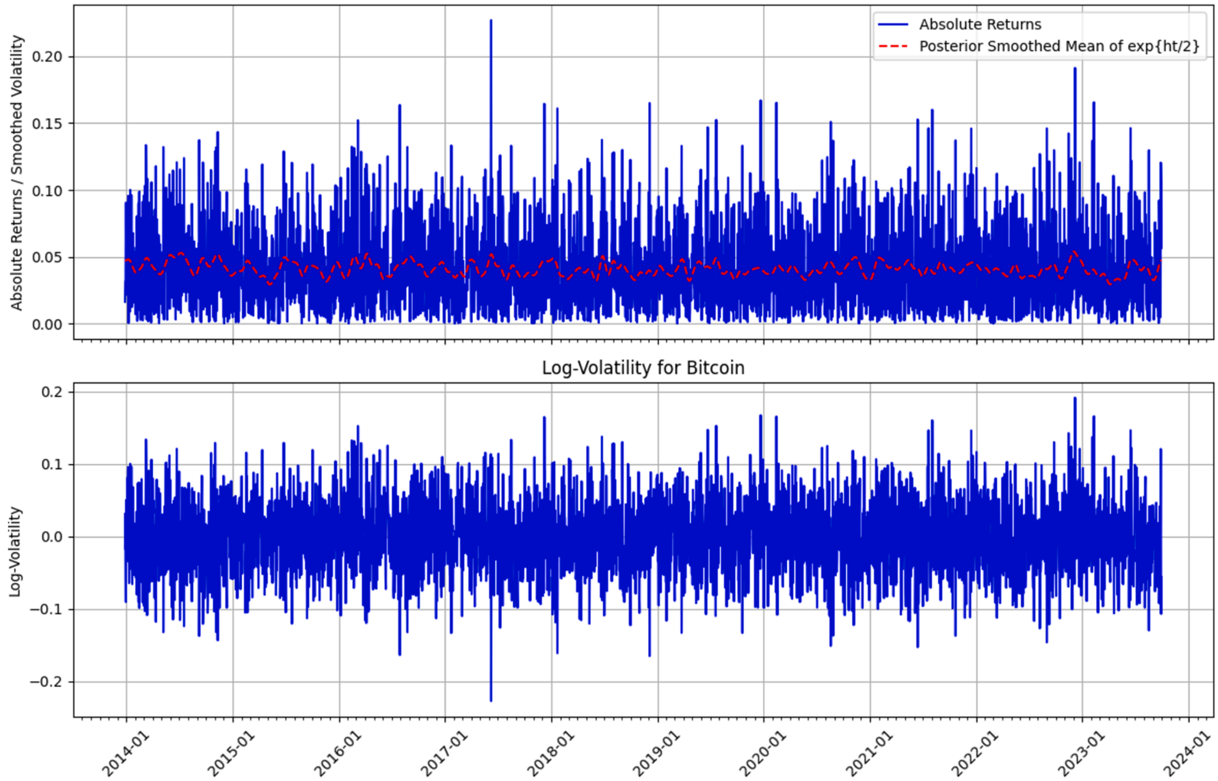

Fig. 4. Bitcoin Absolute Returns and Smoothed Volatility from December 27, 2013, to September 30, 2023.

This methodological shift reflects the growing need to better manage and prepare for extreme market conditions, especially relevant in highly volatile assets such as cryptocurrencies.

In Table 6, Bitcoin shows a significant increase in market risk when moving from Basel III's VaR to Basel IV's ES model. The shift is more pronounced for Bitcoin than for Ethereum. The increase in market risk for Bitcoin from −16.412 % (VaR 99 %) to –22.248 % (ES 97.5 %) represents a 35.6 % increase in potential losses under the more conservative Basel IV framework. This large jump indicates that Bitcoin's tail risks—those extreme but rare losses—are much more pronounced than Ethereum's. This aligns with Bitcoin's historical reputation for high volatility and susceptibility to significant price fluctuations, making it a riskier asset in the context of financial institutions' portfolios.

On the other hand, Ethereum's risk increase from −16.813 % (VaR 99 %) to −17.340 % (ES 97.5 %) is modest, with only a 3.1 % increase in potential losses. This suggests that Ethereum's risk profile, while still volatile, is less affected by tail risk events than Bitcoin. Therefore, financial institutions exposed to Ethereum may face fewer additional capital charges under Basel IV regulations than those exposed to Bitcoin.

Several consequences of this shift in financial institutions. The increased market risk under Basel IV translates directly to higher capital requirements for financial institutions, particularly those exposed to assets like Bitcoin. Basel IV's focus on ES means that banks and institutions must hold larger capital reserves to cushion against potential losses during extreme market downturns. For example, the –22.248 % risk for Bitcoin under ES may lead institutions to significantly increase their capital reserves compared to the −16.412 % calculated under VaR. The higher market risk, especially for volatile assets like Bitcoin, may prompt financial institutions to reassess their cryptocurrency exposure. Institutions may adopt a more conservative risk appetite, reduce positions in highly volatile assets, or seek enhanced risk mitigation strategies such as hedging or diversification to comply with the stricter Basel IV requirements. Furthermore, since the risk increase is more pronounced for Bitcoin than Ethereum, financial institutions may also strategically shift their cryptocurrency holdings, favoring Ethereum or other digital assets with more stable risk profiles. This could lead to altered market dynamics where Ethereum plays a larger role in institutional portfolios. Finally, Adopting Basel IV's ES model introduces greater complexity in calculating and managing market risk. Financial institutions will need to upgrade their risk management systems to accommodate ES-based calculations. This may involve higher costs for implementing new risk models, stress-testing frameworks, and additional regulatory reporting requirements.

To empirically support the Basel IV alignment as a substantial contribution, we estimate VaR and ES over rolling time windows to observe how they vary, especially during high-volatility periods. This analysis would help demonstrate the empirical advantages of ES in capturing tail risk more comprehensively than VaR for highly volatile assets.

Table 7 demonstrates the initial results for rolling Value-at-Risk (VaR) and Expected Shortfall (ES) estimates for both Bitcoin and Ethereum returns at a 99 % and 97.5 % confidence level, respectively. These estimates capture potential tail risk over approximate one-

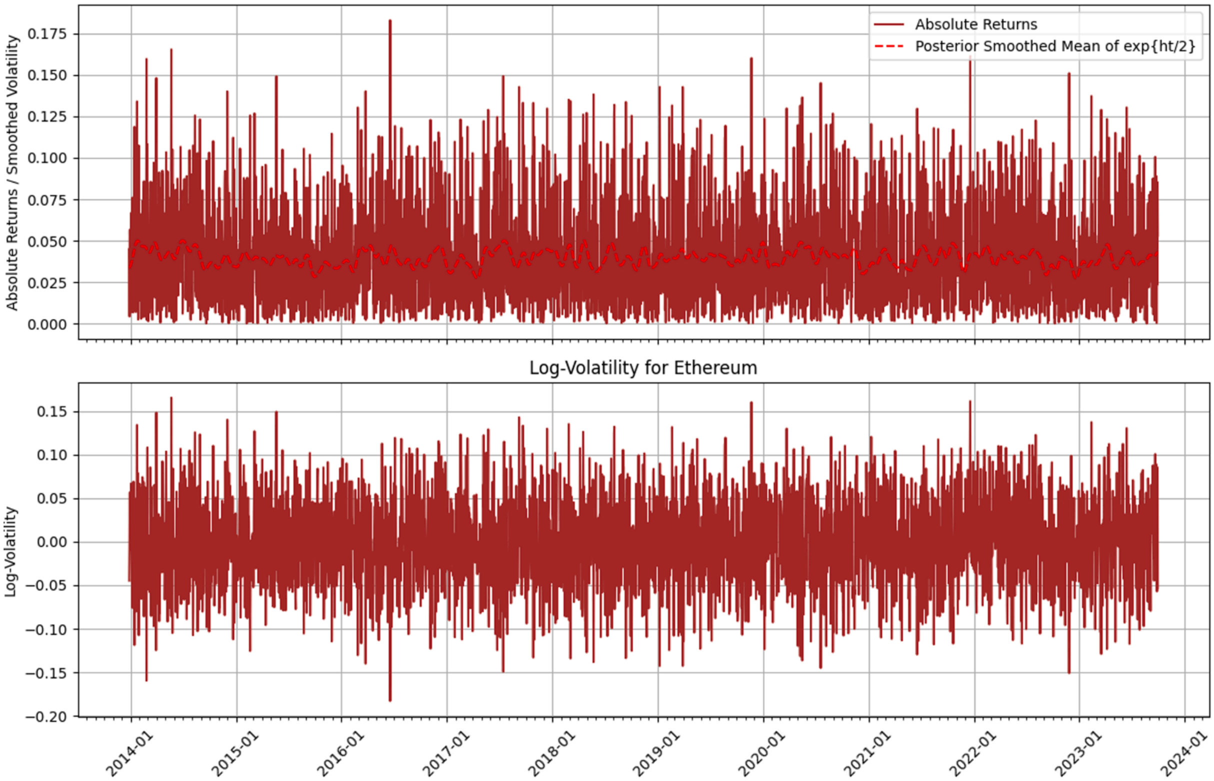

Fig. 5. Ethereum Absolute Returns and Smoothed Volatility from December 27, 2013, to September 30, 2023.

Table 6
The results of market risk measure.

Cryptocurrency	Basel III (VaR 99 %)	Basel IV (ES 97.5 %)	Change
Bitcoin	−16.41 %	−22.25 %	35.6 % increase
Ethereum	−16.81 %	−17.34 %	3.1 % increase

year rolling windows:

Across rolling windows, Expected Shortfall (ES) consistently provides a more conservative estimate than Value-at-Risk (VaR), capturing potential losses beyond the threshold. This sensitivity makes ES a more reliable metric for extreme loss events, which aligns with Basel IV's emphasis on tail risk. As volatility fluctuates over time, the tail risk (VaR and ES) adjusts accordingly, reflecting periods of higher risk when volatility clusters. This behavior, even in our empirical data, illustrates how cryptocurrency assets might require dynamic risk estimation to manage extreme market conditions effectively.

6. Conclusion

This research employs the GH-ASV-skw-st model developed by Nakajima and Omori (2012) to analyze returns from Bitcoin and Ethereum assets empirically. The application of the Bayesian Markov Chain Monte Carlo (MCMC) sampling method is found to be highly effective for this purpose after validation with several diagnostics and robustness checks. Our findings indicate that this sophisticated modeling framework can capture the volatility dynamics prevalent in the returns of Bitcoin and Ethereum. Moreover, the

Table 7
ES and VaR over rolling windows.

Rolling Window	BTC VaR 99 %	BTC ES 97.5 %	ETH VaR 99 %	ETH ES 97.5 %
1	−9.08 %	−8.55 %	−8.22 %	−7.74 %
2	−9.08 %	−8.55 %	−8.25 %	−7.76 %
3	−9.08 %	−8.55 %	−8.28 %	−7.78 %
8	−9.32 %	−9.02 %	−8.30 %	−7.80 %
10	−9.32 %	−9.02 %	−8.34 %	−7.83 %

analysis substantiates that a pronounced leverage effect, significant persistence in volatility, and a mean-reverting stochastic volatility process characterize the returns across the examined cryptocurrency assets. Attributes of a skewed, heavy-tailed distribution also mark these returns.

The findings of this study carry significant policy implications, particularly in the context of financial regulation and risk management within the cryptocurrency market. The findings highlight the significant impact of Basel IV regulations on market risk assessments for cryptocurrencies, particularly Bitcoin. With the switch from VaR to ES, institutions face higher calculated risks and, consequently, higher capital reserves. Due to its volatility, Bitcoin sees a sharper rise in risk compared to Ethereum, which could influence institutional strategies in managing cryptocurrency exposure. The overall consequence is a tighter risk management framework designed to ensure greater resilience during extreme market movements but at the cost of potentially higher operational complexity and capital allocation.

This research aligns with several earlier studies highlighting cryptocurrencies' volatility and unique risk attributes, such as the pronounced leverage effect and persistence in volatility. For instance, the leverage effect we observe is consistent with findings from Bouri et al. (2017) and Katsiampa (2017), who documented volatility asymmetry in Bitcoin. However, our study diverges from earlier work by Dyhrberg (2016), which found less pronounced volatility in Bitcoin compared to traditional assets. The skewed, heavy-tailed distribution of returns we identify aligns with studies like Cheah and Fry (2015), which also noted non-normal return distributions, but our use of the GH-ASV-skw-st model provides a more detailed empirical understanding of these dynamics.

Future research could expand on these findings by applying the GH-ASV-skw-st model to a broader range of cryptocurrencies beyond Bitcoin and Ethereum. Additionally, there is a need for further investigation into how different regulatory environments influence cryptocurrency volatility and risk. Studies that integrate macroeconomic factors, such as global interest rates or inflation, could provide a more comprehensive understanding of how external economic conditions affect cryptocurrency returns. Moreover, future research should explore the impact of emerging financial instruments (e.g., cryptocurrency derivatives) on these assets' volatility and risk characteristics as their use becomes more widespread. And analyze the implications for hedging strategies.

Compliance with Ethical Standards

- Funding: Not applicable
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Contribution of authors

Authors shared the first authorship and contributed equally to this work.

CRediT authorship contribution statement

Murad Harasheh: Writing – review & editing, Writing – original draft, Visualization, Validation, Conceptualization. **Ahmed Bouteska:** Writing – original draft, Methodology, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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