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ORIGINAL RESEARCH

Mapping-Based Accounts of Applicability and Converse Applications

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Abstract
While the problem of the applicability of mathematics in science has been the object of much philosophical discussion, the converse issue of accounting for the successful application of science in mathematics is still in its exploratory stages. In this paper I focus on the latter issue and I discuss it in connection with the mapping view of applied mathematics, which is currently the most influential approach adopted by philosophers to account for the applicability of mathematics in the empirical sciences. More specifically, I address the question of whether the mapping view works for cases of converse applications (i.e., successful applications of the empirical sciences in mathematics). By focusing on some case studies, I argue that the answer to this question is negative and the mapping account of applied mathematics, as it is usually presented in the literature on the applicability of mathematics, does not have the resources to handle the converse applicability issue. To make my point, I will proceed in the following way: first, I will maintain that we can distinguish two types of converse applications, which I name *in-argument* and *in-result* converse applications; next, I will assess the mapping account on these types of converse applications and I will point to the reasons why such view cannot accommodate converse applications within its framework.

1 Introduction

The ‘problem of the applicability of mathematics in science’, or sometimes simply ‘the applicability problem’, is the philosophical issue arising from the successful application of mathematics in the empirical sciences. Surely, such an issue has a long-standing philosophical pedigree and the first reflections on it can be traced back at least to the early Pythagoreanism. Nevertheless, the fact that successful interactions between mathematics and the empirical sciences have been growing massively since the first half of the twentieth century has given new impetus to the

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philosophical study of the applicability of mathematics. Several analyses have been developed by philosophers, and many of these centred around the reaction to Eugene Wigner's paper 'The unreasonable effectiveness of mathematics in the natural sciences' (Wigner, 1960).

Among the accounts that have been proposed, the most influential approach to address the applicability issue remains the so-called 'mapping view of applicability' (Pincock, 2004, 2012; Bueno & Colyvan, 2011; Bueno & French, 2018). The mapping view draws on a structuralist view of models and a 2-place relation account of representation, according to which representation is relation between a model and a target system. In this view, an explanation of the applicability of mathematics in the empirical sciences is given in terms of mappings that are established between mathematics and the empirical system we want to study. These mappings are mathematical and include not only isomorphisms but also other kinds of mathematical mappings such as homomorphisms, epimorphisms and monomorphisms. What mappings ensure is that some crucial features of the empirical system are mirrored in the mathematical model (a 'structure', in the mathematical sense) used to study that system. Thus, according to the mapping view, the applicability of mathematics is fully explained by appreciating the relevant structural similarities between the empirical system under study and the mathematical framework used in the investigation of that system.

Is the applicability problem the end of the story when it comes to the philosophical analysis of the interplay between mathematics and science? This question has been addressed in Molinini (2022) and Molinini (2023), where it is argued that the applicability problem, as usually conceived by philosophers of science and mathematics, does not fully render the philosophical issue that stems from the successful interplay between mathematics and the empirical sciences. The reason is that there also exist "converse applications", namely successful applications of the empirical sciences in mathematics.[1] In these studies, several examples of such applications are offered. Furthermore, some research avenues that are potentially advantageous for tackling the converse applicability issue (i.e., the issue of accounting for the effectiveness of science in mathematics) are outlined. Among these directions for future research, one concerns the analysis of the mapping view in the context of converse applications and is particularly relevant to the present study:

> By offering a novel, although sketchy, picture of application I am not ruling out the possibility that a different account of applicability, as for instance the mapping account view, may be able to provide a unified treatment of direct and converse applications. (Molinini, 2022, p. 19)[2]

Implicit within this remark is the following research suggestion: to investigate whether the mapping view, namely the most influential account that has been

[1] Molinini (2023) offers the first detailed philosophical analysis of converse applications. Nevertheless, it is important to note that other authors had already acknowledged the issue of accounting for the effectiveness of science in mathematics (see, e.g., Levi, 2009; Skow, 2015; Ginammi, 2018).

[2] The expression 'direct applications' refers to successful applications of mathematics in science.

proposed by philosophers to explain the success of applications of mathematics in science, can be used to account for converse applications. The main goal of the present study is to enrich our understanding of converse applications by examining this proposal.

To pursue this objective and assess whether the mapping view of applicability can be used to account for successful applications of science in mathematics, I will proceed in two steps. First, in Sect. 2, I will distinguish two types of converse applications (*in-argument* and *in-result* converse applications). In this section, I will also present two case studies from mathematical practice that bring out the significance of making such type-distinction. Secondly, in Sect. 3, I will focus on the question of whether the mapping view can accommodate converse applications and I will show why this question should be answered in the negative. More precisely, I will draw attention to two issues arising for the mapping account in the context of converse applications: *object*- and *theory*- sensitivity. After presenting my argument, in Sect. 4 I will consider some remarks that can be used to challenge my criticism and I will maintain that these considerations fall short of their goal. Finally, in the concluding section, I will resume the results of my analysis and point to the import that the present work has in the context of the philosophical study of one topic, that of converse applications, which is still largely unexplored.

2 Types of Converse Applications

Before presenting the distinction I make between two types of converse applications, let me introduce a couple of clarifications about what we mean by 'successful applications of science in mathematics' (or 'converse applications').[3]

First, when we speak of 'applications of science in mathematics', we are only considering the application of theoretical parts of science in mathematics. In other words, the applicability in question has to do with the "application of (methods and ideas that are proper to) science in mathematics", with the proviso that these methods and ideas are not empirical in themselves (Molinini, 2023, p. 3). For instance, the use of the alleged mechanical instruments invented by the Pythagorean Archytas of Tarentum to determine mean proportionals, which seems to have angered Plato, would not count as a converse application of mechanics in mathematics,[4] Similarly, Lakatos's use of empirical considerations in his famous discussion of Euler's conjecture for polyhedra, reported in his *Proofs and Refutations* (Lakatos, 1976), would not qualify as a converse application. Or, to use a more modern example, the use of a computer to solve a mathematical problem would not count as a converse

[3] In characterising such applications, I am following Molinini (2023).

[4] The story of Plato's quarrel with Archytas is reported in the *Table Talk*, which is a set of dialogues in Book VIII of Plutarch's *Moralia*. According to Plutarch, Plato criticized Archytas' use of the intelligible realm in geometry and accused him of destroying the value of geometry by appealing to machines to solve geometrical problems.

application, since here too we would not be applying a theoretical bit of science, but rather an empirical piece of it.

Secondly, it is important to explain what we mean by 'successful' applications of science in mathematics. In other words, when can we say that an application of science in mathematics is successful? The answer to this question is, as suggested in Molinini (2023), straightforward: we have a successful (converse) application when we have a purely mathematical proof of the mathematical result that is the object of such (converse) application (*Ibid.*, p. 13). For instance, consider that we are using a physical law within a mathematical argument (this is the scenario that we will encounter in the first example illustrated below). In this case, to say that the application of the physical law in mathematics is successful, we need to know that the mathematical result is correct from a purely mathematical point of view, that is, we need a purely mathematical proof of it. If we do have such proof, then we can answer in the affirmative the question of whether the application of physics in mathematics is successful.

Let me now introduce a new topic for discussion. This topic has gone unnoticed until now and it concerns the distinction of converse applications into two different types: *in-argument* and *in-result* converse applications. I define them in the following way:

> *in-argument* converse applications: converse applications in which non-mathematical considerations are successfully used in the mathematical treatment of a mathematical problem.
>
> *in-result* converse applications: converse applications in which non-mathematical considerations lead to the conclusion of a mathematical argument.

The distinction between *in-argument* and *in-result* converse applications lies in the specific way in which non-mathematical considerations are applied in mathematics (i.e., in their 'point of application').[5] As the name suggests, *in-argument* converse applications are characterised as converse applications in which non-mathematical considerations are applied within the mathematical argument that is used to establish a (mathematical) result. To put it another way, in *in-argument* converse applications the point of application of non-mathematical considerations is within the mathematical argument itself. On the other hand, in *in-result* converse applications the non-mathematical ingredient does not intervene within the mathematical argumentation but at the level of the mathematical result. Once a non-mathematical conclusion is reached, such conclusion can be identified with a mathematical result, and therefore we say that non-mathematical considerations lead to, or 'support', the conclusion of a mathematical argument.

[5] Implicit in my definitions is the assumption that the non-mathematical considerations applied in mathematics are scientific ones. And even if in the present paper I will be focusing on the case of physics, I am leaving open the possibility that the non-mathematical considerations in question come from empirical sciences other than physics.

To illustrate the distinction just introduced, I will consider two examples. Both are drawn from actual mathematical practice, and for each of the two mathematical results relative to them there exists a purely mathematical proof that sanctions the 'success' of the converse application in question (indeed, for each of these results we have more than one mathematical proof). The first example comes from Archimedes' work in geometry and it has already been presented in detail in Molinini (2023). Here I will just recap it to make the following point: the kind of converse application involved in this example is an *in-argument* converse application. The second example, which provides a case for the existence of *in-result* converse applications, deals with an application of mechanics in geometry. This example has also already been used in previous philosophical writing. In fact, it has been discussed by Gila Hanna and Hans Niels Jahnke in the context of a study in mathematics education (Hanna & Jahnke, 2002). Nevertheless, differently from what these authors do in their 2002 paper, here I will examine the example in the context of the philosophical analysis of converse applications.

2.1 An Example of *In-Argument* Converse Application

In the first proposition of his treatise *The Method of Mechanical Theorems*, usually referred to as *The Method*, Archimedes states that the parabolic segment (i.e., the region bounded by a parabola and a line) is four-thirds the triangle it encloses,[6] The diagram in Fig. 1 serves as an illustration of this proposition: if the parabolic segment *ABC* (henceforth ABC_{ps}) is bounded by the straight line *AC* and the parabola γ, and if the point *B* is found by drawing from the middle point of *AC* (the point *D* in figure) a line parallel to the axis of symmetry, or diameter, of the parabola (the dashed line in figure), then the area of the parabolic segment ABC_{ps}

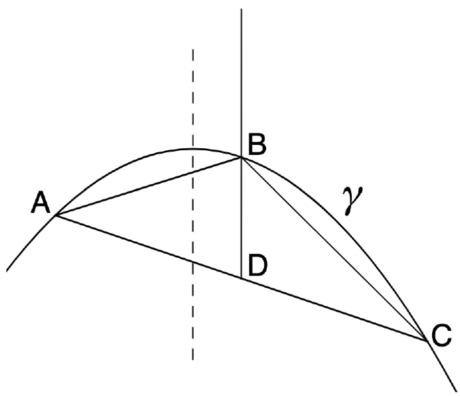

Fig. 1 The parabolic segment *ABC* is bounded by the straight line *AC* and the parabola γ. Point *B* is determined by drawing a line parallel to the axis of symmetry of the parabola from the midpoint *D* of *AC*

[6] In this section, I am reporting only those aspects of Archimedes' argument that are instrumental to make my point. For a more comprehensive presentation of Proposition 1 of *The Method* the reader is referred to Heiberg (1909) and Dijksterhuis (1987). Archimedes' mathematical treatment is also described, in full details, in Molinini (2023).

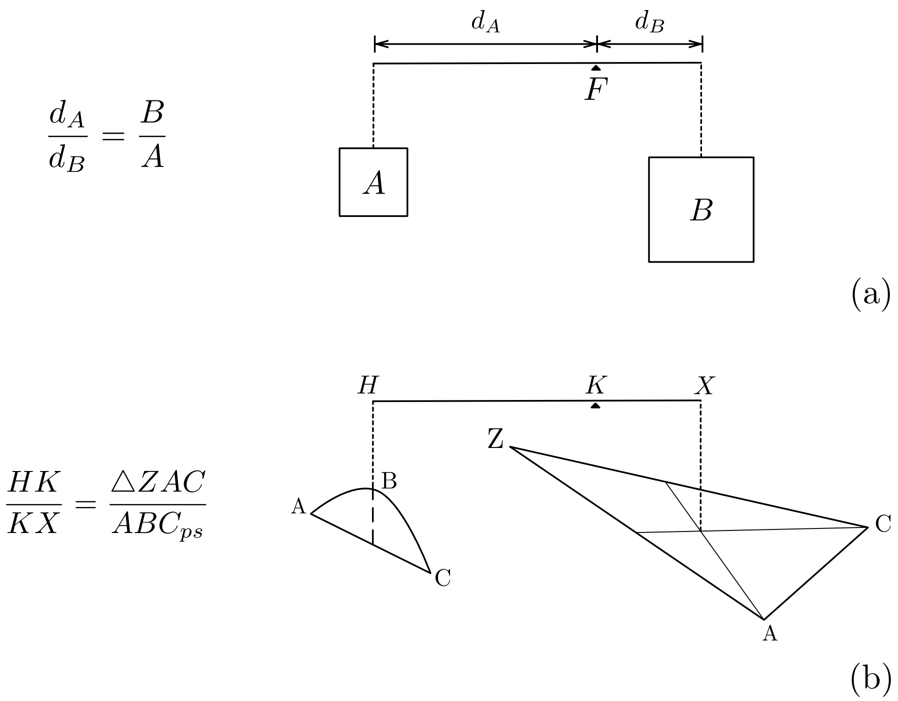

Fig. 2 The law of the lever, established by Archimedes in Propositions 6 and 7 of the treatise *On the Equilibrium of Planes*, is applied in a purely geometrical context

is equal to four-thirds of the area of the triangle $\triangle ABC$ inscribed in the parabola ($ABC_{ps} = \frac{4}{3} \triangle ABC$).

To reach the desired conclusion, Archimedes makes use of physical, and therefore non-mathematical, considerations. More precisely, he uses the law of lever. How? The law of the lever, found by Archimedes in Propositions 6 and 7 of his treatise *On the Equilibrium of Planes*, states that bodies placed on opposite sides of a fulcrum are in equilibrium at distances reciprocally proportional to their weights. For instance, if two bodies of masses A and B are placed on the arms of a straight lever of fulcrum F, and if d_A and d_B are the distances of the bodies' centres of mass from the fulcrum, then the two bodies will balance just in case $d_A/d_B = B/A$ (Fig. 2a). In his mathematical treatment, Archimedes uses this mechanical law in the following way: he takes a straight line, HX, and considers it as an idealized lever, of arms HK and KX, that remains in equilibrium under the influence of two weights; the two 'weights' in question are the parabolic segment ABC_{ps} and the triangle $\triangle AZC$; by considering the geometrical objects in this way, namely as objects that define a lever-system in which longer segments and larger figures have greater weight than shorter segments and smaller figures, Archimedes is able to use the law of the lever and state the following equation: $\frac{HK}{KX} = \frac{\triangle ZAC}{ABC_{ps}}$ (Fig. 2b). Such equation, which results from an application of the law of the lever within a purely geometrical scenario, is what allows Archimedes to proceed in his mathematical treatment and reach the mathematical conclusion.

It has already been noted how Archimede's geometrical treatment of Proposition 1 is an example of converse application, where the success of the application is secured by the existence of a purely mathematical proof of the result (Molinini, 2023). To these considerations, I add a further element: the kind of converse application involved in this example is an *in-argument* converse application, since in this case non-mathematical considerations (the law of the lever) are successfully used in the mathematical treatment of a mathematical problem (Archimedes' geometrical argument).

But do converse applications *always* conform to the type of application described here? In the next subsection I focus on another example, which shows how there exists another type of converse application that is essentially different from an *in-argument* converse application.

2.2 An Example of *In-Result* Converse Application

Let us now move to another converse application: an application of mechanics in geometry to find that given an arbitrary quadrangle the midpoints of its sides form a parallelogram. In mathematics, this result is known as 'Varignon theorem'.

Consider *A*, *B*, *C*, *D* as four weights, each of mass 1, connected by rigid but weightless rods. What we obtain is the physical system represented in Fig. 3, of total mass 4. If we want to determine the centre of gravity of such system, we start from the following consideration: *AB* and *CD*, which are subsystems of the main system, each have weight 2. Next, we observe that the centres of gravity of *AB* and *CD* are their midpoints *W* and *Y*. At this point, we appeal to static considerations and replace *AB* and *CD* by *W* and *Y*, each having mass 2. But *AB* and *CD* constitute the entire system *ABCD*. Therefore, the centre of gravity of *ABCD* is the midpoint *M* of *WY*. By reasoning in the same way, we can consider *ABCD* as constituted by *BC* and *DA*. Hence the centre of gravity of the system *ABCD* must also be the midpoint of *XZ*. Now, since the centre of gravity of the system is unique, this midpoint must be

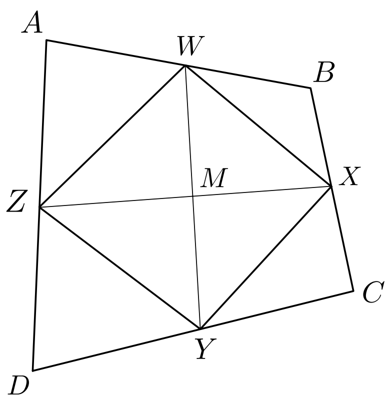

Fig. 3 Diagram representing a physical system *ABCD* composed by four weights (*A*, *B*, *C*, *D*), each of mass 1, connected by rigid but weightless rods. The centre of gravity of *ABCD* is the midpoint *M* of *WY*, which is also the midpoint of *XZ*

M. What this means is that *M* divides both *WY* and *XZ* into equal parts. As a consequence, the object *WXYZ*, whose diagonals are *WY* and *XZ*, is a parallelogram,[7]

What we have found through a physical argument is a result that can be interpreted mathematically. In Euclidean geometry, Varignon theorem deals with the construction of a particular parallelogram ('Varignon parallelogram') from an arbitrary quadrangle. The theorem, which was first proved by the French mathematician Pierre Varignon in 1731 (Varignon, 1731), states that 'The midpoints of the sides of an arbitrary quadrilateral form a parallelogram'. This is exactly the result that we found through the physical argument.

We can now appreciate the difference between the example considered in the previous subsection and the one seen in the previous lines. Both are converse applications (of physics in mathematics). Nevertheless, there is an essential difference between the way in which non-mathematical considerations intervene in the application process. In Archimedes' case, a physical law is applied within a mathematical argument. In the example illustrated in this subsection, on the other hand, physical considerations are used to support a mathematical conclusion (Varignon theorem) and are therefore applied to the conclusion of the mathematical argument. In the latter case, what we have is an *in-result* converse application.

Before proceeding to the next section, and more specifically to the question of whether the mapping view can accommodate *in-argument* and *in-result* converse applications, let me dispel two important doubts that may arise in connection with the last example discussed. First of all, are we sure that the role of physics in this example is not merely a representational one (i.e., that of just representing the mathematical concepts we are interested in)? Secondly, are we applying physics to mathematics in the form of theoretical principles? The doubts in question can be dispelled by introducing some observations. First, it is important to note that the physical considerations used to analyse the system *ABCD* make use of physical laws (e.g., *mechanical laws* about the equilibrium of bodies), which in turn embed concepts that have no mathematical counterpart (e.g., the uniqueness of the *centre of gravity* of a 2-dimensional body). Furthermore, the physical considerations allow one to establish theoretical connections among statements, thus providing us with an argument that can be used to generalise further.[8] For these reasons, the role of physics is not simply that of representing some mathematical concepts, and the application in question should be regarded as an application of theoretical parts of physics.[9] The very same point about the value of physical considerations in this example is made by Hanna and Jahnke:

[7] The argument reported here is taken from the textbook *Some Applications of Mechanics to Mathematics*, written by the Russian mathematician Vladimir Uspenskii (1961, pp. 30–31). The same argument appears in Hanna and Jahnke (2002).

[8] To generalise and extend the physical argument, we can consider a system with a different number of masses and determine its centre of gravity. For the way in which this can be done, see Hanna and Jahnke (2002).

[9] Here I am claiming that physical considerations have a key role in leading to, or supporting, the mathematical result. This, of course, does not mean that the physical argument is the *only* path to the mathematical result. In fact, as I have already noted, Varignon theorem can be proved within pure mathematics.

> The application of physics under discussion here goes well beyond the simple physical representation of mathematical concepts [...] What is being explored here is the classroom use of proofs in which a principle of physics, such as the uniqueness of the centre of gravity, plays an integral role in a proof by being treated as if it were an axiom or a theorem of mathematics. This application of physics is also entirely distinct from experimental mathematics, which purports to employ empirical methods to draw valid general mathematical conclusions from the exploration of a large number of instances. (Hanna & Jahnke, 2002, p. 1)

Having illustrated the distinction between two types of converse applications, and how this distinction is supported by two cases of converse applications taken from actual mathematical practice, in the next section I move to the examination of the mapping view in the context of converse applications. The distinction made between *in-argument* and *in-result* converse applications will play a key role in developing my analysis.

3 Mapping View and Converse Applications

Rather than a single view, the mapping view can be described as a set of approaches. In fact, proponents of this picture of application adopt different stances on what mappings are to be adopted and how these mappings should be determined. Nevertheless, what all these approaches have in common is that they "seek to explain the utility of mathematics in some applied situation by demonstrating the existence of the right kind of map from a mathematical structure to some appropriate physical structure" (Batterman, 2010, p. 8). In the present section, I will consider this idea as the hallmark of the mapping view and my analysis will be directed towards the mapping view in general (rather than towards one or more specific mapping approaches).

Let us now address the following question: Can the mapping account of applied mathematics accommodate *in-argument* and *in-result* converse applications? Below I offer a twofold argument to answer this question in the negative. First, I show that the mapping view falls short in explaining *in-argument* converse application because of a specific issue that I call *object-sensitivity*. Next, I show that the mapping view also faces a challenge when it comes to *in-result* converse applications, since in this context it suffers from another issue that I name *theory-sensitivity*. Taken together, these issues highlight that the mapping view, in the general form discussed here, is not able to handle converse applications.

Before moving to the specific issues just mentioned, it is important to discuss a point that will be instrumental in building my argument. In order to assess the mapping account for a particular scenario, the standard way to proceed is to characterise the mathematical and the empirical content of our setting in structural terms and see whether the (mathematical and empirical) structures are related through one or more suitable mappings. Thus, we can represent the mathematical content by a structure $T = < E, R_t >$, where E is a non-empty set of mathematical objects and R_t is a non-empty indexed set of mathematical relations on E, and the empirical content by a structure $S =< D, R_s >$ that consists of a non-empty set D of physical objects and a non-empty indexed set R_s of relations on D. In this way, we identify two structures (one relative to the mathematical content and the other relative to the physical content). Afterward, we can assess if there exists a structure-preserving mapping that links the two structures. Now, if we proceed in this way, it seems that the mapping account can be said to work for both *in-argument* and *in-result* converse applications. In fact, in both cases, the structural characterisation of the setting allows us to identity the relevant structures and pick out a structure-preserving mapping. The reason for this is that in the case of converse applications it is always possible to say that T is structurally isomorphic to S, that is, there exists a function $f : D \longrightarrow E$ that is bijective and that preserves relations between the elements of D and E. This is easy to see in our examples. In Archimedes' case, the structure $T =< E, R_t >$ is identified by considering as elements of E the particular geometrical objects that appear in the equation $HK/KX = \triangle ZAC/ABC_{ps}$ and as elements of R_t the geometrical relations that are fixed by the equation itself. If we move to the empirical content, we can characterise $S =< D, R_s >$ as the structure in which D is the set of physical objects that appear in the law of the lever $d_A/d_B = B/A$ and R_s is the set of physical relations fixed by the law of the lever itself. Similarly, if we consider our second example, we can identify: a mathematical structure $T =< E, R_t >$, in which the elements of the set E are some geometrical objects (lines and points) and the elements of R_t are the particular relations existing between these objects according to Varignon theorem; an empirical structure $S =< D, R_s >$ in which D is the set of physical objects that appear in the conclusion of the physical argument (weights and rods) and R_s is the set of physical relations that exist between these objects. In both examples, the mathematical and the physical structures are related through a structure-preserving mapping, and more specifically through an isomorphism.

Thus, the mapping view seems to work very well for converse applications. Nevertheless, this conclusion is too quick. In fact, although the mapping account works very well (and maybe *too well*) according to the analysis just sketched, if we examine its viability from a slightly different standpoint we can see how two important issues arise: *object-sensitivity* and *theory-sensitivity*. It is to these issues that I turn my attention in the remaining part of this section.

3.1 The *Object-Sensitivity* Issue

I introduce *object-sensitivity* as the following issue:

> *object-sensitivity*: the mapping account is not sensitive to some features of the physical setting that are embedded in, and essential to, the non-mathematical considerations that are applied in mathematics.

What does this mean? Consider a physical law in the form of a generalisation (e.g., the law of freely falling bodies, which states that the distance a body falls to earth in time t is $(1/2)gt^2$). Such generalisation usually requires a number of provisos that must be met for it to be valid (e.g., the absence of air resistance). Moreover, the sort of information contained in these provisos is information about some features of the physical setting we are interested in (e.g., the physical setting is a physical setting in which the medium is devoid of resistance). Therefore, we can say that the generalisation embeds a number of features of the physical setting that must be 'as such' if we want to consider the law as valid.[10] Now, *if* some of the conditions concerning these features had been different, but the generalisation had retained the same formal structure (e.g., $s(t) = (1/2)gt^2$), it is reasonable to assume that the result of the converse application would also have been different. Nevertheless, under the counterfactual hypothesis that some conditions on the features of the physical setting were different, the mapping account would still have worked (that is, the analysis provided above in terms of structures and structure-preserving mappings would still apply). More precisely, in such a counterfactual scenario, the formal relations that characterise the generalisation will continue to hold for the relevant mathematical objects, regardless of the provisos on the law that do not actually allow its application, and the appropriate mapping would mirror such formal relations. Therefore, the mapping account is not sensitive to some features of the physical setting that are embedded in, and essential to, the non-mathematical considerations that are applied in mathematics.

The object-sensitivity issue clearly emerges in connection with the analysis of *in-argument* converse applications. Take, for instance, Archimedes' example. In the counterfactual scenario that I am considering, the counterfactual changes affect the provisos required for the law to apply, but the law itself remains unaffected. In Archimedes' case, we can focus on some features of the physical underpinning for the equilibrium-preserving assumption, such as the uniform weightless character of the lever arms. Although such features do not explicitly figure in the law of the lever, they are embedded in this law and are also essential to it. In particular, what we can envisage is a counterfactual scenario in which the proviso concerning the uniform, weightless character of the lever arms is different (e.g., the law applies when the lever arms are not uniform and weightless), but the law of the lever (magnitudes are in equilibrium at distances reciprocally proportional to their weights) still holds. In such a scenario, the law of the lever would *not* apply in mathematics because, as observed by Archimedes himself, in mathematics we do consider segments and figures as uniform and weightless. Nevertheless, in the very same scenario, all the structural relations that characterise the law of the lever are still intact and have a mathematical counterpart. And since these relations are the formal (structural) relations captured by the mapping view when demanded to account for the applicability

[10] My claim that some laws require provisos can be elaborated further in terms of the debate on *strict* generalisations and *ceteris-paribus* generalisations (for a presentation of this debate see Lange, 1993; Earman & Roberts, 1999; Reutlinger & Unterhuber, 2014). Such discussion has received much attention in the philosophical literature on laws and it is unquestionably central to the philosophy of science. Nevertheless, it is not necessary to resort to it in order to make the simple point that some laws require provisos. For this reason, I am not addressing it here.

of physics in Archimedes' example, the mapping would still obtain (regardless of the provisos on the law that do not actually allow the application).

This can be seen more formally. Consider the hypothetical scenario described in the previous paragraph, in which the law of the lever applies when the lever arms are not uniform and weightless. In this case, considering the step at which Archimedes uses the law of the lever in his mathematical treatment, we would (still) have: a structure $S =< D, R_s >$ that captures the empirical content, where D = {body 1 of weight W_1, body 2 of weight W_2, arm 1 of length L_1, arm 2 of length L_2} and R_s is a four-place relation holding between W_1, W_2, L_1, L_2 and expressing the law of the lever (the four-place relation expresses the following proportion: 'W_1 is to W_2 as L_2 is to L_1'); a mathematical structure $T =< E, R_t >$, where E = {geometric figure 1 of area A_1, geometric figure 2 of area A_2, segment 1 of length S_1, segment 2 of length S_2} and R_t is a four-place relation holding between A_1, A_2, S_1 and S_2 (the relation is 'A_1 is to A_2 as S_2 is to S_1'). We can now observe that S is structurally isomorphic to T because there exists a function $f : D \longrightarrow E$ that is bijective and that preserves relations between the elements of D and E.[11] This is all the (structural) information required for the mapping account to capture the successful application of the law of the lever in mathematics. Thus, the mapping has survived because no information about the specific conditions required for the law to apply is mirrored in this account. Nevertheless, since in the counterfactual scenario we are considering the provisos on the law do not allow the application of the law of the lever in mathematics, we do not have an application of the physical law in mathematics.

The principal conclusion drawn from the preceding analysis is that since certain provisos do not define the structural relations that characterise the physical law involved in the application, a change in them does *not* affect the way in which the mapping view accounts for the applicability of physics to mathematics. In fact, in a physically possible scenario in which such provisos are different but the physical law retains the same structure, the mapping established between the mathematical and the physical structures is still one that preserves structure, and the mapping account is still deemed successful. Nevertheless, and this is the key point, we do know that the physical features described by the provisos are crucial to the application process. Indeed, if one or more of these features were different, we would have a different application of physics in mathematics or, as it happens in Archimedes' case, no application at all. Thus, there is something that the mapping account picture of applications is missing when it comes to converse applications, and more particularly when such view is used to give an explanation of the success of non-mathematical considerations in those converse applications that I have named *in-argument* converse applications.

[11] Let me note that, on a more rigorous level of analysis, here we should speak of relations between *properties* of the elements of D and E. Nevertheless, this more fine-grained perspective does not undermine my formal analysis and the point I want to make in this paragraph, since discussing relations between properties of the elements of D and E is essentially discussing relations between elements of these sets (e.g., speaking of a relation between the weight of body 1 and the weight of body 2 is speaking of a relation between body 1 and body 2). For this reason, and also for reasons of brevity, I am using 'relations between elements' instead of 'relations between properties of elements'.

3.2 The *Theory-Sensitivity* Issue

Let us now move to the second issue examined here, that is, *theory-sensitivity*. Such issue can be characterised in the following way:

> *theory-sensitivity*: the mapping account is not sensitive to the actual physics used in the converse application process.

What this means is that *if* some physical laws (the actual physics) had been different but had led to the same result, the mapping account would still have worked. Similarly to the previous issue, what we are considering here is a counterfactual scenario. Nevertheless, differently from the previous issue, we are focusing on the physical laws involved in the converse application, rather than on the features of the physical setting.[12] What we are examining is the way in which the mapping view would respond to a change in one or more physical laws used in the converse application.

In the previous subsection, we noted how object-sensitivity arises in the context of *in-argument* converse applications. In the present subsection, we can proceed in a similar way and observe how theory-sensitivity can be appreciated by considering *in-result* converse applications. Take, for instance, our second example (i.e., the example in which we analysed the system *ABCD*). We can imagine a scenario in which the weight *A* has mass 1 and the weight *B* has mass 2, but the centre of gravity of *AB* is still its midpoint. Of course, this is a counterfactual scenario that does not correspond to our actual physics (indeed, if we use the actual physics, the centre of gravity for such system would not be the midpoint of *A* and *B*). But on a contingentist view of laws, some facts about the world are contingent on the way some aspects of the world are, and we can take some laws of physics as contingent (see, e.g., (Sidelle, 2002; Strevens, 2008). For instance, the fact that the centre of gravity of a system made by *A*, of mass 1, and *B*, of mass 2, is *not* their midpoint, is contingent on how gravity is acting in the world. Thus, we can conceive a physically possible scenario in which the weight *A* has mass 1 and the weight *B* has mass 2, and it still holds that the centre of gravity of *AB* is its midpoint *M*. If we acknowledge such a counterfactual scenario, it is possible to consider again the system *ABCD* and build a physical argument whose conclusion coincides with the conclusion we reached when we presented the example of Varignon theorem in Sect. 2.2. And, at this point, we can ask whether the mapping account would still work (even though we know that the physical laws we used are, from the standpoint of our actual science, incorrect).

Not surprisingly, we discover that the mapping account would still work in scenarios like that envisaged in the previous lines. Why? The diagnosis is similar to

[12] Note that I am implicitly making a distinction between the law and the provisos about the state of affairs described by a law-statement. For instance, I am considering that the law of falling bodies $s(t) = (1/2)gt^2$ should be distinguished from those conditions, like the absence of air resistance, that should be met for such law to apply. Such distinction is usually made in the philosophical literature on laws.

what we have already seen in the case of object-sensitivity and can be resumed by saying that the mapping view is not sensitive to (i.e., it does not track) the physics that is relevant to the application process. When we change the physics of the system, and at the same time we have that the conclusion of the physical argument still applies with success in mathematics, the empirical and mathematical structures examined in the application process are unaffected by the change. And the same holds for the structure-preserving mapping between these structures: the mapping account will continue to work, untouched by the changes that we are envisaging, because the mathematical relations will keep mirroring the empirical relations. However, this would be problematic, as we want to say that the applicability in question holds when we use our current physical laws, and not also when we manipulate the actual physical laws used in the application process.

Let me elaborate more on theory-sensitivity and clarify why it presents an issue for the mapping account.[13] In the counterfactual scenario envisioned above, the application of physics to mathematics is still successful. And we may think that what this shows is that, contra to my claims, certain aspects of the physics are not important to the application. Thus, the advocates of the mapping account would stress that what my argument shows is that only some aspects (e.g., the fact that the centre of gravity is at the midpoint between the two masses) are relevant to the success of the application. Nevertheless, my aim in this section is to present an argument that goes beyond the structural aspects considered by the mapping account. More precisely, what I address in this section is the question of whether wrong physical considerations (that is, physical considerations that do not belong to our actual, or best, physics because they are disconfirmed empirically) genuinely lead to a converse application process. My point is that obtaining an in-argument application through physical considerations that –we know– are not right (as the physical consideration envisaged in the counterfactual scenario) does not count as a genuine successful application. And this even if the result of the argument built on these considerations preserves some structural aspects that can be seen as essential to the (converse) application process. How can I support this view? The argument in favour of this standpoint is twofold. First, we can note how working mathematicians do not consider applications of flawed physics in their applications of physics in mathematics. This holds for the examples of converse applications analysed in the literature (see, e.g., Molinini, 2023), and also for the examples reported in the present work. All such cases are taken from mathematical practice. Thus, if we want to give an account of converse applications that mirrors the way in which working mathematicians apply physics in their practice, it seems that we have to consider that a legitimate converse application does not make use of wrong physics. Secondly, and perhaps more significantly, converse applications resulting from a physical argument based on erroneous physical considerations (such as the physical law discussed earlier, specifically 'the centre of gravity of two different weights A and B is the midpoint of AB') should not be considered genuine converse applications because they rely on arguments that use

[13] I am thankful to an anonymous reviewer for pushing me to clarify my presentation of the theory-sensitivity issue and improve my argument.

principles falling outside the scope of what is accepted within our best physics. This is precisely the point with theory-sensitivity. If we have two arguments that lead to the same converse application, where one argument (e.g., that envisioned in the counterfactual example) relies on considerations we deem illegitimate from a physical standpoint, while the other employs principles grounded in our actual physics, we aim to distinguish between them and assert that the only valid converse application arises from sound physical considerations. The mapping view does not discriminate between the two scenarios (it is not sensitive to the counterfactual change). Nevertheless, it seems that we want to draw such a distinction.[14]

The natural conclusion of this section is that, taken together, object- and theory-sensitivity undermine the possibility of using the mapping view of application to account for converse applications. Furthermore, what the analysis presented thus far seems to highlight is that, rather than providing an *explanation* for the success of science in mathematics, the mapping view offers a picture about *how* some empirical terms that are involved in the application process are represented mathematically. And for it to provide an explanation of the converse application process something more than mere correspondence between an empirical and a mathematical structure is required.[15]

Can we rescue the mapping view from the criticism above? The goal of the next section is to discuss some strategies that proponents of the mapping account can employ to defend their view.

4 Possible Responses to the Criticism

In this section I address four concerns that can arise in connection with the criticism just presented. By limiting my discussion to these concerns, I am not suggesting that they are the only worries that can be raised against my argument. Nevertheless, I see them as particularly relevant to my analysis and for this reason I am narrowing my focus.

First, it can be noticed how the conclusion reached in the previous section (i.e., object- and theory- sensitivity undermine the possibility of using the mapping view of application to account for converse applications) rests on the claims

[14] For the unconvinced reader, let me briefly consider an even more distant-from-actuality (counterfactual) scenario. In this scenario, the centre of gravity of two weights A and B is the midpoint of AB only when A and B are perfect spheres (denote this law as L_S, and W_S as the world in which such law holds). In W_S, the application of a physical argument based on L_S in mathematics is successful. However, when we consider our actual world and actual physics, it is reasonable not to regard the converse application of an argument based on L_S as a genuine converse application. This is because L_S is not recognised as a physical law in our actual world. Nevertheless, the mapping account does not distinguish between the counterfactual and the actual scenarios.

[15] This point resonates with what has been already observed by other authors in their discussions of the mapping account in cases of applications of mathematics in science. For instance, in his analysis of the application of the mapping view in the context of a problem of election design (Arrow's voting problem), Davide Rizza has noted how "mapping-based accounts of applications must therefore include mapping-independent moves as part of the application process, if they are to be realistic."(Rizza, 2013, p. 402).

that (a) converse applications are just of two types, namely *in-argument* and *in-result* converse applications, and that (b) these types of converse applications are essentially different. Are these claims questionable? Of course they are. Nevertheless, those who want to deny (a) would need to find other types of converse applications in the actual practice of mathematicians and show that these types of converse applications do not conform to the types of converse applications introduced here, while those who want to deny (b) should show that there is no essential difference between examples like those discussed here. In both cases, the burden of the proof is on those who want to deny (a) and (b).

Let me now address a second, more significant concern: even granting that the mapping account is not sensitive to the features of the physical setting (object-sensitivity) and to the actual physics used in converse applications (theory-sensitivity), is such insensitivity a real issue from the perspective of the mapping view? Supporters of the mapping view might respond to this question with a simple "no", noting that the fact that the mapping-based account works well when we analyse converse applications in terms of structures marks the end of the story. If we endorse this line of reasoning, insensitivity (in both its variants) does not matter. In fact, since the idea behind the mapping view is to show that the application is successful because there exists a structural similarity between the empirical and the mathematical contents, finding such structure similarity *is* the desired result. To this remark, I respond by observing how the application process does not seem to be just a matter of showing that there is a structural similarity between two structures taken in isolation from the whole physical and mathematical contexts. In fact, what my examination of object- and theory- sensitivity suggests is that an analysis of converse applications in terms of purely structural features may not be sufficient to account for these applications. This is because such analysis lacks relevant information about the provisos on the law(s) and the actual character of the physical considerations used in the application. Thus, if non-structural features can matter to converse applications, we must not overlook these elements when explaining the converse application process. Simply asserting a structural similarity between two structures at the stage where we apply physics to mathematics does not appear to adequately capture the richer context in which the relevant application takes place.

To the considerations just raised, we can add an observation that highlights an interesting asymmetry existing between the way in which the mapping account handles object- and theory- sensitivity in the context of converse applications and the way in which the same account deals with these issues in the context of direct applications (that is, in the context of applications of mathematics to science). Although the mapping view is, according to my argument above, not able to accommodate converse applications because of object- and theory- sensitivity, it is not vulnerable to the same issues when it is considered in the context of direct applications. Why? The reason lies in the following remarks. First, if we consider object-sensitivity in the context of direct applications, we can note how the mathematics used in the application is not subject to the same counterfactual treatment that we used, in Sect. 3.1, to reason on the provisos of the physical law. Mathematical laws, in fact, are not contingent on external conditions like physical laws are. And even if we consider that there are contexts in mathematics where certain conditions or assumptions (analogous to provisos in a broader sense) are required for specific results or theorems to hold, such conditions or assumptions are stated explicitly because they guarantee that the conclusion of the theorem is true. Thus, a change in the assumptions of the deductive argument that leads to a result would always produce a change of the result itself, and therefore a different application of mathematics to science or an invalid proof. In this situation, contrary to what happens with converse applications, the mapping account will be able to track the change by incorporating it into its structural framework.[16] Secondly, if we evaluate the issue of theory-sensitivity in the context of direct applications, we can note how such issue does not arise. The reason is that it would be impossible to imagine a counterfactual scenario in which a mathematical law (i.e., a theorem) or axiom is changed and the mathematical result that is obtained through that law or axiom remains the same. A change in the mathematical law(s) or axiom(s) used to reach a particular result would involve a change in the result itself. This is, after all, a key feature of mathematics: even a slight variation of the statements that are used in the deductive chain that drives us from some statements to the conclusion will lead to a different outcome (or to an invalid proof). Therefore, contrary to what happens with converse applications, in this scenario the mapping account is immune to theory-sensitivity because a change in the theorem or axiom used to reach a conclusion is always mirrored in the structural framework of the account.[17]

There is a third remark that seems to me extremely relevant to the criticism presented in the previous section and that can be advanced against my argument, namely: even if in its basic form the mapping account cannot handle converse applications, such account can be supplemented with a *partial* structures framework and this extension may be sufficient to handle the issues raised here. It is to this third remark that I will now turn my attention.

At the beginning of the previous section, I observed how there exist several strategies to implement the mapping conception of application. I also noted how all these approaches share a common idea: the applicability of mathematics can be explained in terms of (mathematical and empirical) structures and mappings between these structures. The partial structures framework, originally introduced by Newton da

[16] It may be observed that there are also cases in which the mathematical result used in the direct application, say *T*, can be obtained through methods that belong to one or more different mathematical theories (see, e.g., Dawson, 2015; Ording, 2019). In these cases, it seems that the same result can be obtained by changing the features of the mathematical setting. Note, however, that when a mathematical result is obtained through a different mathematical theory, the way in which the result is formulated is different (depending on the theory used) and this change in formulation has a consequence on the way in which the result is applied in science. Therefore, also in such scenario, it would be fair to consider that the application context has radically changed and that, because of its capacity to respond to the change, the mapping-account is not challenged by object-sensitivity.

[17] My feeling is that the asymmetry discussed in this paragraph can be explored further by focusing on the contrast between counterfactuals with a physically possible antecedent and counterpossibles whose antecedents are mathematically impossible. The latter are also called 'countermathematicals' (for an overview of the debate over countermathematicals, and counterpossibles in general, see Baker, 2021; Kocurek, 2021). Due to space constraints, I will not get into this subject here and I leave it for future studies.

Costa and Steven French (da Costa & French, 1990, 2003; French, 2003), and later explicitly discussed in the context of the applicability of mathematics in terms of a three-step framework (Bueno & Colyvan, 2011; Bueno & French, 2018), is among these approaches and provides an extension of the mapping view. Such framework has been considered by many philosophers of science as a philosophical device that it sufficiently strong to explain the dynamics of rational theory change over time, as well as the use of abstract mathematical laws and idealisations in direct applications.[18]

The basic idea of the partial structures approach is the following: rather than requiring complete identity of structure between a structure and an empirical target system, we loosen our requirements on representation to allow partial identity of structure between these systems. Unlike a traditional or 'total' structure (i.e., the structure we have already seen in the previous section), a partial structure is a structure $A =< D, R_i >$ in which each relation is identified with an ordered triple $< R_1, R_2, R_3 >$, where: R_1 is the set of ordered pairs that stand in relation R_i; R_2 is the set of ordered pairs that do not stand in relation R_i; and R_3 is the set of ordered pairs for which it is not defined (i.e., it is left open) whether they stand in relation R_i or not.[19] In this approach, an important role is played by the notion of partial mapping (Bueno, 1997; Bueno et al., 2002). To give an example of partial mapping, let me consider the case of (partial) isomorphisms.[20] Given two partial structures $A =< D, R_i >$ and $B =< E, R'_i >$, where $R_i =< R_1, R_2, R_3 >$ and $R'_i =< R'_1, R'_2, R'_3 >$, the function f from D to E (that is, $f : D \longrightarrow E$) is a partial isomorphism between A and B if: (a) f is bijective, and (b) for all x and y in D, $R_{i1}(x, y)$ if and only if $R'_{i1}(f(x), f(y))$ and $R_{i2}(x, y)$ if and only if $R'_{i2}(f(x), f(y))$ (French, 2003).[21]

A fuller account of how the applicability of mathematics can be treated through a partial structures approach is given by Otávio Bueno and Steven French in their recent book *Applying Mathematics: Immersion, Inference, Interpretation* (Bueno & French, 2018). Building on previous work by Bueno and Colyvan (2011), Bueno and French propose a framework for dealing with the applicability of mathematics within the partial structures approach, in terms of a three-step process: (a) immersion, (b) derivation and (c) interpretation. In the immersion step, a correspondence between mathematics and the empirical set up is established via a suitable mapping. In the second step, the derivation step, some consequences are generated from the

[18] Although it has found much favor among philosophers of science, the partial structures view is not free from criticisms. For instance, some authors have raised objections to using this view, as well as the corresponding notion of partial isomorphism, as a suitable account of representation (see, e.g., Cartwright, 1999; Suárez, 2003, 2004; Pincock, 2005; Frigg, 2006; Batterman, 2010).

[19] If each R_3 is empty, a partial structure becomes a total structure by replacing the ordered triples $< R_1, R_2, R_3 >$ by R_1.

[20] For the sake of simplicity, I am considering here only the case of partial isomorphisms, which can be used to spell out the notion of 'partial identity of structure' between structures. Nevertheless, it is important to note that the partial mappings framework can be extended to include a richer variety of partial morphisms (see, e.g., Bueno & French, 2018).

[21] If $R_{i3} = R'_{i3} = \emptyset$, we do not have partial structures but total ones. In this circumstance, what we obtain is the conventional (i.e., not partial) notion of isomorphism (Bueno, 1997).

mathematical formalism. Finally, in the interpretation step, the mathematical consequences obtained in the derivation step are interpreted in terms of the initial empirical set up via a mapping that does not necessarily coincide with the mapping used in the immersion step. In this account, partial structures intervene at the level of the immersion and derivation steps, since according to Bueno and French these steps, and more precisely the mappings involved in them, can be interpreted with partial morphisms.

What is the utility of extending the mapping view through the framework of partial structures (and partial mappings)? As observed by Chris Pincock, "partial models are useful in indicating *what the theory remains silent about*" (Pincock, 2005, p. 1249, my italics). This means that, by being able to incorporate those elements that are not fully characterised by a theory, the partial structures framework is seen as a useful account also in cases where idealizations are involved. Now, if we recall what we saw about the insensitive character of the mapping account in the context of converse application, we can identify those elements (about which the theory, or model, remains silent) with those features and theoretical components of the physical setting to which the mapping account is insensitive. Thus, it seems that by adopting the partial framework there is a way to avoid both the object- and theory- sensitivity issues and take the mapping view as capable to handle converse applications. To this remark, I reply that there is no such solution in terms of partial structures and mappings in the context of converse applications. And this for the following reason. Differently from what happens when we adopt the partial structures framework to explain direct applications, in the context of converse applications we cannot include some empirical elements to which the account is insensitive into the set R_3 of ordered pairs for which it is not defined whether they stand in relation R_i or not. Why? The reason is very simple: these elements do *not* have a mathematical counterpart. Consider, for instance, what we saw in Archimedes' example. When we discussed object-sensitivity, we noted how the mapping account is insensitive to features like the uniform weightless character of the lever arms. Now, if we adopt a partial mappings framework, we should somehow include these features into the set R_3, which represents those pairs for which we are as yet unsure whether R_i obtains. But, and here is the crucial observation, there is nothing in the mathematical domain that can serve as a counterpart (that is, a representation in the mathematical structure) for physical features like the uniform weightless character of the lever arms. Hence, even though the partial structures extension can be considered as an adequate way to handle direct applications, it does not have the same efficacy in the context of converse applications and it cannot save the mapping view from my criticism above.[22]

[22] The argument just set out is enough to weaken the claim that the adoption of partial structures and partial mappings can save the mapping view from my criticism. Nevertheless, as the reader may have noticed, I have not used considerations of theory-sensitivity in it. The reason for this is that when it comes to theory-sensitivity, it is less clear how my response can be elaborated. In fact, physical laws are usually mathematised. And to develop an argument that shows how the partial framework cannot save the mapping account from the theory-sensitivity issue, we should first of all discuss the possibility of having physical laws that cannot have a mathematical counterpart. For reasons of space, and also in view of the fact that such discussion would take us far from the content of the present paper, I am not addressing this discussion here.

Finally, it can be observed that if we assume the heuristic value of converse applications, there is no obligation for the advocates of the partial structures approach to accommodate such heuristic treatments within their account.[23] Consequently, someone who supports the mapping account might reject my claims in this way.[24] Regarding this concern, it should be noted that the heuristic character of converse applications is not settled in a conclusive way, and therefore it remains an open question as to whether or not the mapping approach can be used to account for the success of converse applications. In fact, although converse applications like those discussed here are seen by some as incorporating heuristic moves, there is no conclusive argument in favour of the purely heuristic character of converse applications. It is true that in Sect. 2 I followed the criterion of success (of converse applications) offered in Molinini (2023), and this criterion points to the heuristic character of such applications. Nevertheless, what if (some) converse applications do not fall within the regime of purely heuristic devices? This possibility should be considered. For instance, in the final pages of Molinini (2023), it is suggested that converse applications could involve more than just heuristic methods (the idea advanced is that, in cases like that of Archimedes, what is involved is an application of metaphysically necessary principles). For this reason, and more precisely because the heuristic or non-heuristic character of converse applications is not settled, a discussion of the mapping account in this context is valuable and my claims cannot be dismissed by appealing to the alleged heuristic character of converse applications.

5 Conclusions

In this paper I have focused on converse applications, namely on successful applications of science in mathematics. With the help of two case studies, I have introduced a distinction between two different types of converse applications: *in-argument* and *in-result* converse applications. Furthermore, I have argued that the mapping view of applicability, which is currently the most widely accepted account of the applicability of mathematics, cannot accommodate converse applications because of the following issues: (*object-sensitivity*) the mapping account is not sensitive to some features of the physical setting that are embedded in, and essential to, the non-mathematical considerations that are applied in mathematics; (*theory-sensitivity*) the mapping account is not sensitive to the actual physics used in the converse application process. Finally, I have tackled some potential objections that can be used against my argument and I have maintained that these objections do not pose a serious challenge to my criticism.

The analysis of converse applications is a relatively uncharted field, presenting philosophers with a wide array of potential research directions. Rather than offering a full-fledged account of converse applications, my aim in the present study

[23] Da Costa and French (2003, p. 53), as well as Bueno and French (2018, p. 47), make clear that there is no obligation for the advocates of the partial structures approach to accommodate heuristic treatments within their account.

[24] I would like to thank an anonymous reviewer for raising this concern.

was to explore one of these research paths to lay the ground for future work on this topic. Surely, new analysis of case studies, as well as a deeper examination of how other conceptions of applicability may prove relevant for a better understanding of converse applications, will enrich or even revise the considerations reported here, thus opening promising avenues for further research.

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