Supplementary material for

The estimation of *b*-value of the frequency-magnitude distribution and of its 1σ intervals from binned magnitude data

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Introduction

We include here additional tables showing simulation results with varying number of data and theoretical *b*-values.

Table S1. Estimates from complete simulated sets with *N*=100, 2δ=0.1 and *b*=0.7

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	0.769901	0.084749	100	0.393114
Aki (1965), Utsu (1966)	(4)	0.706577	0.071219	100	0.928112
Bender (1983)	(6)	0.674516	0.074310	100	0.721033
This paper, magnitudes	(7)	0.708190	0.071720	100	0.915437
This paper, absolute differences by Eq.(23)	(15)	0.714940	0.102108	50	0.891027
This paper, trimmed absolute differences by Eq.(23)	(17)	0.716066	0.107306	46	0.887555

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S2. Estimates from complete simulated sets with *N*=1000, 2δ=0.1 and *b*=0.7

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	0.760484	0.026004	1000	0.014043
Aki (1965), Utsu (1966)	(4)	0.699200	0.021978	1000	0.969920
Bender (1983)	(6)	0.696285	0.022347	1000	0.869662
This paper, magnitudes	(7)	0.700721	0.022122	1000	0.974966
This paper, absolute differences by Eq.(23)	(15)	0.701315	0.031301	500	0.963636
This paper, trimmed absolute differences by Eq.(23)	(17)	0.701498	0.033097	460	0.965967

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S3. Estimates from complete simulated sets with *N*=10,000, 2δ=0.1 and *b*=0.7

Estimator		Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)		(2)	0.759567	0.008161	10000	0.000000
Aki (1965), Utsu (1966)		(4)	0.698480	0.006901	10000	0.822639
Bender (1983)		(6)	0.699408	0.006958	10000	0.926509
This paper, magnitudes		(7)	0.699992	0.006946	10000	0.998386
This paper, absolute differences by Eq.(23)		(15)	0.700147	0.009885	5000	0.988880
This paper, trimmed absolute differences by Eq.(23)		(17)	0.700165	0.010372	4598	0.986693

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S4. Estimates from complete simulated sets with *N*=100, 2δ=0.1 and *b*=1.0

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.140602	0.130137	100	0.239814
Aki (1965), Utsu (1966)	(4)	1.006869	0.101070	100	0.949307
Bender (1983)	(6)	0.964875	0.105965	100	0.731851
This paper, magnitudes	(7)	1.011558	0.102523	100	0.914183
This paper, absolute differences by Eq.(23)	(15)	1.021249	0.145522	50	0.870159
This paper, trimmed absolute differences by Eq.(23)	(17)	1.021194	0.142904	88	0.891143

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S5. Estimates from complete simulated sets with *N*=1000, 2δ=0.1 and *b*=1.0

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.125907	0.039867	1000	0.000587
Aki (1965), Utsu (1966)	(4)	0.996582	0.031225	1000	0.913052
Bender (1983)	(6)	0.994843	0.031965	1000	0.869062
This paper, magnitudes	(7)	1.001003	0.031644	1000	0.974374
This paper, absolute differences by Eq.(23)	(15)	1.001331	0.040499	999	0.974819
This paper, trimmed absolute differences by Eq.(23)	(17)	1.001663	0.043709	885	0.970893

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S6. Estimates from complete simulated sets with *N*=10,000, 2δ=0.1 and *b*=1.0

Estimator	**Eq.**	$\overline{b}_K$	S_K	$\overline{N}$	p
Aki (1965)	(2)	1.124494	0.012507	10000	0.000000
Aki (1965), Utsu (1966)	(4)	0.995589	0.009804	10000	0.652051
Bender (1983)	(6)	0.999174	0.009951	10000	0.929549
This paper, magnitudes	(7)	0.999985	0.009934	10000	1.000000
This paper, absolute differences by Eq.(23)	(15)	1.000114	0.012832	9999	0.993860
This paper, trimmed absolute differences by Eq.(23)	(17)	1.000134	0.013762	8853	0.992708

$\overline{b}_K$ and S_K are the average and the standard deviation of b-values computed for the K=10,000 simulated datasets, $\overline{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column “Eq.” shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S7. Estimates from complete simulated sets with *N*=100, 2δ=0.1 and *b*=1.5

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.820546	0.220736	100	0.094665
Aki (1965), Utsu (1966)	(4)	1.501947	0.149377	100	1.000000
Bender (1983)	(6)	1.450695	0.159020	100	0.741691
This paper, magnitudes	(7)	1.517671	0.154244	100	0.919017
This paper, absolute differences by Eq.(23)	(15)	1.526190	0.196491	99	0.870460
This paper, trimmed absolute differences by Eq.(23)	(17)	1.535430	0.224140	82	0.880431

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S8. Estimates from complete simulated sets with *N*=1000, 2δ=0.1 and *b*=1.5

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.794213	0.067283	1000	0.000000
Aki (1965), Utsu (1966)	(4)	1.486744	0.046178	1000	0.775183
Bender (1983)	(6)	1.492692	0.048027	1000	0.879805
This paper, magnitudes	(7)	1.501569	0.047579	1000	0.973995
This paper, absolute differences by Eq.(23)	(15)	1.501768	0.060370	999	0.981150
This paper, trimmed absolute differences by Eq.(23)	(17)	1.502808	0.068133	828	0.970548

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S9. Estimates from complete simulated sets with *N*=10,000, 2δ=0.1 and *b*=1.5

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.791608	0.021142	10000	0.000000
Aki (1965), Utsu (1966)	(4)	1.485221	0.014528	10000	0.319110
Bender (1983)	(6)	1.498797	0.014987	10000	0.934466
This paper, magnitudes	(7)	1.499960	0.014966	10000	1.000000
This paper, absolute differences by Eq.(23)	(15)	1.500145	0.019108	9999	0.994231
This paper, trimmed absolute differences by Eq.(23)	(17)	1.500202	0.021393	8289	0.993121

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c$

Table S10. Estimates from complete simulated sets with *N*=100, 2δ=0.5 and *b*=0.7

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.103234	0.172009	100	0.000357
Aki (1965), Utsu (1966)	(4)	0.670961	0.062500	100	0.665050
Bender (1983)	(6)	0.681989	0.075978	100	0.809473
This paper, magnitudes	(7)	0.708903	0.074119	100	0.934969
This paper, absolute differences by Eq.(23)	(15)	0.712966	0.097374	50	0.916667
This paper, trimmed absolute differences by Eq.(23)	(17)	0.727632	0.142025	31	0.846974

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S11. Estimates from complete simulated sets with *N*=1000, 2δ=0.5 and *b*=0.7

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.078186	0.051090	1000	0.000000
Aki (1965), Utsu (1966)	(4)	0.664927	0.019403	1000	0.077204
Bender (1983)	(6)	0.697119	0.022934	1000	0.900000
This paper, magnitudes	(7)	0.700705	0.022752	1000	0.979323
This paper, absolute differences by Eq.(23)	(15)	0.701016	0.029962	500	0.975049
This paper, trimmed absolute differences by Eq.(23)	(17)	0.702326	0.041544	309	0.955495

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S12. Estimates from complete simulated sets with *N*=10000, 2δ=0.5 and *b*=0.7

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.075979	0.015962	10000	0.000000
Aki (1965), Utsu (1966)	(4)	0.664403	0.006085	10000	0.000000
Bender (1983)	(6)	0.699520	0.007135	10000	0.944511
This paper, magnitudes	(7)	0.699988	0.007128	10000	0.991747
This paper, absolute differences by Eq.(23)	(15)	0.700100	0.009483	5000	0.993259
This paper, trimmed absolute differences by Eq.(23)	(17)	0.700212	0.012882	3088	0.982898

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S13. Estimates from complete simulated sets with *N*=100, 2δ=0.5 and *b*=1.0

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.947475	0.369722	100	0.000000
Aki (1965), Utsu (1966)	(4)	0.910411	0.078116	100	0.252852
Bender (1983)	(6)	0.979091	0.111999	100	0.836223
This paper, magnitudes	(7)	1.014568	0.109744	100	0.910601
This paper, absolute differences by Eq.(23)	(15)	1.017671	0.136411	50	0.882614
This paper, trimmed absolute differences by Eq.(23)	-	-	-	24	-

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the K=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column “Eq.” shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S14. Estimates from complete simulated sets with *N*=1000, 2δ=0.5 and *b*=1.0

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.884281	0.106413	1000	0.000000
Aki (1965), Utsu (1966)	(4)	0.903155	0.024395	1000	0.000000
Bender (1983)	(6)	0.996548	0.033689	1000	0.916258
This paper, magnitudes	(7)	1.001296	0.033480	1000	0.966446
This paper, absolute differences by Eq.(23)	(15)	1.001698	0.041874	500	0.960875
This paper, trimmed absolute differences by Eq.(23)	(17)	1.004231	0.069217	240	0.954589

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S15. Estimates from complete simulated sets with *N*=10000, 2δ=0.5 and *b*=1.0

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.878329	0.033370	10000	0.000000
Aki (1965), Utsu (1966)	(4)	0.902428	0.007700	10000	0.000000
Bender (1983)	(6)	0.999376	0.010561	10000	0.949578
This paper, magnitudes	(7)	0.999999	0.010548	10000	1.000000
This paper, absolute differences by Eq.(23)	(15)	1.000071	0.013320	5000	1.000000
This paper, trimmed absolute differences by Eq.(23)	(17)	1.000436	0.021368	2403	0.981686

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S16. Estimates from complete simulated sets with *N*=100, 2δ=0.5 and *b*=1.5

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	4.286245	1.167907	100	0.000000
Aki (1965), Utsu (1966)	(4)	1.220084	0.086820	100	0.001962
Bender (1983)	(6)	1.478663	0.181355	100	0.877775
This paper, magnitudes	(7)	1.527358	0.179330	100	0.863684
This paper, absolute differences by Eq.(23)	(15)	1.529780	0.206390	50	0.837442
This paper, trimmed absolute differences by Eq.(23)	-	-	-	15	-

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S17. Estimates from complete simulated sets with *N*=1000, 2δ=0.5 and *b*=1.5

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	4.037235	0.307427	1000	0.000000
Aki (1965), Utsu (1966)	(4)	1.213101	0.027563	1000	0.000000
Bender (1983)	(6)	1.495492	0.054343	1000	0.951408
This paper, magnitudes	(7)	1.502108	0.054033	1000	0.954232
This paper, absolute differences by Eq.(23)	(15)	1.501975	0.062520	500	1.000000
This paper, trimmed absolute differences by Eq.(23)	(17)	1.515564	0.141768	151	0.911699

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

Table S18. Estimates from complete simulated sets with *N*=10,000, 2δ=0.5 and *b*=1.5

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	4.016923	0.095118	10000	0.000000
Aki (1965), Utsu (1966)	(4)	1.212576	0.008662	10000	0.000000
Bender (1983)	(6)	1.499139	0.016910	10000	0.959815
This paper, magnitudes	(7)	1.500027	0.016900	10000	1.000000
This paper, absolute differences by Eq.(23)	(15)	1.500213	0.019652	5000	0.983504
This paper, trimmed absolute differences by Eq.(23)	(17)	1.501361	0.042970	1510	0.974344

$\bar{b}_K$ and S_K are the average and the standard deviation of b-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, p is the performance index computed by the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S19. Aftershock sequence with time-dependent incompleteness; parameters: $\boldsymbol{\lambda = 0.2}$, $\boldsymbol{m = 5.6}$, $\boldsymbol{p_O = 1}$, $\boldsymbol{c_O = 0.01}$, $\boldsymbol{T_E = 5}$ days, $\boldsymbol{N = 1600}$ (before thinning), $\boldsymbol{2\delta = 0.1}$, $\boldsymbol{b = 0.7}$, $\boldsymbol{M_c = M_{mxc} + 4\delta = 1.4}$

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	0.590710	0.054564	104	0.066974
Aki (1965), Utsu (1966)	(4)	0.552814	0.047741	104	0.006035
Bender (1983)	(6)	0.512722	0.055816	104	0.001427
This paper, magnitudes	(7)	0.553579	0.047942	104	0.006660
This paper, absolute differences by Eq.(23)	(15)	0.690707	0.095972	52	0.917980
This paper, trimmed absolute differences by Eq.(23)	(17)	0.697615	0.101923	48	0.981070
This paper, trimmed positive differences by Eq.(22)	(17)	0.699880	0.103493	47	0.999785
This paper, trimmed negative differences by Eq.(22)	(17)	0.707574	0.099433	49	0.950931

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S20. Aftershock sequence with time-dependent incompleteness; parameters: $\boldsymbol{\lambda = 0.2}$, $\boldsymbol{m = 5.6}$, $\boldsymbol{p_O = 1}$, $\boldsymbol{c_O = 0.01}$, $\boldsymbol{T_E = 5}$ days, $\boldsymbol{N = 16,000}$ (before thinning), $\boldsymbol{2\delta = 0.1}$, $\boldsymbol{b = 0.7}$, $\boldsymbol{M_c = M_{mxc} + 4\delta = 1.4}$

Estimator	Eq.	$\overline{b}_K$	S_K	$\overline{N}$	p
Aki (1965)	(2)	0.586450	0.017330	1043	0.000000
Aki (1965), Utsu (1966)	(4)	0.549330	0.015205	1043	0.000000
Bender (1983)	(6)	0.541480	0.016273	1043	0.000000
This paper, magnitudes	(7)	0.550066	0.015266	1043	0.000000
This paper, absolute differences by Eq.(23)	(15)	0.681190	0.029103	521	0.521135
This paper, trimmed absolute differences by Eq.(23)	(17)	0.686713	0.030928	484	0.672272
This paper, trimmed positive differences by Eq.(22)	(17)	0.688435	0.043816	241	0.782778
This paper, trimmed negative differences by Eq.(22)	(17)	0.687702	0.043426	253	0.777708

$\overline{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\overline{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S21. Aftershock sequence with time-dependent incompleteness; parameters: $\boldsymbol{\lambda = 0.2}$, $\boldsymbol{m = 5.6}$, $\boldsymbol{p_O = 1}$, $\boldsymbol{c_O = 0.01}$, $\boldsymbol{T_E = 5}$ days, $\boldsymbol{N = 160,000}$ (before thinning), $\boldsymbol{2\delta = 0.1}$, $\boldsymbol{b = 0.7}$, $\boldsymbol{M_c = M_{mxc} + 4\delta = 1.4}$

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	0.586093	0.005392	10426	0.000000
Aki (1965), Utsu (1966)	(4)	0.549042	0.004732	10426	0.000000
Bender (1983)	(6)	0.548015	0.004895	10426	0.000000
This paper, magnitudes	(7)	0.549776	0.004751	10426	0.000000
This paper, absolute differences by Eq.(23)	(15)	0.680279	0.009201	5213	0.036992
This paper, trimmed absolute differences by Eq.(23)	(17)	0.685634	0.009685	4840	0.147866
This paper, trimmed positive differences by Eq.(22)	(17)	0.685753	0.009745	4839	0.146499
This paper, trimmed negative differences by Eq.(22)	(17)	0.685651	0.009713	4841	0.138064

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S22. Aftershock sequence with time-dependent incompleteness; parameters: $\boldsymbol{\lambda = 0.2}$, $\boldsymbol{m = 5.6}$, $\boldsymbol{p_O = 1}$, $\boldsymbol{c_O = 0.01}$, $\boldsymbol{T_E = 5}$ days, $\boldsymbol{N = 4000}$ (before thinning), $\boldsymbol{2\delta = 0.1}$, $\boldsymbol{b = 1.0}$, $\boldsymbol{M_c = M_{mxc} + 4\delta = 1.3}$

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	0.842503	0.082409	104	0.080614
Aki (1965), Utsu (1966)	(4)	0.767418	0.068258	104	0.003810
Bender (1983)	(6)	0.714674	0.078891	104	0.001431
This paper, magnitudes	(7)	0.769473	0.068817	104	0.004450
This paper, absolute differences by Eq.(23)	(15)	0.962419	0.129661	52	0.762849
This paper, trimmed absolute differences by Eq.(23)	(17)	0.977640	0.141878	47	0.863424
This paper, trimmed positive differences by Eq.(22)	(17)	0.985810	0.147207	46	0.922878
This paper, trimmed negative differences by Eq.(22)	(17)	0.989011	0.138066	47	0.923725

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S23. Aftershock sequence with time-dependent incompleteness; parameters: $\boldsymbol{\lambda = 0.2}$, $\boldsymbol{m = 5.6}$, $\boldsymbol{p_O = 1}$, $\boldsymbol{c_O = 0.01}$, $\boldsymbol{T_E = 5}$ days, $\boldsymbol{N = 40,000}$ (before thinning), $\boldsymbol{2\delta = 0.1}$, $\boldsymbol{b = 1.0}$, $\boldsymbol{M_c = M_{mxc} + 4\delta = 1.3}$

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	0.835400	0.025265	1041	0.000000
Aki (1965), Utsu (1966)	(4)	0.762046	0.021019	1041	0.000000
Bender (1983)	(6)	0.752022	0.022715	1041	0.000000
This paper, magnitudes	(7)	0.764015	0.021183	1041	0.000000
This paper, absolute differences by Eq.(23)	(15)	0.952553	0.040146	520	0.244594
This paper, trimmed absolute differences by Eq.(23)	(17)	0.965537	0.043553	469	0.430854
This paper, trimmed positive differences by Eq.(22)	(17)	0.966745	0.043654	468	0.442129
This paper, trimmed negative differences by Eq.(22)	(17)	0.967363	0.043399	470	0.446190

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S24. Aftershock sequence with time-dependent incompleteness; parameters: $\lambda = 0.2$, $m = 5.6$, $p_O = 1$, $c_O = 0.01$, $T_E = 5$ days, $N = 400,000$ (before thinning), $2\delta = 0.1$, $b = 1.0$, $M_c = M_{mxc} + 4\delta = 1.3$

Estimator	Eq.	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	0.835344	0.008044	10409	0.000000
Aki (1965), Utsu (1966)	(4)	0.762050	0.006694	10409	0.000000
Bender (1983)	(6)	0.761431	0.006987	10409	0.000000
This paper, magnitudes	(7)	0.764015	0.006746	10409	0.000000
This paper, absolute differences by Eq.(23)	(15)	0.952057	0.012605	5204	0.000000
This paper, trimmed absolute differences by Eq.(23)	(17)	0.964870	0.013725	4692	0.010606
This paper, trimmed positive differences by Eq.(22)	(17)	0.965066	0.013844	4690	0.013641
This paper, trimmed negative differences by Eq.(22)	(17)	0.965058	0.013778	4693	0.011774

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S25. Aftershock sequence with time-dependent incompleteness; parameters: $\lambda = 0.2$, $m = 5.6$, $p_O = 1$, $c_O = 0.01$, $T_E = 5$ days, $N = 16,000$ (before thinning), $2\delta = 0.1$, $b = 1.5$, $M_c = M_{mxc} + 4\delta = 1.2$

Estimator	**Eq.**	$\bar{b}_K$	S_K	$\bar{N}$	p
Aki (1965)	(2)	1.306131	0.137636	104	0.184424
Aki (1965), Utsu (1966)	(4)	1.133973	0.103384	104	0.004020
Bender (1983)	(6)	1.067075	0.117219	104	0.001640
This paper, magnitudes	(7)	1.140651	0.105262	104	0.005511
This paper, absolute differences by Eq.(23)	(15)	1.393506	0.187085	52	0.558024
This paper, trimmed absolute differences by Eq.(23)	(17)	1.430806	0.215757	44	0.732624
This paper, trimmed positive differences by Eq.(22)	(17)	1.449873	0.221163	43	0.809885
This paper, trimmed negative differences by Eq.(22)	(17)	1.441412	0.207441	45	0.748422

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S26. Aftershock sequence with time-dependent incompleteness; parameters: $\boldsymbol{\lambda = 0.2}$, $\boldsymbol{m = 5.6}$, $\boldsymbol{p_O = 1}$, $\boldsymbol{c_O = 0.01}$, $\boldsymbol{T_E = 5}$ days, $\boldsymbol{N = 160,000}$ (before thinning), $\boldsymbol{2\delta = 0.1}$, $\boldsymbol{b = 1.5}$, $\boldsymbol{M_c = M_{mxc} + 4\delta = 1.2}$

Estimator	**Eq.**	$\boldsymbol{\bar{b}_K}$	$\boldsymbol{S_K}$	$\boldsymbol{\bar{N}}$	$\boldsymbol{p}$
Aki (1965)	(2)	1.294029	0.041611	1037	0.000000
Aki (1965), Utsu (1966)	(4)	1.126110	0.031505	1037	0.000000
Bender (1983)	(6)	1.116660	0.033835	1037	0.000000
This paper, magnitudes	(7)	1.132499	0.032046	1037	0.000000
This paper, absolute differences by Eq.(23)	(15)	1.379312	0.057407	518	0.040204
This paper, trimmed absolute differences by Eq.(23)	(17)	1.412397	0.065514	445	0.194792
This paper, trimmed positive differences by Eq.(22)	(17)	1.415141	0.065797	444	0.201919
This paper, trimmed negative differences by Eq.(22)	(17)	1.413490	0.065098	446	0.188062

$\bar{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\bar{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.

S27. Aftershock sequence with time-dependent incompleteness; parameters: $\boldsymbol{\lambda = 0.2}$, $\boldsymbol{m = 5.6}$, $\boldsymbol{p_O = 1}$, $\boldsymbol{c_O = 0.01}$, $\boldsymbol{T_E = 5}$ days, $\boldsymbol{N = 1,600,000}$ (before thinning), $\boldsymbol{2\delta = 0.1}$, $\boldsymbol{b = 1.5}$, $\boldsymbol{M_c = M_{mxc} + 4\delta = 1.2}$

Estimator	**Eq.**	$\overline{b}_K$	S_K	$\overline{N}$	p
Aki (1965)	(2)	1.292584	0.013211	10367	0.000000
Aki (1965), Utsu (1966)	(4)	1.125133	0.010009	10367	0.000000
Bender (1983)	(6)	1.128046	0.010470	10367	0.000000
This paper, magnitudes	(7)	1.131492	0.010180	10367	0.000000
This paper, absolute differences by Eq.(23)	(15)	1.377884	0.017956	5183	0.000000
This paper, trimmed absolute differences by Eq.(23)	(17)	1.410471	0.020337	4455	0.000000
This paper, trimmed positive differences by Eq.(22)	(17)	1.410944	0.020364	4453	0.000201
This paper, trimmed negative differences by Eq.(22)	(17)	1.410471	0.020351	4456	0.000000

$\overline{b}_K$ and S_K are the average and the standard deviation of *b*-values computed for the *K*=10,000 simulated datasets, $\overline{N}$ is the average number of shocks in simulated datasets, and *p* is the performance index computed by means of the Eq.(24). The column "Eq." shows the equations used to estimate the b-values. Trimming with $\Delta M'_c = 0.1$.