Third-Degree Price Discrimination in Two-Sided Markets
Online Appendix

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A. Non-negative price constraints

We show that achieving Pareto improvement does not necessarily require that the platform subsidizes either buyers or low-type sellers. The complete analysis is rather intricate and is available upon request. Nevertheless, it can be shown that when moving from a scenario where all participation fees are positive under uniform pricing (which needs additional conditions beyond those outlined in Assumption 1), three distinct cases may emerge with the adoption of price discrimination: (i) participation fees turn negative solely for low-type sellers; (ii) participation fees turn negative solely for buyers; (iii) participation fees turn negative for both buyers and low-type sellers.

Consider case (i), for instance, where we have to impose $f_L = 0$ since the beginning and then recompute the equilibrium under price discrimination. We obtain that our welfare results hold true only when network effects are particularly strong. Otherwise, not only are high-type sellers worse off, but total welfare may also diminish with the adoption of price discrimination. The distortion caused by the non-negative price constraint may then overturn our main welfare results.

However, when network effects are high enough, then welfare increases and high-type sellers may also benefit from price discrimination, thus confirming that Pareto improvement is still possible. Take

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the following numerical values, for example: $\theta_H = 1$,$\theta_L = 0.4$, $b = 0.55$, and $v = 0.1$. It is possible to verify that welfare increases ($SW^D - SW^U = 0.019$) and that high-type sellers are better off ($DS_H^D - DS_H^U = 0.005$). Furthermore, all participation fees are non-negative ($P^D = 0.001 < P^U = 0.028$; $f_L^D = 0 < f^U = 0.017 < f_H^D = 0.062$), and the conditions for interior solutions remain satisfied. Similar results can be obtained for cases (ii) and (iii).

B. Seller-specific buyers' benefits

B.1. Proof of Lemma 1

Straightforward computation of the equilibrium under price discrimination gives

$$f_i^D = \frac{v(\theta_i - b(\theta_i))}{4 - (\theta_H + b(\theta_H))^2 - (\theta_L + b(\theta_L))^2}.$$

Comparing the fees offered to the two types of sellers, we have

$$f_L^D - f_H^D = \frac{v(\theta_L - b(\theta_L) - (\theta_H - b(\theta_H)))}{4 - (\theta_H + b(\theta_H))^2 - (\theta_L + b(\theta_L))^2}.$$

The above fee difference is positive when $\theta_L - b(\theta_L) > \theta_H - b(\theta_H)$.

B.2. Proof of Proposition 6

We study the platform's dual problem of choosing the participation level on each side to maximize profit. The aim is again to show that the platform can attract more buyers under price discrimination, and this in turn entices more sellers to join the platform.

Uniform pricing Under uniform pricing, one can view the platform's maximization program as choosing N_B and N_S to maximize profit, without being able to adjust N_L and N_H. For a given fee f, we have

$$N_H = \theta_H N_B - f, \quad \text{and} \quad N_L = \theta_L N_B - f. \tag{A1}$$

Adding these two equations, one gets the market clearing uniform price

$$F^U(N_B, N_S) = \frac{\theta_H + \theta_L}{2} N_B - \frac{N_S}{2}.$$

On the buyer side, demand is given by

$$N_B = \theta_H N_H + \theta_L N_L - p. \tag{A2}$$

The market clearing price thus depends on the allocation N_H and N_L, not only on the aggregate number of sellers N_S. However, using (A1), we know that under uniform pricing N_H and N_L will necessarily satisfy $N_H = N_L + (\theta_H - \theta_L)N_B$. This implies that

$$N_L = \frac{N_S}{2} - (\theta_H - \theta_L)N_B, \quad N_H = \frac{N_S}{2} + (\theta_H - \theta_L)N_B.$$

Plugging this into (A2), we obtain the market-clearing buyer price:

$$P^U(N_B, N_S) = \frac{b(\theta_H) + b(\theta_L)}{2} N_S - (1 + (b(\theta_H) - b(\theta_L))(\theta_H - \theta_L))N_B.$$

The platform's profit is

$$\Pi^U(N_B, N_S) = N_S F^U(N_B, N_S) + N_B P^U(N_B, N_S).$$

It is straightforward to check that $\frac{\partial^2 \Pi^U(N_B,N_S)}{\partial N_B \partial N_S} > 0$, so that $\widetilde{N_S^U}(N_B)$ and $\widetilde{N_B^U}(N_S)$ are increasing.

The first-order conditions are

$$\frac{\partial \Pi^U(N_B, N_S)}{\partial N_S} = 0 \Leftrightarrow \widetilde{N_S^U}(N_B) = \frac{(b(\theta_H) + b(\theta_L) + \theta_H + \theta_L)N_B}{2}, \tag{A3}$$

$$\frac{\partial \Pi^U(N_B, N_S)}{\partial N_B} = 0 \Leftrightarrow 2\left(1 + (b(\theta_H) - b(\theta_L))(\theta_H - \theta_L)\right) \widetilde{N_B^U}(N_S) = \frac{(b(\theta_H) + b(\theta_L) + \theta_H + \theta_L)N_S}{2}. \tag{A4}$$

Price discrimination Under price discrimination the platform can choose N_B, N_L and N_H. Market-clearing prices are given by

$$F_L^D(N_B, N_L) = \theta_L N_B - N_L, \quad F_H^D(N_B, N_H) = \theta_H N_B - N_H,$$

$$P^D(N_B, N_L, N_H) = b(\theta_H)N_H + b(\theta_L)N_L - N_B.$$

The platform's profit is

$$\Pi^D(N_B, N_L, N_H) = N_L F_L^D(N_B, N_L) + F_H^D(N_B, N_H) + N_B P^D(N_B, N_L, N_H).$$

The first-order conditions are

$$\frac{\partial \Pi^D(N_B, N_L, N_H)}{\partial N_B} = 0 \Leftrightarrow 2\widetilde{N_B^D}(N_L, N_H) = (b(\theta_H) + \theta_H)N_H + (b(\theta_L) + \theta_L)N_L, \tag{A5}$$

$$\frac{\partial \Pi^D(N_B, N_L, N_H)}{\partial N_H} = 0 \Leftrightarrow \widetilde{N_H^D}(N_B) = \frac{b(\theta_H) + \theta_H}{2} N_B, \tag{A6}$$

$$\frac{\partial \Pi^D(N_B, N_L, N_H)}{\partial N_L} = 0 \Leftrightarrow \widetilde{N_L^D}(N_B) = \frac{b(\theta_L) + \theta_L}{2} N_B. \tag{A7}$$

Note that adding (A6) and (A7) gives $\widetilde{N_S^D}(N_B) = \frac{b(\theta_H)+\theta_H+b(\theta_L)+\theta_L}{2} N_B = \widetilde{N_S^U}(N_B)$ (by (A3)): for a given buyer participation level N_B, the optimal seller participation level is the same under the two pricing regimes.

Next, using (A6) and (A7), we obtain that:

$$\widetilde{N_H^D}(N_B) = \frac{b(\theta_H) + \theta_H}{b(\theta_H) + \theta_H + b(\theta_L) + \theta_L} \underbrace{(\widetilde{N_H^D}(N_B) + \widetilde{N_L^D}(N_B))}_{=\widetilde{N_S^D}(N_B)},$$

$$\widetilde{N_L^D}(N_B) = \frac{b(\theta_L) + \theta_L}{b(\theta_H) + \theta_H + b(\theta_L) + \theta_L} \widetilde{N_S^D}(N_B).$$

Because the optimal ratios N_H/N_S and N_L/N_S are constant, we can rewrite (A5) as a function of N_S:

$$2\widetilde{N_B^D}(N_S) = \frac{(b(\theta_H) + \theta_H)^2 + (b(\theta_L) + \theta_L)^2}{b(\theta_H) + \theta_H + b(\theta_L) + \theta_L} N_S.$$

Because $b(\theta_H) + \theta_H > b(\theta_L) + \theta_L$, the right-hand side of the previous equation is larger than $\frac{b(\theta_H)+\theta_H+b(\theta_L)+\theta_L}{2} N_S$, which, by (A4), is equal to $2\left(1 + (b(\theta_H) - b(\theta_L))(\theta_H - \theta_L)\right) \widetilde{N_B^U}(N_S)$. This implies that $\widetilde{N_B^D}(N_S) > \widetilde{N_B^U}(N_S)$.

Putting things together, the facts that (i) all the $\widetilde{N_S}$ functions are increasing, (ii) $\widetilde{N_S^U}(N_B) = \widetilde{N_S^D}(N_B)$, and (iii) $\widetilde{N_B^D}(N_S) > \widetilde{N_B^U}(N_S)$, imply that, in equilibrium, $N_S^D > N_S^U$ and $N_B^D > N_B^U$.

C. Ad-Valorem Fees: Proof of Proposition 7

In this extension, we consider the case where the monopolist platform charges sellers ad-valorem fees. As in the benchmark, we compare the uniform pricing regime where the platform charges the same ad-valorem fee to all sellers ($r_H = r_L = r$) to the one where it sets $r_H \neq r_L$.

Sellers' payoffs. Suppose the platform charges ad-valorem fees r_j to sellers of type j. The payoff of a seller from group $j \in \{H, L\}$ with participation cost k^S from affiliating with the platform is

$$\tilde{\pi}_j(k^S) = (1 - r_j)\theta_j N_B^e - k^S,$$

where N_B^e is the sellers' expectations on the total mass of buyers affiliating with the platform.

Sellers affiliate with the platform if and only if they obtain positive utility from participating $\tilde{\pi}_j(k^S) \geq 0 \implies k^S \leq (1 - r_j)\theta_j N_B^e$ for $j \in \{H, L\}$. Thus, the mass of sellers of type j participating in the platform ecosystem are

$$\tilde{N}_j(N_B^e, f_j) = (1 - r_j)\theta_j N_B^e.$$

The total mass of sellers active on the platform under price discrimination is then

$$\tilde{N}_S(N_B^e, r_H, r_L) = ((1 - r_H)\theta_H + (1 - r_L)\theta_L)N_B^e. \tag{A8}$$

Under a uniform pricing regime, the total mass of sellers active on the platform is instead

$$\tilde{N}_S(N_B^e, r, r) = (1 - r)(\theta_H + \theta_L)N_B^e. \tag{A9}$$

Platform payoffs. Platform profit when employing uniform pricing and discriminatory pricing regimes are respectively given as

$$\max_{r,p} \Pi_U = (p + r(\theta_H N_H + \theta_L N_L))N_B, \quad \max_{r_H, r_L, p} \Pi_D = (p + r_H\theta_H N_H + r_L\theta_L N_L)N_B.$$

Timing and equilibrium concept are the same as in the baseline model, and to ensure an interior solution, we make the following assumption.

Assumption A1. *We assume that buyers' and sellers' valuation for participation on the other side as well as buyer intrinsic valuation are not too large, namely:* $0 < v < \frac{4-2b^2-2b\theta_H-\theta_H^2-2b\theta_L-\theta_L^2}{2}$, $b < \frac{\sqrt{8-(\theta_H-\theta_L)^2}-(\theta_H+\theta_L)}{2} = \bar{b}^{ad}(\theta_H, \theta_L)$, *and* $\theta_L^2 + \theta_H^2 < 4$ *and* $\theta_L < \sqrt{2}$.

Uniform pricing. In this pricing regime, recall the buyer and seller participation from equations (1) (in the main text) and (A9), respectively. In a rational expectations equilibrium agents correctly anticipate participation by the other group, so that participation levels $\tilde{N}_B^U$ and $\tilde{N}_S^U$ satisfy

$$\tilde{N}_B^U = v + b\tilde{N}_S^U - p \quad \text{and} \quad \tilde{N}_S^U = (1 - r)(\theta_L + \theta_H)\tilde{N}_B^U.$$

Solving the above system of equations for N_B^U and N_S^U yields buyer participation and seller total participation as functions of prices:

$$\tilde{N}_B^U(p, r) = \frac{v - p}{1 - b(1 - r)(\theta_H + \theta_L)}, \; \tilde{N}_S^U(p, r) = \frac{(\theta_H + \theta_L)(v - p)(1 - r)}{1 - b(1 - r)(\theta_H + \theta_L)}.$$

Seller demand can be further decomposed into

$$\tilde{N}_H^U(p, r) = \frac{\theta_H(v - p)(1 - r)}{1 - b(1 - r)(\theta_H + \theta_L)}, \; \tilde{N}_L^U(p, r) = \frac{\theta_L(v - p)(1 - r)}{1 - b(1 - r)(\theta_H + \theta_L)}.$$

The platform sets prices to

$$\max_{p,r}(p + r(\theta_H \tilde{N}_H^U(p,r) + \theta_L \tilde{N}_L^U(p,r)))\tilde{N}_B^U(p,r).$$

Differentiating platform profits with respect to p and r and solving the system of first order conditions yields the following prices.

$$\begin{aligned}
\tilde{p}^U &= \frac{v(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}{2b\theta_H^3+\theta_H^4+2b\theta_H\theta_L(b+\theta_L)-\theta_L^2(4-(b+\theta_L)^2)-\theta_H^2(4-b^2-2b\theta_L-2\theta_L^2)},\\
\tilde{r}^U &= \frac{\theta_H^2+\theta_L^2-b(\theta_H+\theta_L)}{2(\theta_H^2+\theta_L^2)}.
\end{aligned}$$

The associated equilibrium seller demands for type $j \in \{L, H\}$, buyer demand, and platform profit are respectively given by:

$$\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U) = \frac{\tilde{p}^U\theta_j(\theta_H^2+\theta_L^2+b(\theta_H+\theta_L))}{(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}, \tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U) = \frac{2\tilde{p}^U}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))},$$

$$\tilde{\Pi}^U = \frac{v\tilde{p}^U}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}.$$

Buyer surplus and type $j \in \{L, H\}$ sellers' surplus are respectively given by

$$\begin{aligned}
\widetilde{CS}^U =& \int_0^{\tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U)} (v + b(\tilde{N}_H^U(\tilde{p}^U,\tilde{r}^U) + \tilde{N}_L^U(\tilde{p}^U,\tilde{r}^U)) - \tilde{p}^U - k^B)dk^B\\
=& \frac{2(\tilde{p}^U)^2}{(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))^2},
\end{aligned}$$

$$\widetilde{DS}_j^U = \int_0^{\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U)} ((1-\tilde{r}^U)\theta_j\tilde{N}_B^U(\tilde{p}^U,\tilde{r}^U) - k^S)dk^S = \frac{(\tilde{N}_j^U(\tilde{p}^U,\tilde{r}^U))^2}{2},$$

for a total welfare of

$$\widetilde{SW}^U = \widetilde{CS}^U + \tilde{\Pi}^U + \sum_{i=1,2}\widetilde{DS}_j^U = (\tilde{p}^U)^2\mathcal{X},$$

where

$$\mathcal{X} = \frac{\theta_H^2(2(6-\theta_L^2)-b^2-2b\theta_L)+\theta_L^2(12-(b+\theta_L)^2)-\theta_H^4-2b\theta_H^3-2b\theta_H\theta_L(b+\theta_L)}{2(\theta_H^2+\theta_L^2)(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))^2}.$$

Price discrimination. Under price discrimination, buyer participation is still given as in equation (1) in the main text, while seller participation is given as in equation (A8). Under rational expectations, equilibrium participation thus satisfies the following system:

$$\tilde{N}_B^D = v + b(N_L^D + N_H^D) - p, \quad \tilde{N}_H^D = (1-r_H)\theta_H N_B^D \quad \text{and} \quad \tilde{N}_L^D = (1-r_L)\theta_L N_B^D.$$

Solving the above system of equations for $\tilde{N}_B^D$, $\tilde{N}_H^D$ and $\tilde{N}_L^D$ yields buyer participation and seller participation as functions prices. We present these demands below. The solution is

$$\begin{aligned}
\tilde{N}_B^D(p, r_H, r_L) &= \frac{v-p}{1-b((1-r_H)\theta_H + (1-r_L)\theta_L)}, \\
\tilde{N}_H^D(p, r_H, r_L) &= \frac{(v-p)(1-r_H)\theta_H}{1-b((1-r_H)\theta_H + (1-r_L)\theta_L)}, \\
\tilde{N}_L^D(p, r_H, r_L) &= \frac{(v-p)(1-r_L)\theta_L}{1-b((1-r_H)\theta_H + (1-r_L)\theta_L)}.
\end{aligned}$$

The platform sets prices to maximize profits

$$\max_{p, r_H, r_L} (p + r_H\theta_H \tilde{N}_H^D(p, r_H, r_L) + r_L\theta_L \tilde{N}_L^D(p, r_H, r_L))\tilde{N}_B^D(p, r_H, r_L).$$

Differentiating platform profits with respect to p and r_j, for $j \in \{L, H\}$ and solving the system of first order conditions yields the optimal prices as follows.

$$\tilde{p}^D = \frac{v(2-\theta_H^2-\theta_L^2-b(\theta_H+\theta_L))}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \ \tilde{r}_j^D = \frac{\theta_j - b}{2\theta_j}, \text{ for } j \in \{H, L\},$$

where superscript D indicates the case with price discrimination.[1] The associated equilibrium seller demands for $j \in \{L, H\}$, buyer demand, and platform profit are respectively given as

$$\tilde{N}_j^D = \frac{v(b+\theta_j)}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}, \tilde{N}_B^D = \frac{2v}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)},$$

$$\tilde{\Pi}^D = \frac{v^2}{4-2b^2-\theta_H^2-\theta_L^2-2b(\theta_H+\theta_L)}.$$

Before proceeding further, we make a few observations.

Observation 1. *The following equality holds true.*

- *Under price discrimination, the price charged to buyers remains unchanged regardless of the pricing structure incident on sellers — i.e.,* $p^D = \tilde{p}^D$.

[1]The denominator is positive by Assumption A1.

- *Under price discrimination, the total price charged to sellers remains unchanged regardless of the pricing structure incident on sellers — i.e.,* $\tilde{r}_j^D \theta_H \tilde{N}_B^D(\tilde{p}^D, \tilde{r}^D) = f_j^D$.

The above implies that the mass of buyers, sellers and platform profits are identical under price discrimination regime regardless of whether platforms charge sellers a fixed participation price or an ad-valorem fee. As a consequence, consumer surplus and welfare expressions are identical as well.

Price discrimination vs. uniform pricing. In the following, we show the robustness of the main result obtained in the baseline model.

Comparison of total participation of buyers and sellers. Total participation of both buyers and sellers is higher under price discrimination than under uniform pricing:

$$\tilde{N}_B^D - \tilde{N}_B^U = \frac{2b^2 v(\theta_H - \theta_L)^2}{\begin{array}{l}(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)) \\ \quad (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)\end{array}} > 0;$$

$$\tilde{N}_S^D - \tilde{N}_S^U = \frac{v(\theta_H + \theta_L)(\theta_H^2 + \theta_L^2 + b(\theta_H + \theta_L))}{(\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)} > 0.$$

In both inequalities, the numerators are positive, as it can easily seen, and the denominators are positive under Assumption A1.

Comparison of platform profit. Considering platform profits, we obtain that:

$$\Pi^D - \widetilde{\Pi}^U = \frac{b^2 v^2(\theta_H - \theta_L)^2}{\begin{array}{l}(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)) \\ \quad (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)\end{array}}.$$

We observe that the sign of the difference in platform profit is determined by the sign of the expressions in the denominator. The two terms in the denominator of the difference in profits are positive as they are just the terms in the denominator of the platform profits in the two pricing regimes. Since Assumption A1 guarantees platform profits are positive, they must be positive as well because the numerator of the profits is always positive.

Comparison of consumer surplus. Comparing consumer surplus under price discrimination with the consumer surplus under uniform pricing yields

$$CS^D - \widetilde{CS}^U = \frac{2}{(2 - \theta_H^2 - \theta_L^2 - b(\theta_H + \theta_L))^2}\left((p_1^D)^2 - (\tilde{p}^U)^2\right).$$

Hence, the difference in buyer prices determines the sign of the difference in consumer surplus.

$$(p_1^D)^2 - (\tilde{p}^U)^2 = \mathcal{A}((\theta_L^2 + \theta_H^2)(8 - 3b^2 - 4b\theta_L - 4\theta_L^2) - 2b\theta_H\theta_L(b + 2\theta_L) - 2\theta_H^3(\theta_H + 2b)),$$

where $\mathcal{A}$ is a composite term of squared expressions:

$$\mathcal{A} = \frac{2b^2v^2(\theta_H - \theta_L)^2}{\begin{array}{l}(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L))^2 \\ \quad (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)^2\end{array}} > 0.$$

Therefore, the sign of $(p_1^D)^2 - (\tilde{p}^U)^2$ is determined by the sign of

$$\mathcal{B} = ((\theta_L^2 + \theta_H^2)(8 - 3b^2 - 4b\theta_L - 4\theta_L^2) - 2b\theta_H\theta_L(b + 2\theta_L) - 2\theta_H^3(\theta_H + 2b)).$$

Differentiating $\mathcal{B}$ with respect to b yields

$$\frac{\partial \mathcal{B}}{\partial b} = -2(2(\theta_H + \theta_L)(\theta_H^2 + \theta_L^2) + b(3\theta_H^2 + 3\theta_L^2 + 2\theta_H\theta_L)) < 0.$$

Thus, it is sufficient to show that $\mathcal{B}$ at $b = \bar{b}^{ad}$ is positive.

$$\mathcal{B}\big|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)^2(4 + 2\theta_H\theta_L - (\theta_H + \theta_L)\sqrt{(8 - (\theta_H - \theta_L)^2)})}{2}.$$

The second term in the numerator given by $(4+2\theta_H\theta_L-(\theta_H+\theta_L)\sqrt{(8 - (\theta_H - \theta_L)^2)}$ is always positive for $\theta_H > \theta_L > 0$. Thus, we show that consumer surplus is always higher under the price discrimination regime than under a uniform pricing regime.

Comparison of low-type seller surplus. Turning to the low-type sellers, a sufficient statistic for seller surplus is seller participation:

$$N_L^D - \tilde{N}_L^U = \frac{bv(\theta_H - \theta_L)\mathcal{Z}_L}{\begin{array}{l}(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)) \\ \quad (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)\end{array}}$$

where

$$\mathcal{Z}_L = (\theta_H(4 - (b + \theta_H)^2) - (b + \theta_H)\theta_L(b + \theta_L)).$$

The sign of $N_L^D - \tilde{N}_L^U$ is determined by the sign of the term $\mathcal{Z}_L$ as all other terms are guaranteed to be positive under Assumption A1.

Differentiating $\mathcal{Z}_L$ with respect to b yields

$$\frac{\partial \mathcal{Z}_L}{\partial b} = -(2\theta_H(b + \theta_H) + \theta_L(2b + \theta_H) + \theta_L^2) < 0.$$

Thus, it is sufficient to show that $\mathcal{Z}_L$ at $b = \bar{b}^{ad}(\theta_H, \theta_L)$ is positive.

$$\mathcal{Z}_L\big|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)(4 - \theta_L^2 - \theta_H(\sqrt{(8 - (\theta_H - \theta_L)^2)} - \theta_L)}{2}.$$

The second term in the numerator given by $(4 - \theta_L^2 - \theta_H(\sqrt{(8 - (\theta_H - \theta_L)^2)} - \theta_L)$ is always positive as Assumption A1 ensures $\theta_H > \theta_L > 0$ and $\theta_H^2 + \theta_L^2 < 4.$ Thus, we show that the surplus of low-type sellers is always higher under price discrimination than under uniform pricing.

Comparison of total welfare. Comparing total welfare under price discrimination with the total welfare under uniform pricing yields

$$SW^D - \widetilde{SW}^U = \frac{\mathcal{A}}{4}\mathcal{Y},$$

where $\mathcal{Y} \triangleq 4b\theta_H^5 + \theta_H^6 + 2b\theta_H\theta_L(2\theta_L^3 + 5b\theta_L^2 - \theta_L(24 - 5b^2) - 2b(6 - b^2)) + \theta_L^2(80 - 2b^2(18 - b^2) - 6b\theta_L(8 - b^2) - \theta_L^2(24 - 7b^2) + 4b\theta_L^3 + \theta_L^4) + \theta_H^4(7b^2 + 4b\theta_L - 3(8 - \theta_L^2)) + 2b\theta_H^3(3b^2 + 5b\theta_L - 4(6 - \theta_L^2)) + \theta_H^2(80 + 2b^4 + 10b^3\theta_L - 48\theta_L^2 + 3\theta_L^4 - 8b\theta_L(6 - \theta_L^2) - 2b^2(18 - 7\theta_L^2))$. Differentiating $\mathcal{Y}$ twice with respect to b yields

$$\frac{\partial^3\mathcal{Y}}{\partial b^3} = 12(\theta_H + \theta_L)(3\theta_H^2 + 3\theta_L^2 + 2\theta_H\theta_L + 4b(\theta_H + \theta_L)) > 0.$$

Computing the second derivative of $\mathcal{Y}$ with respect to b at $b = \bar{b}^{ad}$ yields

$$\frac{\partial^2\mathcal{Y}}{\partial b^2}\Big|_{b=\bar{b}^{ad}} = -2(\theta_H - \theta_L)^2\left(12 + 2(\theta_H + \theta_L)^2 + 2\theta_H\theta_L - 3(\theta_H + \theta_L)\sqrt{8 - (\theta_H - \theta_L)^2}\right) < 0.$$

Thus, we confirm that $\frac{\partial^2\mathcal{Y}}{\partial b^2}$ is always negative in the feasible region.

Evaluating the first derivative of $\mathcal{Y}$ with respect to b at $b = 0$ yields

$$\frac{\partial\mathcal{Y}}{\partial b}\Big|_{b=0} = -4(\theta_H + \theta_L)(\theta_H^2 + \theta_L^2)(12 - \theta_H^2 - \theta_L^2) < 0.$$

The above is negative as Assumption A1 ensures that $\theta_H^2 + \theta_L^2 < 4$.

Finally, computing $\mathcal{Y}$ at $b = \bar{b}^{ad}$ yields

$$\mathcal{Y}\big|_{b=\bar{b}^{ad}} = 4(\theta_H - \theta_L)^2\left(4 + 2\theta_L - (\theta_L + \theta_H)\sqrt{8 - (\theta_H - \theta_L)^2}\right) > 0.$$

The above is always positive as Assumption A1 ensures that $\theta_H^2 + \theta_L^2 < 4$. Hence, we show that total welfare is always higher under price discrimination than under uniform pricing.

Comparison of high-type seller surplus. A sufficient statistic for seller surplus is seller participation, which yields:

$$N_H^D - \tilde{N}_H^U = \frac{bv(\theta_H - \theta_L)\mathcal{Z}_H}{\begin{array}{l}(4 - 2b^2 - \theta_H^2 - \theta_L^2 - 2b(\theta_H + \theta_L)) \\ \quad (\theta_H^2(4 - b^2 - 2b\theta_L - 2\theta_L^2) - \theta_L^2(4 - (b + \theta_L)^2) - 2b\theta_H\theta_L(b + \theta_L) - \theta_H^4 - 2b\theta_H^3)\end{array}}$$

where

$$\mathcal{Z}_H = ((b + \theta_L)(\theta_H^2 + \theta_L^2 + b(\theta_H + \theta_L)) - 4\theta_L).$$

The sign of $N_L^D - \tilde{N}_L^U$ is determined by the sign of the term $\mathcal{Z}_H$ as all other terms are positive under Assumption A1.

Differentiating $\mathcal{Z}_H$ with respect to b yields

$$\frac{\partial \mathcal{Z}_H}{\partial b} = \theta_H^2 + 2\theta_L^2 + \theta_H\theta_L + 2b(\theta_H + \theta_L) > 0.$$

Computing $\mathcal{Z}_H$ at $b = 0$, yields

$$\mathcal{Z}_H|_{b=0} = \theta_L(4 - \theta_L^2 - \theta_H^2) > 0.$$

The above is positive as Assumption A1 ensures that $\theta_H^2 + \theta_L^2 < 4$.

Similarly, computing $\mathcal{Z}_H$ at $b = \bar{b}^{ad}$ yields

$$\mathcal{Z}_H\big|_{b=\bar{b}^{ad}} = \frac{(\theta_H - \theta_L)(4 - \theta_H^2 + \theta_H\theta_L - \theta_L\sqrt{(8 - (\theta_H - \theta_L)^2)}}{2}.$$

The second term in the numerator given by $(4 - \theta_L^2 - \theta_H(\sqrt{(8 - (\theta_H - \theta_L)^2)} - \theta_L)$ is always positive as Assumption A1 ensures $\theta_H > \theta_L > 0$ and $\theta_H^2 + \theta_L^2 < 4$.

Thus, by intermediate value theorem, there must exist a critical level of b denoted by

$$\hat{b}^{ad}(\theta_H, \theta_L) = \frac{1}{2}\left(\frac{\sqrt{\theta_H^4 + 2\theta_H\theta_L(8 - \theta_H^2) + \theta_L^2(16 + \theta_H^2)} - 2\theta_L^2}{\theta_H + \theta_L} - \theta_H\right)$$

where $N_H^D - \tilde{N}_H^U = 0$. For $b > \hat{b}^{ad}(\theta_H, \theta_L)$, we must have $N_H^D - \tilde{N}_H^U > 0$ and for $b < \hat{b}^{ad}$, we must have $N_H^D - \tilde{N}_H^U < 0$.

Thus, we show that the surplus of high type sellers can also increase giving us the result that price discrimination can result in Pareto improvement over uniform pricing.

D. One-sided pricing: Proof of Proposition 8

As in the benchmark case, in order to ensure that the maximization problem is concave, we impose the following conditions:

Assumption A2 *Provided buyer intrinsic valuation as well as sellers' valuations are low enough, we consider the region* $\max\{0, \frac{2\theta_H}{\theta_L(\theta_H-\theta_L)}\} < b < \overline{b}'(\theta_H,\theta_L) = \frac{\sqrt{(v(v-8)+8)(\theta_H^2+\theta_L^2)+2v^2\theta_H\theta_L}+(v-2)(\theta_H+\theta_L)}{(\theta_H-\theta_L)^2}$.

Reproducing the analysis carried out in Section 4 (in the main text), we can easily see that Subsection 4.1 does not change, the only caveat being that we have to consider $p = 0$. As per the modification to Subsections 4.2 and 4.3, we obtain the following results.

Uniform pricing. The platform sets the uniform fee to maximize profits $fN_S^U(f)$, which yields the equilibrium fee:

$$f^U = \frac{v(\theta_H + \theta_L)}{4}.$$

The associated equilibrium seller demands for type $j \in \{L, H\}$, buyer demand, and platform profit are respectively given by:

$$N_j^U = \frac{v(\theta_j(3 - b\theta_j) - \theta_{-j}(1 - b\theta_{-j})}{4 - 4b(\theta_H + \theta_L)},\ N_B^U = \frac{v(2 - b(\theta_H + \theta_L))}{2 - 2b(\theta_H + \theta_L)},\ \Pi^U = \frac{v^2(\theta_H + \theta_L)^2}{8 - 8b(\theta_H + \theta_L)}.$$

Total participation of the sellers is then $N_S^U = N_L^U + N_H^U = \frac{v(\theta_H+\theta_L)}{2-2b(\theta_H+\theta_L)}$.

Buyer surplus and type $j \in \{L, H\}$ sellers' surplus is respectively given by

$$CS^U = \frac{v^2(2 - b(\theta_H + \theta_L))^2}{8(1 - b(\theta_H + \theta_L))^2},\ DS_j^U = \frac{v^2(\theta_j(3 - b\theta_j) - \theta_{-j}(1 - b\theta_{-j}))^2}{32(1 - b(\theta_H + \theta_L))^2}.$$

Total welfare amounts to:

$$SW^U = \frac{v^2(b^2(2 + (\theta_H - \theta_L)^2)(\theta_H + \theta_L)^2 - 2b(\theta_H + \theta_L)\Sigma)}{16(1 - b(\theta_H + \theta_L))^2},$$

where $\Sigma = (4 + 3\theta_H^2 - 2\theta_H\theta_L + 3\theta_L^2) + 8 + 7(\theta_H^2 + \theta_L^2) - 2\theta_H\theta_L)$.

Price Discrimination. The platform sets two different fees in order to maximize $f_H N_H^D(f_H, f_L)+ f_L N_L^D(f_H, f_L)$, which yields at equilibrium

$$f_j^D = \frac{v(2\theta_j(1 - b\theta_j) - b\theta_{-j}(\theta_j - \theta_{-j}))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2},\ \text{for } j \in \{H, L\}.$$

The associated equilibrium seller demands for $j \in \{L, H\}$, buyer demand, and platform profit are:

$$N_j^D = \frac{v(\theta_j(2 - b\theta_j) + b\theta_{-j}^2)}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}, \; N_B^D = \frac{2v(2 - b(\theta_H + \theta_L))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2},$$

$$\Pi^D = \frac{v^2(\theta_H + \theta_L)^2}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}.$$

Total seller participation is then given as

$$N_S^D = N_L^D + N_H^D = \frac{v(2(2 + \theta_L) - 2b\theta_L - b\theta_H(2 - \theta_H - \theta_L))}{4 - 4b(\theta_H + \theta_L) - b^2(\theta_H - \theta_L)^2}.$$

Buyer surplus and type $j \in \{L, H\}$ sellers' surplus are respectively given by

$$CS^D = \frac{2v^2(2 - b(\theta_H + \theta_L))^2}{(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2}, \; DS_j^D = \frac{v^2(\theta_j(2 - b\theta_{-j}) + b\theta_{-j}^2)}{2(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2}.$$

Total welfare amounts to:

$$SW^D = \frac{v^2(16 + 12(\theta_H^2 + \theta_L^2) - 8b(\theta_H + \theta_L)(2 + \theta_H^2 + \theta_L^2) - b^2\Delta)}{2(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))^2},$$

where $\Delta = (\theta_H^4 - 2\theta_H^3\theta_L - 4\theta_L^2 + \theta_L^4 - 2\theta_H^2(2 - \theta_L^2) - 2\theta_H\theta_L(4 + \theta_L^2))$.

Price discrimination vs. uniform pricing. Firstly, it is straightforward to show that the platform earns higher profit under price discrimination than under uniform prices.

Before we proceed further, it is informative to keep in mind how seller prices change under price discrimination. Comparing prices, we observe that

$$f^U - f_L^D = \frac{v(2 + b(\theta_H - \theta_L))(\theta_H - \theta_L)(2 - b(\theta_H + \theta_L))}{4(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} > 0$$

and

$$f^U - f_H^D = -\frac{v(\theta_H - \theta_L)((2 - b\theta_H)^2 - b^2\theta_L^2)}{4(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} < 0.$$

A corollary from the above price relations is that the low-type sellers are always better off.

Secondly, we find that total seller participation rises:

$$N_S^D - N_S^U = \frac{bv(\theta_H - \theta_L)^2(2 - b(\theta_H + \theta_L)}{2(1 - b(\theta_H + \theta_L))(4 - b^2(\theta_H - \theta_L)^2 - 4b(\theta_H + \theta_L))} > 0.$$

The above is always positive because both $(2 - b(\theta_H + \theta_L)$ at the numerator and the expressions at the denominator are positive under Assumption A2. A direct consequence of the above is that buyer surplus rises. This is because the buyer price is set at zero and seller participation increases under price discrimination, thus benefiting buyers. Regarding buyers, their total participation increases, as it can be obtained by investigating the sign of:

$$N_B^D - N_B^U = \frac{2v(4b^3(\theta_H+\theta_L) - b^2(8-6(\theta_H^2-\theta_L^2) - 4\theta_H\theta_L) - b(\theta_H+\theta_L)(8-(\theta_H+\theta_L)^2) - 2(\theta_H+\theta_L)^2+8)}{(8-(2b+\theta_H+\theta_L)^2)(4-b^2(\theta_H-\theta_L)^2-4b(\theta_H+\theta_L))},$$

which is always positive under Assumption A2.

Finally, in order to show that Pareto improvement is a possibility, it is sufficient to find conditions under which the high-type sellers can be better off under price discrimination. A sufficient statistic for this result to hold is to show that the participation of high-type sellers is higher under price discrimination than under uniform pricing despite the fact that participation fee to the high-margin type rises. This can be formally demonstrated as follows. Taking the difference of participation of the high type under price discrimination with its participation under uniform prices yields

$$N_H^D - N_H^U = \frac{v(\theta_H-\theta_L)(2-b(\theta_H+\theta_L))\Omega}{4(1-b(\theta_H+\theta_L))(4-b^2(\theta_H-\theta_L)^2-4b(\theta_H+\theta_L))},$$

where $\Omega = (2 - b^2(\theta_H+\theta_L) + b(3\theta_H+\theta_L))$. Note that the sign of $N_H^D - N_H^U$ follows that of Ω as all other terms are positive under the assumption that the problem is concave.

Differentiating Ω with respect to b, we observe that

$$\frac{\partial\Omega}{\partial b} = 3\theta_H + \theta_L + 2b(\theta_H-\theta_L)^2 > 0.$$

Further, computing Ω at the two bounds, we find that

$$\Omega|_{b=0} = -2,\ \Omega|_{b=\bar{b}'} = \frac{4(\theta_H^2+\theta_H\theta_L+2\theta_L^2) - 2(\theta_H+3\theta_L)\sqrt{2(\theta_H^2+\theta_L^2)}}{(\theta_H-\theta_L)^2} > 0.$$

Thus, by the intermediate value theorem, we can state there exists a cut-off denoted by $\hat{b}'$ above which $\Omega > 0$ and negative otherwise.

Equating Ω to zero and solving for b yields the following threshold

$$\hat{b}'(\theta_H,\theta_L) = \frac{\sqrt{17\theta_H^2 - 10\theta_H\theta_L + 9\theta_L^2} - 3\theta_H - \theta_L}{(\theta_H-\theta_L)^2},$$

which is within the admissible parameter bounds, as it can be easily demonstrated.

Finally, comparing social welfare in the two cases, we find that $SW^D > SW^U$ if and only if $b > b_w(\theta_H,\theta_L)$ with $b_w(\theta_H,\theta_L) < \hat{b}'(\theta_H,\theta_L)$; the analytical expression of $b_w(\theta_H,\theta_L)$ is very complex but can be provided upon request.

E. Price-discrimination on both sides: Proof of Proposition 9

We reproduce the analysis of Section 4 (in the main text) and limit our comparison between uniform pricing and price discrimination to the parametric region where second order conditions are satisfied and where participation on both sides can be expanded. However, calculations with two groups of users on each side become more cumbersome than in the benchmark case, and precise conditions cannot be easily written. The same holds for the most relevant comparisons between the two scenarios. For this reason, in this appendix we only write the relevant equilibrium expressions and those calculations that are analytically feasible. Full calculations are however available upon request. Finally, in order to shorten the expressions, we use the following notation:

$$
\begin{aligned}
\Delta\theta &= \theta_H - \theta_L > 0,\ \ \Delta b = b_h - b_l > 0,\ \ \Sigma\theta = \theta_H + \theta_L,\ \ \Sigma b = b_h + b_l,\\
\varphi &= ((b_h^2 + b_l^2)(8 - \Delta\theta^2) + 8b_l\Sigma\theta + 8(\theta_H^2 + \theta_L^2 - 2) + 2b_h(b_l\theta_H^2 + 2\theta_H(2 - b_l\theta_L) + \theta_L(4 + b_l\theta_L)),\\
\Lambda &= 2b_h(b_l + \Sigma\theta)(1 + b_l\Sigma\theta) - b_l^2(1 + \Sigma\theta)(1 - \Sigma\theta) - b_h^2(1 + 2b_l(b_l + \Sigma\theta) - \Sigma\theta^2),\\
\varrho &= b_l^2(28 - 5(\theta_H^2 + \theta_L^2) - 2\theta_H\theta_L) + b_h^3(b_l(12 - \Delta\theta^2) - 6\Sigma\theta),\\
\varsigma &= 3b_l^2(8 - \Delta\theta^2) - 12b_l\Sigma\theta - 2(28 - 5(\theta_H^2 + \theta_L^2) - 2\theta_H\theta_L),\\
\vartheta &= 8\Sigma\theta - b_l(40 - b_l^2(12 - \Delta\theta^2) + 6b_l\Sigma\theta + 2(5(\theta_H^2 + \theta_L^2) + 2\theta_H\theta_L)).
\end{aligned}
$$

The following assumption will be adopted throughput the analysis as they ensure the maximization problem is concave.

Assumption A3 *Provided buyer intrinsic valuation v is sufficiently low, we consider the region where* $(2 - \Sigma b - \Sigma\theta) > 0$ *and* $\varphi > 0$.

The remaining conditions are quite tedious. Nevertheless, we can show that the results of our analysis hold when we impose reasonable parameter values that fulfill all required restrictions such as the second order conditions and interior solutions.

Uniform pricing. The platform sets the uniform fees to maximize profits $fN_S^U(f,p) + pN_B^U(f,p)$, which yields the equilibrium fees:

$$p^U = \frac{v(2 - \Sigma\theta(\Sigma b + \Sigma\theta))}{(2 - \Sigma b - \Sigma\theta)(2 + \Sigma b + \Sigma\theta)},\ f^U = \frac{v(\Sigma\theta - \Sigma b)}{(2 - \Sigma b - \Sigma\theta)(2 + \Sigma b + \Sigma\theta)}.$$

The associated equilibrium seller demands for type $j \in \{L, H\}$, buyer demands for type $i \in \{l, h\}$, and platform profit are respectively given by:

$$N_{S_j}^U = \frac{v(3\theta_j - \theta_{-j} + \Sigma b)}{(2 - \Sigma b - \Sigma\theta)(2 + \Sigma b + \Sigma\theta)},\ N_{B_i}^U = \frac{v(2 + b_i(b_i + \Sigma\theta) - b_{-i}(b_{-i} + \Sigma\theta))}{(2 - \Sigma b - \Sigma\theta)(2 + \Sigma b + \Sigma\theta)},$$

$$\Pi^U = \frac{2v^2}{(2-\Sigma b-\Sigma\theta)(2+\Sigma b+\Sigma\theta)}.$$

Total participation of sellers is then $N_S^U = N_{S_L}^U + N_{S_H}^U$ and that of buyers is $N_B^U = N_{B_h}^U + N_{B_l}^U$; full expressions are straightforward and omitted for brevity.

Seller surplus for type $j \in \{L, H\}$ and buyer surplus for type $i \in \{l, h\}$ are respectively given by $DS_j^U = (N_{S_j}^U)^2/2$ and $CS_i^U = (N_{B_i}^U)^2/2$. Total welfare amounts to:

$$SW^U = \frac{v^2(12 + b_h^4 + b_l^4 + (\theta_H - 3\theta_L)(3\theta_H - \theta_L) + 2(b_h^3 + b_l^3 - b_l)(\Sigma\theta + \Lambda)}{(2-\Sigma b-\Sigma\theta)^2(2+\Sigma b+\Sigma\theta)^2}.$$

Price Discrimination. The platform sets four different fees in order to maximize $f_H N_{S_H}^D(f_H, f_L, p_h, p_l) + f_L N_{S_L}^D(f_H, f_L, p_h, p_l) + p_h N_{B_H}^D(f_H, f_L, p_h, p_l) + p_l N_{B_L}^D(f_H, f_L, p_h, p_l)$, which, for $j \in \{L, H\}$ and $i \in \{l, h\}$, yields at equilibrium:

$$f_j^D = \frac{v(b_h^2(3\theta_j + \theta_{-j}) - 2b_h(b_l(3\theta_j + \theta_{-j}) - 2) + b_l(b_L(3\theta_j + \theta_{-j}) + 4) - 8\theta_j)}{\varphi},$$

$$p_i^D = \frac{v(2b_i(\Sigma\theta + b_{-i}(\Delta\theta - 2)) + 6b_{-i}\Sigma\theta - (b_i^2 + b_{-i}^2)\Delta\theta + 4b_{-i} + 8(\theta_H^2 + \theta_L^2 - 1))}{\varphi}.$$

The associated equilibrium seller demands for $j \in \{L, H\}$, buyer demands for $i \in \{l, h\}$, and platform profit are:

$$N_{S_j}^D = \frac{v((b_h^2 + b_l^2)\theta_j - 2(b_h b_l + 4)\theta_j - \Delta b^2\theta_{-j} - 4\Sigma b)}{\varphi}, N_{B_i}^D = \frac{2v(4 - b_i^2 + 2b_h b_l + (b_i - b_{-i})\Sigma\theta)}{\varphi},$$

$$\Pi^D = \frac{2v^2(4 - \Delta b^2)}{\varphi}.$$

Total seller participation and total buyer participation are respectively given by $N_S^D = N_{S_L}^D + N_{S_H}^D$ and $N_B^D = N_{B_H}^D + N_{B_L}^D$, whose expressions are straightforward and omitted for brevity.

Seller surplus for type $j \in \{L, H\}$ and buyer surplus for type $i \in \{l, h\}$ are respectively given by $DS_j^D = (N_{S_j}^D)^2/2$ and $CS_i^D = (N_{B_i}^D)^2/2$. Finally, total welfare amounts to:

$$SW^D = \frac{v^2((b_h^4 + b_l^4)(24 - \Delta\theta^2) + 8b_l(3b_l^2 - 4)\Sigma\theta + 32(6 - \theta_H^2 - \theta_L^2) - 4\varrho + 2b_h^2\varsigma - 4b_h\vartheta)}{\varphi^2}.$$

Price discrimination vs. uniform pricing. Firstly, we show that the platform earns higher profit under price discrimination than under uniform prices. This arises directly from the fact that it has more tools to extract surplus from the different sides. Formally,.

$$\Pi^D - \Pi^U = \frac{2v^2(4\Delta\theta^2 + \Delta b^2(\Sigma b(\Sigma b + \Sigma\theta) + 4\theta_H\theta_L))}{(2-\Sigma b-\Sigma\theta)(2+\Sigma b+\Sigma\theta)\varphi} > 0$$

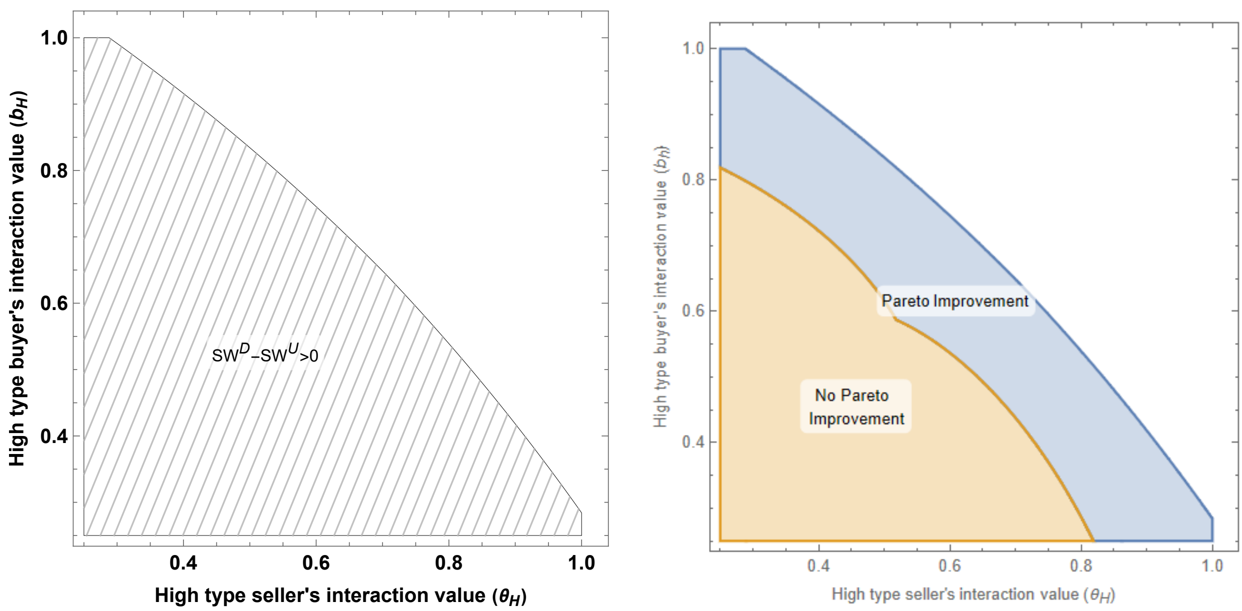
given the conditions on Assumption A3. The same conditions enable us to demonstrate that total participation rises on the seller side:

$$N_S^D - N_S^U = \frac{2v(\Sigma b + \Sigma\theta)(4\Delta\theta^2 + \Delta b(4 - \Delta\theta^2))}{(2 - \Sigma b - \Sigma\theta)(2 + \Sigma b + \Sigma\theta)\varphi} > 0.$$

Similarly, we confirm that total participation rises on the buyer side:

$$N_B^D - N_B^U = \frac{4v(2\Sigma b\Sigma\theta\Delta b^2 + 4(\Delta\theta^2 + \Delta b^2\theta_H\theta_L) + (b_H^2 - b_L^2)^2)}{(2 - \Sigma b - \Sigma\theta)(2 + \Sigma b + \Sigma\theta)\varphi} > 0.$$

We can also demonstrate that low-type buyers and low-type sellers are always better off under price discrimination. Comparing the total welfare in the two regimes, we can then show that it is higher under price discrimination. Finally, we find that Pareto improvement occurs only if both high-type sellers and high-type buyers sufficiently value participation on the other side, i.e. if both θ_H and b_h are high enough. Calculations are however very complex and we simulate our welfare and Pareto improvement results when $\theta_L = b_l = 0.25$ and $v = 0.1$. A graphical representation is provided in Figure E.1: Panel (a) shows that total welfare under these parameters is always higher under price discrimination, whereas Panel (b) shows the presence of a Pareto improvement when the value of interactions of buyers and sellers is high enough.



(a) Total Welfare

(b) Pareto Improvement.

Figure E.1: Total welfare and area of Pareto improvement under price discrimination. Parameter values: $\theta_L = b_l = 0.25$ and $v = 0.1$.