Supplementary Information: Information Theoretical Limits for Quantum Optimal Control Solutions: Error Scaling of Noisy Control Channels

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ABSTRACT

In this supplementary Information we provide the derivation of the master equation under the evolution of a classical stochastic noise field (Eq. (10) of the main article.)

1 Derivation of the master equation (Eq. (10) of the main article)

Let us start the derivation from Eq. (8) in the main text, i.e.,

$$\dot{\rho}(t) = -i[H_s(t),\rho(t)] - i\left[\xi(t)H_n, \left(\rho(0) - i\int_0^t [H(t'),\rho(t')]dt'\right)\right].$$

After some straightforward calculations, one ends-up to the following relation:

$$\dot{\rho}(t) = -i[H_s(t),\rho(t)] - \xi(t)\left\{i\,[H_n,\rho(0)] + \int_0^t \Big[H_n,[H_s(t'),\rho(t')]\Big]dt'\right\} - \left[H_n,\left[H_n,\int_0^t \xi(t)\xi(t')\rho(t')dt'\right]\right].$$

In this way, by averaging over the noise realizations and using the assumption $\langle\xi(t)\rangle = 0$, one gets

$$\langle\dot{\rho}(t)\rangle = -i[H_s(t),\langle\rho(t)\rangle] - \left[H_n,\left[H_n,\int_0^t \langle\xi(t)\xi(t')\rho(t')\rangle dt'\right]\right],$$

where

$$\langle\xi_1(t)\rangle \equiv \int_{\xi_1} p_t(\xi_1)\xi_1\,d\xi_1$$
$$\langle\xi_2(t)\xi_3(t')\rangle \equiv \int_{\xi_2}\int_{\xi_3} p_{t,t'}(\xi_2,\xi_3)\xi_2\xi_3\,d\xi_2\,d\xi_3$$

with ξ_1, ξ_2, ξ_3 here representing generic stochastic processes. Accordingly, Eq. (10) in the main text is recovered under the further assumption to consider ξ and ρ as *uncorrelated* processes, so that the following approximation holds:

$$\int_0^t \langle\xi(t)\xi(t')\rho(t')\rangle dt' \quad\approx\quad \int_0^t \langle\xi(t)\xi(t')\rangle\langle\rho(t')\rangle dt' = \int_0^t R_\xi(t,t')\langle\rho(t')\rangle dt'.$$