# Supporting Information for "A Lagrangian estimate of the Mediterranean outflow's origin"

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## Text S1.

The Lagrangian trajectories are computed using the parcel-tracking software "Ariane", developed by Blanke and Raynaud (1997). Ariane computes parcel trajectories analytically, using an approximation to the exact trajectories which is accurate for periods shorter than the available velocity sampling – daily in this case. The velocity fields need to be defined on a C-grid type mesh. The algorithm is efficient, because it calculates parcel positions only on the edges of grid cells. Additionally, it fully respects the local non-divergence of the flow, allowing volume transport pathways to be mapped.

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At the seeding section, each parcel is assigned a small fraction of the total transport. Because the velocity field is non-divergent, this initally-assigned volumetric transport is conserved along each parcel trajectory and is part of the parcel's "label". Therefore, by using a closed subdomain, it is possible to estimate the transport exchanged between the initial and the final interception sections. Each trajectory integration lasts until the parcel reaches one of the interception sections. When a parcel does not intercept any section within the maximum integration time allowed, it will be considered lost and its transport removed from the total transport.

The parcels are released with the same sampling frequency of the velocity field output, i.e., daily, over a time period shorter than that covered by the reanalysis dataset. The seeding period should cover at least one year to take into account seasonal variability, and several years to account for inter-annual variability.

A homogeneous distribution of parcels in space and time over the seeding section is not appropriate, because regions with strong currents would be relatively under-represented. Therefore, the number of parcels initialized at each release-cycle within each grid-cell is "flux-weighted", i.e. proportional to the transport crossing that grid-cell at that time. The goal of flux-weighting is to assign each parcel an individual transport that is comparable among parcels. In practice, each individual parcel transport is capped to a small, constant value, while limiting the maximum total number of parcels (Blanke & Raynaud, 1997, Appendix A). The net result is that the number of parcels varies among grid-cells and release-cycles, and the individual transport differs among parcels.

Ariane solves the Lagrangian trajectory equations without stochastic terms that would lead to diffusion of the parcels' probability density function. Nevertheless, because the three coupled ordinary differential equations for the parcels' positions are nonlinear, and in our case, also non-autonomous because the velocity is time-dependent, the resulting trajectories are chaotic, i.e. sensitive to initial conditions. Additionally, Ariane linearly interpolates the discrete velocities on the reanalysis grid into piece-wise continuous fields, therefore parcels starting at neighboring points will also experience slightly different initial velocities. The net result is that the trajectories of parcels starting at neighboring points at the same time in the same grid-cell will have trajectories that diverge exponentially after a typical decorrelation time.

Several works have quantified the exponential divergence of Lagrangian trajectories through Finite Time Lyapunov Exponents (FTLE). In the Mediterranean, for the region just east of the Strait of Gibraltar, FTLEs are of the order of two days near the surface (d'Ovidio et al., 2004; García-Olivares et al., 2007). Presumably, FTLE are somewhat longer for the slower subsurface flow, but still much shorter than the typical transit times across the Western Mediterranean, which are of the order of six to eight years. Thus, each grid-cell should ideally be seeded by many parcels. Here, we adopt a spatially uniform seeding distribution within each grid cell, with at least one parcel per cell, and the maximum transport per parcel capped at $10^{-3}$ Sv.

In the following we detail the definition of ensemble-average used in the background shading (colors) of figure 1 of the main text.

The transport-weighted ensemble-average potential density, $< \sigma(x,y) >$, is defined as

$$< \sigma(x,y) > \equiv \frac{\sum_{n=1}^{N_{xy}} t_n \sigma_n(x,y)}{\sum_{n=1}^{N_{xy}} t_n}\,, \tag{1}$$

where $(x,y)$ is the center position of the horizontal grid cell in the longitude – latitude plane, $n$ is the parcel label, $\sigma_n$ its potential density and $t_n$ its transport. The sum is computed over the number of parcels, $N_{xy}$, detected at any time within the grid cell centered at $(x,y)$. In this way, $< \sigma >$ is a measure of the typical potential density experienced by the parcels along their typical trajectories.

## References

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