## $\label{eq:posterior} Pb_9Cu(PO4)_6O \ is \ a \ charge-transfer \ semiconductor \ - \ Supplementary \\ Materials$

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## I. EFFECT OF THE STRUCTURAL RELAXATION ON THE BAND GAP



FIG. 1: Band gap as a function of U. Comparison between fixed and relaxed structure calculations.

**II. SUBSTITUTION AT THE PB2 SITE** 



FIG. 2: Band structure for the spin up and down channels as calculated with VASP ncl using U = 5.5 eV and J = 0.8 eV and Cu at the Pb2 site. The system is insulating with  $E_{gap} = 1.03$  eV.

Calculation performed with U = 5.5 eV and J = 0.8 eV.

$$O^{std} = \begin{pmatrix} 0.8694 & 0.0227 & -0.0000 & -0.1016 & -0.0000 \\ 0.0227 & 0.8636 & -0.0000 & 0.0000 & -0.1016 \\ -0.0000 & -0.0000 & 0.9665 & 0.0000 & 0.0000 \\ -0.1016 & 0.0000 & 0.0000 & 0.8636 & -0.0227 \\ -0.0000 & -0.1016 & 0.0000 & -0.0227 & 0.8694 \end{pmatrix}$$
(1)

$$O^{ncl} = \begin{pmatrix} 0.827 + 0.000i & 0.019 - 0.151i & 0.000 + 0.000i & -0.146 - 0.018i & -0.000 - 0.153i \\ 0.019 + 0.151i & 0.827 + 0.000i & 0.000 + 0.000i & 0.000 + 0.152i & -0.146 + 0.018i \\ 0.000 + 0.000i & 0.000 + 0.000i & 0.971 + 0.000i & -0.000 + 0.000i & -0.000 + 0.000i \\ -0.146 + 0.018i & -0.000 - 0.152i & 0.000 + 0.000i & 0.827 + 0.000i & -0.019 - 0.151i \\ 0.000 + 0.153i & -0.146 - 0.018i & 0.000 + 0.000i & -0.019 + 0.151i & 0.827 + 0.000i \end{pmatrix}$$
(2)



FIG. 3: Graphical representation of the occupation matrices  $O^{std}$  calculated with VASP std. The diagonal elements have been scaled by a factor 0.1 to improve the visibility of the off-diagonal terms.



FIG. 4: Graphical representation of the occupation matrices  $O^{ncl}$  calculated with VASP ncl. The real and imaginary part are respectively given in the right and left panel. The diagonal elements have been scaled by a factor 0.1 to improve the visibility of the off-diagonal terms.



FIG. 5: Comparison between the std (right) and ncl (left) band structures.



## IV. ELECTRONIC STRUCTURE WITH CRPA U AND J VALUES

FIG. 6: Band structure for the spin up and down channels as calculated with VASP ncl using U = 3.5 eV and J = 0.8 eV. The latter correspond to the cRPA values obtained within the d - d model (see Tab. 1 of the main text). The system is insulating with  $E_{gap} = 0.07$  eV.



FIG. 7: Band structure for the spin up and down channels as calculated with VASP ncl using U = 5.5 eV and J = 0.8 eV. The latter correspond to the cRPA values obtained within the p + d model (see Tab. 1 of the main text). The system is insulating with  $E_{gap} = 0.16$  eV.