# $\mathrm{Pb}_{9} \mathrm{Cu}(\mathrm{PO} 4)_{6} \mathrm{O}$ is a charge-transfer semiconductor - Supplementary Materials 

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## I. EFFECT OF THE STRUCTURAL RELAXATION ON THE BAND GAP



FIG. 1: Band gap as a function of U . Comparison between fixed and relaxed structure calculations.

## II. SUBSTITUTION AT THE PB2 SITE



FIG. 2: Band structure for the spin up and down channels as calculated with VASP ncl using $U=5.5 \mathrm{eV}$ and $J=0.8 \mathrm{eV}$ and Cu at the Pb 2 site. The system is insulating with $E_{g a p}=1.03 \mathrm{eV}$.

## III. ELECTRONIC SYMMETRY BREAKING ANALYSIS

Calculation performed with $U=5.5 \mathrm{eV}$ and $J=0.8 \mathrm{eV}$.

$$
O^{s t d}=\left(\begin{array}{rrrrr}
0.8694 & 0.0227 & -0.0000 & -0.1016 & -0.0000  \tag{1}\\
0.0227 & 0.8636 & -0.0000 & 0.0000 & -0.1016 \\
-0.0000 & -0.0000 & 0.9665 & 0.0000 & 0.0000 \\
-0.1016 & 0.0000 & 0.0000 & 0.8636 & -0.0227 \\
-0.0000 & -0.1016 & 0.0000 & -0.0227 & 0.8694
\end{array}\right)
$$

$$
O^{n c l}=\left(\begin{array}{rrrrr}
0.827+0.000 i & 0.019-0.151 i & 0.000+0.000 i & -0.146-0.018 i & -0.000-0.153 i  \tag{2}\\
0.019+0.151 i & 0.827+0.000 i & 0.000+0.000 i & 0.000+0.152 i & -0.146+0.018 i \\
0.000+0.000 i & 0.000+0.000 i & 0.971+0.000 i & -0.000+0.000 i & -0.000+0.000 i \\
-0.146+0.018 i & -0.000-0.152 i & 0.000+0.000 i & 0.827+0.000 i & -0.019-0.151 i \\
0.000+0.153 i & -0.146-0.018 i & 0.000+0.000 i & -0.019+0.151 i & 0.827+0.000 i
\end{array}\right)
$$



FIG. 3: Graphical representation of the occupation matrices $O^{s t d}$ calculated with VASP std. The diagonal elements have been scaled by a factor 0.1 to improve the visibility of the off-diagonal terms.


FIG. 4: Graphical representation of the occupation matrices $O^{\text {ncl }}$ calculated with VASP ncl. The real and imaginary part are respectively given in the right and left panel. The diagonal elements have been scaled by a factor 0.1 to improve the visibility of the off-diagonal terms.


FIG. 5: Comparison between the std (right) and ncl (left) band structures.

## IV. ELECTRONIC STRUCTURE WITH CRPA U AND J VALUES



FIG. 6: Band structure for the spin up and down channels as calculated with VASP ncl using $U=3.5 \mathrm{eV}$ and $J=0.8 \mathrm{eV}$. The latter correspond to the cRPA values obtained within the $d-d$ model (see Tab. 1 of the main text). The system is insulating with $E_{\text {gap }}=0.07 \mathrm{eV}$.


FIG. 7: Band structure for the spin up and down channels as calculated with VASP ncl using $U=5.5 \mathrm{eV}$ and $J=0.8 \mathrm{eV}$. The latter correspond to the cRPA values obtained within the $p+d$ model (see Tab. 1 of the main text). The system is insulating with $E_{\text {gap }}=0.16 \mathrm{eV}$.

