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Condition-based maintenance for a multi-component system subject to heterogeneous failure dependences

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ABSTRACT

Many industrial facilities consisting of multiple components are prone to failure interactions and degradation interactions. In such systems, these interactions are frequently characterized by failure dependences that may accelerate the degradation of components. Due to system layout and functional interactions, not all components have the same failure dependence. In the general context of complex failure dependences in dependent multi-component systems, heterogeneous failure dependences further complicate the maintenance activities during operation. The present study developed a comprehensive framework for evaluating heterogeneous failure dependences and a maintenance optimization model by Markov processes for multi-component systems. The proposed method is applied to a practical case consisting in a parallel subsea transmission system to illustrate the effects of heterogeneous failure dependences. The results show that the heterogeneous failure dependences framework and the maintenance model guide the optimization of maintenance strategies to maximize the system availability and minimize the maintenance cost.

1. Introduction

Modern industrial systems usually consist of several components that need to operate simultaneously to accomplish the overall mission. As the systems become more complex with more interactions among the components, it is essential to pay close attention to the failure dependences existing between them. Failure dependences exist in such systems, meaning that the failure of one component may have influence on the failures of the other components, usually increasing their failure probabilities. The malfunction or degradation of the first component is defined as the triggering event of a failure cascading process. The failed component is defined as the triggering component. In some cases, failure dependence may not manifest as an immediate termination of component functions, but as a gradual degradation in the performance of those components. Thus, the failure dependences can be classified as [1,2]:

- Type I failure dependence: A triggering event results in direct damage. In such a context, a component could fail due to a combined effect of its normally inherent degradation, and the shock from the failures of other components.
- Type II failure dependence: A triggering event redistributes the total working load on the overall system. In such a context, a component could fail due to a combined effect of its normally inherent degradation, and the accelerated degradation caused by the failures or malfunctions of other components.

These two types of failure dependences can take place within the same system [3]. Thus, it is necessary to consider both in reliability analysis and maintenance.

In reliability analysis, degradation models are generally developed based on the performance data of a system or component over time to predict how it will degrade in the future. By considering failure dependence in degradation models, it is possible to have a better understanding on the underlying mechanisms of degradations and failures in complex systems. This can lead to more accurate models reflecting the reality. Therefore, numerous studies have been conducted so far integrating failure dependence in degradation models for reliability analysis and maintenance optimization. These models are roughly divided into three categories: multivariate joint distribution-based models, copula-based models, and degradation rate interaction (DRI) models [4,5]. Multivariate joint distribution-based models use joint probability distribution to present the dependence of degradation paths [6,7].

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Notation

n	Total number of components in a system
k	The number of degradation states of components before failure
x_i	The degradation state of component i
$x_i(t)$	The degradation state of component i at time t
χ	The state space of the n components system, which is taken to be $\{X_0,\ X_1, \dots X_{k^n}\}$
a	The threshold for minor preventive maintenance activity
b	The threshold for major corrective maintenance activity
N_{IMR}	The total number of inspections, maintenances and repairs
s	The number of inspections, maintenances and repairs
$P_{x_i}(t)$	The probability that the component i is in state x_i at time t
$P_{X_i}(t)$	The probability that the entire system is in state X_i at time t
$P(t)$	The sojourn probability of the Markov process at time t.
λ_{x_i}	The degradation rate of component i from state x_i to state x_i $+1$ without failure dependence
$\lambda^i_{x_i,x_j}$	The degradation rate of component i from state x_i to state x_i $+ 1$ when there exists failure dependence between it and another component j whose state is x_j
γ_{ij}	Cascading intensity from component j to component i
φ_j	Degradation level of component j
β_j	Correction coefficient
ϕ_{x_j}	Influencing level from component j
D_{i,x_j}	The failure dependence from component j on component i when component j is in state x_j
$\mathbb{A}$	The transition matrix denoting the transition rates of the entire system
$\mathbb{B}$	The probability matrix of different states after inspection, maintenance and repair actions
$\mathbb{D}$	The matrix of failure dependences among components
t^-	The time immediately before inspection
t^+	The time right after inspection, maintenance and repair actions
T_s	The time when the sth inspections, maintenances and repairs are conducted
X_F	The failed state of the component or the entire system
$\overline{A_s}$	The unavailability of the system
A_S	The availability of the system
c_{in}	The inspection cost of the system for each time
$c_{\mathrm{m1},i}$	The cost of each minor preventive maintenance activity on component i
$c_{\mathrm{m2},i}$	The cost of each major corrective maintenance activity on component i
c_{p}	The planned downtime cost per inspection
c_{u}	The unplanned downtime cost of the system
C_S	The average life-time cost

Abbreviation

DMDM	Degradation model for dependent multi-component system
FD	Failure dependence model
CBM	Condition-based maintenance
CTMC	Continuous-time Markovian chain
IMRs	Inspections, maintenances and repairs
PM	Minor preventive maintenance
CM	Major corrective maintenance
MTTF	Mean time to failure
MTBI	Mean time between inspections
OREDA	Offshore and Onshore Reliability Data

Copula-based method models the dependence between components in the combination of multivariate dependence with univariate marginals [8–10]. These two approaches have one common property that they use a multivariate distribution or copula to describe the joint aging process. Different from the above two methods, the DRI models manifest the degradation process of one component affected by the degradation of other components, which is more in accordance with the actual degradation of a dependent multi-component system [4]. The DRI model was firstly proposed by Bian and Gebraeel [11] to analyze the stochastic degradation process and prognostics of a multi-component system. Hafsa et al. [12] defined a degradation effect coefficient and presented a stochastic methodology by modeling the DRI effects of multi-component interaction in the remaining useful life (RUL) calculation. Considering Based the influence of degradation interaction and uncertainty, Shao et al. [13] contributed a multi-stage model-based framework to better describe the degradation acceleration process and evaluate the system RUL.

Given that the degradation status can be observed or measured, condition-based maintenance (CBM) is applied to many technical systems to keep system reliability while reducing maintenance cost. The Markov chain has been used for modeling the interactions between degradation processes and maintenance activities [14–18]. In these studies, a continuous-time Markovian chain (CTMC) is generally adopted to describe the system degradation behavior and the transitions between states. There are plenty of studies applying CTMC to model the degradation process and maintenance policies of a multi-state system, providing approximate analytical solutions for availability and cost [15, 19-21]. However, CBM for a multi-component system with failure dependences is generally more complicated [22]. Several previous studies on CBM strategies for multi-component system with failure dependence using the CTMC model were carried out by Liang et al. [15,23,24], where the failure dependence is modeled as the accelerated deterioration, and CBMs is optimized in considering multiple dependent deterioration path. Inspired by the above, we intend to build the CBM model by CTMC to present the normal degradation process and accelerated degradation process.

To the best of our knowledge, most of the current modeling approaches consider a two-component system or an n components system with identical failure dependence. For example, the chemical cluster is a system with n components mainly subject to Type I failure dependence, and a road network is a system with n components only subject to Type II failure dependence. However, such approaches are no longer completely aligned with reality, since the failure dependences in a multi-component system are more complex and heterogeneous [4]. Heterogeneous failure dependences occur in the situation where at least two types of non-identical failure dependence exist in a multi-component system. Therefore, a flexible framework to model the heterogeneous failure dependences in the context of maintenance optimization is desired for designing more reasonable CBM policies. In this paper, we focus on modeling the heterogeneous failure dependence within a multi-component system. Compared to alternative modeling approaches applied only in a two-component system, or in a multi-component system with identical components, the presenting work targets a multi-component system with non-identical components. In contrast to the current approaches, heterogeneous failure dependences modeling accounts better for the variety of interactions and dependences among non-identical components in a system. Such work is expected to predict the system behavior more precisely, which helps identify critical components and failure modes that are often overlooked in simpler models. In detail, the degradation model for dependent multi-component system (DMDM) is proposed based on two basic principles: (1) the general degradation process of independent component is depicted by a discrete

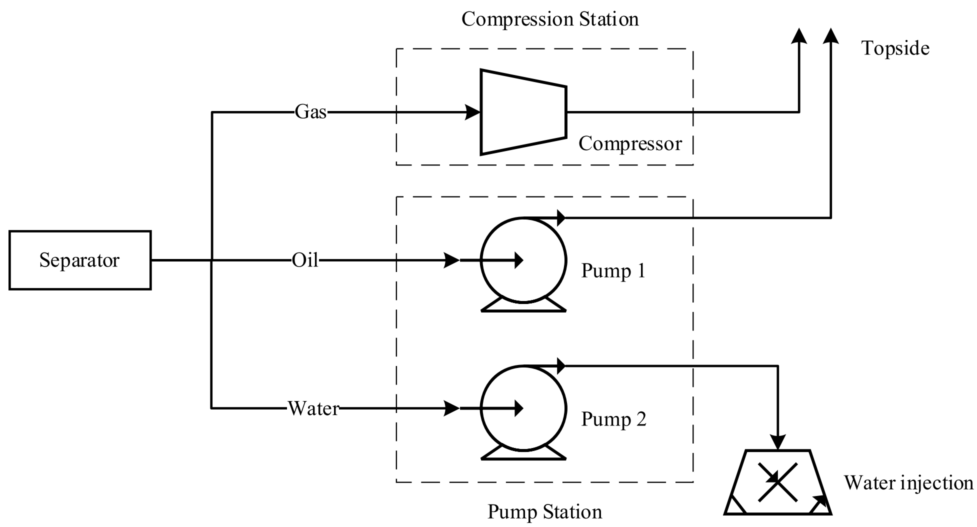


Fig. 1. Scheme of the transmission system considered in the motivating case

state space; (2) the failure dependences among components are characterized and quantified by proposing the failure dependence (FD) model.

Then, we will specify the CBM policy for multi-component systems accordingly. The maintenance policies are depicted considering preventive maintenance (PM) and corrective maintenance (CM). The major contributions of this study are outlined below:

(1) A new mechanism to model the heterogeneity of failure dependences for the degradation process in a multi-component system.
(2) A novel CBM strategy-making method for the multi-component system with heterogeneous failure dependences.
(3) Managerial implications on optimizing maintenances with a case study on a parallel subsea transmission system after the separator.

The rest of this paper is structured as follows. To start, we present a description of the motivating example about the subsea transmission system in Section 2. The degradation models for independent components and the dependent multi-component system are described in Section 3. In Section 4, the maintenance policies are interpreted by the Markov chain. Section 5 applies the overall approach to the practical case study of a three-component system maintenance. Finally, suggested future work and conclusions occur in Section 6.

2. Motivating example and problem description

In our study, we consider that load redistribution and failure-induced damage mainly lead to failure and degradation dependences. In the load redistribution mode, redistributed load determines the strength of failure dependence. In the failure-induce damage mode, the distance between components and safety barrier measures influence failure dependence.

In order to illustrate the problem, we introduce the transmission system of a subsea separation system that is developed to enhance oil recovery, using a horizontal gravity separator to separate bulk water from the hydrocarbon stream. A scheme of the system is reported in Fig. 1. Three pipes transporting gas, oil and water after the outlet of separator [25] are directed to the pump station and compression station, which are located near the separation station. The transmission part of the subsea system, which encompasses the compression station and pump station, can be regarded as a dependent system. In the following, we will refer to this system as the transmission system for the sake of brevity. One compressor and two pumps are installed in parallel in the simplified transmission system model. Wet gas is compressed by a compressor routed to the topside platform. Then, the separated oil and water are respectively pumped following the topside direction or reinjected into a reservoir via the water injection.

The service life of the compressor and pump are generally designed for 5-10 years without any intervention [26] and they are expected to serve 30-50 years with inspections, maintenances and repairs (IMRs) [27]. During their long service lifetime, these devices deteriorate stochastically, and the degradation process may be accelerated by the malfunction or degradation of the other components. The compressor and the two pumps normally transport different substances at the desired power under ideal conditions. In practice, however, devices degrade naturally, resulting in a variety of failure modes such as low output, leakage, vibration, overheating, spurious stop, etc. Some of the failures affect not only their own production efficiency, but also the degradation rates of other devices in the system. For example, the separator cannot separate the three substances completely, and the mutual doping of substances will aggravate the degradation of the compressor and pumps. Similarly, if a component such as compressor malfunctions, but somehow the system cannot be inspected and repaired timely, and it still needs to continue working, gas will enter the pipeline that transports oil or water, and the doping of the gas will compound the damage to the pumps, which is what we call failure dependence. This type of failure dependence can be considered as load redistribution. Another example is that the vibration and overheating of one pump may have a direct impact on the operation and aging process of another pump within a certain distance in the pump station. This kind of failure dependence is related to the safety distance, and the safety barrier measures. Hence, we can find that the compressor and pumps are subject to gradual degradation failure and two types of failure dependences.

Condition of the transmission system is assessed through periodic inspections. Two types of maintenances can be implemented according to the inspection results. The first is minor preventive maintenance which could lower the accumulated damage to a certain level, such as anti-corrosion coating, de-rusting and cleaning. The second type is major corrective maintenance including overhaul and preventive replacement that components are perfectly overhauled or replaced, and their states are reset to “as-good-as-new” state.

IMRs are considered very costly when the accessibility of the item to be maintained is low, such as this system operating in deep water[27]. It is beneficial to conduct a reasonable maintenance strategy for reducing IMRs costs while keeping the system performance acceptable. With the motivating example, a comprehensive approach is proposed to optimize

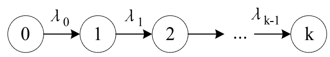


Fig. 2. State transition diagram of individual component

the maintenance activities for dependent multi-component systems. The overall approach developed in this paper can be summarized as follows:

Step 1 Describe the degradation process of the system without failure dependence.
Step 2 Identify the system structure and factors influencing the failure dependence.
Step 3 Evaluate the failure dependences between components based on system and environmental conditions.
Step 4 Describe the degradation process of the system with failure dependence.
Step 5 Construct a CBM model.
Step 6 Calculate the system availability and maintenance cost.
Step 7 Find the optimal maintenance threshold for maintenance activities.

The proposed approach is detailed and discussed in the following sections.

3. Degradation models for a dependent multi-component system

3.1. Independent general degradation model

An independent general degradation model with a general degradation path is developed firstly in Fig. 2 to reflect the inherent independent degradation of components in a dependent system. This model serves as the foundation of DMDM when degradation dependences are taken into account in a dependent multi-component system.

We start with a fully functioning state $x = 0$ at time t=0 and observe the component until failure. State $x = k$ represents the failed state of component i as an absorbing state. Between $x = 0$ and $x = k$ there are $k-1$ intermediate states. Let $P_x(t)$ be the probability that the component is in state x at time t. We could obtain a time dependent probability vector $P(t) = [P_0(t), P_1(t), \dots, P_k(t)]$, denoting the sojourn probability of the Markov process at time t. The initial state probability $P(0) = [1, 0, \dots, 0]$, and the sum of state probabilities is equal to 1 at any time.

Let $\mathbb{A}$ be a $k \times k$ matrix where the element $a_{x,y}$ denotes the transition rates from state x to state y for all $x \neq y$ and $x, y \in \{0, 1, 2, \dots, k\}$. State 0 is the brand-new state. For this simple independent degradation model, we assume that the degradation process proceeds all states chronologically from 0 to k. The degradation rate could be represented by λ_x from state x to state x+1. Then the state equation may be written according to Kolmogorov forward equations[28] in matrix terms as

$$P(t) \cdot \mathbb{A} = \dot{P}(t) \tag{1}$$

from which it follows

$$\dot{P}_y(t) = \sum_{x=0}^{k} a_{x,y} P_x(t) \tag{2}$$

If $P_x(t)$ tends to a constant value when $t \to \infty$, then

$$\lim_{t \to \infty} \dot{P}_y(t) = 0 \tag{3}$$

The steady state probabilities $\boldsymbol{P} = [P_0, P_1, \cdots, P_k]$ must therefore satisfy the matrix equation

$$\boldsymbol{P} \cdot \mathbb{A} = \boldsymbol{0} \tag{4}$$

More basic illustrations and details about how to develop the Markov models are reported in the literature[28].

3.2. Failure dependence model

If the degradation rate of a component is impacted by other degrading or failed components, the state transition can be shown in Fig. 3. The state of two-component system is expressed as $X = (x_1,\ x_2)^T$, and so is the state of the n components system $X = (x_1,\ x_2, \dots\ x_n)^T$, where $x_i \in \{0, 1, 2, \dots, k\}$ could characterize the degradation state of component i in this system. Each component has $k+1$ states, and the state in which the component is depends on how degraded it is in comparison to the failed state. As a result, components in the same state may exhibit varying degrees of degradation. This means that although there may be some components in the same degradation state, they could have distinct levels of degradation. The degradation of the two-component system could be illustrated by $\{X_0,\ X_1, \dots\ X_{(k+1)^2}\}$ since there are states for each component and $(k+1)^2$ states for the whole system. Similarly, the degradation of the n components system is governed by the state space , which is taken to be $\{X_0,\ X_1, \cdots\ X_{(k+1)^n}\}$ since there are $k+1$ states for each component and $(k+1)^n$ states for the n components system in total. State $X_0 = (0, 0, \cdots, 0)^T$ is the brand-new

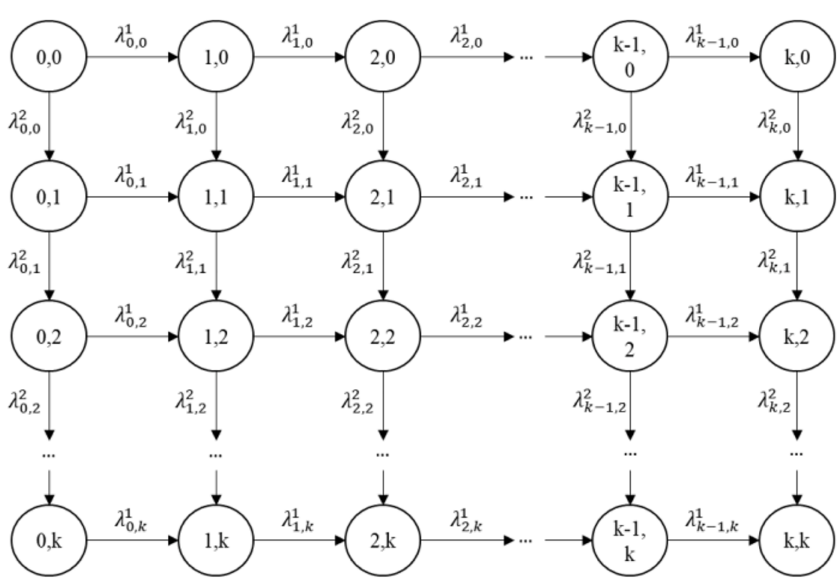


Fig. 3. State transition diagram of a two-component system with failure dependence

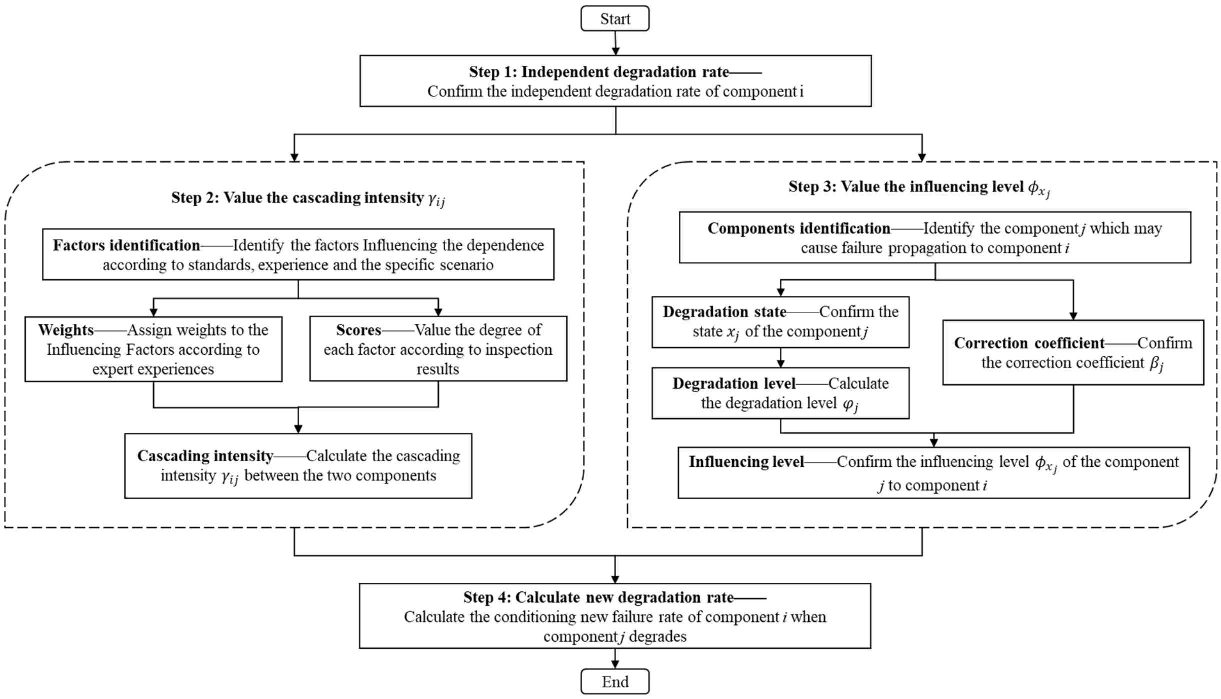


Fig. 4. Flowchart of new degradation rate identification considering failure dependence

state. State $X_{(k+1)^n} = (k, k, \cdots, k)^T$ is an absorbing state.

Now we start with $X = (0, 0)^T$ at time t=0 in Fig. 3. State $X = (k, k)^T$ represents the failed state of the two-component system as an absorbing state. The transition rates in this state transition diagram are illustrated as follows. The $(k+1)^2 \times (k+1)^2$ matrix now represents the transition rates from state (x_1, x_2) to next state. $\lambda^1_{x_1,x_2}$ is the degradation rate of component 1 from state x_1 to state $x_1 + 1$ when there exists failure dependence between it and another component 2 whose state is x_2. Similarly, in an n components system, the $(k+1)^n \times (k+1)^n$ matrix represents the transition rates from state $(x_1, x_2, \cdots, x_i, \cdots, x_n)$ to next state. $\lambda^i_{x_1,x_2,\cdots,x_i,\cdots,x_n}$ is the degradation rate of component i from state x_i to state $x_i + 1$ when there exists failure dependence between it and other components whose states are $(x_1, x_2, \cdots, x_n)$. Observe that events of multiple transitions are not included in the state transition diagram, such as a transition between state (x_i, x_j) and $(x_i + 1, x_j + 1)$, since it is assumed to be impossible for all the components in a system to degrade simultaneously during a short time interval from the point of practical.

We initially examine the failure dependence between two components i and j, and then expand the failure dependence model to conclude n components. For failure dependence between two components i and j, when component j degrades, the degradation rate of component i increased, and the calculation procedure of new degradation rate for component i could be demonstrated by the flowchart in Fig. 4. The new degradation rate for component i from state x_i to state x_i +1 and influenced by degradation of component j is expressed by

$$\lambda^i_{x_i,x_j} = \left(1 + \gamma_{ij}\phi_{x_j}\right)\lambda_{x_i}, \ \forall i \neq j \tag{5}$$

where γ_{ij} is the cascading intensity between components i and j, representing the possibility that failures or degradations are cascaded to components i from component j; ϕ_{x_j} is the influencing level from component j, whose value is determined by the degradation degree of component j compared to its failed state. Detailed explanations about the parameters are provided in the following:

(1) Cascading intensity γ_{ij}

The cascading intensity[29,30] between components is determined based on system layout, material backup, safety redundancy, and other practical constraints. According to industrial standards, expert experience, and practical scenarios, it is possible to obtain γ_{ij}, which characterizes the influence of component j on component i in a probabilistic manner, and the same goes for the influence of component i on component j. Furthermore, the value of cascading intensity is supposed to be between 0 and 1. When γ_{ij} is closer to 0, the degradation of component j has little influence on the degradation of component i. When γ_{ij} is closer to 1, the failure dependence between components is quite strong.

The value of cascading intensity depends on the importance of influencing factors and the situation of each factor in the given circumstances. A simple example is given here to illustrate how to determine γ_{ij}. Assume that there are three factors determining the cascading intensity between two pipelines: distance, load redistribution, and safety barrier. These three factors basically encompass the two types of failure dependences outlined previously, as well as the safety measures to mitigate them. More specifically, the distance between components is an essential factor influencing Type I failure dependence. Similarly, load redistribution is the dominant factor in Type II failure dependence. We assign weights for them based on historical data and expert experience: 0.4, 0.2, and 0.4. The value of distance degree could be scored simply as $1 - (d/D)$, where d is the real distance between two pipelines and D is the safe distance. Thus, the distance degree could be 2/5 if their horizontal clearance is 30mm but the required horizontal clearance is 50mm. For load redistribution, if two pipelines are both required to transfer fluid at 70% capacity, and one pipeline would suffer 10% more when another pipeline fail, we can score the factor load redistribution between the two pipelines at 1/7. We can score the third factor based on whether a safety barrier is in place or not and what is its availability. If there is no safety barrier, the score is set as 1, and the score decreases as the availability and reliability of safety barrier improved. Here we assume there are thermal-protective coating surrounding the pipelines, but its reliability is on the decline, and we can provide a score of 0.7 after evaluation. The examination aforementioned can reach the conclusion that the overall cascade intensity between two pipelines is $0.4 \times 2/5 + 0.2 \times 1/7 +$

$0.4 \times 0.7 = 0.4686$. However, this study intends to develop a universal model to provide some guidance to quantify the failure dependence. Therefore, the methods to assign weighs and score influencing factors are not highly emphasized in our methodology.

(1) Influencing level ϕ_{x_j}

The influencing level ϕ_{x_j} denotes the loading increment or shock from degrading or failed inducing component j to the induced component i. It is considered to be determined by the degradation degree of component j:

$$\phi_{x_j} = \varphi_j \beta_j \tag{6}$$

where φ_j is the degradation degree of inducing component j, which depends solely on its degradation level compared to the failed state. Since the components degrade overtime and may be repaired after maintenance actions, the value of φ_j varies over time.

$$\varphi_j = x_j / k \tag{7}$$

Technical errors or environmental factors can negatively impact the accuracy of engineering data, resulting in obtained data that may not be in accordance with real data. In order to minimize such errors, we introduce a correction coefficient β_j considering that the effect of component degradation degree on other components is not strictly distributed due to unstable factors. Introducing correction coefficients enables adjustments of the influencing level model to better reflect the actual situation based on historical or experimental data. The range of correction coefficient should be [0, 1] empirically. Normally linear regression could be used to estimate the value of β_j based on historical data or experimental data. We assume the correction coefficient is uniformly distributed in [0, 1] in our study.

3.3. Dependent multi-component degradation model

For the degradation process in a system composed of n components, the state transition influenced by degradation of all the other components could be expressed by a Markov model step by step. Since there are n^2 correlations denoting the cascading intensity in the n components system, the cascading intensity among components could be expressed by a matrix $\boldsymbol{\gamma} = \{\gamma_{ij}\}_{n \times n}$. In addition, $\boldsymbol{\phi} = (\varphi_1\beta_1, \varphi_2\beta_2, \cdots, \varphi_k\beta_k)^T$ is a vector of influencing level for all the components $j \neq i$. We can use an $n \times n$ matrix $\mathbb{D}$ to denote the failure dependences among components

$$\mathbb{D} = \begin{pmatrix} \mathbb{D}_1 \\ \mathbb{D}_2 \\ \vdots \\ \mathbb{D}_n \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \boldsymbol{D}_{1,2} & \cdots & \boldsymbol{D}_{1,n-1} & \boldsymbol{D}_{1,n} \\ \boldsymbol{D}_{2,1} & \mathbf{0} & & \boldsymbol{D}_{2,n-1} & \boldsymbol{D}_{2,n} \\ \vdots & & \ddots & \vdots & \\ \boldsymbol{D}_{n,1} & \boldsymbol{D}_{n,2} & \cdots & \boldsymbol{D}_{n,n-1} & \mathbf{0} \end{pmatrix} \tag{8}$$

where $\mathbb{D}_i$ is the matrix of failure dependences from other components to component i following the selection of component i as the target component, which is component 1. As the identification numbers of the components do not carry any significance, any component in the system can be designated as component 1. For this reason, in the section of our study that follows, we presume that component 1 and the identification numbers of the other components have been determined to simplify the issue. $\mathbf{0}$ is the null matrix whose order is corresponding by the dimensions of blocks $\boldsymbol{D}_{i,j}$, indicating that the component suffers no failure dependence from itself. This also means that $D_{i,x_i} = 0$ for any component i. $\boldsymbol{D}_{i,j} = (D_{i,x_j=0}, D_{i,x_j=1}, \cdots, D_{i,x_j=k})$ denoting the failure dependence from component j on component i. Furthermore, the element in the submatrix $D_{i,x_j} = \gamma_{ij}\phi_{x_j}$ is vector to represent the failure dependence from component j on component i when component j is in state x_j. After the qualification of the correlations among components, the new transition rates of the component could be updated as below.

$$\lambda^i_{x_i,x_j} = (1 + D_{i,x_j})\lambda_{x_i} \tag{9}$$

$$\lambda^j_{x_i,x_j} = (1 + D_{j,x_i})\lambda_{x_j} \tag{10}$$

where $\lambda^i_{x_i,x_j}$ is the degradation rate of component i from state x_i to state $x_i + 1$ when there exists failure dependence between it and another component j whose state is x_j, and $\lambda^j_{x_i,x_j}$ is the degradation rate of component j from state x_j to state $x_j + 1$ when there exists failure dependence between it and another component i whose state is x_i.

The states of the system $(x_1, x_2, \cdots, x_n)$, from (0,…,0) to (k,…,k), are divided into $(k+1)^{(n-1)}$ subsets, where only the component 1 (any component could be chosen as component 1) degrades and other components remain the constant states in every subset $\{(0,x_2,\cdots,x_n),(1,x_2,\cdots,x_n),\cdots,(k,x_2,\cdots,x_n)\}$. Consequently, the matrix could be expressed as a $(k+1)^{(n-1)} \times (k+1)^{(n-1)}$ matrix as follows.

$$\mathbb{A} = \begin{pmatrix} \mathbb{A}_n^0 & \boldsymbol{A}_n^0 & & & & \\ & \mathbb{A}_n^1 & \boldsymbol{A}_n^1 & & & \\ & & \mathbb{A}_n^2 & & & \\ & & & \ddots & & \\ & & & & \mathbb{A}_n^{k-1} & \boldsymbol{A}_n^{k-1} \\ & & & & & \mathbb{A}_n^k \end{pmatrix} \tag{11}$$

The blocks $\mathbb{A}_n^{x_n}$ are the sub matrixes of the whole matrix $\mathbb{A}$, denoting the transition rates of the system where the state of component n keeps constant x_n; The blocks $\boldsymbol{A}_n^{x_n}$ are also the sub matrixes of the whole matrix $\mathbb{A}$, denoting the transition rates of the system where only the component n degrades from state x_n to state $x_n + 1$. The other blocks are null, and their orders are corresponding by the dimensions of blocks and $\boldsymbol{A}_n^{x_n}$. The blocks $\mathbb{A}_n^{x_n}$ could also be further represented by smaller sub matrixes and $\boldsymbol{A}_{n-1}^{x_{n-1}}$ using the same recursive way from $\mathbb{A}$ to and $\boldsymbol{A}_n^{x_n}$. The same applies to the general sub matrixes $\mathbb{A}_i^{x_i}$. With the recursive method, the general blocks $\mathbb{A}_i^{x_i}$ could also be further represented by smaller sub matrixes and $\boldsymbol{A}_j^{x_j}$ as below. The blocks $\mathbb{A}_i^{x_i}$ are the sub matrixes denoting the transition rates of the system where the state of component i keeps constant x_i; The blocks $\boldsymbol{A}_j^{x_j}$ are the sub matrixes denoting the transition rates of the system where only the component j degrades from state x_j to state $x_j + 1$. The other blocks are null, and their orders are corresponding by the dimensions of blocks and $\boldsymbol{A}_j^{x_j}$.

$$\mathbb{A}_i^{x_i} = \begin{pmatrix} \mathbb{A}_{\mathbf{i-1}}^{\mathbf{0}} & \boldsymbol{A}_{\mathbf{i-1}}^{\mathbf{0}} & & & & \\ & \mathbb{A}_{\mathbf{i-1}}^{\mathbf{1}} & \boldsymbol{A}_{\mathbf{i-1}}^{\mathbf{1}} & & & \\ & & \mathbb{A}_{\mathbf{i-1}}^{\mathbf{2}} & & & \\ & & & \ddots & & \\ & & & & \mathbb{A}_{\mathbf{i-1}}^{\mathbf{k-1}} & \boldsymbol{A}_{\mathbf{i-1}}^{\mathbf{k-1}} \\ & & & & & \mathbb{A}_{\mathbf{i-1}}^{\mathbf{k}} \end{pmatrix} \tag{12}$$

$$A_j^{x_j} = \begin{pmatrix} A^j_{x_2,\cdots,0,x_{j+1},\cdots,x_n} & & & \\ & A^j_{x_2,\cdots,1,x_{j+1},\cdots,x_n} & & \\ & & \ddots & \\ & & & A^j_{x_2,\cdots,k,x_{j+1},\cdots,x_n} \end{pmatrix} \tag{13}$$

for $j = 2, 3, \cdots, n$.

where the elements $A^j_{x_2,\cdots,x_j,\ \cdots x_n}$ in $A_j^{x_j}$ are also sub matrixes, as shown in equation (14), denoting the transition rates of the system where only the component j degrades from state x_j to state $x_j + 1$ when other components are at state $(x_1, \cdots, x_n)$ for $j \neq 1$. For example, $A^j_{x_2,\cdots,0,x_{j+1},\cdots,x_n}$ refers to the transition rates of the system where only the component j degrades from state 0 to state 1 when other components are at state $(x_1, \cdots, x_n)$ for $j \neq 1$.

$$A^j_{x_2,\cdots,x_j,\ \cdots x_n} = \begin{pmatrix} \lambda^j_{0,\ x_2,\cdots,x_j,\ \cdots x_n} & & & \\ & \lambda^j_{1,\ x_2,\cdots,x_j,\ \cdots x_n} & & \\ & & \ddots & \\ & & & \lambda^j_{k,\ x_2,\cdots,x_j,\ \cdots x_n} \end{pmatrix} \tag{14}$$

As mentioned before, the whole matrix $\mathbb{A}$ is recursed to the blocks $\mathbb{A}_n^{x_n}$, and further recursed to the blocks for $i = 3, \cdots, n-1$. The recursive process stops when i equals 3, and at this point we have

$$\mathbb{A}_3^{x_3} = \begin{pmatrix} \mathbb{A}_{0,\ x_3,\cdots,\ x_n} & A^2_{0,\ x_3,\cdots,\ x_n} & & & & \\ & \mathbb{A}_{1,\ x_3,\cdots,\ x_n} & A^2_{1,\ x_3,\cdots,\ x_n} & & & \\ & & \mathbb{A}_{2,\ x_3,\cdots,\ x_n} & & & \\ & & & \ddots & & \\ & & & & \mathbb{A}_{k-1,\ x_3,\cdots,\ x_n} & A^2_{k-1,\ x_3,\cdots,\ x_n} \\ & & & & & \mathbb{A}_{k,\ x_3,\cdots,\ x_n} \end{pmatrix} \tag{15}$$

The block $A^j_{x_2,\cdots x_j,\ \cdots x_n}$ could be obtained by equation (14). The block $\mathbb{A}_{\ x_2,\cdots,\ x_n}$ is the submatrix denoting the transition rates of components 1 when other components are at state $(x_2, \cdots, x_n)$.

$$\mathbb{A}_{\ x_2,\cdots,x_n} = \begin{pmatrix} -\sum_{j=1}^{n}\lambda^j_{0,x_2,\cdots,x_n} & \lambda^1_{0,x_2,\cdots,x_n} & & & \\ & -\sum_{j=1}^{n}\lambda^j_{1,x_2,\cdots,x_n} & & & \\ & & \ddots & & \\ & & & -\sum_{j=1}^{n}\lambda^j_{k-1,x_2,\cdots,x_n} & \lambda^1_{k-1,x_2,\cdots,x_n} \\ & & & & -\sum_{j=2}^{n}\lambda^j_{k,x_2,\cdots,x_n} \end{pmatrix} \tag{16}$$

where $\lambda^1_{x_1,x_2,\cdots,x_n}$ is transition rate of the component 1 from state from state x_1 to state $x_1 + 1$ when there exist failure dependences between it and other components whose state are $(x_2, \cdots, x_n)$

$$\lambda^1_{x_1,x_2,\cdots,x_n} = \lambda_{x_1} \cdot \prod_{j=1}^{n}\left(1 + D_{1,x_j}\right) \tag{17}$$

and $\lambda^j_{\ x_1,\cdots,x_n}$ is transition rate of the component j from state from state x_j to state $x_j + 1$ when there exist failure dependences between it and other components whose state are $(x_1, \cdots, x_n)$ for $j \neq 1$.

$$\lambda^j_{x_1,\cdots,x_n} = \lambda_{x_j} \cdot \prod_{i=1}^{n}\left(1 + D_{j,x_i}\right) \tag{18}$$

In this situation we let $P_{x_i}(t)$ be the probability that the component i is in state x_i at time t and $P_{X_i}(t)$ be the probability that the entire system is in state X_i at time t. The vector $\boldsymbol{P}(t) = [P_{X_0}(t), P_{X_1}(t), \cdots, P_{\ X_{k^n-1}}(t)]$ denotes the time dependent state probability, and the initial state probability $\boldsymbol{P}(0) = [1, 0, \ldots, 0]$.

4. Modeling and formulation of condition-based maintenances

In this section we describe the general maintenance policies for multi-component systems with heterogeneous failure dependences. We consider a system with n components. The system state transition process is modeled with a Markov model. In the model, the following assumptions are introduced:

- The states of components are revealed upon periodic inspections.
- The maintenance policies are based on the detected state of system.
- At inspection, a maintenance action can begin without any delay.
- The inspection and repair time could be ignored compared to its long service lifetime.

4.1. Inspections and maintenances

Regular inspections are conducted for many passive items such as valves, pipelines, vessels, and pumps in the process industry. As assumed above, the inspection interval is $(s-1)\tau \leq t \leq s\tau$ for $s = 1, 2, \cdots, N_{IMR}$, where τ is a constant value independent of the component state and the time. Suppose every inspection for the system could reveal the states of all components. The inspections durations are assumed to be neglected and the state of components are revealed immediately. The inspection intervals are recounted after each inspection, maintenance, or repair in the overall lifecycle of the system, and could be modeled as $[0, T_1]$, $[T_1, T_2]$, $\cdots$, $[T_{N_{IMR}-1}, T_{N_{IMR}}]$ If the states of components are found to reach the thresholds of maintenance measures, then a corresponding maintenance task will be carried out timely. The time immediately before inspection is denoted by t^- and the time right after IMRs is denoted by t^+ When the state of the system when $t = T_s^-$ is given, the maintenance activities for the system could be then decided. Note that CBM is a maintenance strategy that involves monitoring the actual condition of systems in order to determine the maintenance activities. Based upon the maintenance policy, the possible maintenance actions and the state of the system just after IMRs are assumed to depend on the state of the system when $t = T_s^-$, but independent of all transitions of the system before T_s The effect of IMRs at time $t = T_s$ could be illustrated by

$$\begin{aligned} &Pr\left(X\left(T_s^+\right) = X_j \middle| X\left(T_s^-\right) = X_i\right) = b_{X_i,X_j}, \\ &\text{for all } X_i, X_j \in \boldsymbol{\chi} \end{aligned} \tag{19}$$

where b_{X_i,X_j} is the probability that the system is in state X_j after IMRs, given that it was in state X_i before inspection.

Considering the aforementioned inspection strategies, several maintenance strategies are proposed. PM and CM are implemented according to the inspection results. The maintenance strategies are illustrated in the Fig. 5.

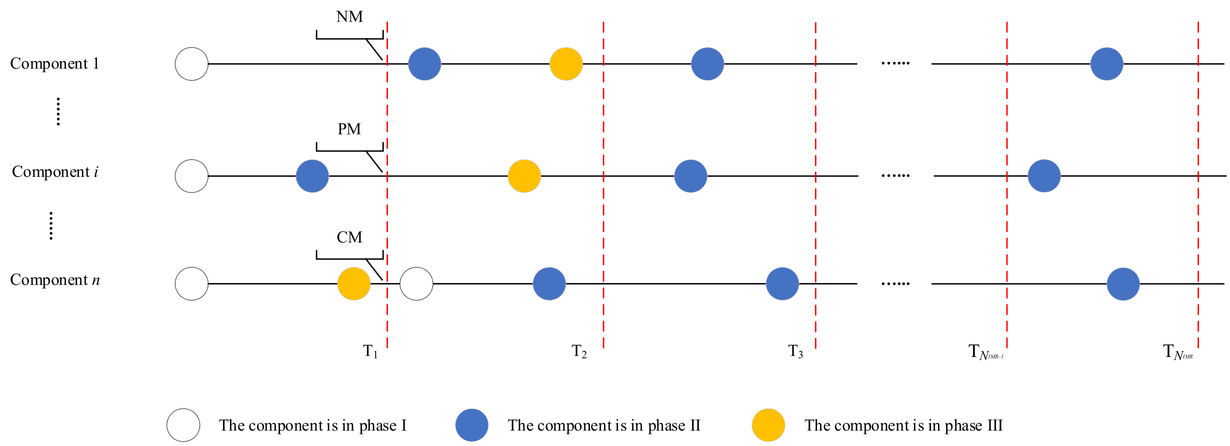


Fig. 5. Illustration of maintenance policies

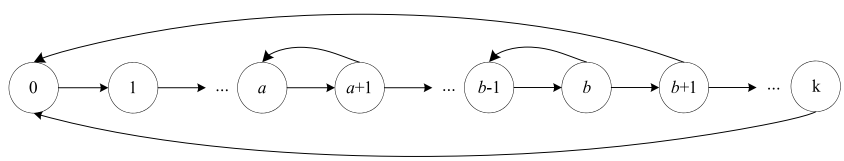


Fig. 6. Markov model of an individual component

For an independent component, the maintenance policy is classified into three phases.

In phase I ($x \le a$), the component is in an acceptable state, and no maintenance activities (NM) are required.

In phase II ($a + 1 \le x \le b$), the component is operating in a degrading state, and PM will be performed to improve the component condition by one state.

In phase III ($b + 1 \le x \le k$), CM is needed to restore the component to an as good as new state.

After applying the maintenance actions, the state transitions could be denoted by degradation transitions, repair transitions, and combinations of those, as seen in Fig. 6. Assume that the components have constant transition rate between two states.

Let $\mathbb{B}$ describes the corresponding maintenance transition matrix of the system, then

$$\boldsymbol{P}(T_s^+) = \boldsymbol{P}(T_s^-) \cdot \mathbb{B} \tag{20}$$

The corresponding maintenance matrix $\mathbb{B}$ is expressed by the sub matrixes

$$\mathbb{B} = \begin{pmatrix} \mathbb{B}_{\mathbf{n}}^{\mathbf{I}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbb{B}_{\mathbf{n}}^{\mathbf{a}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbb{B}_{\mathbf{n}}^{\mathbf{a+1}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbb{B}_{\mathbf{n}}^{II} & \mathbf{0} \\ \mathbb{B}_{\mathbf{n}}^{III} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{21}$$

The blocks $\mathbb{B}_n^I$ and $\mathbb{B}_n^a$ are the sub matrixes, separately denote the maintenance transition of the system where the component n is in phase I or in state a. The explanation for other sub matrixes in equation (21) can be obtained similarly. These sub matrixes could be generalized and recursively defined using equation (22)

$$\mathbb{B}_i^{x_i} = \begin{pmatrix} \mathbf{B}_{\mathbf{i-1}}^{\mathbf{I}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\mathbf{i-1}}^{\mathbf{a}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\mathbf{i-1}}^{\mathbf{a+1}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{i-1}}^{II} & \mathbf{0} \\ \mathbf{B}_{\mathbf{i-1}}^{III} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{22}$$

The blocks $\mathbf{0}$ are null, and their orders are corresponding by the dimensions of sub matrixes.

Recurse the matrix until reaching the matrix of component 2.

$$\mathbb{B}_2^{x_2} = \begin{pmatrix} 1 & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & 1 & & & & & & & \\ & & & 1 & 0 & & & & & & \\ & & & & & \ddots & & & & & \\ & & & & & & 0 & & & & \\ & & & & & & 1 & 0 & & & \\ 1 & & & & & & & & 0 & & \\ \vdots & & & & & & & & & \ddots & \\ 1 & & & & & & & & & & 0 \end{pmatrix} \tag{23}$$

Since the process has no steady state and could be divided into several time units on a given finite time horizon, the time-dependent state probability vector $\boldsymbol{P}(t)$ at time t could be then given by

$$\boldsymbol{P}(t) = \boldsymbol{P}(0) \cdot \left(\prod_{s=1}^{s=N_{IMR}} \exp(\mathbb{A}(T_s - T_{s-1})) \cdot \mathbb{B} \right) \cdot \exp(\mathbb{A}(t - T_{N_{IMR}})) \tag{24}$$

Table 1
Parameter setting of the subsea system considered in the case-study.

Parameter	Value (/year) Compressor	Pumps	Parameter	Value (€) Compressor	Pumps	Parameter	Value (€)
λ_0	0.046	0.104	c_{m1}	1.93×10^6	2.41×10^6	c_{in}	1.21×10^6
λ_1	0.021	0.105	c_{m2}	2.89×10^6	3.86×10^6	c_p	7.23×10^5
λ_2	0.041	0.056				c_u	6.51×10^7

4.2. System availability analysis

In this subsection, the developed state probabilities formulas are applied to quantify the system availability, which refers to the percentage of time that the system remains operational under normal circumstances in order to perform its intended function. Suppose that the system is not available only when it fails, the mean value of the system failure probability over a period of time could then be used to represent the unavailability of the system

$$\overline{A_s} = \frac{1}{T}\int_0^T P_{X_F}(t)dt \tag{25}$$

where X_F denotes that the component or the entire system is in the failed states immediately at time t. $P_{X_F}(t)$ represents the probability that the entire system is in the failed states at time t. Based on the identification of all the failed states and the probabilities that the system is in various states at time t included in vector $\boldsymbol{P}(t)$, $P_{X_F}(t)$ could be calculated by summing up all the probabilities that the entire system is in the failed state at time t.

The availability of the system is the probability of being operational given by

$$A_S = 1 - \overline{A_s} \tag{26}$$

The model in this subsection is proposed to seek for the optimal value of the maintenance threshold to increase the system availability to an acceptable level.

4.3. Maintenance cost

Here we consider that the maintenance cost consists of the inspection cost, the downtime cost, and the repair cost.

Suppose that the inspection cost is c_{in} for each time. The downtime cost contains the planned downtime cost c_p caused by the scheduled maintenance activities and the unplanned downtime cost c_u induced by unexpected failures.

The cumulative maintenance cost between two inspections in the time interval $(T_{s-1}, T_s]$ accounts for the maintenance cost at time $t = T_s$. The repair cost is supposed to includes $c_{m1,i}$ and $c_{m2,i}$ respectively for maintenance activities PM and CM to component i. Therefore, the cumulative maintenance cost for the system in $(T_{s-1}, T_s]$ is

$$\begin{aligned} C((T_{s-1}, T_s]) &= \sum_{i=1}^{n}\left[c_{m1,i}Pr(a+1 \le x_i(T_s) \le b) + c_{m2,i}Pr(b+1 \le x_i(T_s) \le k)\right] \\ &= \sum_{i=1}^{n}\left[c_{m1,i}P_{a+1\le x_i \le b}(T_s) + c_{m2,i}P_{b+1 \le x_i \le k}(T_s)\right] \end{aligned} \tag{27}$$

where $x_i(t)$ is the degradation state of component i at time t.

The average life-time cost during the period T could be given by

$$C_S = \left[c_{in}N_{IMR} + c_pN_{IMR} + \sum_{s=1}^{N_{IMR}} C((T_{s-1}, T_s])\right] \Big/ T + c_u\overline{A_s} \tag{28}$$

The model in this subsection is proposed to seek for the optimal value of the maintenance threshold to minimize the maintenance cost.

Table 2
Parameter setting of the failure dependences.

Parameter	Value	Parameter	Value
γ_{12}	0.34	ϕ_0	0
γ_{13}	0.24	ϕ_1	1/3
γ_{23}	0.66	ϕ_2	2/3
γ_{21}	0.44	ϕ_3	1
γ_{31}	0.34		
γ_{32}	0.56		

5. Case-Study: assessment of the motivating example

The motivating example of a subsea transmission system is explored to illustrate the advantages of the proposed maintenance policies. To reveal the hidden failures, inspections are performed regularly to examine the system to confirm compliance with the performance requirements. The parameter setting of the degradation, inspection and maintenance are provided Table 1. The failure rates values are obtained from the existing literature[25,31] and from the application of the Cox model [32], using the data derived from OREDA database[33]. The service life and repair cost were obtained from the article[34] and thesis [27].

We assume that there are only four states for each component: normally operating, moderately degraded, severely degraded, and failed. The initial state probability . The states of the system $X = (x_1, x_2, x_3)$ from (0, 0, 0) to (3, 3, 3), are divided into 4^2 subsets: (0,0,0), (1,0,0), (2,0,0), (3,0,0); (0,1,0), (1,1,0), (2,1,0) (3,1,0); …; (0,3,0), (1,3,0), (2,3,0), (3,3,0); ……; (0,0,3), (1,0,3), (2,0,3), (3,0,3); (0,1,3), (1,1,3), (2,1,3), (3,1,3); …; (0,3,3), (1,3,3), (2,3,3), (3,3,3).

As illustrated before, three key factors are generally considered to impact on the failure dependences between components: load redistribution, distance, and safety barrier. In this system, weights of the factors are assigned according to experts' experience: distance (2), load redistribution (5), and safety barrier (3). Here load redistribution denotes the material transfer and doping. After expert experience, the parameters of the failure dependences could be evaluated as Table 2 based on the method proposed in Subsection 3.2. Since the states of all the components are expressed as $x_i \in \{0, 1, 2, 3\}$, the degradation level of the components could be estimated as $\varphi_j \in \{0, 1/3, 2/3, 1\}$. It is plausible to conclude that $\boldsymbol{\phi} = (\phi_0, \phi_1, \phi_1, \phi_3)^T = (0, 1/3, 2/3, 1)^T$ is the vector of influencing level for all the components when correction coefficient β_j is assumed to be 1. To address the necessity of considering failure dependence, we also set all the parameters in Table 2 as 0 or other values to imitate the scenario when failure dependence is neglected or varied in this example. With modifying the values in the table after assessing the failure dependences of differing levels, the proposed model could be applied to computing the system under various conditions.

Based on the data from Table 2, we could obtain a 3×3 matrix $\mathbb{D}$ to denote the failure dependences among components

$$\mathbb{D} = \begin{pmatrix} \boldsymbol{E} & \boldsymbol{D}_{1,2} & \boldsymbol{D}_{1,3} \\ \boldsymbol{D}_{2,1} & \boldsymbol{E} & \boldsymbol{D}_{2,3} \\ \boldsymbol{D}_{3,1} & \boldsymbol{D}_{3,2} & \boldsymbol{E} \end{pmatrix} \tag{29}$$

Taking the failure dependence from component 2 on component 1 as a simple example to illustrate the calculation of failure dependences, we have $\boldsymbol{D}_{1,2} = (D_{1,x_2=0}, D_{1,x_2=1}, \cdots, D_{1,x_2=3}) = (0, 0.103, 0.207, 0.31)$ if the correction coefficient β_2 is supposed to be 1. The same calculation method can be applied to other submatrices of failure dependences. With the confirmation of failure dependences among components, the DMDM could be denoted by the transition matrix $\mathbb{A}$. The transition matrix $\mathbb{A}$ could be expressed as a $4^2 \times 4^2$ matrix

$$\mathbb{A} = \begin{pmatrix} \mathbf{A}_3^0 & \mathbf{A}_3^0 & & \\ & \mathbf{A}_3^1 & \mathbf{A}_3^1 & \\ & & \mathbf{A}_3^2 & \mathbf{A}_3^2 \\ & & & \mathbf{A}_3^3 \end{pmatrix} \tag{30}$$

$$\mathbb{A}_3^{x_3} = \begin{pmatrix} \mathbb{A}_{0,x_3} & \boldsymbol{A}^2_{0,x_3} & & \\ & \mathbb{A}_{1,x_3} & \boldsymbol{A}^2_{1,x_3} & \\ & & \mathbb{A}_{2,x_3} & \boldsymbol{A}^2_{2,x_3} \\ & & & \mathbb{A}_{3,x_3} \end{pmatrix} \tag{31}$$

for $x_3 = 0, 1, 2, 3$.

$$\boldsymbol{A}_3^{x_3} = \begin{pmatrix} \boldsymbol{A}^3_{x_2,x_3} & & & \\ & \boldsymbol{A}^3_{x_2,x_3} & & \\ & & \boldsymbol{A}^3_{x_2,x_3} & \\ & & & \boldsymbol{A}^3_{x_2,x_3} \end{pmatrix} \tag{32}$$

for $x_3 = 0, 1, 2$.

$$\boldsymbol{A}_2^{x_2} = \begin{pmatrix} \boldsymbol{A}^2_{x_2,x_3} & & & \\ & \boldsymbol{A}^2_{x_2,x_3} & & \\ & & \boldsymbol{A}^2_{x_2,x_3} & \\ & & & \boldsymbol{A}^2_{x_2,x_3} \end{pmatrix} \tag{33}$$

for $x_2 = 0, 1, 2$.

where the blocks $\mathbb{A}_{x_2,\, x_3}$ and $\boldsymbol{A}^j_{x_2,\, x_3}$ are as follows

$$\mathbb{A}_{x_2,\, x_3} = \begin{pmatrix} -\sum_{i=1}^{3}\lambda^i_{0,\, x_2,\, x_3} & \lambda^1_{0,\, x_2,\, x_3} & & \\ & -\sum_{i=1}^{3}\lambda^i_{1,\, x_2,\, x_3} & \lambda^1_{1,\, x_2,\, x_3} & \\ & & -\sum_{i=1}^{3}\lambda^i_{2,\, x_2,\, x_3} & \lambda^1_{2,\, x_2,\, x_3} \\ & & & -\sum_{i=2}^{3}\lambda^i_{3,\, x_2,\, x_3} \end{pmatrix} \tag{34}$$

$$\boldsymbol{A}^j_{x_2,\, x_3} = \begin{pmatrix} \lambda^j_{0,\, x_2,\, x_3} & & & \\ & \lambda^j_{1,\, x_2,\, x_3} & & \\ & & \lambda^j_{2,\, x_2,\, x_3} & \\ & & & \lambda^j_{3,\, x_2,\, x_3} \end{pmatrix} \tag{35}$$

for $j = 2, 3$.

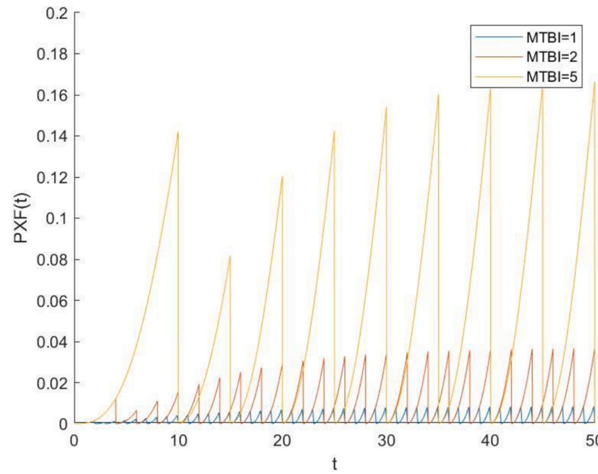


Fig. 7. Failure probabilities for different MTBI

The matrix $\mathbb{B}$ could be expressed as a $4^2 \times 4^2$ matrix.

$$\mathbb{B} = \begin{pmatrix} \mathbf{B}_3^0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_3^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_3^2 & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_3^3 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{36}$$

$$\mathbb{B}_3^{x_3} = \begin{pmatrix} \mathbf{B}_2^0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2^2 & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_2^3 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix},\ \mathbb{B}_2^{x_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{37}$$

$\mathbb{B}_3^{x_3}$ and represents the maintenance matrix when components 3 or 2 are respectively in the state x_3 or x_2.

The whole degradation transition matrix $\mathbb{A}$ and the whole maintenance transition matrix could be obtained after splicing the matrices. The state probability vector $\boldsymbol{P}(t)$ could then be calculated by equation (24). The time dependent failure probabilities $P_{X_F}(t)$ are obtained by summing the probabilities of different failure states of the system.

The maintenance cost for an independent component in $(T_{s-1}, T_s]$ is

$$\begin{aligned} C((T_{s-1}, T_s]) &= c_{m1,comp}P_{x_1=2}(T_s) + c_{m2,comp}P_{x_1=3}(T_s) \\ &\quad + \sum_{i=2}^{3}\left[c_{m1,pump}P_{x_i=2}(T_s) + c_{m2,pump}P_{x_i=3}(T_s)\right] \end{aligned} \tag{38}$$

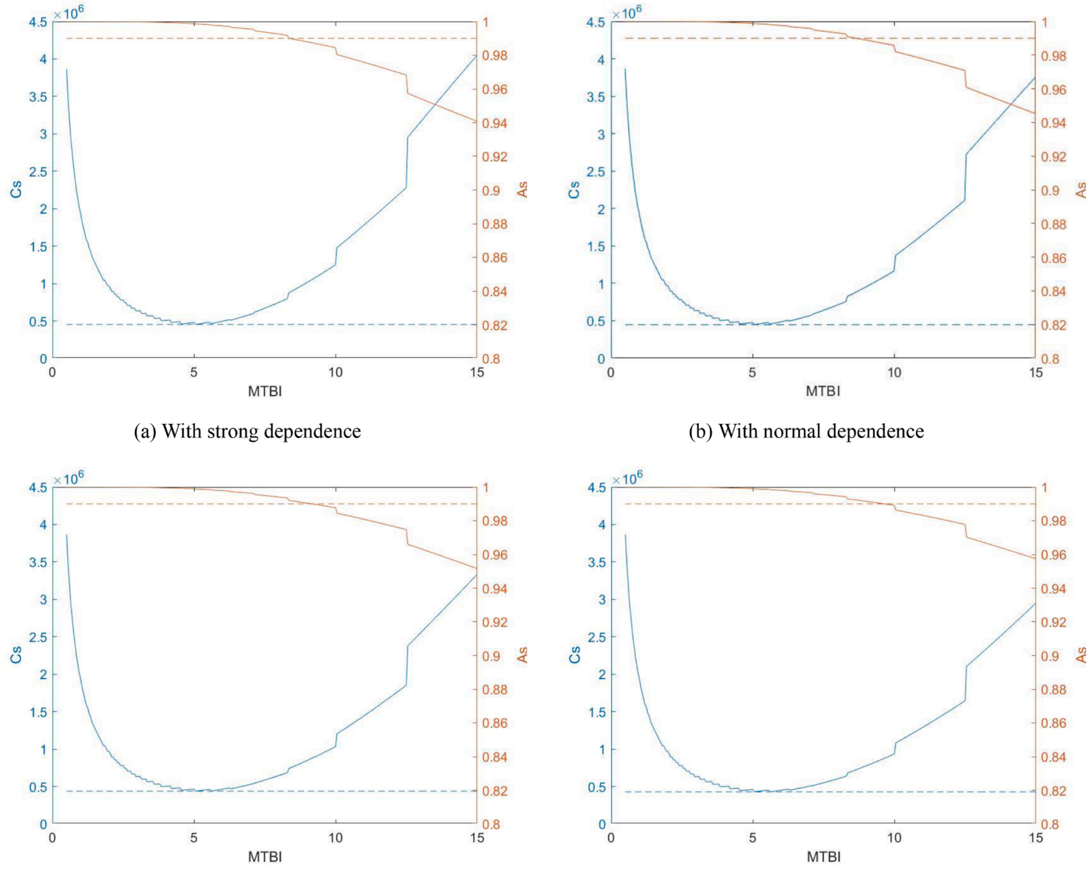
where the $c_{m1,comp}$ and $c_{m2,comp}$ are respectively the PM and CM for a compressor, $c_{m1,pump}$ and $c_{m2,comp}$ are respectively the PM and CM for a pump.

For this kind of system, the availability of the system and its average life-time cost could be obtained by equations (28) and (38).

5.1. Failure probabilities

The time dependent failure probabilities could be found by CTMC simulation. To evaluate the effect of mean time between inspections (MTBI) on the system conditions, observe the difference of $P_{X_F}(t)$ curves in the log-run horizon by testing various MTBI ($MTBI =$ 1 *year*; 2 *years*; 5 *years*.). The results are shown in Fig. 7.

It can be found that the failure probability increases with time and decreases suddenly at the IMRs timepoints under varying MTBI. This is because the system and its components degrade over time. When



(a) With strong dependence

(b) With normal dependence

(c) With weak dependence

(d) Without dependence

Fig. 8. Availability and average life-time cost of the transition system under different MTBI

approaching the IMRs timepoints, the failure probability peaks in this interval of time. However, after the IMRs timepoints, the status of the system and its components could be noticeably improved, and the system failure probability is close to zero, indicating a peak value at the IMRs timepoints. Another finding can be obtained by comparing the failure probability curves under different MTBI. It is obvious that with smaller MTBI, the maximum values of failure probabilities are expected to be lower. On the contrary, the maximum values of failure probabilities tend to be higher when the MTBI increases, which means that the system tends to be less reliable. In this regard, the reliability and availability of the system can be improved by reducing the value of MTBI, that is, shortening the IMRs interval. However, a lower MTBI is not always preferable. The following subsections will go through how to achieve the optimum MTBI value in practical applications.

5.2. *Maintenance strategies with various failure dependences*

Fig. 8 shows the availability and average life-time cost of the transition system under the condition of with various failure dependence respectively. The actual failure dependence in Table 2 is denoted as normal dependence. The failure dependences of the system under other circumstances are also accounted for: The strong dependence is set when all the γ take the maximum value (0.66) in Table 2; the case that all the γ take the minimum value (0.24) in Table 2 is weak dependence; there is no dependence when all the γ take the value of 0.

The figures show that the availability of the system decreases with the increase of MTBI. One interesting observation is that these curves are not smooth, but rather contain distinct breaking lines. It is found by examining these fold points that they are always located at certain MTBI values that enable the IMRs number to be an integer. For the A_s-MTBI curves, the smaller the MTBI is, the larger number of inspections and maintenance activities are needed, the higher the availability reached, and vice versa. This trend is consistent with the conclusion of the previous subsection. At each fold point, the IMRs frequency drops by one, which leads to a sudden increase in system failure probability and steady state probability of failure, resulting in a sudden decrease in system availability. Besides, the curves C_s-MTBI show a similar trend that the average life-time cost falls initially and subsequently climbs as MTBI grows, indicating that there is a point to minimize the cost. A reasonable explanation is that when the MTBI is relatively small, more inspections and maintenance are undertaken, which may lower the failure probability of system and the unexpected downtime cost, also may impose considerable IMRs costs. However, when the MTBI is greatly increased, the IMRs costs can be accordingly decreased; but the system unavailability rises, inevitably leading to more production loss due to unplanned downtime. Similarly, before the cost reaches the lowest value, the variation of IMRs cost dominates the trend of C_s-MTBI curves. As MTBI increases, the amount of IMRs may drop by one, causing the immediate drop of total IMRs cost and the average life-time cost. After the lowest value, the variation of unexpected downtime cost dominates the trend of C_s-MTBI curves. Hence the effect of drop amount of IMRs on the unexpected downtime cost is stronger than the effect on the IMRs cost. As the amount of IMRs drops, the failure probability increases suddenly, as well as the downtime cost, which is strongly proportional to it.

In practical engineering applications, an acceptable availability threshold is generally determined since it is too costly to pursue extensive system availability. In this case the average life-time cost should be minimized while ensuring system availability over 0.99. The optimum of the maintenance policy could be achieved by adjusting the parameter MTBI. From the figures of Fig. 8, the minimal cost appears in the range of

Table 3
Results for the transition system with various failure dependence.

	Availability When A_s is 0.99	When MTBI=15	Average life-time cost When C_s is minimized	When MTBI=15
With strong dependence	(8.35, 0.99)	(15, 0.9408)	(4.55, 454525)	(15, 4047150)
With normal dependence	(8.65, 0.99)	(15, 0.9452)	(5.05, 448474)	(15, 3755460)
With weak dependence	(9.15, 0.99)	(15, 0.9516)	(5.6, 438202)	(15, 3333170)
Without dependence	(9.65, 0.99)	(15, 0.9574)	(5.6, 428870)	(15, 2948640)

system availability greater than 0.99, thus the point of this minimal value could be considered as the ideal option of the maintenance strategy.

Notable distinctions between the findings with various failure dependence could also be observed. Table 3 displays the comparison of the results. In terms of the impact of MTBI on system availability, the availability of the system with stronger failure dependence is generally lower than that of the system with weaker failure dependence and that of the system without failure dependence. The thresholds of MTBI for system availability under 0.99 increases as system failure dependence weakens: 8.35 (strong), 8.65 (normal), 9.15 (weak), 9.65 (without). This means that when there is stronger failure dependence, the system should be inspected and maintained more regularly to keep its availability. From a financial standpoint, the minimal average life-time cost considering strong, normal, and weak failure dependence are respectively 454525€, 448474€, and 438202€, higher than the minimal average life-time cost without failure dependence (428870€). This also supports a similar result that a higher investment is required when stronger failure dependence is considered. The comparison of these graphs reveals the necessity of highlighting the failure dependence of complex systems while implementing CBM.

5.3. Maintenance strategies for various initial costs input

In the following, the variation of some cost parameters setting on the average life-time cost is investigated and the other parameters remain unchanged. By resetting the inspection cost $c_{in} = [1.21 \times 10^5, 1.21 \times 10^6, 1.21 \times 10^7]$, the planned downtime cost $c_p = [7.23 \times 10^4, 7.23 \times 10^5, 7.23 \times 10^6]$, the unplanned downtime cost $c_u = [6.51 \times 10^6, 6.51 \times 10^7, 6.51 \times 10^8]$, the influence of costs input on the average life-time cost is explored in Fig. 9. The A_s-MTBI curves are not depicted in this figure because the costs input hardly imposes effect on availability of the system.

Fig. 9 shows that the average life-time cost basically increases as the three kinds of cost increase. However, the impact of inspection cost and the planned downtime cost are most prominent when the MTBI value is small, whereas the impact of unplanned downtime cost is most pronounced when the MTBI value is high. This finding can serve as a guideline for adjusting the cost in accordance with the existing maintenance strategy. For example, when the MTBI is small and the amount of IMRs is high, the inspection cost can be appropriately decreased to control the average life-time cost. When the value of MTBI is high and the amount of IMRs is low, the unplanned downtime cost is preferred to be lowered by implementing some safety measures to minimize the average life-time cost.

6. Conclusions

Focusing on the heterogeneous failure dependences of component degradation process in a multi-component system, this paper proposed a framework to quantify the failure dependences between components and optimized the policy of condition-based maintenance. By taking the reasonable system availability and minimal average life-time cost in the long-run as the objectives, the Markov process is implemented to with varying MTBI. The impact of the heterogeneous failure dependences on the system maintenance strategies were discussed examining a practical subsea transmission system. The practical implementation of the proposed model in a case study demonstrates its effectiveness and potential for widespread adoption in managing complex multi-component systems, particularly those with heterogeneous failure dependences. The combination of theoretical modeling and its application in a practical case study validates the usefulness of the proposed model. The results of the practical case indicate that the system tends to be more reliable with smaller MTBI. Furthermore, the availability of the system would be overestimated and the annual IMRs costs would be underestimated if we neglect the influence of heterogeneous failure dependences. For various values of MTBI, the inspection cost and planned downtime cost have significant effect on the average life-time cost for low MTBI values, while the impact of unplanned downtime cost is prominent for high MTBI values.

The paper presents managerial actions as references for the decision makers on when to implement the maintenance strategies for complex multi-component system with heterogeneous failure dependences. Based on the finding that a certain system with higher failure dependence is more likely to experience unavailability, one implication could be to address the dependence or to increase the frequency of inspections and maintenance checks. In addition, the system can be assessed to identify the different MTBI ranges and determine the optimal type of cost that maintenance crews could manage to improve the system availability. By optimizing condition-based maintenance strategies,

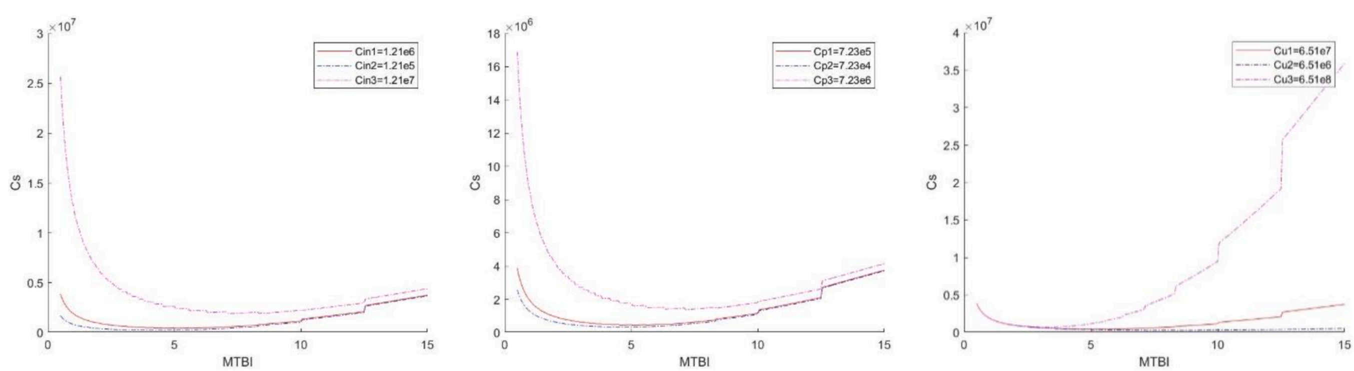


Fig. 9. Maintenance cost for different initial costs input.

organizations can minimize their maintenance costs while ensuring the system remains highly available.

Some other perspectives may be worth to investigating in future work. Firstly, the applicability of the given method may be further verified by applying the proposed model to the maintenance strategies of systems in other configurations. In addition, comparisons with other maintenance models, such as Age-based Maintenance or Opportunistic Maintenance, could be investigated to seek for the optimal maintenance policies for such complex systems.

CRediT authorship contribution statement

Yixin Zhao: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Valerio Cozzani:** Writing – review & editing, Supervision, Methodology. **Tianqi Sun:** Software, Conceptualization. **Jørn Vatn:** Supervision, Conceptualization. **Yiliu Liu:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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