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Closing the Loop in a Duopolistic Circular Economy Model[1]

Elettra Agliardi, Department of Economics, University of Bologna (Italy),
e-mail: elettra.agliardi@unibo.it
Myrto Kasioumi, Department of Economics, University of Guelph (Canada)
e-mail: mkasioum@uoguelph.ca

Abstract. We analyze recycling as firm's optimal strategy in a duopolistic market, under the cases of a linear economy (no recycling technology), semi-circular economy (one firm recycles) and a circular economy (both firms recycle). Furthermore, our analysis will be separated into two cases: when only one firm recycles, then it can either choose to use the recycled material in its production or sell part (or all) of it to the other firm; similarly, if both firms can recycle, they can either use the recycled material they produced, or sell all of it or just part of it to the other firm. We characterize the resulting equilibria showing under which circumstances the economy is linear, semi-circular, or when it "closes the loop", being fully circular (both firms recycle under alternative organizational structures). A few extensions of the same problem under alternative realistic assumptions are finally discussed.

Key words: Circular economies; recycling; emerging equilibria.

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1 Introduction

In recent years, the traditional linear economy is being substituted by a newer and more advanced paradigm called 'circular' economy, in which both production and consumption are occurring in a sustainable and environmentally friendly way. A circular economy involves concepts such as repairing, reselling, reusing, refurbishing, leasing, recycling materials and products as much as possible, enabling them to be reused in the economy, as well as the ideas of reducing waste, remanufacturing products and buildings, repurposing goods or components offering a new life cycle to them, recovering energy and, finally, re-mining or retrieving materials after the landfilling phase (Reike et al., 2018). Countries are encouraged to adopt this way of production and consumption, as it allows the societies to be more resilient for the future (see, e.g., EEA, 2019, 2020). In addition, a circular economy has benefits on the climate, as there will be a lower need to extract and create new products, hence a lower production of greenhouse gas emissions. Moreover, circular economies, compared to linear ones, can help achieve sustainable development without sacrificing economic growth (Zhong and Pearce, 2018, Geissdoerfer et al., 2018). Hence, the importance to study the concept of a circular economy under an economic framework is seen as highly important and vital for the future of the planet.

The two main strategies of the circular economy that are widely used by firms, recommended by governments, and studied in the literature are recycling and reusing (Sudusinghe and Seuring, 2022). Reusing is the action of using a product again or more than once, while recycling is the action of converting unusable products and materials into something reusable. Reuse reduces pollution and waste, as there is no need for new materials or processing the products. On the other hand, the recycling process needs additional energy to remanufacture and dismantle products. In our analysis, we will focus on recycling which is a more complicated process compared to reusing products.

Specifically, recycling is a tool that can be used to reduce the waste pile of the economies and to produce inputs that can be reused in the production process. In this way, economies will not only face pollution reduction coming through recycling directly, but also indirect pollution reduction, as fewer resources will be extracted from the earth and less processing of inputs will take place. Because of its economic and environmental importance, recycling is a substantial pillar in the creation of circular economies.

There are many papers studying the effects of recycling on the environment, climate change, resource usage, energy usage, as well as the socioeconomic impact of recycling. However, there are limited studies focusing on this topic in a market economy framework. The purpose of this paper is to fill in this gap of the literature by introducing a theoretical duopolistic circular economy model. Our paper will provide an answer to questions like these: Under which circumstances will the market "close the loop", becoming fully circular? Will the nonexistence of a re-cycling economy arise simply because the genuine economic costs of undertaking re-cycling are so high, and the price of the recovered raw material is so low that no profitable recycling activity can take place? We will study a market economy in the simplified form of a duopoly model, offering a novel idea on the way firms can achieve (Nash) equilibria in a circular economy and the role of the parameters affecting the equilibria. In particular, we will extend the model of Conrad (1999) to capture the effects of recycling in a duopolistic market, under the cases of a linear economy (no recycling), semi-circular economy (one firm recycles), and a circular economy (both firms recycle). Furthermore, our analysis will be separated into two cases where firms choose to

use the recycled input in their own production, or they can sell it to the other firm. Our main results, summarized in Propositions 1 and 2, allow us to characterize the resulting equilibria showing under which circumstances the economy is linear, semi-circular, or when it "closes the loop" being fully circular, where both firms recycle under alternative organizational models.

The structure of the paper is as follows. Section 2 presents the existing literature review on the topic. Section 3 introduces the general framework of our model, examining the three different cases above (i.e., no firm recycles, only one firm recycles, and both firms recycle). Section 4 presents the equilibrium outcomes (Propositions 1 and 2), and Section 5 offers a discussion of our main results and an evaluation of them under the different model parameters. Section 6 addresses a few extensions of the same problem under alternative realistic assumptions. Specifically, Section 6.1 discusses the case where the price of the recycled input is set by the recycling firm and is not exogenously given, while Section 6.2 explores the case of having environmentally friendly consumers in the market who may prefer to buy products from firms that have adopted recycling technologies and studies how that would affect the results. Finally, Section 7 shows our concluding remarks.

2 Literature Review

The benefits of a circular economy compared to a linear one are highly supported by the literature, which shows that there are various opportunities related to the first, under both economic and social levels. In particular, the circular economy is associated with a lower operational cost for firms through end-of-life management and sustainable supply chains, a reduction in environmental pollution through lower greenhouse gas emissions, additional job openings, and improvement in the trade balance (Wijkman and Skånberg, 2015, Kumar et al., 2019). In addition, recycling is a tool that can also lead to environmental benefits and a sustainable development (Brennan et al., 2015, Zhong and Pearce, 2018). Since the effects of recycling have an impact on different aspects of the world, as the environment (Di Maio et al. (2015), Boyden et al. (2016)), natural resources and energy saving (Gubanova, 2019, Costa, 2021), as well as positive socioeconomic impact (van Beukering et al., 2014, Hanjra et al., 2015), it is important to study this topic under different frameworks.

Bonviu (2014) indicates that businesses in the European Union are more vulnerable under a linear economy as resources might be exhausted in the near future and their prices might increase, leading to lower supply and lower profits. That, along with the available technology that EU has, are the main reasons why firms have already started their transition to a circular economy. On the other hand, Zink and Geyer (2007) showed that both primary and secondary productions are increased when circular activities are engaged in the economy, which sometimes leads to fully offsetting the benefits of the circular economy. Moreover, firms are not willing to adopt strategies that will help avoiding that problem, because that will reduce their profits.

The economic literature on circular economy and, more specifically, on recycling has been developed along three main approaches: a macroeconomic, a microeconomic, and an empirical one. Starting off with the macroeconomic literature, which focuses more on the relationship between economic growth and recycling, Selden and Song (1995), with a neoclassical environmental growth model, showed that when economic growth increases recycling tends to increase as well. This increasing pattern between recycling and economic growth was also confirmed by Kasioumi and Stengos (2022) who focused on a circular closed economy that uses

recycled material in the production of the final good, as well as Kasioumi (2021) who included habit formation on recycling, allowing previous recycling habits to influence the decision on current recycling level. Bongers and Casas (2022) showed that under a neoclassical model with recycling activities, circularity is a crucial condition to increase the social welfare of the economy. In addition, Pittel et al. (2010) indicated that when recycled material is reused in the production process in a decentralized economy, optimality can be achieved. Welfare improvement is an additional effect of waste recycling, as showed by Di Vita (2001). Banionienea et al. (2021) specified the shocks that can lead to a more circular and faster growing economy which, at the same time, affect the environmental quality positively. They found that positive shocks to the refurbished productivity, improved technology, and higher price for the polluting input are the main variables that affect the circularity of the economy. Lastly, Zhou and Smulders (2021) based their model on an economy with two sectors, one for producing the final output and one for refurbishing waste, and showed that in some extreme cases where innovation is mainly focused on resource-saving, production leads to economic loss.

Our paper is most closely related to the microeconomic approach, where contributions focus on firm level and game theory strategies. Under a Cournot duopoly market with a multidimensional cooperative agreement, Carfi and Donato (2021) found that recycling leads to economic growth, better management of the environmental resources, protection of the environment, and improvement of both the quality of life and the public health. However, their results are based on an analysis specifically focused on recycling of paper waste and not on recycling in general, as our paper does. Some papers study the dynamics and stability properties of equilibria where recycling is allowed. Li et al. (2020) with a price game model that focuses on two recyclers who consider fair pricing, show that the fairness factor can significantly reduce the stability of the equilibrium in the marker, while the commonly used strategy of speeding up the price adjustment will lead to lower profits for the firm that adjusted the price adjustment rate and increase them for the other firm. These results are also confirmed by Li et al (2019) who showed that a fast adjustment speed for the recycling price leads the market into chaos and profit losses. However, the market faces maximum profits when the recycling price is stable at the Nash equilibrium. Additionally, Gaudet and Long (2003) created a model where there is a time lag between the production of the final good and recycling which is a product arising from many Cournot rivals' recyclers. Their results suggest that when the time lag between the two productions is increased, the profits of the final good producer will increase but its market power will fall, while when the two production decisions are taken simultaneously, the market power of the final good producer increases as a result of including recycling in the model.

The above-mentioned papers focus on the dynamics and stability properties of equilibria where recycling is allowed. Our model instead is a static game theoretical one and we are mainly concerned with a characterization of the different equilibria outcomes under alternative setups. While we study different effects under competitive setups, other papers focus on cooperative agreements among firms. Bernard (2011) studies a circular economy with a duopoly market and shows that if firms decide to collude, then both consumer surplus and the profits of the firms increase. However, that holds only when environmental regulation is not present. Modak et al. (2015) with a comparison between Cournot and collusion games of a cooperative and non-cooperative model of a closed loop economy, suggested that a system with recycling and collaboration generates more profits than a system without them, a result that is somewhat consistent with one case we considered in our study. Johari and Hosseini-Motlagh (2019) focused on a Stackelberg game and proposed a sustainable closed-loop supply chain model which can lead

to better environmental, economic, and social performances if a two-way two-part tariff is enforced.

Lastly, there is a growing literature which studies the circular economy, and particularly recycling, empirically. Biswas et al. (2000) showed that waste recycling behavior is positively correlated with recycling shopping behavior. In addition, Hage et al. (2009) found that households' recycling level in Sweden is influenced by both moral and economic motives. Furthermore, the results of Kostakis and Tsagarakis (2021) suggest that the main factors that affect recycling in the European Union are the environmental taxes, fertility rate, urbanization, economic wealth, and R&D expenditures. Under a partially linear semiparametric model Kasioumi and Stengos (2020) showed that recycling increases with output in U.S. On the other hand, Razzaq (2021) focused on how recycling affects economic growth, and their results suggest that higher recycling not only leads to higher economic growth for the U.S., but also improves the environmental quality through a reduction of carbon emissions. Blomberg and Söderholm (2009) focused on some big European countries and the market of secondary aluminium and found that even though price policies affect the supply side of the market, they do not significantly affect the recycling rates of aluminium. Additionally, Zhang et al. (2018) showed that industrial waste recycling improves the corporate reputation, which is accompanied by higher corporate sales, an effect that is present only during the recycling adoption year though. Their findings also suggest that recycling is not as profitable as they were expecting, as they do not reduce the cost expenditure notably. Finally, Schlosser et al. (2021), through a non-structural parametric approach, showed that firms will always want to invest in recycling if the recycled resource used in the production of the final good is cheaper than the virgin resource, a result connected to our duopolistic market analysis. However, their analysis does not incorporate firm competition or waste taxation, both of which are included in our model, which can generate different results.

3 The Basic Model

We consider a duopoly, where each firm can take the decision to recycle or not and compete in quantities. More specifically, we extend Conrad (1999) by including two firms in a circular economy model and focus on the resulting Cournot-Nash equilibria, comparing the profits of each firm under three different cases, that is, when no firms recycle, when only one firm decides to recycle and when both firms recycle. The last two cases have two subcategories each. When only one firm recycles, then it can either choose to use the recycled material in its production or sell part (or all) of it to the other firm. Similarly, under the case that both firms recycle, each one can either use the recycled material they produced, or they can choose to sell all of it or just part of it to the other firm[2].

The analysis of these different cases allows us to compare the resulting equilibria when the

[2] Some literature assumes that firms do not directly carry out the recycling process of waste, but they enter a wholesale contract with a third party to recycle their products (e.g., Hu et al. (2016), Yang et al. (2018)). The resulting game between the parties may pose interesting questions on the coordination of supply chain and reverse supply chain with contracts, which have been studied under standard principal agent contract design. Yang et al (2018) study the problem when there is a cost information asymmetry between the recycler and the "bio-firm" which opted out to contract out the recycling process and study under which circumstances quantity discount contracts stimulate recycling. Hu et al (2016) design five typical contracts to coordinate the reverse supply chain with strategic recycling behaviours of consumers and study which contract achieves a better performance. In our case, we do not aim to focus on this problem, as we want to study what the firms' choices will be under different scenarios regarding recycling. This is something that, to our best knowledge, has not been studied previously in the literature, so we focus on a simpler case where the firms recycle the waste they produce.

economy is non-circular, or linear (i.e., no recycling), when it is semi-circular, or when the economy "closes the loop", being fully circular (both firms recycle under alternative organizational structures).

We suppose there are two firms producing a homogeneous final good (Q), using labour (L), capital (K), and gross resources (GR). Part of the resources is useful material that can be used directly in the production process of the final good (R), while the rest is waste and is not suitable for the production (W^0)[3]. As in Conrad (1999), we suppose that waste is produced solely by the gross resource. Although this is a simplification, since waste can be generated by consumption, investment, production activities, or all of them, our assumption allows us to present the basic implications for the circular economy[4]. We suppose that the government can tax firms on waste and the tax rate on waste faced by each firm is τ. The expressions for the above assumptions are the following:

$$\text{Total Waste:} \quad W^0 = \alpha GR \tag{1}$$
$$\text{Net Resources:} \quad R = (1-\alpha)GR \tag{2}$$

where α is the waste coefficient and $0 \leq \alpha \leq 1$. Rewriting the above expressions for net resources, we can get an expression for the gross resources, given by $GR = \frac{1}{1-\alpha}R$. To have the most efficient outcome, the production should be a function of labour, capital and net resources. If gross resources are used instead, the production is not efficient as it includes waste as well. For simplicity we will assume that the production function is characterized by a Cobb Douglas function of the form: $Q(L,K,R) = AK^{\gamma}L^{\zeta}R^{\theta}$, were A is a constant and γ, ζ, θ show how productive a unit of each input is relative to a unit of the other two inputs, for which it holds that $\gamma + \zeta + \theta = 1$ and $\gamma, \zeta, \theta > 0$. A Cobb Douglas production function is widely used in this literature (e.g., Bongers and Casas, 2022). It implies that the elasticity of substitution between inputs is unitary. Alternatively, one could assume a CES production function in which inputs are gross complements. In this case, the implications for circularity would be very similar, however, the computation would be more involved, and therefore we preferred to stick with a Cobb Douglas function. Under this framework, we can build three different cases.

3.1 The Linear Case: No Recycling

Let us start with the first case where no firms recycle. The cost minimization problem that each firm faces is the following:

$$\begin{aligned} &\min_{L_i, K_i, R_i} TC_i = wL_i + rK_i + cGR_i + \tau W_i^0 \\ &\text{s.t. } \bar{Q}_i = AK_i^{\gamma}L_i^{\zeta}R_i^{\theta} \end{aligned} \tag{3}$$

where w, r, c are the unit costs of labour, capital, and gross resources respectively, and i represents the firm, that is $i = 1,2$. By substituting expression (1) and the expression for the gross

[3] Wood can be an example of a resource, as it is a combination of the main living tree and the bark, with the second being useless for production. Waste in this case can be recycled and bark can be used in other productions.

[4] It is also consistent with some empirical evidence about industrial waste, for example, in the extraction industry sector which is one of the largest sources of waste in the EU (EEA, 2019)

resources in (3), we get the following minimization problem as a function of labour, capital, and net resources:

$$\begin{aligned} &\min_{L_i,K_i,R_i} TC_i = wL_i + rK_i + \widetilde{\widetilde{P}}_{Rt}R_i \\ &\text{s.t. } \bar{Q}_i = AK_i^{\gamma}L_i^{\zeta}R_i^{\theta} \end{aligned} \tag{4}$$

where $\widetilde{\widetilde{P}}_{Rt}$ is the price of net resources and is equal to $\widetilde{\widetilde{P}}_{Rt} = \frac{1}{1-\alpha_i}(c + \tau\alpha_i)$. The solution of the above minimization problem gives the optimal capital, net resources, and labour as a function of all the parameters and the constant quantity Q_i, that is:

$$K_i^* = \frac{\bar{Q}_i}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma-1}\left(\frac{\widetilde{\widetilde{P}}_{Rt}}{\theta}\right)^{\theta} \tag{5}$$

$$R_i^* = \frac{\bar{Q}_i}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{\widetilde{\widetilde{P}}_{Rt}}{\theta}\right)^{\theta-1} \tag{6}$$

$$L_i^* = \frac{\bar{Q}_i}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{\widetilde{\widetilde{P}}_{Rt}}{\theta}\right)^{\theta} \tag{7}$$

With these optimal inputs, we can derive the total costs and solve the profit maximization problem of each firm. Suppose that the price function of the final good is: $P = d - b(Q_i + Q_j)$, where d is the constant demand intercept and b is the slope. Under these assumptions, firms have to solve the following profit maximization problem simultaneously:

$$\max_{Q_i} \quad \Pi_i = [d - b(Q_i + Q_j)]Q_i - MC_iQ_i \tag{8}$$

where $MC_i = \frac{1}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{\widetilde{\widetilde{P}}_{Rt}}{\theta}\right)^{\theta}$ is the marginal cost of firm i. In this case, the optimal production of each firm Q_i^* as well as their optimal profit functions Π_i^* at equilibrium are as follows:

$$\Pi_i^* = \left[\frac{d+MC_j-2MC_i}{3}\right]\left[\frac{d+MC_j-2MC_i}{3b}\right] \tag{9}$$

$$Q_i^* = \frac{d+MC_j-2MC_i}{3b} \tag{10}$$

$$where \begin{cases} MC_i = \frac{1}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{1}{\theta}\right)^{\theta}\left(\frac{c+\alpha_i\tau}{1-\alpha_i}\right)^{\theta} \\ MC_j = \frac{1}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{1}{\theta}\right)^{\theta}\left(\frac{c+\alpha_j\tau}{1-\alpha_j}\right)^{\theta} \end{cases}$$

3.2 Recycling is available - One firm recycles

Let us suppose now that recycling is chosen by one firm only, which decides to either sell the recycled product to the other firm or use it in its own production. Let's consider the case where

firm *i* uses the recycled input in its own production and firm *j* faces a similar problem as in Section 3.1. The problem of firm *i* is slightly different though.

To begin with, total waste consists of two parts: one that can be recycled and one that cannot. The total recyclable proportion of waste is equal to $W_{i1} = \beta W_i^0 = \beta \frac{\alpha_i}{1-\alpha_i} R_i$ where β is the recycling coefficient where $0 \leq \beta \leq 1$. That means that the unrecyclable part of waste is equal to $W_{i0} = (1-\beta)\frac{\alpha}{1-\alpha_i} R_i$. Firm can process the recyclable part of waste and get a useful resource which can be reused in the production process. The technology that characterizes this process is assumed to be a Leontief function given by $RW_i = min[oW_{i1}, \xi L_{iw}]$, where o and ξ are technologically determined constants.

In this case, the recycling firm faces two additional costs. One for recycling (v) and one for labour assigned to recycling (L_{iw}). We assume that the unit cost for recycling is affected positively by the recycling quantity, and for simplicity we assume a linear function characterizing that relationship given by $v(\beta) = k + m\beta$. Additionally, the unit cost for the labour units assigned to recycling is the same as the unit cost for labour assigned to the production of the final good (w).

On the other hand, the firm faces a reduction on its cost stemming from two sources. Firstly, it is assumed that the firm has to pay tax for producing waste and polluting the environment. Since the firm is reducing that waste pile due to recycling, the total tax will be reduced. Specifically, firm pays tax on the remaining amount of waste only which is equal to $TW_{i0} = (1-o\beta)\frac{\alpha_i}{1-\alpha_i} R_i$. Moreover, because the firm uses that recycled material in its own production of the final good, the need of new resources is reduced, lowering additionally its total cost. Hence, the minimization problem that the recycling firm faces is given by the following expression:

$$\begin{aligned} &\min_{L_i, K_i, R_i} TC_i = wL_i + rK_i + cGR_i + \tau TW_{i0} + v(\beta)W_{i1} + wL_{iw} - \tilde{P}_{Ri} RW_i \\ &\text{s.t. } \bar{Q}_i = AK_i^{\gamma} L_i^{\zeta} R_i^{\theta} \end{aligned} \tag{11}$$

After substituting the expressions for gross resources (GR_i), total remaining waste (TW_{i0}), the total cost for recycling ($v(\beta)W_{i1}$), and after using the information given from the Leontief technology for the reusable waste (RW_i), we are getting the final form of the cost minimization problem that the firm faces, given by:

$$\begin{aligned} &\min_{L_i, K_i, R_i} TC_i = wL_i + rK_i + P_{Ri} R_i \\ &\text{s.t. } \bar{Q}_i = AK_i^{\gamma} L_i^{\zeta} R_i^{\theta} \end{aligned} \tag{12}$$

Here P_{Ri} is the new resource price after expressing everything with respect to R_i and is equal to $\tilde{P}_{Ri}\left[1 - o\beta\frac{\alpha_i}{1-\alpha_i}\right]$, while $\tilde{P}_{Ri}$ is the new price of net resources before we take into account the reduction on total waste coming through reusing the recycled material and not selling it, and it is equal to $\tilde{P}_{Ri} = \frac{\alpha_i}{1-\alpha_i}\left[\frac{c}{\alpha_i} + \tau(1-o\beta) + (k+m\beta)\beta + \frac{w\beta o}{\xi}\right]$.

Solving problem (12) we can find the optimal quantities of labour, capital, and resources needed to produce a constant amount of the final output ($\bar{Q}_i$). Those formulas are characterized by expressions similar to (5)-(7) if we substitute the new resource price P_{Ri} into them. Now the firm will maximize its profits, given the optimal quantities of the three inputs found earlier (K_i, R_i,

L_i). Hence, the profit maximization problem is given by the following expression, where we maximize with respect to Q_i:

$$\max_{Q_i} \quad \Pi_i = [d - b(Q_i + Q_j)]Q_i - MC_i Q_i \tag{13}$$

where $MC_i = \frac{1}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{P_{Ri}}{\theta}\right)^{\theta}$ is the marginal cost of firm i. Differentiating the above equation with respect to Q_i and combining that equation with the one arising from the profit maximization problem of the second firm which is not recycling, we can calculate the two unknown variables (Q_i, and Q_j) and find the profits of each firm as well.

Moving on to the second category, where firm *i* decides to sell part of the recycled input to the other firm *j*, the minimization and maximization problems of each firm change, as one firm faces higher profit and the other higher cost. We suppose that the price of the recycled input is s, which is exogenously given for simplicity. We denote by z the proportion of the recycled material that the second firm buys from the recycling firm, and $(1 - z)$ is the remaining input that the recycling firm uses in its own production. We assume for simplicity that the proportion z is exogenously determined by regulation or by the policy maker. The cost minimization problems for both firms can be expressed as follows:

$$\begin{aligned} &\min_{L_i,K_i,R_i} TC_i = wL_i + rK_i + \hat{P}_{Ri}R_i \\ &\text{s. t. } \bar{Q}_i = AK_i^{\gamma}L_i^{\zeta}R_i^{\theta} \end{aligned} \tag{14}$$

$$\begin{aligned} &\min_{L_j,K_j,R_j} TC_j = wL_j + rK_j + \widetilde{\widetilde{P_{RJ}}}R_j + szRW_i \\ &\text{s. t. } \bar{Q}_j = AK_j^{\gamma}L_j^{\zeta}R_j^{\theta} \end{aligned} \tag{15}$$

where $\hat{P}_{Ri}$ is the resource price that the recycling firm faces and is equal to $\hat{P}_{Ri} = \tilde{P}_{Ri}\left[1 - (1 - z)o\beta\frac{\alpha_i}{1-\alpha_i}\right]$, and $\widetilde{\widetilde{P_{RJ}}}$ is the resource price that the second firm faces and is equal to $\widetilde{\widetilde{P_{RJ}}} = \frac{1}{1-\alpha_j}(c + \alpha_j\tau)$, as we found in Section 3.1. Solving the above problems, we can get the optimal quantities of labour, capital, and resources as a function of a constant quantity of the final good for both firms. Using those into the following maximization problems, and maximizing with respect to Q_i for the first firm and to Q_j for the second firm, we are able to get the optimal level of profits for both firms:

$$\max_{Q_i} \quad \Pi_i = [d - b(Q_i + Q_j)]Q_i + zsRW_i - MC_i Q_i \tag{16}$$

$$\max_{Q_j} \Pi_j = [d - b(Q_i + Q_j)]Q_j - MC_j Q_j \tag{17}$$

where MC_i and MC_j are the marginal costs of each firm respectively, and they are given by $MC_i = \frac{1}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{\hat{P}_{Ri}}{\theta}\right)^{\theta}$ and $MC_j = \frac{1}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{\widetilde{\widetilde{P_{RJ}}}}{\theta}\right)^{\theta}$.

3.3 Recycling is available - Both firms recycle

Suppose now that recycling is an available technology to both firms and both can choose to either sell part or all of the recycled resource to the other firm, or use it into their own production of the final good. Hence, there are two different scenarios. Let us start with the scenario where firms use their own resource and do not sell it. This problem is characterized as follows:

$$\begin{aligned} &\min_{L_i,K_i,R_i} TC_i = wL_i + rK_i + P_{Ri}R_i \\ &\text{s.t. } \bar{Q}_i = AK_i^{\gamma}L_i^{\zeta}R_i^{\theta} \end{aligned} \tag{18}$$

$$\max_{Q_i} \Pi_i = [d - b(Q_i + Q_j)]Q_i - MC_iQ_i \tag{19}$$

where P_{Ri} is the final resource price and is equal to $P_{Ri} = \frac{\alpha_i}{1-\alpha_i}\left[\frac{c}{\alpha_i} + \tau(1 - o\beta_i) + (k + m\beta_i)\beta_i + \frac{w\beta_i o}{\xi}\right]\left[1 - o\beta_i\frac{\alpha_i}{1-\alpha_i}\right]$, where β_i is the recycling parameter, which we suppose, more generally, to be different for the two firms, i=1,2. We get $MC_i = \frac{1}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{P_{Ri}}{\theta}\right)^{\theta}$ similarly to the expression of previous sections.

Finally, we can write the problem for the case where both firms recycle and both sell part of the recycled input to the other firm. For simplicity, let us suppose that the price at which each firm sells the recycled resource are s_i and s_j and are exogenously given. The proportion of the recycled material that firm j buys from firm i is z_j and vice versa. Both z_i and z_j are considered to be determined by regulation or the social planner and are exogenous. Now each firm has an extra cost from buying the recycled input from the other firm, and a cost reduction from not having to buy it directly on the market of virgin materials. Hence, the minimization problem of firm i is:

$$\begin{aligned} &\min_{L_i,K_i,R_i} TC_i = wL_i + rK_i + \tilde{P}_{Ri}R_i - \tilde{P}_{Ri}(1 - z_j)RW_i + s_jz_iRW_j \\ &\text{s.t. } \bar{Q}_i = AK_i^{\gamma}L_i^{\zeta}R_i^{\theta} \end{aligned} \tag{20}$$

and after substituting both RW_i and RW_j, we end up having the following problem:

$$\begin{aligned} &\min_{L_i,K_i,R_i} TC_i = wL_i + rK_i + \hat{P}_{Ri}R_i + s_jz_io\beta_j\frac{\alpha_j}{1-\alpha_j}R_j \\ &\text{s.t. } \bar{Q}_i = AK_i^{\gamma}L_i^{\zeta}R_i^{\theta} \end{aligned} \tag{21}$$

Moving on to the profit maximization problem, firms have an additional revenue source, as they are selling their recycled material to the other one. The problem is described by the following expression:

$$\max_{Q_i} \Pi_i = P_iQ_i + z_js_iRW_i - TC_i \tag{22}$$

and after substituting P_i, RW_i and TC_i which are the price of the final good, the quantity of the reusable waste, and the total cost function respectively, we get:

$$\max_{Q_i}\Pi_i = [d - b(Q_i + Q_j)]Q_i + z_j s_i o\beta_i \frac{\alpha_i}{1-\alpha_i} R_i - MC_i Q_i - s_j z_i o\beta_j \frac{\alpha_j}{1-\alpha_j} R_j \quad (23)$$

where $MC_i = \frac{1}{A}\left(\frac{\zeta}{w}\right)^{\gamma+\theta-1}\left(\frac{r}{\gamma}\right)^{\gamma}\left(\frac{\hat{P}_{Ri}}{\theta}\right)^{\theta}$ is the marginal cost of the firm. The last term of the above maximization problem does not affect the differentiation with respect to neither Q_i, as it does not include this variable. However, it affects the final value of the profits.

4. Under which circumstances will the market reach a linear, semi-circular or fully circular equilibrium?

In this section we compute the resulting market equilibria. That is, we suppose that firms have the option to choose whether or not to adopt a recycling technology and in such case four scenarios may arise, that is: (NR, NR) if no firms adopt; (NR, R) or (R, NR) if only one firm adopts; and (R, R) if both adopt. The outcomes of the duopolistic interactions can be represented in Table 1. We can compute the resulting Cournot-Nash equilibria, which turn out to depend on the values of parameter o, that is, on the effectiveness of the available recycling technology.

When o is sufficiently low and for the scenario where firms do not sell the recycled product to the other firm but decide to use it in their own production, the optimal strategy for both firms is not to recycle, that is (NR, NR) is the unique Nash equilibrium. In fact, our analysis shows that for firm i it holds $\Pi_i(NR, R) > \Pi_i(NR, NR) > \Pi_i(R, R) > \Pi_i(R, NR)$, with $i = 1,2$, for any value of the parameter β[5]. Although recycling is an available technology, if firms recycle, then their profits are reduced for both of them compared to the no recycling case. Finally, when just one firm recycles, we find that the profits of this firm become the least among all combinations, while the profits for the other firm becomes the highest. The results do not change under different values of the recycling coefficient (β) nor any of the other parameter values. In all three cases, when taxation increases, the profits of all firms decrease as expected. See Figures (1)-(3)[6] and Table 1[7].

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[5] From Table 1 we can see that when firm *i* decides not to recycle *(NR)*, then the best option for firm *j* is not to recycle too, as it gets higher profits compared to the case that firm *i* does not recycle and firm *j* decides to recycle *(R)*, since $\Pi(NR, NR) > \Pi(\mathrm{R}, NR)$. For example, when τ=0.5 we get 46.48>46.4 as from Figures 1-3. When firm *j* decides not to recycle, then the optimal option for firm *i* is not to recycle either to get higher profits. That means that when both firms decide not to recycle, that is optimal for both, and they do not have an incentive to deviate from that decision. In other words, that is the Nash equilibrium of the game. On the other hand, when firm *i* decides to recycle, the best decision for firm *j* is not to recycle as it will get higher profits ($\Pi_j(NR, R) > \Pi_j(R, R)$). For example, if τ=0.5, we get 46.6>46.45, as from Figures 1-3. If firm *j* decides not to recycle, then firm *i* will not recycle either as this is more profitable for it ($\Pi_i(NR, NR) > \Pi_i(R, NR)$). That means, that in that case we do not have a Nash equilibrium as firm *i* will deviate from its initial decision to recycle. The same analysis holds for firm *j* as well. Hence, we see that we have only one Nash equilibrium in the above game, that is both firms decide not to recycle (NR, NR).

[6] Note that Figure 1 corresponds to the case where recycling is not available, hence it will be the same for the entire analysis and all the cases of o as well as the scenarios where firms decide to sell or not to sell the recycled material.

[7] The plots are examples based on parameter values taken from EPA, OECD, and some predetermined by us parameter values. Particularly, the values of α and β are taken from EPA for the United States for 2018, m is also taken from EPA for the United States but for the year 2019, while the value of c is taken from the spend journal for the United Kingdom. Specifically, we used: $d = 30$, $b = 0.3$, $k = 2$, $m = 0.5$, $\beta = 0.7$, $A = 3$, $\alpha_1 = 0.3$, $\alpha_2 = 0.3$, $c = 13.4$, $w = 14.25$, $r = 7$, $\theta = 0.4$, $\zeta = 0.3$, $\gamma = 0.3$, $o = 0.3$, and $\xi = 1$. In addition, for the entire analysis, we focus on the case where $t < \frac{w}{\xi}$ which is a reasonable

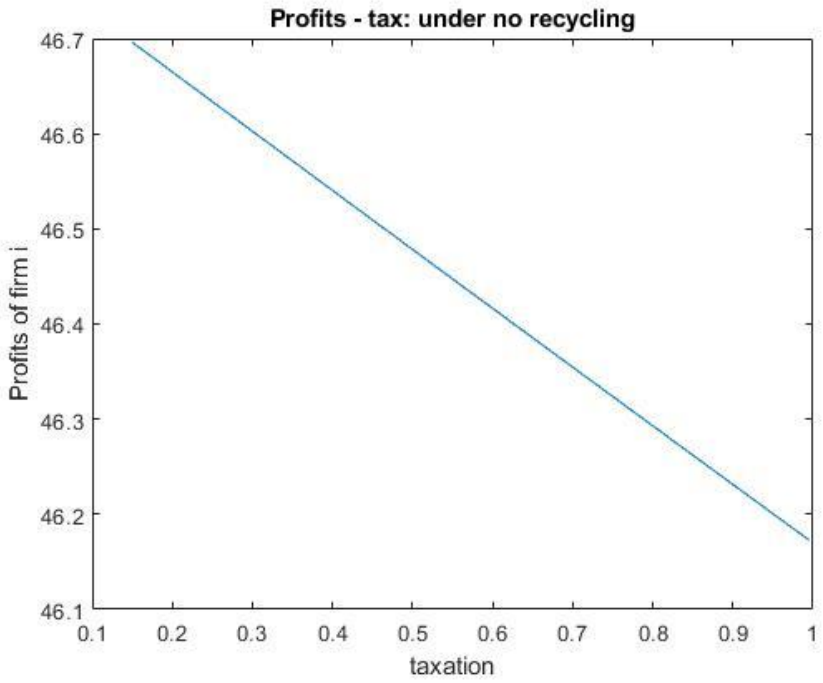

Figure 1: Recycling is not available

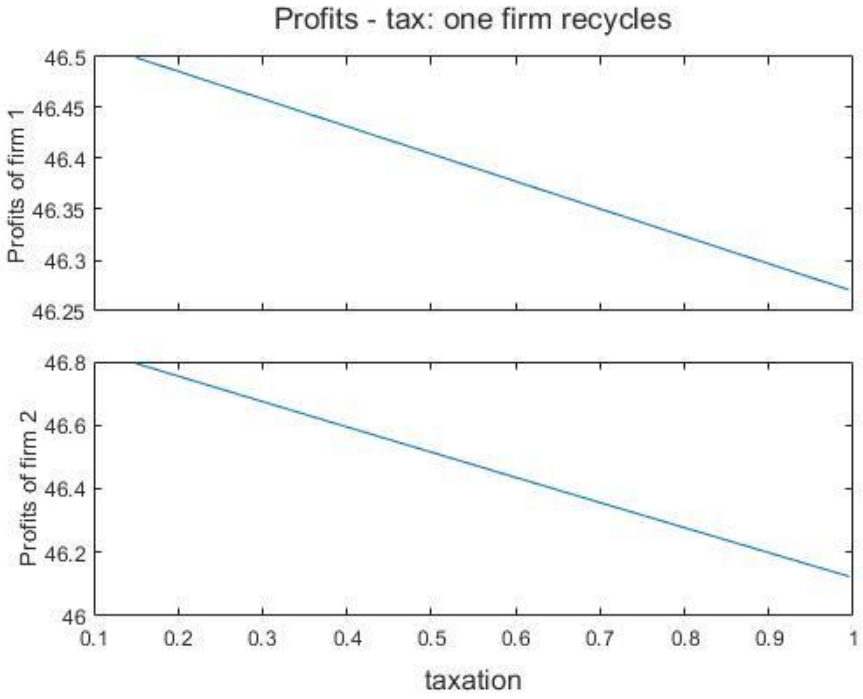

Figure 2: Recycling is available to one firm - low recycling technology

assumption. For simplicity, we show simulations with symmetric parameter values for the two firms. In Section 5 we discuss robustness of our outcomes to the case of asymmetric parameter values.

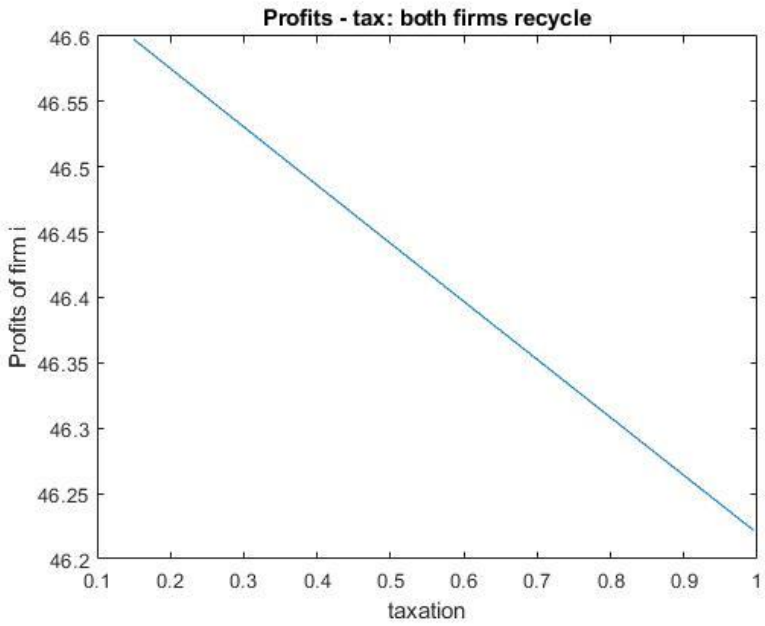

Figure 3: Recycling is available to both firms - low recycling technology

		Firm j	
		NR	R
Firm i	NR	$\Pi_i(NR,NR), \Pi_j(NR,NR)$	$\Pi_i(NR,R), \Pi_j(NR,R)$
	R	$\Pi_i(R,NR), \Pi_j(R,NR)$	$\Pi_i(R,R), \Pi_j(R,R)$

Table 1: Firms' Profit Matrix

Alternatively, in the case o is sufficiently large, the results differ significantly. Specifically, we find that $\Pi_i(R,NR) > \Pi_i(R,R) > \Pi_i(NR,NR) > \Pi_i(NR,R)$, i=1,2 for any value of the parameter β[8]. Hence, under this scenario, we get a unique Nash equilibrium where both firms decide to recycle (R,R). Figures 4 and 5 correspond to this part of our analysis[9]. By continuity with respect to o, we can summarize our results in Proposition 1:

Proposition 1: Under a duopoly, when firms are not selling the recycled material, there exists a value $\bar{o}$ such that for o smaller that $\bar{o}$ the unique Nash equilibrium is (NR,NR), while for o larger than $\bar{o}$ it is (R,R).

[8] From Table 1 we see that when firm *i* decides not to recycle *(NR)*, the best option for firm *j* is to recycle *(R)* since it receives higher profits compared to the case where firm *i* does not recycle and firm *j* does not recycle either since $\Pi_j(R,NR) > \Pi_j(NR,NR)$. However, when firm *j* recycles, firm *i* decides to recycle to get higher profits. On the other hand, when firm *i* decides to recycle, the best choice for firm *j* is to recycle as well since $\Pi_j(R,R) > \Pi_j(NR,R)$. In that case, firm *i* will not deviate from their initial decision and they will decide to recycle since $\Pi_i(R,R) > \Pi_i(NR,R)$, leading to a Nash equilibrium.

[9] The parameter values for this case are the same as the ones used for Figure 1 to 3 assuming that o is equal to 0.7 now instead of 0.3 as in the previous case.

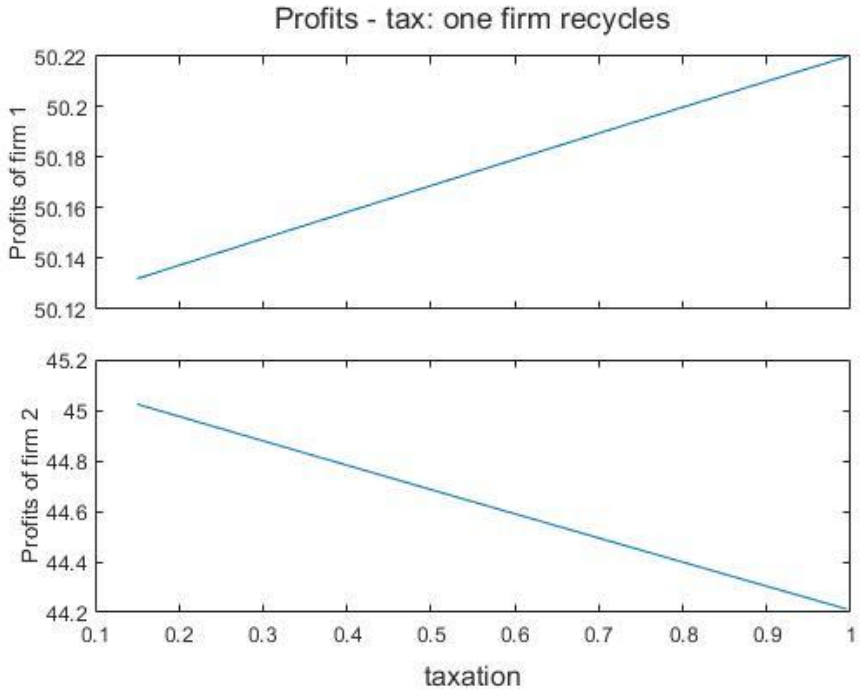

Figure 4: Recycling is available to one firm - high recycling technology

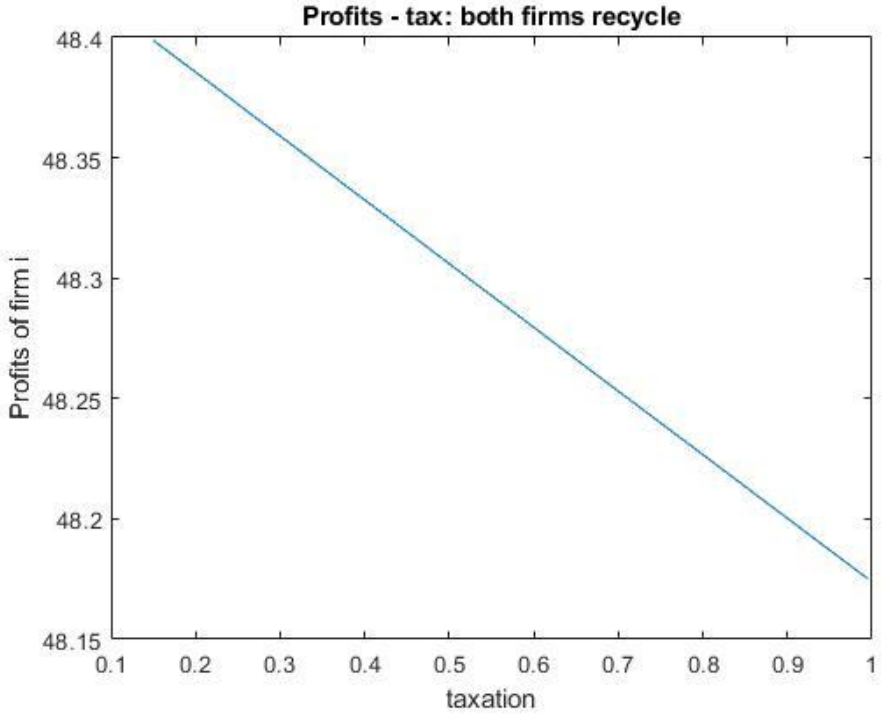

Figure 5: Recycling is available to both firms - high recycling technology

Moving on to the case where firms can sell part of the recycled input to the other firm, our results do change. We considered again the two cases for the value of parameter o. However, the results change when we focus the analysis on further parameters.

We find that when c is significantly smaller than s, ($c<<s$), no matter the values of o, β, z, we get: $\Pi_i(R,R) > \Pi_i(NR,R) > \Pi_i(R,NR) > \Pi_i(\mathrm{N}R,NR)$, leading to a unique Nash equilibrium, where both firms are recycling, that is (R,R).

Notice that this holds even if o is small, implying that if selling the recycled item is a possibility and the price of the recycled item is significantly larger than the price of the virgin material ($c<<s$), then (R,R) emerges as the market equilibrium, even if the recycling technology is not that effective. Therefore, adding the possibility of reselling recycled material may overcome the non-existence of a recycling equilibrium, even if the genuine economic costs of undertaking recycling are such that no profitable recycling activity could have taken place.

However, if $c>>s$, depending on the other parameters, we can get two Nash equilibria, where none firm recycles, that is (NR,NR), or both recycle (R,R). In fact, if z is sufficiently large, the inequalities of the profits of firm i are: $\Pi_i(R,R) > \Pi_i(NR,R) > \Pi_i(NR,NR) > \Pi_i(R,NR)$, leading to two Nash equilibria where both firms decide not to recycle or both firms decide to recycle. Figure (6)-(7) represents this case where firms can sell their recycled input to the other firm, $c>>s$, z is large, and o is low[10], while Figures (8)-(9) represents this case if o is high (o =0.7). These results can be summarized in Proposition 2.

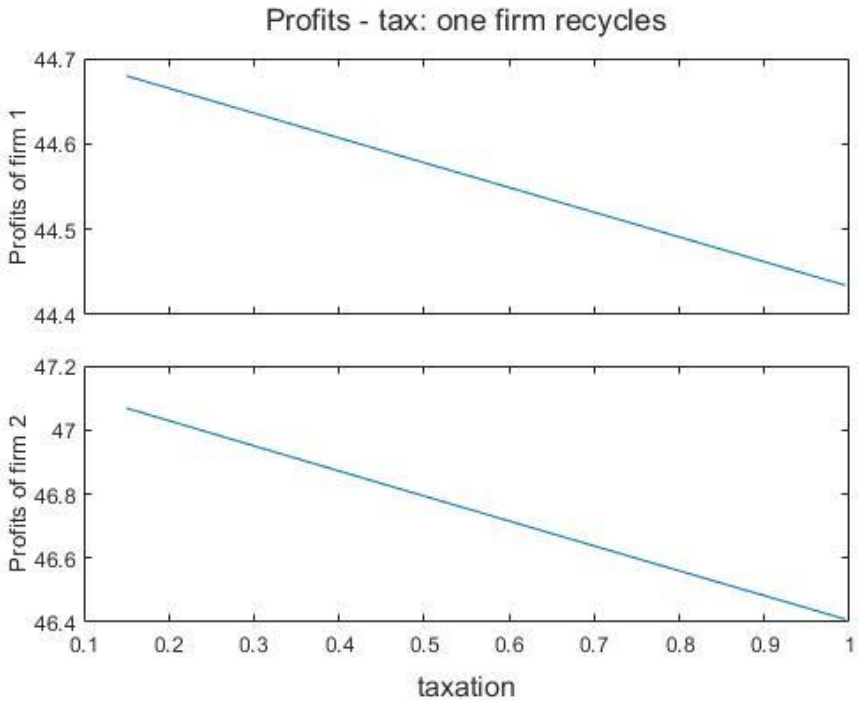

Figure 6: One firm recycles and sells the input to the other firm - low recycling technology

[10] The parameter values for this case are the same as before, assuming also that $o = 0.3$, $s_i = s_j = 7$, and $z_i = z_j = 0.5$

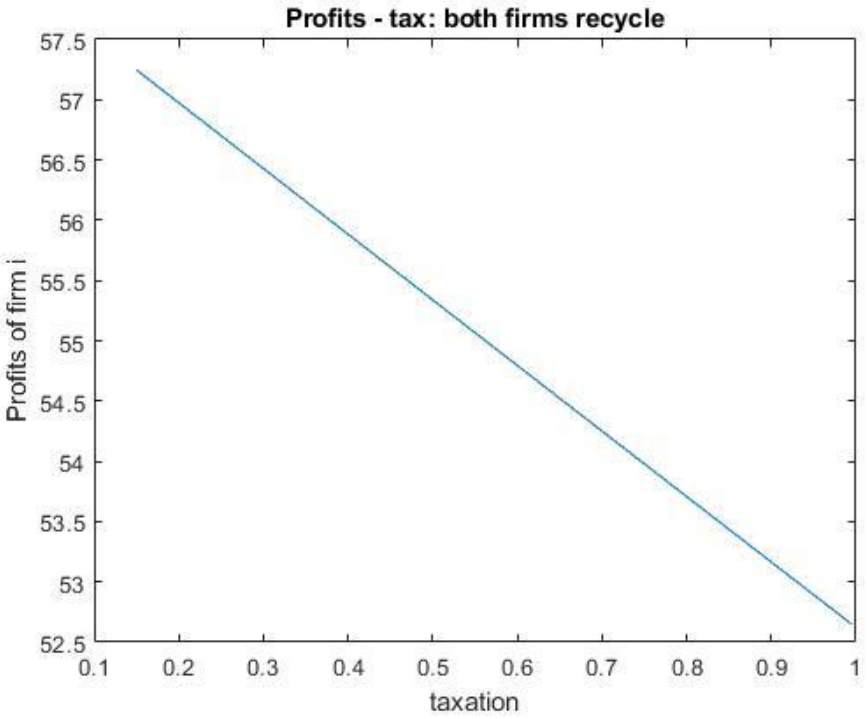

Figure 7: Both firms recycle and sell the input to the other firm - low recycling technology

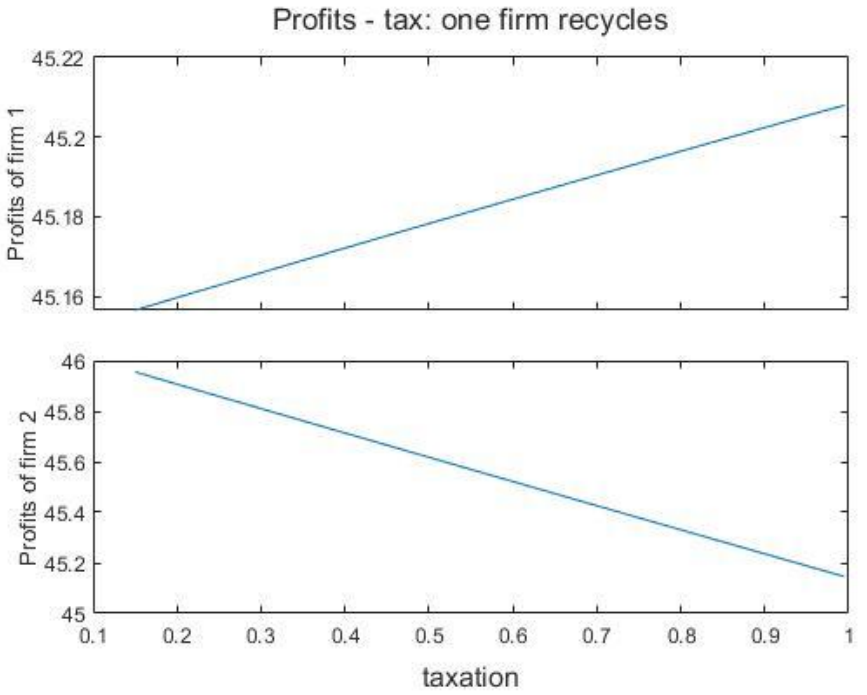

Figure 8: One firm recycles and sells the input to the other firm - high recycling technology

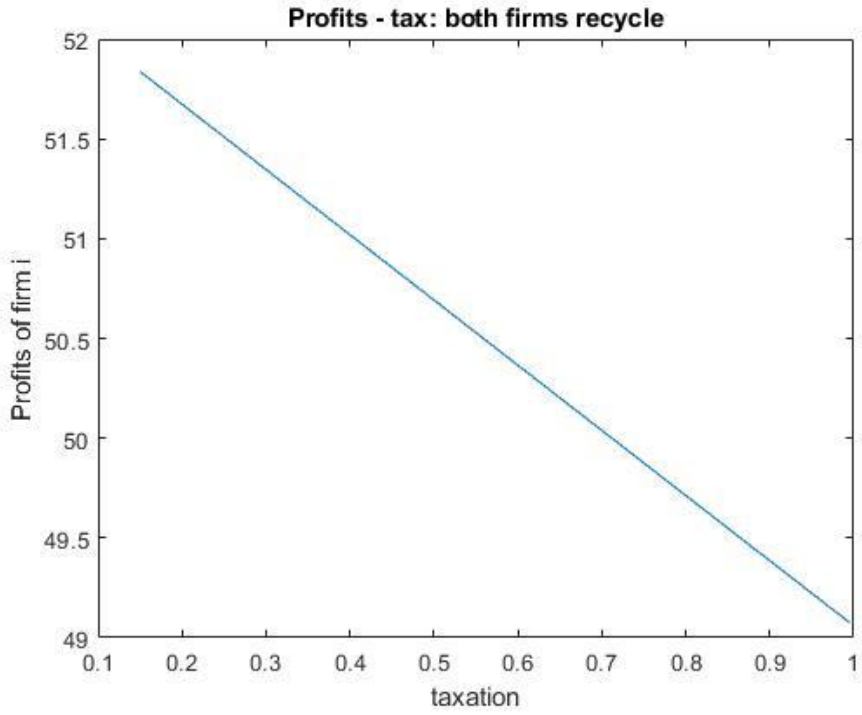

Figure 9: Both firms recycle and sell the input to the other firm - high recycling technology

Proposition 2: Under a duopoly, when firms are selling the recycled material, s, c and z are determining the final equilibrium. Specifically, when s is significantly greater than c, there is a unique Nash equilibrium (R, R), while in the opposite case, when c is significantly greater than s, we get two Nash equilibria (NR, NR) and (R, R), if z is sufficiently large, a result that does not depend on the value of the available recycling technology o.

5 Discussion

Following on the above analysis, we see that the firms' decisions depend on various parameters and specifically on the available recycling technology (o), the price set for the recycled material (s), the amount of the recycled material that each firm buys from the other firm (z), and the unit cost of gross resources (c). We get different Nash equilibria under different parameter values but in all cases both firms decide to take the same action, leading to cases where both firms recycle (R, R) or both firms decide not to recycle (NR, NR). In addition, the results depend on the scenarios we are comparing and particularly we are focusing on the case where recycling is not available to none of the firms and when recycling is available to one or both firms, with the last two categories including two further cases each: the recycling firm(s) can decide whether to sell part or all of the recycled material to the other firm, or use it into its own production.

When firms decide to reuse the recycled material in their own production, our results suggest that the main parameter affecting their decision is the recycling technology available to them. Specifically, if the recycling technology available to firms is sufficiently effective (parameter $o > \bar{o}$), then firm's optimal strategy is to recycle. This will be done by all firms in the

market, that is, it is not the case that one firm recycles and the other is not. Therefore competition leads to a fully circular economy (Proposition 1). Alternatively, when the available recycling technology is low, it is more profitable for both firms not to recycle, as in that case, recycling waste involves a higher cost for them leading to a lower profit compared to the case where the firms do not recycle. This is a result to be expected and it is also a very rational decision taken by the firms.

When firms decide to sell part or the entire amount of the recycled input to the other firm, the results are affected by other parameters. In particular, the firms' decisions in this case are influenced by the price at which each firm sells the recycled material to the other one, the unit cost of the gross resources, and the amount of the recycled material that each firm buys from the other one. We find that irrespective of the effectiveness of the recycling technology, we get a circular economy if we allow for a market to resell recycled materials, where the price of the recycled item is sufficiently high ($s>>c$), in this way creating a market for recycled products (Proposition 2). Specifically, we prove that when the price of the gross resources is lower than the price of the new recycled input, then both firms decide to recycle as their profits are greater in that case. This is something unexpected as recycling is a more expensive resource in that case compared to the simpler polluting gross resource used in the production of the final good, and one would believe that the opposite result would hold. However, in that case, each firm receives the extra revenue from selling the recycled input to the other firm, and since the price of the input is high, their revenues will be high as well, covering their costs and allowing them to have a high profit. On the other hand, when the recycled material price is lower than the price of the gross resources, the results also depend on the quantity that each firm buys of the recycled input. Particularly, if the quantity of the cheaper input bought is small, that does not affect firms' decision to a high degree, as the cost to recycle is almost fully covered by the revenue received from selling that input. However, if the quantity of the recycled material is high, firms face higher costs by doing recycling which are not compensated by higher revenues if the price of recycled material is low, so (NR, NR) may eventually become an equilibrium outcome.

To get a full picture of the model and the results, we also tested how other parameters affect the main variables of our analysis. Specifically, we wanted to see how the inputs cost (c, w, and r), the tax rate (τ), the waste coefficient (β), the price and the proportion of recycled material sold (s and z), and more, affect some important variables such as the profits of each firm (Π_i, Π_j), the final output produced by each firm (Q_i, Q_j), the price of the final good (P), optimal labour (L_i, L_j), capital (K_i, K_j), gross resources (GR_i, GR_j), amount of waste (W_i^0, W_j^0), recyclable waste (W_{1i}, W_{1j}), and so on. The results of this analysis can be seen in the following five tables (Table 2 up to Table 6).

	Π_1	Π_2	Q_1	Q_2	K_1	K_2	L_1	L_2	R_1	R_2	GR_1	GR_2	P	W_1^0	W_2^0
τ	-	-	-	-	+	+	+	+	-	-	-	-	+	-	-
c	-	-	-	-	+	+	+	+	-	-	-	-	+	-	-
w	-	-	-	-	+	+	-	-	+	+	+	+	+	+	+
r	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+

Table 2: Recycling is not available

Table 2 describes the case of a linear economy. The price of net resources increases if c, τ, α increase since it is given by $\widetilde{\widetilde{P_{Ri}}} = \frac{1}{1-\alpha_i}(c + \tau\alpha_i)$. As expected, profits and outputs decrease as

the inputs costs increase (*c*, *w*, *r*), or when the tax rate on waste (τ) increases. In addition, optimal inputs (K_i, L_i, and GR_i) decrease as their prices increase. The effects on optimal K_i, L_i, GR_i, and R_i (since $R = (1-\alpha)GR$) are consistent with substitutability of inputs, following from our assumption that the production function is a Cobb Douglas, as in the Cobb Douglas function the elasticity of substitution between inputs is unitary[11]. The demand price (*P*) increases with *c*, *w*, *r*, and τ and is inversely related to total output. Finally, total waste ($W^0 = \alpha GR$) decreases if the tax rate on waste (τ) increases or the cost of gross resources (*c*) increases (since here we consider that waste can be produced by gross resources only), while, because of substitutability, total waste increases with *w* and *r*.

	Π_1	Π_2	Q_1	Q_2	K_1	K_2	L_1	L_2	R_1	R_2	GR_1	GR_2	P	W_1^0	W_2^0	W_{11}
τ	-	-	-	-	+	+	+	+	-	-	-	-	+	-	-	-
β	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+
o	+	-	+	-	-	-	-	-	+	-	+	-	-	+	-	+
c	+	-	+	-	+	+	+	+	-	-	-	-	+	-	-	-

Table 3: Only one firm recycles and reuses the input in its own production

Table 3 shows the case of a semi-circular economy as only one firm (say firm 1) recycles. In this case, the price of net resources for firm 1 (the recycling firm) becomes expression (13), that is, $\tilde{P}_{R1}\left[1 - o\beta\frac{\alpha_1}{1-\alpha_1}\right]$, where $\tilde{P}_{R1} = \frac{\alpha_1}{1-\alpha_1}\left[\frac{c}{\alpha_1} + \tau(1-o\beta) + (k+m\beta)\beta + \frac{w\beta o}{\xi}\right]$. This price includes the components from the recycling technology in the factor $\tilde{P}_{R1}$, which among others, increases with β because the cost of recycling per unit is a convex function of the degree of recycling β, but also the components representing the benefit side in the factor $\left[1 - o\beta\frac{\alpha_1}{1-\alpha_1}\right]$, which is decreasing with *o* and β.

Furthermore, the relationship between recycled waste and gross waste is described by $W_{11} = \beta W_1^0$. So, as we can see from the table, recycled waste is increasing in the degree of recycling β. As β increases, however, the cost of recycling from the benefit factor decreases, while the cost of recycling from the factor $\tilde{P}_{R1}$ increases. Overall, the effective cost of resources increases for our parameters, which leads to a decrease in profits and outputs.

As *o* increases, the recycling firm increases its benefits from recycling, hence increases optimal R_1, GR_1, W_1^0, W_{11}, Q_1, and Π_1 with a comparative advantage over firm 2 which is not recycling. An increase in the cost of gross resources *c* should boost circular economy. In fact, we observe that profit and output of firm 1 increase as *c* increases, where R_1 and GR_1 decrease as their cost relative to recycled material increases. Because of recycling, optimal waste is reduced even though output increases, contrarily to what one could expect. We also observe that the demand price increases in τ and *c*, as in Table 2, while it decreases with β and *o*, capturing a beneficial effect of recycling.

[11] An alternative production function, such as the CES production function, would imply that inputs are gross complements. The main implications for circularity would not be that different, but results would be more involved.

	Π_1	Π_2	Q_1	Q_2	K_1	K_2	L_1	L_2	R_1	R_2	GR_1	GR_2	P	W_1^0	W_2^0	W_{11}	W_{21}
τ	-	-	-	-	+	+	+	+	-	-	-	-	+	-	-	-	-
β	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+
o	+	+	+	+	-	-	-	-	+	+	+	+	-	+	+	+	+
c	-	-	-	-	+	+	+	+	-	-	-	-	+	-	-	-	-

Table 4: Both firms recycle and reuse the input in their own production

Table 4 shows the case of a circular economy and it is obtained under the assumption of symmetric firms. Here, we consider symmetric parameter values since asymmetric values of *o* and β provide us with results similar to Table 3, where firm 1 recycles (*o* positive, β positive) and firm 2 does not ($o = 0$, $\beta = 0$). Hence, if both firms recycle and are symmetric, the comparative advantage of one firm recycling over the other (as from Table 3) disappears and results are consistent with the above comments. For example, as *o* increases, and therefore benefits from recycling increase, profits and outputs increase for both firms, together with *R*, *GR*, W_i^0 and W_{i1}.

	Π_1	Π_2	Q_1	Q_2	K_1	K_2	L_1	L_2	R_1	R_2	GR_1	GR_2	P	W_1^0	W_2^0	W_{11}
τ	-	-	-	-	+	+	+	+	-	-	-	-	+	-	-	-
β	-	-	-	-	-	-	-	-	-	-	-	-	+	-	-	-
c	+	-	+	-	+	+	+	+	-	-	-	-	+	-	-	-
s	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-	+
z	-	+	-	+	+	+	+	+	-	+	-	+	+	-	+	-

Table 5: One firm recycles and resells part of the recycled material to the other firm

Table 5 describes the case where only one firm recycles (firm 1) and decides to resell part of the recycled material to the other firm (firm 2). We see that as *s* increases, revenues from selling the recycled inputs increase, hence we observe an increase in profits (Π_1) and outputs (Q_1) of firm 1. In addition, we see that R_1, GR_1, W_1^0, and W_{11} increase as well. Now, let us consider the effects as *z* increases. Table 5 is elaborated under the assumption that the recycled material is cheaper than the gross resource. Firm 2 is the firm that buys the recycled material, saving money by not buying the more expensive material and, at the same time, without facing a cost to recycle waste. So, firm 2 has benefits in this case: buy the inexpensive recycled material and avoid the cost to recycle. So, it is logical to see an increase in the profits of firm 2 as *z* increases. Firm 1 is the recycling firm, so it has an additional cost to recycle waste. In addition, since increasing *z* implies that firm 2 buys higher amounts of the recycled material, firm 1 faces higher costs by doing recycling, and at the same time it has to buy a large amount of the expensive materials, so its profits fall. This argument does not consider any restriction imposed on the amount of the recycled material that firm 2 can buy, which would lead to different outcomes. The analysis for the remaining signs are pretty much consistent with prior tables.

Finally, we wanted to further test the robustness of our results, so we repetead the entire process for each case, by including in our analysis some of the calibrated parameter values that Bongers and Casas (2022) showed in their paper. Specifically, we set total factor prodcustivity (A) to be equal to 1, the output-capital elasticity (γ) equal to 0.30, the output-material elasticity (θ) equal to 0.05, the output-labour elasticity (ζ) equal to 0.65, the waste coefficient (α) equal to 0.375,

and m to be equal to 1. In addition, τ was set equal to 0.19 according to OECD, o and β equal to 0.3 and 0.7 respectively according to EPA. The results under these parameter values support the results presented in our previous analysis.

6 A few extensions

In this section we present a couple of extensions of the same problem under alternative realistic assumptions. In particular, in Section 6.1 we consider the case where a firm sells its recycled product to the other firm but chooses to set a price per unit of recycled product, instead of assuming it as exogenously given, as we did in Section 4. Then in Section 6.2 we consider the case of environmentally conscious consumers who may prefer to buy products from firms that have adopted recycling technologies. Firms will have to consider the impact on their consumer base and demand when deciding whether to recycle or not.

6.1 What happens when the price of the recycled material is not exogenously given?

In Section 4 we supposed that *s*, the price of recycled material, is exogenously given. In the real world, it is common for firms selling a recycled input to another firm to set a price per unit of the recycled material. In this section we examine the scenario where one firm recycles and chooses to set the price of the recycled input, when it sells its recycled input to another firm. Hence, we assume a price function that is described by the following form:

$$P_{Rec,i} = \delta - \sigma\, RW_i \;\; (that\ is,\;\; P_{Rec,i} = \delta - \sigma\, \tfrac{o\beta\alpha_i}{1-\alpha_i}\, R_i) \tag{24}$$

In this case the recycling firm (firm *i*) faces a total cost similar to the problem described in the second part of section 3.2. That is, the firm faces labor cost for the final product (wLi), labor cost for recycling waste (wL_{iw}), capital cost (rK_i), cost for materials (cGR_i), taxation on the remaining unrecycled waste (τTW_{i0}), and cost for recycling ($v(\beta)W_{i1}$). At the same time, its cost is being reduced, compared to the no reycling case, as the firm is able to use in its own production the part of the recycling input that it does not sell to the other firm, instead of buying new materials at the price $\widetilde{P_{Ri}}$. Thus, its cost is reduced by $(1-z)\widetilde{P_{Ri}}\, RW_i$. After the necessary substitutions, the minimization problem of the recycling firm is described by:

$$\begin{aligned} &\min_{L_i,K_i,R_i} TC_i = wL_i + rK_i + \hat{P}_{Ri}R_i \\ &\text{s.t. } \bar{Q}_i = AK_i^{\gamma}L_i^{\zeta}R_i^{\theta} \end{aligned} \tag{25}$$

where $\hat{P}_{Ri}$ is the resource price that the recycling firm faces and is equal to $\hat{P}_{Ri} = \frac{\alpha_i}{1-\alpha_i}\left(\frac{c}{\alpha_i} + \tau\,(1-o\beta) + v(\beta)\beta + \frac{w\beta o}{\xi}\right)[1-(1-z)o\beta\frac{\alpha_i}{1-\alpha_i}]$.

However, the minimization problem of the second firm (firm *j*) changes in this scenario. Now, the firm will face labor cost for the production of the final good, capital cost, cost for materials, taxation on the waste produced and cost for purchasing the recycled material from the other firm at price $P_{Rec,i}$. Hence, the total cost of firm j can be expressed as follows:

$$TC_j = wL_j + rK_J + cGR_j + \tau W_j^0 + zP_{Rec,i}RW_i \tag{26}$$

After doing the necessary substitutions in the above expression, we get the minimization problem of the second firm, which is given by:

$$\begin{aligned} &\min_{L_j,K_j,R_j} TC_j = wL_j + rK_j + P_1R_j - P_2R^2 \\ &\text{s. t. } \bar{Q}_j = AK_j^{\gamma}L_j^{\zeta}R_j^{\theta} \end{aligned} \tag{27}$$

Where $P_1 = \frac{\alpha_i}{1-\alpha_i}\left(\frac{c}{\alpha_i} + \tau + z\delta\beta o\right)$ and $P_2 = \sigma z\left(\frac{o\beta\alpha_i}{1-\alpha_i}\right)^2$ [12].

On the other hand, the maximization problem of each firm is similar to the problems described in the second half of Section 3.2 but with the new recycling price. Hence, after all the necessary substitutions the two problems are given below:

$$\max_{Q_i} \quad \Pi_i = \left[d - b\left(Q_i + Q_j\right)\right]Q_i + P_3R_i - P_2R_i^2 - MC_iQ_i \tag{28}$$

$$\max_{Q_j} \Pi_j = [d - b(Q_i + Q_j)]Q_j - MC_jQ_j \tag{29}$$

Where $P_3 = \frac{\delta zo\beta\alpha_i}{1-\alpha_i}$ and P_2 has been specified above.

In this case, the recycling firm has a benefit from selling the recycled input if $P_3 > P_2R_i$, or when $RW_i < \frac{\delta}{\sigma}$. In addition, the second firm will have a greater benefit than cost from buying the recycling material if $P_1 < P_2R_j$ or when $\delta < \sigma o\beta\alpha_i - \frac{c+\alpha_i\tau}{zo\beta\alpha_i}$. Moreover, it will be willing to buy the recycled input only when its price is lower than the price of the rest available materials, which means that at the same time it should also hold that $RW_i > \frac{\delta}{\sigma} - \frac{c+\alpha_i\tau}{(1-\alpha_i)\sigma}$, which is true when $\delta > \frac{c+\alpha_i\tau}{1-\alpha_i}$.

Hence, when the recycling firm sets a price per quantity for the recycled material, it is profitable for both firms to take part in the transaction when $\frac{\delta}{\sigma} - \frac{c+\alpha_i\tau}{(1-\alpha_i)\sigma} < RW_i < \frac{\delta}{\sigma}$.

6.2 The effect of environmentally friendly consumers

Consumers' preferences over goods and services are affected by many different factors such as age, social media, family, culture, social class, beliefs, pollution and more. In this section we will focus on environmentally friendly consumers who are concerned about the environment and study how their preferences may affect our previously discussed results.

It is common for environmentally friendly consumers to be willing to pay a premium for goods that are friendly towards the environment. In our setting, environmentally conscious

12 Please note that, without loss of generality, we assumed that $\alpha_i = \alpha_j$ for simplicity, which is a reasonable assumption in our framework.

consumers may prefer to buy products from firms that have adopted recycling technologies. Hence, we suppose that they are willing to pay a higher price for such products, that is, the price of the final good is increased for the cases where firms use the recycled input in the production of the final good. In other words, we assume that the initial problem discussed in Section 3.1 remains the same, while the rest changes to reflect the increase in price and demand of the final good, due to the new preferences of the consumers. The following two tables (Table 6 and 7) show the profits of the two firms under the three cases (no recycling, one firm recycles, both recycle) and under a new higher price[13]. Specifically, Table 6 focuses on the case where the firm(s) recycle waste but use the recycled input in their own production, while Table 7 focused on the case where the firms decide to sell part of the recycled material:

		Firm j	
		NR	R
Firm i	NR	(46.2, 46.2)	$(201, 254.3)$
	R	(254.3, 201)	(223, 233)

Table 6: Firms' Profit Matrix With Environmentally Friendly Customers

		Firm j	
		NR	R
Firm i	NR	(46.2, 46.2)	$(218.5, 214.2)$
	R	(214.2, 218.5)	(610, 610)

Table 7: Firms' Profit Matrix With Environmentally Friendly Customers – Firms sell the recycled material

From the above tables we can clearly see that the profits of the firms increase significantly when they face the change in the price of the final good due to the new preferences of the consumers. In particular, when firms decide to use the recycled material in their own production, we end up having a unique Nash equilibrium where both firms decide to recycle (R,R). The results are in keeping with Proposition 1, however, now it is more profitable for the firms to recycle. In addition, when firms decide to sell part of the recycled material, we end up having again a unique Nash equilibrium where both firms decide to recycle (R,R). Thus, in this case, the results change compared to the case where consumers did not have a preference over environmentally friendly goods, in that (R,R) becomes the optimal strategy. We see that in both cases, when the price of the final good increases, firms have always an extra motivation to recycle waste to get the extra profit.

7 Concluding remarks

A circular economy is an economic system in which resources are used, reused, and recycled in a closed loop, minimizing waste and pollution. This approach contrasts with the traditional linear economy, in which resources are extracted, used, and then discarded as waste. In

13 Note that Table 6 is based on specific values, however the equilibrium outcomes remain the same under different prices.

a circular economy, the emphasis is on designing products and systems that are easily repairable, reusable, and recyclable, and on finding new ways to use waste materials. Recycling is an important component of a circular economy, as it allows for the recovery and reuse of valuable materials and reduces the need for virgin resources.

There are many benefits associated with a circular economy and specifically recycling, both for the society and the environment. Recycling can lead to the reduction of pollution and the greenhouse gas emissions which are caused from waste, reduce the need for virgin materials, create new job opportunities, and save energy. However, there are also some negative effects linked to recycling, one of which is the high investment cost to infrastructure and technology, quality concerns, and potential contamination if the materials are not properly separated and cleaned bfore recycling.

Our analysis focuses on a duopolistic market and on three different cases regarding the availability of recycling to the firms. We are trying to investigate under which assumptions the firms will decide to recycle, and under which circumstances recycling is not a profitable option for them. In particular, we study a linear economy where no firm recycles, a semi-circular case where only one of the firms recycles, and finally a fully-circular economy where both firms decide to recycle. Under the assumption that a recycling technology is available, we study two more cases: when firms reuse the recycled input in their own production process and when they decide to sell part of the recycled product to the other firm. In addition, we assume that the resources used in the production of the final good have two parts: a useful part that can be used in the production and a useless part that increases the waste pile of the economy. Furthermore, part of the waste can be recycled, which is how the circularity arises in this model.

We compare the profits of the two firms under the three different scenarios and their subcategories, and we also perform a sensitivity analysis based on all the important variables and parameters of the model. We see that when the firms reuse the recycled material in their own production, their decision regarding recycling depends on the available recycling technology. If the technology is updated and sofisticated, then both firms decide to recycle and vice versa. On the other hand, when firms are able to sell part of the input to the other firm, things become more complicated. Specifically, in this case we see that the recycling technology does not influence the decisions of the firms, and that the price of the raw material, the price of the recycled material, and the quantity of this material bought from the other firm affect the result. Propositions 1 and 2 summarize the equilibrium outcomes, while Section 5 discusses in details the results and the sensitivity analysis.

Our analysis focuses on two extra cases as well, that is, when the recycling firm decides to set a price per quantity of the recycling output sold, which transforms the price into an endogenous variable, and the case of having environmentally friendly consumers. For the first case, we analyze the conditions that should hold to be profitable for both firms to recycle when the price of the input is not exogenously given. Finally, the results clearly show that when consumers care about the environment and are willing to pay a higher price for products which have been produced by firms that adopted a recycling technology, firms are always gaining greater profits if they decide to recycle.

As a future research, it would be interesting to maximize the profit function of each firm with respect to the recycling coefficient β. This approach will complicate the solution to a great degree, but at the same time, will also give great insight on the behaviour and decisions of each firm. In addition, one could also solve the problem assuming that the firms can decide how much recycled input they will buy/sell. In other words, solving the problem taking into account that both

z_i and z_j (or z in the case that only one firm recycles) are endogenous, would offer additional information on the behaviour of the firms.

Finally, our result in Proposition 2 about a market for recycled materials presupposes that users are willing to buy recycled items. The cooperation of consumers/users is therefore indispensable to use recycled products. In a sense, they will have to change their mindset to consciously choose recycled materials. This will have major effects on the business models to be adopted by firms. This aspect should be expanded in further studies.

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