



## Correction to: Fundamental functions for local interpolation of quadrilateral meshes with extraordinary vertices

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**In the original publication of the article, some figures and their descriptions are incorrect. This erratum corrects the same and adjusts the meaning of the notation  $\mathcal{R}_\ell^k$  in Definition 5.**

In [1], Figure 2 and the text description at the beginning of page 374 should be replaced with the ones given below.

*The illustrative example in Fig. 2 left (respectively, right) shows the degree-5 (respectively, degree-7) fundamental function with compact support  $[-3, 3]$  (respectively,  $[-4, 4]$ ), that is obtained when selecting  $\Phi_\ell$  as a degree-2 (respectively, degree-3) polynomial B-spline and  $\mathcal{P}_\ell$  as a degree-3 (respectively, degree-4) polynomial interpolating a subset of four (respectively, five) consecutive points  $(x_k, \delta_{k,j})$ ,  $k \geq \ell$ . Since the assumption of Proposition 1 is fulfilled,  $\Psi_j$  is a fundamental function for interpolation. Moreover, according to Proposition 2, since  $m = 3$  and  $w = 3$  (respectively,  $m = 4$  and  $w = 4$ ), then the support width of  $\Psi_j$  is 6 (respectively, 8). Finally, according to Proposition 3, since  $\Phi_\ell$  is  $C^1$  (respectively,  $C^2$ ) then  $\Psi_j$  is  $C^2$  (respectively,  $C^3$ ).*

In [1], the following changes should be considered when reading pages 375–379:

- in the caption of Table 1 replace  $\mathcal{R}_\ell^1$  with  $\mathcal{R}_\ell^0$ ;
- in Definition 5 replace  $k$ -ring neighbourhood with  $(k + 1)$ -ring neighbourhood;
- on page 375, two lines below eq. (3), replace  $\mathcal{R}_\ell^{n+1}$  with  $\mathcal{R}_\ell^n$ ;

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The original article can be found online at <https://doi.org/10.1007/s11565-022-00423-8>.

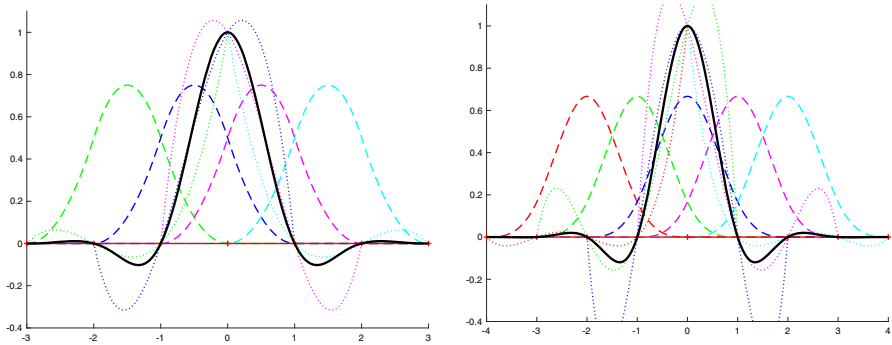
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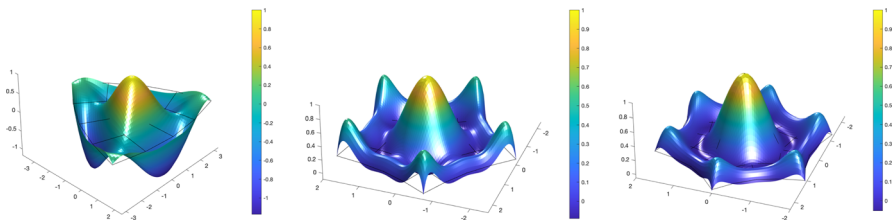
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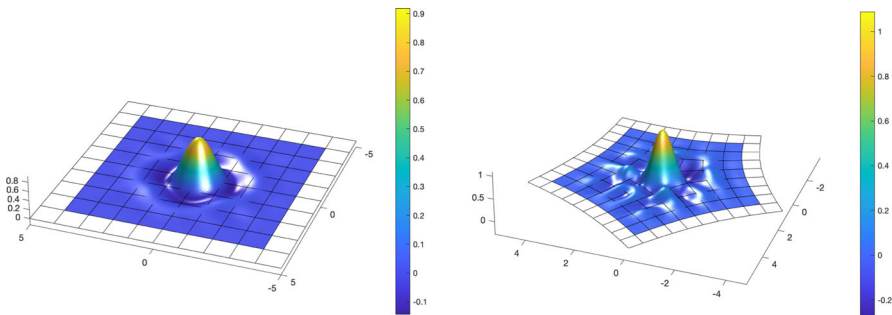
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**Fig. 2** Left: the  $C^2$  fundamental function supported on  $[-3, 3]$  that is obtained by using our constructive approach with the quadratic B-splines as blending functions. Right: the  $C^3$  fundamental function supported on  $[-4, 4]$  that is obtained by using our constructive approach with the cubic B-splines as blending functions. In both figures the interpolating polynomials blended by the B-splines (dashed graphs) are the dotted graphs



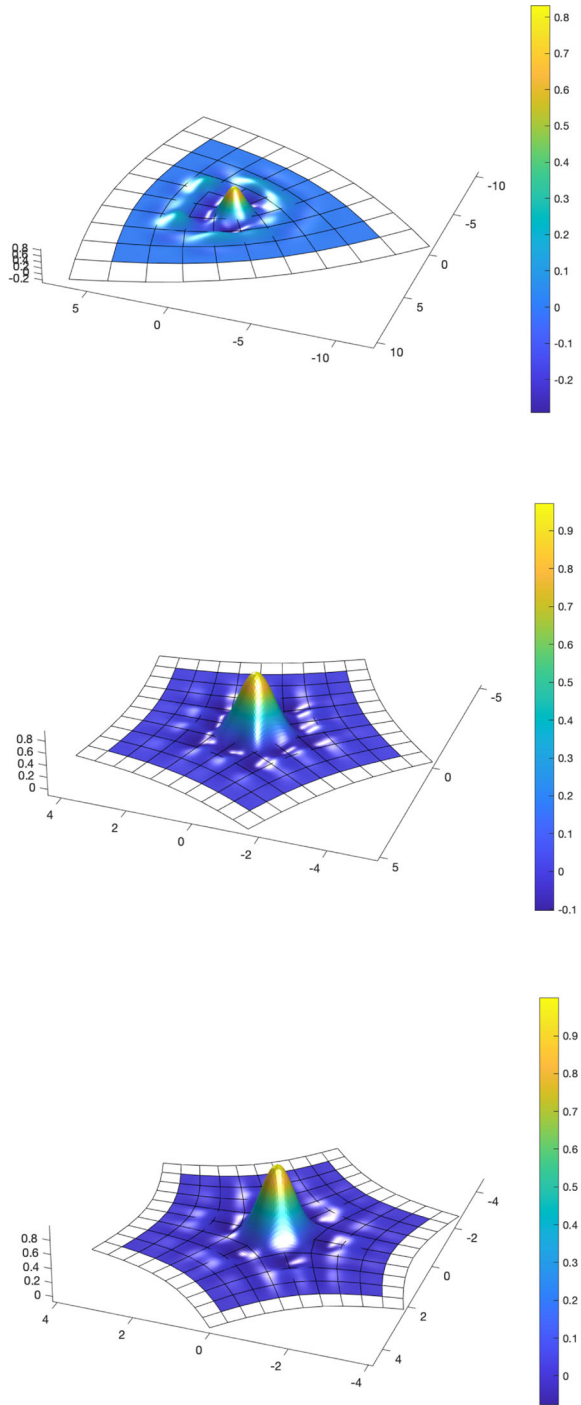
**Fig. 3** Degree-6,7,8 bivariate polynomials that interpolate the  $6N_\ell + 1$  points  $(\mathbf{x}_k, \delta_{k,\ell})$ ,  $\mathbf{x}_k \in \mathcal{R}_\ell^1$  and approximate in the least-squares sense the  $6N_\ell$  points  $(\mathbf{x}_k, 0)$ ,  $\mathbf{x}_k \in \mathcal{R}_\ell^2 \setminus \mathcal{R}_\ell^1$  when the valence of  $\mathbf{x}_\ell$  is  $N_\ell \in \{3, 5, 6\}$



**Fig. 4** Left: fundamental function for local interpolation that is defined on a regular subregion of  $\Omega$  and is  $C^3$  everywhere. Right: globally  $C^2$  fundamental function that is centered at a valence-4 vertex and contains a valence-5 vertex in its support

- on page 376, line 7, replace  $\mathcal{R}_\ell^{n+1}$  with  $\mathcal{R}_\ell^n$ ;
- on page 376, line 16, replace  $\mathcal{R}_\ell^2$  with  $\mathcal{R}_\ell^1$ ;
- in Proposition 4, second line, replace  $\mathcal{R}_\ell^{n+1}$  with  $\mathcal{R}_\ell^n$ ;
- on page 379, fifth line from the bottom, replace 2-ring with 3-ring.

**Fig. 5** First column: globally  $C^2$  fundamental functions for local interpolation centered at an extraordinary vertex of valence 3, 5, 6



Additionally, the second paragraph of [1, Section 5] should be substituted with the following one:

*As to the polynomial interpolants, since when  $n = 1$  condition (5) must hold for  $\#\mathcal{L}_\ell^1 = 6N_\ell + 1$  with  $N_\ell \in \{3, 4, 5, 6\}$  denoting the valence of  $\mathbf{x}_\ell$ , we should require that  $\dim(\Pi_d) \geq \max\{6N_\ell\} + 1$ . In our experiments we have worked with polynomials  $\mathcal{P}_{\ell,1}$  of total degree  $6 \leq d \leq 8$ , as it is a reasonably low degree that allows us to satisfy the condition (5) for all valences up to 6. Hence, for a vertex  $\mathbf{x}_\ell$  of valence  $N_\ell \leq 6$ , we have computed the coefficients of the degree- $d$  polynomial  $\mathcal{P}_{\ell,1}$  by solving a weighted least-squares fitting problem with big weights assigned to the  $6N_\ell + 1$  interpolation points with parameter values in  $\mathcal{R}_\ell^1$ . Examples of bivariate polynomials that interpolate the  $6N_\ell + 1$  points  $(\mathbf{x}_k, \delta_{k,\ell})$ ,  $\mathbf{x}_k \in \mathcal{R}_\ell^1$  and approximate in the least-squares sense the  $6N_\ell$  points  $(\mathbf{x}_k, 0)$ ,  $\mathbf{x}_k \in \mathcal{R}_\ell^2 \setminus \mathcal{R}_\ell^1$  when the valence of  $\mathbf{x}_\ell$  is  $N_\ell \in \{3, 5, 6\}$ , are shown in Fig. 3.*

Finally, Figure 4, 5 and the first column of Figure 5 in [1] should be replaced with the ones given below.

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## Reference

1. Beccari, C.V., Casciola, G., Romani, L.: Fundamental functions for local interpolation of quadrilateral meshes with extraordinary vertices. *Annali dell'Università di Ferrara* **68**, 369–383 (2022)

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