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PRESCHOOL CHILDREN'S REPRESENTATION OF DIVISION WORD PROBLEMS THROUGH DRAWINGS

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Research has shown that preschool children can make sense of multiplication and division situations. However, researchers suggest that sharing is the most intuitive way for young children to solve division problems using manipulatives. Starting from the hypothesis that drawing might be another means by which young children can represent division situations, we conducted a task-based interview with a small group of Australian and Italian 6-year-olds about measurement and partitive division situations. Results indicate that the children could interpret both types of division and their drawings and gestures captured the strategies used. We contend that children's drawings capture their thinking more effectively than their use of manipulatives.

INTRODUCTION

Research has highlighted that children can represent multiplicative situations prior to commencing formal schooling (Bakker et al., 2014; Vanluydt et al., 2022). For instance, Bakker and colleagues (2014) found that first graders (6-7 year olds) could solve multiplicative word problems even when still unaware of how it is formally represented. Others reported that kindergarten children could detect a multiplicative relation (the ratio) in proportionality problems (Vanluydt et al., 2022) and after specific instruction could solve multiplication and division problem by modelling the situation with tally marks, counting, or recall of facts (Carpenter et al., 1993). Although the children in Carpenter et al.'s study were permitted to use counters or pencil and paper to help them solve the problems, many used counters only while modelling division problems.

We wondered if these findings indicate that preschool children can model division problems only with manipulative materials, or if drawings may be another means of representation for division situations. The study presented in this report explored this idea, as several authors consider drawing as a powerful tool for problem solving (e.g., Soundy & Drucker, 2009).

THEORETICAL FRAMEWORK

Young children's drawing in mathematics

Carruthers and Worthington (2006) investigated the development of young children's (3 to 8 year-olds) mathematical graphics. They defined graphics as the full range of marks children make when exploring mathematical ideas. These included dynamic, pictographic, iconic, written, and symbolic marks. They claimed that exploring mathematics through their own intuitive marks helps young children to make sense of standard symbols and bridges the gap between informal mathematics and the abstract

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mathematics of school. Also, children's drawings, and narrative about them, are a "window into the mind of child" (Woleck, 2001, p. 215) allowing teachers and researchers insight into children's mathematical thinking.

Young children's drawings generally serve two purposes in the mathematics classroom. Drawings may support the process of mathematical work and/or represent the product of mathematical work (e.g. Smith, 2003). Smith (2003) described these purposes *as drawing as problem solving* and *drawing of problem-solving*. Similarly, MacDonald (2013) argued that drawings are "not just a procedure by which children record their knowledge about a concept; it is also a process through which understandings can be constructed, re-considered and applied in new ways" (p. 72). Furthermore, Woleck (2001) identified that first graders used drawings in mathematical problem solving "as if they were manipulatives" (p. 216) to carry out the steps of organising and counting, that supported their problem-solving efforts. She also reported that children might use drawings as a prewriting tool to communicate their mathematical thinking to others.

In the context of multiplication and division, Mulligan (2002) found that children's drawings and their explanations of the drawings could be used to identify how they notice multiplicative structures. She reported that the images drawn by low attaining children in the primary years tended to lack structure and were poorly organised. This was attributed to an underlying lack of awareness of the equal groups structure and a reliance on using counting by ones when solving problems.

Division of integers

While it is acknowledged that division is more than just sharing (Squire & Bryant, 2002), the physical act of sharing a quantity equally is division, in that to share a quantity successfully one divides a dividend into equal quotients. Previous research has found that young children (4 to 5-year-olds) can share out quantities using one-to-one correspondence, and model division problems using concrete materials long before any formal introduction to division (Carpenter et al. 1993; Frydman & Bryant, 1988). Furthermore, these initial strategies tend to reflect the action described in the problem (Marton, 1996). These findings led many to suggest that sharing is the schema for action from which an understanding of division develops (e.g. Correa et al., 1998; Squire & Bryant, 2002). Frydman and Bryant (1988) found that although most 4-year-olds in their study could share items equally between two groups, only half of the children (10 out of 24) were able to infer the number of items in each set. This suggests that these children have an understanding of the numerical significance of sharing. It also suggests the developmental nature of this concept.

Division word problems can be interpreted and represented in two different ways, namely division by the multiplier (partition division) and division by the multiplicand (quotitive or measurement division) (e.g., Correa et al., 1998; Greer, 1992; Verschaffel et al., 2007). According to Greer (1992, p. 276):

> Dividing the total by the number of groups to find the number in each group is called partitive division, which corresponds to the familiar practice of equal sharing […]. Dividing the total by the number in each group to find the number of groups is called quotitive division (sometimes termed measurement division, reflecting its conceptual links with the operation of measurement).

The difference between quotition and partition problems relates to the textual structure of the problem (Nesher, 1988). For example, the expression 12 ÷ 4 could be interpreted as a partitive problem, such as: Twelve lollies are shared equally among 4 children. How many did they each receive? In solving the problem the action is one of sharing or distributing the twelve lollies equally between the four children. Interpreted as a quotitive problem, using the same context: There are 12 lollies and each child receives 4. How many children will receive lollies? While the quotient is the same for each, the model is quite different, so is the action. Rather than an action of sharing it is a grouping or count of the twelve lollies into groups of fours.

In summary, critical ideas for students to construct are: that collections/objects can be divided into equal groups; division involves part-whole relations that include three elements: the size of the whole, number of parts, and size of the parts; there is a relationship between three values represented by the dividend, divisor, and quotient; and that division is the inverse of multiplication, in which case multiplication can be used to solve division problems.

Our focus in this report is on four division problems (two measurement and two partitive) (see Table 1), which are part of an interview in our larger study.

Measurement division	Partitive division
P1. Tad fished 12 tadpoles. He put 4 tadpoles in each jar. How many jars did Tad put tadpoles in?	P3. Mr. Gomez had 12 cookies. He put the cookies into 3 boxes so there was the same number of cookies in each box. How many cookies did Mr. Gomez put in each box?
P2. John had 6 crayons. He put 2 crayons in each box. How many boxes did John need?	P4. You have 12 marbles and you give the same amount of marbles to three friends. How many marbles do they each get?

Table 1: Division problems used in the interviews.

In the context of using manipulatives, Carpenter et al. (1993) noted that young children enact different solution strategies when solving a division problem. When solving measurement division of the type M:N, some children counted M items and then arranged them in sets of N items each (*measuring* M in terms of Ns). Others made sets of N items until they reached a total of M items and then counted the number of sets. Some students counted by Ns (*double count* according to Kouba, 1989). In the context of partitive division A:B, children arranged A in B sets with the same number of counters in each set (*grouping*). Some children distributed the counters one by one into the B sets (*sharing*). Others put a number of items in each B set then adjusted the number in each (*trial-and-error* strategy) or used *known or derived facts*.

METHODS

Since we were interested in understanding how children represent division situations using drawings, we needed suitable methods. In particular, we wanted to identify the elements that allow understanding if the situation is modelled as expected, that means if a measurement or partitive division is modelled as such. We needed to distinguish between drawing-as-problem-solving and drawing-of-problem-solving (Smith, 2003). We expected some of the strategies described by Carpenter et al. (1993) in the context of manipulatives to appear in the context of drawings as well.

Research shows how children can produce drawings together with other means of communication like talk, writing, movement, and sound (Soundy & Drucker, 2009). With the aim of taking into consideration all the possible means of communication involved in the production of the drawings – including gestures, spoken words, and the use of manipulatives – we decided to adopt a multimodal semiotic approach by referring to the construct of a *semiotic bundle* as presented by Arzarello et al. (2009). A semiotic bundle is:

> a system of signs […] that is produced by one or more interacting subjects and that evolves in time. Typically, a semiotic bundle is made of the signs that are produced by a student or by a group of students while solving a problem and/or discussing a mathematical question. (Arzarello et al., 2009, p. 100)

In particular, the *synchronic* and *diachronic* analysis of the semiotic bundle may give hints about how the children represent the proposed situations, what are the relations between the different representations and how such representations (do or do not) help the child during the process. The diachronic analysis focuses on the evolution of signs over time, and the transformation of their relationships; the synchronic analysis, instead, focuses on the relations among the signs used in a certain moment. In particular, we will speak of *genetic conversion* (Arzarello, 2006) when the conversion rules between semiotic sets have a genetic nature, namely, one semiotic set is generated by another one, so enlarging the bundle. Our research questions can be then rephrased: Do the same strategies observed for manipulatives appear in the case of graphics? How do the different components of the semiotic bundle correspond with preschool children's mathematical graphics of the division situations?

We conducted our research in countries with different languages (Australia and Italy, so English and Italian languages). In particular, these two languages differ in the way in which multiplication is verbally represented. In English (as in many other languages) the word 'times' is used to read multiplications (like expressions such as 3×4. This is not the case in Italian, where the symbol × is read 'per' is unrelated to the word 'volte' ('times' in Italian) but refers only to the name of the symbol itself. In the Australian context, children start preschool (kindergarten) when they are 4 year-olds; from there they transition to their first year of primary school at the age of 5. In Italy children move directly from kindergarten to primary at the age 6.

Prior to commencing the main study, the interview tasks and recording sheet were trialled, following which refinements were made to the language and the number range. We interviewed Australian children during their Foundation year, while Italian children were interviewed at the very beginning of first grade. Our sample consisted of 19 children with an average age of 6 (6 males and 4 females were Australian; 5 females and 4 males were Italian). The researcher interviewed the children individually using an interview script to ensure consistency. The task could be repeated as many times as needed. Each interview took for approximately 15 minutes. Each interview was video recorded with parent consent. Video recordings were transcribed verbatim, and the transcript was enriched with images of gestures and drawings to describe the semiotic bundle (Radford & Sabena, 2015).

RESULTS

The interview data were categorized in terms of strategies used for measurement or partitive division situations. The video analysis took into account the different modalities of representations, not only the drawing, but also the relations between the different signs both synchronically and diachronically, exemplified below. Table 2 includes the strategies we observed in the children's drawings, which correspond to those observed by Carpenter et al. (1993) when children used manipulatives (see Theoretical Framework section).

Table 2: Occurrences of each strategy for addressing the proposed division situations

	Measurement division		Partitive division	
	P1 (tadpoles)	P2 (crayons)	P3 (cookies)	P4 (marbles)
measuring	✓	✓		
double count	✓			
grouping			✓	
sharing			✓	
trial-and-error			✓	✓
known/derived facts		✓		✓

The diachronic analysis of the process of drawing (including speech and gestures) allowed us to distinguish between the different categories. For instance, Figure 1 shows the final graphics from two different children. The graphics depict P1 (the tadpole's problem) and in both cases it is possible to see a human character and three jars, each containing four tadpoles. However, the processes behind these two drawings are completely different and, we contend, highlight two different strategies of interpretation of this measurement division situation. The Italian child started by drawing one jar with four tadpoles inside (Figure 1a). In contrast, the Australian child drew a long fishing pole and the twelve tadpoles; the jars (represented by C-like lines) were added as last element (Figure 1b). He is measuring the number of tadpoles (12) in terms of the number of tadpoles-per-jar (4) We noticed that this second procedure follows the same order of appearance of the information in the verbal presentation of the situation: the character first, then the number of tadpoles, and finally the number of tadpoles per jar. The first procedure did not present, at least initially, the information about the total number of fished tadpoles while the sequence of drawing appears as a genetic conversion of the sentence 'He put 4 tadpoles in each jar'. The character was added as last element.

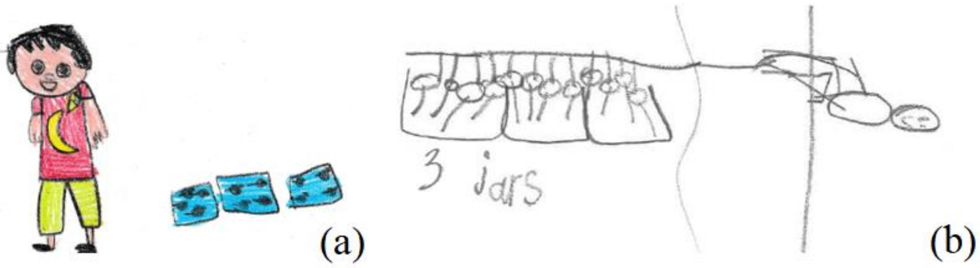

Figure 1: Two children's mathematical graphics of the tadpoles' problem (P1).

Indeed, after drawing the first jar, the child whose graphic is depicted in Figure 1a, drew a second jar with four tadpoles inside (Figure 2a). Then, she counted the drawn tadpoles pointing to them one by one (Figure 2b) After realizing there were eight tadpoles, she stated that another jar was needed and drew the last jar (Figure 2c). We can see that she is using a double count: she is keeping the count of the number of tadpoles-per-jar in each jar and, at the same time, checking that the total amount of tadpoles reaches the expected quantity of twelve.

Figure 2: Video screenshots of the process of generation of the graphic in Figure 1a.

DISCUSSION AND CONCLUSION

Our results show that when young children represent division situations through drawings, they adopt solution strategies similar to those who used manipulatives in earlier studies (Carpenter et al., 1993). A key finding from this study is that drawing may be an efficient tool not just in the sense of *drawing of problem-solving* but *as problem solving* (Smith, 2003) as well. Through the diachronic analysis of the semiotic bundle, we were able to exemplify how different processes involved in the creation of mathematical graphics may correspond to different strategies. Also, that some of these strategies only appear as genetic conversion of the words used to present the situation into inscriptions.

Our results suggest that drawing can be an efficient tool for problem solving in preschool as much as the use of manipulatives. We contend that our analysis allows us to argue that drawings may help researchers (and possibly teachers) capture children's thinking more effectively than manipulatives. Indeed, the diachronic analysis of the relationships between the different inscriptions clearly shows the model of division adopted by the child while allowing the researchers to distinguish the presence of those that we have called the critical ideas related to division (see Theoretical Framework section). Observing the sequence of each child's drawings and gestures they used enabled us to notice whether the child had a sense of the equal group structure and or relied on counting by ones to solve the problems, as observed by Mulligan (2002). Also, we observed grouping strategies for both partitive, and measurement division situations. This result contradicts the commonly held view that sharing is the schema for action from which an understanding of division develops (e.g., Correa et al., 1998; Frydman & Bryant, 1988; Squire & Bryant, 2002).

It is possible that through the production of mathematical graphics that children are able to adopt such strategies: while drawing they produce a permanent record of their previous thinking which may allow them to reconsider it or reason about it – by themselves or with the scaffolding of an adult. Also, symbols like connection-lines and arrows may help in re-tracing the enacted processes of moving, grouping, or sharing. Such strategies are not possible with manipulatives, as only the final product of the process remains visible.

We acknowledge further research is needed to provide large-scale evidence for these speculations. In particular, the study presented in this research report was explorative in its nature and involved a very small sample. The fact that the sample was constituted of children with different schooling experiences and different languages supports the possibility of generalization of the obtained results. In the future, we plan to repeat the same study on a larger scale to support our conjecture.

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