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Quantum dust cores of black holes

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ABSTRACT

We describe the ground state for a gravitationally collapsed ball of dust as the direct product of wavefunctions for dust particles distributed over an arbitrary number of nested layers. This allows us to estimate the expectation value of the global radius as well as the effective energy density and pressures for the dust core of quantum black holes. In particular, the size of the quantum core does not depend on the number of layers and the mass function is shown to grow linearly with the areal radius up to the outermost layer.

1. Introduction

The singularities of known black hole solutions of the Einstein equations [1] can be removed by imposing regularity conditions on the (effective) energy density and scalar invariants inspired by classical physics [2]. This procedure usually induces the appearance (or fails to remove) an inner Cauchy horizon. A different framework, invoked for example in Ref. [3], can be implemented based on the possibility that black hole interiors and the collapsed matter therein are described more accurately by quantum physics. One can then consider an effective energy density $\rho \sim |\psi|^2$, where $\psi = \psi(r)$ is the wavefunction of the fully collapsed matter source, such that the Misner-Sharp-Hernandez mass function [4,5] satisfies

$$m(r) \equiv 4\pi \int_0^r \rho(x)\, x^2\, \mathrm{d}x \sim 4\pi \int_0^r |\psi(x)|^2\, x^2\, \mathrm{d}x < \infty \quad \text{for } r > 0\,. \tag{1.1}$$

This accommodates for $\rho \sim r^{-2}$ and $m \sim r$, which ensures that $m(0) = 0$ and replaces the central singularity with an integrable one [6,7], that is a region where the curvature invariants and the effective energy-momentum tensor diverge but their volume integrals remain finite [8].

An explicit realisation for the inner core of a quantum black hole based on the Oppenheimer-Snyder model of dust collapse [9] was analysed in Refs. [10–12], where only the outermost layer of dust was explicitly considered in an effective one-body approach. This does not allow to estimate uniquely the size of the core or to obtain an effective energy density inside the completely collapsed core itself, which are the main objectives of the present work. To this end, we will here describe the ball as a sequence of layers [13] containing dust particles, whose trajectories are individually quantised as in Ref. [10]. A condition is then imposed to ensure that the fuzzy quantum layers defined by the positions of these particles remain orderly nested in the global quantum ground state. This approach will confirm the expected quantum behaviour in Eq. (1.1) for the effective energy density and mass function.

It is important to stress that the above procedure differs from the canonical quantisation of the Oppenheimer-Snyder model employed, for example, in Refs. [14–18], in which one starts from a reduced Einstein-Hilbert action for the areal radius of the ball. Instead, we here quantise the trajectories of dust particles, which of course follow geodesics in the classical theory, as more physically relevant degrees of freedom of the system, similarly to what is done in the quantum mechanical description of the hydrogen atom. We will then find that there exist ground states for the dust particles in each layer, and a collective ground state for the whole core will be built self-consistently, starting from the quantum ground states of single dust particles. We remark that no dynamical process will be analysed here which could possibly lead to the formation of such a collective ground state, or of other quantum effects, like the Hawking evaporation.

We will introduce the dynamical equation for dust particles in layers of the dust ball and derive the single layer quantum states in the next Section; the global ground state is then constructed in Section 3, where its main features are also analysed; concluding remarks and outlook will be given in Section 4.

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2. Quantum dust in a ball

Let us consider a perfectly isotropic ball of dust with total ADM [19] mass M and areal radius $R=R(\tau)$, where τ is the proper time measured by a clock comoving with the dust. Dust particles, which we assume have the same proper mass $\mu \ll M$, inside this collapsing ball will follow radial geodesics $r=r(\tau)$ in the Schwarzschild spacetime metric[1]

$$\mathrm{d}s^2 = -\left(1-\frac{2\,G_\mathrm{N}\,m}{r}\right)\mathrm{d}t^2 + \left(1-\frac{2\,G_\mathrm{N}\,m}{r}\right)^{-1}\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2 \ , \tag{2.1}$$

where $m=m(r)$ is the (constant Misner-Sharp-Hernandez) fraction of ADM mass inside the sphere of radius $r=r(\tau)$ and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2$. Irrespectively of the mass profile $m=m(r)$, the classical dynamics predicts that an event horizon forms when the surface areal radius $R(\tau)=2\,G_\mathrm{N}\,M\equiv R_\mathrm{H}$ and the collapse will further proceed towards a singularity in a finite proper time.

We can discretise this ball by considering a spherical core of mass $\mu_0=\nu_0\,\mu=\epsilon_0\,M$ and radius $r=R_1(\tau)$ surrounded by N comoving layers of inner radius $r=R_i(\tau)$, thickness $\Delta R_i = R_{i+1}-R_i$, and mass $\mu_i=\epsilon_i\,M$, where ϵ_i is the fraction of ADM mass carried by the $\nu_i=\mu_i/\mu$ dust particles in the $i^{\rm th}$ layer. The gravitational mass in the ball $r<R_i$ will be denoted by

$$M_i = \sum_{j=0}^{i-1}\mu_j = M\sum_{j=0}^{i-1}\epsilon_j = \mu\sum_{i=0}^{i-1}\nu_j \ , \tag{2.2}$$

with $M_1=\mu_0$ and $M_{N+1}=M$. We also note that the radius R_1 and mass $M_1=\mu_0$ of the innermost core,[2] as well as the thickness ΔR_i of each layer, can be made arbitrarily small by increasing the number N of layers in the classical picture.

The evolution of each layer can be derived by noting that dust particles located on the sphere of radius $r=R_i(\tau)$ will follow the radial geodesic equation

$$H_i \equiv \frac{P_i^2}{2\,\mu} - \frac{G_\mathrm{N}\,\mu\,M_i}{R_i} = \frac{\mu}{2}\left(\frac{E_i^2}{\mu^2}-1\right) \equiv \mathcal{E}_i \ , \tag{2.3}$$

where $P_i=\mu\,\mathrm{d}R_i/\mathrm{d}\tau$ is the radial momentum conjugated to $R=R_i(\tau)$, E_i the conserved momentum conjugated to $t=t_i(\tau)$ and the angular momentum conjugated to $\phi=\phi_i(\tau)$ was of course set to zero for dust particles in a non-spinning ball [12]. Notice that Eq. (2.3) depends on the (classically arbitrary) distribution of dust among the layers of mass $\mu_{i\ge 1}=M_{i+1}-M_i$ and the innermost spherical core of mass $\mu_0=M_1$. This is the kind of improvement over previous works that we need in order to estimate the core profile in the quantum ground state.[3]

With the canonical quantization prescription $P_i\mapsto \hat{P}_i = -i\,\hbar\,\partial_{R_i}$, Eq. (2.3) becomes the time-independent Schrödinger equation

$$\hat{H}_i\,\psi_{n_i} = \left[-\frac{\hbar^2}{2\,\mu}\left(\frac{\mathrm{d}^2}{\mathrm{d}R_i^2}+\frac{2}{R_i}\,\frac{\mathrm{d}}{\mathrm{d}R_i}\right) - \frac{G_\mathrm{N}\,\mu\,M_i}{R_i}\right]\psi_{n_i} = \mathcal{E}_{n_i}\,\psi_{n_i} \ . \tag{2.4}$$

The above is formally the same as the equation for s-states of the hydrogen atom, so that one can read out a Bohr radius

$$a_i = \frac{\ell_\mathrm{p}\,m_\mathrm{p}^2}{\mu\,M_i} \tag{2.5}$$

and the solutions are given by the Hamiltonian eigenfunctions

$$\psi_{n_i}(R_i) = \sqrt{\frac{\mu^6\,M_i^3}{\pi\,\ell_\mathrm{p}^3\,m_\mathrm{p}^9\,n_i^5}}\,\exp\left(-\frac{\mu^2\,M_i\,R_i}{n_i\,m_\mathrm{p}^3\,\ell_\mathrm{p}}\right) L_{n_i-1}^1\left(\frac{2\,\mu^2\,M_i\,R_i}{n_i\,m_\mathrm{p}^3\,\ell_\mathrm{p}}\right) \ , \tag{2.6}$$

where L_{n-1}^1 are Laguerre polynomials and $n_i=1,2\,\ldots$, corresponding to the eigenvalues

$$\mathcal{E}_{n_i} = -\frac{\mu^3\,M_i^2}{2\,m_\mathrm{p}^4\,n_i^2} \ . \tag{2.7}$$

The wavefunctions (2.6) are normalised in the scalar product which makes $\hat{H}_i$ Hermitian, that is

$$\langle\, n_i \mid n_i'\,\rangle = 4\,\pi\int_0^\infty R_i^2\,\psi_{n_i}^*(R_i)\,\psi_{n_i'}(R_i)\,\mathrm{d}R_i = \delta_{n_i n_i'} \ . \tag{2.8}$$

The expectation value of the areal radius on these eigenstates is given by

$$\bar{R}_{n_i} \equiv \langle n_i|\,\hat{R}_i\,|n_i\rangle = \frac{3\,m_\mathrm{p}^3\,\ell_\mathrm{p}\,n_i^2}{2\,\mu^2\,M_i} \ , \tag{2.9}$$

with relative uncertainty

$$\frac{\overline{\Delta R}_{n_i}}{\bar{R}_{n_i}} \equiv \frac{\sqrt{\langle n_i|\,\hat{R}_i^2\,|n_i\rangle - \bar{R}_{n_i}^2}}{\bar{R}_{n_i}} = \frac{\sqrt{n_i^2+2}}{3\,n_i} \ , \tag{2.10}$$

which approaches the minimum $\overline{\Delta R}_{n_i} \simeq \bar{R}_{n_i}/3$ for $n_i \gg 1$.

By assuming that the conserved quantity E_i remains well-defined for all the dust particles in the allowed quantum states, we obtain the fundamental condition [10]

$$0 \le \frac{E_i^2}{\mu^2} = 1 + \frac{2\,\mathcal{E}_i}{\mu} = 1 - \frac{\mu^2\,M_i^2}{m_\mathrm{p}^4\,n_i^2} \ , \tag{2.11}$$

which yields the lower bound for the single particle principal quantum numbers

$$n_i \ge N_i \equiv \frac{\mu\,M_i}{m_\mathrm{p}^2} \ . \tag{2.12}$$

Upon saturating the above bound, one then finds

$$\bar{R}_{N_i} = \frac{3}{2}\,G_\mathrm{N}\,M_i \ , \tag{2.13}$$

and the wavefunction for the ν_i particles in each layer is given by the same ground state

$$\psi_{N_i}(R_i) = \sqrt{\frac{\mu\,m_\mathrm{p}}{\pi\,\ell_\mathrm{p}^3\,M_i^2}}\,\exp\left(-\frac{\mu\,R_i}{m_\mathrm{p}\,\ell_\mathrm{p}}\right) L^1_{\frac{\mu\,M_i}{m_\mathrm{p}^2}-1}\left(\frac{2\,\mu\,R_i}{m_\mathrm{p}\,\ell_\mathrm{p}}\right) \ , \tag{2.14}$$

where the values of M_i, hence N_i in Eq. (2.12), must be such that $\bar{R}_i \lesssim \bar{R}_{i+1}$.

From the above wavefunction, one can in principle determine the effective energy density inside each layer as

$$\rho_i = \mu\,\nu_i\,|\psi_{N_i}(r)|^2 \simeq \mu\,\nu_i\,|\psi_{N_i}(3\,G_\mathrm{N}\,M_i/2)|^2 \ , \tag{2.15}$$

[1] We shall always use units with $c=1$ and often write the Planck constant $\hbar = \ell_\mathrm{p}\,m_\mathrm{p}$ and the Newton constant $G_\mathrm{N}=\ell_\mathrm{p}/m_\mathrm{p}$, where ℓ_p and m_p are the Planck length and mass, respectively.

[2] The radius R_1 can be interpreted as the size of the innermost core or the inner radius of the first layer around it.

[3] The effective one-body approach in Ref. [10] is obtained by assuming $\mu\sim M$, which introduces undetermined numerical coefficients [11] but leaves the final results qualitatively unaltered, as we shall duly comment in the following.

in which we approximated $r \simeq \bar{R}_{N_i}$ and used Eq. (2.13). Clearly, the above expression depends on the number ν_i of dust particles in the $i^{\rm th}$ layer and the number $\sum_{j=0}^{i} \nu_j$ of dust particles in the mass M_i (and N_i), which are yet to be determined. We shall see in the next Section how to estimate the distribution of particles ν_i and the corresponding energy density self-consistently.

3. Multilayered ground state

Since dust only interacts gravitationally, we can assume that the Hilbert space for the complete ball of mass M is given by the direct product $\mathcal{H} = \otimes_{i=1}^{N} \left(\otimes_{k=1}^{\nu_i} \mathcal{H}_i\right)$ of bound eigenstates (2.6) for the ν_i dust particles in each layer with $\sum_{i=0}^{N} \nu_i = M/\mu$.

We are here interested in the global ground state given by the product[4]

$$|\{\nu_1, N_1\}, \ldots, \{\nu_N, N_N\}\rangle = \bigotimes_{i=1}^{N} |N_i\rangle^{\nu_i} \tag{3.1}$$

of single layer ground states, each one containing ν_i dust particles in the state $|N_i\rangle$, and which further satisfy $\bar{R}_i \lesssim \bar{R}_{i+1}$. Given the uncertainty (2.10), the minimum thickness of the $i^{\rm th}$ layer of inner radius $\bar{R}_i$ must be of the order of $\overline{\Delta R}_i$ and the finest layering of the dust ball compatible with this quantum description is given by $\bar{R}_{i+1} \simeq \bar{R}_i + \overline{\Delta R}_i \gtrsim 4\,\bar{R}_i/3$. On assuming $N_i \gg 1$ for all $i = 1, \ldots, N$, one finds

$$2\,G_{\rm N}\,M_i = \frac{4}{3}\,\bar{R}_{N_i} \simeq \bar{R}_{N_i} + \overline{\Delta R}_{N_i} \simeq \bar{R}_{N_{i+1}} = \frac{3}{2}\,G_{\rm N}\,M_{i+1}\ , \tag{3.2}$$

or $M_{i+1} \simeq 4\,M_i/3$, which implies that the mass of each layer $\mu_i \simeq M_i/3$. The quantum numbers for the relevant single particle ground states in Eq. (3.1) are therefore given by

$$N_i \simeq \left(\frac{3}{4}\right)^{N-i+1} \frac{\mu\,M}{m_{\rm p}^2}\ . \tag{3.3}$$

In particular, the quantum number for dust particles in the ground state of the outermost layer (with $i = N$) is given by

$$N_{\rm s} \equiv N_N \simeq \frac{3\,\mu\,M}{4\,m_{\rm p}^2}\ , \tag{3.4}$$

which yields the global ball radius

$$R_{\rm s} \equiv \bar{R}_{N_{\rm s}} + \overline{\Delta R}_{N_{\rm s}} \simeq \frac{3}{2}\,G_{\rm N}\,M\ . \tag{3.5}$$

Since $R_{\rm s} < R_{\rm H}$, the ground state of the dust ball can indeed be the core of a black hole, like it was found in Refs. [10–12]. It is remarkable that the radius (3.5) does not depend on the number N of layers, or any other quantity, except M. In fact, N only determines how finely we describe the central region of the ball. In particular, the innermost core has radius $\bar{R}_1 \simeq (3/4)^N R_{\rm s}$ and mass $\mu_0 = M_1 \simeq (3/4)^{N+1} M$. Furthermore, it is interesting to note that multiplying $N_{\rm s}$ by the total number of particles M/μ recovers the black hole area quantisation[5]

$$\frac{M}{\mu}\,N_{\rm s} \equiv N_{\rm G} \sim \frac{M^2}{m_{\rm p}^2} \sim \frac{R_{\rm H}^2}{\ell_{\rm p}^2}\ , \tag{3.6}$$

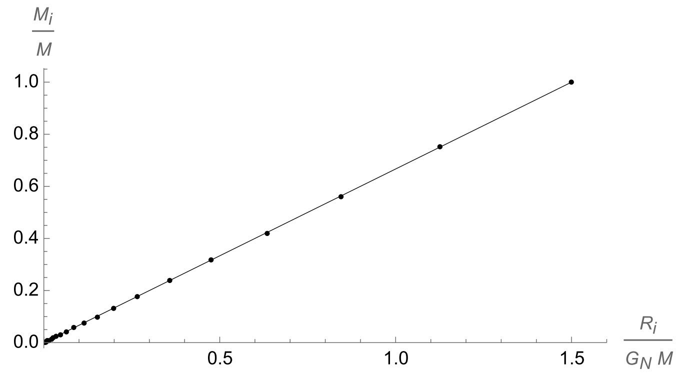

Fig. 1. Mass function M_i (dots) for $N = 100$ layers and its continuous approximation (3.7) (thin solid line). The innermost core has radius $R_1 \simeq 3 \cdot 10^{-13}\,R_{\rm s}$ and mass $M_1 \simeq 2 \cdot 10^{-13}\,M$.

which again agrees with the results for the dust ball described as a single quantum object [10–12] and with the coherent state description of the effective metric [6]. We remark that the numerical prefactor in Eq. (3.6) is not the same of the Bekenstein-Hawking entropy [20], but $N_{\rm G}$ is an integer nonetheless, which suggests that mass and horizon area are quantised.[6] It is certainly intriguing that this scaling of the black hole mass M appears in several different approaches to quantum collapse and black holes (see also, e.g. Refs. [22–24]).

The crucial result for our present purpose is that the discrete mass function M_i grows linearly with the areal radius $R_i = \bar{R}_{N_i}$ in the collective ground state, irrespectively of the number of layers N we employ to describe it. One can therefore introduce a continuous effective energy density

$$\rho \simeq \frac{M}{4\,\pi\,R_{\rm s}\,r^2} \simeq \frac{m_{\rm p}}{6\,\pi\,\ell_{\rm p}\,r^2}\ , \tag{3.7}$$

such that the Misner-Sharp-Hernandez mass function

$$m(r) = 4\,\pi \int_0^r r^2\,\rho(r)\,{\rm d}r = \frac{2\,m_{\rm p}\,r}{3\,\ell_{\rm p}} \tag{3.8}$$

equals the total ADM mass M for $r = R_{\rm s}$ (see Fig. 1).

Since dust particles in the ground state cannot collapse any further, the quantum core is necessarily in equilibrium and one can determine the corresponding effective pressures from the isotropic metric

$$\begin{aligned} {\rm d}s^2 &= -\left(1 - \frac{2\,G_{\rm N}\,m}{r}\right){\rm d}t^2 + \left(1 - \frac{2\,G_{\rm N}\,m}{r}\right)^{-1}{\rm d}r^2 + r^2\,{\rm d}\Omega^2 \\ &\simeq \frac{{\rm d}t^2}{3} - 3\,{\rm d}r^2 + r^2\,{\rm d}\Omega^2\ , \end{aligned} \tag{3.9}$$

for $0 \le r \le R_{\rm s}$. From Eq. (3.9), it is clear that there is no inner horizon inside the ground state core, in agreement with the general results for spherical symmetry presented in Ref. [3]. We should further remark that the effective metric (3.9) cannot be used to describe any meaningful motion inside the core, since matter is fully collapsed and cannot further evolve (except for the Hawking evaporation which we neglect here). The usual analysis of geodesics and geometric invariants therefore remains of purely formal value, as is perhaps the notion of Lorentzian signature inside the quantum core [21]. Nonetheless, the Ricci and Kretschmann scalars are

[4] For our purpose, we do not assume any specific statistics for the dust particles. We expect that Pauli's exclusion principle will affect the analysis for fermions.

[5] For a solar mass ball made of neutrons, the quantum number $N_{\rm s} \simeq 10^{19}$ (corresponding to $N_{\rm G} \simeq 10^{76}$), which makes it practically impossible to study the wave-functions (2.14) for realistic cases.

[6] The connection of Eq. (3.6) with the configurational entropy of the single dust core was investigated in Ref. [11] and the present case is work in progress.

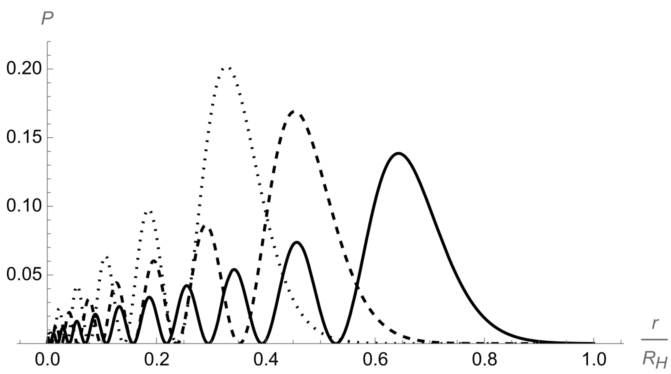

Fig. 2. Probability density (3.13) for $N = 3$, $\mu = m_p/10$, $M_3 = 1100\,\mu$, corresponding to a black hole with $M \simeq 150\,m_p$ and $R_H \simeq 300\,\ell_p$.

given by $\mathcal{R}^2 \simeq \mathcal{R}_{\alpha\beta\gamma\delta}\,\mathcal{R}^{\alpha\beta\gamma\delta} \simeq 64/9\,r^4$, whose square roots are integrable as anticipated in the Introduction.

From the Einstein tensor of the metric (3.9), one readily obtains the effective radial pressure

$$p_r \simeq -\frac{m'}{4\pi r^2} \simeq -\rho \,, \tag{3.10}$$

similar to other constructions for the black hole interior (see [2,25–28] and references therein), and the tangential pressure (or tension)

$$p_\perp \simeq -\frac{m''}{8\pi r} \simeq 0 \,, \tag{3.11}$$

where primes denote derivatives with respect to r. The vanishing of the tension inside each layer suggests that the system should be made to (differentially) rotate easily [12] and is therefore likely unstable under perturbations of the angular momentum.

Note that the outermost layer has an estimated thickness $\overline{\Delta R}_N \simeq R_s/4 \simeq 3\,G_N\,M/8$ and contains $\mu_N/M \simeq 1/4$ of the total mass. A more accurate description near the surface of the core that matches smoothly with the outer Schwarzschild geometry of ADM mass M can then be obtained from the effective energy density

$$\rho \simeq \frac{M}{4}\,|\psi_{N_s}(r)|^2 \,. \tag{3.12}$$

The mass function is therefore not expected to remain linear for $r \gtrsim \bar{R}_N$ and the tension will not vanish near the surface of the core. To clarify this point, we plot the probability densities

$$\mathcal{P}_i = 4\pi r^2\,|\psi_{N_i}(r)|^2 = 4\pi\,\mu_i^{-1}\,\rho_i(r) \tag{3.13}$$

for an example with $N = 3$ layers in Fig. 2, from which it appears that the probability of finding a particle of the $i^{\rm th}$ layer inside both narrower and broader layers with $j \neq i$ is not zero. In particular, the wavefunction of dust particles in the $i^{\rm th}$ layer overlaps with those in all the layers $j < i$. Since this fact was neglected in the derivation of the continuous approximation (3.7) from the discrete mass function M_i, we expect that the actual density decreases somewhat faster from the centre, which could particularly affect the amount of dust in the outermost layer. A more accurate description of the effective energy density of the outermost layer is left for future investigations.

4. Conclusions and outlook

We have improved on the quantum (one-body) description of the ball of dust introduced in Ref. [10] by dividing the ball into N layers, each of which contains dust particles described by quantum states of the same general relativistic dynamics. By requiring that the thickness of each layer be given by the quantum uncertainty in the location of the particles therein, we have obtained a unique collective ground state whereby the radius of the ball is determined by the total ADM mass M, irrespectively of N, and the horizon area is quantised according to Eq. (3.6), in qualitative agreement with Bekenstein's conjecture of the black hole area quantisation [20]. We should remark that these features follow from assuming that the number of dust particles in each layer is large and departures are expected to occur when this condition is violated (that is, for small black holes).

A continuous effective energy density was also estimated to match the discrete mass distribution found for N layers, which turns out to be precisely of the form in Eq. (1.1) almost everywhere inside the core. This latter result lends further support to the picture of quantum black holes described in Ref. [3]. An effective radial pressure of quantum origin exactly opposite the energy density sustains the ground state, whereas the tangential pressure is found to vanish, again almost everywhere inside the core, thus suggesting that the nature of dust is unaffected by quantum gravity in a perfectly spherical configuration.

Of course, the present work is not free from shortcomings and limiting assumptions. As we mentioned in the Introduction, we *a priori* considered static configurations for the dust particles and did not even attempt at analysing the time evolution that could lead to the formation of the collective ground state. In principle, such an evolution should occur as dust particles progressively jump from higher excited states to lower levels [11], a clearly very complex process given the huge number of particles in an astrophysical object. Furthermore, we considered dust particles when one would eventually like to describe matter by means of quantum excitations of standard model fields and all of their interactions. The necessary existence of other interactions will give rise to additional pressure terms and could very significantly influence both the global size of the core and the effective energy density. Even without the inclusion of pressure terms, we argued that the energy density $\rho \sim |\psi_{N_s}|^2$ in the outermost layer implies a different behaviour for the mass function and a non-vanishing tension at the surface of the core. A more accurate description of the core surface will become particularly relevant to understanding what happens when more matter accretes or the Hawking effect evaporates the core. We leave all of these complex issues for future investigations.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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