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The beneficial role of green bonds as a new strategic asset class: Dynamic dependencies, allocation and diversification before and during the pandemic era

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ABSTRACT

The paper proposes a full comprehensive analysis of green bond diversification benefits, their co-movement with multiple market indices, and the corresponding implications for portfolio allocation. Based on a time frame of seven years, divided into four sub-periods, the co-movements of green-bond indices, i.e. Solactive Green Bond Index and Bloomberg Barclays MSCI Green Bond Index, and the stock/bond market have been described, shedding light on the connections with sectors most affected by the Covid-19 pandemic. The Solactive Green Bond Index is found to provide the greater diversification benefit of the two green-bond indices, on average during the seven years and also during the pandemic. Allocation strategies and risk performances have also been analyzed to assess the impact of green-bond indices on otherwise traditional portfolios; their diversification power is discussed by use of traditional measures and an additional behavioral approach, drawing attention to its evolution in time and its consistency in terms of diminished risks and increased returns. Portfolios constructed with the inclusion of green bonds prove preferable in terms of risk, in all periods and for all strategies, while the superiority of returns depends on the allocation strategy.

1. Introduction

The green bond market has enjoyed a rapid expansion over the last decade and recently moved from the previous global record issuance of $ 269.5 billion in 2020 (Jones, 2021), to a new record: $522.7 billion in 2021 (Jones, 2022). The interest in this type of fixed-income security, the proceeds of which are committed to climate and environmental projects, has surged together with environmental, social and governance (ESG) concerns, and with the more widespread awareness of environmental risks following the COVID-19 pandemic. On 21 October 2021, the world's largest green bond issue took place, raising a total of €12 billion. It was the issuance of the NextGenerationEU bonds, the first ever by the European Commission, and was met with an 11-fold demand surplus, as reported in the European Commission Press Release (2021). The supranational entity plans the issue of up to €250 billion in green bonds by the end of 2026. A further development of this market has been the creation of a EU green bond standard (European Commission, 2021), a currently voluntary tool which allows investors and issuers to ensure the green status of the funded projects. Interest in green bonds has come from individual investors, as well as from private financial institutions and funds. The motivations behind it are not limited to a personal sensitivity towards green issues: the clear direction of national and supranational investments and policies signal rapid development and the potentially great impact of this area. The behavior of the green bond market in moments of market growth and downturn is of great importance, because evidence of diversification benefits or safe-haven properties of this asset class could offer a further incentive for its inclusion in more and more portfolios. This would drive up demand (and funding) for green investments and accelerate the transition to a sustainable economy.

The goal of this paper is to investigate the potential diversification benefit provided by green bonds, in order to find for which financial assets, investment strategies and risk-aversion levels it is strongest, based on asset co-movement and portfolio performance. We study the Bloomberg Barclays MSCI Green Bond Index and the Solactive Green Bond Index from October 2014 to June 2021, tracking the change in their dependence with a variety of sectors, with a special focus on the Covid-19 pandemic. We build on the extant literature on the relationship between green bonds and other asset classes by not only considering indices of global equity and of corporate bonds, but also of the key over- and under-performing industries during the pandemic:

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energy commodities, airlines, technology and healthcare. We additionally divide the analysis in different periods, according to the prevalent equity market conditions (bull, bear, pre-pandemic and pandemic), in order to highlight any fundamental changes in the dependence structure across time. We furthermore go beyond the theoretical analysis of correlations and co-movements and create portfolios based on a variety of allocation strategies, tracking their performance during the different sub-periods. We consider portfolios which include green bonds and others which exclude them, in order to highlight their differences along a number of risk measures. Among them, we insert a diversification index and a spectral risk measure, the latter of which extends the analysis to an area yet to be explored by the existing literature: the behavioral dimensions of risk-aversion and utility. This enables us to identify the allocation strategies and the investor risk preferences which lead to a larger request for investment in green bonds. It also highlights which of the two green-bond indices offers the greater diversification benefit.

The work is organized as follows: Section 2 introduces the previous literature and Section 3 describes the sample and performs preliminary analyses on the asset returns. Section 4 fits DCC-GARCH models and dynamic copulas to each pair of assets, for each sub-period, in order to highlight the values and the changes in pairwise dynamic conditional correlation between them. Pairwise empirical tail dependence is also computed and the sectors for which green bonds provide the greater diversification benefit are highlighted. The benefit offered by the two green indices, in terms of magnitude of the correlations, is also compared. Section 5 tackles portfolio construction with and without green bonds, carrying out in-sample and out-of-sample performance analysis, based on a number of indicators. The profile of the investors who would benefit from green bonds can then be deduced, in terms of their preferred asset allocation strategy and risk-aversion level. Finally, Section 6 concludes the paper with a summary of the findings.

2. Literature review

Over the last few years, a number of studies has emerged, focusing on the financial properties of green bonds. The positive effect of this asset class on the issuer's stock price as well as on its environmental and financial performances has been highlighted in Flammer (2021) and in Tang and Zhang (2020). The existing literature on the subject of green bond and market co-movement includes Reboredo (2018) and Reboredo and Ugolini (2020), that find a strong link with the corporate and treasury bond markets and with currencies, and weak links with a number of traditional commodities, including energy prices. Elsayed et al. (2022) perform a time–frequency analysis before and until the early stages of the COVID-19 pandemic (June 2020). They uncover evidence suggesting that the diversification benefits provided by the Bloomberg Barclays MSCI Green Bond Index with respect to the financial market considered in Reboredo (2018), are stronger for shorter-term horizons, while they are weaker for longer ones. In our analysis of co-dependencies, we expand on this in a number of ways: we include the Solactive Green Bond Index, which proves to have a greater diversification benefit than the Bloomberg Barclays MSCI Green Bond Index, and the indices of the sectors most affected by the Covid-19 pandemic, in order to consider a pandemic-specific range of benchmarks. We then extend the analysis by focusing on pairs of indices, composed of one green-bond and one non-green index, and analyzing them not only over the entire window of observation but also within different sub-periods, with pandemic data reaching June 2021. In Naeem et al. (2021), the green bond market has been found to be more efficient than that of conventional corporate bonds during the Covid-19 pandemic, while less efficient in the full sample period. This shows the diversification potential of green bonds in times of extreme market turmoil. Arif et al. (2022) focus on the hedging and safe haven properties of green bonds during the pandemic, by measuring their association with a number of conventional investments, and finding them to be good hedging and safe haven instruments for currency and commodity investments. A less analyzed area is that of the potential benefits derived from the actual, practical inclusion of green bonds in a portfolio. Initial evidence has been presented in Han and Li (2020), where performance is shown to be enhanced by the inclusion of green bonds in an otherwise traditional portfolio of Chinese assets. Our work has the goal of testing a hypothesis affine to the finding of Han and P. Li (2020), but extended to the global market. Additionally, it seeks to identify which financial assets, investment strategies and risk-aversion levels enjoy the strongest diversification benefit from green bonds, based on asset co-movement and portfolio performance, and distinguishing between sub-periods, before and during the Pandemic era.

3. Preliminary analysis

3.1. Sample description

The analysis in this paper has been carried out on the time series of eight market indices.

The global equity market is represented by the MSCI World Index (denoted by MSCI), which includes large and mid-cap stocks across twenty-three developed-market countries, while the traditional bond market is represented by the Bloomberg Barclays Global Aggregate Total Return Index (BBBOND), which is a measure of global investment grade debt including treasury, government-related, corporate and securitized fixed-rate bonds.

The global green bond market is represented by two indices: the Bloomberg Barclays MSCI Green Bond Index (BBGB) and the Solactive Green Bond Index (SOLGB). Both are in line with the Climate Bond Taxonomy, a guide to climate-aligned assets and projects developed by the Climate Bonds Initiative, and have been introduced in 2014 (The GBP Databases and Indices Working Group, 2017).

Both have been used as benchmarks for the green-bond market in past literature: BBGB in Elsayed et al. (2022), Reboredo (2018), and Reboredo and Ugolini (2020), whereas SOLGB was used in Arif et al. (2022) and Naeem et al. (2021). We consider both indices simultaneously in order to enrich the characterization of the green-bond market, by investigating whether they provide different potential diversification benefits to investors.

The two indices differ in the following ways: BBGB is calculated by Bloomberg Index Services Limited, the same entity behind BBBOND, and includes its same local currency debt markets — leading to an expected high correlation with that index. This overlap in markets is not present for SOLGB, which is furthermore calculated by a different provider: Solactive AG. Additionally, BBGB includes corporate, treasury, and securitized bonds, but its largest portion is made up of government-related bonds (Bloomberg Barclays Indices, 2021). SOLGB is instead mostly composed of corporate bonds. Moreover, BBGB only considers investment-grade bonds, whereas SOLGB also includes non-investment grade bonds (Solactive, 2021). While there is no minimum time to maturity for the bonds in BBGB, SOLGB imposes one of six months (Solactive, 2022). Both indices are market-value weighted, but there is a cap of 5% to the weight of each bond in SOLGB (which is lifted in case of technical unfeasibility). Furthermore, the two indices impose different minimum outstanding issue size requirements to bonds: 300 million for the US dollar market for BBGB (but differing across currency markets) and $100 million for SOLGB. Finally, BBGB imposes additional sustainability requirements, beyond the Climate Bond Taxonomy. Bonds are included only if they satisfy the criteria of MSCI ESG Research: the use of proceeds must fall within six eligible environmental categories,[1] which are broadly aligned with the Green

[1] The MSCI ESG Research environmental categories include alternative energy, energy efficiency, pollution prevention, sustainable water, green building, climate adaptation.

Table 1
Descriptive statistics of returns for the full period (October 2014–June 2021).

	BBGB	SOLGB	BBBOND	MSCI	SPGSEN
Mean	0.00008	0.00011	0.00009	0.00037	−0.00009
Minimum	−0.01938	−0.01823	−0.02200	−0.10442	−0.30173
Maximum	0.00828	0.01340	0.01526	0.08406	0.17376
Std. Deviation	0.00185	0.00256	0.00294	0.00962	0.02465
Skewness	−1.14164	−0.55897	−0.53765	−1.49760	−1.24810
Kurtosis	9.71114	4.80359	5.26860	23.83639	23.15864
J-B Test P-Value	0.00000	0.00000	0.00000	0.00000	0.00000

Bond Principles (GBP).[2] The bonds must also be aligned with the four dimensions defined by MSCI ESG Research[3] (Bloomberg Barclays Indices, 2021).

The energy commodity market is represented by the S&P GSCI Energy Index (SPGSEN), a sub-index of the S&P GSCI, which includes futures traded on commodity exchanges and which reflects the dynamics of energy prices. Finally, three additional market indices have been considered in the analysis, going beyond Reboredo (2018) and Elsayed et al. (2022). They have been selected because of their behavior during the pandemic, as they represent sectors which performed very negatively or positively during that time. These indices are:

- S&P 500 Airlines (SP5IAIR) for the airline sector;
- S&P 500 Health Care (SP5EHCR) for the healthcare sector;
- S&P 500 Euro Information Technology (SPEUIT) for the technology sector.

The overall sample is composed of closing daily prices from 15/10/2014 to 07/06/2021, for a total of 1727 observations per asset.

In order to study the features of the green-bond indices and their relationship with other financial sectors under different market conditions, the full sample period has been divided into four sub-periods according to the performance of the MSCI Wold Index.

The sub-periods are the following:

- the "bear market" sub-period, from June 2015 to April 2016, a time of stock market downturn;
- the "bull market" sub-period, from December 2016 to January 2018, during which the stock market exhibited strong growth;
- the "pre-pandemic" sub-period, from January 2019 to 3rd March 2020;
- the "pandemic" sub-period, from the 4th of March 2020 to the 7th of June 2021.

3.2. Return analysis

3.2.1. Full sample 2014–2021

The analysis of market indices starts with their log-returns, as represented in the following formula:

$$X_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

where P_t is the price in t.

Table 1 contains the main descriptive statistics of the five indices of interest for the full period analysis, which are the two green-bond indices, the bond market index, the stock market index and the energy commodity index.

[2] The GBP is a 2014 agreement, published by a consortium of banks, on a set of standards designed to allow investors to assess the green credentials of green-labeled bonds.

[3] The MSCI ESG Research dimensions are: criteria for the use of proceeds, process for project evaluation and selection, management of proceeds, and reporting on the actual use of funds.

Table 2
Descriptive statistics of returns during the four sub-periods.

	BBGB	SOLGB	BBBOND	MSCI	SPGSEN
Bear period					
Mean	0.00014	0.00008	0.00025	−0.00043	−0.00171
Minimum	−0.00721	−0.01823	−0.01109	−0.03794	−0.07726
Maximum	0.00443	0.00965	0.01526	0.02565	0.08853
Std. Dev.	0.00185	0.00345	0.00354	0.00937	0.02597
Bull period					
Mean	0.00004	−0.00013	0.00026	0.00085	0.00084
Minimum	−0.00464	−0.00754	−0.01650	−0.01246	−0.04344
Maximum	0.00353	0.00682	0.01053	0.01594	0.03913
Std. Dev.	0.00150	0.00226	0.00307	0.00373	0.01326
Pre-pandemic period					
Mean	0.00031	0.00037	0.00031	0.00049	−0.00015
Minimum	−0.00662	−0.00751	−0.00604	−0.03669	−0.06817
Maximum	0.00605	0.00727	0.00689	0.03274	0.12142
Std. Dev.	0.00194	0.00212	0.00214	0.00738	0.01823
Pandemic period					
Mean	−0.00004	−0.00009	0.00011	0.00098	0.00107
Minimum	−0.01938	−0.01617	−0.02200	−0.10442	−0.30173
Maximum	0.00828	0.00884	0.01490	0.08406	0.17376
Std. Dev.	0.00250	0.00260	0.00324	0.01626	0.03928

Daily average returns are very small for all considered time series. The standard deviations obtained indicate that the green-bond indices are the least volatile: they are equal to 0.185% for the Bloomberg Barclays MSCI Green Bond Index (BBGB) and 0.256% for the Solactive Green Bond Index (SOLGB). The energy commodity market index (SPGSEN) shows the greatest volatility among the five indices, as it is equal to 2.465%, followed by the stock market index (MSCI) which shows a standard deviation equal to 0.962%. Skewness and kurtosis coefficients suggest left-skewed and leptokurtic distributions for all time series: the skewness value is always negative and the kurtosis is greater than 3. The infinitesimal p-values of the Jarque–Bera test indicate that the returns are not normally distributed.

3.2.2. Sub-periods

Table 2 shows basic descriptive statistics of the five indices during the four sub-periods. The conclusions drawn from the observation of full-period values are confirmed by the values of sub-period statistics: the average returns of all indices remain around 0 and green-bond indices show the smallest standard deviations. In particular, the two indices show low volatility also during stock market stress periods, such as the bear and the pandemic sub-periods.

Finally, Table 3 shows basic descriptive statistics of the three indices which are of interest during the pre-pandemic and the pandemic sub-periods: the S&P500 Airlines (SP5IAIR), S&P500 Health Care (SP5EHCR) and the S&P500 Euro Information Technology (SPEUIT). These sectors are chosen because the first suffered losses and instability leading up to and during the pandemic sub-period, due to travel bans and to a general decrease in the number of flights, while the other two experienced extraordinary growth with respect to the rest of the market.

4. Analysis of market relationships

The objective of this analysis is to explore the relationships between the green-bond market, on the one hand, and the stock market, the corporate bond market, and the indices of some key sectors which stood out during the pandemic, on the other. This is done in order to identify the sectors for which green bonds provide the greater diversification benefit, and during which sub-periods. The benefit offered by the two green indices, in terms of magnitude of the correlations, is also compared. The final aim is to evaluate the implications in terms of portfolio management.

Table 3
Descriptive statistics of sector indices during the Pre-pandemic and Pandemic sub-periods.

	SP5IAIR	SP5EHCR	SPEUIT
Pre-pandemic period			
Mean	−0.00066	0.00035	0.00086
Minimum	−0.07442	−0.03388	−0.04360
Maximum	0.04548	0.04713	0.05071
Std. Dev.	0.01472	0.00932	0.01283
Pandemic period			
Mean	0.00043	0.00079	0.00137
Minimum	−0.22438	−0.10528	−0.12358
Maximum	0.17756	0.07314	0.09847
Std. Dev.	0.04310	0.01683	0.02094

Examining the potential co-movement of green bonds and financial markets is important, both in order to understand the impact of this link, as well as to highlight the potential diversification benefits derived from the inclusion of green bonds in a portfolio.

We expand on the analysis of Reboredo (2018) and Elsayed et al. (2022) by including the data of the pandemic (about one additional year with respect to Elsayed et al. (2022)) and the sector indices that had outstanding negative and positive performances in that time. Additionally, we divide the timeline in different sub-periods, depending on overall market conditions, in order to study the corresponding changes in relationships. Furthermore, we expand on those papers by including a focus on portfolio allocation in the next section, with the aim of identifying the risk profile and the strategy of the investors that could benefit more from the inclusion of green bonds in their portfolios. The division of the time window enables the identification of the indices for which green bonds act as diversifiers or as safe heaven assets — thus deepening the work of Arif et al. (2022) and that of Naeem et al. (2021). The former concentrates on this topic but analyzes the more traditional investment sectors of equity, fixed income, commodity, and currency investments, while the latter only considers the traditional corporate bond market.

In order to characterize the market dynamics between green bonds and financial indices, the DCC (Dynamic Conditional Correlation) GARCH model is used. This setting allows for the modeling of relationships between two or more assets in terms of dynamic correlation and thus yields a time-varying measure of dependence with an intuitive interpretation.

4.1. The model

The relationship between the time series of one green-bond index, on the one hand, and the time series of each of the other considered indices, on the other, is modeled through a bi-variate dynamic copula (Meucci, 2011). The dynamic copula is conditionally elliptical and dependent on a time-varying correlation matrix $\boldsymbol{R}_{t+1}$, which is completely known at time t, and that evolves following the DCC model introduced in Engle (2002).

The starting point of the analysis are the log-returns of each index, defined as:

$$\Delta ln(P_t) = \Delta X_t = \mu + \sqrt{h_t}\epsilon_t;$$

flexible probabilities with exponential decay are then set, with a prior half-life of 120 days. The flexible probabilities approach attributes a time-dependent relative weight to each scenario in the empirical distribution function. Exponential decay probabilities, which are represented as:

$$p_t \mid \tau_{HL} \equiv pe^{-\frac{\ln(2)}{\tau_{HL}}|t^*-t|},$$

weigh the scenarios as $p \equiv 1/\sum_s e^{-\frac{\ln(2)}{\tau_{HL}}|t^*-s|}$, where $\tau_{HL} > 0$ is called half-life and determines the time after which the decaying probabilities become one half of a target time t^*, which in this setting is the most recent observation.

Autoregressive conditional heteroskedasticity in the log-returns is detected through Engle (1982)'s ARCH-LM test. A GARCH(1,1) model is then fit to each time series, the adequacy of the number of lags being confirmed through the Li and Mak (1994) test, and the innovations ϵ_t are recovered. The null hypothesis of the Li and Mak (1994) test is that there are no remaining ARCH effects in the data, which the GARCH(1,1) model was not able to capture. The hypothesis is not rejected at the 99% significance level, for all indices and all sub-periods, therefore the GARCH(1,1) model is considered to be adequate.

In order to assess the best-fitting copula based on the empirical tail dependence, the scatter-plots displaying the innovations of each green-bond index against the innovations of each one of the other indices are analyzed. Pairs displaying little to no empirical tail dependence are modeled with Gaussian copulas, while pairs displaying a higher level of tail dependence are modeled with Student t copulas. The corresponding marginal distributions are used.

The marginal distributions of the innovations of each index i are estimated by fitting the selected Gaussian or Student t distributions via weighted maximum likelihood (using the previously obtained flexible probabilities), with known degrees of freedom ν. The appropriate ν is set to be the same as the degrees of freedom of the copula. The estimated location parameters of the marginal distributions are close to 0 in size, taking slightly positive or negative values. The scale parameters are all larger than 0.5 and only rarely exceed 1.

Each ith marginal time series of innovations is then mapped into standard t or Gaussian realizations, by first computing the estimated cumulative distribution function at each point and then finding the corresponding standard t (with ν degrees of freedom) or Gaussian quantile. This yields the standardized $\{\xi_{i,t}\}_{t=0}^{t=T}$, i.e.

$$\xi_{i,t} \equiv \Phi_\nu^{-1}\left(F_{\epsilon_i}\left(\epsilon_{i,t}\right)\right), \tag{1}$$

where Φ_ν^{-1} is the quantile function of a standard Student t distribution with ν degrees of freedom, F_{ϵ_i} is the marginal cumulative distribution function of the ith time series of innovations ϵ_i, and $\epsilon_{i,t}$ is the value of ϵ_i at time t. The unconditional variance–covariance matrix is estimated from the standardized $\{\xi_{i,t}\}_{t=0}^{t=T}$ via weighted maximum likelihood and the unconditional correlation matrix is obtained from it. The estimate is improved via factor analysis shrinkage, and the DCC model is fit to the data.

The degrees of freedom of the bi-variate copulas are calibrated through a number of steps. First, an analysis of empirical tail dependence is carried out, and pairs of indices displaying zero dependence in at least one tail (and a negligible one in the other) are modeled with Gaussian copulas. Afterwards, the entire model is re-run with a variety of possible degrees of freedom, including the Gaussian case, for all remaining index couples. The optimal ν is selected as the one maximizing the number of innovation pairs falling within the corresponding bi-variate expectation-unconditional covariance ellipsoid, which is defined in detail in Section 4.2.3. The different values of ν impact the estimated unconditional variance–covariance matrix of standardized $\{\xi_{i,t}\}_{t=0}^{t=T}$ on which the ellipsoid is based. They do so by altering the marginal distributions of the innovations (as the location and scale parameters are estimated by taking the degrees of freedom as given) and the quantile functions which are used to obtain the standardized $\{\xi_{i,t}\}_{t=0}^{t=T}$.

The estimated parameters of the marginal distribution of innovations are reported in Appendix A, while a more detailed explanation of the calibration steps can be found in Appendix B.

4.1.1. DCC GARCH model

The DCC GARCH model, introduced by Engle (2002), allows for the quantification of time-varying correlations. There are two steps in the estimation procedure for dynamic conditional correlation:

1. Estimation of a univariate GARCH model for conditional volatility for each of the n time series under consideration;
2. Estimation of the DCC model for conditional correlation, using the standardized residuals resulting from the first step.

The general model equation is:

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t, \tag{2}$$

where $\boldsymbol{H}_t$ is the conditional covariance matrix, $\boldsymbol{D}_t$ is a diagonal matrix with conditional variances $h_{i,t}$ on the diagonal and $\boldsymbol{R}_t$ is the time-varying correlation matrix. The conditional variance $h_{i,t}$ is estimated with a univariate GARCH(Q_i, P_i) model as:

$$h_{i,t} = \omega_i + \sum_{p=1}^{P_i} \alpha_{i,p}(\Delta x_{i,t-p} - \mu_i)^2 + \sum_{q=1}^{Q_i} \beta_{i,q} h_{i,t-q}, \tag{3}$$

where $i = 1, 2, \dots, n$ are the market indices, $\Delta x_{i,t-p}$ is the value of the log return of index i at time $t-p$, μ_i is the mean return of index i, $h_{i,t-q}$ is the value of the conditional variance of index i at time $t-q$, $\omega_i, \alpha_{i,p}$ and $\beta_{i,q}$ are non-negative, and $\sum_{p=1}^{P_i} \alpha_{i,p} + \sum_{q=1}^{Q_i} \beta_{i,q} < 1 \ \forall i$.

The residuals and the conditional variances $h_{i,t}$ are thus obtained and then used in order to recover the standardized residuals $\boldsymbol{u}_t$, with individual elements:

$$u_{i,t} = \frac{\Delta x_{i,t} - \mu_i}{\sqrt{h_{i,t}}},$$

where $i = 1, 2, \dots, n$. These are then used for the estimation of the dynamic conditional correlation matrix $\boldsymbol{R}_t$, given by:

$$\boldsymbol{R}_t = diag\left\{\boldsymbol{Q}_t\right\}^{-1/2} \boldsymbol{Q}_t diag\left\{\boldsymbol{Q}_t\right\}^{-1/2}, \tag{4}$$

where $\boldsymbol{Q}_t$ is a positive-definite quasi correlation matrix. The structure of the dynamic correlation depends from the following equation:

$$\boldsymbol{Q}_t = (1 - a - b)\bar{\boldsymbol{Q}} + a\boldsymbol{u}_{t-1}\boldsymbol{u}'_{t-1} + b\boldsymbol{Q}_{t-1}, \tag{5}$$

where $\bar{\boldsymbol{Q}}$ is the unconditional correlation matrix of the standardized residuals, $\boldsymbol{Q}_{t-1}$ is the one-period lagged value of the quasi correlation matrix, and $\boldsymbol{u}_{t-1}$ is the one-period lagged value of the standardized residuals. Coefficients a and b measure, respectively, the short- and long-term persistence of dynamic conditional correlation.

Element $\rho_{ij,t}$ of the conditional correlation matrix $\boldsymbol{R}_t$ can be expressed with the following typical representation for correlations: $\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ij,t} q_{ij,t}}}$. In our setting, the standardized residuals are set to be $\{\boldsymbol{u}_t\}_{t=0}^{t=T} = \{\xi_t\}_{t=0}^{t=T}$, as defined in Eq. (1).

4.2. Results

For all indices and across all periods, the estimated GARCH β coefficients are larger than the estimated GARCH α coefficients: all returns are thus more sensitive to past volatility than to their own past behavior, regardless of the prevailing market conditions. As for the DCC coefficients, across all periods and for all pairs of indices, a coefficients are substantially more moderate in size than b coefficients, indicating that the time-varying correlation has low sensitivity to previous standardized shocks, but has a high degree of persistence. Differences among same-period coefficients are very small. The estimates of GARCH and DCC parameters are reported in Appendix C, for all sub-periods.

The following paragraphs present an analysis of the dynamic conditional correlation (DCC) values between pairs of indices composed of a green bond index, on the one hand, and each remaining index, on the other. This is done to shed light on the directional dependence between the pairs of time series, which is taken as a proxy of the diversification potential of each green bond index. The goal is to identify the sectors and the sub-periods for which green bonds provide the greater diversification benefit. The benefit offered by the two green indices, in terms of magnitude of the correlations, is also compared.

Table 4
DCC summary of BBGB pairs.

All periods		Average	Minimum	Maximum	Std deviation
	SOLGB	0.752473	0.380372	0.928359	0.127813
	BBBOND	0.578999	0.406788	0.731149	0.072957
	MSCI	−0.078677	−0.347012	0.165301	0.101870
	SPGSEN	−0.136682	−0.363508	0.052892	0.070578
	SP5IAIR	−0.155266	−0.357271	0.070916	0.080379
	SP5EHCR	−0.044269	−0.234361	0.154384	0.069731
	SPEUIT	−0.009498	−0.220451	0.172524	0.072298
Bear					
	SOLGB	0.532480	0.354279	0.738887	0.112694
	BBBOND	0.414948	0.064887	0.638074	0.137683
	MSCI	−0.381140	−0.575365	0.042416	0.121471
	SPGSEN	−0.274066	−0.373865	−0.060968	0.056496
	SP5IAIR	−0.199939	−0.360271	−0.011433	0.068980
	SP5EHCR	−0.233404	−0.336478	0.020414	0.069384
	SPEUIT	−0.375234	−0.615490	−0.124222	0.129710
Bull					
	SOLGB	0.530387	0.228464	0.692805	0.132767
	BBBOND	0.617802	0.414612	0.734160	0.055230
	MSCI	−0.088421	−0.315148	0.159594	0.108690
	SPGSEN	0.005143	−0.249888	0.184793	0.101964
	SP5IAIR	−0.190637	−0.291634	−0.091162	0.046846
	SP5EHCR	−0.090297	−0.295530	0.228860	0.117507
	SPEUIT	0.069187	−0.140331	0.269416	0.107077
Pre-pandemic					
	SOLGB	0.901530	0.874081	0.926579	0.012567
	BBBOND	0.753216	0.717051	0.786991	0.018722
	MSCI	−0.314823	−0.406214	−0.129949	0.065294
	SPGSEN	−0.175859	−0.317256	0.024311	0.050090
	SP5IAIR	−0.336718	−0.442960	−0.188054	0.064937
	SP5EHCR	−0.220933	−0.321952	−0.107759	0.046900
	SPEUIT	−0.185771	−0.306168	−0.030873	0.068398
Pandemic					
	SOLGB	0.879867	0.841127	0.904335	0.013973
	BBBOND	0.555698	0.459872	0.671627	0.050131
	MSCI	0.045741	−0.058213	0.194189	0.070451
	SPGSEN	−0.108563	−0.196312	0.053603	0.060421
	SP5IAIR	−0.104631	−0.313799	0.183844	0.123704
	SP5EHCR	0.052432	−0.068429	0.180663	0.066763
	SPEUIT	0.060733	−0.035040	0.286485	0.073431

4.2.1. Dynamic conditional correlations: BBGB vs all other indices

Table 4 reports the standard deviations and the average, minimum, and maximum values of the estimated dynamic conditional correlations (DCC) between pairs of indices composed of BBGB, on one hand, and each of the other indices, on the other. The values are presented for the entire time period and for each sub-period.

The DCC between the two green-bond indices always takes positive values, but it reaches a minimum during the bull sub-period. During this time, SOLGB shows a faster growth than BBGB, but is also affected by a large number of drastic changes in value, in contrast to the greater stability of BBGB. During the pandemic, the two indices have a very similar behavior, with their DCC maintaining one of the highest average values of all sub-periods.

The values of the DCC between BBGB and BBBOND are also always positive. This is not surprising, as the underlying debt markets considered by the two indices has a significant overlap. This particular property allows us to focus on the effect of the "green" component in relation to the traditional corporate bond market. From Table 4, it can be seen that the DCC reaches a minimum during the bear sub-period, when its standard deviation is the highest of all considered sub-periods. During this time, the corporate bond index displays substantial volatility, compared to the much lower one of BBGB. The correlation increases substantially during the bull sub-period, when both indices grow with the market. It decreases substantially during the pandemic, when the behavior of the two indices is less strongly linked. Overall, the diversification benefit offered by BBGB for investors in traditional corporate bonds is very limited.

As for the relationship between BBGB and MSCI, their DCC is lowest during the bear sub-period, when it is negative. It remains negative but decreases in absolute terms during the bull sub-period, when BBGB remains stable compared to the consistent growth of MSCI. The correlation then moves towards zero during the pandemic, when it takes the only positive average value. This evidence indicates a potential diversification benefit of BBGB for investors in MSCI, which decreases during the pandemic.

Concerning the energy sector, the DCC between BBGB and the relevant index, SPGSEN, is lowest during the bear sub-period, when it is negative. It then moves towards zero during the bull sub-period, when it takes the only positive average value. It returns to the negative domain and remains substantially negative during the pandemic. This suggests a substantial diversification benefit of BBGB for investors in the energy sector represented by SPGSEN.

The airline industry is represented by the SP5IAIR index. Its DCC with BBGB barely reaches any positive value, as it is on average always negative. It is lowest during the pre-pandemic sub-period and highest during the pandemic, but it still remains negative, indicating a substantial diversification benefit of BBGB with respect to the airline industry throughout the entire period, including the pandemic.

As for the healthcare sector, the DCC between SP5HCR and BBGB is lowest during the pre-pandemic sub-period, as in the case of SP5IAIR. It increases and becomes slightly positive, on average, during the pandemic. This evidence points to a potential diversification benefit offered by BBGB to investors in the healthcare industry, but it also highlights an increase in dependence in times of growth for SP5HCR.

From Table 4, it can be seen that the DCC between BBGB and SPEUIT is on average lowest during the bear sub-period, as in the cases of BBBOND, MSCI and SPGSEN. It increases during the pandemic, when it reaches a slightly positive value, comparable to that of the bull sub-period. This indicates a potential diversification benefit offered by BBGB to investors in the Information Technology (IT) sector, but it also points to an increase in dependence in times of growth for SPEUIT.

The plots of the dynamic conditional correlations in Table 4 can be seen in Appendix D.

4.2.2. *Dynamic conditional correlations: SOLGB vs all other indices*

Table 5 reports the standard deviations and the average, minimum, and maximum values of the estimated dynamic conditional correlations (DCC) between pairs of indices composed of SOLGB, on one side, and each of the other indices, on the other. The values are presented for the entire time period and for each sub-period.

In contrast with the behavior of BBGB, the DCC between SOLGB and BBBOND does not always take positive values. However, as for BBGB, it reaches a minimum during the bear sub-period, as displayed in Table 5. It is highest before the pandemic and decreases during the pandemic, but remains strongly positive. This suggests a very limited diversification benefit offered by SOLGB to investors in traditional corporate bonds.

As for the stock market index MSCI, its DCC with SOLGB is always negative, on average, as shown in Table 5. It reaches its maximum, closely approaching zero, during the bear sub-period. During the pandemic it takes one of its highest average values, but in contrast to BBGB it remains negative. This evidence indicates that the diversification benefit offered by SOLGB is potentially higher than that offered by BBGB.

The DCC between SOLGB and SPGSEN is predominantly in the negative domain. It is negative during the bear sub-period and moves towards zero during the bull sub-period, when it takes its highest (but still negative) value. It is lowest before the pandemic, and remains substantially negative during the pandemic. This behavior at times mirrors that of BBGB and, similarly, indicates a substantial diversification benefit on the part of SOLGB for investors in the energy sector represented by SPGSEN.

Table 5
DCC summary of SOLGB pairs.

All periods		Average	Minimum	Maximum	Std deviation
	BBBOND	0.054893	−0.293653	0.498024	0.208038
	MSCI	−0.135248	−0.400437	0.073147	0.078112
	SPGSEN	−0.111442	−0.283788	0.076749	0.068029
	SP5IAIR	−0.073601	−0.284706	0.137399	0.088203
	SP5EHCR	−0.032574	−0.234578	0.221627	0.079263
	SPEUIT	−0.113902	−0.363757	0.144061	0.071850
Bear					
	BBBOND	−0.305091	−0.465057	−0.185101	0.065114
	MSCI	−0.033187	−0.281348	0.186121	0.113504
	SPGSEN	−0.076103	−0.200063	0.088367	0.068213
	SP5IAIR	0.065397	−0.073305	0.227355	0.064815
	SP5EHCR	0.093114	−0.054810	0.279569	0.068606
	SPEUIT	−0.195288	−0.540565	0.055795	0.134199
Bull					
	BBBOND	−0.102658	−0.386768	0.089695	0.105434
	MSCI	−0.221323	−0.445757	0.066699	0.136182
	SPGSEN	−0.014148	−0.187108	0.212821	0.101541
	SP5IAIR	−0.027320	−0.305607	0.151676	0.106862
	SP5EHCR	−0.019550	−0.197531	0.216876	0.091843
	SPEUIT	−0.213783	−0.502987	0.095523	0.155727
Pre-pandemic					
	BBBOND	0.478017	0.417367	0.567178	0.037237
	MSCI	−0.260982	−0.338715	−0.141952	0.048638
	SPGSEN	−0.130023	−0.259831	−0.012087	0.045550
	SP5IAIR	−0.230601	−0.321426	−0.124185	0.050250
	SP5EHCR	−0.157956	−0.266187	−0.076765	0.041039
	SPEUIT	−0.155969	−0.256142	−0.021365	0.059628
Pandemic					
	BBBOND	0.225995	0.069748	0.399815	0.080338
	MSCI	−0.080376	−0.137088	0.028467	0.042150
	SPGSEN	−0.092203	−0.205871	0.101736	0.071854
	SP5IAIR	−0.096729	−0.260967	0.083136	0.080179
	SP5EHCR	−0.025528	−0.125109	0.107510	0.063964
	SPEUIT	−0.054864	−0.123986	0.139274	0.059118

Similarly to the relationship with BBGB, the DCC between SP5IAIR and SOLGB is predominantly in the negative domain. It is lowest, on average, before the pandemic. It increases during the pandemic period, but it remains substantially negative, suggesting a substantial diversification benefit as in the case of BBGB.

As for the healthcare industry, represented by the index SP5EHCR, its DCC with SOLGB reaches its lowest average value before the pandemic. It increases and narrowly approaches zero, only remaining slightly negative, during the pandemic. This evidence indicates a potential diversification benefit offered by SOLGB to investors in SP5EHCR, and, in contrast to BBGB, the benefit persists during the pandemic.

The IT sector is another one for which the diversification potential of SOLGB during the pandemic differs from that of BBGB. The DCC between SOLGB and SPEUIT is predominantly in the negative domain and is strongly negative before the pandemic. It then moves towards zero, but remains noticeably negative, during the pandemic. The evidence suggests a substantial diversification benefit offered by SOLGB to investors in SPEUIT, and, in contrast to BBGB, the benefit persists during the pandemic.

The plots of the dynamic conditional correlations in Table 5 can be seen in Appendix D.

4.2.3. *Expectation-unconditional covariance ellipsoids*

The multivariate expectation–covariance ellipsoid of a random vector $\boldsymbol{X}$ is the set of all points such that

$$\partial\mathcal{E}(\mathbb{E}\{\boldsymbol{X}\},\mathbb{C}v\{\boldsymbol{X}\}) =$$
$$= \Big\{ \boldsymbol{x} \in \mathbb{R}^{\bar{n}} : \|\boldsymbol{z}_{\boldsymbol{X}}(\boldsymbol{x})\|^2 \equiv$$
$$\equiv (\boldsymbol{x} - \mathbb{E}\{\boldsymbol{X}\})'(\mathbb{C}v\{\boldsymbol{X}\})^{-1}(\boldsymbol{x} - \mathbb{E}\{\boldsymbol{X}\}) = 1 \Big\},$$

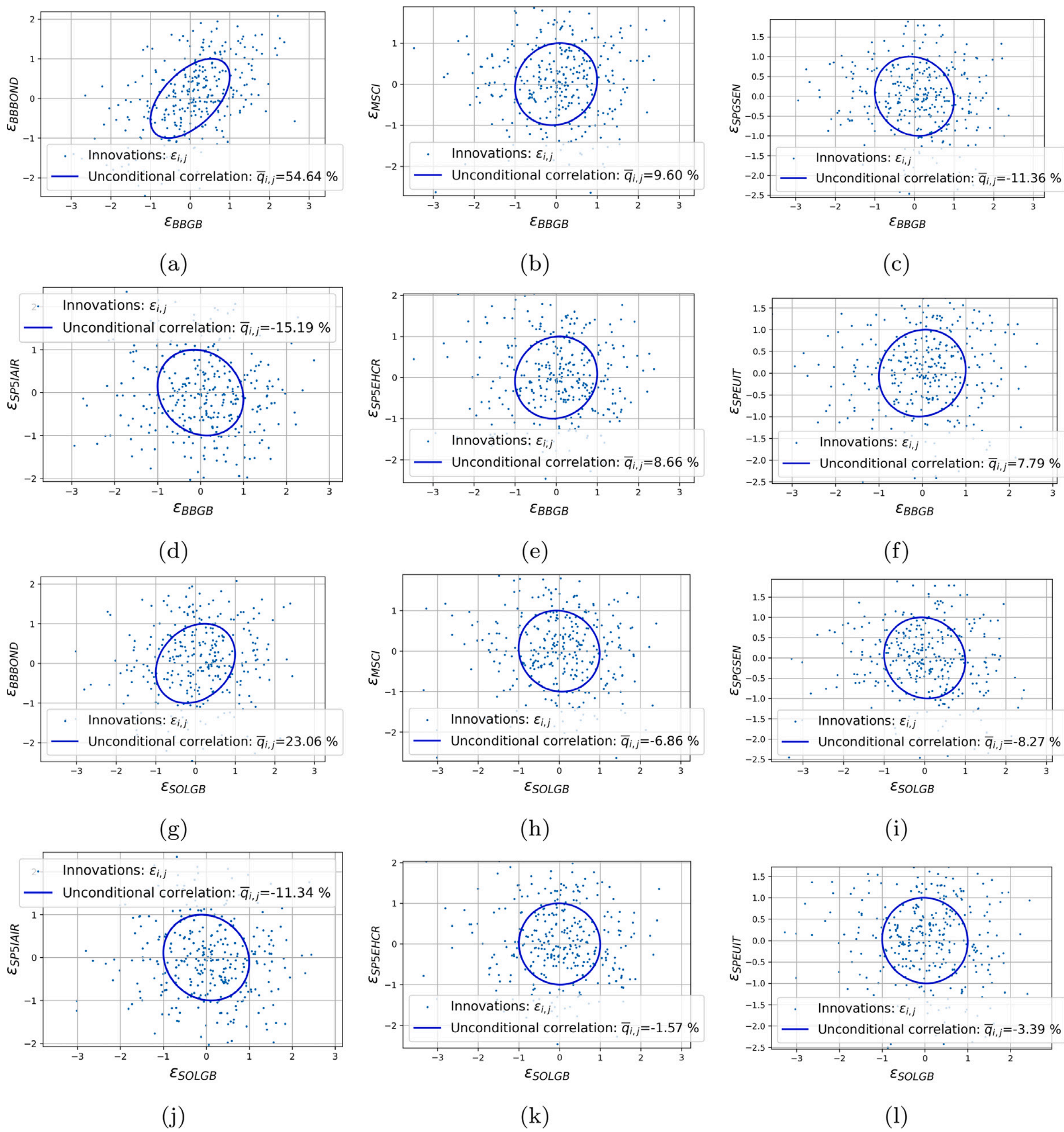

Fig. 1. Pandemic sub-period ellipsoids.

where $\mathbb{E}\{\boldsymbol{X}\}$ is the expected value of $\boldsymbol{X}$, $\mathbb{C}v\{\boldsymbol{X}\}$ is the variance–covariance matrix of $\boldsymbol{X}$, n is the size of the random vector, and $\|\boldsymbol{z}_{\boldsymbol{X}}(\boldsymbol{x})\|^2$ is the square z-score of vector $\boldsymbol{X}$.

The plots of the ellipsoids for each pair of indices (i,j) are shown below for the pandemic sub-period. They display the $\{\epsilon_{i,t}\}_{t=0}^{t=T}$ and the $\{\epsilon_{j,t}\}_{t=0}^{t=T}$ with the corresponding ellipsoid overlaid on top. The ellipsoid is computed based on the unconditional correlation between the two time series, element $\overline{q}_{ij}$ of the matrix $\bar{Q}$. Larger correlations lead to more oblong ellipsoids, while values closer to zero make them more circular. Additionally, the ellipsoid is tilted towards the right if the correlation is positive, and tilted towards the left for negative values.

From the previous paragraphs we recall that, during the pandemic, there is an overall increase in the dynamic conditional correlation between each green-bond index and all other indices with respect to the other sub-periods. The same happens for the unconditional correlation which, in many cases, moves away from the negative domain and reaches null or positive values. Consequently, the ellipsoids in Fig. 1 take a noticeably more rounded shape than before. However, the airline and the energy sectors, which experienced a strong drop during the beginning of the pandemic, maintain a negative relationship with the green-bond market, as during the pre-pandemic sub-period. This is evidenced by Subfigures (c) and (d), for BBGB, and (i) and (j), for SOLGB.

The unconditional correlation of SOLGB with MSCI also remains slightly negative, as can be seen in Subfigure (h), while BBGB's moves into the positive domain. This is shown in Subfigure (b) and confirmed by the average dynamic conditional correlation values previously shown in Tables 4 and 5. As for the stand-out sectors of this time period, healthcare and IT, BBGB changes to a low but positive unconditional correlation, represented in Subfigures (e) and (f), while SOLGB maintains a negative – although moderate – one, represented in Subfigures (k) and (l).

Over the entire window of observation, the diversification benefit provided by green bonds is smallest for corporate bonds. This result is in line with the findings of Reboredo (2018), whose analysis only considers BBGB, with data up to August 2017, and is based on the shape and parameters of the bi-variate non-parametric copula densities. As for the energetic and the airline sectors, SOLGB offers the larger diversification benefit of the two green-bond indices, due to the consistently more negative correlation values. Additionally, and in contrast with BBGB, SOLGB displays a negative unconditional correlation also with the world stock index and the healthcare and IT sectors. SOLGB thus appears to be the superior green-bond option, on average, for investors in those indices. In fact, SOLGB negatively co-moves with all remaining sectors of the analysis also in the pre-pandemic sub-period, when BBGB displays a weak co-movement with the global stock market and with the healthcare and IT sectors.

The plots of the ellipsoids of the remaining periods, together with a more detailed breakdown of them, are in Appendix B.

5. Asset allocation analysis

We select six different allocation strategies and build the corresponding portfolios. For each strategy, one portfolio is built from the entire selection of assets (from hereon referred to as the “green portfolio”) and one only considering non-green assets (from hereon referred to as the “non-green portfolio”). The performance of both kinds of portfolios is then compared along a variety of measures. Our procedure is first tested with the in-sample approach, in which portfolios are built only once, in an omniscient way, on the basis of the entire sample data. Next, the out-of-sample approach is used, in which portfolios are built every 7 days on the basis of the performance of the assets during the previous 50 days (DeMiguel et al., 2007). The length of the window is calculated as the average number of days between negative shocks in the MSCI index, in the time frame of the sample under consideration.

The remainder of this section is structured as follows: in the first part the different allocation strategies and performance measures are explained, then an analysis of the average asset weights in each portfolio is performed with the in-sample and out-of-sample approaches. Afterwards, the performance of each of the portfolios is analyzed over the different sub-periods, for both green and non-green portfolios. Finally, the portfolios are compared in terms of the spectral risk measure. The goal of this section is to identify the allocation strategies and the investor risk preferences which lead to a larger request for investment in green bonds. Additionally, the two green-bond indices are compared, by recognizing which strategy assigns a greater weight to which, and what portfolio performance results from that allocation.

5.1. Allocation strategies

The selected allocation strategies determine the optimal portfolio composition on the basis of optimization problems centered around different aspects of asset allocation. Optimization is restricted to non-negative weights.

5.1.1. Mean–variance strategy

The mean–variance allocation strategy, from Markowitz (1952), determines the optimal portfolio weights by maximizing the expected portfolio return and minimizing the portfolio variance. The optimal portfolio is selected by assuming zero interest rates.

The optimization problem can be expressed in the following way:

$$
\begin{aligned}
&\max\ w'\mu - w'\Sigma w \\
&\quad \text{s.t } w'\mathbf{1} = 1 \\
&\qquad w \geq 0,
\end{aligned}
$$

where w is the Nx1 vector of asset weights, Σ is the NxN variance–covariance matrix, μ is the vector of expected asset returns and **1** is an Nx1 vector of ones. N is the number of assets considered in the allocation problem.

5.1.2. Minimum-variance strategy

The minimum-variance allocation strategy determines the optimal weights with the aim of minimizing portfolio variance. The optimization problem can be expressed in the following way:

$$
\begin{aligned}
&\min\ w'\Sigma w \\
&\quad \text{s.t } w'\mathbf{1} = 1 \\
&\qquad w \geq 0,
\end{aligned}
$$

where w is the Nx1 vector of asset weights, Σ is the NxN variance–covariance matrix, and **1** is an Nx1 vector of ones. N is the number of assets considered in the allocation problem.

5.1.3. Risk-parity strategy

The risk-contribution-parity approach from Maillard et al. (2010) consists in determining optimal portfolio allocation in such a way that each asset contributes equally to total portfolio risk. If a portfolio is considered, with weights given by $w = (w_1, w_2, \ldots, w_N)$ and volatility given by $\sigma(w) = \sqrt{w'\Sigma w}$, the marginal contribution of the ith asset to portfolio risk is defined as:

$$
\partial_{w_i}\sigma(w) = \frac{\partial(\sigma(w))}{\partial(w_i)} = \frac{\partial(\sqrt{w'\Sigma w})}{\partial(w_i)} = \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}} \tag{6}
$$

and represents the variation in portfolio volatility given an infinitesimal variation in the weight of one component. If the total risk contribution of the ith asset is denoted as $\sigma_i(w) = w_i \partial_{w_i}\sigma(w)$, then portfolio volatility can be rewritten as:

$$
\sigma(w) = \sum_{i=1}^{N} \sigma_i(w). \tag{7}
$$

Therefore, the risk-parity strategy requires the solution of the following minimization problem:

$$
\begin{aligned}
&\min\ \sum_{i=1}^{N}\left[w_i - \frac{\sigma(w)^2}{(\Sigma w)_i}\right]^2 \\
&\quad \text{s.t } w'\mathbf{1} = 1 \\
&\qquad w \geq 0.
\end{aligned}
$$

5.1.4. CVaR-optimization strategy

The Conditional-Value-at-Risk (CVaR) optimization strategy determines the portfolio weights such that the Conditional Value at Risk of the portfolio is minimized. The CVaR possesses the appealing features of sub-additivity and convexity. The weights are determined by solving the following minimization problem:

$$
\begin{aligned}
&\min\ CVaR_\alpha \\
&\quad \text{s.t } w'\mathbf{1} = 1 \\
&\qquad w \geq 0,
\end{aligned}
$$

where $CVaR_\alpha$ is the Conditional Value at Risk of order α, defined in Section 5.1.6.

The linear programming solution proposed in Rockafellar and Uryasev (2000) is used.

5.1.5. Maximum-diversification strategy

The maximum-diversification strategy determines portfolio weights by maximizing the diversification ratio introduced in Choueifaty and Coignard (2008):

$$
DR = \frac{w'\sigma}{\sqrt{w'\Sigma w}},
$$

where σ is the vector of asset volatilities, w is the vector of portfolio weights and Σ is the variance–covariance matrix. The optimization problem then becomes:

$$
\max\ DR
$$

$$\text{s.t } w'\mathbf{1} = 1$$
$$w \geq 0.$$

More specifically, the numerator is a weighted average of asset volatilities and the denominator is the portfolio volatility. Larger values of DR are given by lower levels of the latter.

5.1.6. Performance indicators

In order to compare the different portfolios, the following performance measures are used:

- annualized portfolio log-returns, assuming 252 trading days in a year;
- annualized volatility, assuming 252 trading days in a year;
- downside risk, which measures risk associated to losses:

  $$D = \left(E[(X-0)^2 \mathbb{1}_{X \leq 0}]\right)^{1/2},$$

  where X are daily log-returns. Larger values of this measure represent a greater risk;
- maximum drawdown, an indicator of loss risk which measures the maximum percentage loss with respect to the local maximum in the observation period of length $(0, T)$:

  $$MDD(T) = \max_{\tau \in (0,T)} \left[\max_{t \in (0,\tau)} x_t - x_\tau \right],$$

  where x_t are the realized log-returns of day t. Larger absolute values of this measure entail a greater risk.

In order to evaluate portfolio risk in terms of potential losses, the following measures are used:

- 5% daily Value at Risk, which expresses the maximum daily loss which will not be exceeded with a 95% probability:

  $$VaR_{0.05}(X) = F^{-1}(0.05);$$

  where F is the cumulative probability distribution function of daily log-returns X. Larger absolute values of this measure entail a greater risk;
- 5% Conditional Value at Risk which captures the expected daily loss in case it exceeds the 5% daily Value at Risk:

  $$CVaR_{0.05}(X) = -\frac{1}{0.05}\int_0^{0.05} VaR_\gamma(X)d\gamma,$$

  where $VaR_{0.05}(X)$ is the 5% Value at Risk of daily log-returns X. Larger absolute values of this measure entail a greater risk.

Finally, the risk-adjusted performance is evaluated through:

- Sharpe ratio, which measures excess return (over the market risk-free rate here assumed to be 0) per unit of risk:

  $$SR = \frac{E[X]}{\sigma},$$

  where X are daily log-returns and σ is their volatility. Larger values of this measure are preferable;
- Omega ratio, which is defined as the probability-weighted ratio of returns to losses with respect to a specific benchmark return, which is set here to zero:

  $$\Omega = \frac{\int_0^{+\infty}[1-F(r)]dr}{\int_{-\infty}^0 F(r)dr},$$

  where F is the cumulative probability distribution function of daily log-returns X. This indicator, in contrast to the Sharpe ratio, considers all moments of the return distribution. It was introduced in Keating and Shadwick (2002). A larger ratio is preferable, as it indicates that the asset provides more gains than losses, relative to the threshold.

5.2. Average asset weights overtime

Table 6 shows the in-sample asset weights of green portfolios for each asset allocation strategy, except for the equal weight strategy (where they are constant at 12.5%) over the entire sample period.

It can be noticed that all strategies attribute the highest weights to at least one of the two green-bond indices and sometimes to both. More specifically, the maximum-diversification and mean–variance strategies select the SOLGB index exclusively. Given the high correlation between the two green-bond indices and the fact that the SOLGB index is less correlated with the traditional bond market than the BBGB index, this result can be expected of a strategy seeking to maximize diversification. Similarly, since mean–variance portfolio optimization is achieved by a combination of return maximization and variance minimization, we understand how the higher historical return of SOLGB and the lower covariance with other assets favor its selection over the other green-bond index. Both strategies suggest an investment of about half of the portfolio in green bonds and they favor the green-bond index over the corporate bond index.

The minimum-variance and the CVaR-optimization strategies assign the greatest weight to the green-bond indices (an impressive 80.6% and 81.7% of the corresponding portfolios), signaling the high usefulness of the green market whenever the goal is the minimization of a measure of risk, be it volatility or CVaR. In both cases, BBGB – with its lower standard deviation – is favored over SOLGB.

Table 7 shows the average out-of-sample weights of green portfolios, for each asset allocation strategy and for each sub-period. The equal weight strategy, where the weights are constant at 12.5%, is again excluded from the table.

It can be noticed that all strategies attribute the highest weights to either one of the two green-bond indices or to the corporate bond index, BBBOND. In fact, the out-of-sample allocation, in contrast to the in-sample one, is not omniscient, but depends on a moving window of previous observations. This can give rise to less than optimal allocation choices for the future.

The mean–variance allocation strategy gives a larger weight to SOLGB than to BBGB over each sub-period, which is consistent with the in-sample approach. The higher historical return of SOLGB and the lower covariance with other assets favor its selection over the other green-bond index. In periods of general market downturn, meaning the bear and pandemic periods, the strategy also allocates a relevant portion of the portfolio to BBGB: about 15%. This can be due to its consistently lower standard deviation, which renders this index useful for the purposes of variance optimization. The corporate bond index is attributed a larger weight than SOLGB in all sub-periods except for the pre-pandemic one. However, its weight is always smaller than the sum of the weights given to the green-bond indices. This shows the key role played by the green-bond market in the mean–variance-optimization setting.

The minimum-variance allocation strategy is among those that give the highest weight to the green-bond market, up to about 80% of the entire portfolio. This is consistent with the in-sample approach. It gives a larger weight to BBGB than to SOLGB in the bear and bull sub-periods, when the standard deviation of BBGB is noticeably lower, and a smaller weight to BBGB in the pre-pandemic and pandemic sub-periods, when the difference between the two standard deviations is very small. Over these last two sub-periods, the average standard deviation of BBGB is still slightly lower than that of SOLGB, however this is not consistently true across all 50-day calibration windows. This, together with time-variations in asset covariances, could explain why the average weight attributed to SOLGB is higher than that attributed to BBGB. In the last two sub-periods, the corporate bond market increases in relative importance and reaches 41.7% and 33.5%. However, its weight is always smaller than the sum of the weights given to the green-bond indices. The green-bond market has a central role also in the variance-minimization setting.

Table 6
In-sample % weights.

Strategy	BBGB	SOLGB	BBBOND	MSCI	SPGSEN	SP5IAIR	SP5EHCR	SPEUIT
Mean Variance	0	52.3	37.3	0.6	0	0	4	5.8
Minimum Variance	60.8	19.8	14.7	3.9	0.4	0	0.4	0
Risk Parity	27.9	29.6	25	4.4	2.7	2.3	4.5	3.6
CVaR Opt.	59.8	21.9	14.1	1.1	0.4	0	2.7	0
Maximum Divers.	0	47.4	39.3	0	3.4	2.3	4.4	3.2

Table 7
Out-of-sample average % weights.

	BBGB	SOLGB	BBBOND	MSCI	SPGSEN	SP5IAIR	SP5EHCR	SPEUIT
Mean–Variance								
Bear	16.6	22.2	25.5	0.5	7.1	5.9	14.7	7.4
Bull	4.6	21.3	22.1	16.3	6.3	6.4	10.4	12.7
Pre-pandemic	4.2	41.3	26.8	15.9	1.3	0.9	8.7	0.9
Pandemic	14.4	20.9	35.1	3.3	5.9	3.6	12.8	4
Minimum Variance								
Bear	72.4	6.1	11.7	3.2	1.5	0.7	0.8	3.6
Bull	61.6	17	4.8	10.5	1.3	1.1	2.9	0.9
Pre-pandemic	16.6	29.5	41.7	6.7	1	2.9	1.3	0.4
Pandemic	15.4	47.6	33.5	0.4	0.7	0.7	1.1	0.5
Risk Parity								
Bear	29.2	21.6	29.2	4.8	3.3	2.7	4	5.1
Bull	23.3	27.7	18.6	9.1	4.3	3.7	7.9	5.4
Pre-pandemic	24.3	24.7	29.8	5.7	3.1	4.2	4.9	3.3
Pandemic	27.6	31.1	25.5	3.8	2.7	2	4.4	2.9
CVaR Optimization								
Bear	55.4	8.9	19.6	5.1	3	2.1	2.2	3.7
Bull	42.2	19.5	12	12.1	3.5	2.8	4.5	3.6
Pre-pandemic	18.6	25	41.3	8.1	2.1	1.9	2.5	0.6
Pandemic	19.9	37	36.9	0.7	1	1.5	2.1	0.9
Max Diversification								
Bear	0	39	44.7	0	4.4	2.8	1.5	7.4
Bull	0	44.4	30.5	0.7	5	4.9	8.4	6.2
Pre-pandemic	0	32.9	49.2	0.8	3.9	6.8	3.8	2.5
Pandemic	0	48.2	39.2	0	3.3	2.3	4.6	2.3

The risk-parity allocation strategy gives comparable weights to SOLGB and BBGB over each sub-period, but slightly higher to the former rather than the latter. This is consistent with the in-sample approach. This result tells us that the marginal contribution of SOLGB to portfolio risk over the calibration windows is lower than that of BBGB and thus a larger proportion of this index is required to achieve risk parity. The only exception is the bear period, when a greater weight is given to BBGB. Variance and covariance are the measures of risk considered in this allocation strategy and the bear period is also the time in which the difference between the two standard deviations is largest, with that of BBGB being the lower of the two. This translates to the choice of a larger investment in BBGB during the sub-period in order to obtain risk parity. The corporate bond market is attributed a weight comparable to that of the two green-bond indices. Its weight is always smaller than the greater of the weights given to the green-bond indices, except for the pre-pandemic period, when it is slightly larger. The average standard deviations of the three indices are extremely similar during this time, which means that any differences that lead to the preference of one index over the other manifest in terms of marginal changes in time. The weight given to BBBOND is always smaller than the sum of the weights of the green-bond indices. Again, the green-bond market has an important role also in the risk-parity setting.

The CVaR-optimization strategy is among those that give the largest weight to the green-bond market, about 60% of the entire portfolio. This is consistent with the in-sample approach. It gives a greater weight to BBGB than to SOLGB in the bear and bull sub-periods, when the standard deviation of BBGB is noticeably lower and its average return is higher, and a larger weight to SOLGB in the pre-pandemic and pandemic sub-periods, when the difference between the two standard deviations is very small and SOLGB's return is higher. These results are akin to those obtained in the minimum-variance setting. Similarly, in the last two sub-periods, the corporate bond market increases in relative importance and reaches 41.3% and 36.9%. However, its weight is always smaller than the sum of the weights given to the green-bond indices.

The portfolios built with the maximum-diversification approach attribute zero average weight to the BBGB index and a weight of about 40% to SOLGB across all time periods. This is consistent with results obtained with the in-sample approach. The diversification index being maximized is dependent on the asset variance and covariance matrix. Given the high correlation between the two green-bond indices and the fact that SOLGB is less correlated with the other assets than BBGB, this result can be expected of a strategy seeking to maximize diversification. The diversification potential associated with BBGB is absorbed by SOLGB and by the other assets. Additionally, the "corporate" diversification potential offered by BBGB is likely absorbed by BBBOND since – as can be recalled – the two indices refer to overlapping debt markets. This asset allocation strategy also attributes a great weight to the traditional corporate bond market, comparable to that of SOLGB across all sub-periods. This is a key indication that the diversification benefit provided by the green-bond market and the traditional corporate-bond market is different, meaning that neither can be taken as a substitute for the other.

5.3. Portfolio performance

This section tackles the analysis of portfolio performance across time periods. When analyzing the behavior of average weights within green portfolios, the change in the relative importance of green assets during each sub-period and for each strategy has already been stressed. Comparing the performance and the risk of green and non-green portfolios will give more insight into those considerations.

Tables 8, 9, 10, and 11 report the value of the performance measures for the bear, bull, pre-pandemic and pandemic sub-periods, respectively. Within each table, the measures are computed for each allocation strategy, for in-sample and out-of-sample portfolios and for both green and non-green portfolios. The findings relative to the in-sample approach are reported for the sake of completeness, although it is not a realistic approach, as it is omniscient. In the written analysis, the focus is on the out-of-sample approach and, in particular, on the difference between the performance of green portfolios and of non-green portfolios. Emphasis is placed on which kind performs best for each allocation strategy, and along which measures, within each time period.

5.3.1. Bear sub-period

During the bear sub-period, a time of market downturn, green portfolios always prove less risky than non-green ones, along all measures of risk and loss: annualized volatility, maximum drawdown, downside risk, VaR and CVaR. As for the performance, Table 8 shows that green portfolios built with strategies focusing on risk reduction but disregarding return optimization lead to lower losses than non-green portfolios, with the exception of the 5% CVaR optimization strategy and the minimum-variance strategy. The 5% CVaR-optimized non-green portfolio has a positive annualized return while the green portfolio has a negative one, although their difference is small. As for the minimum-variance portfolios, they are able to achieve positive annualized returns and Sharpe ratios – the highest of any strategy – but the non-green-portfolio values are substantially larger.

The equal-weight strategy gives insight into whether a naive allocation strategy would benefit from the inclusion of the two green-bond indices in the portfolio. Both the green and the non-green portfolios perform very negatively, but, similarly to the portfolios based on pure risk-reduction strategies, the green performs less badly than the latter. The simple inclusion of the green-bond indices in the portfolio is useful in reducing the average loss, but it is not enough in order to achieve a satisfactory (positive) performance.

The mean–variance allocation strategy, the only one which not only minimizes risk but aims at the maximization of returns per unit of risk, shows the green portfolio to be preferable to non-green one not only over all measures of risk, but also of performance. Both kinds have a negative annualized return, but losses are less than half for the green portfolio. Similar results are given by the risk-adjusted performance measures: the Sharpe ratio is less strongly negative, meaning that a smaller loss per unit of risk is incurred, and the Omega ratio is higher, indicating that the cumulative return distribution of the green portfolio is larger than that of the non-green portfolio in the area of gains.

5.3.2. Bull sub-period

During the bull sub-period, when stock market growth is high, Table 9 shows that all considered strategies report the same result: the non-green portfolios perform better in terms of returns, but green-portfolio returns are still quite high. The green indices confirm their risk-reduction benefit by rendering the corresponding portfolios substantially less risky than the non-green ones along all measures of risk, all the while being able to capture some market growth, although to a lower degree than the non-green ones.

The 5% CVaR optimization strategy is of particular note, as the corresponding green portfolio has a high annualized return, despite allocating an average weight of more than 60% to the green indices, as can be seen in Table 7.

5.3.3. Pre-pandemic sub-period

The pre-pandemic sub-period, which is characterized by the most relevant presence of green bonds, sees superior performance of the green portfolios over the non-green ones for almost all allocation strategies and in terms of all measures: greater annualized returns, better risk-adjusted return measures, and lower risk.

This is an interesting result: the pre-pandemic sub-period is characterized by general stock market growth, as is the bull sub-period. However, in the former, a consistently better performance of the green portfolios is seen for all strategies, while in the latter a better performance of the green portfolios is seen only in terms of risk, but not in terms of returns. However, an explanation can be found for this by noticing that the average performance of the green indices is substantially better before the pandemic than during the bull sub-period. Still, in both time windows, the risk-reduction benefit of green bonds is always substantial.

The risk-parity strategy is the only one to yield a slightly better performance for the non-green portfolio during the pre-pandemic period, at least in terms of annualized returns. The difference can be seen in Table 10. It is very small, however, and the risk-adjusted performance measures (Sharpe and Omega ratios) clearly favor the green portfolio. This portfolio is also preferable in terms of risk measures. The inclusion of green assets before the pandemic allows the investor to achieve comparably high returns without facing as much risk.

5.3.4. Pandemic sub-period

During the pandemic, again considering the out-of-sample approach, the risk-reduction benefit provided by the inclusion of green indices in the portfolios is also consistent across all strategies, while the difference between the returns of green portfolios and non-green ones depends on the strategy.

Strategies which focus on risk-reduction and disregard the optimization of returns (risk parity, minimum variance, CVaR optimization, and maximum diversification) are able to produce green portfolios which are less risky than the non-green ones. However, the corresponding green portfolios perform worse than the non-green ones in terms of returns. Nevertheless, as can be seen in Table 11, in the case of the risk-parity and maximum-diversification strategies, all portfolios achieve positive returns. It is therefore very interesting to note that, in those cases, the inclusion of the green indices in the portfolios is able to not only stabilize their behavior during the pandemic, a time of market turmoil, but also to provide an overall positive return.

On the other hand, the mean–variance strategy, the only one optimizing not only in terms of risk but also in terms of returns, produces a green portfolio which is preferable to the non-green one in terms of both lower risk and higher returns. The annualized return and the Sharpe ratio of the green portfolio are positive, while the non-green portfolio has markedly negative ones.

Finally, the equal-weight strategy leads to the highest average returns out of any other strategy. This is due to the fact that the selection of stock indices in this study includes some of the best-performing indices during the pandemic, which receive a largely superior weight through this strategy than through any other. The annualized volatility of both portfolios reaches the highest values obtained in this study.

5.4. The spectral risk measure

5.4.1. Definition

We undertake one final evaluation of the performance of the different portfolios by using the Spectral Risk Measure (SRM). The SRM is a distortion risk measure, as introduced in Denneberg (1994) and Wang et al. (1997). A distortion risk measure is the expected loss under a transformation of the underlying cumulative probability distribution by means of a distortion function. It is linked to an agent's risk aversion, as it takes a weighted average of the distribution quantiles in which the weights depend on a risk-aversion parameter. In the present paper,

Table 8
Portfolio performance — Bear sub-period.

Performance measures	Green Portfolios		Non-green Portfolios	
	In-sample	Out-of-sample	In-sample	Out-of-sample
Mean–Variance				
Annualized return (%)	2.643	−3.561	2.602	−8.525
Annualized volatility (%)	2.889	10.727	4.933	13.761
Sharpe ratio	0.917	−0.284	0.545	−0.578
Downside risk (%)	2.147	8.635	3.361	10.920
Omega ratio	1.172	0.939	1.095	0.894
Maximum Drawdown (%)	−2.472	−9.891	−4.381	−14.190
VaR 5% (%)	−0.276	−1.023	−0.450	−1.402
CVaR 5% (%)	−0.443	−1.839	−0.659	−2.373
Minimum Variance				
Annualized return (%)	2.809	1.281	4.713	4.825
Annualized volatility (%)	2.728	2.684	4.875	4.645
Sharpe ratio	1.029	0.488	0.969	1.038
Downside risk (%)	2.074	2.033	3.373	3.044
Omega ratio	1.198	1.087	1.185	1.197
Maximum Drawdown (%)	−1.946	−2.809	−2.816	−3.160
VaR 5% (%)	−0.273	−0.292	−0.468	−0.461
CVaR 5% (%)	−0.459	−0.422	−0.714	−0.612
Equal Weight				
Annualized return (%)	−7.224	−7.224	−10.600	−10.600
Annualized volatility (%)	10.778	10.778	14.544	14.544
Sharpe ratio	−0.642	−0.642	−0.698	−0.698
Downside risk (%)	8.038	8.038	10.869	10.869
Omega ratio	0.899	0.899	0.890	0.890
Maximum Drawdown (%)	−15.247	−15.247	−20.427	−20.427
VaR 5% (%)	−1.192	−1.192	−1.638	−1.638
CVaR 5% (%)	−1.581	−1.581	−2.136	−2.136
Risk Parity				
Annualized return (%)	0.890	−0.638	−0.753	−2.478
Annualized volatility (%)	3.190	3.376	5.973	6.380
Sharpe ratio	0.294	−0.173	−0.097	−0.362
Downside risk (%)	2.333	2.576	4.188	4.631
Omega ratio	1.049	0.973	0.985	0.943
Maximum Drawdown (%)	−3.293	−4.715	−7.987	−9.560
VaR 5% (%)	−0.297	−0.381	−0.596	−0.646
CVaR 5% (%)	−0.472	−0.497	−0.745	−0.871
CVaR Optimization				
Annualized return (%)	2.892	−1.213	4.792	0.540
Annualized volatility (%)	2.823	3.613	4.858	5.459
Sharpe ratio	1.024	−0.320	0.988	0.126
Downside risk (%)	2.149	2.690	3.369	3.725
Omega ratio	1.197	0.947	1.189	1.021
Maximum Drawdown (%)	−1.997	−5.716	−2.760	−6.093
VaR 5% (%)	−0.272	−0.395	−0.465	−0.588
CVaR 5% (%)	−0.475	−0.554	−0.717	−0.719
Maximum Diversification				
Annualized return (%)	1.448	−0.885	1.005	−1.920
Annualized volatility (%)	3.148	3.464	5.188	6.348
Sharpe ratio	0.472	−0.239	0.219	−0.274
Downside risk (%)	2.317	2.588	3.576	4.644
Omega ratio	1.080	0.961	1.036	0.955
Maximum Drawdown (%)	−3.152	−5.624	−6.493	−9.243
VaR 5% (%)	−0.314	−0.360	−0.508	−0.604
CVaR 5% (%)	−0.459	−0.503	−0.648	−0.902

the Exponential Risk Measure (ERM) is used, a particular kind of SRM which implies constant absolute risk aversion. It has already been used in a number of optimization problems in finance, among which optimal futures hedge ratio determination, as in Barbi and Romagnoli (2016). We employ their same notation and define the ERM as:

$$ERM = -\int_0^1 \frac{ke^{-ks}}{1-e^{-k}} q_X(s)ds,$$

where $k > 0$ is the Arrow–Pratt absolute risk-aversion coefficient, X is the distribution of portfolio log-returns and $q_X(s)$ denotes the s-quantile of X.

The spectral risk measure is computed in each period for all portfolio allocation strategies and for different values of the risk-aversion parameter k. Only risk-averse investors are considered and thus only positive values of k, which are larger for higher levels of risk aversion. We do not consider the case of risk-loving investors, due to the low-risk nature of green bonds as an asset class. We aim to check whether risk-averse investors, those most interested in fixed-income instruments, display a preference towards portfolios which include green bonds and, in general, under which allocation strategy.

The empirical quantile is considered and discretized over intervals of length 1/1000. Lower values of the risk measure are preferable, as they indicate a lower perceived risk on the part of the investor. Figures relative to the in-sample portfolios and tables relative to the in-sample and out-of-sample ERM values are included in Appendix E.

Table 9
Portfolio performance — Bull sub-period.

Performance measures	Green Portfolios		Non-green Portfolios	
	In-sample	Out-of-sample	In-sample	Out-of-sample
Mean–Variance				
Annualized return (%)	3.932	13.613	13.162	19.754
Annualized volatility (%)	2.398	5.553	4.447	6.532
Sharpe ratio	1.620	2.327	2.803	2.793
Downside risk (%)	1.682	3.491	2.849	3.883
Omega ratio	1.318	1.513	1.599	1.601
Maximum Drawdown (%)	−1.692	−2.103	−1.767	−2.348
VaR 5% (%)	−0.272	−0.512	−0.407	−0.543
CVaR 5% (%)	−0.366	−0.733	−0.580	−0.776
Minimum Variance				
Annualized return (%)	1.862	4.289	8.038	12.848
Annualized volatility (%)	2.277	2.179	4.417	3.815
Sharpe ratio	0.822	1.939	1.773	3.188
Downside risk (%)	1.620	1.482	2.981	2.380
Omega ratio	1.147	1.381	1.355	1.705
Maximum Drawdown (%)	−1.581	−1.436	−2.498	−1.444
VaR 5% (%)	−0.264	−0.217	−0.402	−0.335
CVaR 5% (%)	−0.337	−0.308	−0.606	−0.479
Equal Weight				
Annualized return (%)	15.507	15.507	21.600	21.600
Annualized volatility (%)	5.003	5.003	6.706	6.706
Sharpe ratio	2.907	2.907	2.951	2.951
Downside risk (%)	3.076	3.076	4.107	4.107
Omega ratio	1.603	1.603	1.613	1.613
Maximum Drawdown (%)	−2.693	−2.693	−3.284	−3.284
VaR 5% (%)	−0.481	−0.481	−0.657	−0.657
CVaR 5% (%)	−0.630	−0.630	−0.837	−0.837
Risk Parity				
Annualized return (%)	5.097	7.622	13.329	16.229
Annualized volatility (%)	2.267	2.628	3.968	4.584
Sharpe ratio	2.204	2.808	3.174	3.305
Downside risk (%)	1.528	1.666	2.429	2.760
Omega ratio	1.449	1.595	1.686	1.707
Maximum Drawdown (%)	−1.487	−1.372	−1.725	−1.943
VaR 5% (%)	−0.239	−0.270	−0.346	−0.415
CVaR 5% (%)	−0.322	−0.353	−0.499	−0.568
CVaR Optimization				
Annualized return (%)	1.605	8.948	8.138	14.752
Annualized volatility (%)	2.281	2.871	4.393	4.448
Sharpe ratio	0.710	3.000	1.803	3.116
Downside risk (%)	1.633	1.768	2.962	2.694
Omega ratio	1.126	1.657	1.363	1.688
Maximum Drawdown (%)	−1.581	−1.755	−2.498	−1.881
VaR 5% (%)	−0.256	−0.240	−0.403	−0.422
CVaR 5% (%)	−0.336	−0.369	−0.602	−0.566
Maximum Diversification				
Annualized return (%)	4.168	5.967	10.566	12.773
Annualized volatility (%)	2.346	2.852	4.002	4.763
Sharpe ratio	1.752	2.047	2.530	2.548
Downside risk (%)	1.592	1.865	2.559	2.988
Omega ratio	1.345	1.399	1.528	1.514
Maximum Drawdown (%)	−1.737	−1.659	−2.052	−2.450
VaR 5% (%)	−0.256	−0.291	−0.353	−0.481
CVaR 5% (%)	−0.330	−0.375	−0.529	−0.600

In this section, the focus is on the out-of-sample values, which reflect a more realistic (non-omniscient) portfolio construction.

5.4.2. Results

When the Arrow–Pratt risk-aversion coefficient is equal to 10, the lowest value in this analysis, the hypothetical investor is moderately risk averse.

In Fig. 2 it can be seen that, in this setting, green portfolios have overall lower ERM values than non-green ones. The relative position of portfolios built with different strategies is very similar for green and non-green portfolios. In both cases, the minimum-variance, the maximum-diversification and the CVaR-optimization strategies minimize the ERM. Their values are almost overlapping. The risk parity portfolio scores very similarly to them except for the non-green portfolio during the pandemic, when its ERM raises noticeably. From a careful look at the data in Table 11, insight into this issue can be gained. The pandemic is the sub-period during which the risk-parity strategy yields its largest value in terms of 5% VaR. This indicates a quantile distribution taking on values that are large in magnitude in the left tail, thus increasing $q_r(s)$, while the weight $\frac{ke^{-ks}}{1-e^{-k}}$ is the same for every strategy.

The mean–variance and the equal-weight strategies are the ones with the highest ERM. This means that their return distributions, when balanced by a weight relating risk-aversion to each percentile s, are considered more risky. The equal-weight strategy is the riskiest during

Table 10
Portfolio performance - Pre-pandemic sub-period.

Performance measures	Green Portfolios		Non-green Portfolios	
	In-sample	Out-of-sample	In-sample	Out-of-sample
Mean–Variance				
Annualized return (%)	9.827	11.522	10.178	11.119
Annualized volatility (%)	2.661	3.622	4.057	3.841
Sharpe ratio	3.537	3.030	2.410	2.765
Downside risk (%)	1.615	2.130	2.542	2.228
Omega ratio	1.825	1.691	1.506	1.616
Maximum Drawdown (%)	−1.393	−1.902	−2.105	−1.889
VaR 5% (%)	−0.246	−0.298	−0.393	−0.320
CVaR 5% (%)	−0.342	−0.444	−0.524	−0.457
Minimum Variance				
Annualized return (%)	8.552	7.825	8.149	6.491
Annualized volatility (%)	2.736	2.620	2.993	2.919
Sharpe ratio	3.014	2.889	2.632	2.170
Downside risk (%)	1.734	1.611	1.790	1.753
Omega ratio	1.643	1.620	1.557	1.447
Maximum Drawdown (%)	−2.296	−1.360	−1.454	−1.521
VaR 5% (%)	−0.256	−0.249	−0.254	−0.263
CVaR 5% (%)	−0.358	−0.331	−0.365	−0.348
Equal Weight				
Annualized return (%)	5.682	5.682	4.423	4.423
Annualized volatility (%)	8.749	8.749	11.899	11.899
Sharpe ratio	0.675	0.675	0.423	0.423
Downside risk (%)	6.438	6.438	8.837	8.837
Omega ratio	1.129	1.129	1.079	1.079
Maximum Drawdown (%)	−9.826	−9.826	−13.700	−13.700
VaR 5% (%)	−0.952	−0.952	−1.329	−1.329
CVaR 5% (%)	−1.456	−1.456	−2.002	−2.002
Risk Parity				
Annualized return (%)	8.416	7.087	7.687	7.211
Annualized volatility (%)	2.789	2.987	4.704	3.965
Sharpe ratio	2.912	2.308	1.598	1.776
Downside risk (%)	1.741	1.976	3.135	2.620
Omega ratio	1.628	1.480	1.324	1.360
Maximum Drawdown (%)	−1.833	−2.040	−4.476	−2.913
VaR 5% (%)	−0.257	−0.310	−0.464	−0.378
CVaR 5% (%)	−0.367	−0.423	−0.656	−0.550
CVaR Optimization				
Annualized return (%)	8.479	7.003	8.268	6.546
Annualized volatility (%)	2.788	2.800	2.984	2.898
Sharpe ratio	2.934	2.432	2.678	2.203
Downside risk (%)	1.788	1.766	1.776	1.798
Omega ratio	1.620	1.508	1.571	1.459
Maximum Drawdown (%)	−2.384	−1.673	−1.397	−1.546
VaR 5% (%)	−0.254	−0.268	−0.259	−0.277
CVaR 5% (%)	−0.366	−0.357	−0.364	−0.361
Maximum Diversification				
Annualized return (%)	8.289	6.125	6.631	5.867
Annualized volatility (%)	2.645	2.993	3.624	3.436
Sharpe ratio	3.024	2.002	1.790	1.676
Downside risk (%)	1.603	1.944	2.259	2.201
Omega ratio	1.663	1.396	1.358	1.328
Maximum Drawdown (%)	−1.319	−2.220	−3.158	−2.539
VaR 5% (%)	−0.266	−0.311	−0.297	−0.326
CVaR 5% (%)	−0.334	−0.393	−0.447	−0.443

the pandemic, for both green and non-green portfolios. It is in fact the strategy with by far the most extreme value in terms of 5% VaR. As for the mean–variance strategy, it reaches its highest ERM score during the bear sub-period, for both green and non-green portfolios. The 5% VaR in Table 8 confirms our findings, as it is maximum for this strategy during the bear period, for both green and non-green portfolios.

Barbi and Romagnoli (2016) provide insight into the change in weight linked to a change in the risk-aversion parameter. Larger values of k assign a greater weight to the left tail of the distribution of returns, and a progressively smaller weight to non-tail quantiles. For $k = 10$, they find that approximately 63% of the total weighting mass lies in the left 10% tail of the probability distribution, while the weighting mass increases to 99% for $k = 50$, the case of an extremely risk-averse agent. When increasing k, it can thus be expected for the ERM to increase, because of the concentration of weights towards the most extreme quantiles. Additionally, the differences between strategies should be exacerbated by highlighting their most extreme tail behavior, thus penalizing even further the two outliers in terms of tail risk: the equal-weight and the mean–variance strategies. These considerations are verified by Figs. 3, 4, and 5, which represent out-of-sample ERMs for $k = 20$, $k = 30$, and $k = 50$, respectively.

Section 5.3 has shown green portfolios to be consistently less risky than their non-green counterparts, ceteris paribus. This section goes further, showing that portfolios which include green bonds are systematically preferable for a risk-averse investor also under a risk measure which is tailored to behavioral preferences.

Table 11
Portfolio performance — Pandemic sub-period.

Performance measures	Green Portfolios		Non-green Portfolios	
	In-sample	Out-of-sample	In-sample	Out-of-sample
Mean–Variance				
Annualized return (%)	2.612	1.334	9.014	−3.021
Annualized volatility (%)	4.735	7.846	8.416	11.053
Sharpe ratio	0.568	0.208	1.068	−0.221
Downside risk (%)	3.741	5.948	6.298	9.413
Omega ratio	1.120	1.042	1.230	0.952
Maximum Drawdown (%)	−8.462	−8.312	−11.742	−13.448
VaR 5% (%)	−0.413	−0.876	−0.811	−0.973
CVaR 5% (%)	−0.748	−1.338	−1.292	−1.967
Minimum Variance				
Annualized return (%)	0.419	−0.910	4.291	0.819
Annualized volatility (%)	4.039	4.093	5.294	5.664
Sharpe ratio	0.124	−0.203	0.820	0.172
Downside risk (%)	3.291	3.372	4.069	4.693
Omega ratio	1.025	0.961	1.179	1.037
Maximum Drawdown (%)	−7.503	−7.733	−9.127	−10.700
VaR 5% (%)	−0.333	−0.338	−0.442	−0.465
CVaR 5% (%)	−0.658	−0.702	−0.831	−0.932
Equal Weight				
Annualized return (%)	13.600	13.600	18.091	18.091
Annualized volatility (%)	20.649	20.649	27.354	27.354
Sharpe ratio	0.722	0.722	0.747	0.747
Downside risk (%)	16.328	16.328	21.440	21.440
Omega ratio	1.165	1.165	1.170	1.170
Maximum Drawdown (%)	−25.786	−25.786	−31.852	−31.852
VaR 5% (%)	−1.619	−1.619	−2.239	−2.239
CVaR 5% (%)	−3.668	−3.668	−4.860	−4.860
Risk Parity				
Annualized return (%)	3.842	2.885	10.367	10.059
Annualized volatility (%)	6.414	5.844	11.732	9.735
Sharpe ratio	0.620	0.516	0.900	1.034
Downside risk (%)	5.279	4.821	9.284	7.634
Omega ratio	1.141	1.113	1.208	1.233
Maximum Drawdown (%)	−11.066	−10.253	−16.785	−13.203
VaR 5% (%)	−0.484	−0.482	−0.875	−0.889
CVaR 5% (%)	−1.100	−0.990	−2.028	−1.656
CVaR Optimization				
Annualized return (%)	0.104	−0.474	4.402	1.823
Annualized volatility (%)	3.971	4.584	5.352	5.748
Sharpe ratio	0.046	−0.081	0.832	0.343
Downside risk (%)	3.203	3.790	4.080	4.558
Omega ratio	1.009	0.984	1.182	1.072
Maximum Drawdown (%)	−7.223	−8.030	−9.150	−10.063
VaR 5% (%)	−0.307	−0.427	−0.435	−0.503
CVaR 5% (%)	−0.639	−0.798	−0.832	−0.956
Maximum Diversification				
Annualized return (%)	3.024	1.756	7.336	5.360
Annualized volatility (%)	5.798	6.347	9.386	7.726
Sharpe ratio	0.543	0.306	0.802	0.715
Downside risk (%)	4.795	5.243	7.510	6.278
Omega ratio	1.122	1.069	1.185	1.158
Maximum Drawdown (%)	−10.375	−10.980	−14.134	−12.282
VaR 5% (%)	−0.449	−0.517	−0.688	−0.648
CVaR 5% (%)	−0.994	−1.075	−1.646	−1.352

6. Conclusions

The purpose of this paper was to analyze the market co-movements and diversification benefits of green bonds and the corresponding implications on portfolio allocation, when considering a number of relevant market indices and with a focus on the Covid-19 pandemic. With this goal, we identified for which assets green bonds provided the largest diversification benefits, and during which sub-period. Additionally, we highlighted the allocation strategies and the investor risk preferences which lead to a larger request for the inclusion of green bonds in portfolios. During the analysis, differences between the two green-bond indices, and the diversification benefits provided by each one, emerged.

Concerning the co-movement, our results indicate that the two green-bond indices considered, the Bloomberg Barclays MSCI Green Bond Index and the Solactive Green Bond Index, show a significantly positive dynamic conditional correlation with the traditional corporate bond market in all sub-periods. They thus do not appear to be particularly helpful for diversification in this sector, but their even lower volatility makes them an appealing new asset class for conservative investors.

The Solactive Green Bond Index negatively co-moves with all remaining sectors of the analysis: the global stock market, the energy commodity index, the airline industry, the healthcare sector, and the IT index, when considering the entire time window and when analyzing the pre-pandemic and the pandemic sub-periods separately. The

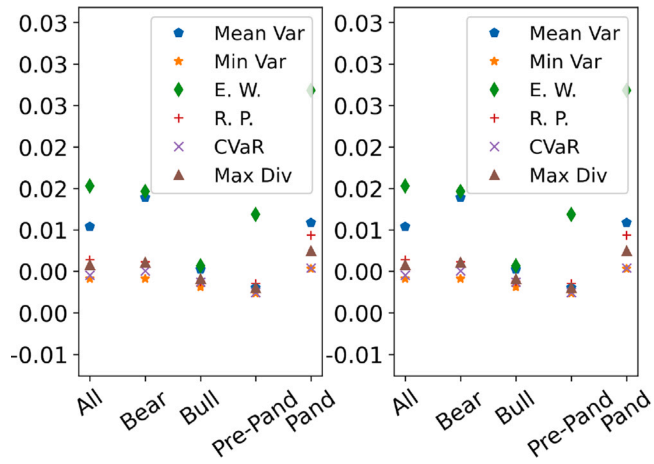

Fig. 2. Out-of-sample Exponential Risk Measures, $k = 10$.

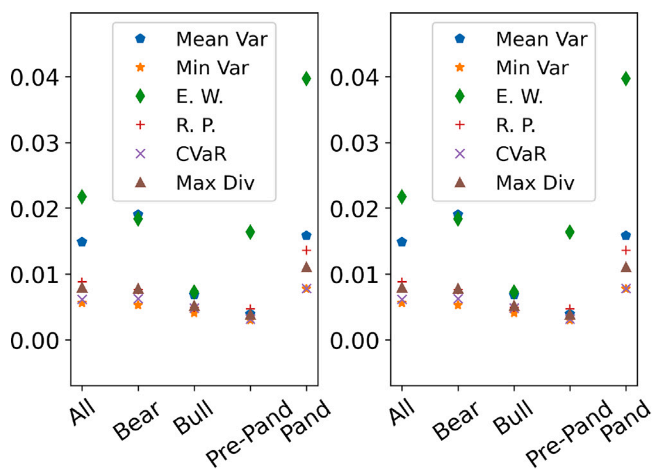

Fig. 3. Out-of-sample Exponential Risk Measures, $k = 20$.

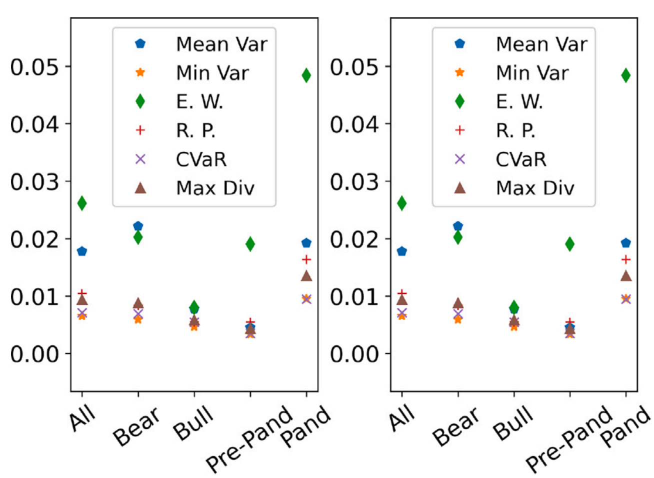

Fig. 4. Out-of-sample Exponential Risk Measures, $k = 30$.

Bloomberg Barclays MSCI Green Bond Index, on the other hand, displays a weak co-movement with the global stock market and with the healthcare and IT sectors, especially before and during the pandemic. Therefore, the Solactive Green Bond Index seems to be the better green option for investors in these industries, in terms of diversification potential. Interestingly, the Bloomberg Barclays MSCI Green Bond Index

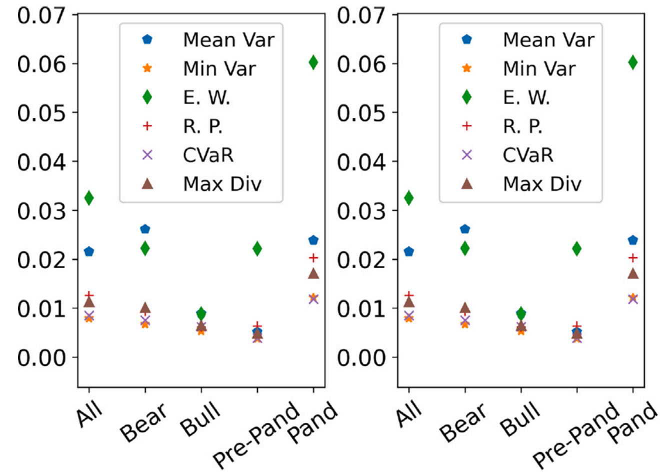

Fig. 5. Out-of-sample Exponential Risk Measures, $k = 50$.

is slightly preferable to Solactive in terms of volatility and displays a weak positive co-movement with the sectors which had an outstanding positive performance during the pandemic.

We then considered a variety of portfolio allocation strategies and of risk and performance measures, with the aim of assessing the impact of the inclusion of the green-bond indices in otherwise traditional portfolios. The difference between the two green indices, in terms of diversification potential and volatility, was confirmed by the weights attributed by the various allocation strategies. More weight was attributed to the Bloomberg Barclays MSCI Green Bond Index by strategies which prioritized variance reduction, while the Solactive Green Bond Index was selected exclusively when the aim was diversification maximization. This suggests that the diversification potential associated with the other green-bond index was absorbed by Solactive and by the non-green assets.

Green portfolios always proved preferable to non-green portfolios in terms of risk, in all periods and for all strategies. They also displayed lower losses in the bear period, a time of market downturn, for almost all considered strategies, while they had positive but lower returns during the bull sub-period, a time of market growth. The difference in the returns of green portfolios and non-green portfolios was emphasized by strategies which prioritized risk reduction over return optimization and thus attributed larger weights to the least risky assets: the two green bond indices. The inclusion of green bonds in the pre-pandemic period allowed the corresponding portfolios to achieve returns higher than or comparable to the non-green ones, and without facing as much risk. During the pandemic, the strategies which placed all focus on portfolio variance, thus neglecting to consider returns, lead to a substantially better performance of non-green portfolios in terms of increases in value. On the contrary, the mean–variance optimization strategy, which maximizes returns per unit of risk, lead to largely superior returns for the green portfolios. When adding to the analysis a further risk measure, in order to account for behavioral components by incorporating investor risk aversion, green portfolios were again shown to be consistently preferable, ceteris paribus, to their non-green counterparts. The risk-reduction and the diversification benefits provided by the inclusion of green indices was consistent across all strategies and for all periods, including the extreme scenario of the Covid-19 pandemic.

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Appendix A-E. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.eneco.2023.106587.

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