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# Workers' Firm in Mixed Duopoly\*

Flavio Delbono<sup>†</sup>    Diego Lanzi<sup>‡</sup>    Carlo Reggiani<sup>§</sup>

## Abstract

Cooperatives, including those owned and run by workers (Workers Firms, WFs), compete with capitalist firms in oligopolistic industries (mixed oligopolies). We rationalize several facts emerging from the empirical research as: the concern of WFs for their employment; the interplay between membership and workplace safeguard within WFs; the different reaction to shocks between WFs and capitalist enterprises. We do so by means of a new model of WFs' short-run behavior in a mixed duopoly. We innovate in modelling the WF's objective function by including both profits and employment, and characterize the resulting Nash equilibrium.

*JEL Classification:* L13, L21, P13.

*Keywords:* workers' firm, labor-managed firms, employment, oligopoly.

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<sup>†</sup>Department of Economics, University of Bologna, Italy. Email: flavio.delbono@unibo.it. Corresponding author.

<sup>‡</sup>Department of Statistical Sciences and Department of Management, University of Bologna, Italy. E-mail: diego.lanzi@unibo.it.

<sup>§</sup>Department of Economics, University of Manchester, United Kingdom. Email: carlo.reggiani@manchester.ac.uk.

# Workers' Firm in Mixed Duopoly

## Abstract

Cooperatives, including those owned and run by workers (Workers Firms, WFs), compete with capitalist firms in oligopolistic industries (mixed oligopolies). We rationalize several facts emerging from the empirical research as: the concern of WFs for their employment; the interplay between membership and workplace safeguard within WFs; the different reaction to shocks between WFs and capitalist enterprises. We do so by means of a new model of WFs' short-run behavior in a mixed duopoly. We innovate in modelling the WF's objective function by including both profits and employment, and characterize the resulting Nash equilibrium.

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# 1 Introduction

The last thirty years have witnessed an increasing volume of empirical research aimed at understanding the actual behavior of cooperative firms. The exponential growth of such an empirical literature is well illustrated and classified in Dow (2018), Jones (2018) and Mirabel (2021). This interest is likely fueled by the diffusion of the cooperative movement worldwide: according to the International Cooperative Alliance reports, in 2020, at least 12% of humanity is a cooperator of any of the 3 million cooperatives on earth (Euricse, 2020).

In what follows, we concentrate on workers' firms (WF, hereafter), i.e., a type of cooperative firm, in the past often named *labor-managed firms*, that has the following characteristics. "All, or most of, the capital is owned by employees (members) whether individually and/or collectively (capital ownership arrangements vary). All categories of employees can become members; and most employees are members.<sup>1</sup> Following international cooperative principles, members each have one vote, regardless of the amount of capital they have invested in the business. Members vote on strategic issues" (Pérotin, 2016, p. 2). Such enterprises have received a great deal of attention in the economic literature since John Stuart Mill as a model of enterprise alternative to the capitalist one.<sup>2</sup> They operate more in service industries (transport, catering, facility management, logistics, tourism, cultural activities, professionals) than in manufacturing.<sup>3</sup>

A lasting issue in comparative economics deals with the differences between WFs and conventional, i.e., capitalist firms (CFs, hereafter). To tackle this issue, the traditional approach pioneered by Ward (1958) is unsatisfactory. The economic system modelled by Ward, also known as Illyria, is an idealized economy in which workers-owned companies produce and sell in a perfectly competitive market environment. He assumes that a WF maximizes added value, net of non-labor costs, per member,<sup>4</sup> and its results raise two severe objections. On theoretical

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<sup>1</sup>We will refer to the *membership ratio* as to the ratio between (working) members and total employment at the firm or industry level. Obviously, the membership ratio deals with firms where members confer their work to the company that they co-own, whereas it would be meaningless for, say, users' cooperatives where members are customers as in retail trade, utilities, credit, insurance, housing. See Zamagni (2015) for a classification of cooperatives.

<sup>2</sup>Mill seemed fairly optimistic about the success of the cooperative form, as it transpires from his *Principles of Political Economy*, published in 1848. He thought that such worker-run cooperative organizations would eventually crowd capitalist enterprises out of the market because of their major efficiency and other benefits for the working owners.

<sup>3</sup>For an order of magnitude on the diffusion and relevance of WFs, see Bonin *et al.* (1993), Dow (2003, 2018) and the updated Euricse (2020). For the rich Italian experience, see Zamagni and Zamagni (2010) and the detailed map of Italian cooperatives in Cori *et al.* (2021).

<sup>4</sup>Under price taking behavior and unitary membership ratio, this is equivalent to maximize

grounds, in a competitive economy, such formulation entails a strange negative relationship between output price shock and output response.<sup>5</sup> Moreover, such an approach finds a limited empirical support. One may arguably claim that perfectly competitive market structures are rare and, in reality, WFs normally operate in oligopolistic product markets,<sup>6</sup> more precisely, in *mixed* oligopolies, i.e., concentrated industries hosting companies pursuing different goals.<sup>7</sup>

We propose a simple model, which does not aim to capture the subtleties of workers' firms; however, it allows us to realign the theoretical predictions with the extant empirical evidence. More precisely, we go beyond Ward's approach in several directions. First, including employment, in addition to profits, in the WF's maximand. This is also because, in the words of a recognised authority in this field: "...there is little doubt that LMFs have smaller output elasticity than CFs and are reluctant to layoff members. When they depart from profit maximisation, it is in the direction of employment maximisation, rather than maximisation of income per worker" (Dow, 2018, p.113). Second, employment, in turn, is split between members and non-member workers. Third, the weight assigned to profits and employment is made to depend on the market size. Indeed, we propose a new model of mixed duopoly in which a WF aims at maximizing a weighted sum of total profits and its employment. Moreover, we emphasize the different concern of the WF for working members and non-member workers. This captures the fact that WFs do not exhibit a unitary membership ratio (as it is usually assumed in the theoretical literature, since Ward, 1958), and there are reasons to believe that members be more protected than hired workers during downturns. In doing so, we bridge theory and the robust empirical evidence collected in the last decades (see Section 2).

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profit per worker-member.

<sup>5</sup>This is well-known as the perverse effect, and it is not the only one: another one is that the short-run output adjustment would be positive as a response to an increase in fixed costs. Moreover, as shown in Delbono and Lambertini (2014), in an infinite supergame among Ward-like players, in equilibrium tacit collusion is increasing in the number of participants, as opposed to the familiar conclusion under profit-maximizing behaviour. Furthermore, Delbono and Lambertini (2016) show that horizontal mergers between labor-managed firms entail very different consequences with respect to similar arrangements between CFs.

<sup>6</sup>A remarkable exception comes from local markets for childcare services, disadvantaged people, elderly: here the buyers are often local public institutions auctioning the provision of such services to groups of social cooperatives (active in Italy since the early '90s of the last century). Such markets often echo oligopsonistic types of competition. In Italy, the social cooperatives represent an increasingly large subset of WFs.

<sup>7</sup>We analyse a mixed duopoly setting in which a conventional firm competes with a workers' firm. To avoid any possible confusion we note that in recent years, and unlikely their original connotation (De Fraja and Delbono, 1990), the term mixed oligopoly has largely been employed for markets in which private firms compete with a public firm.

The results of our analysis succeed to capture and rationalize the following *stylized facts*. First, WFs operate in oligopolistic product markets where they compete with capitalist enterprises. Second, WFs care about their employees. Third, WFs protect their employment with different intensity between working members and non member workers. Finally, during downturns, WFs may prefer to sacrifice profits if required to safeguard employment.

The rest of the paper is organized as follows. In Section 2, we summarize the two relevant streams of theoretical literature that we bridge in our model also on the basis of insights stemming from the empirical research. In Section 3 we draw some results from a fairly general mixed duopoly model and, to gain further insights, we specialize the model. Section 4 contains a discussion of the results. Section 5 concludes.

## 2 Theory and empirical evidence

An alternative *theoretical* formulation of the WF objective function with respect to Ward (1958) is in Kahana and Nitzan (1989), who proceed along the path suggested by Fellner (1947) and Law (1977). Under price-taking behavior, a workers' enterprise chooses inputs and output to maximize income per worker/member subject to an employment constraint or, alternatively, the level of employment subject to a profit per worker/member constraint (bounded below by the collective wage). Standard duality arguments show the equivalence between these two formulations, which try to consider the concern for employment that should shape the decisions of firms owned and run by workers-members according to democratic governance (one head-one vote). The inspiring paper by Law actually considered an "augmented" utility function of the representative working-member in which, in addition to income per worker, there is room also for "employment". While interesting, these attempts (see also Miyazaki and Neary, 1985) to model the objective function of a WF are confined to price-taking behavior in the product market.

Another group of papers has tackled the behavior of WFs within models of mixed oligopoly. To the best of our knowledge, the first research investigating the strategic interaction between a CF and a WF has been proposed by Miyamoto (1982) in the wake of Meade (1974). He models a homogeneous duopoly where a CF plays a Cournot game with a labor-managed one, i.e., a firm which maximizes net income per worker. Miyamoto (1982) also provides a taxonomy of the properties of the Cournot equilibrium of such a mixed oligopoly.

Especially in the early '90s, several papers have then dealt with mixed oligopolies: for instance, Mai and Hwang (1989), Horowitz (1991), Cremer and Crémer (1992), Delbono and Rossini (1992). In these last papers the comparative statics properties of the Cournot equilibrium all fit the taxonomy in Miyamoto (1982) quite neatly.

Starting from the early 90's, major attention has been dedicated to the *empirical* analysis of cooperatives. In a number of papers, Craig and Pencavel (1992, 1993) and Craig *et al.* (1995) investigate the plywood industry in the US Pacific Northwest between the late '60s and mid '80s of the last century. They conclude that, with respect to conventional firms, a WF "is more likely to adjust earnings and less likely to adjust employment" (Craig and Pencavel, 1992, p. 1103) as a reaction to changes in their market conditions.

In another paper, they estimate the parameters of a general objective function for WFs which nests dividend maximization and employment maximization as special cases, and they conclude that "employment seems figure more prominently than earnings in the cooperatives' objectives" (Craig and Pencavel, 1993, p. 307). They reach this finding within a model where the product market is a mixed oligopoly in which price-taking cooperatives choose wages, hours, employment and the level of a non-labor input.

The same methodology of Craig and Pencavel (1993) is shared by Burdín and Dean (2012) using a panel of Uruguayan firms between 1996 and 2005, including the entire population of WFs. Burdín and Dean (2012) conclude that WFs put some weight on both employment and income per worker (close to profit maximization, as we know), and estimate the weight assigned to profit. Using the same database as in their 2012 paper, Burdín and Dean (2009), compare employment and wage decisions within workers' cooperatives. They show, *inter alia*, that the employment adjustment is larger in CFs than in WFs.

The institutional settings considered in these empirical papers vary across countries and periods in terms, for example, of labor market rules, collective contracts, and civil and fiscal legislation. However, overall, the evidence suggests that while CFs tend to adjust employment to follow fluctuations in demand, WFs adjust pay to protect employment levels, at least for their members (Pérotin, 2012).<sup>8</sup>

This conclusion has been confirmed, for instance, by Delbono and Reggiani (2013) for a large group of Italian WFs immediately after the 2008 financial crisis; Euricse (2013, pp. 87-102) for a large sample of medium-large Italian cooperatives between 2006 and 2010; Navarra (2016) for a small sample of Italian WFs between

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<sup>8</sup>Note that, since the wage is frequently set through national collective bargaining, such adjustment may regard the number of working hours as well as the distribution of the rebates.

2000 and 2005; Istat-Euricse (2019, pp. 22-26) comparing employment in Italian cooperatives (not only WFs) with respect to other firms in 2007 and 2015; Caselli *et al.* (2022) for all cooperatives and cooperative controlled firms in the Emilia-Romagna region between 2010 and 2018.

Figure 1: Value added of all cooperatives (not only WFs) and CFs in Emilia-Romagna, million euros. Source: Caselli *et al.* (2022).

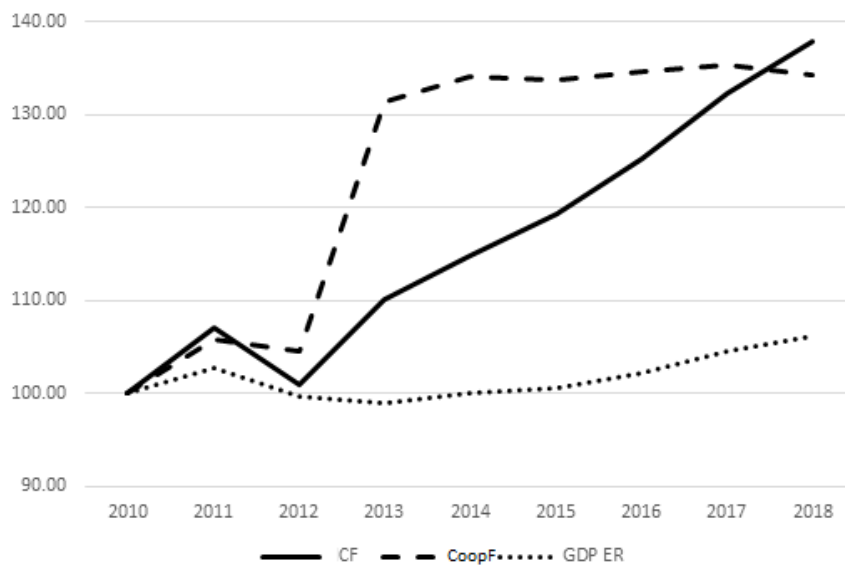
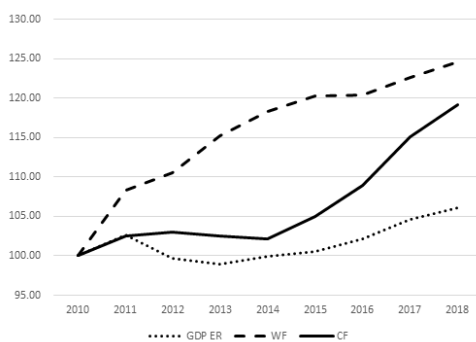
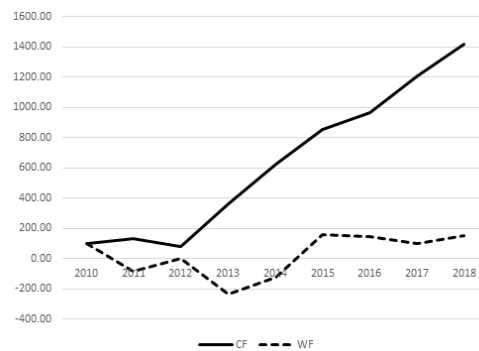


Figure 2: Employment (100 in 2010) and profits (million euros) of firms in Emilia-Romagna. Source: Caselli *et al.* (2022).



(a) Employment



(b) Profits

These findings hint at a WF's objective function along the lines suggested by Craig and Pencavel (1993) and Burdín and Dean (2012). According to these authors,



the WFs' implicit maximand is a weighted average of profits and employment, the weight assigned to the latter becoming larger during slumps, even at the cost of incurring temporary losses.

Figures 1 and 2, both from Caselli *et al.* (2022), show the pattern of GDP in the Italian region Emilia-Romagna, the world's most sizable cooperative district, the added value of all cooperatives (not only WFs) and of CFs (Figure 1). In Figures 2a and 2b, we plot employment and profits, respectively, in the same time span. Unfortunately, data are not yet available for the period since 2019 and this lack prevents one from detecting the consequences of COVID-19 on the relevant variables.

The differences between cooperatives and non cooperative firms are striking. The added value increases in both groups of firms. However, such a similar expansion yields drastically diverging consequences: profits grow fourteen-fold in CFs and only 53% in cooperatives, whereas the number of employees increase by 19% in CFs and almost by 25% in cooperatives. While CFs tend to be pro-cyclical, cooperatives seem to stabilize their employment and, given their critical mass, they contribute to flatter also the overall regional employment level, even by giving up profits (Caselli *et al.*, 2022).

### 3 The model analysis

#### 3.1 The general model

We consider a mixed duopoly in which a workers' firm (labeled W) and a capitalist one (labeled C) produce an homogeneous good, and compete in a game where labor as the choice variable. Workers are homogeneous in skills and abilities; the nominal wage,  $\omega > 0$ , and the length of the workday are institutionally fixed. Labor supply is unconstrained at the market wage  $\omega$ .<sup>9</sup> Both firms have a short run production function,  $f$ , defined as:  $q = F(L, \bar{K}) = f(L)$ , in which the amount of capital is fixed. Regarding  $f(L)$  we assume:

$$(i) f(0) = 0, (ii) f' > 0, (iii) f'' < 0, (iv) \lim_{L \rightarrow 0} f' = \infty, (v) \lim_{L \rightarrow \infty} f' = 0.$$

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<sup>9</sup>The value of market wage can be thought of as emerging from either national collective bargaining - this is the case in Italy, for instance - or competitive interaction in the labor market. As a consequence, there is no asymmetry between the two firms as for technology and costs. Indeed, the empirical research does not support either view about the presence of productivity gaps: "there are enough instances in which co-ops seem no less efficient than capitalist firms that a presumption of co-ops relative inefficiency is not warranted" (Pencavel, 2012, p.26). In our model the only difference between the two types of firm lies in the objective function.

Assumptions (iii)-(iv)-(v) are the well-known Inada conditions ensuring an interior solution. In our setting, these are sufficient to guarantee the existence and uniqueness of a Nash equilibrium. Moreover, both producers have short run fixed costs ( $\Gamma$ ). Market price is strictly decreasing with respect to total quantity:  $p = p(Q)$ ,  $\frac{dp}{dQ} < 0$ ,  $Q = q_W + q_C$ . There is a finite upper bound on demand when price approaches zero. Given the above monotonic production function, the market game we are going to analyze is equivalent to a quantity setting game that has the amount of labor as players' strategy.

For each enterprise, CF and WF, the profit is given by:

$$\Pi_i(L_i, L_j) = p[(f(L_i) + f(L_j))]f(L_i) - \omega L_i - \Gamma, \quad i \neq j = C, W. \quad (1)$$

The CF maximizes (1) with respect to  $L_C$ , whereas the WF maximizes the following objective function with respect to  $L_W$ :

$$V = \phi \Pi_W(L_W, L_C) + (1 - \phi) [m + \beta(1 - m)] L_W = \phi [p(f(L_C) + f(L_W))f(L_W) - \omega L_W - \Gamma] + (1 - \phi) [m + \beta(1 - m)] L_W. \quad (2)$$

Notice that equation (2) is a weighted-average of the WF's profits and its employment, where  $\phi \in (0, 1]$  is the weight assigned to profits. Assuming that  $\phi$  is strictly positive ensures the concavity of (2) with respect to  $L_W$ . At any rate, if  $\phi$  goes to zero, the optimal  $L_W$  is bounded above by the maximum quantity that can be sold. Burdín and Dean (2012), discussed in Section 2, estimate the value of  $\phi$  ranging between 0.70 and 0.91. Expression (2) also encompasses the presence of corporate stock companies controlled by WFs. Indeed, in many industries we observe subsidiaries controlled by cooperative firms or cooperative groups (for the case of Italy, see Istat-Euricse, 2019).

Our model assumes that the number of workers can be linearly transformed into profit-equivalent values. From a purely theoretical point of view, rationales may exist to justify convex or concave functions. For instance, an argument may be put forward in favour of concavity by suggesting that the larger the number of workers, the smaller the voting power of the additional worker (and thus the weaker conditions and value of that worker). Alternatively, an argument in favour of convexity may rely upon suggesting that the number of social links of the additional worker increases with their number. Whereas the exact functional form is ultimately an empirical issue, assuming a linear relation may be justified on the ground of analytical tractability.

Note that the WF considers its wage bill a cost rather than one of the goals to be maximized. This looks at odds with the traditional Ward's formulation, but it is what we observe in reality as the wage rate is often set through a national collective bargaining.

Clearly, if  $\phi = 1$  we obtain the standard duopoly model between CFs. For the moment, we consider  $\phi$  as a fully exogenous parameter, but we will make it dependent on the parameters of the demand function to illustrate the anti-cyclical behavior of the workers' firm.

Moreover, in equation (2) workers of the WF are divided into members,  $L_M$ , and non-members,  $L_{NM}$ , with  $L_W = L_M + L_{NM}$ . Hence, the membership ratio,  $m$ , is:

$$m = \frac{L_M}{L_W} = 1 - \frac{L_{NM}}{L_W}. \quad (3)$$

Only members share WF's profits, if any, in the form of rebates. If  $m = 1$  all workers are members. If  $m < 1$ , the WF distinguishes between labor supplied by members and by non-members, and the latter may receive less protection than the former in case of negative shocks. Indeed, the parameter  $\beta \in [0, 1]$  in (2) measures how the WF internalizes the employment of non-members in its overall payoff. Anecdotal evidence suggests that  $\beta < 1$  at least during slums: indeed, the layoffs of non-member workers seem to exceed those of working members in downturns (Burdín and Dean, 2009).

To recap, according to expression (2), the WF experiences a constant marginal benefit  $(1 - \phi)$  in hiring workers. Such a positive reward from employment depends on both the WF's membership ratio  $m$ , i.e., the percentage of workers who are also members, and the weight assigned to non-member employees,  $\beta$ .

In our model, the source of divergence between market players' behavior stems from the value assigned to employment by the WF. The less important and inclusive such an aim is, the lower will be the *employment-enhancing* effect of the WF. This claim is proved in Proposition 1, where, for ease of notation, we set  $\gamma \equiv m + (1 - m)\beta$ ,  $\gamma \in (0, 1]$ . The parameter  $\gamma$  summarizes the importance of membership within the WF's objective function. Indeed, an increase in  $m$  corresponds to a larger number of working members, while an increase in  $\beta$  amounts to treating non-member workers more similarly to members in the WF's concern for employment.

Notice that  $m$  must be strictly positive: in Italy, for instance, there must be at least three members to register a co-operative. Since our model is short-run in nature, we take  $m$  as given: this a reasonable assumption as such ratio is usually set with a long-run perspective. The value of  $m$  is not registered in the balance

sheets of WFs, but in Italy, the national average value was around 0.7 ten years ago (Delbono and Reggiani, 2013) and it seems unchanged according to one of the major cooperative associations (Legacoop) in 2019. Moreover, as  $\gamma$  is strictly increasing in  $m$ , if  $m$  shrinks, the WF degenerates into a CF. This happens when non-member workers replace departing member workers, a phenomenon often observed in large WFs.<sup>10</sup>

**Proposition 1.** *In the unique Nash equilibrium of this mixed duopoly, the WF hires more workers and produces more output than the CF.*

**Proof:** From expressions (1) and (2), it is apparent that the marginal revenue functions of the two firms, absent any concern for employment by the WF, are strictly decreasing in their own  $L$ . Indeed, after dividing (2) by  $\phi$ , we can write:

$$MR_C = MR_W = \frac{\partial p f(L_i)}{\partial L_i}, \quad i = C, W.$$

Hence, the optimal quantity of labor  $L_i^*$  for each firm is determined by the following conditions:

$$L_C^* | MR_C(L_C) = \omega$$

$$L_W^* | MR_W(L_W) = \omega - \frac{1 - \phi}{\phi} \gamma$$

Straightforwardly, given that:  $\frac{1 - \phi}{\phi} \gamma > 0$  for  $\phi \neq 1$  and  $MR_i(L_i)$  is decreasing in  $L_i$ , it is true that  $L_W^* > L_C^*$ . Since output is strictly increasing in labor, it follows that in equilibrium  $q_W^* > q_C^*$ . Q.E.D.

Note that in our model, both in the  $L_C - L_W$  space and in the  $Q_C - Q_W$  one, the reaction functions are monotonically decreasing as in the textbook version of the Cournot model. This feature is driven by our formulation of the WF's objective function, (2). In contrast, had the WF been as in Ward (1958), then its reaction function would be upward sloping and its equilibrium output lower than the CF's (Delbono and Rossini, 1992).

The following corollary can be stated:

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<sup>10</sup>If the membership ratio progressively shrinks, the original WF tends to mimic a CF. This phenomenon has been stigmatized as the *degeneration* of the WF. For a thorough analysis, see Pencavel (2012) and Dow (2018), chapters 7 and 9. Notice that in a number of countries (e.g., Italy and Uruguay, among others) WFs lose tax benefits if  $m$  falls below specific thresholds.

**Corollary 1.** *In the mixed duopoly, the equilibrium price is lower than in a purely capitalist duopoly.*

The corollary descends from the total output in the mixed duopoly being larger than in the conventional profit-making setting, and the downward sloping demand function. Larger quantities, and a lower price, make the market equilibrium more competitive than the one with only profit-maximizing firms. Hence, consumers are better off in presence of a workers' firm in the industry. Such a result is reminiscent of the effect emerging in a Cournot-Nash equilibrium of a mixed oligopoly where one company maximizes the industry social welfare (De Fraja and Delbono, 1989).

This same result obtains also when a profit maximizing company competes against a consumer-friendly public firm (maximizing its profit and consumer surplus). The latter can be thought of as a consumer cooperative, i.e., one in which members are consumers (see Garcia *et al.*, 2018).<sup>11</sup> Another analysis related to ours considers the interplay between a conventional enterprise and a social (subsidised) firm as in Cho and Lee (2017). The ultimate goal of a such social enterprise is consumer surplus augmented by its weighted output, under a break even constraint.

Moreover:

**Corollary 2.** *In the mixed duopoly, the equilibrium output of the WF decreases with  $\phi$  and increases in  $m$  as well as in  $\beta$ .*

Unsurprisingly, the employment-enhancing effect of the WF increases when the weight of labor in its objective function increases, i.e., for lower  $\phi$ . On the other hand, an increase in  $m$  and/or  $\beta$  raises the relative importance of labor *vis-à-vis* profits in equation (2). In the case of  $m$ , the share of members increases, whereas in the case of  $\beta$  it harmonizes the concern for the employment of members and non-member workers. Both these changes expand the optimal level of employment and output of the WF.

### 3.2 The specialized model

In order to further study the properties of the mixed duopoly, we specialize the previous model as follows. We assume a quadratic production function:

$$q_i = \sqrt{L_i}, \quad i = C, W, \quad (4)$$

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<sup>11</sup>On a formal model of consumer cooperatives, see Marini and Zevi (2011).

and a linear inverse demand function:

$$p = a - Q, \quad (5)$$

where  $a \in (\omega, \bar{a}]$  is the maximum quantity when price goes to zero. The finite parameter  $\bar{a}$  can be understood as the maximum potential quantity, for example, resulting from a positive demand shock.

Plugging (4) and (5) into the objective functions (1)-(2) and solving for the labor demands, we then obtain the following optimal output:

$$\begin{aligned} q_C^* &= \frac{a[(2\omega + 1)\phi - 2\gamma(1 - \phi)]}{(4\omega^2 + 8\omega + 3)\phi - 4\gamma(\omega + 1)(1 - \phi)} \\ q_W^* &= \frac{a(2\omega + 1)\phi}{(4\omega^2 + 8\omega + 3)\phi - 4\gamma(\omega + 1)(1 - \phi)} \end{aligned} \quad (6)$$

In line with Proposition 1, it is easy to verify that the WF produces more than the CF. The equilibrium can now be fully characterized, and all the expressions can be found in Appendix A. More precisely, we can now compare the equilibrium profits of the two firms.

**Proposition 2.** *If the weight of profits in the WF's objective function is large enough, the WF profits are larger than the CF ones.*

**Proof:** We start by noting that the non-negativity of the equilibrium profits of both firms requires the fixed cost  $\Gamma$  being not too large. Alternatively, for a given fixed cost  $\Gamma$ , there is a minimum value of  $\phi$ , the weight of profits in the WF's objective function, ensuring that profits are non-negative. We identify such minimum values, as a function of the parameters of the model with  $\underline{\phi}_W(a, \omega, \gamma, \Gamma)$  and  $\underline{\phi}_C(a, \omega, \gamma, \Gamma)$  for the WF and CF, respectively. Their explicit expressions can be found in Appendix A, equations (12) and (13). Hence, it must be that:

$$\phi \geq \underline{\phi} = \max \left\{ \underline{\phi}_W, \underline{\phi}_C \right\}.$$

Given the equilibrium quantities derived above, we can compute the corresponding profits. The difference between the profits of the CF and the WF is:

$$\Delta\pi = -\frac{2a^2\gamma(1 - \phi)[2\gamma(\omega + 1)(1 - \phi) - (2\omega + 1)\phi]}{[(4\omega^2 + 8\omega + 3)\phi - 4\gamma(\omega + 1)(1 - \phi)]^2}.$$

The denominator is always positive. The numerator is negative if the term  $2\gamma(\omega + 1)(1 - \phi) - (2\omega + 1)\phi$

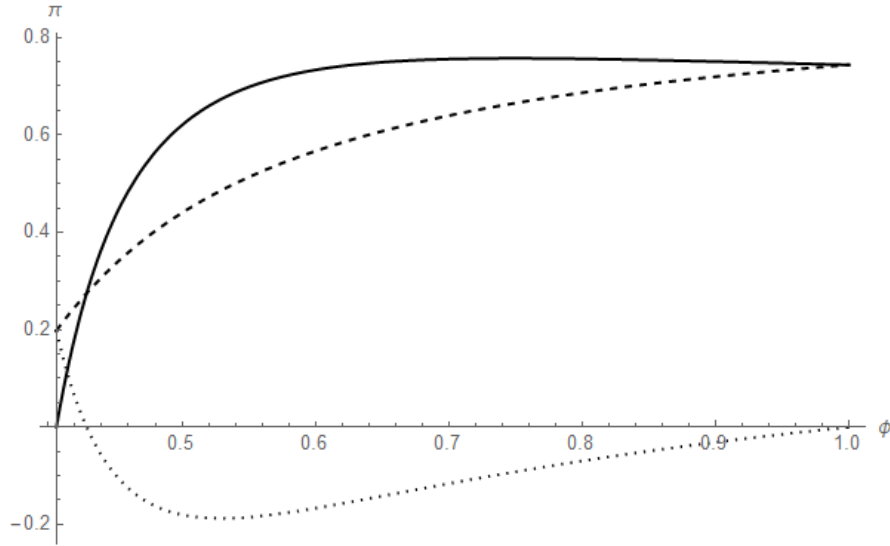
1)(1 -  $\phi$ ) - (2 $\omega$  + 1) $\phi$  is positive. This is the case for:

$$\phi > \phi^* = \frac{8\gamma(\omega + 1)^2}{8\gamma + 8\gamma\omega^2 + 4\omega^2 + 16\gamma\omega + 8\omega + 3}.$$

It can be verified that:  $\underline{\phi} < \phi^* < 1$ . This completes the proof. Q.E.D.

In the rest of the paper (including the relevant figures), consistently with Proposition 2, we restrict the attention on parameters' constellations such that  $\phi \geq \underline{\phi}$ . The contents of Proposition 2 are illustrated in Figure 3, where the origin of the horizontal axis is indeed set at  $\underline{\phi}$ , i.e., the minimum value that guarantees non-negative profits to both firms. For values of  $\phi$  in the region on the left of  $\phi^*$ , CF makes more profits than the WF. In correspondence of the parameters in the example of Figure 3, it turns out that  $\underline{\phi} = 0.406$  and  $\phi^* = 0.429$ . For values of  $\phi$  greater than  $\phi^*$ , the WF is more profitable than the CF.

Figure 3: Equilibrium profits of the WF, CF and their difference ( $a = 3$ ,  $\omega = 0.5$ ,  $\Gamma = 0.1$ ,  $\gamma = 0.5$ ). Profit of WF: solid line; profit of CF: dashed; profit difference: dotted.



The intuition for these findings is as follows. Start from the limit case of capitalist duopoly, i.e.,  $\phi = 1$ . Moving left means that the WF gives increasing weight to employment. As we know, this entails a greater output, and greater profits. Because of the decreasing returns to scale in production, as the weight keeps increasing, the profit gap shrinks and ends in correspondence of  $\phi^*$ . Such profit gap is then non-monotonic and it reaches a maximum in our example at  $\phi = 0.529$ . We note,

however, that this is not the value of  $\phi$  that maximizes WF's profits, which is concave in  $\phi$  and, in the example in Figure 3, reaches its peak at  $\phi = 0.75$ . CF's instead are monotonically increasing in  $\phi$ . This latter finding is not surprising as the CF's profit is higher when the CF is competing with another profit maximizer than with a company concerned also with employment.

A notable feature of Proposition 2 is that, in a mixed duopoly under quadratic technology and linear demand, the WF can earn higher profits than the CF even by pursuing *not only* profits. This result evokes the conclusion of the literature pioneered by Vickers (1985, p. 138) that, in markets where firms are interdependent, "it is not necessarily true that maximum profits are earned by firms whose objective is profit maximization". In Vickers (1985)'s model such a finding is obtained in an oligopolistic model of managerial incentives where managers may be asked to maximize a mix of firm's profits and output. Under Cournot rules in the product market, this arrangement ultimately yields an outward shift of the reaction function of the managerial company, a larger market share and higher profits (as it would happen because of a reduction in its marginal costs).

**WFs duopoly.** Besides the previously discussed capitalist duopoly ( $\phi = 1$ ), an even more interesting benchmark is a duopolistic market in which both firms are WF. The equilibrium output and price of the WFs' duopoly are:

$$q^d = \frac{a\phi}{2\gamma\phi - 2\gamma + 2\omega\phi + 3\phi}$$

$$p^d = \frac{a(2\gamma\phi - 2\gamma + 2\omega\phi + \phi)}{2\gamma\phi - 2\gamma + 2\omega\phi + 3\phi}$$

The output of each firm in this "pure" WFs duopoly lies in between the output of the CF and the WF in the mixed duopoly. Clearly, the three quantities tend to the same value as  $\phi$  tends to one. The overall quantity, however, is larger under the pure duopoly, implying a lower equilibrium price and a higher consumer surplus. As for profits, it turns out that now each WF obtains less than in a mixed duopoly, but more than a CF in such a market, provided that the weight of profits is large enough. For more details, see Appendix B.

**Welfare analysis.** The novelty of our model lies in the relative weight of profit vis-à-vis employment, captured by the parameter  $\phi$ . Hence, it is of interest to study how changes in  $\phi$  affect consumer and producer surplus, and welfare.

As for consumer surplus, which in our model is proportional to the aggregate output (equation 10 in Appendix A), it turns out that it is monotonically decreasing

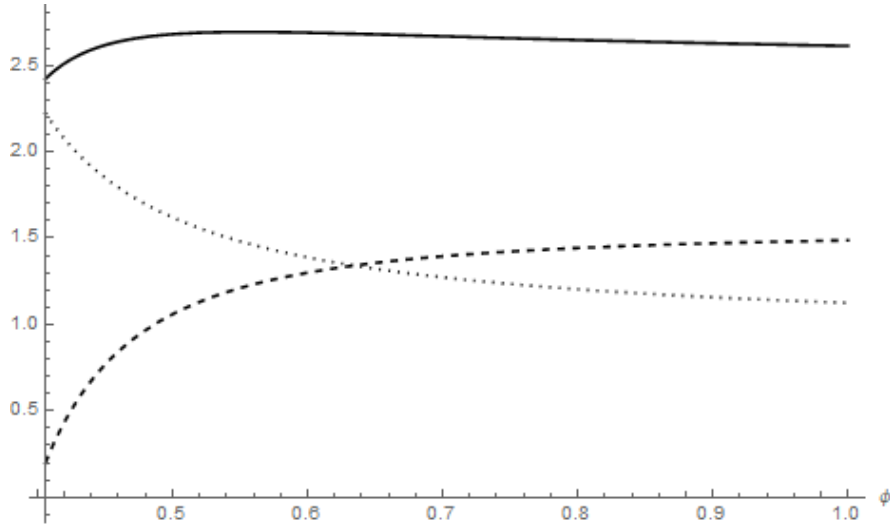


in  $\phi$ . The intuition is as follows. A greater weight to profit of the WF entails a decrease in its equilibrium output. At the same time, the CF expands its output, as reaction functions are downward sloping. The first effect dominates the second, and consumer surplus shrinks as a result.

Turning to the producer surplus, defined as the sum of the profits of all firms, it is monotonically increasing in  $\phi$ . Despite the fact that the WF's profit reaches a maximum at an interior value of  $\phi$ , as noted before, the producer surplus keeps increasing, driven by the CF's profit.

As a result of the previous findings, the market welfare needs not to be monotonic. Indeed, in correspondence of the parameter constellation of Figure 3, welfare reaches a maximum for  $\phi = 0.56$ . The previous discussion is graphically illustrated in Figure 4. A formal proof of the comparative statics with respect to  $\phi$  of producer surplus, consumer surplus and welfare is contained in Appendix A.

Figure 4: Equilibrium producer surplus, consumer surplus and welfare ( $a = 3$ ,  $\omega = 0.5$ ,  $\Gamma = 0.1$ ,  $\gamma = 0.5$ ). Producer surplus: dashed; consumer surplus: dotted; welfare: solid.



### 3.3 The anti-cyclical behavior of the workers' firm

In order to address the well documented anti-cyclical behavior of the WFs, we relate the weight the WF assigns to profits,  $\phi$ , to the position parameter of the demand function,  $a$ . In particular, we set

$$\phi(a) = \frac{a - \omega}{\bar{a} - \omega} \quad (7)$$

By construction,  $\phi(a) \in (0, 1]$ ; this derives from the fact that  $\bar{a}$  has been defined as the largest possible market size and  $(\bar{a} - \omega)$  may be interpreted as such potential market size net of the marginal cost. As claimed previously, a reduction of  $a$  may be interpreted as a negative demand shock.

Clearly, we are aware that capturing macroeconomic shocks within a partial equilibrium model is a challenging task. Our choice of modelling the dependence of the weight of profit in the WF maximand from the demand parameter as in (7) is driven by the sake of analytical tractability. Nevertheless, this formulation displays some intuitive properties and is conducive to insights apparently consistent with the empirical evidence, as we remark below.

Through this extended version of the model, we can show the following:

**Proposition 3.** *If the market size  $a$  is small enough, in the Nash equilibrium of the mixed duopoly, the WF behaves anti-cyclically.*

**Proof:** The equilibrium output levels of the extended model with (7) are given by:

$$\begin{aligned} q_C^{**} &= \frac{a(2a\gamma + 2a\omega + a - 2\gamma\bar{a} - 2\omega^2 - \omega)}{4a\gamma + 4a\omega^2 + 4a\gamma\omega + 8a\omega + 3a - 4\gamma\bar{a} - 4\gamma\bar{a}\omega - 4\omega^3 - 8\omega^2 - 3\omega}, \\ q_W^{**} &= \frac{a(2\omega + 1)(a - \omega)}{4\gamma(\omega + 1)(a - \bar{a}) + (4\omega^2 + 8\omega + 3)(a - \omega)}. \end{aligned} \quad (8)$$

By taking the derivative of  $q_W^{**}$  with respect to  $a$  we obtain:

$$\frac{\partial q_W^{**}}{\partial a} = \frac{(2\omega + 1)[4\gamma(\omega + 1)(a^2 - 2a\bar{a} + \bar{a}\omega) + (2\omega + 1)(2\omega + 3)(a - \omega)^2]}{[4\gamma(\omega + 1)(a - \bar{a}) + (4\omega^2 + 8\omega + 3)(a - \omega)]^2}.$$

The sign of the derivative is the same as the sign of the numerator. It can be shown that it is negative for  $a$  below a critical threshold, reported in Appendix C, equation (17). Q.E.D.

Proposition 3 provides an interpretation of the reaction of the WF to demand shocks. The intuition for this finding stems from the formulation of the WFs objective function, as well as equation (7). The objective function of the WF attaches some weight to its employment; such weight, according to equation (7), increases in a downturn and it entails that the WF mitigates the consequences of shocks. More precisely, it establishes that, if the market size  $a$  is not too large, the labor demand and the corresponding output move in the opposite direction as compared to the demand shock. For instance, in recessionary period when demand

shrinks the WF expands its employment and then, as observed in a number of studies surveyed in Section 2, its output.

## 4 Discussion

In this paper, we have tried to innovate upon Ward's workers' firm approach. First of all, we considered an oligopolistic product market. We then embed employment, in addition to profits, in the WF's objective function. Moreover, we split employees between members and non-member. Finally, we made the weight assigned to profits and employment dependent on the demand parameters.

It turns out that in the Nash equilibrium of our mixed duopoly, the WF employs more workers and, as capital is fixed, utilizes more labor intensive production processes than capitalist firms. Moreover, it may behave anti-cyclically in front of demand shocks hitting the industry. These traits of WFs make the market equilibrium of the mixed duopoly more competitive than a standard Cournot-Nash duopolistic equilibrium.

It is worth stressing that also our specification of the WF's objective function may yield what the literature has stigmatized as "perverse effects" of the WF's supply curve, although we have apparently ennobled them as anti-cyclical responses. However, such comparative statics finds in our model a very different explanation with respect to Ward's approach. In Ward (1958)'s model the WF always increases output and then the number of workers-members as a reaction to a fall in output price, and vice-versa. Since the WF maximizes net income per member, it restricts the workforce and then memberships by using fewer workers than a CF if profits increase. If the output price augments, the WF has an incentive to shrink the workforce, opening the door to a negatively sloped output supply curve. By the same token, the WF increases its labor demand as a response to higher fixed costs, in order to split it among a larger number of members-workers.

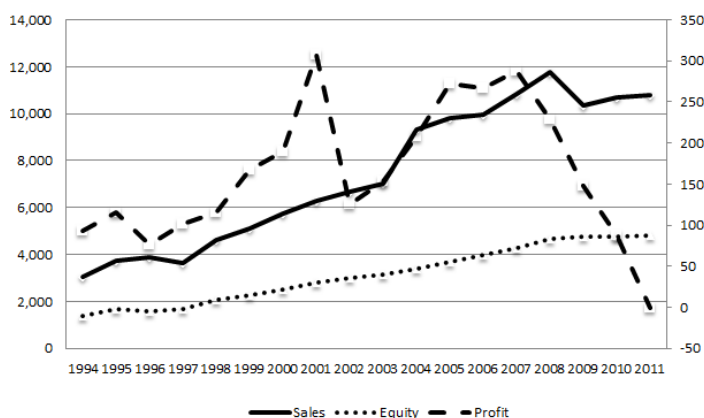
Our model too may predict such responses, but they emerge only under some circumstances and, above all, they are driven by the explicit concern for employment (in some proportion between members and non-members) featuring the strategies of the WF in a mixed oligopoly.

A further comment is worthwhile regarding profits. Notwithstanding that our analysis is static and short-run in nature, the empirical evidence indicates that a WF is better equipped to resist temporary losses than a capitalist one. Although in our model we rule out that the WF makes negative profits in equilibrium, it might

be able to absorb them if needed to protect employment. Empirically this has been detected as in the case visualized in Figure 2(b).

In this respect, Figure 5 is revealing. It shows neatly the forward-looking policy of a large sample of Italian WFs. On average, they distributed about 5% of profits to members. In the same period, the largest Italian capitalist companies distribute more than two thirds of their profits in the form of dividends. This is the basic reason why the Italian WFs have been more resilient than the profit-making enterprises during the downturn following the 2008 financial crisis. Our model can easily accommodate an amended profit constraint that allows for temporary losses for the WF.

Figure 5: Sales, equity (left axis) and profits (right axis) of a sample of Italian workers' firms (million euros). Source: Delbono and Reggiani (2013).



## 5 Conclusions

In a Ward (1958)'s economy, firms are supposed to be under the control of worker councils elected on a democratic, one-member/one vote basis, which select managers running the firm in a perfectly competitive product market, absent any constraint from the (unmodelled) labor market. Illyrian firms were of obvious interest in the debate about market socialism, as emphasized by Ward (1967), because they are one of the simplest organizational forms satisfying the requirements for a decentralized and socialist economy. In this paper, we have dropped the assumption of perfect competition in the product market, and the market syndicalism embedded into Ward (1958)'s objective function, and focused our attention

on mixed duopolies in which a labor-concerned WF and a CF compete in labor demand and, then, in output levels. Our analysis shows that, even beyond market socialism, the role of WFs can be relevant in shaping the equilibrium of imperfectly competitive markets.

Our simple model could be extended in several directions: we mention some of them. First, our game may be seen as the last stage of a multi-stage game where the WF initially chooses the membership ratio and/or the concern for non-member workers. For example, the membership ratio may be made endogenous as the choice entails a trade-off for the initial members. Indeed, allowing new membership to workers yields an increase in the assets of the company (because of the entry fee) but also a larger number of recipients of the distributable profits. Second, the representation of the technology may be enhanced with a parameter capturing the productivity of labor, possibly different across types of firms. For instance, an increase of such parameter may be thought of as resulting from labor-saving technical progress. Third, as previously discussed, the functional specification of the WFs objective function might be generalized to encompass also non-linear combinations of profit and employment. Fourth, one may try to generalise the relationship between the weight of profits in the WF's objective function and macroeconomic shocks.

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## A The equilibrium of the specialized model

This appendix provides a full characterization of the equilibrium of the specialized model presented in Section 3.2. From the simultaneous maximization of the firms' objective functions, the Nash equilibrium employment levels are:

$$\begin{aligned} L_C^* &= \frac{a^2[(2\omega + 1)\phi - 2\gamma(1 - \phi)]^2}{[(4\omega^2 + 8\omega + 3)\phi - 4\gamma(\omega + 1)(1 - \phi)]^2}, \\ L_W^* &= \frac{a^2(2\omega + 1)^2\phi^2}{[(4\omega^2 + 8\omega + 3)\phi - 4\gamma(\omega + 1)(1 - \phi)]^2}. \end{aligned} \quad (9)$$

As reported in the text, the equilibrium output of each firm is:

$$\begin{aligned} q_C^* &= \frac{a[(2\omega + 1)\phi - 2\gamma(1 - \phi)]}{(4\omega^2 + 8\omega + 3)\phi - 4\gamma(\omega + 1)(1 - \phi)}, \\ q_W^* &= \frac{a(2\omega + 1)\phi}{(4\omega^2 + 8\omega + 3)\phi - 4\gamma(\omega + 1)(1 - \phi)}. \end{aligned}$$

The total output is then:

$$Q^* = \frac{2a[2(1 + \omega)\phi - \gamma(1 - \phi)]}{(4\omega^2 + 8\omega + 3)\phi - 4\gamma(\omega + 1)(1 - \phi)}, \quad (10)$$

and the corresponding equilibrium price:

$$p^* = \frac{a(2\omega + 1)[(2\omega + 1)\phi - 2\gamma(1 - \phi)]}{4\phi(\omega^2 + \gamma\omega + 8\omega) + 4\gamma(\phi + \omega - 1) + 3\phi}. \quad (11)$$

The equilibrium profits can be obtained as  $\pi_i^* = p_i^*q_i^* - \omega L_i^* - \Gamma$ . The consumer and producer surplus are, respectively,  $CS^* = Q^{*2}/2$  and  $PS^* = \pi_C^* + \pi_W^*$ , and welfare is  $W^* = PS^* + CS^*$ .

The non-negativity of the equilibrium profits of both firms,  $\pi_i^*$ , requires the fixed cost  $\Gamma$  being not too large. As for the WF, the minimum value of  $\phi$ , which depends on  $\Gamma$ , ensuring that it is the case is the following:

$$\begin{aligned} \underline{\phi}_W &= \frac{a\gamma(2\omega + 1)\sqrt{(2a\omega + a)^2 - 8\Gamma(\omega + 1)(2\omega(\omega + 2) + 1) + (2a\omega + a)^2}}{(2a\omega + a)^2(2\gamma + \omega + 1) - \Gamma(4\gamma + 4\omega(\gamma + \omega + 2) + 3)^2} + \\ &\frac{(2a\omega + a)^2 - 4\gamma\Gamma(\omega + 1)(4\gamma + 4\omega^2 + 4\gamma\omega + 8\omega + 3)}{(2a\omega + a)^2(2\gamma + \omega + 1) - \Gamma(4\gamma + 4\omega(\gamma + \omega + 2) + 3)^2} \end{aligned} \quad (12)$$

The equivalent threshold for the CF is:

$$\underline{\phi}_C = \frac{2\gamma^2(\omega + 1) [4\Gamma(\omega + 1) - a^2]}{\sqrt{a^2\gamma^2\Gamma(\omega + 1)(2\omega + 1)^2 + \gamma(\omega + 1) [2\Gamma(4(\gamma + \omega^2 + \Gamma\omega + 2\Gamma\omega) + 3) - a^2(2(\gamma + \omega) - 1)]}}. \quad (13)$$

In the main text, we focus on parameters' constellations such that  $\phi \geq \underline{\phi} = \max \{ \underline{\phi}_W, \underline{\phi}_C \}$ .

The partial derivative of the producer surplus with respect to  $\phi$  is:

$$\frac{\partial PS^*}{\partial \phi} = - \frac{2a^2\gamma(2\omega + 1)[4\omega(\omega + 1)(\phi + 2\gamma(1 - \phi)) + \phi]}{[4\gamma(\omega + 1)(1 - \phi) - (4\omega(\omega + 2) + 3)\phi]^3}. \quad (14)$$

As all the expressions at the numerator are positive, the sign of (14) coincides with the negative of the sign of the denominator, i.e.:

$$\text{sign} \left[ \frac{\partial PS^*}{\partial \phi} \right] = - \text{sign} [4\gamma(\omega + 1)(1 - \phi) - (4\omega(\omega + 2) + 3)\phi].$$

As  $\text{sign} [4\gamma(\omega + 1)(1 - \phi) - (4\omega(\omega + 2) + 3)\phi] < 0$  for  $\phi > \frac{4\gamma(\omega+1)}{4(\gamma+\omega^2+\gamma\omega+2\omega)+3} = \phi^{PS}$  and  $\phi^{PS} \leq \underline{\phi}$ , it follows that the producer surplus is monotonically increasing in  $\phi$ .

As consumer surplus is a monotonic transformation of total output, it is sufficient to consider the partial derivative of total output with respect to  $\phi$ :

$$\frac{\partial Q^*}{\partial \phi} = - \frac{2a\gamma(2\omega + 1)^2}{[4\gamma(\omega + 1)(1 - \phi) - (4\omega(\omega + 2) + 3)\phi]^2} < 0. \quad (15)$$

Finally, as welfare is the sum of the two previous functions, both monotonic in  $\phi$  but in opposite directions, the sign of its partial derivative needs to be studied. This is:

$$\frac{\partial W^*}{\partial \phi} = - \frac{2a^2\gamma(2\omega + 1) [2\gamma(4\omega^2 + 6\omega + 1)(1 - \phi) - 4\omega(\omega + 1)\phi - \phi]}{[4\gamma(\omega + 1)(1 - \phi) - (4\omega(\omega + 2) + 3)\phi]^3}, \quad (16)$$

which only has one zero at:

$$\phi^W = \frac{2(\gamma + 4\gamma\omega^2 + 6\gamma\omega)}{2(\gamma + 4\gamma\omega^2 + 2\omega^2 + 6\gamma\omega + 2\omega) + 1}.$$

As  $\frac{\partial W^*}{\partial \phi} \Big|_{\phi=1} = - \frac{2a^2\gamma}{(2\omega+3)^3} < 0$  and  $\phi^W > \underline{\phi}$ , we can conclude that welfare first increases in  $\phi$ , reaches a maximum at  $\phi^W$ , and then decreases.

## B Pure WFs duopoly

This appendix provides a characterization of the equilibrium of a pure WFs duopoly benchmark and a comparison with the mixed duopoly, as discussed at the end of Section 3.2.

The objective function of one of the competitors  $i$ , with  $i = 1, 2$  in this case, is:

$$V_i = \phi \left[ \left( a - \sqrt{L_i} - \sqrt{L_{-i}} \right) \sqrt{L_i} - \omega L_i - \Gamma \right] + (1 - \phi) \gamma L_i.$$

Simultaneously maximizing profits in  $L_i$  leads to the following symmetric equilibrium employment:

$$L_i^d = \frac{a^2 \phi^2}{(2\gamma\phi - 2\gamma + 2\omega\phi + 3\phi)^2}.$$

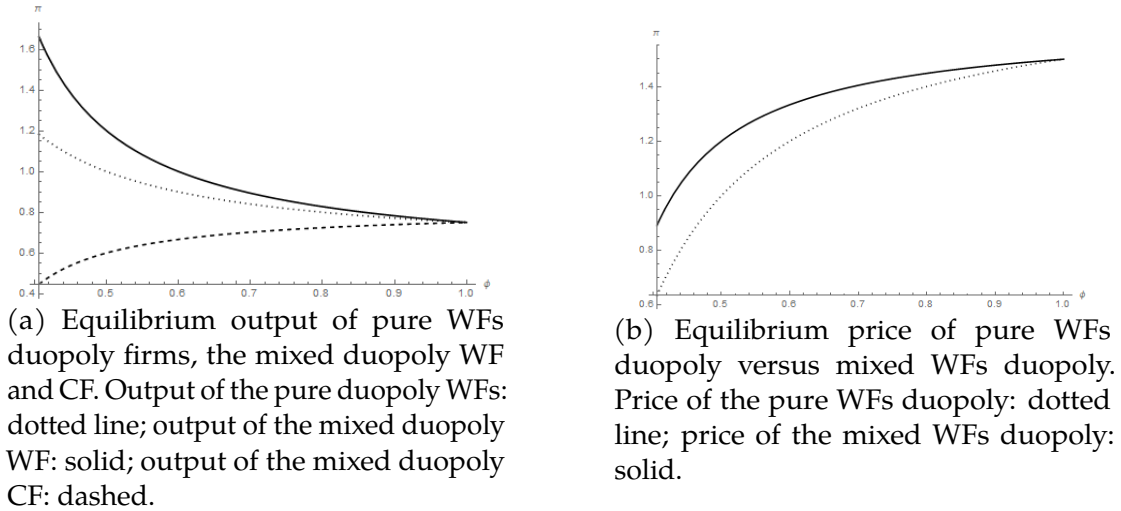
The corresponding equilibrium individual and aggregate output and price are as follows:

$$q_i^d = \frac{a\phi}{2\gamma\phi - 2\gamma + 2\omega\phi + 3\phi}$$

$$Q^d = \frac{2a\phi}{2\gamma\phi - 2\gamma + 2\omega\phi + 3\phi}$$

$$p^d = \frac{a(2\gamma\phi - 2\gamma + 2\omega\phi + \phi)}{2\gamma\phi - 2\gamma + 2\omega\phi + 3\phi}$$

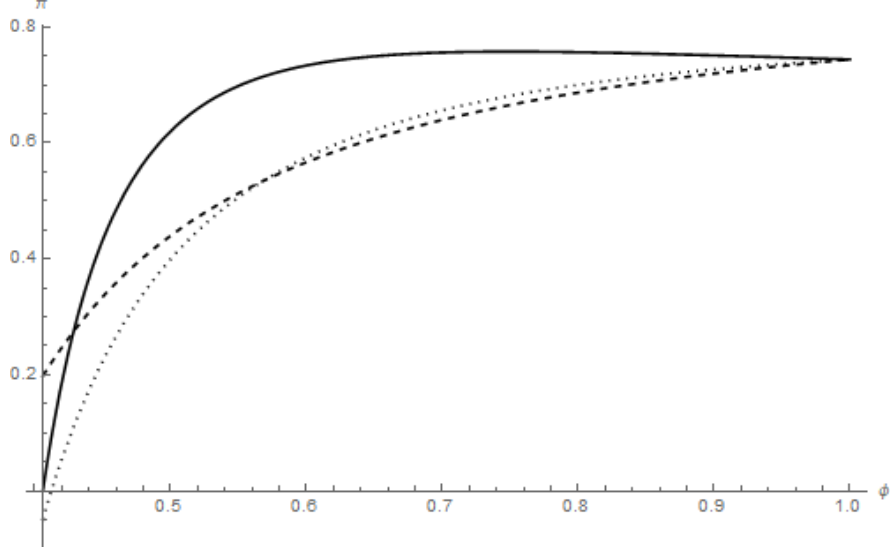
Figure B.1: Output and price comparison between a pure and a mixed WFs duopoly ( $a = 3, \omega = 0.5, \Gamma = 0.1, \gamma = 0.5$ ).



Turning to the profits, it can be seen from Figure B.2 that each duopoly WF obtains less than in a mixed duopoly but, provided that the weight of profits is

large enough, profits exceeds those of a CF in such a market.

Figure B.2: Equilibrium profits of pure WFs duopoly firms, mixed duopoly WF and CF ( $a = 3, \omega = 0.5, \Gamma = 0.1, \gamma = 0.5$ ). Profit of pure WFs duopoly: dotted line; profit of mixed duopoly WF: solid; profit of mixed duopoly CF: dashed.



## C Demand and the countercyclicity of the WF's behavior

In the main text, we showed that the optimal output of the WF,  $q_W^{**}$  may be increasing if the demand parameter  $a$  decreases (negative shock). This happens if the sign of derivative  $\partial q_W^{**}/\partial a$  is negative, i.e.:

$$\frac{\partial q_W^{**}}{\partial a} = \frac{(2\omega + 1) [4\gamma(\omega + 1) (a^2 - 2a\bar{a} + \bar{a}\omega) + (2\omega + 1)(2\omega + 3)(a - \omega)^2]}{[4\gamma(\omega + 1)(a - \bar{a}) + (4\omega^2 + 8\omega + 3)(a - \omega)]^2} < 0.$$

The critical value is, then:

$$a^{**} = \frac{4\gamma\bar{a}(\omega + 1) + 4\omega^3 + 8\omega^2 + 3\omega + 2\sqrt{\gamma(1 + \omega)(\bar{a} - \omega) [4\gamma\bar{a}(\omega + 1) + 4\omega^3 + 8\omega^2 + 3\omega]}}{4\gamma(\omega + 1) + 4\omega^2 + 8\omega + 3}, \quad (17)$$

and the output increases as  $a$  decreases as long as  $a < a^{**}$ .