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Optimal Terminations of 2D Meta-Surfaces for Uniform Magnetic Field Applications

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This paper investigates the possibility of obtaining a uniform magnetic field close to a 2D metamaterial made of magnetically coupled resonant circuits. The magnetic field is controlled by terminating the boundary of the meta-surface with proper additional impedances, whose values are calculated by means of an optimization procedure. The adopted circuit model has been discussed and compared with others presented in the literature, highlighting its advantages and disadvantages with reference to the considered application. Furthermore, the effect of the quality factor on the current distribution generating the field has been addressed. The uniformity of the magnetic field is then discussed, with reference to possible applications e.g. in magnetic resonance imaging or energy harvesting for wireless power transfer.

***Index Terms*—Metasurface, metamaterial, optimization, wireless power transfer, magnetic resonance imaging, MRI.**

I. Introduction

METAMATERIALS are assemblies of multiple elements, often called ?unit cells? or ?meta-atoms?, arranged in 2D or 3D patterns to form regular lattices. Shape, size, orientation of the unit cells give the material specific properties. They are widely exploited in areas such microwaves and optics, and have been recently employed also in wireless power transfer (WPT) systems with different goals, such as magnetic field focusing and electric field shielding [1]–[4]. Metamaterial structures are usually analysed by means of circuit theory, offering the possibility of evaluating the magnitudes of currents and voltages in each unit cell. Another possible approach is based on the theory of magneto-inductive waves (MIW) [5], [6], which has been developed for 1D, 2D and 3D lattices. In particular, for 1D structures it allows a better understanding of the matching conditions, as shown in [7].

In this contribution we investigate the possibility of optimizing the meta-surface terminations (lumped impedances) considering a full coupling model of the metamaterial, in order to obtain a uniform magnetic field on a plane at a given distance from the meta-surface. The results show that the magnetic field obtained by such optimization may be considered sufficiently uniform also in other planes parallel to the meta-surface at different distances, a characteristic that is fundamental in applications like, for instance, magnetic resonance imaging (MRI) or energy harvesting for WPT.

II. Magnetic Coupling Between Cells of the 2D Meta-Surface

The considered meta-surface is composed of $N\times N$ resonant RLC circuits immersed in a linear medium and arranged to form a square lattice, as represented in Fig. 1, as well as the termination impedances $\hat{Z}_{T1}$, $\hat{Z}_{T2}$ and $\hat{Z}_{T3}$ used for the optimization of the magnetic field, as described in Sec. IV.

The relations between the currents in each loop can be expressed by applying the Kirchhoff voltage law (KVL) to

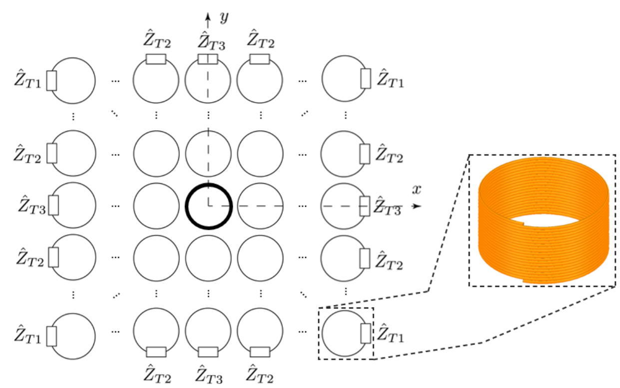

Fig. 1: Schematic representation of a terminated 2D meta-surface fed through the central resonator (bold circle). A detail of the unit cell is also depicted.

all elements of the meta-surface, which results in a linear algebraic system $\hat{\mathbf{V}} = \hat{\mathbf{Z}}_{\mathbf{m}}\hat{\mathbf{I}}$ representing the meta-surface; $\hat{\mathbf{V}} = [0 \dots 0\, \hat{V}_s\, 0 \dots 0]^T$ represents the resonator phasor voltage vector ($\hat{V}_s$ is the phasor supply voltage of the transmitter), $\hat{\mathbf{I}}$ is the vector of the resonator phasor currents and $\hat{\mathbf{Z}}_{\mathbf{m}}$ is the impedance matrix of the system. The meta-surface is considered excited in the central resonator. As regards $\hat{\mathbf{Z}}_{\mathbf{m}}$, it can be built considering all the couplings between each cell and the other cells of the meta-surface (full coupling model), or only the couplings between each cell and the others in the immediate proximity.The coupling models are described as follows:

a) Full Coupling Model: The impedance matrix $\hat{\mathbf{Z}}_{\mathbf{m}}$ is a full matrix since it includes the terms associated to the mutual coupling between all the loops. Being a full matrix, when implemented in a computer code it is characterized by a larger memory occupation and is associated to a larger CPU time for the system's solution. However, usual sizes of practical meta-surfaces do not make the previous characteristics important drawbacks. On the contrary, the calculation of all the coupling

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coefficients (from analytical or numerical procedures) can be cumbersome.

b) Nearest-Neighbour Approximation Model: In [5] the authors study systems with high number of unit cells with the nearest neighbour approximation, i.e. the couplings between nonadjacent resonators and between resonators along the diagonal directions are neglected; in this way, each single unit cell is coupled with the nearest four cells (along the x and y axes of Fig. 1). In addition, the hypothesis of a solution in terms of a MIW is used, in which propagation in the x and y directions is assumed, resulting in a dispersion equation [8]. Although for 1D structures the MIW theory can be efficiently used to define a matching termination (similar to the concept of characteristic impedance in a transmission line), in case of 2D structures (as in the case of a meta-surface) the definition of the matching terminations is not straightforward. When a KVL approach is used, the nearest neighbour approximation obviously leads to a sparser impedance matrix as only one coupling coefficient (if the lattice is regular) needs to be calculated.

c) Nearest-Neighbour Approximation and Adjacent-Diagonal Couplings: A less rough approximation than the previous one consists in considering also the mutual impedances between resonators adjacent along the diagonals of the lattice.

In order to show how meta-surfaces of small dimensions behave, a set of simulations with the three above mentioned coupling models were performed.

The meta-surface is composed of 25 circular resonators of radius $d = 2$ cm which are characterized by a resistance $R = 0.01\,\Omega$ and a self-inductance $L = 88$ nH; a lumped capacitance is connected to each loop to tune the system at the frequency $f_0 = 1$ MHz. The mutual inductances between the unit-cells were calculated numerically with a FEM electromagnetic computer code. The strongest couplings are the ones between adjacent cells in the x and y directions of the lattice and, in our example, we have $M_{adj} = -4.9$ nH. Figure 2 shows the results of the analytical solutions obtained using the three coupling models. It is evident that, for small-size meta-surfaces, the approximations shown in Figs. 2 (b) and in (c), i.e. nearest-neighbour plus adjacent-diagonal and nearest-neighbour only, may give less accurate results if compared with those obtained with the full coupling model. In particular, Fig. 2 c) shows that the current values calculated in small size meta-surfaces can be significantly different from those obtained with the full coupling model, shown in Fig. 2 a). The nearest-neighbour approximation has an effect on the current propagation similar to that experienced by a meta-surface with a low quality factor, as it is shown in the next section.

III. Effect of the Resonator Quality Factor on the Current Distribution

In real resonators, all quantities are affected by the operating frequency, because of skin and proximity effects. However, by using Litz wires the circuits can be considered close to ideal ones with a reasonable approximation and the dominating quantity is the quality factor $Q = \omega_0 L/R$. As a matter of fact, currents are strongly affected by the resistance R, which basically determines the attenuation; at a fixed frequency, different quality factors lead to different current distributions and this effect can be particularly appreciated as the extension of the meta-surface increases. Considering a full impedance matrix $\hat{\mathbf{Z}}_{\mathbf{m}}$, for low quality factors (i.e., $Q < 100$) the resonators laying on the diagonals of the lattice experience higher currents, similarly to when adopting the nearest-neighbour approximation (see Fig. 3(a)). Differently, as Q increases it is difficult to predict and control the current distribution, as shown in Fig. 3(b).

The above mentioned behaviour could be heuristically explained as follows: a lower quality factor leads to stronger attenuation of the currents far away from the source at the centre of the lattice; the nearest-neighbour approximation influences the current propagation so that each resonator influences the four nearest ones only, somehow reducing propagation, leading to a similar effect.

IV. Magnetic Field Generated by a Current Distribution in the meta-surface

The distribution of the magnetic field generated by the meta-surface is directly linked to the value of the currents in the resonators and to their geometry and arrangement. Assuming a linear medium, the superposition principle for the magnetic field can hold. Thus, the magnetic flux density generated by the whole meta-surface at a generic point $\bar{P}$ of the three-dimensional space can be expressed as:

$$\hat{\bar{B}}(\bar{P}) = \sum_{i}^{N\times N} \hat{\bar{B}}_i(\bar{P}) \tag{1}$$

where $\hat{\bar{B}}_i(\bar{P})$ is the magnetic field at point $\bar{P}$ generated by the ith meta-surface cell. It is then possible to write:

$$\hat{\bar{B}}(\bar{P}) = \sum_{i}^{N\times N} \hat{I}_i \bar{G}_i(\bar{P}) \tag{2}$$

where $\hat{I}_i$ is the phasor current of the ith cell and $\bar{G}_i(\bar{P})$ is a function of the coil geometry and material only, and it is referred to the point $\bar{P}$. For a fixed field point, the functions $\bar{G}_i$ of each cell are the same, being all identical, and are obtained by implementing the formulas given in [9].

Once the system geometry is defined, the magnetic field can then be controlled by acting on the metamaterial currents. As shown in sec. II, an accurate approach to analyze the currents of a meta-surface consists in analytically determine the full impedance matrix and solve it by using KVL. Considering a single source (for example, the central meta-surface resonator), the cell currents can be varied by acting on the termination impedances inserted in the boundary resonators. The uniform magnetic field above the meta-surface requires then the optimal distribution of currents in the resonators to be found and this, in turn, entails the optimal triplet of terminations impedances to be obtained.

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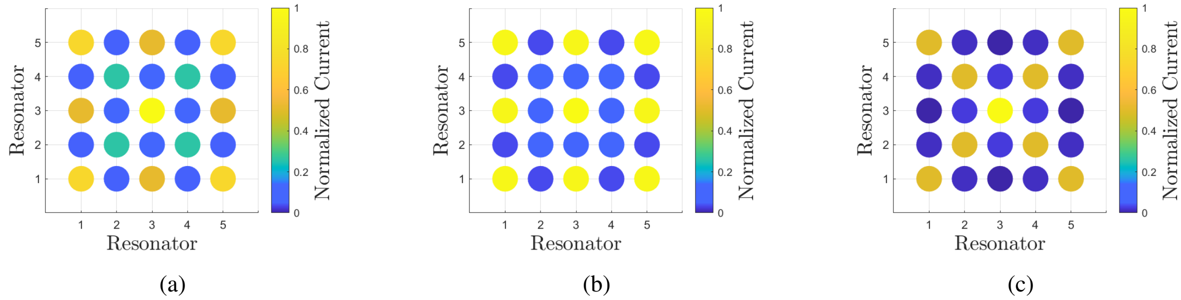

Fig. 2: Current distribution of a 5×5 meta-surface excited in the central resonator considering: (a) all couplings between coils, (b) the couplings between adjacent coils in the x, y and diagonal directions (nearest-neighbour approximation plus adjacent couplings) and (c) the nearest-neighbour approximation. The values are normalized to the current of the central resonator.

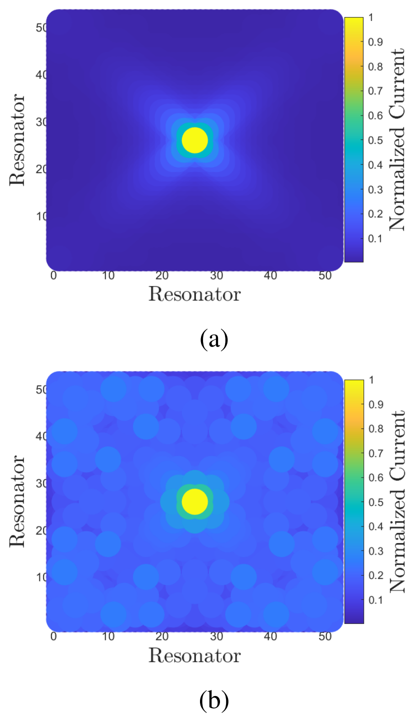

Fig. 3: Current distribution of a 51×51 meta-surface excited in the central resonator considering the interaction of each coil in case of (a) $Q = 40$ and (b) $Q = 260$.

V. Optimal Terminations for Uniform Magnetic Field

The uniformity of the magnetic field is assessed by evaluating the magnetic flux density at a number of field points on a xy plane at a given distance from the meta-surface.

Considering the metasurface described in Sec. II, the magnetic flux density values are calculated according to (2) on a regular grid of points with a step of $d/4$ centred on the central meta-surface resonator. The grid is on a plane distant 5 cm from the meta-surface. As a first analysis, all the unit cells are considered in resonant condition.

The optimization procedure is driven by a genetic optimization algorithm and aims at minimizing the difference between the values of the magnetic flux density in the field points and the average one. Introducing the average magnetic field magnitude as

$$B_{AVG} = \frac{1}{N_{FP}} \sum_{k}^{N_{FP}} |\hat{\tilde{B}}_k| \tag{3}$$

where N_{FP} is the number of field points, the cost function F can be written as:

$$F = \max_{k} \left\{ |\hat{\tilde{B}}_k| - B_{AVG} \right\}. \tag{4}$$

The reason why a genetic optimization procedure is chosen (and not, for instance, a gradient based algorithm) simply lays in the fact that the optimization process is not the main object of the paper, and the genetic algorithm is one of the simplest methods for a practical implementation. Considering the symmetry of the meta-surface, we chose to have at most three different values of the termination impedances: $\hat{Z}_{T1}$ in the corners, $\hat{Z}_{T2}$ in the generic edge resonators and $\hat{Z}_{T3}$ in the middle-edge cells, as shown in Fig. 1. For each triplet of tentative values of the terminations, the currents are calculated by solving the resonator KVL as:

$$\hat{\mathbf{I}} = \hat{\mathbf{Z}}^{-1}\hat{\mathbf{V}}. \tag{5}$$

In particular,

$$\hat{\mathbf{Z}} = \hat{\mathbf{Z}}_{\mathbf{m}} + \hat{\mathbf{Z}}_{\mathbf{T}} \tag{6}$$

where $\hat{\mathbf{Z}}_{\mathbf{T}}$ is an impedance matrix with the values of the termination impedances $\hat{Z}_{T1}$, $\hat{Z}_{T2}$ and $\hat{Z}_{T3}$ on the diagonal

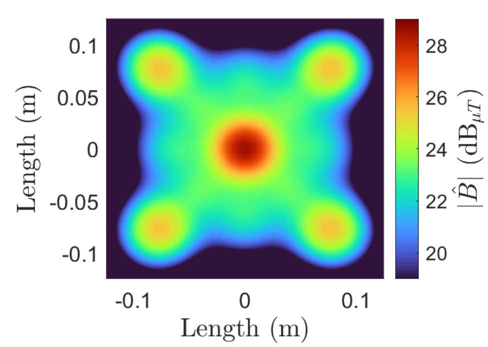

Fig. 4: Magnetic flux density generated by a metasurface without terminations on the $z = 50$ mm plane.

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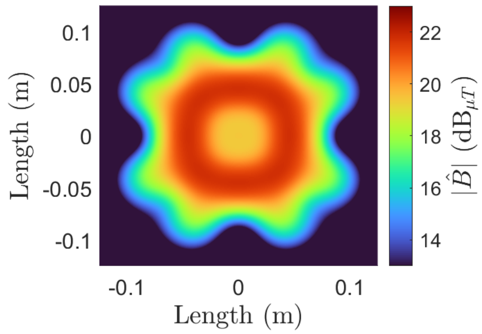

Fig. 5: Magnetic flux density generated by a metasurface with optimal terminations on the $z = 50$ mm plane.

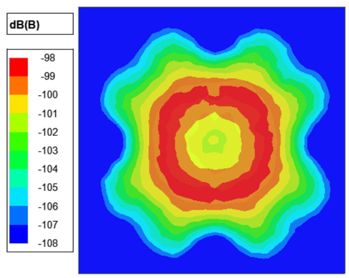

Fig. 6: Magnetic field from a terminated meta-surface calculated numerically on the $z = 50$ mm plane with an electromagnetic FEM code.

and zeros elsewhere. The optimized terminations are found as follows: $\hat{Z}_{T1} = 3.70 + j1.07$, $\hat{Z}_{T2} = 7.22 \cdot 10^{-5} - j0.027$, $\hat{Z}_{T3} = 3.51 - j3.58$. The optimization requires 20400 iterations to converge and 102 genetic generations, resulting in a residual value of the cost function of 3.4e-6. To complete the optimization, the algorithm takes 16 seconds.

Fig. 4 shows the magnetic flux density on the $z = 50$ mm plane for the center-fed nonterminated meta-surface. As it can be seen, in the central area of the plane the variation between the magnetic flux density values is about 6 dB. With the optimal terminations obtained with the optimization procedure, the field uniformity is greatly enhanced, as Fig. 5 shows: in the central area of the plane the variation of the field is within 3 dB

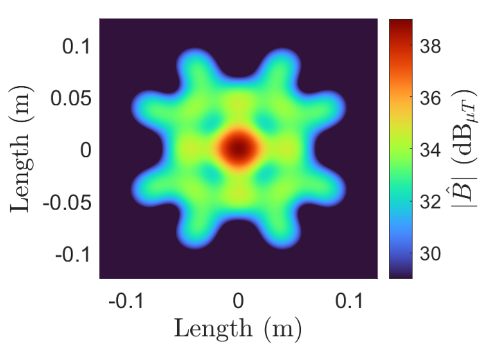

Fig. 7: Magnetic flux density on the $z = 25$ mm plane with the optimal terminations for $z = 50$ mm.

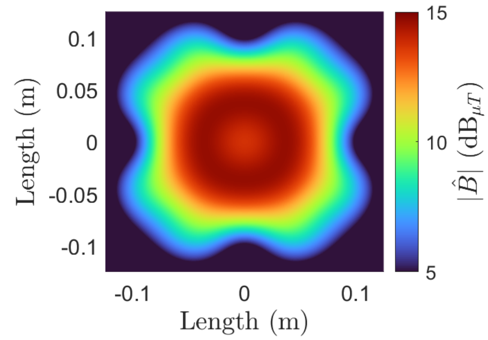

Fig. 8: Magnetic flux density on the $z = 70$ mm plane with the optimal terminations for $z = 50$ mm.

only. In order to validate this result, the magnetic flux density was calculated with an electromagnetic field code and is shown in Fig. 6. These results are in very good agreement with those of Fig. 5. The magnetic flux density was also calculated, for the same optimal terminations, on the two additional planes $z = 25$ mm and $z = 70$ mm; the results are shown in Figs. 7 and 8, respectively. For both distances it can be noticed that in the central area of the planes the variation of the magnetic flux density is within 6 dB. The magnetic field can then be considered uniform for a range of distances from the meta-surface, making then this procedure appealing for a number of applications, e.g. MRI or energy harvesting for WPT.

VI. Conclusions

This paper shows that a uniform magnetic field at a given distance from a centre-fed meta-surface can be obtained by controlling the termination impedances of the meta-surface. It is shown also that with these optimal terminations the magnetic field remains sufficiently uniform also on other planes parallel to the metasurface at different distances, without any additional active circuitry.

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