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The interdisciplinarity of Special Relativity: A historical analysis

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Summary. — This paper is framed within the Erasmus+ project titled IDENTITIES, whose aim is to develop interdisciplinary teaching materials for pre-service teacher education. In collaboration with the research group in STEM education of Crete, a blended module on special relativity has been developed. The module is based on an analysis of the original texts by Lorentz, Poincaré, Einstein and Minkowski (written between 1904 and 1908), aimed to recognise the interplay between mathematics and physics implemented in the four papers. The analysis has been carried out by applying the "Boundary Crossing and Boundary Object" research framework developed in 2011 by Akkerman and Bakker. The results of the analysis show that Lorentz Transformations can be read as a Boundary Object and this lens allows for different nuances of the interplay between mathematics and physics to be recognised in the four papers. A series of activities to be conducted in blended mode in a pre-service teacher education course have been designed with the goal of exploiting special relativity as a context to develop interdisciplinary skills.

1. – Introduction

Bridging the gap between science and society is crucial, if not mandatory, in this age of uncertainty and in what Rosa defined "society of acceleration" [1]. The great changes that have taken place in society in recent years, such as the advent of social networks and the evolution of new technologies, have enlarged the gap between the academic world (schools, universities, and research) and everyday life. The disconnection and misalignment between what is taught and what students need to address daily challenges is increasing every day. This disconnection depends on several factors. One of the most important is how school curricula are strictly structured and organised in disciplines, while modern society and research require a more open vision that is intra-multi-trans-disciplinary.

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Adopting an interdisciplinary approach in teaching is therefore paramount, because "an interdisciplinary learning approach integrates the disciplines and diffuses their limits, passing through different levels of cognitive ability in pursuit of developing a holistic thought process. In this manner, students can make meaningful connections that allow them to process knowledge to produce an interdisciplinary understanding that is applicable to reality" ([2], p. 802). Interdisciplinarity is necessary to understand the challenges of today's society due to their complexity and their multifaceted nature. In doing so, it is fundamental to focus on the role that disciplinary epistemologies, methodologies and practices play in the shift from a disciplinary approach to an interdisciplinary one.

An analysis of the specialised literature on STEM (Science Technology Engineering and Mathematics) Education shows that in recent years there has been an increased commitment to interdisciplinary and transdisciplinary STEM integration [3-8]. Different epistemological views on STEM disciplines and their integration can have implications for general education and on the construction of future society as a whole [8]. Due to its "young age", in the STEM integration field more research remains to be conducted, such as investigating the complexity of making crosscutting STEM connections, preparing teachers to deal with STEM topics, and designing materials, for instructional and assessment support and guidance [5]. Educational researchers indicate that teachers struggle to make connections across the STEM disciplines [7]. Secondary teachers have expressed difficulty in using frameworks from other disciplines or other teachers and felt that STEM curricula may be inflexible and not be able to impart meaningful learning [9]. Also, teachers express the belief that the availability of a quality curriculum would enhance the likelihood of success of STEM initiatives [9]. In their words, "Teachers need quality curriculum that aligns with district and state guidelines and includes formative assessment techniques teachers can use to assess their students' conceptual understandings" ([2], p. 14). Teachers request instructional materials that both exemplify the innovation and are easy for them to implement [4]. A focus on connecting core content knowledge and processes across the disciplines still appears limited [3]. Further research is required on ways of assisting teachers to foster the STEM connections across disciplines, especially when appropriate curriculum frameworks and resources might be lacking [3]. Considering the transition between disciplines, "disciplinary boundaries are often blurred and there is much research to show that disciplines are not static, instead, knowledge, methods and theories are constantly moving across disciplinary boundaries and the disciplines are constantly shifting [...] Interdisciplinarity presupposes disciplinarity such that one is integral to the other" ([6], p. 939). As stated by Lenoir and Hasni [10], "there can be no interdisciplinarity without disciplinarity" (p. 2448).

The last statement is the core idea behind the project IDENTITIES, an ERASMUS + project coordinated by the research group in Physics Education from the University of Bologna. In this project, indeed, the partners assume that the search for the meaning of interdisciplinarity cannot ignore the meaning of disciplines and their epistemological identities. The project IDENTITIES started in September 2019 (`identitiesproject.eu`). Together with the group from Bologna, there are other 4 different universities: Barcelona, Crete, Montpellier, and Parma. The main goal of the project is to build innovative and transferable teaching modules and courses to be used in contexts of pre-service teacher education (*e.g.*, curricula in Physics Education, Mathematics Education or Computer Science Education within master's degree courses). The central theme of the modules is interdisciplinarity in the S-T-E-M fields, with a focus on the links and interweaving between physics, mathematics, and computer science.

This paper is part of the project and has been developed together with the research group in Science Education at the University of Crete and the research group in Physics Education from the University of Bologna. The focus of this paper is the Special Theory of Relativity (STR). Starting from a literature review on the teaching of STR, an interdisciplinary analysis of 4 historical papers that build the theory has been conducted with the aim of highlighting the relation between mathematics and physics in the theory and its implications from an epistemological and educational point of view.

2. – The case of Special Relativity

As already stressed, interdisciplinarity is fundamental to start from disciplines. The separation of knowledge into current disciplines has found its peak in the 20th century, when the hyper-specialisation requested from the labour market and society in general shaped schools curricula into today's situation. In this paper, the perspective of Tzanakis [11] on the interrelation between disciplines is considered, in particular between mathematics and physics: "In teaching and learning mathematics or physics, neither history can be ignored, nor their close interrelation can be circumvented or bypassed." (p. 2). This line of thought stresses how important is to focus on the "ontological status of mathematics and physics, their historical interrelation and their epistemological affinity as scientific disciplines" (p. 2).

As a field of application to study this interrelation, STR has been chosen. As Tzanakis stated, Special Relativity works as a great historical example to highlight the deep ties between mathematics and physics, in that many scientists coming from different fields have worked and contributed to its development. In particular, Tzanakis focuses on the works of four authors: Lorentz, Poincaré, Einstein and Minkowski. He does that because each of them contributed in a different way to the construction of the theory, both from a mathematical point of view and from a physical one [11]. In particular, Lorentz obtained the Lorentz Transformations (from now on, "LT") while looking for those equations that keep unaltered Maxwell's equations without giving them any physical implication. Poincaré gave mathematical meaning to the transformations, unveiling their group structure but still using concepts like ether. Einstein made a great step forward starting from two basic principles (invariance of physics laws and constancy of c) shaping the theory as we know it today, while Minkowski introduced a new way of looking at the transformations using a geometrical language, setting the stage for the works on space-time of the following years [11]. The theory of Special Relativity plays a very important role in the history of scientific progress because of its temporal location. The period in which it was developed, the end of the 19th century, was a period marked by profound uncertainty, due to the questioning of those historical theories that had marked the previous centuries (Newtonian theory, or Euclidean geometry) and the birth of new theories such as quantum mechanics, statistical thermodynamics, or non-Euclidean geometries, that would have marked the near future of science [12-14].

In addition to its important role in the physics of the 20th century, STR plays a special role in school curricula, as it is one of the last topics of classical physics and the first of modern physics. Working on this theory is therefore essential to allow a meaningful understanding of the subsequent changes within that discipline. Because of its educational relevance, many studies have been carried out in Physics Education Research (PER) "to understand how this theory is taught in High School and University, what are the difficulties that students encounter while studying these topics and how the teaching of this topic is changing due to new technologies and new pedagogic and epistemological approaches" [15]. That topic has been deeply investigated in the learning sciences as a case study for developing theories of learning and of conceptual change [16-18]. In the next section, the main results achieved by these studies in PER and in the learning sciences are reported. They represent the framework to position the "interdisciplinary analysis" that has been carried out on the four historical papers. To complete the framework, I also considered the papers related to space-time geometry carried out in Mathematics Education Research (MER).

3. – Literature review

The review has been carried out to identify what are the main research topics, methodologies and research techniques related to STR in PER and MER, what are the main difficulties encountered by students and teachers in addressing the topic and what are the less considered aspects of the theory. For the review, the databases of the main scientific journals dealing with physics and mathematics education were considered. A total number of 55 papers was found, 49 in the field of PER and 6 in the field of MER. The papers were analysed by extracting information from: Title, Abstract, Introduction, and Conclusions. To identify the major research topics, a thematic analysis of each paper has been produced, to cluster them and point out the main topics and research focuses. That process led to the identification of seven different categories in the PER field and only one in the MER field. The more covered topic is "Student Difficulties", followed by "Digital Tools Development", "History and Epistemology", "Curricula Development" and "Conceptual Change". The last two categories, the less treated ones, were "In/Pre-service Teachers Formation" and "Interdisciplinarity". In the MER field, the only strand found focused on the geometrical aspects of STR. The specific results of the analysis are reported in Miani [15]. For the purpose of this article, the main result that it is worth mentioning is the poorness of educational research on the role of mathematics in teaching/learning STR. This result motivated the analysis presented below. The only works that focused on the interdisciplinarity of STR were the ones from Tzanakis [11, 19-21] and Galili [22]. In their opinion, STR can be an excellent historical example useful to demonstrate the deep connection existing between mathematics and physics. Bondi [23] used a different method, deriving relativity from Newtonian ideas and obtaining LT using simple algebra after having established the concepts and effect of STR.

From the review, it is evident that LT have usually been used in teaching as a useful tool to derive, mathematically, the relativistic effects, and not as a conceptual tool for deep comprehension of the concepts, or for highlighting the epistemological value of the theory [24-29]. The analysis presented in the next section was realised looking closely at the works of Tzanakis: in particular, Tzanakis designed a didactical implementation aimed at obtaining LT using reasoning based on plane rotations in 2D-analytic geometry [11].

Starting from the didactical potential that a historical analysis like Tzanakis' can have on the understanding theory, I focused on which visions of the interdisciplinarity between mathematics and physics emerge from the role of LT in the theory. The research questions that guided the analysis are:

1) How to characterise the role of mathematics in the different articles?

2) What visions and facets of the relationship between mathematics and physics emerge?

3) In what way can this analysis help to understand the links between the disciplines?

4. – Papers analysis

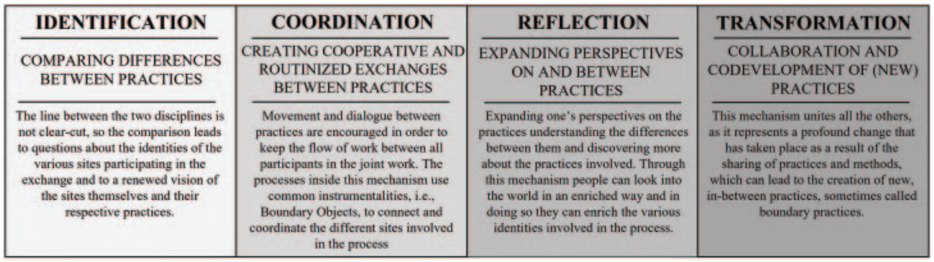
The interdisciplinary analysis that I present has been carried out using the interdisciplinary framework of "Boundary Crossing and Boundary Object" from Akkerman and Bakker [30]. This framework is widely used in science and mathematics education to point out and analyse the learning mechanisms that can take place at boundaries between disciplines. These mechanisms are unpacked by leveraging the boundary metaphor to show different modalities in which "socio-cultural [*i.e.*, disciplinary] differences and resulting discontinuities in action and interaction can come to function as resources for development of intersecting identities and practices" ([30], p. 132).

The framework has been adapted within the IDENTITIES project to highlight how exchanges between different disciplines can be analysed from a content, methodological, and epistemological point of view. In this analysis, I applied the "Boundary crossing and Boundary object" framework on the four historical texts that laid the foundations for STR, that are: "Electromagnetic phenomena in a system moving with any velocity smaller than that of light" by Lorentz [31], "On the dynamic of the electron" by Poincaré [32], "On the Electrodynamics of moving bodies" by Einstein [33] and "Space and Time" by Minkowski [34]. The analysis aimed at highlighting the mechanisms at the boundary between physics and mathematics in these papers.

Specifically, I used the four learning mechanisms provided by the framework to describe how the boundaries between physics and mathematics are crossed in the historical papers. The four mechanisms are: Identification, Coordination, Reflection, and Transformation (fig. 1).

Together with these four mechanisms, in the framework from Akkerman and Bakker we found the concept of Boundary Object, *i.e.*, "those objects that both inhabit several intersecting worlds and satisfy the informal requirements of each of them... [They are] both plastic enough to adapt to local needs and the constraints of the several parties employing them, yet robust enough to maintain a common identity across sites. They are weakly structured in common use, and become strongly structured in individual site use" ([35], p. 393).

Applying these interdisciplinary lenses to the papers allowed the identification of the boundary mechanisms and the boundary objects inside them. The analysis has been realised on two levels: a first general level that focused on the papers as a whole,



IDENTIFICATION	COORDINATION	REFLECTION	TRANSFORMATION
COMPARING DIFFERENCES BETWEEN PRACTICES	CREATING COOPERATIVE AND ROUTINIZED EXCHANGES BETWEEN PRACTICES	EXPANDING PERSPECTIVES ON AND BETWEEN PRACTICES	COLLABORATION AND CODEVELOPMENT OF (NEW) PRACTICES
The line between the two disciplines is not clear-cut, so the comparison leads to questions about the identities of the various sites participating in the exchange and to a renewed vision of the sites themselves and their respective practices.	Movement and dialogue between practices are encouraged in order to keep the flow of work between all participants in the joint work. The processes inside this mechanism use common instrumentalities, i.e., Boundary Objects, to connect and coordinate the different sites involved in the process	Expanding one's perspectives on the practices understanding the differences between them and discovering more about the practices involved. Through this mechanism people can look into the world in an enriched way and in doing so they can enrich the various identities involved in the process.	This mechanism unites all the others, as it represents a profound change that has taken place as a result of the sharing of practices and methods, which can lead to the creation of new, in-between practices, sometimes called boundary practices.

Fig. 1. – The four Boundary Crossing mechanisms [30].

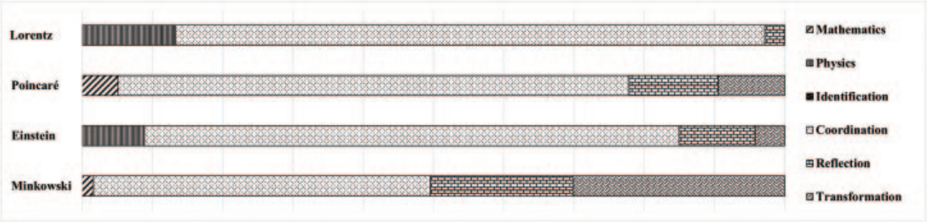


Fig. 2. – Spectrum of boundary crossing mechanisms [36].

and a second more detailed level that focused on some particular excerpts considered particularly interesting. This analysis is currently being carried out on a deeper level together with Lucia Modica. An analytic grid with operational markers has been produced to recognise, in the texts, the boundary crossing mechanisms, described by Akkerman and Bakker [36]. The following representation, elaborated in collaboration with Modica, shows the results of the first general level of analysis. It highlights the different ways in which the authors have addressed the boundary between physics and mathematics. To represent those mechanisms, each line of the papers has been numbered and mechanism by mechanism summed together to represent its weight in each paper (fig. 2). The main mechanism that can be detected in the four papers is, as expected, Coordination. It refers, very often, to the use of mathematics in an instrumental way, to explain physical phenomena formally treatable. This is what almost entirely characterizes Lorentz's paper. Here, the specificity is that, once he has obtained the results using mathematics, he does not give them a physical meaning. This is also why we do not see many other mechanisms like Reflection or Transformation here.

In Poincaré's work Reflection is also observable. Reflection is a mechanism that implies taking a perspective and using one discipline to "see" something new in the other, in a dynamic and back-and-forth process of boundary crossing. Poincaré uses mathematics as a lens, a perspective to interpret Lorentz's laws and extract new physical meaning. Following his intention to give a solid logical and rigorous basis to Lorentz's demonstrations and suppositions, Poincaré demonstrates that Lorentz's transformations form a group. Thanks to this mathematical perspective, he is able to give a rigorous justification for Lorentz's assumptions, and in doing so he is forced to introduce a potential proportional to the volume of the electron, the consequences of which, namely the deformation of the electron, are dictated by the conditions imposed by the group transformations.

As far as Einstein's work is concerned, Coordination is still prevalent, with some traits of Reflection and Transformation. This series of mechanisms are used by Einstein to develop a solid theoretical structure that adheres to macrophysical principles, elevating them to the rank of postulates. The whole argumentation grounds on a formal axiomatic structure, from which the foundations of the conceptions of space and time are re-analysed and transformed. As a result, a transformative theory is obtained, and concepts are developed, moving continuously on the boundary between the two disciplines.

The Transformation mechanism plays a key role in Minkowski's paper. The approach that Minkowski applies in his work has a profoundly transformative basis: his intentions, clear from the beginning of the paper, are to unite the concepts of space and time, through a theory in which these two ideas are only a projection of the general concept that is space-time. Although Minkowski's approach is geometrical and profoundly mathematical, it is grounded in experimental physics and addresses concepts that have a solid physical basis. The result is the creation of a theory that stands on the boundary, constantly moving between the two disciplines and even creating its own zone, a new reality on the boundary.

A second level of the analysis highlights how LT takes on the role of Boundary Objects, as they give a different picture of the theory depending on the type of approach used to explain them.

In Lorentz, they assume the role of a mathematical instrument necessary to give an explanation to the experimental results obtained in those years on the existence of the ether. The pure process of Coordination that we have identified demonstrates this, highlighting how from the mathematical result one does not draw results or implications that go to modify the physical reasoning. Poincaré gives to LT a double identity, in that he provides them with a physical demonstration through the principle of least action, and a mathematical one, showing that they form a group. This type of approach makes it possible to look at LT differently from the previous one, and at the same time gives the theory itself a different appearance depending on which side we look at it. Einstein's approach adds meaning and vision to the theory, showing that it is not necessary to start from experimental evidence but rather from principles to obtain the same transformation rules. His continuous boundary work through the processes of Coordination and Reflection charges his method of obtaining LT with meaning, a meaning that needs a physical as well as a mathematical vision. It is here evident how the instrumental use of LT to demonstrate relativistic effects does not restore the true value of LT.

This value takes on an even different role if we use the approach that Minkowski uses to arrive at the demonstration of LT. In his article, we see how the transformations are obtained through a geometric process necessary to move from one reference system to another through the calibration hyperbola, which then leads to the identification of the Minkowski metric that lays the foundations for subsequent work on space-time. Focusing on their multifaceted nature and their different meanings is possible to understand better the role that LT have in the theory and therefore comprehend the theory more deeply, understanding how interconnected the two disciplines are. In this way, the inner interdisciplinarity of STR emerges. This analysis is the basis for the design of a module, consisting of a set of teaching activities.

5. – Design of the activities

The module, developed on the interdisciplinary analysis, consists of four activities. It aims to highlight the different mechanisms of learning at the boundary in the four papers and the role that LT have as a Boundary Object, highlighting how the connections, methods, and epistemologies of the different disciplines can merge to produce a result that takes its cue from the disciplines and transforms their realities. The module has been designed to be conducted in a blended modality, following the definitions of those who worked in this field in the last 20 years [37-39] and the MIX taxonomy [40]. Following this body of work, the four activities are designed to be held on three different levels of blending, *i.e.*, low-impact blend, medium-impact blend, and high-impact blend. The place in which these activities will be realised is a pre-service teacher formation course about physics, mathematics, computer science, and more in general science education.

The first activity consists of four videos: one for the historical panorama of the end of the 19th century from the point of view of physical discoveries (electromagnetism, statistical mechanics, thermodynamics, etc.), one for the historical panorama from the point of view of mathematics (Non-Euclidean Geometries, the Erlangen Programm, etc.), one for the genesis of the four articles that will be analysed (Lorentz, Poincaré, Einstein, Minkowski) and one to introduce the Boundary Framework of Akkerman and Bakker as understood by the authors and our research group. This activity will be realised with a Flipped approach, *i.e.*, a high-impact blend. That means the students will watch the videos in an a-synchronous way, to discuss later what they saw all together. The videos will last 10/15 minutes each.

The second activity involves students analysing the four papers, applying the boundary framework lenses presented earlier to some excerpts of the original papers. Analysis will be carried out in groups of 3/5 people, who will analyse at least two excerpts from two different authors. A guide made beforehand will allow them to highlight the differences in the respective approaches and to detect the four boundary mechanisms. The activity will last approximately two hours and will end with an oral discussion between the groups of at least one hour to discuss how the lenses have been applied. It is categorised as a low-impact blend, in that digital resources are added to the materials normally used in the classroom.

The third activity focuses on LT and their role in the theory. During this activity, the students, this time divided into groups of 2/3 people, will have to derive LT by tracing the steps of Einstein and Minkowski. In detail, the students will follow activities in which they will be guided step by step to obtain the transformations using software such as GeoGebra or Maple. There is also the possibility to obtain the LT as a group from Maxwell's equations, following Poincaré's steps, but its applicability depends on the level of the students. The activity should last about 2 hours and due to its design is categorised as a medium-impact blend, as the activity involves intensive use of digital resources but still in class and in synchronous mode.

The fourth and last activity involves a long discussion among the students in which the teacher takes the role of a moderator, giving the input to start the discussion but following it laterally. The discussion aims to compare the results obtained by individual students during the activities to see if a deeper understanding of the theory of special relativity and the processes at work between disciplines within the theory emerge from the course. The discussion will last an hour and includes filling in a final questionnaire with closed and open questions to obtain useful feedback for remodelling the activities.

6. – Conclusions

From the analysis, it is possible to recognise different styles and approaches when dealing with the boundary between disciplines. These styles and approaches give a very varied view of the role and connection that mathematics has, and can have, with physics. In addition, the analysis allows for in-depth analysis of the relationship between the disciplines within STR: mathematics can be used in an instrumental way to coordinate the two disciplines and motivate steps (*e.g.*, Lorentz or Poincaré), but allows through its reasoning schemes to generalise results (*e.g.*, Einstein) or apply abstraction processes that help physics to develop its theories (*e.g.*, Minkowski). The designed module will help both students and teachers in focusing on the different ways of working at the boundary between mathematics and physics in the theory, bringing out the multiple nature of Lorentz Transformations, and in this way allowing a deeper understanding of the theory itself.

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