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Evaluation of mining projects subjected to economic uncertainties using the Monte Carlo simulation and the binomial tree method: Case study in a phosphate mine in Egypt

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1 Evaluation of mining projects subjected to economic uncertainties using the 2 Monte Carlo simulation and the binomial tree method: case study in a 3 phosphate mine in Egypt

4 ABSTRACT

5 Mining project evaluation is a very complex process and time-consuming task. Any mining
6 project passes through several study stages, like conceptual, preliminary feasibility, and
7 feasibility studies, using a systematic and standardized approach. These studies allow investors
8 to evaluate the exploitations alternatives according to economic and technical criteria to make
9 a better decision. This research aims to study a model for investment risk analysis developed
10 for the case study of "Abu Tartur mining project, Egypt." Abu Tartur plateau is rich with
11 phosphate rock, turned into phosphoric acid and fertilizers. This study used site-specific data
12 that has been taken from the literature to build evaluation models using the Monte Carlo
13 Simulation (MCS) and the Binomial Decision Tree (BDT) methods. The Discounted Cash
14 Flow (DCF) is considered a benchmark method for all other evaluation methods. The MCS
15 method uses a set of parameters that, in some cases, will give a higher Net Present Value of the
16 project and some others with lower values, but this depends on the probability of the input
17 parameters. The developed evaluation models allowed a specific range of values, with
18 confidence intervals as the MCS model. The advantage of using a probabilistic approach helps
19 in the decisional phase, allowing for a more precise overview of the variability of the final
20 economic value of the mine. Deterministic methods, like the DCF method, offer solution that
21 is limited to uncertainty analysis.

22 **KEYWORDS:** Risk Analysis; Monte Carlo Simulation; DCF; Abu Tartur Plateau; phosphate;
23 Mining; Binomial Tree.

24 1. Introduction

25 The mining industry is a very risky activity involving very high capital investment. The
26 decision-makers have to deal with many kinds of risks, from geologic, economic,
27 environmental, and political risks (Dehghani, 2018). Among the others, the primary sources of
28 uncertainty in the mining industry are: (i) the amount of overburden and reserve estimation;
29 (ii) ore grade estimation; (iii) capital and operating costs; (iv) selling prices, (v) recovery ratio;
30 (vi) dilution; (vii) environmental issues. Each parameter should be evaluated and determined
31 by its effects on the project, as investigated by (Topal, 2008) (Dehghani and Ataee-Pour, 2013)
32 (Kopacz et al., 2019).

33 Each risk should be evaluated, estimated, and determined by its effects on the project and
34 community. The Discounted Cash Flow - Rate of Return (DCF-ROR) (Trigeorgis, 2000) is the
35 most used project evaluation method in mining project evaluation. It is possible to classify it
36 as a deterministic method, while the Monte Carlo Simulation (MCS) (Amico, 2003) and the
37 Binomial Decision Tree (BDT) (Brandao et al., 2005) are classified as probabilistic methods.
38 The mining ventures can be estimated using the Net Present Value (NPV) method (Gardner,
39 2015). Nonetheless, inputs are unpredictable, and their risks should be identified before
40 estimating the NPV of ore reserves (Erdem, 2008). This kind of calculation does not give any
41 information about the probability of occurrence of the estimated NPV.

42 Uncertainty and risk management are critical for making decisions in the extraction of mineral
43 resources in a sustainable manner. Uncertainty in mine planning can be evaluated, for example,
44 with a geostatistical simulation of the variables influencing the decision process, which
45 generates a series of equally likely models. Then, for each model, a transfer function is applied.
46 The outcome is a response function distribution that determines the likelihood (or risk) that a
47 particular area meets the product quality requirements. High-risk operations frequently fail to
48 meet ore quality requirements, lowering the value of mineral resources (Kloeckner et al., 2021).

49 The preliminary investigation stage of any project is known as the pre-feasibility study. The
50 pre-feasibility study is performed to identify, assess, and choose the best business scenarios
51 technically and financially. In this early stage, shortcut methods are permissible and can be
52 applied to determine small investment components and production costs. If the chosen scenario
53 is deemed viable, it is advised that the research be extended to feasibility to gain a more in-
54 depth understanding of the project scenario. In estimating the project's chances of success, a
55 feasibility study considers all the project's essential components, including economic,
56 technical, legal, and scheduling difficulties. It is an engineering study that uses test work and
57 engineering analysis to assess whether or not the project should proceed to the final engineering
58 and construction stage. It is a "go/no-go" decision point, meaning that the answer is sometimes
59 "no" The second difference between these two stages is that once a project has progressed to
60 the detailed feasibility study stage, corporations have typically invested significant resources
61 and built a professional reputation assuming that the project would be practical. Several
62 elements influence whether a project is worthwhile, including the project's cost and return on
63 investment or if the enterprise earned sufficient money or sales from customers (Dou et al.,
64 2020).

65 The DCF-ROR evaluation method evaluates if a mineral resource can be considered a reserve
66 and, consequently, a mine. It needs a considerable study and works not only a complete
67 assessment and feasibility studies of the deposit but also, needs a complete design and
68 simulation of the mine. If the simulated parameters show promising and positive results, it is
69 possible to exploit the mineral deposit properly. The detailed feasibility study should contain
70 four main fields: economic, financial, environmental, and political studies. Our study will focus
71 on the economic effects due to price volatility and other financial parameters. Political and
72 environmental constraints exist in all countries, especially in Egypt, but they are out of the
73 scope of this research. The economic evaluation of a mining project can be divided into two
74 main quantities, the potential earnings, and the investment costs. If the difference between the
75 two quantities is high enough, the project will be feasible. From the above explanation, the
76 cash flow analysis is defined as the sum of net profit, depreciation, depletion, deferred
77 deduction, and amortization, or the sales revenue minus operating costs and income taxes.

78
79 The Binomial Decision Tree (BDT), introduced firstly by (Cox et al., 1979), becomes
80 nowadays one of the most used techniques in economic evaluation, which presents a simple
81 option valuation discrete-time model. The basic economic principles of arbitrage methods for
82 option pricing are apparent in this setting. Its creation involves only basic mathematics, but it
83 includes the well-known Black-Scholes model (Black and Scholes, 1973) as a particular limiting
84 case, historically obtained only from much more complicated methods. In several cases, the
85 basic concept is readily committing itself to generalization. Black and Scholes considered
86 continuous trading and expected stock values to be lognormal distribution. Their approach
87 relied on some very complex mathematics, while it was needed for using a much simpler

88 method (Dehghani et al., 2014). So, a simple option valuation discrete-time model is developed
89 with basic mathematics. It gave rise to a simple and efficient numerical method to evaluate a
90 mining project efficiently (Ganguly et al., 2021) (Azimi et al., 2013) (Ikhsani and Nainggolan,
91 2021) (Amini et al., 2015) (Dehghani and Ataee-pour, 2012).

92 MCS is a standard extension of the deterministic NPV base case since it accounts for the
93 possibility that variables are subjected to uncertainty. Generic statistical models classify the
94 input parameters, such as regular, lognormal, uniform, and triangular models. Although it is
95 possible to make associations between variables, they are most generally viewed as separate.
96 In our approach, we will use the MCS jointly with the Black and Scholes time series models
97 (Black and Scholes, 1973) to integrate the association of successive parameter values (e.g., metal
98 price). These expanded simulations of Monte Carlo are somewhat close to those used for the
99 pricing of European options (Kopacz et al., 2017) (Chen, 2011). The duration of the mining
100 project is set in all situations, and the tests provide us with the histogram of potential
101 consequences and the estimated benefit. The fundamental distinction resides in how money's
102 time worth is handled. The discount rate in the deterministic DCF-ROR is called in MCS the
103 "risk-free rate," and in the option pricing method is called "risk-neutral odds of options." The
104 most common method to measure the uncertainty is the sensitivity analysis, while the Monte
105 Carlo Simulation is another way to do that with more consistency (Fontes, Marcelio et al., 2020).
106 Mining project evaluation is a dynamic process, so it needs new evaluation methods, like Real
107 Options Valuation (ROV) (Shafiee et al., 2009).

108 According to the previous works in mining project evaluation, we found that most economic
109 evaluation is applied to the mineral's ores like Iron, Copper, Gold, etc., which are more
110 valuable, of course (Tleubergen et al., 2017). At the same time, the phosphate ores and other
111 stone ores did not find their way into such models. In this paper, a comparison of the DCF,
112 MCS, and BDT methods is presented and applied to a real case study. We analyze their
113 similarities and differences critically from three points of view: how they treat volatility in
114 parameter values, such as metal price and cost; how they integrate the time value of money;
115 and how they require managerial flexibility. We demonstrate that, given their apparent
116 distinctions, they are simply different features of a general project evaluation system which has
117 its most straightforward type, the deterministic base case scenario. It is worth mentioning that
118 modeling the natural world involves simplification and loss of accuracy. From the general
119 perspective, all three methods can be achieved by dealing with validation elements (Galli and
120 Armstrong, 1999). We emphasized this research as a case study of the (Abu Tartur plateau,
121 Egypt) phosphate mining project.

122 Among the others, the most important parameters in economic uncertainty are metal price
123 uncertainty and operating cost uncertainty. The NPV was calculated using the three methods
124 (DCF, MCS, and BDT) applied to the Abu Tartur phosphate mining project in four scenarios:
125 (1) assuming certainty for both pricing and operating costs, (2) assuming uncertainty for the
126 price, (3) assuming uncertainty for both price and operational costs, and (4) assuming
127 uncertainty for the price, operational costs, interest rate, inflation rate, taxes' rate, grade, and
128 recovery. When both pricing and operating cost uncertainties were considered, the mine
129 evaluation presented a higher NPV (Dehghani & Ataee-pour, 2012). The evaluation method
130 based on the multidimensional binomial tree method was developed by (Dehghani et al., 2014):
131 the pyramid technique. The NPV has been computed under economic uncertainties to
132 determine the efficiency of the pyramid technique. Finally, the obtained results were compared
133 to the outcomes of other evaluation methods, such as the binomial tree. To the best knowledge

134 of the authors, no previous research performing a comparison between DCF, MCS, and BDT
135 applied to the evaluation of phosphate mining projects with the same four scenarios is present
136 in the scientific literature. Therefore, scenario four is unique, where seven uncertain parameters
137 have been studied. Software to model the binomial tree approach is rare, therefore we
138 developed the binomial lattice algorithm, and it has been implemented and coded in macros in
139 the Excel © environment to build the model. It was a unique effort to handle the problem of
140 the complicated lattice. The price lattice of this mining project has nearly 253 cells and the cash
141 flow and discounted cash flow lattices are as well. In addition, phosphate mining has lacked
142 economic studies. Therefore, with this paper, we contribute a progression to the previous
143 research in the field of mining project economic evaluation.

144 The paper discusses briefly the nature of phosphate ore chemical composing and its existence
145 in Egypt. Then the developed models have explained in detail the different techniques of
146 mining project evaluation. Results and discussion results are presented, followed by the
147 conclusion.

148 **2. Phosphate ore in Egypt**

149 Phosphates are essential elements for life on Earth (Mitra et al., 2020). Phosphates are used in
150 agriculture, industrial, and chemical process as well. Phosphorous in phosphate form is one of
151 the macro-elements necessary for plant growth (Nitrogen-Phosphorous-Potassium). Farmers
152 have long been concerned with nitrogen fertilizers only for increasing their crops, but with the
153 improvement of agriculture research, it has been established that phosphorus plays a significant
154 role in natural plant growth and therefore increases productivity (Elwageeh, 2017). The growing
155 number of people across the globe implies the growth of the demand for phosphorous.

156 The world consumption of phosphate rocks is more than 150 million tons per year, where the
157 commercial grade is about 30% of P_2O_5 or higher, as stated (Abouzeid, 2008). Egypt was granted
158 a considerable reserve of phosphates. The Egyptian phosphate ore reserves are usually of low
159 grade, so it needs mineral processing and removing gangue minerals.

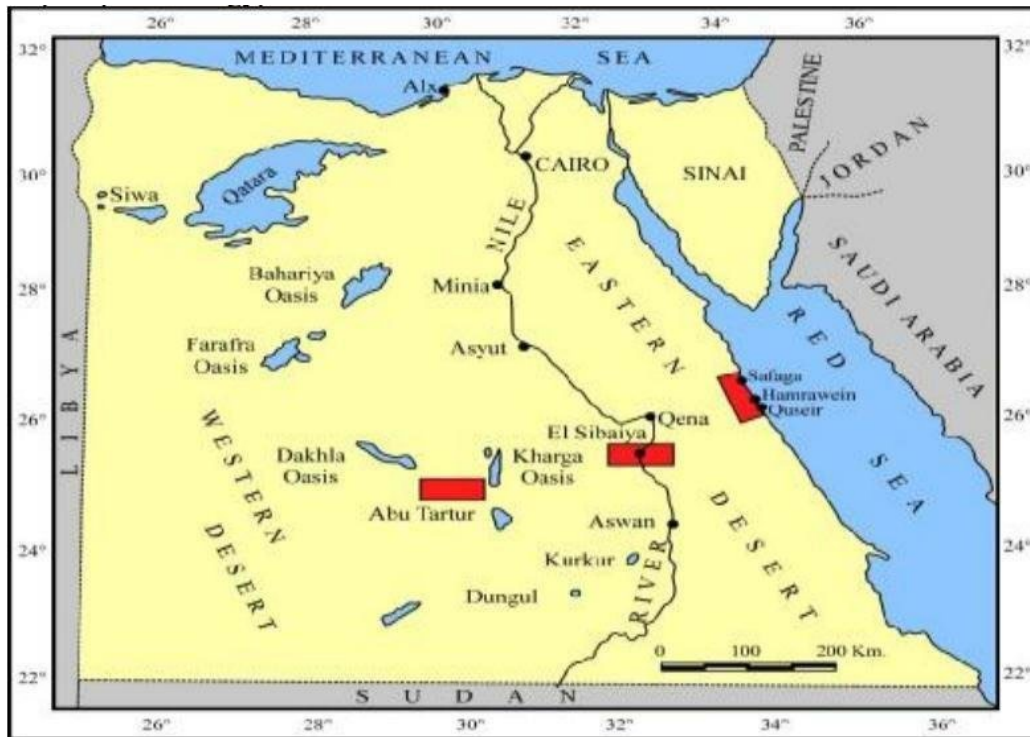


Fig. 1. Location map of phosphorus deposits belt in Egypt, (EMRA, 2021).

160 Egypt is regarded as one of the world's leading phosphate rock exporters. Nevertheless, by
 161 refining this treasure to highly evaluated items, the need to add significant value to Egyptian
 162 mineral wealth must be accomplished. The unprocessed Phosphate rock can be used directly
 163 in agriculture as soil fertilizer, and the processed mineral can be used in different applications,
 164 like pharmaceuticals, personal care products, and polishing agents in toothpaste with calcium.
 165 According to (Elwageeh, 2017), Phosphate is composed of apatite and a group of calcium
 166 phosphate minerals that is the primary source of phosphoric acid in phosphate fertilizers. Egypt
 167 has three main areas of phosphate, which is the belt of phosphate Fig. 1 from the east red
 168 seacoast to the New Valley in the western desert. This belt of phosphate is almost of deposition
 169 origin.

170 Abu Tartur plateau lies 50 km west of Kharga City, the capital of the New Valley Governorate
 171 in southwestern Egypt. The Abu Tartur plateau is perfectly connected to the urban through a
 172 first-class network of a heavy-load asphaltic road. It takes 45 minutes to go from the airport to
 173 the mine. The phosphate extracted from the Abu Tartur mine ranges from 24-31% P_2O_5 . It is
 174 ideal for producing Triple Super Phosphate (TSP), Single Super Phosphate (SSP), Nitrogen
 175 Phosphorus and Potassium (NPK), Di-Ammonium Phosphate (DAP), and Mono-Ammonium
 176 Phosphate (MAP) fertilizers and direct application. It is as well ideal for producing Phosphoric
 177 Acid after using special treatment. It also exhibits significantly low toxic components like As,
 178 Cd, Pb, Cr, and Hg. The area of the Abu Tartur plateau is about 1200 km², and a raw estimation
 179 establishes that the total reserve is more than 5 billion tons of phosphate rock (EMRA, 2021).

180 The richest phosphate rock in Egypt has been discovered in Sebaiya (Nile Valley), Safaga (Red
 181 Sea), as well as Abu Tartur (the Western Desert). The higher the amount of organic matter in
 182 the soil, the faster the phosphate rock dissolves. Phosphate rock biological solubilization is
 183 more environmentally friendly than acidulation. Phosphate rock injected with bacteria and

184 Mycrohizae has proven to be a practical approximation of using phosphate rock for continuous
185 crop production (Hellal et al., 2019).

186 Mine reclamation is the process of restoring mined land to an ecologically functional or
187 economically viable state. Although mine reclamation occurs after mining is completed, mine
188 reclamation activities are planned before a mine is permitted or started. Liquidation is the
189 process of converting assets into cash or cash equivalents by selling them on the open market.
190 An asset is a value-added component in finance. In our case study, the reclamation and
191 liquidation costs are not considered in the NPV calculations, therefore it is recommended to
192 reassess the project implementing these costs.

193 **3. Methods**

194 The mining project evaluation process involves uncertainty. The economic evaluation of
195 mining projects can be classified into four scenarios. The First scenario uses Parameters With
196 Fixed Values (PWFV). These values are considered “trusted values” that have been known
197 before from the first stages of the pre-feasibility and feasibility studies (Barnes, 1980).
198 Therefore, the economic evaluation process will use the DCF method. The second scenario
199 assumes one uncertain parameter, while the others are fixed as the first scenario which is the
200 ore price. In this scenario, the representation model is the BDT model. The third scenario
201 assumes two uncertain parameters, and the MCS method is the most common method used to
202 deal with this kind of problem. The fourth scenario of the evaluation process implements seven
203 uncertain parameters to the evaluation model using the MCS method.

204 The deterministic method, like the DCF-ROR, uses the input parameters as PWFV. The BDT
205 method mainly depends on predicting mineral price and operating cost parameters. Prediction
206 means that we know the present value of such parameters and need to know its value in the
207 future and, in turn, predict its effect on the NPV, the Internal Rate of Return (IRR), and the
208 Payback Period (PP). On the other side, the MCS will also start from present values, but it will
209 depend on the distribution of such parameters according to the variable nature of the
210 parameters. This makes turbulence of parameters like price (Dehghani and Bogdanovic, 2018)
211 reverse the investment from profit to loss. So, the MCS technique is beneficial for decision-
212 makers. MCS has been used in this study to evaluate the NPV value considering parameter
213 uncertainty (Mohapatra, 2009).

214 **3.1. The Net Present Value approach (NPV)**

215 The NPV must be computed to assess the value of the mine. In DCF, the most popular approach
216 is NPV, BDT, and MCS. NPV is defined as the cash inflow discounted to the present minus
217 the investment cost. According to Eq. (3), the NPV shows the initial capital cost and the
218 discounted cash flow. So, the NPV is defined as the difference between the present value of
219 cash inflows and the present value of cash outflows over some time. NPV is used in capital
220 budgeting and investment planning to analyze the profitability of investment projects. NPV is
221 the result of calculations used to find today’s value of a future stream of payments and costs.
222 Most decision-makers utilize a single-figure output to determine the project's financial worth.
223 However, the NPV should always be calculated based on the project's financial scale. There
224 are two types of expenditures Operating expenses (OPEX) and capital expenditures (CAPEX)
225 that a company incurs on an ongoing basis.

226 CAPEX is a company's major, long-term expense, whereas OPEX is day-to-day costs.
 227 Equipment, machines, and trucks are examples of CAPEX. Overhead expenses such as wages,
 228 utilities, and property taxes are all examples of OPEX. For tax purposes, capital expenditures
 229 cannot be deducted from income, whereas operating expenses can (Chen, 2021). Companies
 230 should be cautious about investing in projects with high CAPEX and OPEX but only a
 231 moderately positive NPV.

232 To calculate the NPV, the analyst should discount the future cash flow in addition to the
 233 investment cost, as shown in Eq. (3). The Cash Flow is calculated through Eq. (1) and Eq. (2):

$$234 \quad X_t = (P_t - C_t) \times Q_t - F_t - D_t \quad (1)$$

$$235 \quad \left\{ \begin{array}{ll} CF_t = X_t(1 - T_t) + D_t & \text{if } X_t > 0 \\ CF_t = X_t + D_t & \text{if } X_t \leq 0 \end{array} \right\} \quad (2)$$

237 Where:

- 238 X_t : taxable income at time t
- 239 P_t : price at time t
- 240 C_t : operating cost at time t
- 241 Q_t : tonnage of production at time t
- 242 F_t : fixed cost at time t
- 243 D_t : the depreciation at time t
- 244 CF_t : cash flow of the project
- 245 T_t : tax rate at time t.

$$246 \quad NPV = -CI + \sum_{t=1}^n \frac{CF}{(1+i)^t} \quad (3)$$

247 Where:

- 248 NPV: is the net present value.
- 249 CI: is the capital investment.
- 250 CF: is the cash flow.
- 251 t: is the time in years.
- 252 n: is the project lifetime.
- 253 i: is the discount rate.

254 3.2. The Discount Cash Flow method (DCF)

255 The classical deterministic method of economic analysis of a mining project is the Discounted
 256 Cash Flow (DCF), derived by (Fisher, 1930). DCF is the most reliable and popular method
 257 with three main measures NPV, IRR, and PP.

258 The DCF analysis is commonly used to estimate the discounted potential cash flows over the
 259 project life to get its present value. The PWFV is applied, so uncertainty and risk are not
 260 represented. The economic study's main aim is to help investors understand the estimated
 261 present value of the potential profit and expenses relative to the investment's costs (Stermole
 262 and Stermole, 1999).

263 **3.3. Monte Carlo Simulation method (MCS)**

264 The MCS method is designed to model the likelihood of various outcomes in a procedure that
 265 cannot be easily forecasted due to random variables. It is a method used in statistical and
 266 forecasting models to study the effects of risk and uncertainty (Brandimarte, 2014). A random
 267 value is chosen from a given range between min. and max. For each input parameter. These
 268 values are involved in the equation of the NPV computation. A standard simulation in MCS
 269 calculates many solutions, using different parameter values chosen each time randomly. When

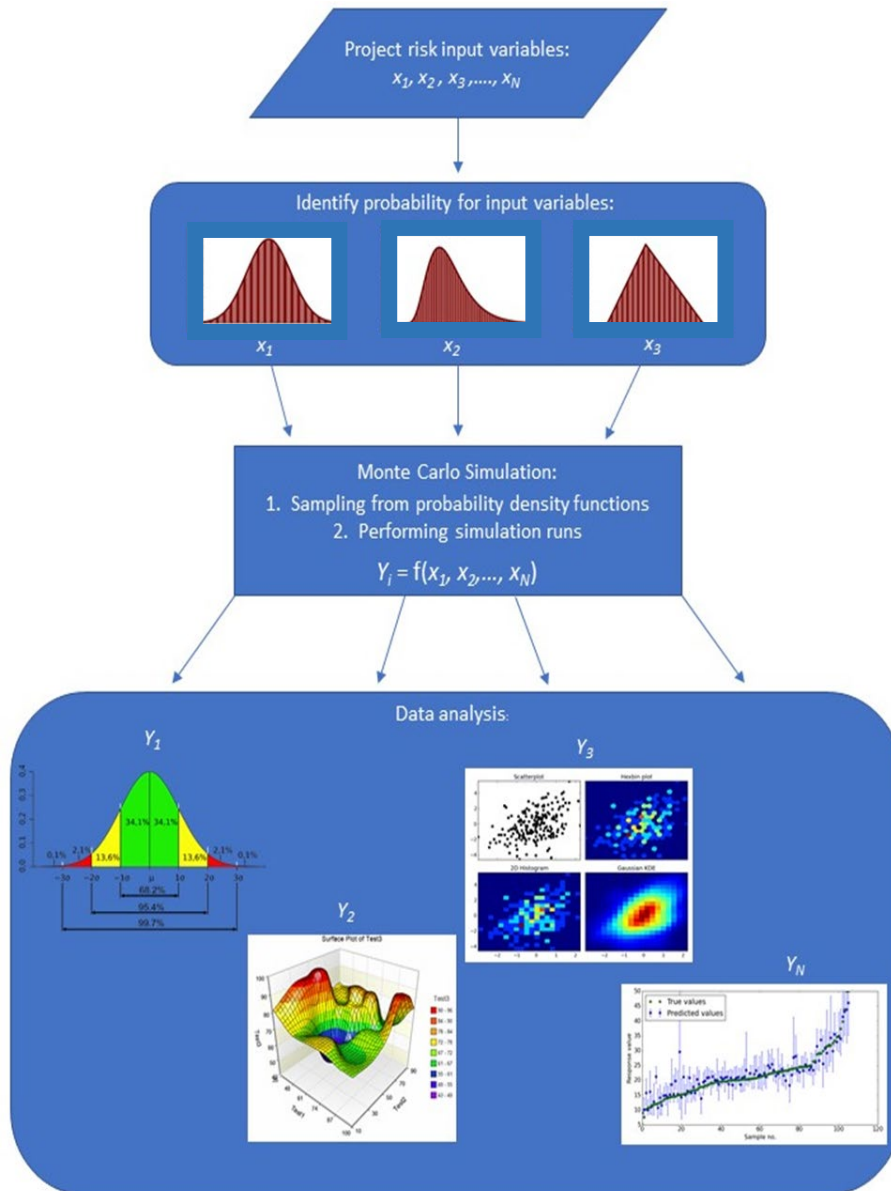


Fig. 2. The procedure of Monte Carlo Simulations from input to output (NASA, 2017).

270 the simulation is completed, as indicated in Fig. 2, we have many model outputs, each
 271 dependent on random input values. These results are used to characterize the probability of the
 272 model producing different outcomes (Samis and Davis, 2014), (Gamba, 2005) (NASA, 2017).

273 **3.4. The Binomial Decision Tree method (BDT)**

274 The binomial model is a well-known, discrete alternative time developed by (Cox et al., 1979).
 275 The BDT is a graph structure that maps all possible metal price trajectories over time, as the
 276 model permits calculating the NPV approach. The structure is made up of nodes and branches.
 277 Each node in each layer corresponds to a possible metal price at a specific point in time. Nodes
 278 are marked with both traversal probabilities and metal prices. A convenient indexing scheme
 279 has the layer or time level defined by j (number between 1 and n , number of layers or time
 280 steps), and the nodes within each layer (possible metal prices) by i (number between 1 and m ,
 281 number of nodes in the layer). The node count m for any given layer will range from j to twice
 282 the number of nodes in the previous layer, depending on whether the tree is recombining or
 283 not. Each branch or path in a binomial pricing tree represents a potential transition from one
 284 node to another later in the tree and is associated with a likelihood and a ratio. Higher node
 285 connections represent probabilistic value (P_0) and up factor (u), whereas lower node branches
 286 represent probabilistic value (P_0) and down factor (d). Fig. 3 shows a schematic binomial tree
 287 with three measures on the first metal price at time zero (P_0). The factors up (u) and down (d)
 288 and the likelihood of occurrence (P_r) were calculated using the following formulation
 289 (Dehghani, 2018)(Brandao et al., 2005).

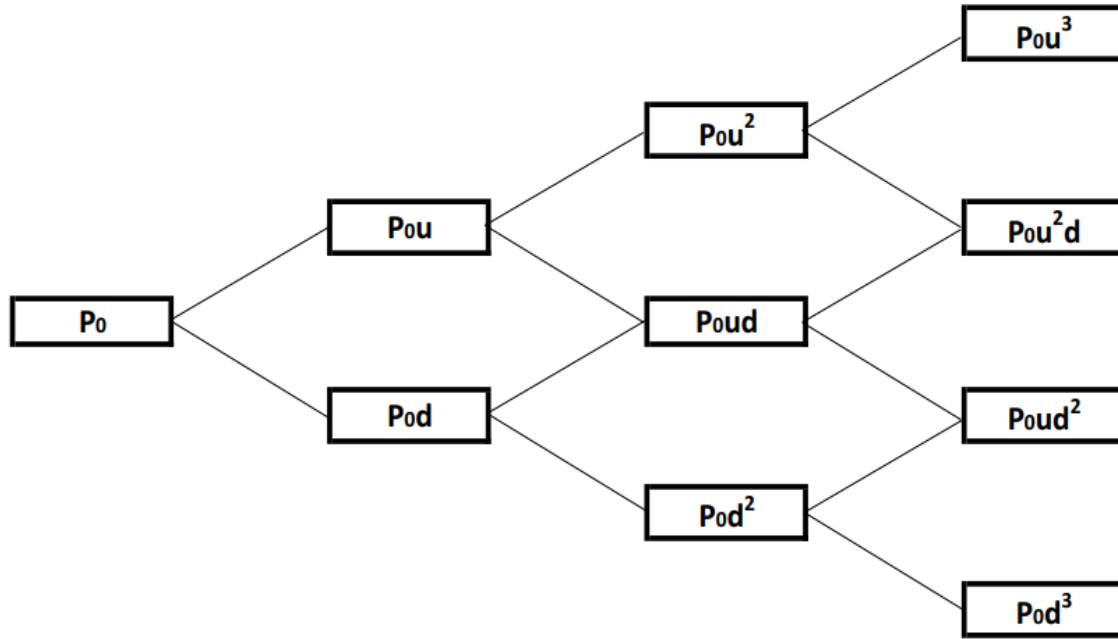


Fig. 3. Three-time step binomial tree

290
$$u = e^{\sigma \sqrt{\delta t}} \tag{4}$$

291
$$d = e^{-\sigma \sqrt{\delta t}} = \frac{1}{u} \tag{5}$$

292
$$p_r = \frac{(1+r)-d}{u-d} \tag{6}$$

293 The volatility of the metal price (σ), the risk-free rate (r), and the stepping time (δt) are the
 294 primary inputs. In evaluating the proposed Abu Tartur area project, we select three different
 295 scenarios, assuming a combination of deterministic and probabilistic parameters.

296 **3.5. Computation methods**

297 To evaluate the feasibility study of a hypothetical phosphate mine project, we implemented
 298 several Excel worksheets for the three presented methods, DCF, BDT, and MCS method. From
 299 a computational point of view, the DCF method is quite simple and easy to implement. The
 300 BDT algorithm complexity has been implemented using the Visual Basic for Applications
 301 (VBA) programming language in an Excel environment. The MCS method has been applied
 302 to our scenario using the software (@Risk, Crystal ball).

303 In the following, we compute the pre-feasibility study using the deterministic and probabilistic
 304 methods, as presented in Table 1.

305 **Table 1**
 306 Computation methods assumptions

Computation method number	Comp. Method	Variable's type	Variable's uncertain assumptions
1	DCF	deterministic	None
2	CF, BDT	deterministic and probabilistic	ore price
3	MCS	deterministic and probabilistic	ore price and operating cost
4	MCS	probabilistic	ore price, operating cost, grade, recovery, ROR, inflation rate, and taxes rate

307 In computation method 1, we assumed deterministic metal price and operating cost. In this
 308 way, we can estimate the DCF value of the mine. The purpose of these calculations is to make
 309 investors aware of the expected present value of future cash flows compared to investment
 310 costs. Eq. (1), Eq. (2), and Eq. (3) can be used to get the base case NPV.

311 In computation method 2, it is considered that metal price is uncertain using the Binomial
 312 Decision Tree (BDT). A binomial tree is constructed depending on the historical data and Eq.
 313 (4), Eq. (5), and Eq. (6). It is the method to forecast future prices. The future operating cost
 314 data is determined using regression analysis depending on the inflation parameter. So, a new
 315 Cash Flow (CF) binomial tree is estimated depending on the previous lattice, annual estimated
 316 operating cost, and Eq. (1) and Eq. (2). Finally, the CF is discounted in another tree that is
 317 constructed. The DCF will be estimated using Eq. (7) and opposite the previous lattices (from
 318 right to left).

319
$$DCF_{t,k} = CF_{t,k} + \frac{p_r \times DCF_{t+1,k} + (1-p_r) \times DCF_{t+1,k+1}}{(1+r)} \quad (7)$$

320 Where k is the node number at time t.

321 In computation method 3, the price and operating cost have been assumed as uncertain
 322 parameters, while the other parameters are PWFV. For each parameter, a probability
 323 distribution is chosen depending on historical data. A close fit to the distribution is supposed

324 to result in good predictions. In distribution fitting, it is necessary to use a distribution that is
325 well-suited to the data. The project evaluation is obtained by the MCS method.

326 In computation method 4, all dynamic input parameters are uncertain, including (price,
327 operating cost, grade, recovery, interest rate, inflation rate, and Taxes rate), while the other
328 parameters are PWFV. After the global economic crisis in 2009 and the revolutions that
329 occurred in Egypt in 2011 and 2016; Egypt has floated its currency in a move that has reduced
330 its value by almost 50% against the USD dollar in 2016. The Egyptian Central Bank considered
331 this move as one of a list of reforms designed to strengthen confidence in the economy. The
332 Central Bank has also increased interest rates by 3% reaching 14.75%. Therefore, we aimed to
333 study the effect of variability of the interest rate by using a probability distribution.

334 The proposed method of project estimation that consider the variable uncertainty is the MCS.
335 To perform the MCS, the first step is to develop an analytical model to be evaluated. The
336 second step is to generate a probability distribution for each variable of the model, using
337 subjective or historical data. The MCS model computes the outcomes of the project using the
338 statistical distribution of all the parameters in the NPV equation. The statistical distributions,
339 such as normal, lognormal, triangular, and uniform, are used to evaluate the project considering
340 the uncertainty in parameters. We used the statistical distributions of the variables according
341 to the available data and we assumed the statistical distribution of parameters for variables with
342 poor information. In every simulation, the values are selected randomly from each parameter
343 distribution for every period and substituted into the NPV equation to generate one possible
344 outcome of the project. We noted that the price and operating cost had identical distributions
345 and values as in scenario 3. Finally, many model outcomes are generated, and a corresponding
346 model of probabilities vs. model outcomes is generated.

347 4. Results

348 This study investigates the uncertainty of prices and costs of the ore and the equipment.

349 4.1. Input Data

350 The input data that have been used in this work are literature data from (Elwageeh, 2017). In
351 Table 2, a resume of the technical parameter is reported. The Phosphate average grade (30%)
352 and the cutoff grade (25%) improve the company's profit (Afify and Zaghloul, 2014) (Hellal et al.,
353 2019). The stripping ratio is defined as the amount of overburden that should be removed to
354 extract a given ore quantity. The hypothetical mine life is determined by dividing the reserve
355 tonnage at the cutoff (million tons) (Azimi et al., 2013) over the ore production rate planned
356 (million tons/year), which gives 22 years for the production rate of 3.5 million tons/year. Table
357 3 is resumed the financial parameters used in this study.

358 **Table 2**

359 Technical parameters related to mine

Technical Parameters	
Average Grade	30.0%
Cutoff Grade	25.0%
Reserve Level at Cutoff (million tons)	77.50
Contained Value (million tons of P ₂ O ₅)	23.25

Stripping Ratio	8.5
Ore Production Rate (t/d)	11,667
Mill Recovery	95.0%
Operating days/year	300
Mine Life (year)	22
Ore Production Rate planned (million tons/year)	3.50

360

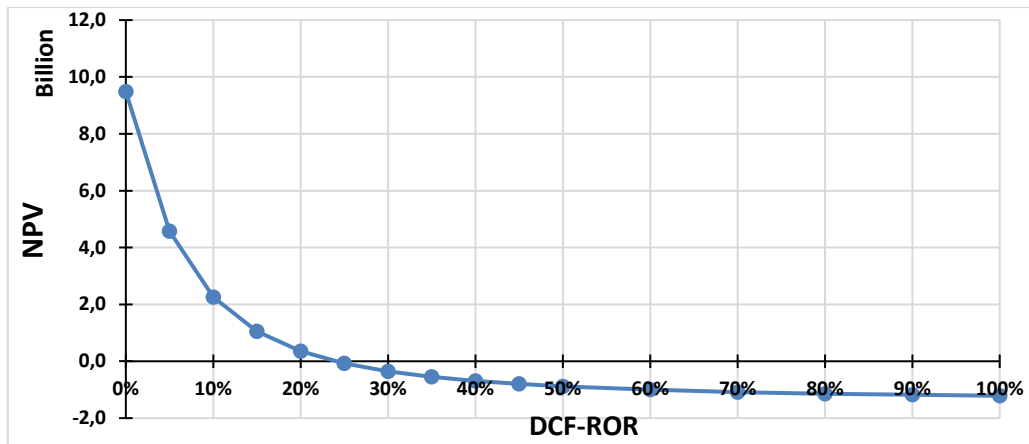
361 **Table 3**

362 Financial parameters

Financial Parameters	
Average (HPO4) Price (\$/t)	766.0
Mine Operating Cost (\$/t)	15.0
Mill Operating Cost (\$/t)	45.0
Total Operating Cost (\$/t)	60.0
Mine Capital Cost (\$)	580,000,000
Mill Capital Cost (\$)	800,000,000
Total Capital Cost (\$)	1,380,000,000
Working Capital (\$)	60,000,000
Capitalized Exploration Cost (\$)	1,000,000
Depletion Allowance (%)	15.0%
Royalty (% Net Smelter Return)	5.0%
Income Tax Rate (%)	46.0%
Salvage Value (% of Capital Costs)	10.0%
Real Risk-adjusted Discount Rate (ROR) (%)	10.0%
Inflation (%)	3.0%

363 **4.2. Output Data**

364 For computation method 1, considering the deterministic metal price and operating cost, Fig.
365 4 represents the relation between NPV and DCF-ROR. The figure indicates that the NPV
366 decreases with an increase in the ROR. We notice that, at DCF-ROR=0%, the NPV is \$9.5
367 Billion, which is an impossible case, of course. While at DCF-ROR 24%, NPV vanishes (i.e.,
368 NPV=\$0), and in this case, the DCF-ROR is called the IRR. By increasing the ROR more than
369 the IRR limit (24%), we obtain a negative NPV which means that a risk case benefits the mining
370 project.



371

Fig. 4. The relation between net present values (NPV) versus DCF-ROR.

372 The NPV, IRR, and payback period are presented in Table 4. The NPV, calculated at a 3%
 373 inflation rate and 10% rate of return, is estimated at \$2.25 billion. NPV is a positive value
 374 which is a good indication for the project. The second measure is the IRR; in our case, we get
 375 nearly 24% greater than the introductory rate ROR=10%. It is also a good indication and
 376 confidence in our progress in the project's evaluation process. The payback period is estimated
 377 at five years, less than a quarter of the project's lifetime (22 years). So, it is a very positive
 378 duration for payback.

379 **Table 4**
 380 Results from the DCF-ROR method.

Results	
Average NPV (\$)	2,258,968,007
IRR (%)	24.00
Payback Period (years)	5.0

381 Computation method 2 (uncertain metal price) is used to predict the mineral price and calculate
 382 the free cash flow, the DCF, and the NPV. Each of them has its decision tree and equations.
 383 So, in this scenario, one of the input data is of uncertain value while the others are PWFV; this
 384 parameter is the ore price.

385 Table 5 contains the input data for each decision tree described previously. The first price in
 386 the price tree is 766 \$/ton, which is used in the model of DCF. Volatility (σ) represents the
 387 change rate of historical price data. In the BDT method, the risk-free rate of return is the
 388 theoretical rate of return of an investment with zero risk. In a risk-free investment, the risk-free
 389 rate is the interest an investor would incur over a specified period of time. The so-called "real"
 390 risk-free rate can be calculated by subtracting the current inflation rate from the yield of the
 391 treasury bond matching the investment duration.

392 **Table 5**
 393 Input parameters related to price for the BDT method and BDT-NPV result.

Input data	
Volatility (σ)	28.5%
Up (u)	1.33

Down (d)	0.75
Risk-free rate (rf)	7%
Stepping time (t)	1.0 year
Probability (Pr)	55%
First price in the Binomial lattice (P₀)	766\$/ton
Average NPV	\$2,126,849,233

394 The parameters for the computation method 3 are reported in Table 6 and Fig. 5. It assumes a
395 distribution function for parameters subject to uncertainty. Table 6 reports the basic statistics
396 and the histogram for the seven parameters used in calculations. The historical data for the
397 seven parameters have been analyzed and the statistical distribution has been assumed using
398 the best-fitting theoretical distribution. The model of MCS generates random numbers from
399 the range (0-100%) for all parameters of the theoretical statistical distribution, then applied in
400 Eq. (1), Eq. (2), and Eq. (3) to get the NPV value. This process has been repeated 10000 times
401 and finally, was possible to generate the probability distribution of the NPV.

402 Table 7 shows the correlation matrix between the input variables which is an important factor
403 to consider in Monte Carlo simulation. When the input random variables are considered
404 independent, the uncertainty can be underestimated or overestimated. In the case of a positive
405 correlation, the value for each should be relatively high in one simulation iteration and
406 relatively low in next simulation. For negatively correlated inputs, one variable should be at
407 the high end of the possible range for a given iteration, while the other should be at the low
408 end. In most cases, the variables are assumed to be independent to simplify the calculation. In
409 reality, the majority of the variables are frequently correlated. In mining, for example, ore
410 grades are positively correlated with ore recovery. In addition, it is shown that the prices and
411 operating costs are correlated with the inflation rate.

412 **Table 6**
413 Input data about phosphoric acid in the MCS technique.

Parameter	Distribution	Mean	Standard Deviation	Skewness	Kurtosis	Histogram of the parameter
Average Price	Normal	766.5 \$/ton	16.5 \$/ton	0.186	-0.521	

Operating Cost	Normal	63 \$/ton	2 \$/ton	0.182	-0.207	
Average Grade	Normal	29.4 %	8 %	0.121	-0.756	
Interest Rate	Log-Normal	11%	3%	1.315	0.619	
Inflation	Log-Normal	10.4%	6%	0.895	0.143	

Income Tax Rate	Uniform	44.55%	0%	-	-	
Mill Recovery	Normal	95%	0%	-	-	

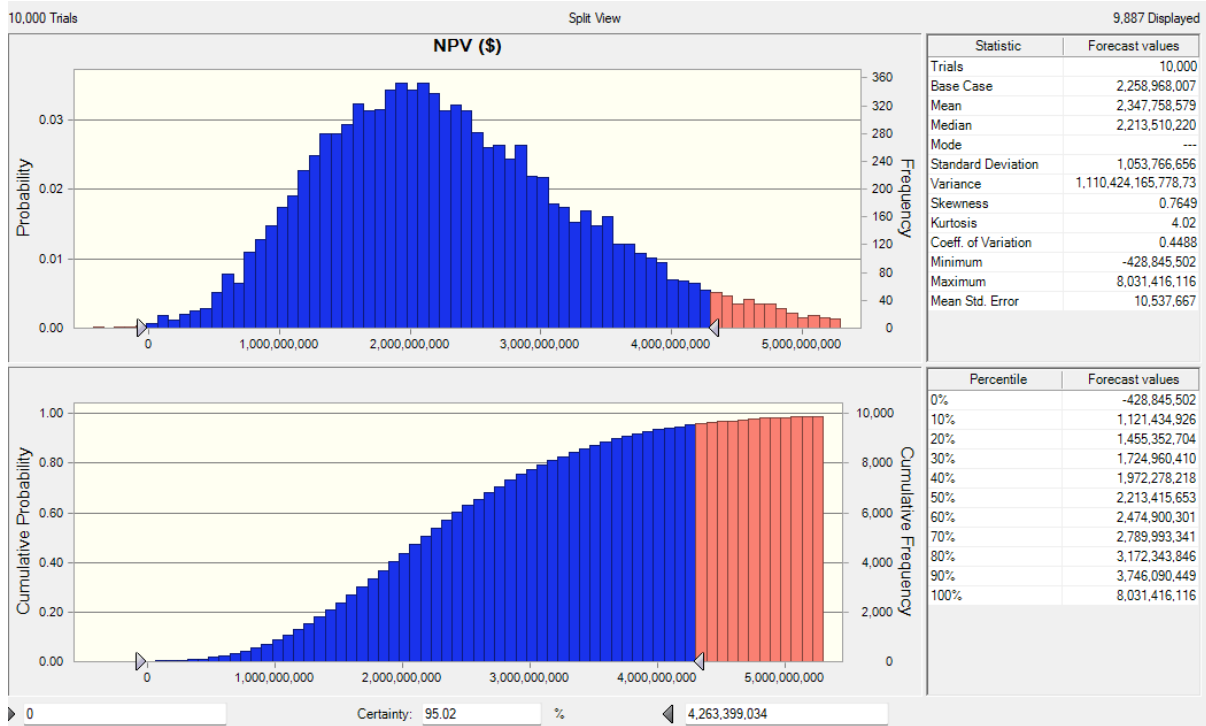
414 **Table 7**

415 Correlation index between input data in the MCS technique.

Parameter	Average Price	Operating Cost	Average Grade	Interest Rate	Inflation Rate	Income Tax Rate	Mill Recovery
Average Price	1.0	-	-	-	-	-	-
Operating Cost	0.77	1.0	-	-	-	-	-
Average Grade	0.80	0.50	1.0	-	-	-	-
Interest Rate	-0.45	-0.22	0.03	1.0	-	-	-
Inflation Rate	0.94	0.88	0.28	0.80	1.0	-	-
Income Tax Rate	0.66	-0.40	-0.08	0.01	0.01	1.0	-
Mill Recovery	0.30	0.60	0.05	0.01	0.01	0.01	1.0

416 Due to the character of this preliminary feasibility study of the mine project, the statistical
417 distribution of variables has been fitted to the histogram distribution. The mean and the
418 standard deviation have been used to define the variable distribution in the Crystal Ball
419 software. The mean values are corresponding to values used in scenario 1, for all the
420 parameters. The statistics of data have been estimated using SPSS software (Farahani and
421 Bayazidi, 2018) from the historical data fluctuation for those parameters (AFA, 2021).
422 Depending on the characteristics of the phenomenon and the distribution, some probability
423 distributions can be fitted more closely to the observed frequency of the data rather than others.
424 The distributions of the discount rate, income tax, and inflation parameters are taken based on
425 the official websites of the Central Bank of Egypt (Egypt, 2022a), the Ministry of Finance of
426 Egypt (Egypt, 2022b), and the Central Agency for Public Mobilization and Statistics (Egypt,

427 2022c). These parameters are distributed according to the guidance values published on such
 428 websites, and they were considered unrelated.

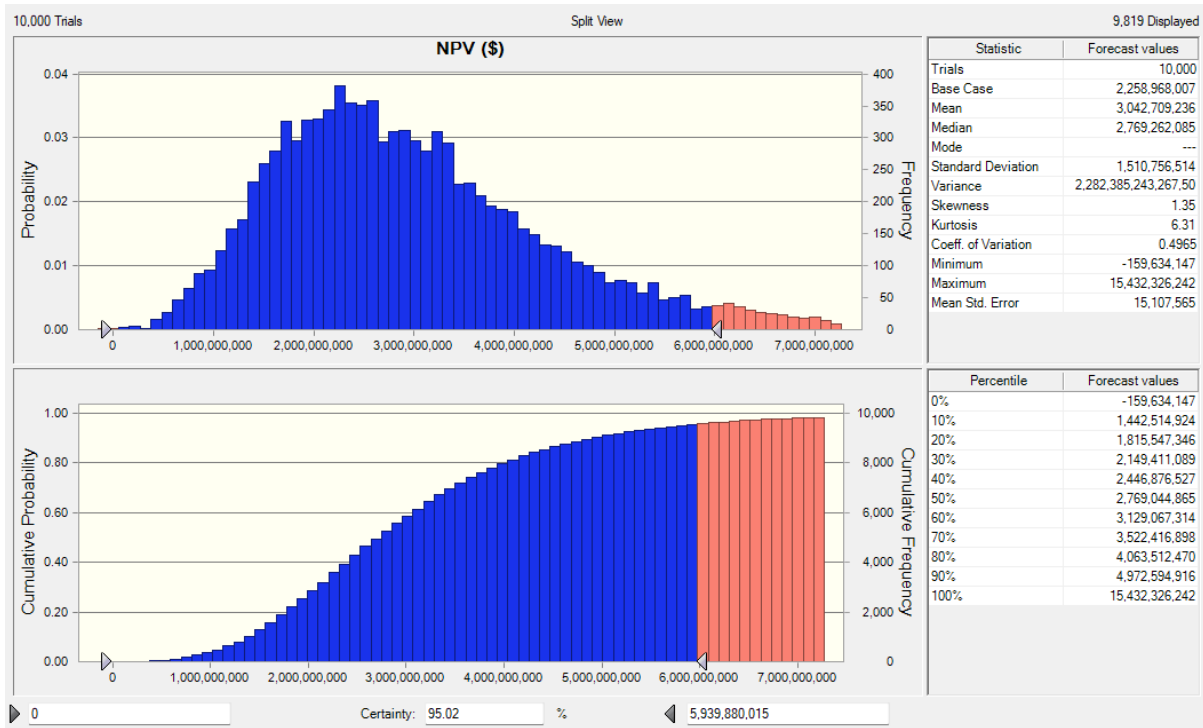


429

Fig. 5. Statistical distribution of NPV and its parameters for scenario 3.

430

431 Fig. 5 represents the results of the computation method 3. It reports the NPV as a function of
 432 the parameter's probability. It indicates that the mean value of MCS is near the base case, under
 433 the effect of uncertainty in the parameters (price and operating cost). The graph reveals that for
 434 the uncertainty of the input parameter of 50 % (Median value), the expected value of the NPV
 435 was 2.35 billion dollars. If we consider a confidence interval of 95%, the expected value of the
 436 NPV will be between (0.00-4.26) billion dollars.



437

Fig. 6. Statistical distribution of NPV and its parameters for scenario 4.

438 Computation method 4 is represented in Table 6 and Fig. 6. It considers the case in which most
 439 input parameters are subject to uncertainty. As shown in Table 6, we have seven parameters
 440 under the effect of uncertainty. Fig. 6 shows the results of the MCS run obtained using 10000
 441 iterations. The graph reveals that for the uncertainty of the input parameter of 50 % (Median value),
 442 the expected value of the NPV was 3.04 billion dollars. If we consider a confidence interval of 95%,
 443 the expected value of the NPV will be between (0.00-5.94) billion dollars. In MCS, assuming that
 444 the ore price, operating cost, grade, and other parameters are subjected to uncertainty, the NPV
 445 is calculated considering uncertainty and offers more realistic values, than the other methods.
 446 As shown in Fig. 6, the distribution of the output of the NPV is lognormally skewed to the left.
 447 This effect comes from considering the lognormal distribution of some variables used in the
 448 simulation.

449 5. Discussion

450 The DCF method deals with technical and financial parameters in a deterministic manner. BDT
 451 technique handles the parameters uncertainty with a discrete probability for each node, but the
 452 tree becomes exponentially larger with increasing the time step compared to the mine life. The
 453 MCS was better than the DCF and the BDT methods of handling the uncertainty using
 454 continuous distribution for technical and financial parameters.

455 The DCF method assumes that the scenario used PWFV, so the decision-maker cannot have a
 456 clear overview of the final results of the parameter change. The BDT method deals with
 457 different strategies and shows all the probable outcomes for the NPV obtained from these
 458 strategies. The MCS method focuses on modeling the uncertainties and neglects managerial
 459 flexibility, but it shows the distribution of the NPV before starting the project.

460 MCS is a natural extension for the base case of NPV since the variables are not known with
 461 certainty. These variables are described using statistical distributions such as normal, log
 462 normal, and uniform distribution. Although it is allowed to make correlations between
 463 variables, they are viewed as independent. In MCS, the project life is fixed in all cases, and the
 464 results have the histogram of future results and the expected value. The underlying distinction
 465 lies in how the time value of money is treated. In the BDT method, the maximum expected
 466 value is estimated by folding the tree back from the outer end branches toward the start. They
 467 have some characteristics such as: (i) how they handle the time value of money (discount rate
 468 vs. risk-free rate plus change of probability), (ii) how they allow for uncertainty in parameter
 469 values, and (iii) whether they incorporate managerial flexibility.

470 Scenario 1 indicates that using the PWFV, the mine project is feasible. Scenario 2 considers
 471 the variability of the mineral price, indicating that in some cases, the project is not feasible, and
 472 in others, the NPV is greater than the NPV obtained by Scenario 1. Scenarios 3 and 4 allow for
 473 obtaining similar results, with the added value to give jointly with the NPV a measure of the
 474 probability of success of the mining project. In addition, the MCS method allows re-generating
 475 during the mine exploitation by updating the random used values with the actual values,
 476 allowing for back analysis.

477 **Table 8**
 478 Summary of the results.

Case No.	Average NPV (B \$)	Change of the base case	Confidence Interval	Notes
Scenario 1	2.26	0%	0%	DCF (Base Case)
Scenario 2	2.13	-6%	<50%	BDT (uncertain mineral price)
Scenario 3	2.35	4%	95% (0-4.26 B\$)	MCS (uncertain price and operating cost)
Scenario 4	3.04	34%	95% (0-5.94 B\$)	MCS (uncertain multi-input parameters)

479 Table 8 shows the NPV obtained using the four different computation methods. The reference
 480 case was the DCF method, which used the deterministic value of the technical and financial
 481 parameters. These parameters mainly come from the feasibility and pre-feasibility studies made
 482 on the project. According to the computation of the NPV, it was estimated that NPV was \$2.26
 483 billion, which is an encouraging value (NPV>0) for the feasibility of the project. The IRR was
 484 24%, which is an encouraging result as well. Encouraging results for mining projects are
 485 considered when IRR>ROR10%. Finally, the computed payback period was five years. So, it
 486 is also fine because the Payback Period is less than the mine life, which was established in 22
 487 years (as in Table 2).

488 The second scenario has been computed using the BDT method. The average value for this
 489 scenario was NPV=\$2.13 billion, which is lower than the reference case of the DCF by -6%.

490 The MCS technique for the third computation method was obtained considering the uncertain
 491 mineral price and operating cost parameters. Results show that the NPV was higher than the
 492 reference scenario by 4%, which is a good evaluation result of the method.

493 The fourth scenario worked for seven uncertain parameters using the MCS method. Results
 494 show that the computed average of the computed NPVs was higher than the reference scenario
 495 by 34%. The confidence interval was higher in scenarios 3 and 4.

496 These project evaluation results were hopeful and can encourage the investor to consider the
497 mining project with more confidence. Fluctuations of the ore price and the cost are not
498 represented in the deterministic method, while in the probabilistic methods they are taken into
499 consideration. Therefore, the MCS method can study thousands of trials or scenarios that may
500 happen in the real world and give the probability of the results and their confidence intervals
501 value. The MCS can consider the real-world parameter variability, in order to compute
502 thousands of scenarios and compute the probability of the obtained NPV. The issue here is how
503 to represent the parameters well so that we can evaluate the project accurately. In our case
504 study, we found that the project is profitable from the DCF calculations, but what if the price
505 decreases, and what if the cost increases? The sensitivity analysis or what-if analysis can
506 answer these questions for limited cases and limited parameters. The MCS aims to consider
507 these problems, therefore we can represent the fluctuation of multi-parameters in one model
508 and at any stage of the project.

509 **6. Conclusion**

510 The work aims to assess phosphate mining project scenarios in the Abu-Tartur region, Egypt.
511 Different methods have been used to estimate the NPV, IRR, and PP in the present work. The
512 latter parameters have been computed using different computation methods under various
513 scenarios. Discounted Cash Flow (DCF) is commonly used to evaluate mining projects, while
514 other methods like Binomial Decision Tree (BDT) and Monte Carlo Simulation (MCS) enable
515 the decision-maker to evaluate the project clearly and consider the risk in the project due to
516 uncertainty in parameters. Finally, the BDT method results in a specific value of NPV
517 depending on the probability of input parameters. We can conclude that the MCS method is
518 favorable compared to other methods. The MCS method applies a set of parameters that, in
519 some situations, produce better results than others, although this is dependent on the likelihood
520 of the input parameters. It is the best way since it aids in considering the uncertainty of the
521 parameters and variability assessment and determining which parameters greater influence the
522 outcomes.

523 In order to consider uncertainty, the distributions of the input variables were tested for goodness
524 of fit with the observed data using histograms. The parameters of the normal, lognormal, and
525 uniform distributions, i.e., the mean and standard deviation were also calculated from the data
526 for each variable. The MCS for project evaluation overplays the importance of the choice of
527 which distribution to use to represent input variables and underplays the importance of
528 assessing and including correlations between the variables. Simulation runs of the evaluation
529 model including correlations showed that correlations must be included in Monte Carlo
530 Simulation otherwise the analysis leads to an underestimate of risk. It is relatively more
531 important to investigate and allow correlations between the variables than to select the best fit
532 distribution to represent the parameters in the MCS.

533 The MCS technique allows to improve project estimation and comprehension of the variability
534 of the results. So, the MCS method allows for a better estimation of the project and an
535 understanding of the variability of the results. Real validation of the proposed method will be
536 possible when the mining project will start. Future work can consider a comparison of the
537 results obtained in this study with other project evaluation emerging methods, like the Real
538 option Valuation and Cash Flow at Risk (Sauvageau and Kumral, 2018), as well as considering
539 the liquidation/ reclamation cost of the project and the mineral grade modeling using, for
540 example, geostatistical methods.

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