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VALUE CREATION AND INVESTMENT PROJECTS: AN APPLICATION OF FUZZY SENSITIVITY ANALYSIS TO PROJECT FINANCING TRANSACTIONS

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This paper presents a methodology which blends sensitivity analysis and fuzzy arithmetic for managing uncertainty in project financing transactions. Specifically, we adopt the perspective of the equityholders and use the average Return On Equity (ROE) to measure shareholder value creation and, in particular, the financial efficiency of the equity investment. We cope with uncertainty via global and local sensitivity analysis and fuzzy arithmetic; we use the fuzzy version of the well known (global) *Gamma* indicator and we introduce the fuzzy versions of two (local) importance measures, the *Differential Importance Measures* (DIM). We then apply them to the pro forma financial statements drawn up by the analyst for measuring and ranking the impact of the key accounting parameters on the resulting values and we show how the uncertain accounting and financial magnitudes of the project company affect the financial efficiency. Among the advantages of this analysis, aimed to enhance the managerial insights generated by the financial model and to lead to appropriate managerial actions, we focus on the attractiveness of fuzzy calculus and possibility theory to represent and compute with all relevant financial data that appear in project financing and budgeting, where available information is characterised by incompleteness or non-statistical uncertainty. In this context, fuzzy computing and appropriate sensitivity analysis techniques, based on application of the extension principle, allow complete investigation of the project characteristics.

Keywords and phrases: Investment analysis, project finance, financial efficiency, local and global sensitivity analyses, fuzzy numbers, extension principle.

Introduction

Project finance is a form of financing whereby a group of sponsoring firms creates a new economic entity – the Special Purpose Vehicle (SPV) or *project company* – for the sole purpose of undertaking a specific project. The project is usually a large-scale, long-term one, so typical projects where this form of financing is found are power generation facilities, oil and natural gas pipelines, electric utilities, chemical plants, water and waste water treatment facilities, renewable energy and green technologies, hospitals, railways, roads, internet and e-commerce projects (see ref.[1], ref.[2], ref.[3]).

The funds needed for designing, building, and executing the project consists of (i) equity capital provided by the sponsors, which are the shareholders of the SPV, and (ii) loans granted by a group of banks, which constitute the debtholders (see ref.[4], ref.[5], ref.[2]).

As compared with corporate financing, project financing transactions are riskier enterprises because lenders have no (or limited) recourse to the sponsors' assets, and the loans' only collateral are represented by the project company's assets: "By segregating risky assets in a project company, managers can prevent a failing project from dragging the parent firm into default. Project finance allows the firm to isolate asset risk in a separate entity where it has limited ability to inflict collateral damage on the sponsoring firm." (see ref.[5] p. 217). For this reason, debtholders impose a large set of covenants, aimed at limiting their own risk by reducing the flexibility of the use of funds by the SPV's shareholders (see ref.[6]).

The particular features of project financing imply that an extremely careful analysis of the project's financial efficiency must be conducted in order to verify whether the project is able to cover the operating costs, service the debt, and yield a minimum acceptable return to equityholders. This implies that time and effort are devoted by sponsors and lenders to modelling the accounting and financial magnitudes, since shareholders and debtholders must agree on the estimations of the key inputs (otherwise the transaction will not take place).

As a result, financial modelling and pro forma financial statements play a pivotal role in project finance. Specifically, analysts are involved with the delicate task of constructing a sequence of pro forma financial statements (income statements, balance sheets, cash-flow statements) which aim to single out the key drivers and measure their impact on the project company's expected economic profitability. The key drivers consist of accounting and financial magnitudes which relate to both estimated variables and decision variables: operating costs, sales revenues, use of plant and machinery, amount of working capital, financing mix, etc. They result from

(1) operating decisions (e.g. credit policy, marketing policy, cost management)
(2) financing decisions (e.g., borrowing policy, payout policy)
(3) economic prospects (potential market demand, expected prices of raw material or finished goods, etc.).

The accuracy of this process is essential and is guaranteed by the intervention of several parties with conflicting interests: the sponsoring firms and the banks, along with advisers and auditors, must certify the key inputs and the pro forma financial statements.

Traditionally, in engineering economics and corporate finance, shareholder value creation is either expressed as an absolute measure such as the Net Present Value (NPV) or as a relative measure quantifying financial efficiency. Among others, relative measures include the internal rate of return (IRR), the modified internal rate of return (MIRR), some accounting rates such as the return on investment (ROI) or the return on equity (ROE) (see ref.[7], ref.[8] and ref.[9]; see also ref.[10] for a recent account of absolute and relative measures and the role of accounting rates of return for measuring economic profitability). An accept-reject decision is usually based on either types of measure or both, as long as they are consistent (see ref.[10] on a reconciliation and a unified approach to capital budgeting decisions).

In this paper, we measure shareholder value creation and the project's financial efficiency by employing the ROE as derived from Magni's Average-Internal-Rate-of-Return (AIRR) approach (see ref.[11], ref.[12] for details). We net it out of the cost of equity capital and call this efficiency measure the *above-normal ROE*. The AIRR approach has already been applied in industrial applications, real estate assets, portfolio management, savings and credit transactions, firm valuation (e.g., ref.[13], ref.[14], ref.[15], ref.[16], ref.[17] and ref.[18]; see also ref.[10] for a detailed treatment) and is particularly suited for project finance, precisely because it makes direct use of the pro forma financial statements built by sponsors and lenders. In them, prospective capital amounts and incomes are reported, whence the SPV's ROEs are easily computed for each period.

Applying the AIRR approach to the ROEs, the above-normal ROE is found, measuring the project's financial efficiency and indicating whether the project is worth undertaking. Contrary to the traditional IRR approach, this approach is consistent with the estimated incomes and capital values as reported in the pro forma financial statements and has the advantage of setting a direct relations between the accounting rates of return, which are massively used in practice, and shareholder value creation as measured by the Net Present Value (NPV). In particular, consistency between the above-normal ROE and the NPV derives from the fact that the NPV may be obtained as the product of the total capital invested by the equityholders (which measures the investment size) and the above-normal ROE (which measures the investment's financial efficiency) (see ref.[12], ref.[19] and ref.[10] for details).

Mathematically, such an above-normal ROE is equal to the ratio of the discounted sum of the net incomes to the discounted sum of book values; as such, it does not suffer from problems of existence and uniqueness, which sometimes impair the IRR. As opposed to the IRR, it can easily cope with time-varying costs of capital (and, therefore, with a time-varying term structure of interest rates) and can be easily expressed as a function of the key inputs estimated by the analyst, which makes it easier to analyse the robustness of the financial efficiency under changes of one or more inputs of the model, (see ref.[20] on the problematic use of IRR when inputs change; see also ref.[21] on the use of IRR in capital investment projects and ref.[22], ref.[23] on the impact of inputs' changes on both NPV and IRR).

The process of drawing suitable pro forma financial statement ends up in prospective accounting and financial outcomes (book values, incomes, and cash flows). However, notwithstanding the accuracy with which financial statements are built and agreed upon (a task which may take several months), the realized outcomes may well deviate from the expected outcomes. In this respect, the project company's pro forma financial statements only represent the *base case*, which reflects the base (i.e., most likely) values of the key inputs. Both sponsors and banks may then benefit from some managerial tool which, starting from the financial statements, is capable of detecting the major sources of uncertainty in a more sophisticated way and measure their impact on the project company's financial efficiency. This may lead to a more informed decision on whether to incorporate the SVP (i.e., accept the project) or not.

The project's risk may be treated in several different ways. In applied corporate finance, risk is often managed with a risk-adjusted cost of capital. A most widely used cost of capital is the weighted average cost of capital, which takes account of the various sources of financing (e.g., see ref.[24], ref.[25], ref.[16]).

Less common in project appraisal and investment decision-making is the use of sensitivity analysis (SA) for managing uncertainty in capital budgeting projects. SA can assess the robustness of the project to changes in the value drivers, measure the relevance of the key parameters and rank them in terms of their impact on the model output. The use of SA enables extracting relevant information and triggering managerial insights, possibly leading to further investigations into the sources of risk and suggesting proper managerial actions aimed to perturb some key inputs in order to reduce the project's uncertainty or increase its economic efficiency (see ref.[26], ref.[1], ref.[27], ref.[28] on SA techniques). SA has been applied in ref.[1] for a project financing transaction to assess the degree of coherence between the NPV and the debt service coverage ratio, from an equity perspective; ref.[29] applied sensitivity analysis to the IRR of photovoltaic grid-connected systems; ref.[21] employed sensitivity analysis to measure the consistency between IRR and NPV in industrial projects. ref.[20] applied sensitivity analysis to measure the degree of consistency between the average ROI and the NPV of industrial projects. Real-option theory is also sometimes invoked if some kind of managerial flexibility is present (e.g., see ref.[30] for uncertainty in the time-to-build, and ref.[31] for an extension of real-option analysis.

Fuzzy logic approaches in financial, economic, and decision making modelling started after 1990, with few publications before that year (e.g., ref.[32], ref.[33], ref.[34], ref.[35], ref.[36]); a substantial amount of contributions has been published in the last decades related to capital budgeting problems which extend traditional and non-traditional measures to fuzzy numbers (see, e.g., ref.[37], ref.[38], ref.[39], ref.[40], ref.[41]), including possibility theory and its applications (see ref.[42], ref.[43], ref.[44], ref.[45], ref.[46], ref.[47]). An extended literature survey on project investment is ref.[48]. For a comparison between fuzzy and probabilistic approaches to project investment see ref.[49], ref.[50], ref.[51]. In ref.[53] a nice survey of the state of the art is presented, along with modifications of the traditional NPV and IRR. Recent studies in fuzzy decision making include applications to renewable energy investment projects and risk (see ref.[54], ref.[55]), to bank investments (e.g., ref.[56], ref.[57]), among many others (see the recent special issue ref.[58] which includes twenty-four papers on fuzzy systems in management and information sciences).

In this paper, we cope with uncertainty by blending SA and fuzzy mathematics. In particular, we use two types of sensitivity measures for assessing the importance of value drivers on the project's financial efficiency: we compute the fuzzy version of Borgonovo's δ measure, which is a global SA indicator (see ref.[59], ref.[60]) and the fuzzy version of the *Differential Importance Measure* (DIM), a local SA indicator developed in ref.[61] (see also ref.[1]).

We calculate the project's efficiency by using the fuzzy AIRR, developed in ref.[62] and ref.[40], netting it out of the cost of equity capital, thereby finding the *above-normal ROE*, a measure of financial efficiency of the equity investment. The sign of this metric directly informs on whether shareholder value will be created or not. The fuzzy DIM is applied to the project's economic profitability, as defined by the project's above-normal ROE, which represents the model output and, in general, is affected by key drivers such as operating costs, sales revenues, fixed assets, working capital, outstanding debt, interest rates, etc.

Armed with the above-normal ROE as the output of the model and the corresponding fuzzy DIM, we show how the uncertain accounting and financial magnitudes of the project company, reported in the pro forma financial statements, affect the financial efficiency when the variability of the accounting inputs is taken into account. This analysis brings about helpful managerial insights which may trigger further inspection of the most relevant key parameters and appropriate managerial actions aiming to enhance the robustness of the project company.

The paper is structured into four sections. Section 1 describes the AIRR approach, the project ROE, and the scale-efficiency breakdown of the NPV. Section 2 describes the relevant accounting and financial magnitudes and the key inputs from which the efficiency of a project financing transaction is derived. Section 2 briefly presents the extension principle for modelling uncertainty. Section 2 presents the Gamma indicator and the fuzzified DIM1 and DIM2, which measure the uncertainty propagation and enable ranking the key inputs in terms of their impact on the project's financial efficiency. The methods presented are applied in Section 3, where an illustrative example is analysed; we obtain a ranking of the value drivers on the project's ROE in terms of their importance, measured by the SA indicators described in Section 2. Section 4 concludes the paper.

1. The AIRR approach and the project ROE

The Average-Internal-Rate-of-Return approach (ref.[11], ref.[12] and ref.[19]) provides a direct relation between the financial efficiency of a project and the shareholder value created, based on the actual economic referents of the project. More precisely, it makes explicit use of the estimated costs, revenues, and assets as they are reported on pro forma financial statements. The arithmetic mean of the accounting rates, weighted by the invested capital amounts (pro forma book values), summarizes the project's performance and correctly signals value creation or destruction when compared to the required rate of return, also known as cost of capital, expressing a cutoff rate for project acceptability.

Consider an n-period project P; let $\mathbf{c} = (c_0, c_1, \ldots, c_{n-1})$ be the vector of capital amounts invested in the various periods (the value of c_n, by definition, is assumed to be zero and is not referenced) and let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ be the corresponding sequence of incomes. Letting r be the cost of capital (normal rate of return) and $v_t = 1/(1+r)^t$ be the discount factor (i.e., the present value of \$1 available at time t),[a] the AIRR is the ratio of the aggregate income to the aggregate invested capital (where aggregation takes into account the time value of money):

$$\bar{\jmath} = \frac{\sum_{t=1}^{n} x_t \cdot v_t}{\sum_{t=1}^{n} c_{t-1} \cdot v_t}. \tag{1}$$

If $c_t \neq 0$ for all $t = 0, 1, \ldots n-1$, then (see ref.[11])

$$NPV = \sum_{t=0}^{n} f_t v_t > 0 \iff \bar{\jmath} > r \tag{2}$$

where f_t denotes the estimated cash flow at time t. If the term structure of interest rates is time-varying, then the cost of capital is time-varying. Letting $\mathbf{r} = (r_1, r_2, \ldots, r_n)$ be the vector of costs of capital, the cut-off rate is

$$\bar{r} = \frac{\sum_{t=1}^{n} x_t^* \cdot v_t}{\sum_{t=1}^{n} c_{t-1} v_{t-1,0}} \tag{3}$$

where $x_t^* = r_t c_{t-1}$ is the market return which is foregone in period t if the project is undertaken. Then, eq. (2) becomes

$$NPV = \sum_{t=1}^{n} f_t v_{t,0} > 0 \iff \bar{\jmath} > \bar{r} \tag{4}$$

where $v_{t,0} := \prod_{k=1}^{t} (1 + r_k)^{-1}$ (see ref.[19], eq. (17)).[b] This implies the following scale-efficiency breakdown of NPV:

$$NPV = C \cdot (\bar{\jmath} - \bar{r}) \tag{5}$$

where $C = \sum_{t=1}^{n} c_{t-1} v_{t,0}$ represents the scale of the project (total capital invested) and $(\bar{\jmath} - \bar{r})$ is the project's above-normal ROE, that is, the rate of return over and above the cost of capital. It represents the project's financial efficiency and, as

[a]For risky projects, the rate r is often obtained as the sum of the risk-free interest rate prevailing in the security market and a risk premium taking into account the uncertainty of the project.
[b]If $\sum_{t=1}^{n} c_{t-1} v_{t-1,0} > 0$, then the project is a net financing and $\bar{\jmath}$ is a borrowing rate, so the rule is reversed: $NPV = \sum_{t=0}^{n} f_t v_{t,0} > 0 \iff \bar{\jmath} < \bar{r}$.

such, supplies information on the economic value created for each dollar of invested capital. Its sign determines project acceptance or rejection (note that it is positive if and only if the project NPV is positive). AIRR and NPV are then two sides of the same medal: NPV measures the amount of value created (it is an absolute measure), while the above-normal AIRR measures the financial efficiency of the project (it is a relative measure).

In the context of project financing transactions, the estimated capital amounts c_t are the pro forma book values of equity and the income x_t represents net earnings (net income). Therefore, $j_t = x_t/c_{t-1}$ is the well-known Return On Equity (ROE) and the AIRR, $\bar{\jmath}$, in a project financing transaction is an average ROE (total estimated net income divided by total estimated book value or, equivalently, book-value-weighted mean of ROEs). We call this mean the *project ROE*; we net it out to give rise to the *project above-normal ROE*, which expresses the financial efficiency of the equity investment. The use of an average accounting rate makes the analysis of value creation and efficiency in a project financing transaction simpler and more intuitive as opposed to a traditional IRR analysis, since (i) it directly makes use of available data (reported in the pro forma financial statements) and is explicitly represented as a function of the accounting and financial key inputs, (ii) does not require the calculation of cash flows (the sign of the project above-normal ROE suffices to signal value creation or destruction), (iii) complements the NPV with additional information and insights regarding a precise measurement of how good or bad the invested capital is expected to be managed, (iv) reconciles NPV and accounting rates of return by providing a natural scale-efficiency breakdown of the NPV in terms of accounting magnitudes.

Furthermore, the average ROE always exists and is unique, whereas the IRR may not exist or multiple IRRs may arise, since it is the solution of equation[c]

$$f_0 + \frac{f_1}{1+y} + \ldots + \frac{f_n}{(1+y)^n} = 0. \tag{6}$$

Moreover, if the IRR exists and is unique, the relation between IRR and NPV is implicit, whereas (5) establishes a direct, explicit relation between the project ROE and the NPV. Finally, there is no significant relation whatsoever between the invested capital estimated by the analyst, here denoted as c_t, and the IRR, since $y \neq j_t$; finally, the use of IRR is problematic with a structure of time-varying costs of capital (e.g., because the structure of interest rate is non-flat or the risk premium is time-varying) (see ref.[12]).

We now focus on the equity NPV and the above-normal ROE as derived from the pro forma financial statements.

[c]Inexistence of IRR springs up even in simple cases (see ref.[10] for examples).

Pro forma financial statements, financial efficiency, and shareholder value creation

Consider a Special Purpose Vehicle (SPV), that is, a company which is incorporated by a panel of sponsoring firms (SPV's equityholders) for undertaking a capital asset investment. Let V be a valuation criterion and let $\mathbf{x} = (x_1, x_2, \ldots, x_m)$ be the vector of value drivers. $V(\mathbf{x})$ establishes a relation between the value drivers and the valuation criterion:

$$V = V(\mathbf{x}) \qquad V : X \subseteq \mathbb{R}^m \to \mathbb{R}.$$

We take the perspective of the project company's equityholders and analyse two related valuation criteria: the equity NPV, which we denote as $V_1(\mathbf{x})$, and the project above-normal ROE.

Both metrics depend on a set of accounting, financial, business, and fiscal first-order variables that are reflected in the pro forma financial statements. They are also known as *value drivers* or *key performance drivers*, as they ultimately affect the economic profitability of the project. As previously noted, the project ROE can be expressed as a function of the value drivers, whereas the IRR is not susceptible to be expressed in terms of the value drivers in an explicit way. More precisely, the pro forma income statements supplies the net income:

$$NI_t = (EBIT_t - I_t)(1 - \tau)$$

where $EBIT_t = R_t - \ OE_t - \text{Dep}_t$ is the earning before interest and taxes, with R_t denoting the revenues and Œ$_t$ the operating expenses, while Dep_t denotes the depreciation expense, and τ is the tax rate. Letting i be the interest rate on debt and D_t be the book value of debt, interest on debt is found as $I_t = i \cdot D_{t-1}$. As is well-known, the relation between cash flow to equity (CFE) and net income is

$$CFE_t = NI_t - \Delta E_t$$

where E_t is the book value of equity of the project and Δ denotes variation from year $t-1$ to year t. The equity book value is a residual balance sheet item, obtained from the accounting identity $WC_t + FA_t = E_t + D_t$, where WC_t, FA_t, denote, respectively, the working capital (inventories and accounts receivable net of accounts payable), and the fixed assets (property, plant and equipment) net of accumulated depreciation. Therefore, the equity capital is calculated as

$$E_t = WC_t + FA_t - D_t$$

so that

$$CFE_t = \overbrace{(R_t - OE_t - \text{Dep}_t - i \cdot D_{t-1})(1-\tau)}^{NI_t} - \overbrace{(\Delta WC_t + \Delta FA_t - \Delta D_t)}^{\Delta E_t}. \quad (7)$$

The (equity) NPV depends on the CFEs, which are discounted at the required return on equity r, representing the minimum attractive rate of return for the equityholders (equity cost of capital). Allowing for time-varying equity costs of capital, let $\mathbf{r} = (r_1, r_2, \ldots, r_n)$ be the vector of equity costs of capital. Hence, the vector of value drivers is

$$\mathbf{x} = (\mathbf{R}, \mathbf{Dep}, \mathbf{OE}, \mathbf{WC}, \mathbf{FA}, \mathbf{r}, D_0, \tau, i). \tag{8}$$

Since the vectors **R**, **Dep**, **OE**, **WC**, **FA**, **r** have n components each, the vector **x** has $6n+3$ variables. The equity NPV is the difference between the equity value of the project and the initial equity investment, E_0. This implies that the NPV is a function V_1 such that

$$\begin{aligned} NPV &= V_1(\mathbf{x}) \\ &= -E_0 + \sum_{t=1}^{n} \frac{CFE_t}{(1+r_1)\cdot(1+r_2)\cdot\ldots\cdot(1+r_t)} \\ &= -E_0 + \sum_{t=1}^{n} \frac{(R_t - OE_t - \text{Dep}_t - i\cdot D_{t-1})(1-\tau) - \Delta WC_t - \Delta FA_t + \Delta D_t}{(1+r_1)\cdot(1+r_2)\cdot\ldots\cdot(1+r_t)} \end{aligned} \tag{9}$$

The above-normal ROE is a function V_2 defined on the same set of variables as the NPV: from (1),

$$\begin{aligned} V_2(\mathbf{x}) &= \overline{ROE} - \overline{r} \\ &= \frac{\sum_{t=1}^{n}(NI_t - r_t E_{t-1})\cdot v_{t,0}}{\sum_{t=1}^{n} E_{t-1}\cdot v_{t,0}} \\ &= \frac{\sum_{t=1}^{n}\left((R_t - OE_t - \text{Dep}_t - i\cdot D_{t-1})(1-\tau) - r_t(WC_{t-1} + FA_{t-1} - D_{t-1})\right)\cdot v_{t,0}}{\sum_{t=1}^{n}(WC_{t-1} + FA_{t-1} - D_{t-1})\cdot v_{t,0}} \end{aligned} \tag{10}$$

where $v_{t,0} = \prod_{k=1}^{t}(1+r_k)$. It may be shown that the average ROE is a weighted mean of the company's ROEs:

$$\overline{ROE} = w_1 \cdot ROE_1 + w_2 \cdot ROE_2 + \ldots + w_n \cdot ROE_n \tag{11}$$

where

$$w_t := \frac{E_{t-1}\cdot v_{t,0}}{\sum_{t=1}^{n} E_{t-1}\cdot v_{t,0}} = \frac{(WC_{t-1} + FA_{t-1} - D_{t-1})\cdot v_{t,0}}{\sum_{t=1}^{n}(WC_{t-1} + FA_{t-1} - D_{t-1})\cdot v_{t,0}}$$

and $ROE_t = NI_t/E_{t-1}$ is the return on equity (see ref.[11]).

In this setting, the capital base is project's total invested capital as measured by the working capital and the fixed assets: $C = \sum_{t=1}^{n}(WC_{t-1} + FA_{t-1} - D_{t-1})v_{t,0}$. Therefore, (5) may be written as

$$V_2(\mathbf{x}) = \frac{V_1(\mathbf{x})}{\sum_{t=1}^{n}(WC_{t-1} + FA_{t-1} - D_{t-1})v_{t,0}} \tag{12}$$

or, which is the same,

$$\overline{ROE} = \overline{r} + \frac{V_1(\mathbf{x})}{\sum_{t=1}^{n}(WC_{t-1} + FA_{t-1} - D_{t-1})v_{t,0}} = \overline{r} + V_2(\mathbf{x}). \tag{13}$$

The use of accounting measures is common in the engineering economy literature and operations research. A widespread metric is the Return On Investment (ROI), and decision criteria based on the use of ROI are studied in the literature and adopted in practice (e.g., ref.[63], ref.[64], ref.[65], ref.[66],ref.[67], ref.[68], ref.[16]). However, strictly speaking, the ROI measures the project return, the ROE measures the shareholder return, and the two coincide only if the project is unlevered (no debt) or if the interest rate on debt is equal to the required return on debt (e.g., see ref.[69]). We then prefer to use the ROE, which is a specific measure of the economic value created for the sponsoring firms, and, in particular, the AIRR approach, which takes the book-value-weighted mean of all the project's ROEs.

This approach is also useful as opposed to the traditional Internal Rate of Return. Contrary to the latter, the former has the compelling property of uniqueness, its financial nature does not depend on the cost of capital and it is univocally determined as investment rate (if $\sum_{t=1}^{n}(WC_{t-1} + FA_{t-1} - D_{t-1})v_{t,0} > 0$) or financing rate (if $\sum_{t=1}^{n}(WC_{t-1} + FA_{t-1} - D_{t-1})v_{t,0} < 0$). Furthermore, if the WC is either exogenous (i.e., it does not change under changes in the value drivers) or not uncertain (as in those projects where WC is not used)[d], then the above-normal ROE is NPV-consistent in a strong sense, as shown in ref.[20].

2. Modelling fuzzy uncertainty and fuzzy sensitivity analysis

Uncertainty is intrinsically embedded in models that cope with real-life applications and the decisions are highly dependent on the chosen uncertainty frame (see ref.[46]). On the other hand, the *'uncertainty is intrinsically related to the process of assessing projects in which inaccurate or incomplete information is available'* (see ref.[53] p. 27) and fuzzy numbers can be used to quantify inexact information in cases where the decision-maker's knowledge is quantitatively imprecise or inaccurate or described in terms of possible numerical values between specified ranges or intervals.

Here we model uncertainty through fuzzy numbers as introduced in ref.[62] and detailed in ref.[40], where the authors developed the fuzzy versions of the AIRR.

Real fuzzy intervals u are characterized by a compact support $[a, b]$ and a compact nonempty core $[c, d] \subset [a, b]$ where $a \leq c \leq d \leq b \in \mathbb{R}$; they are defined in terms of a quasi-concave, upper-semicontinuous function $u : \mathbb{R} \longrightarrow [0, 1]$ such that $[a, b] = cl\,(\{x|u(x) > 0\})$ is the support and $[c, d] = \{x|u(x) = 1\}$ is the core (here, $cl\,(A)$ is the closure of set A). The space of fuzzy intervals will be denoted by $\mathbb{R}_{\mathcal{F}}$.

[d]Typically, WC is zero if sales are made on a cash basis, suppliers are paid on cash, and no inventory is required.

When $a < c \leq d < b$, the membership function of $u \in \mathbb{R}_{\mathcal{F}}$ has the form

$$u(x) = \begin{cases} 0 & \text{if} \quad x < a \\ u^L(x) & \text{if } a \leq x < c \\ 1 & \text{if } c \leq x \leq d \\ u^R(x) & \text{if } d < x \leq b \\ 0 & \text{if} \quad x > b \end{cases} \tag{14}$$

where $u^L : [a, c] \longrightarrow [0, 1[$ is a nondecreasing right-continuous function, $u^L(x) > 0$ for $x \in]a, c]$, called the *left side* of the fuzzy interval and $u^R : [d, b] \longrightarrow [0, 1]$ is a nonincreasing left-continuous function, $u^R(x) > 0$ for $x \in [d, b[$, called the *right side* of the fuzzy interval. If $c = d$ then u is called a fuzzy number and $\{c\}$ is the core or u.

The α-cuts of u are defined to be the compact intervals $[u]_\alpha = \{x|u(x) \geq \alpha\}$ with $\alpha \in]0, 1]$ and $[u]_0 = cl(supp(u))$, where $cl(.)$ denotes the closure and $supp(u) = \{x|u(x) > 0\}$ is the support of the membership function u; in such a way, the fuzzy number u is uniquely determined by a pair $u = (u^-, u^+)$ of functions $u^-, u^+ : [0, 1] \longrightarrow \mathbb{R}$, defining the end-points of the α-cuts, such that u^- is bounded increasing, u^+ is bounded decreasing with $u^-(1) \leq u^+(1)$, u^- and u^+ are left continuous at any $\alpha \in]0, 1]$ and are right continuous at $\alpha = 0$; furthermore, the α-cuts of u are given by intervals $[u^-(\alpha), u^+(\alpha)]$ for all $\alpha \in [0, 1]$ if and only if its membership values are (see ref.[70])

$$u(x) = sup\left\{\alpha|u^-(\alpha) \leq x \leq u^+(\alpha)\right\}. \tag{15}$$

The two functions u^-, u^+ are called lower and upper end-point functions of u and their values will be denoted by u^-_α, u^+_α according to the notation $[u]_\alpha = [u^-_\alpha, u^+_\alpha] \subset \mathbb{R}$.

When the shape functions u^L and u^R are both linear, we obtain a trapezoidal fuzzy number (if $c < d$), usually denoted $u = (a, c, d, b)$, or a triangular one (if $c = d$), denoted $u = (a, c, b)$. Based on the AIRR approach and the project ROE described in the previous section, we suggest to handle fuzzy creation analysis with the SPV financial quantities, collected in vector x in (8), all expressed by trapezoidal or triangular fuzzy numbers (see section 3 and Table 2 for their description).

In order to perform arithmetic operations with fuzzy numbers or, more generally, to extend an ordinary function $f : X \subseteq \mathbb{R}^d \to \mathbb{R}$ of d real arguments $x_1, ...x_d \in \mathbb{R}$ and real values $f(x_1, ..., x_d) \in \mathbb{R}$ to fuzzy arguments $u_1, ..., u_d \in \mathbb{R}_{\mathcal{F}}$ and fuzzy values $f(u_1, ..., u_d) \in \mathbb{R}_{\mathcal{F}}$, we apply the well known Zadeh's Extension Principle (EP), introduced in ref.[71]. For a continuous function f, its EP-extension $v = f(u_1, ..., u_d)$, for fixed d fuzzy numbers $u_1, ..., u_d$, is the fuzzy number $v \in\in \mathbb{R}_{\mathcal{F}}$ having α-cuts $[v]_\alpha = [v^-_\alpha, v^+_\alpha]$ for all $\alpha \in [0, 1]$ given by solving the box-constrained min/max optimization problems:

$$(EP)_\alpha : \begin{cases} v^-_\alpha = \min\left\{f(x_1, ..., x_d)|x_j \in [u_j]_\alpha,\ j = 1, ..., d\right\} \\ v^+_\alpha = \max\left\{f(x_1, ..., x_d)|x_j \in [u_j]_\alpha,\ j = 1, ..., d\right\} \end{cases} \tag{16}$$

Solving the optimization problems in (16) is not always an easy task as, in general, we face two difficulties (see ref.[72], ref.[73] and the references therein):
1. we have to find global min and max points $p^{(m)}(\alpha) \in \mathbb{R}^d$ such that $f(p_1^{(m)}(\alpha),...,p_d^{(m)}(\alpha)) \leq f(x_1,...,x_d)$ and $p^{(M)}(\alpha) \in \mathbb{R}^d$ satisfying $f(p_1^{(M)}(\alpha),...,p_d^{(M)}(\alpha)) \geq f(x_1,...,x_d)$ for all $x_j \in [u_j]_\alpha$, $j = 1,...,d$;
2. we should solve an infinite number of global optimizations to obtain all the α-cuts of the EP-extension for $\alpha \in [0,1]$; in general, we fix a finite subset of $n+1$ values $0 = \alpha_0 < \alpha_1 < ... < \alpha_n = 1$ and solve (16) for these α_k, $k = 0,1,...,n$.
It is well known that one can find the exact solutions only for specific (elementary) functions or when we know that the continuous function $f(x_1,...,x_d)$ is monotonic (increasing or decreasing) with respect to each variable x_j on its interval $[u_j]_\alpha$ and for all possible values of the other variables x_i on the corresponding intervals $[u_i]_{\alpha_k}$. In these cases, the points $p^{(m)}(\alpha)$ and $p^{(M)}(\alpha)$ are located at the vertices of boxes

$$U_\alpha = [u_1]_\alpha \times [u_2]_\alpha \times ... \times [u_d]_\alpha \tag{17}$$

with components, for $j = 1,...,d$,

$$p_j^{(m)}(\alpha) = \begin{cases} u_{j,\alpha}^- & \text{if } f \text{ is increasing w.r.t. } x_j \\ u_{j,\alpha}^+ & \text{if } f \text{ is decreasing w.r.t. } x_j \end{cases}$$

and

$$p_j^{(M)}(\alpha) = \begin{cases} u_{j,\alpha}^+ & \text{if } f \text{ is increasing w.r.t. } x_j \\ u_{j,\alpha}^- & \text{if } f \text{ is decreasing w.r.t. } x_j. \end{cases}$$

This is true for basic fuzzy arithmetic operations
$u \oplus v = f_{EP}(u,v)$ with $f(x,y) = x + y$,
$u \ominus v = f_{EP}(u,v)$ with $f(x,y) = x - y$,
$u \otimes v = f_{EP}(u,v)$ with $f(x,y) = xy$,
$u \oslash v = f_{EP}(u,v)$ with $f(x,y) = x/y$
obtaining the closed forms, in terms of α-cuts,
$[u \oplus v]_\alpha = [u_\alpha^- + v_\alpha^-, u_\alpha^+ + v_\alpha^+]\,, \alpha \in [0,1]$,
$[u \ominus v]_\alpha = [u_\alpha^- - v_\alpha^+, u_\alpha^+ - v_\alpha^-]\,, \alpha \in [0,1]$,
$[u \otimes v]_\alpha = [minP_\alpha, maxP_\alpha]$ where $P_\alpha = \{u_\alpha^- v_\alpha^-, u_\alpha^- v_\alpha^+, u_\alpha^+ v_\alpha^-, u_\alpha^+ v_\alpha^+\}$, $\alpha \in [0,1]$,
$[u \oslash v]_\alpha = [minQ_\alpha, maxQ_\alpha]$ where $Q_\alpha = \left\{\frac{u_\alpha^-}{v_\alpha^-}, \frac{u_\alpha^-}{v_\alpha^+}, \frac{u_\alpha^+}{v_\alpha^-}, \frac{u_\alpha^+}{v_\alpha^+}\right\}$, $\alpha \in [0,1]$.
But properties such as distributivity or arithmetic cancellation are not valid and simple fuzzy extension like $f(x,y) = 3x + y(2x - y)$ cannot be computed by basic operations $3u \oplus 2u \otimes v \ominus v \otimes v$ without the risk of producing over-estimated α-cuts and, consequently, undesired over-propagation of uncertainty.
For these reasons, even relatively simple fuzzy EP-based computations will require a global optimization procedure to solve (16) for a finite set of α-cuts. In this paper, we use an implementation of a modified version of the Differential Evolution (DE) method, adapted to the min and max problems in (16), which takes into account that α-cuts are nested, that is, $U_{\alpha_{k+1}} \subseteq U_{\alpha_k}$ if $0 = \alpha_0 < \alpha_1 < ... < \alpha_n = 1$ are

the chosen α's and consequently, for $k = 0, 1, ..., n-1$ we have $v^-_{\alpha_k} \leq v^-_{\alpha_{k+1}}$ and $v^+_{\alpha_k} \geq v^+_{\alpha_{k+1}}$. It follows that both min and max values, for a given α_k, are to be found on the same box U_{α_k}. This allows an efficient implementation of DE method that simultaneously solves (16) for all required values $\{\alpha_k | k = 1, ..., n\}$ (see Algorithm 9 in ref.[73]).

Gamma indicator and fuzzy DIM for measuring uncertainty propagation

In order to analyse which parameters are the most influential in uncertainty propagation we apply three sensitivity analysis (SA) indicators, one is global and two are local ones.

The first technique is based on the δ indicator introduced in ref.[61]; we will denote it as *Gamma* (Γ).

The other two are based on the Differential Importance Measure (DIM), introduced in ref.[59] and utilized in ref.[1] precisely for a project financing transaction. The fuzzy *Gamma* indicator, presented in ref.[60], is an extension of the above mentioned δ indicator. Let $x_j = t$ be a fixed value of the j-th variable with membership value $u_j(t)$; consider the fuzzy extension of $f(x_1, x_2, \dots, x_d)$ to the parameters $x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_d$ taken as fuzzy and x_j, taken as crisp. In other terms:

$$v_{j,t} = f_{EP}(u_1, \dots, u_{j-1}, t, u_{j+1}, \dots, u_d)$$

so that $v_{j,t}$ is a fuzzy number $v_{j,t} : \mathbb{R} \to [0, 1]$ for each $j = 1, 2, \dots, d$ and $t \in [u_j]_0$; more precisely $v_{j,t}(z)$ is the membership value of $v_{j,t}$ at $z \in \mathbb{R}$ obtained by extension principle.

The cardinalities of the fuzzy numbers v_{EP} and v_{j,x_j} (with $t = x_j \in \mathbb{R}$) take the following form:

$$N(v_{EP}) = \int_{-\infty}^{+\infty} v_{EP}(z)\, dz$$

$$N\left(v_{j,x_j}\right) = \int_{-\infty}^{+\infty} v_{j,x_j}(z)\, dz$$

and we can represent the (absolute) difference between the fuzzy number v_{EP} and the fuzzy number v_{j,x_j} as the quantity

$$S_j(x_j) = \int_{-\infty}^{+\infty} \left| \frac{v_{EP}(z)}{N(v_{EP})} - \frac{v_{j,x_j}(z)}{N\left(v_{j,x_j}\right)} \right| dz$$

which can be interpreted as a measure of the effect of the j-th variable fixed to the crisp value x_j on the model function expressed by f.

In particular, when $S_j(x_j)$ is large then the value x_j of the j-th variable is important in the propagation of uncertainty, the contrary holds when $S_j(x_j)$ is small. The uncertainty importance measure of the j-th variable with respect to v_{EP} is then given by:

$$\Gamma_j = \frac{\int_{-\infty}^{+\infty} u_j(x) S_j(x)\, dx}{\int_{-\infty}^{+\infty} u_j(x)\, dx}. \tag{18}$$

The value Γ_j can be used to rank the variables in terms of their relevance in uncertainty propagation.

In practice, all the integrals are computed (approximated) through a finite decomposition of the support of each $u_j \in \mathbb{R}_{\mathcal{F}}$ and a unique discrete subdivision of the support of v_{EP} will suffice; this benefits from the fact that supports of v_{j,x_j} are all included in the support of v_{EP}.

The *Differential Importance Measure* (DIM) of parameter x_j, $j = 1, ..., d$ for function $f(x_1, ..., x_d)$ at point $x^* = (x_1^*, ..., x_d^*)$ is defined by

$$DIM_j(f; x^*) = \frac{f'_{x_j}(x^*)\, dx_j}{\sum_{i=1}^{d} f'_{x_i}(x^*)\, dx_i} \tag{19}$$

and distinguishing the uniform and the proportional cases (as in ref.[61] and ref.[1]) we obtain DIM1 and DIM2:

1) uniform changes in parameter values ($dx_j = h$ for all j)

$$DIM1_j(f; x^*) = \frac{f'_{x_j}(x^*)}{\sum_{i=1}^{d} f'_{x_i}(x^*)} \tag{20}$$

2) proportional changes of the parameters ($dx_j = \omega x_j^*$ for all j)

$$DIM2_j(f; x^*) = \frac{E_j(x^*)}{\sum_{i=1}^{d} E_i(x^*)} \tag{21}$$

where $E_i(x^*) = x_i^* \frac{f'_{x_i}(x^*)}{f(x^*)}$, $i = 1, ..., d$, are the partial elasticities of f at x^* (see also ref.[74]).

To obtain the fuzzy versions of DIM1 and DIM2 we apply the extension principle (EP). The computed fuzzy importance measures DIM1 and DIM2 are then ranked in decreasing order of their possibilistic average (see ref.[70] and chapter 3 of ref.[46]) defined, for a fuzzy number u with α-cuts $[u_\alpha^- \, u_\alpha^+]$, as

$$\widehat{u} = \int_0^1 \alpha(u_\alpha^- + u_\alpha^+) d\alpha. \tag{22}$$

3. A complete example with comments and discussion

We illustrate the application of the fuzzy importance measures to a generic project finance transaction, where pro forma financial statements are built starting from the accounting and financial input data. A Special Purpose Vehicle is incorporated on December 31, 2022 by some sponsoring firms for undertaking a 10-year project whereby a new facility is built, producing revenues R_t and operating expenses OE_t at time t.

Suppose a capital expenditure occurs at the time of incorporation and no other capital expenditures will occur after year 2022, so that $\text{Dep}_t = -\Delta FA_t$. We assume that the construction phase is shorter than one year, and the project company starts collecting sales revenues R_t in 2023. The amount of such revenues is expected to grow at an annual rate of g_R. After estimating sales revenues, the next step is to forecast operating expenses. Expenses OE_t are expected to vary with sales. They are usually projected to remain at a constant percentage of sales. However, we allow for a greater generality and here assume that the ratio of operating expenses to revenues in year 2023 is α_O and the operating expenses will grow at an annual rate of g_O. Letting $t = 1, 2, \ldots, 10$ be the number of years after 2022, r_1 is the equity cost of capital (COC) in 2023, which is expected to grow at an annual rate of g_r so that $r_t = r_1 \left(1 + g_r\right)^{t-1}$. The net fixed assets are depreciated evenly, so that $\text{Dep}_t = FA_0/n$ and $FA_t = FA_0 \left(1 - \frac{t}{n}\right)$ for $t = 0, 1, \ldots, n$. The working capital is expected to vary with sales; denoting as α_{WC} the (assumed constant) ratio of working capital to revenues, we have $WC_t = \alpha_{WC} R_1 \left(1 + g_r\right)^{t-1}$ for $t = 1, 2, \ldots, n$. We also assume that the debt tenor is repaid with flat payments. Since the flat payment is equal to $iD_0/(1-(1+i)^{-n})$, this assumption implies $D_t = D_0 \frac{1-(1+i)^{-n+t}}{1-(1+i)^{-n}}$ for $t = 0, 1, \ldots, n$.

With such assumptions, the vector of value drivers x becomes

$$x = (R_1, g_R, \alpha_O, g_O, WC_0, \alpha_{WC}, FA_0, r_1, g_r, D_0, \tau, i).$$

The first CFE is given by $CFE_0 = -\text{E}_0$. For $t = 1, 2, \ldots, n-1$, (7) becomes

$$\begin{aligned} CFE_t = {} & \left[R_1(1+g_R)^{t-1} - \alpha_O R_1(1+g_O)^{t-1} - i \cdot D_{t-1}\right](1-\tau) + \\ & - \alpha_{WC} \cdot g_R R_1 (1+g)^{t-2} + \frac{\tau \cdot FA_0}{n} + \Delta D_t \end{aligned} \tag{23}$$

whereas, for $t = n$

$$CFE_n = \left[R_1(1+g_R)^{n-1} - \alpha_O(1+g_O)^{n-1} - i \cdot D_{n-1}\right](1-\tau) - WC_{n-1} + \frac{\tau \cdot FA_0}{n} - D_{n-1}.$$

Some constraints serve the reliability of the model:

(i) $r_t > i$ (interest rate on debt is smaller than the equity COC, since equity is riskier than debt)
(ii) $E_0 > 0$ (the sponsors invest some equity at time 0)
(iii) $D_t \leq FA_t + WC_t$ (the SPV's debt is smaller than the SPV's assets; that is, equity is nonnegative for all $t \geq 1$)
(iv) $g_O < g_R$ (growth rate in the operating costs is smaller than growth rate in the sales revenues)
(v) $WC_t < R_t$ (net working capital is smaller than revenues)

Requirements (i)-(ii) are necessary for theoretical validity, while (iii)-(v) are common in real-life applications. The symbols for the input parameters are summarized in Table 1 and the data assumptions are collected in Table 2.

The assumptions made enable rewriting the above-normal ROE as

$$V_2(x) = \frac{\displaystyle\sum_{t=1}^{n} \frac{\left(R_1\left(G_R^{t-1} - \alpha_0 G_O^{t-1}\right) - \frac{FA_0}{n} - iD_{t-1}\right)(1-\tau) - r_t\left(FA_{t-1} + WC_{t-1} - D_{t-1}\right)}{(1+r_1)(1+r_2)\cdot\ldots\cdot(1+r_t)}}{\displaystyle\sum_{t=1}^{n} \frac{FA_{t-1} + WC_{t-1} - D_{t-1}}{(1+r_1)(1+r_2)\ldots(1+r_t)}} \tag{24}$$

where $G_R = (1 + g_R)$, $G_O = (1 + g_O)$. This is the output model.

Table 1. Used symbols of input parameters

Value Driver	**Symbol**
Revenues (year 2022)	R_1
Revenues growth rate (2023-2032)	g_R
Operating Costs (for unit of sales) (2023)	α_O
Operating Costs growth rate (2024-2032)	g_O
Net Working Capital (2022)	WC_0
Net Working Capital (for unit of sales) (2023-2032)	α_{WC}
Net Fixed Assets (2022)	FA_0
Required Return on Equity (2023)	r_1
Required Return on Equity growth rate (2024-2032)	g_r
Debt (2022)	D_0
Tax Rate	τ
Interest Rate on Debt	i

Tables 3 and 4 present the pro forma financial statements for the base case $\hat{x}$ and the computation of shareholder value creation (NPV) and financial efficiency. In the base case, the average ROE is 19.64%, higher than the cost of capital by 8.18%. The latter is the project's financial efficiency. The total equity investment

Table 2. Values of input fuzzy parameters

Value Driver	Mid-point	Lowest	Most likely interval		Highest
		a	c	d	b
R_1	\$210,000	\$168,000	\$189,000	\$231,000	\$294,000
g_R	8.0 %	5.6 %	7.4 %	8.6 %	12.0 %
α_O	25.0 %	24.8 %	24.8 %	25.2 %	30.0 %
g_O	2.0 %	1.6 %	2.0 %	2.0 %	2.8 %
WC_0	\$140,000	\$120,000	\$129,500	\$150,500	\$170,000
α_{WC}	30.0 %	0.0 %	30.0 %	30.0%	34.5 %
FA_0	\$950,000	\$902,000	\$950,000	\$950,000	\$980,000
r_1	10.0%	9.5 %	10.0%	10.0%	11.0 %
g_r	4.0 %	3.9 %	3.9 %	4.1 %	4.1 %
D_0	\$654,000	\$654,000	\$654,000	\$654,000	\$654,000
τ	38.0 %	37.24 %	37.24 %	38.76 %	38.76 %
i	6.0 %	5.5 %	5.86 %	6.14 %	6.5 %

is $C = \$401,514.7$. From the point of view of the equityholders, the incorporation of the SVP is equivalent, in the base case, to an overall investment of \$401,514.7 at an above-normal rate of 8.18%.

The shapes of the fuzzy numbers, representing the twelve parameters in the SPV, can be triangles, rectangles and trapezoids; for each value driver, they are indicated on the basis of three scenarios: worst scenario (lowest value), most-likely scenario (base value), best scenario (highest value). Choices about shapes and values should be justified in terms of coherence with real data (informed judgment is then essential).

Table 3. Pro forma financial statements and CFEs − base case

	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
BALANCE SHEET											
Net Working Capital WC_t ($)	140,000	63,000	68,040	73,483	79,362	85,711	92,568	99,973	107,971	116,609	0
Gross Fixed Assets ($)	950,000	950,000	950,000	950,000	950,000	950,000	950,000	950,000	950,000	950,000	950,000
Less: Acc. Depreciation		−95,000	−190,000	−285,000	−380,000	−475,000	−570,000	−665,000	−760,000	−855,000	−950,000
Net Fixed Assets FA_t ($)	950,000	855,000	760,000	665,000	570,000	475,000	380,000	285,000	190,000	95,000	0
Total Assets ($)	1,090,000	918,000	828,040	738,483	649,362	560,711	472,568	384,973	297,971	211,609	0
Debt D_t ($)	654,000	576,086	493,497	405,952	313,155	214,790	110,523	0	0	0	0
Equity E_t ($)	436,000	341,914	334,543	332,531	336,207	345,921	362,045	384,973	297,971	211,609	0
Total Liabilities ($)	1,090,000	918,000	828,040	738,483	649,362	560,711	472,568	384,973	297,971	211,609	0
INCOME STATEMENT											
Revenues R_t ($)		210,000	226,800	244,944	264,540	285,703	308,559	333,244	359,903	388,695	419,791
Less: Operating Expenses (OE_t)		−52,500	−53,550	−54,621	−55,713	−56,828	−57,964	−59,124	−60,306	−61,512	−62,742
EBITDA ($)		157,500	173,250	190,323	208,826	228,875	250,595	274,120	299,597	327,183	357,049
Less: Depreciation (Dep_t)		−95,000	−95,000	−95,000	−95,000	−95,000	−95,000	−95,000	−95,000	−95,000	−95,000
EBIT ($)		62,500	78,250	95,323	113,826	133,875	155,595	179,120	204,597	232,183	262,049
Less: Interest (I_t)		−39,240	−34,565	−29,610	−24,357	−18,789	−12,887	−6,631	0	0	0
Profit Before Taxes $EBIT_t - I_t$ ($)		23,260	43,685	65,713	89,469	115,086	142,707	172,489	204,597	232,183	262,049
Less: Taxes ($\tau(EBIT_t - I_t)$)		−8,839	−16,600	−24,971	−33,998	−43,733	−54,229	−65,546	−77,747	−88,230	−99,578
Net Income NI_t ($)		14,421	27,085	40,742	55,471	71,353	88,479	106,943	126,850	143,954	162,470
PROJECT CASH FLOWS ($)											
Net Income (NI_t)	0.00	14,421.2	27,084.6	40,742.2	55,470.8	71,353.1	88,478.5	106,943.0	126,850.2	143,953.6	162,470.1
Less: $\Delta\mathrm{WC}_t$	(140,000.0)	77,000.0	(5,040.0)	(5,443.2)	(5,878.7)	(6,348.9)	(6,856.9)	(7,405.4)	(7,997.8)	(8,637.7)	116,608.6
Less: $\Delta\mathrm{FA}_t$	(950,000.0)	95,000.0	95,000.0	95,000.0	95,000.0	95,000.0	95,000.0	95,000.0	95,000.0	95,000.0	95,000.0
Add: ΔD_t	654000.0.0	(77,914.3)	(82,589.2)	(87,544.5)	(92,797.2)	(98,365.0)	(104,266.9)	(110,522.9)	0.0	0.0	0.0
Cash Flow to Equity CFE_t ($)	(436,000.0)	108,506.9	34,455.5	42,754.5	51,794.9	61,639.2	72,354.7	84,014.7	213,852.4	230,315.9	374,078.7

Table 4. Financial efficiency ($V_2(\mathbf{x})$) − base case

		2022	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**
Net income ($)			14,421	27,085	40,742	55,471	71,353	88,479	106,943	126,850	143,954	162,470
Equity ($)		436,000	341,914	334,543	332,531	336,207	345,921	362,045	384,973	297,971	211,609	0
ROE			3.31%	7.92%	12.18%	16.68%	21.22%	25.58%	29.54%	32.95%	48.31%	76.78%
Discount factor		1	0.90909	0.82345	0.74308	0.66795	0.59799	0.53313	0.47325	0.41821	0.36787	0.32203
Total net income ($)	401,514.7											
Total equity capital C ($)	2,044,181.4											
Project ROE ($\bar{\jmath}$)	**19.64%**											
Market returns $r_t c_{t-1}$ ($)			43,600	35,559	36,184	37,405	39,331	42,087	45,810	50,660	40,779	30,119
Equity ($)		436,000	341,914	334,543	332,531	336,207	345,921	362,045	384,973	297,971	211,609	0
Discount factor		1	0.90909	0.82345	0.74308	0.66795	0.59799	0.53313	0.47325	0.41821	0.36787	0.32203
Total market return ($)	234,313.8											
Total equity capital C ($)	2,044,181.4											
Project COC ($\bar{r}$)	**11.46%**											
Financial efficiency above-normal ROE ($V_2(\mathbf{x}) = \bar{\jmath} - \bar{r}$)	**8.18%**											
Equity NPV $V_1(\mathbf{x}) = C \cdot V_2(\mathbf{x})$ ($)	167,200.90											

Table 5. Vector (E_t, NI_t, CFE_t): interval-valued equity, net income, and cash flow to equity

	2022	**2023**	**2024**	**2025**	**2026**	**2027**	**2028**	**2029**	**2030**	**2031**	**2032**
Left-support Equity ($)	368,000	236,914	230,175	228,029	230,724	238,529	251,724	270,606	180,407	90,207	0
Left-core Equity ($)	425,500	335,282	326,824	323,732	326,340	335,003	350,095	372,018	283,457	195,373	0
Midpoint Equity E_t ($)	436,000	341,914	334,543	332,531	336,207	345,921	362,045	384,973	297,971	211,609	0
Right-core Equity ($)	446,500	348,548	342,340	341,500	346,354	357,252	374,566	398,687	313,465	229,082	0
Right-support Equity ($)	496,000	362,695	349,663	342,125	340,436	344,975	356,144	374,373	280,874	287,627	0
Left-support Net Income ($)	0	−14,030	−6,078	2,370	11,346	20,882	31,011	41,769	53,196	60,584	68,415
Left-core Net Income ($)	0	3,807	14,706	26,406	38,965	52,445	66,912	82,436	99,095	112,543	127,021
Midpoint Net Income NI_t ($)	0	14,421	27,084	40,742	55,471	71,353	88,479	106,943	126,850	143,954	162,470
Right-core Net Income ($)	0	25,347	39,974	55,827	73,008	91,628	111,806	133,672	157,366	178,753	202,033
Right-support Net Income ($)	0	59,570	83,710	110,647	140,705	174,251	211,694	253,492	300,156	348,286	402,277
Left-support Cash Flow to Equity ($)	−496,000	102,300	6,953	9,908	13,035	16,343	19,841	23,540	146,695	153,831	256,042
Left-core Cash Flow to Equity ($)	−446,500	102,300	23,164	29,498	36,357	43,783	51,819	60,514	187,656	200,627	322,395
Midpoint Cash Flow to Equity CFE_t ($)	−436,000	108,507	34,456	42,754	51,795	61,639	72,355	84,015	213,852	230,315	374,078
Right-core Cash Flow to Equity ($)	−425,500	115,025	46,181	56,668	68,154	80,730	94,493	109,550	242,589	263,135	431,116
Right-support Cash Flow to Equity ($)	−368,000	190,657	90,449	112,794	138,009	166,446	198,499	234,610	390,355	438,485	492,484

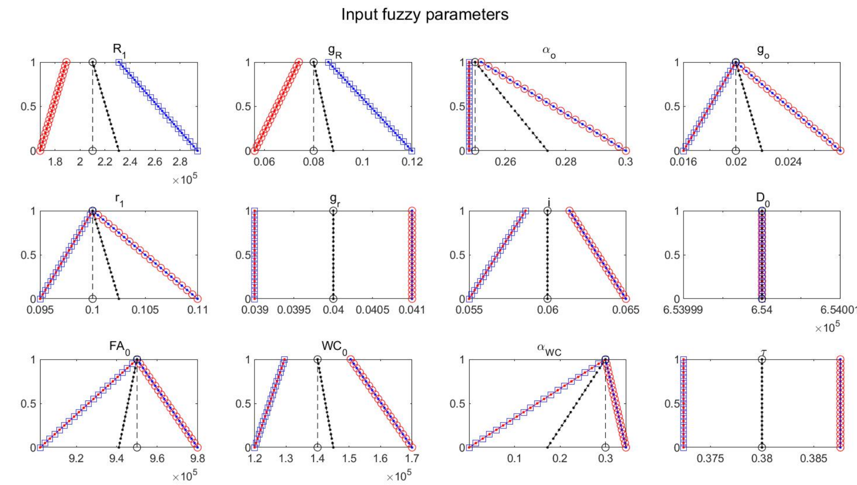

Fig. 1. All fuzzy parameters are represented by linear left (red lines) and right (blue lines) branches (triangular or trapezoidal fuzzy numbers); D_0 is crisp, g_r and τ are proper intervals. The black pointed lines give the central values of the α-cuts; only i is symmetric. The red circles mark the values of the parameters at which, for the corresponding α-cut, the EP value (16) is minimal; the blue squares mark the points with maximum value.

For simplicity, we will denote the *above-normal ROE* by the shortcut $anROE$.

The shapes of twelve input fuzzy parameters are collected in Figure 1; for each parameter, red lines describe the left side and blue lines show the right side of the membership function. Red circles and blue squares indicate, for each α-cut, the minimum and the maximum values in (16), respectively. Consequently, when the red circles lay on the red line and blue squares lay on the blue line, then $anROE$ is monotonically increasing with respect to the parameter; when the red circles lay on the blue line and blue squares lay on the red line, then $anROE$ is monotonically decreasing with respect to the parameter; otherwise, $anROE$ is not monotonic (the min or max values are internal to the α-cut, but in the chosen range of parameters this situation does not occur).

All the computations are performed with 21 α-cuts, corresponding to the values $\alpha \in \left\{ \dfrac{k}{20} | k = 0, 1, ..., 20 \right\}$ ($k = 0$ gives the support, $k = 20$ produces the core); the obtained numerical results are precise up to six decimal digits.

Figure 2 shows the resulting fuzzy $anROE$ obtained through the described extension principle.

The support interval of the percentage $anROE$ (by extension principle) is $[-7.717, 60.616]$, with central value 26.45 and radius 34.166. The value -7.717 gives the most unfavourable result and the values 60.616 corresponds to the most favourable combination of the input parameters; they represent the extremal pos-

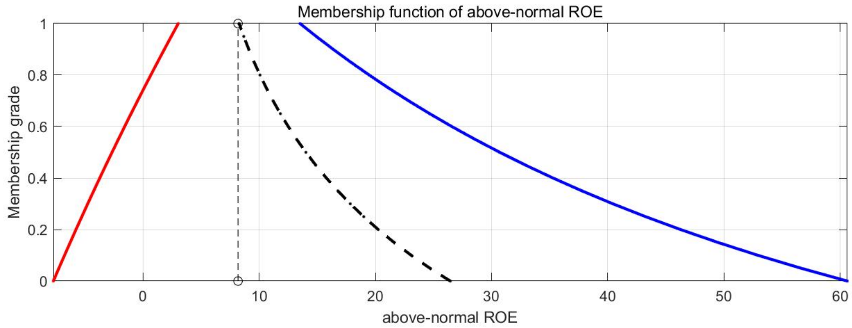

Fig. 2. The resulting fuzzy above-normal ROE has a non-linear membership function; it is strongly asymmetric, in favour of values on the right of (i.e., greater than) the crisp solution (dashed vertical line).

sible values, compatible with the supports of the (uncertain) data.

The resulting core of the percentage $anROE$ is the interval $[3.034, 13.515]$ with central value 8.274 and radius 5.24. The core gives the range of possible output values compatible with each input parameter in the respective core. If we consider the input cores as the most plausible (but uncertain) set of values for the data, we can deduce that the output core-interval contains the most plausible values of the resulting $anROE$; if we are reasonably optimistic, we can be attracted by the right value of the core (13.515), while the left value (3.034) gives instead a reasonably pessimistic (and plausible) result.

Also the α-cut intervals corresponding to decreasing α are of interest; e.g., we have $[anROE]_{.95} = [2.433, 14.912]$, $[anROE]_{.90} = [1.839, 16.368]$, $[anROE]_{.80} = [0.673, 19.470]$ with increasing central values (8.274 for the core, then 8.673, 9.103, 10.071 and so up to 26.45 for the support).

It is also interesting to compute the relevant magnitudes produced by the parameter values which generate the α-cuts $[anROE]_\alpha$. Consider, for example, the *equity* E_t, the *net income* NI_t and the *cash flow to equity* CFE_t. In Table 3 we can see their values, for the years 2022-2032, corresponding to the mid-point values of the input parameters (given in the second column of Table 2). For each α-cut $[anROE]_\alpha = [anROE^-_\alpha, anROE^+_\alpha]$, $\alpha \in \left\{\frac{k}{20} | k = 0, 1, ..., 20\right\}$, there exist two vectors of parameters, one to obtain the left value $anROE^-_\alpha$ and the other giving $anROE^+_\alpha$; clearly, with the two vectors we are able to reproduce corresponding left values $(E_t)^-_\alpha$, $(NI_t)^-_\alpha$, $(anROE_t)^-_\alpha$ and right values $(E_t)^+_\alpha$, $(NI_t)^+_\alpha$, $(anROE_t)^+_\alpha$.

The computed fuzzy-valued magnitudes E_t, NI_t and $anROE_t$ are pictured in Figure 3, where the red lines correspond to the left values and the blue lines reproduce the right ones. Table 5 reproduces the three magnitudes obtained with the midpoint parameters, with the left and right core parameters (the ones giving $[anROE]_1 = [3.034, 13.515]$) and with the left and right support parameters

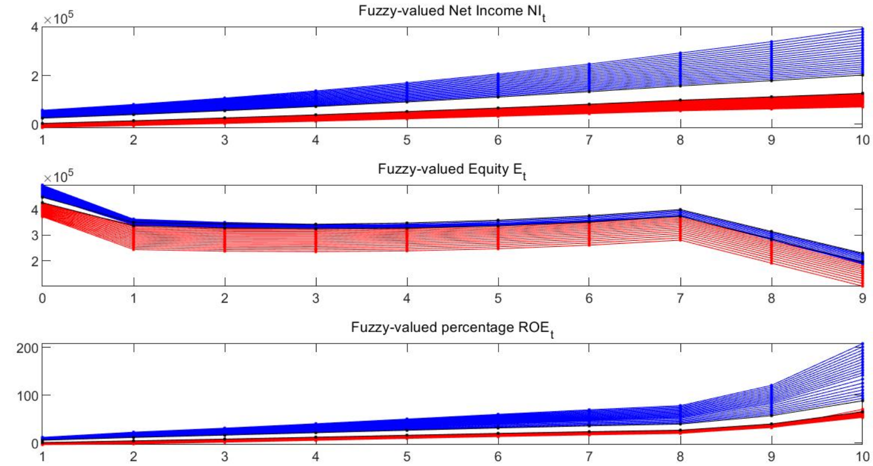

Fig. 3. Computed fuzzy-valued E_t, NI_t and $anROE_t$; red lines correspond to the left side of the membership while blue lines are the right side values.

(the ones giving $[anROE]_0 = [-7.717, 60.616]$). For example, the *Left-support* E_t, NI_t and CFE_t in Table 5 are associated with the most unfavourable result value $anROE = -7.717$ while the Right-support ones give the most favourable $anROE = 60.616$; analogously, the Left-core values are associated with the reasonably optimistic value $anROE = 13.515$, while the Right-core values give instead the reasonably pessimistic $anROE = 3.034$.

More generally, we can easily construct the analogous of Tables 1 and 4 for each of the fuzzy $anROE$ α-cuts.

In Figure 4 the membership function of v_{EP} is shown together with the membership of each v_{j,x_j} when x_j ranges in the support of the fuzzy j-th parameter. Simulations are based on 101 points for each parameter and 21 α-cuts for the membership function.

In particular, Figure 4 graphically shows the effect of changes in the revenues' drivers (R_1 and g_R) on one hand and the cost drivers (α_O and g_O) on the other hand. As noted above, efficiency is strongly dependent on the uncertainty of sales rather than costs, as is evidenced by the bandwidth (bandwidth is proportional to the uncertainty propagation).

Figure 5 shows the functions $S_j(x_j)$ that measure the distance between v_{EP} and v_{j,x_j} for the j-th parameter, as described in section 4; in each graphic, the red lines describes the j-th parameter in the 101 points of its support and the blue lines are the parameter's membership.

In the described example of the Special Purpose Vehicle, the fuzzy differential importance measures $DIM1_j$ and $DIM2_j$ produce results in Figures 6 and 7. The

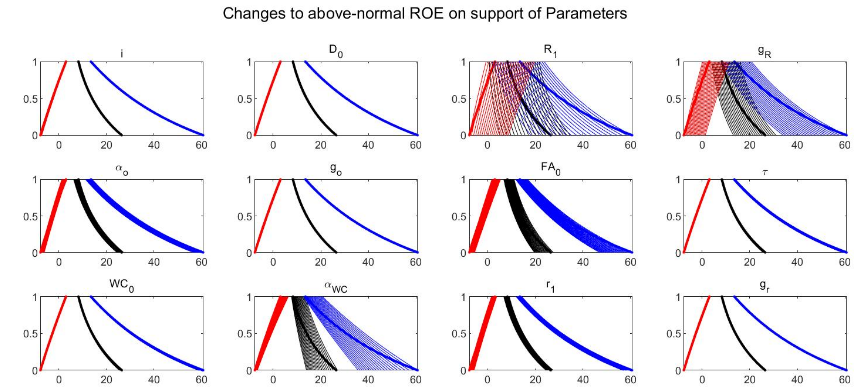

Fig. 4. For each fuzzy parameter u_j, the curves give the family of functions $v_{j,t}$. From the low or high scattering of the curves, we can deduce the effect of u_j on the uncertainty of $anROE$; the parameters R_1, g_R, α_{WC}, FA_0 have a great effect on $anROE$.

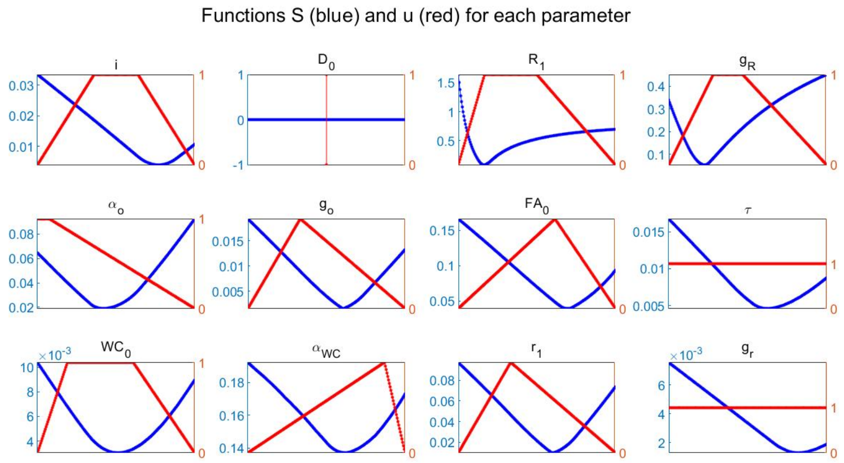

Fig. 5. For each u_j the functions $S_j(x_j)$ (blue lines) represent the fuzzy distance between $anROE_{EP}$ and v_{j,x_j}, with x_j ranging on the support of u_j (red lines).

values of the Gamma indicator, Γ_j (absolute and normalized), and the (defuzzified) differential importance measures $DIM1_j$ and $DIM2_j$ are collected in Table 6.

We first comment on some facts from the uncertainty importance measures Γ_j (Table 6, columns 2,3,4): there is evidence about the importance hierarchy in the computation of fuzzy anROE; the contribution of all twelve parameters to the total uncertainty of the fuzzy above-normal ROE is ranked in column 2 in terms of the

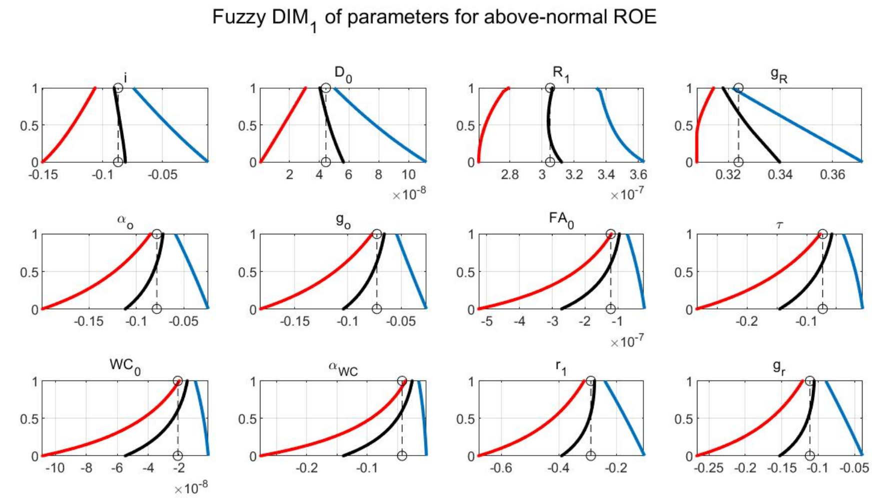

Fig. 6. Fuzzy-valued differential importance measure DIM1 (uniform changes in parameter values) for each parameter. Derivatives are approximated by numerical difference ratios.

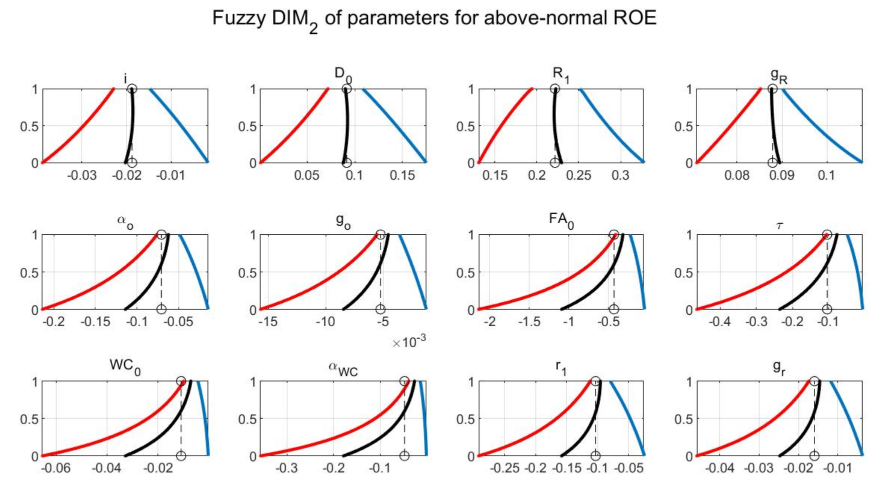

Fig. 7. Fuzzy-valued differential importance measure DIM2 (proportional changes in parameter values) for each parameter. Elasticities are estimated numerically.

values Γ_j (position 1 is the most important).

Table 6. Gamma indicator, Γ_j, and Differential Importance Measures, $DIM1$ and $DIM2$; they quantify the impact on $anROE$ of each input parameter. A positive sign of $DIM1_j$ or $DIM2_j$ means that the resulting $anROE$ is increasing with the value of the j-th parameter; if negative, it is decreasing. Normalized values are the ratios $\frac{\Gamma_j}{\sum_{i=1}^{d} \Gamma_i}$.

Parameter	Γ-rank	absolute Γ value	normalized Γ value	DIM_1-rank	DIM_1 value	DIM_2-rank	DIM_2 value
R_1	1	0.4374	1.0000	9	$3.1e-7$	2	0.2217
g_R	2	0.2086	0.4769	1	0.3240	6	0.0879
α_{WC}	3	0.1501	0.3433	8	-0.0421	8	-0.0482
FA_0	4	0.0726	0.1659	10	$-1.2e-7$	1	-0.4295
r_1	5	0.0394	0.0902	2	-0.2889	4	-0.1033
α_O	6	0.0385	0.0881	5	-0.0784	7	-0.0707
i	7	0.0136	0.0311	4	-0.0872	9	-0.0187
τ	8	0.0085	0.0195	6	-0.0732	3	-0.1038
g_O	9	0.0075	0.0172	7	-0.0727	12	-0.0052
WC_0	10	0.0048	0.0110	12	$-2.1e-8$	11	-0.0110
g_r	11	0.0037	0.0084	3	-0.1113	10	-0.0160
D_0	12	0.0000	0.0000	11	$4.4e-8$	5	0.0916

In terms of managerial insights, the magnitude of the impact of the inputs on the financial efficiency may change depending on whether the input is believed to have a potentially small or large perturbation with respect to the base case.

As can be gleaned from inspection of Table 6, according to the Gamma indicator (which measures the impact of larger deviations),the uncertainty of revenues has by far the greatest impact on the uncertainty of financial efficiency. This can be seen also graphically: the four parameters R_1, g_R, α_{WC},FA_0 have the most scattered membership functions v_{j,x_j} (from Figure 4) and the biggest values of $S(x_j)$ functions (from Figure 5).

In particular, the first-year revenues (R_1) have rank 1 and the growth rate of revenues (g_R) has rank 2. This implies that sales revenues are the main source of uncertainty in this project financing transaction. The direction of impact is positive, as expected: higher revenues brings about higher efficiency.

The other parameters have a considerably smaller impact; e.g., operating costs depend on α_O and g_O, which have rank 6 and 9 respectively. In normalized terms, their importance amounts to, respectively, 8.81% and 1.72% of the importance of R_1. Therefore, their uncertainty do not play a major role.

In other words, the financial efficiency of the project company will depend on its ability to sell products. This suggests that the management should conduct further investigation on the sales prospects and possibly take appropriate managerial actions to reduce the uncertainty via credit policy and/or marketing policy. The former is particularly important because a managerial action on credit policy may also affect α_{WC}, which is the third most important value driver (it may even be that, after a further investigation on these value drivers, the analyst revises the forecasts or that the new estimates turns out to have a smaller degree of uncertainty).

It is worth noting that changes in the cost of capital play a negligible role: the first-year COC, r_1, and the growth rate, g_r, have rank 6 and 11, respectively, and their normalized importance only amounts to 9.02% and 0.84% of the importance of R_1.

The DIM measures show a somewhat different impact of the inputs, as they refer to the case of small perturbations. $DIM1$ is not scale invariant with respect to the possible ranges of the various parameters; nevertheless, we can subdivide them into two scale-homogeneous groups $\{R_1, FA_0, WC_0, D_0\}$ and $\{g_R, \alpha_{WC}, r_1, \alpha_O, i, \tau, g_O, g_r\}$ so that the values of $DIM1$, within each group independently, can be compared.

For the first group of parameters we have the following orders of importance, based on the three types of measures:

$$\begin{aligned} \Gamma_j \text{ rank:} \quad & R_1 > FA_0 > WC_0 > D_0; \\ DIM1_j \text{ rank:} \quad & R_1 > FA_0 > D_0 > WC_0; \\ DIM2_j \text{ rank:} \quad & FA_0 > R_1 > D_0 > WC_0. \end{aligned}$$

The three ranking rules provide the same piece of information on revenues and fixed assets, namely, that the pair R_1, FA_0 is significantly more important than the pair D_0, WC_0, with D_0 becoming more important (position 5) in the case of proportional changes in the value of parameters, as captured by $DIM2$.

Among the parameters of the second group, g_R and r_1 are confirmed to be the most important ones with respect to the sum of scores in the three rankings, followed at some distance by α_O and τ, then by α_{WC}, i and g_r, g_O in final position (e.g., the sums of the three rank scores below are, respectively, 5 for g_R, 7 for r_1, 13 for α_O and τ, 15 for α_{WC} and i, 18 for g_r and 22 for g_O):

$$\begin{array}{rl} \Gamma_j \text{ rank:} & g_R > \alpha_{WC} > r_1 > \alpha_O > i > \tau > g_O > g_r; \\ DIM1_j \text{ rank:} & g_R > r_1 > g_r > i > \alpha_O > \tau > g_O > \alpha_{WC}; \\ DIM2_j \text{ rank:} & \tau > r_1 > g_R > \alpha_O > \alpha_{WC} > i > g_r > g_O. \end{array}$$

The managerial recommendation derived from the three indicators is not univocal. This should not come as a surprise, since the managerial actions will depend on the likelihood of the base case reported in pro forma financial statements and the likelihood of extreme events (worst and best case) as opposed to the likelihood of small perturbations. This may lead to different recommendations about the group of parameters which deserve further study and related actions to improve the financial efficiency. Such decisions may only be the result of informed judgment and soft skills.

As discussed in ref.[76], ref.[23], where the identification of the most influential parameters is highlighted, we complete this section by showing a simple way to identify an overall (unique) *ranking* of the input data, based on the three indicators in Table 6, by using a Multiple Criteria Decision Making (MCDM) tool such as the well known *Technique for Order of Preference by Similarity to Ideal Solution* (TOPSIS) method, introduced in ref.[77] (see also ref.[78] and ref.[79] for adjustments and modifications). Indeed, we can identify both a virtual most-influencing (ideal) input and a less-influencing (anti-ideal) input and find an overall ranking of all inputs, according to a bipolar comparison of inputs with both the virtual (ideal and anti-ideal) ones; the distances to the virtual cases are calculated for each parameter and then the aggregated criterion is built. A similar approach is suggested in ref.[80], were TOPSIS is compared with other MCDM tools (see also the interesting literature review contained in ref.[56])

To conduct TOPSIS analysis we start by presenting the ranking problem in the form of a 12×3 matrix, where each row represents a model input and each column describes a normalized indicator (absolute values):

$$X = \begin{bmatrix} 1.0000 & 9.6E^{-7} & 0.5162 \\ 0.4769 & 1.0000 & 0.2047 \\ 0.3433 & 0.1299 & 0.1122 \\ 0.1659 & 3.7E^{-7} & 1.0000 \\ 0.0902 & 0.8917 & 0.2405 \\ 0.0881 & 0.2419 & 0.1646 \\ 0.0311 & 0.2691 & 0.0435 \\ 0.0195 & 0.2259 & 0.2417 \\ 0.0172 & 0.2244 & 0.0121 \\ 0.0110 & 0.6E^{-8} & 0.0256 \\ 0.0084 & 0.3435 & 0.0373 \\ 0.0000 & 1.4E^{-7} & 0.2127 \end{bmatrix}. \tag{25}$$

In the language of MCDM, each row (model input, in the same order as in Table 6) describes the alternative $i = 1, 2, ..., 12$ under consideration, each column describes the criterion $j = 1, 2, 3$ (corresponding to the three indicators $\Gamma, DIM1, DIM2$, respectively) for measuring the alternatives' performance (in terms of sensitivity importance) and $x_{i,j}$ is the normalized absolute value (level) of alternative A_i with respect to criterion j. Furthermore, we can assign non-negative weights to the indicators (columns) such that, without loss of generality, a bigger weight value corresponds to higher indicator attention in the overall analysis.
In our computations, the normalization is performed by dividing the (absolute) values in Table 6 by their column maximum (L_∞-normalization) and the weights are 0.5 for Γ and 0.25 for $DIM1$ and $DIM2$, in order to take into account that we have one global indicator ((Γ) and two local indicators ($DIM1, DIM2$), so that the two groups (global/local) are equally weighted. Denote by $W = diag(0.5, 0.25, 0.25)$ the diagonal matrix of weights and let $A = XW$ be the matrix X with the columns multiplied by the $w_j, j = 1, 2, 3$.
The remaining steps consist in
1. computing the virtual ideal and anti-ideal rows a^+, a^-, given, for $j = 1, 2, 3$, by

$$a_j^+ = max(a_{i,j}; i = 1, 2, ..., 12) \text{ and } a_j^- = min(a_{i,j}; i = 1, 2, ..., 12); \tag{26}$$

2. calculating the separations d_i^+ and d_i^- (e.g. by euclidean distances) between each row of matrix A and the virtual rows a^+, a^-, where d_i^- is the distance between row i of A and a^- and d_i^+ is the distance between row i of A and a^+;
3. determining the relative closeness $C_i, i = 1, 2, ..., 12$ of each row in A to the ideal virtual row a^+:

$$C_i = \frac{d_i^+}{d_i^- + d_i^+}, i = 1, 2, ..., 12. \tag{27}$$

The value of C_i represents the bipolar *distance* of the indicators $Gamma_i$, $DIM1_i$

and $DIM2_i$ of the i-th uncertain input, to the ideal and anti-ideal virtual indicators; the overall ranking of the indicators is obtained in descending order of the scores C_i.
From the data in Table 6, we obtained

$$d^+ = (0.278, 0.329, 0.452, 0.486, 0.494, 0.536, 0.570, 0.560, 0.583, 0.605, 0.575, 0.593),$$

$$d^- = (0.516, 0.349, 0.176, 0.261, 0.234, 0.084, 0.070, 0.081, 0.057, 0.006, 0.086, 0.050)$$

and the vector of scores

$$C = (0.350, 0.485, 0.719, 0.651, 0.678, 0.865, 0.891, 0.874, 0.911, 0.989, 0.870, 0.922).$$

The resulting overall importance ranking of the inputs is expressed in the following vector $rank = (1, 2, 5, 3, 4, 6, 9, 8, 10, 12, 7, 11)$, corresponding to
$R_1 > g_R > r_1 > \alpha_{WC} > FA_0 > \alpha_O > g_O > \tau > WC_0 > D_0 > i > g_r$, according to which the estimated revenues in the first year, the growth rate of revenues, and the first-year cost of equity have paramount importance, whereas the interest rate on debt and the growth rate of the cost of equity have a negligible impact on the valuation and the decision.

4. Conclusions

This papers deals with investment decisions and related uncertainty. Knowledge of the main sources of uncertainty and their ranking is relevant, since it may trigger further analysis on the key inputs, possibly leading to revision of forecasts, following some appropriate managerial actions aimed at reducing uncertainty and/or increasing the project's financial efficiency. While scenario analysis is often used in practice, we rest on a more sophisticated treatment and management of risk. In particular, we use fuzzy sensitivity analysis to assess the impact on uncertainty propagation on shareholder value creation and, in particular, on the financial efficiency of project financed transactions. These operations are characterized by a substantial amount of uncertainty, leading to possible wide deviations from the estimated accounting and financial magnitudes as reported in the base-case pro forma financial statements, so that a scenario analysis, widely applied in practice, may be not sufficient. We measure financial efficiency in terms of a fuzzy average ROE over and above a normal rate of return (cost of capital), recently developed; then we rank the key parameters in terms of their impact on the financial efficiency via a global uncertainty importance measure, the Gamma indicator, which recently extended ref.[59], and two local fuzzy-valued differential importance measures (DIM), which we compute via the extension principle. The joint use of average ROE and three importance measures, along with informed judgment and managerial skills, should lead to the appropriate decision on whether accept or reject the project as such or spend further time and effort to retrieve some more information about the set of most relevant input parameters.

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