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Production delays, supply distortions and endogenous price dynamics

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Abstract

It takes time to produce commodities, and different production technologies may take different lengths of time. Suppose that firms may switch between different production technologies that take different lengths of time. A natural implication of such a scenario is that not all firms would then offer their commodities in every period, i.e. firms' total supply schedule would become a time-varying quantity. Based on a behavioral cobweb framework, we analytically demonstrate that commodity markets become unstable when firms switch too rapidly between production technologies that take different lengths of time. In particular, we observe that supply distortions lead to endogenous commodity price dynamics due to a mismatch between supply and demand.

Keywords

Cobweb models; production delays; endogenous price dynamics; bounded rationality and learning; nonlinear maps; stability and bifurcation analysis.

JEL classification

D24; E32; Q11.

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1 Introduction

Commodity prices display a remarkable variability. As made clear by Cashin and McDermott (2002), Drechsel and Tenreyro (2018) and Kohn et al. (2021), this is an important issue. For instance, a number of countries are heavily dependent on the export revenues they obtain from a few crucial commodities; other countries habitually suffer from volatile food prices. The goal of our paper thus is to identify a new explanation that may contribute to the excessive variability of commodity prices. Let us start by previewing the basic idea underlying our explanation. Our point of departure is that the production of commodities takes time, and that different production technologies may take different lengths of time. Now, suppose that firms may switch between different production technologies that take different lengths of time. One implication of such a scenario is that not all firms would then offer their commodities in every period, i.e. firms' total supply schedule would become a time-varying quantity. From here on it is easy to imagine that such supply distortions may result in endogenous commodity price dynamics.

Based on this insight, we develop a simple behavioral model in which firms select production technologies with respect to their past profitability. We analytically demonstrate that commodity markets become unstable when firms switch too rapidly between production technologies that take different lengths of time. In particular, we show that supply distortions lead to endogenous commodity price dynamics due to mismatches between supply and demand. In addition, we find that these price fluctuations originate either from a flip bifurcation scenario, leading to irregular zigzag price dynamics, or from a Neimark-Sacker bifurcation scenario, leading to more orderly cyclical price dynamics.

We use a cobweb framework to formalize our line of reasoning, as put forward by Tinbergen (1930), Leontief (1934) and Ezekiel (1938). Recall that cobweb models describe a dynamic price adjustment process on a competitive market for a single nonstorable commodity with a supply response lag. Due to this supply response lag, firms must form price expectations. Most cobweb models assume that firms face a fixed production delay of one period and that firms have naïve price expectations. As a result, these cobweb models correspond to onedimensional maps, and their dynamics are typically characterized by irregular zigzag price fluctuations that emerge via flip bifurcation scenarios.

As early as 1938, however, Ezekiel (1938) compared the dynamics of cobweb models in which firms either face fixed production delays of one, two or three periods. He demonstrated that these cobweb models may create a stable period-two cycle, a stable period-four cycle or a stable period-six cycle, respectively. Of course, additional production lags increase the dimension of the cobweb model's underlying map, a prerequisite for the emergence of a Neimark-Sacker bifurcation scenario and the onset of endogenous cyclical prices dynamics. Unfortunately, the economic literature has largely ignored this part of his work.

One exception is the recent contribution by Dieci et al. (2022), who develop a cobweb model in which firms, facing a fixed production delay of two periods, have access to two different production technologies. In particular, they can select between an inflexible production technology, requiring them to start the production process immediately, and a flexible production technology, allowing them to delay the production process by one period. The dynamics of their cobweb model is due to a four-dimensional nonlinear map, which is able to produce endogenous cyclical price dynamics via a Neimark-Sacker bifurcation. Note that a major difference between our paper and the work by Ezekiel (1938) and Dieci et al. (2022) is that we consider that firms may switch between different production technologies that take different lengths of time.[1]

There are only a few other cobweb models that may produce endogenous cyclical price dynamics. Dieci and Westerhoff (2010) consider a cobweb model in which

[1] Our goal is to make a general contribution to the literature on production time, supply distortions and endogenous price dynamics. However, what we have in mind here is that the fast production technology relies on machinery that is more efficient or is associated with quicker transportation possibilities that reduce the time from producing the commodity to bringing it to the market by one period. This line of reasoning is also consistent with the theoretical literature on technology adoption (Murphy et al. 1989, Desmet 2000) in which some producers switch to a new and more efficient technology to produce a certain good that is also produced with an older technology by some other producers at the same time. In other words, two production technologies – a more efficient and a less efficient one – coexist for the same good at the same time, where the more efficient production technology is associated with a faster production process.

firms can offer their commodities in two different markets, implying that the number of firms in the two markets, and thus the two markets' total supply, may vary over time. Using a mix of analytical and numerical tools, they show that their cobweb model may generate endogenous cyclical price dynamics. In Hommes (1998), firms rely on a linear backward-looking expectation rule, involving a weighted average of past prices. In this way, he obtains a higher-order dynamical system that is capable of producing endogenous cyclical price dynamics. Cavalli et al. (2021) develop a cobweb model in which a firm's supply depends on the season in which it is manufactured. For instance, a firm's supply may be high in the summer and low in the winter, resulting, for a time-independent demand schedule, in low summer prices and high winter prices.

Finally, we stress that the seminal paper by Brock and Hommes (1997) heavily influenced our work, too. Within their model, firms adapt their price expectations by switching between naïve and rational expectation rules, subject to their past performance. Most importantly, Brock and Hommes (1997) show that fixed-point dynamics may turn into chaotic dynamics when firms switch too rapidly between the two expectation rules. We refer the reader to Hommes (2018) for a survey of the literature.

Our paper continues as follows. In Section 2, we introduce our basic model setup, entailing two different model specifications. In Sections 3 and 4, we study the implications of these two model specifications. In Section 5, we conclude our paper. A number of robustness checks are presented in Appendices A and B.

2 Setup of two model specifications

Cobweb models, as pioneered by Tinbergen (1930), Leontief (1934) and Ezekiel (1938), allow us to study the price dynamics of a nonstorable commodity whose production takes time. These models are thus ideally suited for the purpose of our paper. Let us start with a preview of the basic structure of our model. We assume that firms face two consecutive decision tasks. In the first task, firms have to select one of two possible production technologies: a fast production technology with a fixed production lag of one period, and a slow production technology with a fixed production lag of two periods. Based on their choice of production technology, the firms' second task is to determine their optimal supply. Here we assume that firms maximize their expected profits subject to a quadratic cost function. Due to the supply response lag of the production technologies, firms must form price expectations. Since the goal of our paper is to explore the effects of production delays, we assume that firms have naïve price expectations. Besides helping us to investigate the effects of production delays with greater clarity, this simplifying assumption also preserves the comparability of our results with those obtained from traditional cobweb models.

To illustrate the model's underlying time structure in more detail, let us consider the options a firm faces in period $t-1$. As already mentioned, a firm can opt for a fast production technology that allows it to produce the commodity within one period. In period t, such a firm, indexed by F, first sells its supply and then reconsiders its production technology choice. Alternatively, a firm can opt for a slow production technology. Since the slow production technology involves a production delay of two periods, a firm that opts for the slow production technology offers its supply in period $t+1$. In that period, it can also reconsider its production technology choice. Firms opting for the slow production technology in period $t-1$ are indexed by M in period t and by S in period $t+1$. For ease of exposition, we call firms that chose the fast (slow) production technology to offer their commodities "fast" ("slow") firms. Slow production technology adopters in the manufacturing process are called "manufacturing" firms. For simplicity, we assume that firms do not revise their production plans during the manufacturing process.

Three model implications should be highlighted at the outset. One implication of our model is that not all firms are able to adjust their production technology choice in period $t-1$. In fact, only firms that have opted for the fast production technology in period $t-2$ and firms that have opted for the slow production technology in period $t-3$ can change their production technology choice in period $t-1$. Motivated by Brock and Hommes (1997), we assume that firms do this by comparing the past profitability of the two production technologies. While firms are boundedly rational, we stress that they display a profit-dependent learning behavior. Clearly, manufacturing firms cannot change their production technology in period $t-1$.

Another implication of our model is that not all firms offer their commodities every period. For instance, the total supply by firms in period t depends on the supply provided by firms that opted for the fast production technology in period $t-1$ and on the supply provided by firms that opted for the slow production technology in period $t-2$, i.e. firms labeled M in period $t-1$. Put differently, firms that opted for the slow production technology in period $t-1$ are absent from the market in period t. It is therefore easy to imagine that firms' switching between production technologies with different delays may lead to supply distortions, which, in turn, may induce mismatches between supply and demand that have an impact on the formation of commodity prices. This is the main topic of our paper.

A final implication of our model results from firms' naïve price expectations. To be precise, the supply provided by fast technology adopters in period t depends on the price they observe in period $t-1$, while the supply provided by slow technology adopters in period t depends on the price they observe in period $t-2$. When the market is at rest, firms' price expectations are homogenous and correct. Out of equilibrium, however, firms' price expectations are typically heterogeneous and incorrect. Hence, when the market shares of the two production technologies vary over time, the history of past prices affects firms' total supply in a nontrivial way. This adds a further layer of complexity to the dynamics of our model. Of course, this layer of complexity would also be present in our model if firms used other expectation rules, say extrapolative or regressive expectation rules. Naïve expectations have the advantage that they come without additional model parameters and that they keep the dimension of the model's underlying map low.

Let us now formalize the details of our model. We consider a fixed number N of firms and denote the market shares of firms offering their supply in period t after opting for the slow or fast production technology by W_t^S and W_t^F, respectively. Obviously, the market share of firms that are still busy manufacturing the commodity is equal to $W_t^M = 1 - W_t^F - W_t^S$. Firms' total supply of the commodity in period t therefore amounts to

$$S_t = N(W_t^F S_t^F + W_t^S S_t^S), \tag{1}$$

where S_t^F and S_t^S denote the supply provided by firms that relied on the fast and the slow production technology, respectively, for trading occurring in period t. Consumers' commodity demand depends negatively on the current commodity price P_t. Using a linear demand schedule, we express their demand by

$$D_t = a - bP_t, \tag{2}$$

where a and b are positive parameters. Needless to say, parameters a and b have to ensure that consumers' commodity demand remains non-negative. Since the commodity cannot be stored, the market clearing condition for the market that operates in period t, i.e.

$$D_t = S_t, \tag{3}$$

implies that the commodity price is

$$P_t = \frac{a-(W_t^F S_t^F + W_t^S S_t^S)}{b}, \tag{4}$$

where, for ease of exposition, we have normalized the mass of firms to $N = 1$.

Each firm maximizes its expected profits subject to a quadratic cost function. Let $P_t^{e,i}$ stand for the commodity price that firm i expects to receive in period t. Moreover, let $C_t^i = \frac{1}{2c}(S_t^i)^2 + d^i$ reflect its cost function, with $c > 0$ and $d^i \geq 0$. We can write the profits that firm i expects to make in period t as

$$\pi_t^{e,i} = P_t^{e,i} S_t^i - \frac{1}{2c}(S_t^i)^2 - d^i, \tag{5}$$

yielding its optimal supply decision

$$S_t^i = cP_t^{e,i}. \tag{6}$$

Hence, firm i's supply of the commodity depends positively on its price expectations. Since we assume that firms have naïve price expectations, the optimal supply provided by a firm that relies on the fast production technology, requiring it to determine its supply in period $t-1$, is equal to

$$S_t^F = cP_{t-1}, \tag{7}$$

while the optimal supply provided by a firm that applies the slow production technology, requiring it to determine its supply in period $t-2$, is equal to

$$S_t^S = cP_{t-2}. \tag{8}$$

Firm i's fixed costs depend on its choice of production technology. We assume that the fast production technology invokes higher fixed costs than the slow production technology, i.e. $d^F > d^S \geq 0$.[2] In addition, note that the fast production technology rests on naïve one-period-ahead predictions, i.e. $P_t^{e,F} = P_{t-1}$, while the slow production technology relies on naïve two-periods-ahead predictions, i.e. $P_t^{e,S} = P_{t-2}$.

How do firms select production technologies? Note first that the fixed costs disadvantage of the fast production technology may pay off via two channels. Most importantly, the fast production technology allows firms to produce the commodity twice as often as the slow production technology. Relative to the slow production technology, the fast production technology furthermore allows firms to monitor the commodity price one period closer to the actual trading period. In this paper, we assume that firms' choice of production technology depends on realized profits. The profits realized by the production technologies are determined by

$$\pi_t^F = P_t S_t^F - C_t^F = \frac{c}{2} P_{t-1}(2P_t - P_{t-1}) - d^F \tag{9}$$

and

$$\pi_t^S = P_t S_t^S - C_t^S = \frac{c}{2} P_{t-2}(2P_t - P_{t-2}) - d^S, \tag{10}$$

respectively. Since the profits of a firm that opts for the slow production technology arise only every second period, we study two possible specifications of fitness functions. The idea behind these two specifications is to make the profitability of the two production technologies comparable.

First, firms may compare the profits they make with the fast production technology with half of the profits they make with the slow production technology. In this case, labeled as Model Specification A, the fitness functions of the fast and slow production technology read as

$$U_t^{F,A} = \pi_t^F \tag{11}$$

and

$$U_t^{S,A} = \frac{1}{2}\pi_t^S, \tag{12}$$

respectively. Second, firms may compare the profits they make using the fast production technology in periods t and $t-1$ with the profits they make using the

[2] In Appendix A, we consider that the two production technologies may also differ with respect to their marginal costs.

slow production technology in period t. In this case, labeled as Model Specification B, the fitness functions read as

$$U_t^{F,B} = \pi_t^F + \pi_{t-1}^F \tag{13}$$

and

$$U_t^{S,B} = \pi_t^S, \tag{14}$$

respectively. Note that, in both cases, we ignore any possible intertemporal effects that may occur when firms have access to capital markets allowing them to invest their profits in, say, safe government bonds. Moreover, we assume that there is complete reversibility of production technologies and that no costs arise when firms switch between them.[3] We explore Model Specification A in Section 3 and Model Specification B in Section 4.

We capture firms' choice of production technology via the discrete choice approach, as propagated by Brock and Hommes (1997). Let index $k \in \{A, B\}$ refer to Model Specifications A and B, respectively. Since the market share of firms that reconsider their choice of production technology in period $t-1$ is given by $W_{t-1}^F + W_{t-1}^S$, the market share of firms that select the fast production technology in period $t-1$ so as to offer their supply in period t is equivalent to

$$W_t^F = (W_{t-1}^F + W_{t-1}^S)\frac{exp[\beta U_{t-1}^{F,k}]}{exp\left[\beta U_{t-1}^{F,k}\right] + exp[\beta U_{t-1}^{S,k}]} = (W_{t-1}^F + W_{t-1}^S)\frac{1}{1+exp[\beta(U_{t-1}^{S,k} - U_{t-1}^{F,k})]}, \tag{15}$$

while the market share of firms that opt for the slow production technology in period $t-1$ so as to offer their supply in period $t+1$, being equal to the market share of firms in the manufacturing process in period t, is equivalent to

$$W_t^M = (W_{t-1}^F + W_{t-1}^S)\frac{exp[\beta U_{t-1}^{S,k}]}{exp\left[\beta U_{t-1}^{F,k}\right] + exp[\beta U_{t-1}^{S,k}]} = (W_{t-1}^F + W_{t-1}^S)\frac{1}{1+exp[\beta(U_{t-1}^{F,k} - U_{t-1}^{S,k})]}, \tag{16}$$

where parameter $\beta > 0$ controls how sensitive the mass of firms is to selecting the fittest production technology. Moreover, the market share of firms that selected the slow production technology in period $t-2$, i.e. the market share of firms that offer their supply in period t using the slow production technology, is equal to the market share of firms that are still manufacturing in period $t-1$ using the slow production technology. Hence,

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[3] Without going too much into details, such a view may be justified by the assumption that firms are renting the necessary equipment they need to realize their production decisions.

$$W_t^S = W_{t-1}^M = 1 - W_{t-1}^F - W_{t-1}^S. \tag{17}$$

The discrete choice expressions (15) and (16) capture firms' learning behavior twofold. First, they ensure that the higher the fitness of a production technology, the more firms will select it. Second, they imply that the higher the intensity of choice parameter β, the more firms will select the fitter production technology. In this respect, it is helpful to consider two extreme cases. For $\beta \to 0$, firms do not observe differences in the fitness between the two production technologies and are divided equally among them. For $\beta \to \infty$, firms perfectly observe differences in the fitness between the two production technologies and all firms opt for the fitter production technology. Roughly speaking, firms' degree of bounded rationality increases with their intensity of choice, which is why we may regard the case in which $\beta \to \infty$ as the neoclassical limit.[4]

3 Model Specification A

In this section, we first derive the map that governs the dynamics of Model Specification A. We then analytically discuss its steady state and stability implications under a certain parameter restriction. Finally, we conduct a number of numerical experiments to study its out-of-equilibrium behavior.

Combining (4)-(12), (15) and (17) enables us to express the first specification of our model in the form of a five-dimensional nonlinear map, represented by

$$MA := \begin{cases} P_t = \dfrac{a - \left((W_{t-1}^F + W_{t-1}^S) \frac{1}{1+exp\left[\beta\left(\frac{1}{2}\pi_{t-1}^S - \pi_{t-1}^F\right)\right]} cP_{t-1} + \left(1 - W_{t-1}^F - W_{t-1}^S\right) cX_{t-1} \right)}{b} \\ X_t = P_{t-1} \\ Y_t = X_{t-1} \\ W_t^F = (W_{t-1}^F + W_{t-1}^S) \frac{1}{1+exp\left[\beta\left(\frac{1}{2}\pi_{t-1}^S - \pi_{t-1}^F\right)\right]} \\ W_t^S = 1 - W_{t-1}^F - W_{t-1}^S \end{cases}, \tag{18}$$

where $\frac{1}{2}\pi_{t-1}^S - \pi_{t-1}^F = \frac{1}{2}(\frac{c}{2} Y_{t-1}(2P_{t-1} - Y_{t-1}) - d^S) - (\frac{c}{2} X_{t-1}(2P_{t-1} - X_{t-1}) - d^F)$

[4] Note that the discrete choice approach has been successfully used to explain the dynamics of commodity prices (Brock and Hommes 1997), risky asset prices (Brock and Hommes 1998), exchange rates (de Grauwe and Grimaldi 2006), business cycles (Anufriev et al. 2013), laboratory experiments (Anufriev et al. 2016) and housing markets (Martin et al. 2021). However, there are alternative schemes to describe discrete choice decisions of boundedly rational agents. In Appendix B, we consider the case of exponential replicator dynamics.

and X_t and Y_t are auxiliary variables.[5] Note that the market share of manufacturing firms, i.e. W_t^M, does not enter map MA.

For analytical reasons, we introduce the following assumption.

Assumption A1: Fixed cost parameters d^F and d^S ensure that the relation $d^F = \frac{c}{4}(\frac{3a}{3b+2c})^2 + \frac{1}{2}d^S$ holds.

As we will see, Assumption A1 guarantees that firms are indifferent between the slow and the fast production technology when the dynamics of Model Specification A is at rest, i.e. $\overline{U^{F,A}} = \overline{U^{S,A}}$. The economic rationale behind this assumption is that the fixed costs for the machinery of the two production technologies, expressed by parameters d^F and d^S, are not exogenously given, but adjust in the long run such that the above relation is met. While we do not explicitly model the formation of these two parameters, we have in mind that the supply and demand for the two competing production technologies implicitly yield Assumption A1. Numerically, we discuss digressions from Assumption A1 in the sequel, too.

Given Assumption A1, we find that map MA has the unique steady state

$$SSA = \left(\bar{P}, \bar{X}, \bar{Y}, \overline{W^S}, \overline{W^F}\right) = \left(\frac{3a}{3b+2c}, \frac{3a}{3b+2c}, \frac{3a}{3b+2c}, \frac{1}{3}, \frac{1}{3}\right), \tag{19}$$

where the steady-state commodity price may also be expressed as $\bar{P} = \frac{a}{b+c(\overline{W^F}+\overline{W^S})}$. Furthermore, interpreting $\tilde{c} = c\left(\overline{W^F} + \overline{W^S}\right)$ as the aggregate (effective) slope parameter of firms' aggregate supply schedule, this solution is reminiscent to the steady state commodity price we encounter in traditional (linear) cobweb models, given by $\bar{P} = \frac{a}{b+\tilde{c}}$. Obviously, $\bar{P}$ increases with parameter a and decreases with parameters b and c. Moreover, the corresponding steady-state profits associated with the fast and slow production technology are given by $\overline{\pi^F} = \frac{c}{2}\bar{P}^2 - d^F$ and $\overline{\pi^S} = \frac{c}{2}\bar{P}^2 - d^S$. Since Assumption A1 implies that $d^F = \frac{c}{4}\bar{P}^2 + \frac{1}{2}d^S$, we have that $\overline{\pi^F} = d^F - d^S = \frac{1}{2}\overline{\pi^S}$. Finally, it follows from $\overline{U^{F,A}} = \overline{U^{S,A}}$ that $\overline{W^F} = \overline{W^S} = \overline{W^M} = \frac{1}{3}$. At the steady state, one-third of the firms thus employ

[5] Note that $U_{t-1}^{S,A} - U_{t-1}^{F,A} = \frac{1}{2}\pi_{t-1}^S - \pi_{t-1}^F = \frac{1}{2}(\frac{c}{2}\, P_{t-3}(2P_{t-1} - P_{t-3}) - d^S) - (\frac{c}{2}\, P_{t-2}(2P_{t-1} - P_{t-2}) - d^F)$.

the fast production technology, while two-thirds of them rely on the slow production technology, of which half offer their supply while the other half is still in the manufacturing process. Needless to say, all firms have homogenous and correct price expectations, and the supply of fast and slow firms is given by $\overline{S^F} = \overline{S^S} = c\bar{P}$.[6]

To facilitate the stability analysis of map MA's steady state, we introduce the following assumption.

Assumption A2: Marginal cost parameter c and demand parameter b ensure that $c < 3b$.

As we will see, Assumption A2 guarantees that the steady state SSA is locally stable when firms do not switch between production technologies, i.e. when the intensity of choice parameter β approaches zero.

Tedious computations reveal that we may express the characteristic polynomial of the Jacobian matrix of map MA at the steady state SSA as follows:

$$P(\lambda) = \lambda^2 \left(\lambda^3 + (\tfrac{1}{2} + \tfrac{c}{3b} + \tfrac{3a^2c^2}{4b(3b+2c)^2}\beta)\lambda^2 + \tfrac{c}{2b}\lambda + \tfrac{c}{6b}\right). \tag{20}$$

Since two eigenvalues of (20) are obviously always equal to zero, say $\lambda_1 = 0$ and $\lambda_2 = 0$, the local stability of the steady state SSA depends on the remaining three eigenvalues, say λ_3, λ_4 and λ_5, determined by the term in brackets on the right-hand side of (20). Let us rewrite the characteristic polynomial as

$$P(\lambda) = \lambda^2(\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3), \tag{21}$$

where $a_1 = \frac{1}{2} + \frac{c}{3b} + \frac{3a^2c^2}{4b(3b+2c)^2}\beta$, $a_2 = \frac{c}{2b}$ and $a_3 = \frac{c}{6b}$. Using the stability and bifurcation results derived in Lines et al. (2020) and Gardini et al. (2021), we can conclude that the steady state SSA loses its local stability when one of the following three inequalities becomes broken: (i) $1 + a_1 + a_2 + a_3 > 0$, (ii) $1 - a_1 + a_2 - a_3 > 0$ and (iii) $1 - a_2 + a_1a_3 - {a_3}^2 > 0$, where a separate violation of the first, second and third inequality, while the other two inequalities hold, is

[6] Note also that the steady state price $\bar{P}$ obtained under Assumption A1 corresponds to that of model (18) under the limiting case $\beta \to 0$ (resulting in a four-dimensional map), in which fixed cost parameters d^F and d^S no longer matter for the dynamics. The steady state of this reduced map is stable if and only if $c < 3b$.

associated with a fold, a flip, and a Neimark-Sacker bifurcation, respectively.

Obviously, stability condition (i) is always satisfied. Stability condition (ii) requires that $\frac{1}{2} - \frac{3a^2c^2\beta}{4b(3b+2c)^2} > 0$. We are particularly interested in the role played by firms' intensity of choice. Solving for parameter β results in

$$\beta < \beta_{crit,A}^{flip} := \frac{2b(3b+2c)^2}{3a^2c^2} = \frac{6b}{c^2\bar{P}^2}. \tag{22}$$

If the steady state SSA becomes unstable via a violation of (22), we observe a flip bifurcation and the birth of a period-two cycle. For this to occur, parameter β has to exceed $\beta_{crit,A}^{flip}$, i.e. firms have to react strong enough to the fitness difference between the fast and slow production technology. Note that the critical bifurcation value $\beta_{crit,A}^{flip}$ increases with parameter b and decreases with parameters a and c. We remark that traditional (linear) cobweb models predict that the commodity price will converge towards its steady state when the slope parameter of the demand function is larger than the slope parameter of the supply function (see, e.g. Gandolfo 2009). Interestingly, stability condition (22) mirrors these results.

Stability condition (iii) necessitates that $2(3b-c)(12b-c) + \frac{9a^2c^3}{(3b+2c)^2}\beta > 0$. Solving again for parameter β yields

$$\beta > \frac{2(c-3b)(12b-c)(3b+2c)^2}{9a^2c^3} = \frac{2(c-3b)(12b-c)}{c^3\bar{P}^2}. \tag{23}$$

By assumption, parameter β is positive. Hence, (23) can only be violated for $3b < c < 12b$, which is excluded by Assumption A2.[7]

The following proposition summarizes our main analytical results.

Proposition A: Under Assumptions A1 and A2, map MA possesses the unique steady state $SSA = \left(\bar{P}, \bar{X}, \bar{Y}, \overline{W^S}, \overline{W^F}\right) = \left(\frac{3a}{3b+2c}, \frac{3a}{3b+2c}, \frac{3a}{3b+2c}, \frac{1}{3}, \frac{1}{3}\right)$. This steady state loses its local stability if $\beta < \frac{2b(3b+2c)^2}{3a^2c^2}$ becomes violated. The stability loss is associated with a flip bifurcation.

[7] According to the stability results established in Gardini et al. (2021), necessary and sufficient conditions for the steady state of Model Specification A to be locally stable require that conditions (i), (ii) and (iii) hold together with $|a_3| < 1$, where the latter implies that $c < 6b$. Due to Assumption A2, this condition is always met.

We continue with a numerical analysis of Model Specification A, assuming the following base parameter setting: $a = 5/3$, $b = 1$, $c = 1$, $d^F = 0.25$, $d^S = 0$ and $\beta = 18$. Under this parameter setting, Assumptions A1 and A2 are true. Moreover, the steady state SSA implies that $\bar{P} = 1$, $\overline{\pi^F} = 0.25$, $\overline{\pi^S} = 0.5$ and $\overline{W^F} = \overline{W^S} = \overline{W^M} = 1/3$. Since firms' intensity of choice parameter is equal to $\beta = 18$, stability condition (22), implying that $\beta_{crit,A}^{flip} = 6$, does not hold and the commodity market is unstable. Figures 1 to 3 present our main results. Figure 1 shows an example of the dynamics of Model Specification A, Figure 2 displays a number of bifurcation diagrams of key model variables with respect to parameter β, and Figure 3 explores the possibility of coexisting attractors.

Let us start with Figure 1. Apparently, Model Specification A may yield chaotic dynamics. The panels show from top to bottom the evolution of the commodity price, the corresponding market shares of fast (black), slow (white) and manufacturing (gray) firms, respectively, the fitness difference between the fast and slow production technology, and the commodity price in period $t + 1$ versus the commodity price in period t. Indeed, the intricate behavior of prices, market shares and fitness differences, together with the emergence of a strange attractor, suggest that the dynamics is chaotic. Note that the commodity price is on average above its steady state level $\bar{P} = 1$, given by the gray line in the top panel of Figure 1, implying that firms' out-of-equilibrium supply falls short of their steady-state supply.

******* Figure 1 about here *******

We may understand the functioning of Model Specification A, as depicted in Figure 1, as follows. To be able to appreciate the out-of-equilibrium properties of Model Specification A, let us first reconsider its main steady-state implications. At the steady state, all firms have correct price expectations, the supply of fast and slow firms is identical and amounts to $\overline{S^F} = \overline{S^S} = c\bar{P} = \bar{P}$, and, since firms are equally distributed across production technologies, i.e. $\overline{W^F} = \overline{W^S} = \overline{W^M} = \frac{1}{3}$, and have mass $N = 1$, their total supply is equal to $\bar{S} = \frac{2}{3}\bar{P}$, yielding the market clearing price $\bar{P} = 1$. At the steady state, firms' production processes are in sync: there are no supply distortions, no mismatches between supply and demand and no price changes. Out of equilibrium, however, firms' production processes are not in sync. With some liberty, we can identify two competing regimes, say, Regime R1 and Regime R2.

- In Regime R1, which is present, for instance, in the first 20 periods of the time series plots depicted in Figure 1, the commodity price describes a zigzag behavior that is near to a period-4 cycle. However, closer inspection reveals that the amplitudes of the price dynamics slowly increase up to around period 20, where Regime R1 ends. Note that only a few firms opt for the fast production technology during these periods for the following reason: every second period, slow production technology adopters must reconsider their choice of production technology. Due to the zigzag behavior of the commodity price, the naïve two-period-ahead forecast required by the slow production technology is more precise than the naïve one-period-ahead forecast needed for the fast production technology. Hence, the slow production technology is fitter than the fast production technology in these periods, which is why the vast majority of firms selects it.[8] Consequently, the supply in the next period is relatively low, resulting in a high commodity price, and the supply in the next but one period is relatively high, resulting in a low commodity price. Clearly, too many firms rely on the slow production technology in Regime R1, causing supply distortions, a mismatch between supply and demand, and a strong price variability.
- In Regime R2, say around period 25 and period 55 in the time series plots depicted in Figure 1, the fast production technology is more popular than the slow production technology, reaching market shares up to 70 percent. Now, slow firms and fast firms reconsider their choice of production technology, and, given the past price sequences of the commodity, the fast production technology may outperform the slow one, at least for a few consecutive time

[8] Ironically, the third panel of Figure 1 reveals that in Regime 1 the fast production technology is fitter than the slow production technology in every fourth period, yet only a few firms reconsider their choice of production technology in these periods. Apparently, the commodity market is temporarily captured in a state in which too many firms prefer the slow production technology. Economically, this highly undesirable outcome, associated with, on average, higher commodity prices and lower commodity supplies, emerges when firms switch too rapidly between production technologies, i.e. when parameter β is too high.

steps. Nevertheless, Regime R2 also yields supply distortions and mismatches between supply and demand that cause erratic price dynamics.

As is evident from Figure 1, these two regimes repeat themselves in a complex manner, leading to chaotic price dynamics.

Figure 2 presents bifurcation diagrams for Model Specification A. The top left (right) panel of Figure 2 shows the price (market share of manufacturing firms) versus parameter β for our base parameter setting. As predicted by Proposition A, a flip bifurcation sets in at $\beta_{crit,A}^{flip} = 6$. Note furthermore that the amplitude of the dynamics increases with parameter β. We can thus conclude that the dynamics of Model Specification A becomes wilder when firms switch more rapidly between the fast and slow production technology.

So far, we have focused on the case in which Assumption A1 holds. However, what happens if Assumption A1 becomes violated? The middle panels in Figure 2 depict the same experiment as before, except that $d^F = 0.15$. Accordingly, a decrease in the fixed costs disadvantage of the fast production technology implies that more firms will use the fast production technology at the steady state. As a result, firms' total supply increases at the steady state, which, in turn, drives down the commodity price at the steady state. Note that these two effects increase with parameter β. The bottom panels of Figure 2 repeat this experiment for $d^F = 0.35$. As can be seen, an increase in the fixed costs disadvantage makes the fast production technology less popular at the steady state. Hence, firms' total supply decreases at the steady state and the commodity price increases at the steady state. Note that deviations from Assumption A1 in the form of a lower or higher value of parameter d^F do not destroy the emergence of a flip bifurcation and the eventual onset of chaotic price dynamics. Put differently, the simulations reported in Figure 2 demonstrate the robustness of Proposition A with respect to violations of Assumption A1.

***** **Figure 2 about here** *****

Interestingly, Model Specification A may give rise to coexisting attractors, an aspect that we illustrate with the help of Figure 3. Figure 3 is based on our base parameter setting for Model Specification A, except that $\beta = 10.6$. The simulated commodity prices visible in black and red in the top and middle panels of Figure

3 only differ with respect to their initial conditions. While one set of initial conditions leads to a period-four cycle, the other set yields a period-sixteen cycle. In the presence of exogenous noise, such a constellation may easily lead to intricate attractor switching dynamics. The bottom panel of Figure 3 shows two overlapping bifurcation diagrams. The bifurcation diagram in black is identical to the one plotted in the top left panel of Figure 2. Here we have selected the initial conditions for each new value of parameter β in the neighborhood of the attractor that emerged for the previous value of parameter β. In contrast, the bifurcation diagram in red is based on random initial conditions. Obviously, there are ranges of parameter β that are associated with coexisting attractors.

******* Figure 3 about here *******

4 Model Specification B

This section is organized in the same way as the previous section. For better readability, we repeat the key steps in the derivation of our main results. Although Model Specifications A and B display the same steady state, they differ in their stability and out-of-equilibrium properties.

Combining (4)-(10), (13)-(15) and (17) reveals that the dynamics of our second model specification is driven by the following five-dimensional nonlinear map:

$$MB := \begin{cases} P_t = \dfrac{a-\left((W_{t-1}^F+W_{t-1}^S)\frac{1}{1+exp\left[\beta\left(\pi_{t-1}^S-\left(\pi_{t-1}^F+\pi_{t-2}^F\right)\right)\right]}cP_{t-1}+\left(1-W_{t-1}^F-W_{t-1}^S\right)cX_{t-1}\right)}{b} \\ X_t = P_{t-1} \\ Y_t = X_{t-1} \\ W_t^F = (W_{t-1}^F + W_{t-1}^S)\frac{1}{1+exp\left[\beta\left(\pi_{t-1}^S-\left(\pi_{t-1}^F+\pi_{t-2}^F\right)\right)\right]} \\ W_t^S = 1 - W_{t-1}^F - W_{t-1}^S \end{cases}, \qquad (24)$$

where $\pi_{t-1}^S - (\pi_{t-1}^F + \pi_{t-2}^F) = (\frac{c}{2}\, Y_{t-1}(2P_{t-1} - Y_{t-1}) - d^S) - ((\frac{c}{2}\, X_{t-1}(2P_{t-1} - X_{t-1}) - d^F) + (\frac{c}{2}\, Y_{t-1}(2X_{t-1} - Y_{t-1}) - d^F))$ and X_t and Y_t are auxiliary variables.[9] Similar to Model Specification A, the market share of manufacturing firms, i.e. W_t^M, does not enter map MB.

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[9] Note that $U_{t-1}^{S,B} - U_{t-1}^{F,B} = \pi_{t-1}^S - (\pi_{t-1}^F + \pi_{t-2}^F) = (\frac{c}{2}\, P_{t-3}(2P_{t-1} - P_{t-3}) - d^S) - ((\frac{c}{2}\, P_{t-2}(2P_{t-1} - P_{t-2}) - d^F) + (\frac{c}{2}\, P_{t-3}(2P_{t-2} - P_{t-3}) - d^F))$.

For analytical reasons, we introduce the following assumption.

Assumption B1: Fixed cost parameters d^F *and* d^S *ensure that the relation* $d^F = \frac{c}{4}(\frac{3a}{3b+2c})^2 + \frac{1}{2}d^S$ *holds.*

Note that Assumption A1 is equivalent to Assumption B1. The reason for this is that both model specifications share the same steady state. In fact, straightforward computations reveal that map MB possesses the unique steady state

$$SSB = (\bar{P}, \bar{X}, \bar{Y}, \overline{W^S}, \overline{W^F}) = \left(\frac{3a}{3b+2c}, \frac{3a}{3b+2c}, \frac{3a}{3b+2c}, \frac{1}{3}, \frac{1}{3}\right) \tag{25}$$

under Assumption B1, which is identical to the steady state SSA of map MA. Moreover, the corresponding steady-state profits of the two production technologies are given by $\overline{\pi^F} = (d^F - d^S) = \frac{1}{2}\overline{\pi^S}$. Clearly, the different setups for the production technologies' fitness functions do not alter our model's steady state implications.

However, the characteristic polynomial of the Jacobian matrix of map MB at the steady state SSB now reads

$$P(\lambda) = \lambda^2(\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3), \tag{26}$$

where $a_1 = \frac{1}{2} + \frac{c}{3b}$, $a_2 = \frac{c}{2b} + \frac{3a^2c^2}{2b(3b+2c)^2}\beta$ and $a_3 = \frac{c}{6b}$. Obviously, two eigenvalues of (26) are always equal to zero, say $\lambda_1 = 0$ and $\lambda_2 = 0$, while the remaining three eigenvalues, say λ_3, λ_4 and λ_5, are determined by the term in brackets on the right-hand side of (26). Based on the stability and bifurcation results established in Lines et al. (2020) and Gardini et al. (2021), we can conclude that the model's steady state loses its local stability when one of the three inequalities (i) $1 + a_1 + a_2 + a_3 > 0$, (ii) $1 - a_1 + a_2 - a_3 > 0$ and (iii) $1 - a_2 + a_1a_3 - {a_3}^2 > 0$ is not met anymore.

Before we continue, let us introduce the following assumption.

Assumption B2: Marginal cost parameter c *and demand parameter* b *ensure that* $c < 3b$*.*

Note that Assumption A2 is equivalent to Assumption B2.

As in the previous section, stability condition (i) is always satisfied. Stability condition (ii) now requires that $\frac{1}{2}+\frac{3a^2c^2\beta}{2b(3b+2c)^2}>0$ and is always satisfied, too. A crucial difference between Model Specifications A and B is thus that Model Specification B does not give rise to a flip bifurcation. Stability condition (iii) is equivalent to $(3b-c)(12b-c)-\frac{54a^2bc^2}{(3b+2c)^2}\beta>0$. Solving this inequality for firms' intensity of choice parameter β results in

$$\beta<\beta_{crit,B}^{NS}:=\frac{(3b+2c)^2(3b-c)(12b-c)}{54a^2bc^2}=\frac{(3b-c)(12b-c)}{6\bar{P}^2bc^2}. \quad (27)$$

Accordingly, the steady state SSB becomes unstable in the form of a Neimark-Sacker bifurcation, giving rise to cyclical dynamics when (27) becomes violated.[10]

We have thus proven the following proposition.

Proposition B: Under Assumptions B1 and B2, map MB possesses the unique steady state $SSB=\left(\bar{P},\bar{X},\bar{Y},\overline{W^S},\overline{W^F}\right)=\left(\frac{3a}{3b+2c},\frac{3a}{3b+2c},\frac{3a}{3b+2c},\frac{1}{3},\frac{1}{3}\right)$. This steady state loses its local stability if $\beta<\frac{(3b+2c)^2(3b-c)(12b-c)}{54a^2bc^2}$ becomes violated. The stability loss is associated with a Neimark-Sacker bifurcation.

Let us assume the following base parameter setting to illustrate the dynamics of Model Specification B: $a=5/3$, $b=1$, $c=1$, $d^F=0.25$, $d^S=0$ and $\beta=4$.[11] Accordingly, Assumptions B1 and B2 hold. Moreover, we have that $\bar{P}=1$, $\overline{\pi^F}=0.25$, $\overline{\pi^S}=0.5$ and $\overline{W^F}=\overline{W^S}=\overline{W^M}=1/3$. Since firms' intensity of choice parameter is equal to $\beta=4$ and thus larger than $\beta_{crit,B}^{NS}=\frac{22}{6}=3.66$, stability condition (27) does not hold and the steady state SSB is unstable due to a Neimark-Sacker bifurcation. In fact, Figure 4 suggests that the violation of stability condition (27) gives rise to cyclical dynamics. Its first panel shows the evolution of the commodity price; the second panel depicts the corresponding market shares of fast (black), slow (white) and manufacturing (gray) firms, respectively; and the third panel reports the fitness difference between the fast and slow production technology. The fourth panel of Figure 4, showing the commodity price

[10] As reported in footnote 7, necessary and sufficient conditions for the steady state of Model Specification B to be locally stable require not only that conditions (i), (ii) and (iii) hold, but also that $|a_3|<1$ is met, where the latter restriction implies again that $c<6b$. Assumption B2 ensures that this condition always holds.

[11] Except parameter β, we use the same parameter setting as in the previous section.

in period $t + 1$ versus the commodity price in period t, visualizes the existence of a limit cycle.

From an economic perspective, we may understand the functioning of Model Specification B as follows. As is clear from the third panel of Figure 4, there is a permanent evolutionary competition between the fast and slow production technology, and each production technology repeatedly possesses a higher fitness for a few consecutive periods. As a result, there are periods where more firms opt for the fast production technology and periods where more firms opt for the slow one, as is evident from the second panel of Figure 4. Importantly, however, the number of firms that offer their commodities (fast and slow firms, represented by black and white in the second panel of Figure 4) is subject to short-run waves, too. As a result, firms' total supply oscillates, as do commodity prices.

We stress that the dynamics of Model Specifications A and B have in common that the production processes of firms is not in sync. For both model specifications, we observe supply distortions, mismatches between supply and demand, and endogenous commodity price dynamics, irregular ones for Model Specification A and regular ones for Model Specification B. In a sense, one may argue that there is a coordination failure, brought about by firms' too rapid switching between production technologies. As already mentioned, both model specifications differ with respect to the nature of their steady state's primary bifurcation, an aspect that obviously has to do with the formalization of the fitness functions, as expressed by (11) to (14). It seems that the reason why Model Specification B gives rise to a Neimark-Sacker bifurcation instead of a flip bifurcation is that its fitness functions place more weight on past (price) observations relative to those of Model Specification A.

******* Figure 4 about here *******

Figure 5 presents bifurcation diagrams for Model Specification B. The top left (right) panel of Figure 5 shows the price (market share of manufacturing firms) versus parameter β for our base parameter setting. As predicted by our analytical results, a Neimark-Sacker bifurcation sets in at $\beta_{crit,B}^{NS} = \frac{22}{6} \approx 3.66$. Note that the amplitude of the dynamics increases with parameter β. We can thus conclude that the dynamics of Model Specification B becomes more volatile when firms switch more rapidly between the fast and slow production technology. The middle panels in Figure 5 depict the same experiments, except that $d^F = 0.18$. Note that a decrease in the fixed costs disadvantage of the fast production technology implies that more firms will use the fast production technology at the steady state. As a result, firms' total supply increases at the steady state, which, in turn, drives down the commodity price at the steady state. Note that these two effects increase with parameter β. The bottom panels of Figure 5 repeat this experiment for $d^F = 0.32$. As can be seen, an increase in the fixed costs disadvantage makes the fast production technology less popular at the steady state. Hence, firms' total supply decreases at the steady state and the commodity price increases at the steady state. While deviations from Assumption B1, e.g. via a higher or lower value of parameter d^F, shift the steady state's stability frontier, we observe in all three scenarios depicted in Figure 5 the emergence of a Neimark-Sacker bifurcation, and thus the onset of cyclical price dynamics. Put differently, the simulations reported in Figure 5 demonstrate the robustness of Proposition B with respect to violations of Assumption B1.

******* Figure 5 about here *******

5 Conclusions

The goal of our paper is to offer a new explanation for the excessive variability of commodity prices. Based on a behavioral cobweb model, we show that supply distortions may lead to endogenous price dynamics due to mismatches between supply and demand. To be more precise, we assume that firms have the choice between production technologies that take different lengths of time: a fast production technology with a fixed supply-response lag of one period, and a slow production technology with a fixed supply-response lag of two periods. Firms choose between production technologies according to an evolutionary fitness measure that takes into account their past profitability. Firms thereby display a boundedly rational learning behavior. Endogenous commodity price dynamics emerges when firms switch too rapidly between production technologies, either via a flip bifurcation scenario, leading to irregular zigzag commodity price dynamics, or via a Neimark-Sacker bifurcation scenario, leading to more orderly cyclical commodity price dynamics.

A final economic remark is in order. Since the slow production technology requires two periods to manufacture the commodity, not all firms offer their supply every period. Put differently, firms' total supply schedule is a time-varying quantity in our model. When firms react only weakly to the production technologies' fitness differences, the commodity price is stable. Firms' production processes are then in sync. When firms react strongly to the production technologies' fitness differences, the commodity price is unstable. Firms' production processes are then not in sync. Out of equilibrium, too many firms opt for the fast or slow production technology, a coordination failure that results in an excessive price variability.

We conclude our paper by pointing out four avenues for future research. First, we assume that firms reconsider their choice of production technology whenever they have the opportunity of doing so. However, it may be worthwhile to explore the case in which firms reconsider their choice of production technology less frequently, e.g. by using the asynchronous updating procedure, as employed by Diks and van der Weide (2005), Hommes et al. (2005) and Anufriev and Hommes (2012). Second, we assume that no costs arise when firms switch between production technologies. In reality, switching production technologies may be associated with additional costs, an aspect that may tame firms' willingness to adapt their production technology. Third, it may also be interesting to assume that manufacturing firms may be able to adjust their production quantity when they observe new information about the commodity price. See Dieci et al. (2022) for a starter in this direction. Fourth, we assume that firms have naïve expectations. Alternatively, one may use the framework by Brock and Hommes (1997, 1998) and assume that firms select between heterogeneous expectation rules. It is important to understand the interplay between supply distortions and commodity price fluctuations. We hope that our paper stimulates more work in this direction.

Appendix A

For ease of exposition, we assume in the main body of our paper that the two production technologies differ with respect to their fixed costs but not with respect to their marginal costs. In this appendix, we show that our main results are robust when we relax this simplifying assumption. In particular, we focus on the scenario $d^F > d^S$ and $c^F > c^S$, i.e. the fast production technology has higher fixed costs and lower marginal costs relative to the slow production technology. In the following, we limit ourselves to Model Specification B. The dynamics of the generalized model is still driven by a five-dimensional nonlinear map. We adjust Assumption B1 as follows.

Assumption B1A: Fixed cost parameters d^F and d^S ensure that the relation $d^F = \frac{1}{4}\left(\frac{3a}{3b+c^F+c^S}\right)^2 (2c^F - c^S) + \frac{1}{2}d^S$ holds.

Note that if $c^F \to c^S$, then assumption B1A implies that $d^F \to \frac{c^S}{4}\left(\frac{3a}{3b+2c^S}\right)^2 + \frac{1}{2}d^S$, as required by Assumption B1. Based on this assumption, the unique steady state results as

$$SSBA = \left(\overline{P}, \overline{X}, \overline{Y}, \overline{W^S}, \overline{W^F}\right) = \left(\frac{3a}{3b+c^F+c^S}, \frac{3a}{3b+c^F+c^S}, \frac{3a}{3b+c^F+c^S}, \frac{1}{3}, \frac{1}{3}\right). \tag{A1}$$

Assumption B2 needs to be modified as follows.

Assumption B2A: Marginal cost parameters c^F and c^S and demand parameter b ensure that $c^F < 3b$ and $c^S < 3b$.

From the characteristic polynomial of the Jacobian matrix of the underlying map at the steady state $SSBA$, i.e. $P(\lambda) = \lambda^2\,(\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3)$, where $a_1 = \frac{1}{2} + \frac{c^F}{3b} + \frac{c^F(c^F - c^S)}{6b}\left(\frac{3a}{3b+c^F+c^S}\right)^2 \beta$, $\quad a_2 = \frac{c^F}{6b} + \frac{c^S}{3b} + \frac{\left(\sqrt{2}c^F - c^S\right)^2 + 2\left(\sqrt{2}c^F c^S - c^F c^S\right)}{6b}\left(\frac{3a}{3b+c^F+c^S}\right)^2 \beta$ and $a_3 = \frac{c^S}{6b} + \frac{(c^F - c^S)}{6b}\left(\frac{3a}{3b+c^F+c^S}\right)^2 \beta$, it follows that two eigenvalues are equal to zero. The remaining three eigenvalues depend on by the term in brackets on the right-hand side of the characteristic polynomial. Accordingly, stability conditions $1 + a_1 + a_2 + a_3 > 0$ and $1 - a_1 + a_2 - a_3 > 0$ are always fulfilled. However, stability condition $1 - a_2 + a_1 a_3 - {a_3}^2 > 0$, associated with the emergence of a Neimark-Sacker bifurcation, requires that

$$\beta < \beta^{NS}_{crit,BA} := \frac{(3b+c^F+c^S)^2(3b-c^S)(12b-2c^F+c^S)}{9a^2\left(c^F(c^F-c^S)\left(9b-(2c^F-c^S)\right)+6bc^Sc^S\right)}. \tag{A2}$$

For $c^F \to c^S$, the above expression simplifies to $\beta < \frac{(3b+2c^S)^2(3b-c^S)(12b-c^S)}{54a^2bc^Sc^S}$. We can thus state the following proposition.

Proposition BA: Under Assumptions B1A and B2A, the unique steady state $SSBA = \left(\bar{P}, \bar{X}, \bar{Y}, \overline{W^S}, \overline{W^F}\right) = \left(\frac{3a}{3b+c^F+c^S}, \frac{3a}{3b+c^F+c^S}, \frac{3a}{3b+c^F+c^S}, \frac{1}{3}, \frac{1}{3}\right)$ *loses its local stability if* $\beta < \frac{(3b+c^F+c^S)^2(3b-c^S)(12b-2c^F+c^S)}{9a^2\left(c^F(c^F-c^S)\left(9b-(2c^F-c^S)\right)+6bc^Sc^S\right)}$ *becomes violated. The stability loss is associated with a Neimark-Sacker bifurcation.*

Figure A1 illustrates the dynamics of the generalized Model Specification B, assuming the base parameter setting $a = 5/3$, $b = 1$, $c^F = 1.1$, $c^S = 0.9$, $d^F = 0.325$, $d^S = 0$ and $\beta = 3.7$. The panels show from top to bottom the evolution of the commodity price, the corresponding market shares of fast (black), slow (white) and manufacturing (gray) firms, respectively, and the commodity price versus parameter β. Note that Assumptions B1A and B2A hold. Since firms' intensity of choice parameter is larger than $\beta^{NS}_{crit,BA} = 3.43$, the commodity price circles around $\bar{P} = 1$. Overall, we may thus conclude that our main results are robust with respect to the simplifying assumption that both production technologies have identical marginal costs.

***** **Figure A1 about here** *****

Appendix B

The switching scheme we use in the main body of our paper relies on the discrete choice approach, as popularized by Brock and Hommes (1997). In this appendix, we employ the exponential replicator dynamics approach, as put forward by Taylor and Jonker (1978), Hofbauer and Sigmund (1988) and Hofbauer and Weibull (1996), and applied by Droste et al. (2002), Dindo and Tuinstra (2011), Bischi et al. (2015), Kopel et al. (2014) and Schmitt et al. (2017). Our attention rests on Model Specification B. We show that our main results are robust when we replace the discrete choice approach with the exponential replicator dynamics approach.

According to the exponential replicator dynamics approach, the market share of firms that select the fast production technology in period $t-1$ so as to offer their supply in period t is determined by

$$W_t^F = (W_{t-1}^F + W_{t-1}^S)\frac{W_{t-1}^F}{W_{t-1}^F + W_{t-1}^S exp[\beta(U_{t-1}^{S,B} - U_{t-1}^{F,B})]}, \quad \text{(B1)}$$

while the market share of firms that opt for the slow production technology in period $t-1$, being equal to the market share of firms in the manufacturing process in period t, is equivalent to

$$W_t^M = (W_{t-1}^F + W_{t-1}^S)\frac{W_{t-1}^S}{W_{t-1}^S + W_{t-1}^F exp[\beta(U_{t-1}^{F,B} - U_{t-1}^{S,B})]}. \quad \text{(B2)}$$

The market share of firms that offer their supply in period t using the slow production technology remains equal to

$$W_t^S = W_{t-1}^M = 1 - W_{t-1}^F - W_{t-1}^S. \quad \text{(B3)}$$

As in the discrete choice approach, parameter $\beta > 0$ controls how sensitive the mass of firms is to selecting the fittest production technology. In contrast to the discrete choice approach, however, the exponential replicator dynamics approach entails a herding component. Due to firms' herding behavior, a very high (low) market share of fast firms in period $t-1$ tends to lead to a high (low) market share of fast firms in period t, even if the fitness of the fast production technology is below (above) the fitness of the slow production technology. In this sense, one may argue that herding behavior introduces a modest level of inertia in the evolutionary dynamics of the underlying model. Another consequence of the herding component is that the exponential replicator dynamics approach may have up to three steady states, namely two border steady states and one inner steady state.[12] In the following, we assume that the inner steady state exists and focus on its properties.

One appealing property of the exponential replicator dynamics approach in our context is that it automatically guarantees that firms are indifferent between the slow and the fast production technology at the inner steady state, i.e. there is no need for introducing an assumption about the relation between the fixed costs of

[12] At the border steady states, we either have $\overline{W^F} = 0$ or $\overline{W^F} = 1$, i.e. all firms either rely on the slow or on the fast production technology. This follows immediately from (B1).

the two production technologies. In fact, we obtain from $\overline{U^{S,B}} = \overline{U^{F,B}}$ that $\overline{P} = \frac{\sqrt{2(2d^F - d^S)}}{\sqrt{c}}$ and, consequently, $\overline{W^S} = 1 + \frac{b}{c} - \frac{a}{\sqrt{2c(2d^F - d^S)}}$ and $\overline{W^F} = \frac{2a}{\sqrt{2c(2d^F - d^S)}} - 1 - \frac{2b}{c}$.[13]

The characteristic polynomial of the Jacobian matrix of the underlying map at the inner steady state is now given by $P(\lambda) = \lambda(\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4)$, where $a_1 = \frac{c\overline{W^F}}{b}$, $a_2 = -1 + \frac{c\overline{W^S}}{b} + \frac{(c\overline{P})^2\overline{W^F}\overline{W^S}\beta}{b(\overline{W^F} + \overline{W^S})}$, $a_3 = -\frac{c\overline{W^F}}{b}$ and $a_4 = -\frac{c\overline{W^S}}{b}$. Since one eigenvalue is always equal to zero, the local stability of the inner steady state depends on the remaining four eigenvalues. Applying the necessary and sufficient conditions derived by Farebrother (1973), we can conclude that the inner steady state loses its local stability when one of the following five inequalities (i) $1 + a_1 + a_2 + a_3 + a_4 > 0$, (ii) $1 - a_1 + a_2 - a_3 + a_4 > 0$, (iii) $a_4 < 1$, (iv) $3 + 3a_4 - a_2 > 0$ and (v) $(1 - a_4)(1 - {a_4}^2) - a_2(1 - a_4)^2 + (a_1 - a_3)(a_3 - a_1a_4) > 0$ becomes violated. Inequalities (i) to (iii) are always fulfilled. Let us again assume that $c < 3b$. Then inequality (v) is more binding than inequality (iv). Solving inequality (v) for parameter β yields

$$\beta < \frac{\overline{W^F} + \overline{W^S}}{c^2\overline{P}^2\left(b + c\overline{W^S}\right)^2\overline{W^F}\overline{W^S}} 2\left(b - c\overline{W^S}\right)\left(b + c\left(\overline{W^S} - \overline{W^F}\right)\right)\left(b + c\left(\overline{W^S} + \overline{W^F}\right)\right). \qquad \text{(B4)}$$

Hence, if the inner steady state is stable, it eventually becomes unstable when firms' intensity of choice increases.

Of course, imposing Assumption B1, i.e. setting $d^F = \frac{c}{4}\left(\frac{3a}{3b+2c}\right)^2 + \frac{1}{2}d^S$, reveals that $\overline{W^F} = \overline{W^S} = \overline{W^M} = \frac{1}{3}$ and, consequently, that $\overline{P} = \frac{3a}{3b+2c}$. In this case, the steady state coordinates of the exponential replicator dynamics approach are equivalent to those of the discrete choice approach. Furthermore, stability condition (B4) simplifies to

$$\beta < \frac{4b(3b-c)(3b+2c)^3}{3a^2c^2(3b+c)^2}. \qquad \text{(B5)}$$

A comparison of stability conditions (B5) and (27) indicates that the exponential replicator dynamics approach necessitates a higher value for the intensity of

[13] Hence, $0 < \overline{W^F} < 1$ requires that $\frac{1}{2}\frac{a^2\,c}{(b+c)^2} < 2d^F - d^S < 2\frac{a^2\,c}{(2b+c)^2}$. If this condition holds, there is an inner steady state.

choice parameter to set endogenous commodity price dynamics in motion than the discrete choice approach. Put differently, the herding component of the exponential replicator dynamics approaches has a stabilizing effect on the dynamics but does not necessarily prevent the emergence of endogenous commodity market dynamics.

Figures B1 and B2 illustrate the dynamics of Model Specification B in connection with the exponential replicator dynamics approach. The simulations depicted in Figure B1 are based on $a = 5/3,\ b = 1,\ c = 1,\ d^F = 0.25,\ d^S = 0$ and $\beta = 8.5$, implying that Assumption B1 holds. Accordingly, $\bar{P} = 1$ and $\overline{W^F} = \overline{W^S} = \overline{W^M} = 1/3$. As predicted by stability condtion (B5), the dynamics becomes unstable when the intensity of choice parameter exceeds 7.5. As discussed in Section 4, we observe a Neimark-Sacker bifurcation for the original Model Specification B when β exceeds 3.66. The simulations reported in Figure B2 are based on $a = 5/3,\ b = 1,\ c = 1,\ d^F = 0.275,\ d^S = 0$ and $\beta = 8.5$, implying that Assumption B1 does not hold. Since the fast production technology entails higher fixed costs, less firms rely on it. As a result, we have that $\bar{P} = 1.048$, $\overline{W^F} = 0.178$ and $\overline{W^S} = \overline{W^M} = 0.411$. In line with stability condition (B4), the steady state becomes unstable when parameter β becomes larger than 8.48. Importantly, however, we may conclude that our main results are robust with respect to the updating scheme of the production technologies' market shares.

******* Figures B1 and B2 about here *******

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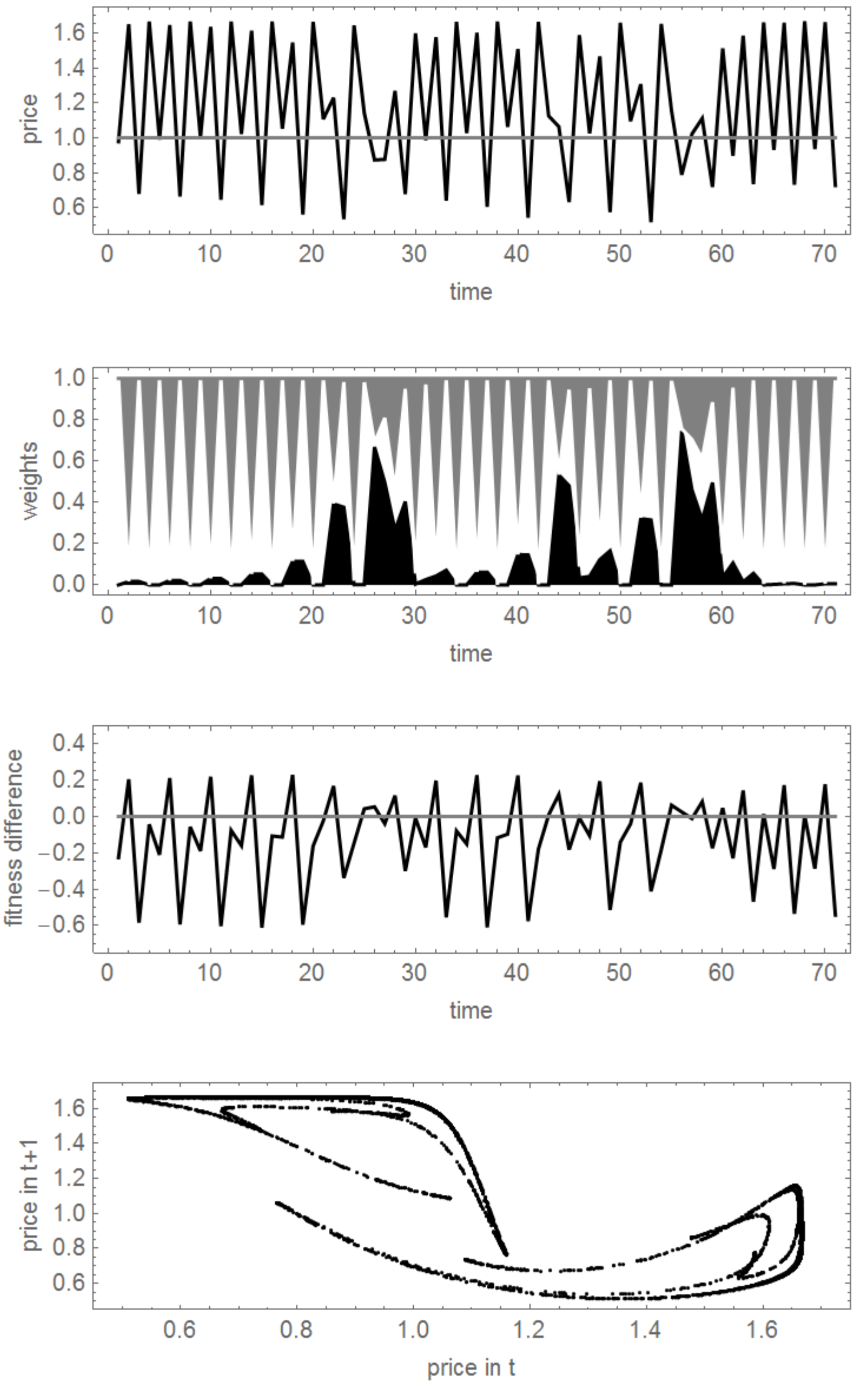

Figure 1: Example of dynamics for Model Specification A. The panels show from top to bottom the evolution of the commodity price, the market shares of fast (black), slow (white) and manufacturing (gray) firms, respectively, the fitness difference between the fast and slow production technology, and the commodity price in period $t+1$ versus the commodity price in period t. A longer transient period has been erased from all simulations. Parameter setting: $a = 5/3$, $b = 1$, $c = 1$, $d^F = 0.25$, $d^S = 0$ and $\beta = 18$.

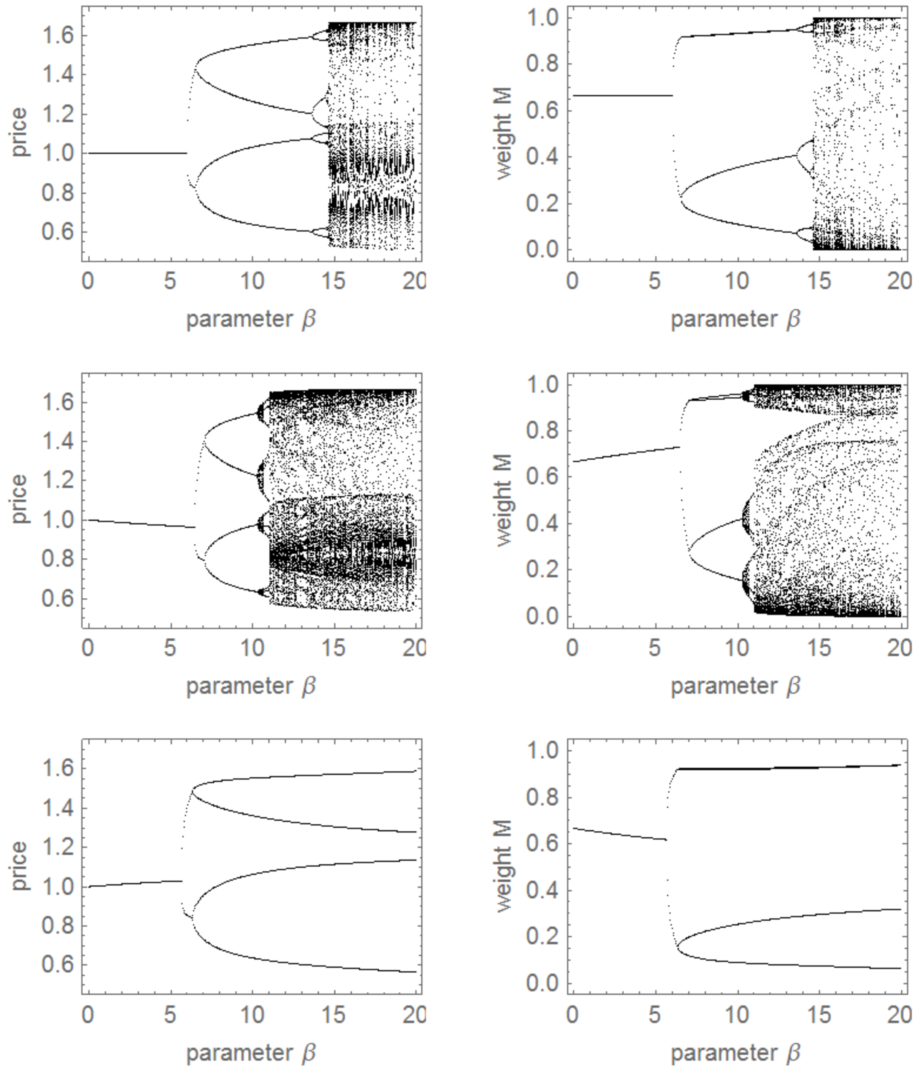

Figure 2: Bifurcation diagrams for Model Specification A. The left (right) panel at the top shows the price (market share of manufacturing firms) versus parameter β, assuming that $a = 5/3$, $b = 1$, $c = 1$, $d^F = 0.25$ and $d^S = 0$. The middle and bottom panels show the same, except that $d^F = 0.15$ and $d^F = 0.35$, respectively.

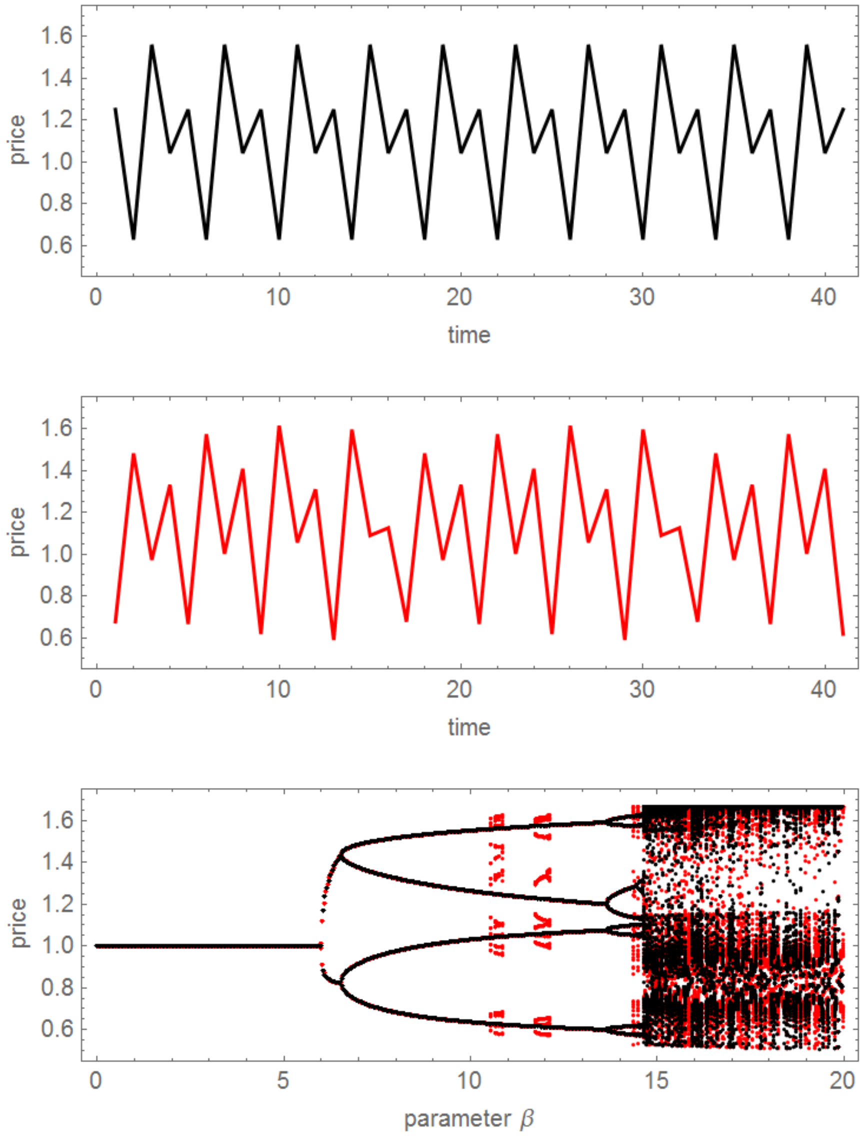

Figure 3: Coexisting attractors and Model Specification A. The simulations of the commodity price in the top and middle panels and the two overlapping bifurcation diagrams in the bottom panel are based on two different sets of initial conditions. Parameter setting: $a = 5/3$, $b = 1$, $c = 1$, $d^F = 0.25$, $d^S = 0$ and $\beta = 10.6$.

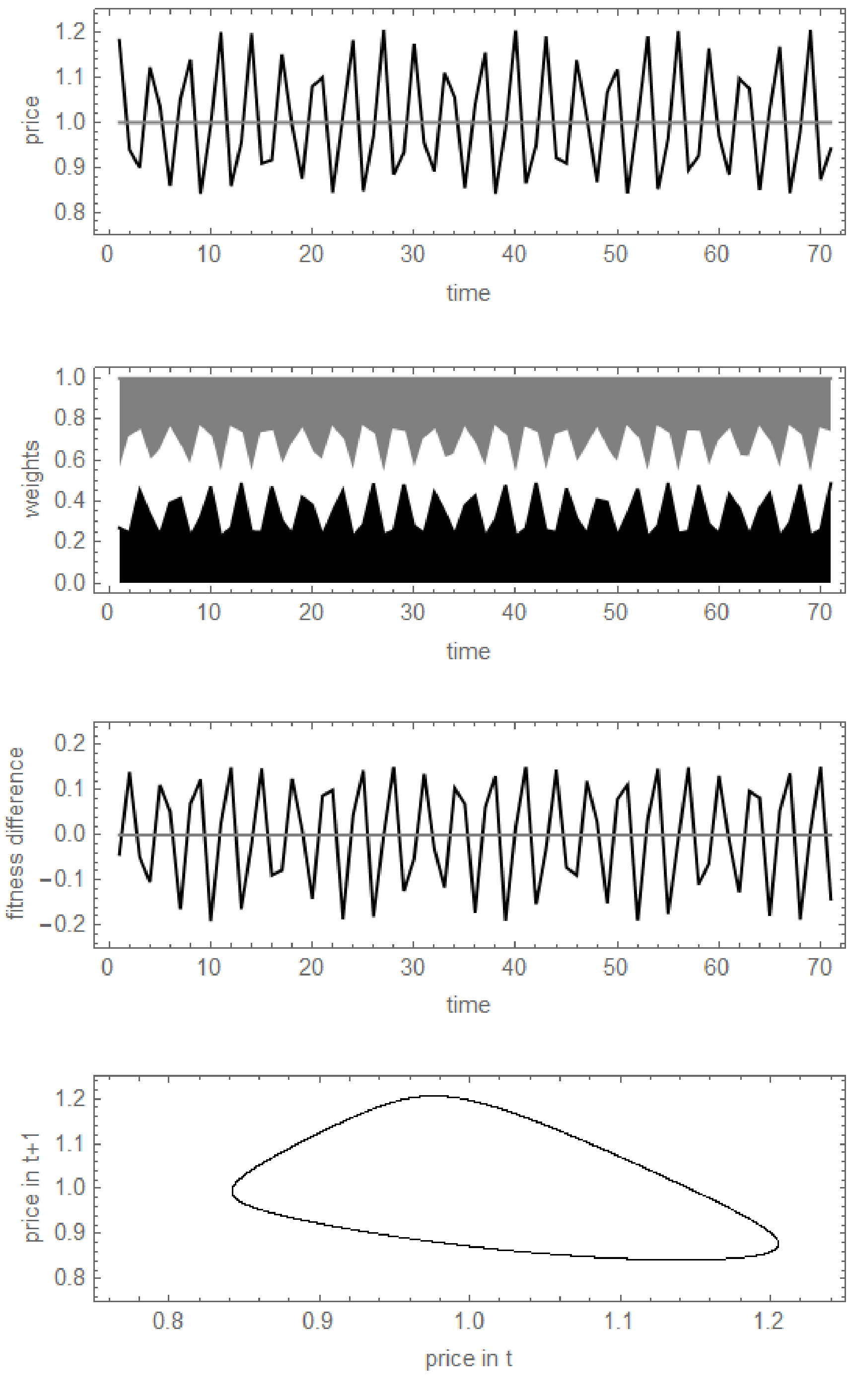

Figure 4: Example of dynamics for Model Specification B. The panels show from top to bottom the evolution of the commodity price, the corresponding market shares of fast (black), slow (white) and manufacturing (gray) firms, respectively, the fitness difference between the fast and slow production technology, and the commodity price in period $t + 1$ versus the commodity price in period t. A longer transient period has been erased from all simulations. Parameter setting: $a = 5/3$, $b = 1$, $c = 1$, $d^F = 0.25$, $d^S = 0$ and $\beta = 4$.

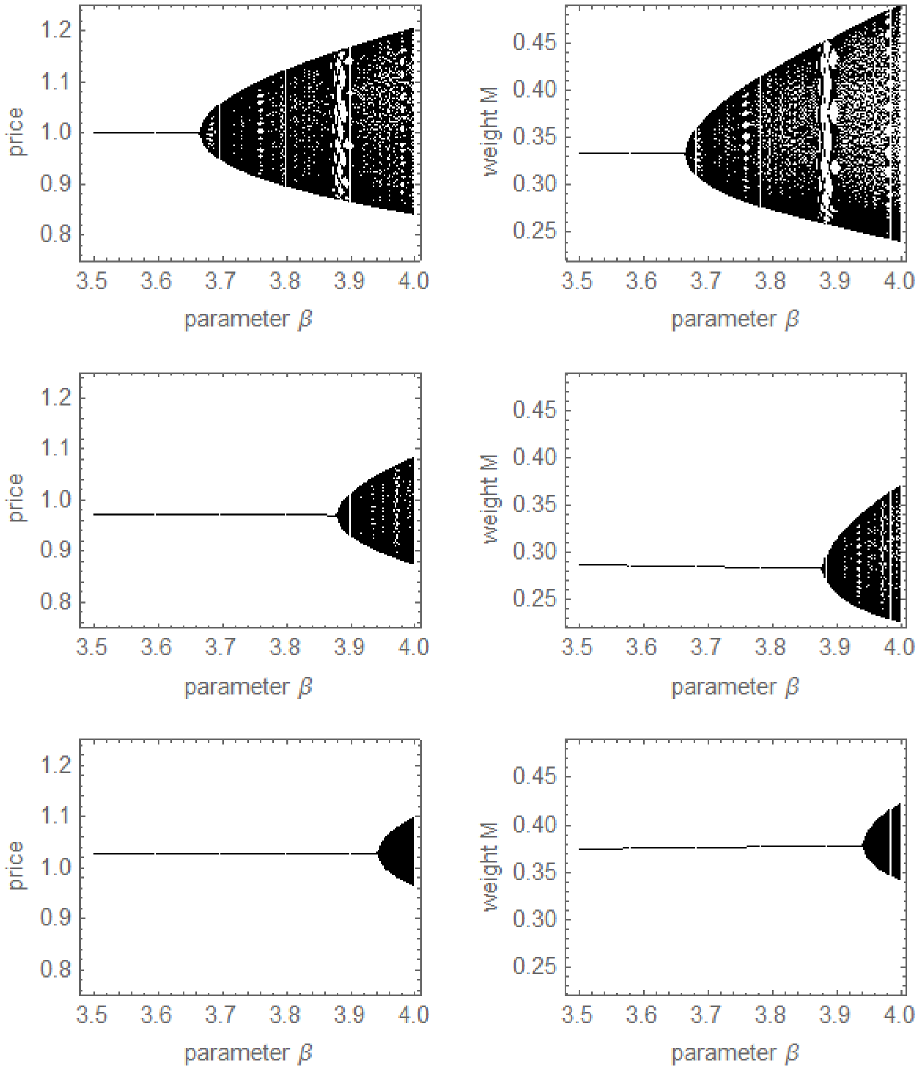

Figure 5: Bifurcation diagrams for Model Specification B. The left (right) panel at the top shows the price (market share of manufacturing firms) versus parameter β, assuming that $a = 5/3$, $b = 1$, $c = 1$, $d^F = 0.25$ and $d^S = 0$. The middle and bottom panels show the same, except that $d^F = 0.18$ and $d^F = 0.32$, respectively.

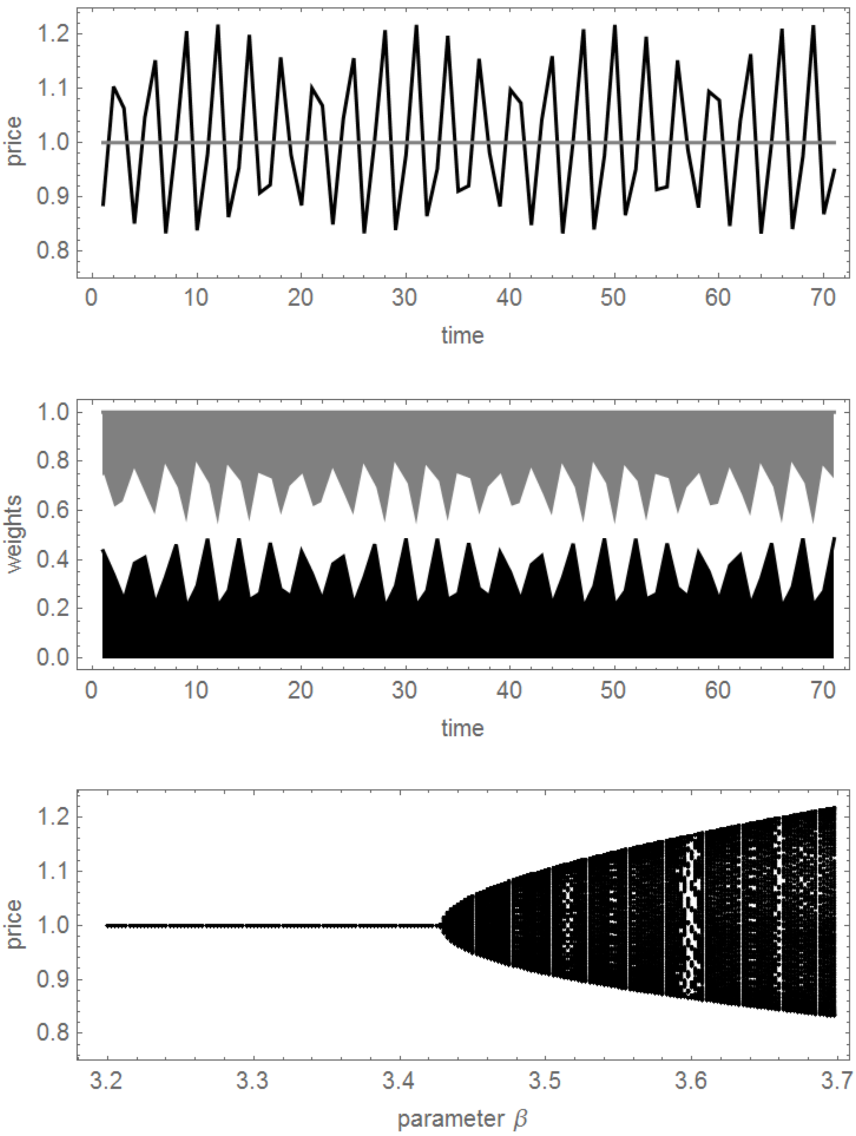

Figure A1: Overview of dynamics of generalized Model Specification B. The panels show from top to bottom the evolution of the commodity price, the corresponding market shares of fast (black), slow (white) and manufacturing (gray) firms, respectively, and the commodity price versus parameter β. Parameter setting: $a = 5/3$, $b = 1$, $c^F = 1.1$, $c^S = 0.9$, $d^F = 0.325$, $d^S = 0$ and $\beta = 3.7$.

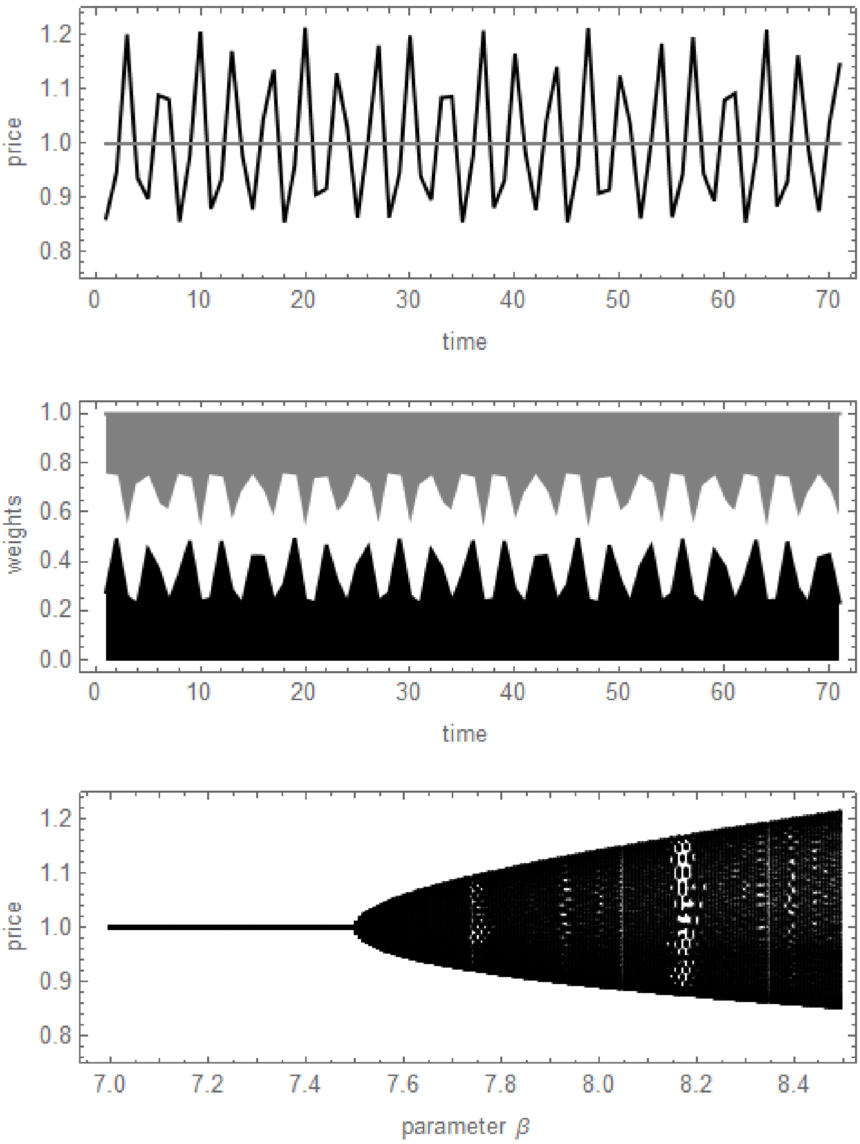

Figure B1: Exponential replicator dynamics approach with Assumption B1. The panels show from top to bottom the evolution of the commodity price, the corresponding market shares of fast (black), slow (white) and manufacturing (gray) firms, respectively, and the commodity price versus parameter β. Parameter setting: $a = 5/3$, $b = 1$, $c = 1$, $d^F = 0.25$, $d^S = 0$ and $\beta = 8.5$.

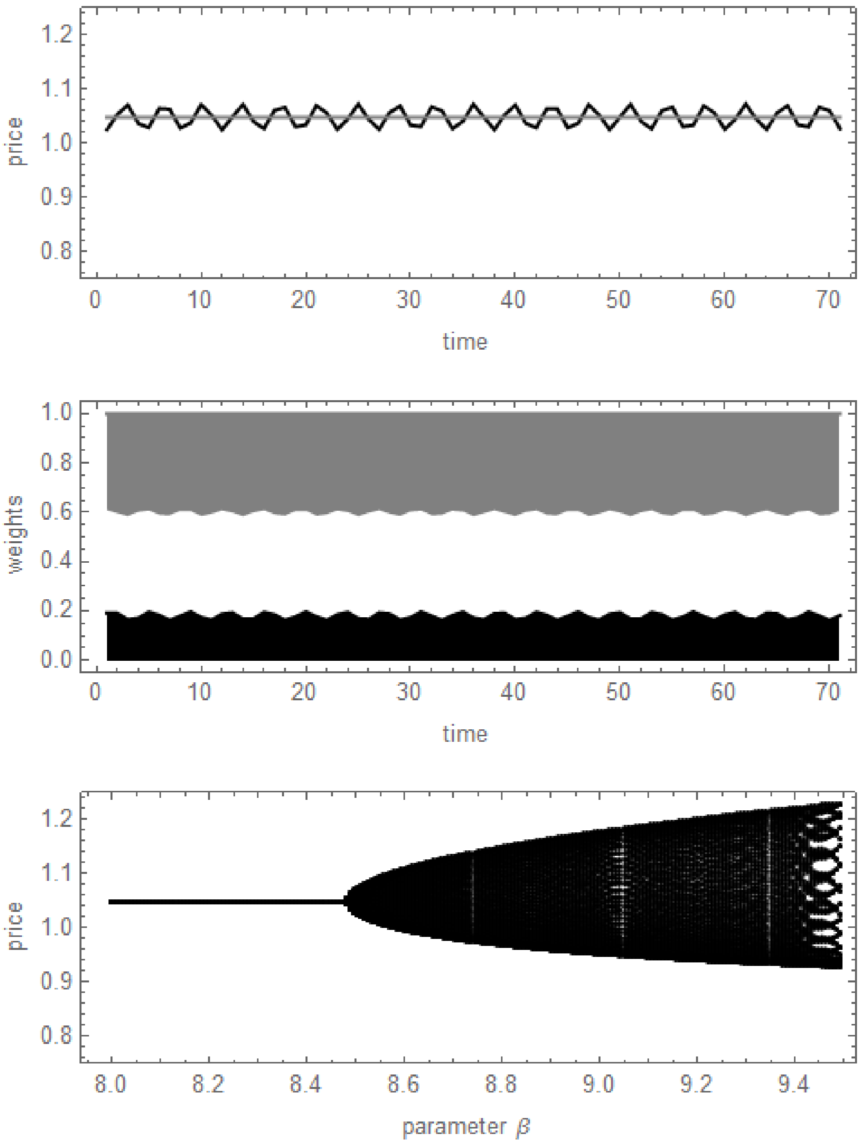

Figure B2: Exponential replicator dynamics approach without Assumption B1. The panels show from top to bottom the evolution of the commodity price, the corresponding market shares of fast (black), slow (white) and manufacturing (gray) firms, respectively, and the commodity price versus parameter β. Parameter setting: $a = 5/3$, $b = 1$, $c = 1$, $d^F = 0.275$, $d^S = 0$ and $\beta = 8.5$.