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Disturbance observer-based nonlinear feedback control for position tracking of electro-hydraulic systems in a finite time

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Abstract

In this paper, a disturbance observer-based backstepping tracking control is designed for an electro-hydraulic actuator (EHA) system to estimate and track reference signals in a finite time. It is assumed that the system is uncertain with unknown upper bounds. Different from the existing ones, the proposed observer can deal with strong uncertainties in which the estimation error converges into an arbitrarily small neighborhood of zero in a finite time. Then, the disturbance observer-based backstepping tracking control is provided to compensate the uncertainties and estimation errors and to guarantee the finite-time tracking of the piston position toward the desired time-varying reference signal. The key idea is to employ a monotonically increasing function associated with the control objective to improve the control performance, where the finite-time boundedness criterion is guaranteed using Lyapunov stability analysis. Finally, the efficacy of the proposed robust scheme for the EHA system with unknown measurement noise is illustrated in numerical simulations as compared to a leading observer-based control strategy in the literature. It is shown that the proposed approach results in more accuracy and faster convergence compared to that technique, which makes it a qualified alternative approach with noteworthy potential.

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Keywords: Backstepping-based control, disturbance observer, electro-hydraulic actuator systems, finite-time stability, time-varying transformation.

1. Introduction

Electro-Hydraulic Actuator (EHA) systems provide a high power-to-weight ratio, high stiffness, and high load efficiency and are widely used in mechatronic systems [1, 2]. Control and tracking have been fundamental challenges in EHAs, where precision tracking performance in a finite time has frequently been required [3, 4]. EHAs include nonlinearities and several uncertainties such as unmodeled dynamics, disturbances, and frictions with dynamics that are not exactly known [5, 6]. Due to these challenges, there have been restrictions in accomplishment of finite-time tracking and high-precision control tasks in EHAs, and hence the concept of finite-time ultimate boundedness (FTUB) for tracking has been considered instead [7, 8]. Different techniques considering robustness have been employed to suppress nonlinearities and uncertainties, where the convergence of the tracking errors to a neighborhood of zero is achieved over a finite time interval [9]. Within the robust control context, feedback linearization is a helpful approach to cope with the problem of nonlinearities by eliminating the undesired nonlinear terms [10, 11]. In addition, to overcome the challenge of the uncertainties, sliding mode control (SMC) approach has been well presented [11, 12], where compensate gains are designed to be larger than the upper bounds of the uncertainties. One of the main drawbacks of these reviewed strategies is that information on uncertainties is assumed to be known and available online, which is not feasible in many practical situations [13, 14]. In real-world settings, the upper bound is partly known or even completely unknown, so the compensate gains must be sufficiently large to suppress the influence of the uncertainties. In practice, to overcome this restriction, an alternative approach is to equip the systems with sensors and design an observation algorithm to process incomplete information collected by the sensors and construct a reliable estimation of the uncertainties [13-17]. In this regard, the disturbance observer-based control strategies have been proposed [18], where controllers are updated according to the estimates obtained by the observers.

Within the disturbance observer-based control context, some schemes have already been presented in [19-23] to handle unknown terms. For instance, SMC-based approaches have been one of the leading methods with widespread applications [19, 20]. However, despite their acceptable efficiency, standard SMC-based approaches can potentially lead to destructive chattering phenomena in the closed-loop response, due to the use of the sign function in those methods. As a result, they cannot be always relied upon in real-world settings [21]. This has been overcome via using high-order SMC strategies to reduce the chattering phenomenon [22, 23], where finite-time convergence can be achieved with a relatively slow convergence rate for the initial conditions far from the equilibrium point. Therefore, it is desirable for control approaches to guarantee the convergence in a finite time interval regardless of initial conditions [24]. Fixed-time approaches using the concept of bi-limit homogeneity and high-gain differentiators were introduced in [25-27] and are more powerful than the finite-time approaches that guarantee the boundedness of the convergence time independently of the initial conditions. However, in addition to structural limitations, the gains are not easily computable. Also, defining a proper Lyapunov function to ensure the stability condition is not straightforward for high-order systems.

So far, despite several studies, the subject of disturbance observer-based finite-time control has not been fully recognized for uncertain real-world systems and it has potential for further improvement. Some of the leading strategies recently presented in this area are summarized below. Time-varying approaches were presented in [28]-[30], where the system's response is only valid in a short time interval, while, in many applications, it is important to have a valid response for a longer duration. Also, the estimation errors have not explicitly been considered in the proof, but they can act as uncertainties in the values of the state variables. To overcome these restrictions, another time-varying technique was introduced in [31, 32], where, by considering the estimation errors, the system's response is valid for a longer duration. However, despite its high efficiency and straightforward design, it can encounter singularity when time goes to infinity. Hence, although that time-varying technique could be a valid scheme for stabilization and estimation, it is applicable only for single-input single-output canonical systems. Another powerful nonlinear tool is the implementation of high-gain observers [11], which can be used for a wide class of uncertain nonlinear systems and guarantees a proper efficiency when the observer gain is sufficiently high. The efficacy of a high-gain disturbance observer combined with an extended state observer (ESO) was highlighted in [33], where ESO was used to estimate the unmeasurable system states and the external matched disturbances, while the mismatched disturbances were estimated by the high-gain disturbance observer. According to that strategy, the disturbance estimation performance depends not only on the physical disturbance in the real system but also the unmeasurable state estimation that is concurrently estimated by the ESO. Since high-gain observers are basically approximate differentiators, their practicality is limited by measurement noise and unmodeled high-frequency dynamics [11]. This approach was successfully developed in [34], where one ESO was employed to estimate various types of uncertainties. Note that the estimation of ESOs in [11] and [34] is essentially based on the Levant differentiator [27] with a highly accurate version of it reported in [35]. According to the studies above, it can be concluded that the output feedback controller with disturbances rejected (OFCDR) in [35] has superior properties such as high tracking accuracy without any singularity problem, finite-time convergence, chattering phenomenon reduction, and robustness against the effects of the uncertainties and external disturbances. That is, in that study, the estimations of the Levant differentiator are shown to be very close to the actual data indicating its high accuracy in estimating the uncertainties and external disturbances. Also, various comparative simulations were provided to analyze the efficiency of the OFCDR method as compared to a proportional-integral-differential (PID) controller that has been widely used in industrial applications, a SMC method with strong robustness, and a robust backstepping controller (RBC) method, and it was shown that OFCDR can provide higher tracking precision and less chattering in control input than the other techniques studied in that work. Therefore, with the exception of the leading strategy proposed in [35], the subject of finite-time observer-based tracking control still remains an open problem for further improvement in EHA systems.

Inspired by the above considerations, in this paper, a disturbance observer-based backstepping tracking control is designed for a class of EHA systems in a simple and straightforward manner. For this purpose, first, in order to successfully achieve the position and velocity tracking in a finite time, the state-space model of the EHA system is partially transformed into a time-varying form, associated with the control objective. Then, assuming the upper bounds of uncertainties are known, a control law is designed using the backstepping technique to guarantee the finite-time tracking. Next, for realistic analysis, this assumption is relaxed by designing a finite-time disturbance observer based tracking control (FT-DOTC), where the uncertainties and disturbances are estimated using the proposed finite-time disturbance observer (FTDO). Distinct from the existing observers, this observer can deal with strong uncertainties with unknown upper bounds, where the estimation error converges into an arbitrarily small neighborhood around zero in finite time with the size of that neighborhood being adjustable through tuning the system parameters. Finally, the FT-DOTC law is designed to compensate the uncertainties and estimation errors and guarantee the finite-time tracking of the piston position and velocity toward the desired time-varying reference signals. To highlight the efficacy of the proposed framework, several comparative simulations are reported between the proposed scheme and the OFCDR method introduced in [35]. The main contributions of this paper follow: a) To improve the tracking control performance, a positive time-varying increasing function associated with the control objective is used, where the stabilization with finite-time boundedness properties is guaranteed in straightforward manners. b) A novel adaptive disturbance observer is proposed to finite time estimation of uncertainties. c) A reduced-order observer represents an improvement on previous works of the authors to estimate the full states of system in a finite time that its time-varying gains computed based on straightforward algebraic equations. d) Finally, without any knowledge about the upper bound of uncertainties, a continuous and chattering-free FT-DOTC law is designed, which makes it a qualified alternative approach with noteworthy potential.

The remainder of this paper is organized as follows. In Section 2, a class of EHA systems that consist of a double-rod hydraulic cylinder and a proportional valve is introduced. Then, a preliminary of the finite-time boundedness concepts is presented. In Section 3, the FT-DOTC law is designed to guarantee finite-time tracking in the presence of different unknown terms. Furthermore, a guideline for tuning parameters is presented based on the trade-off between the convergence time and the control effort. In Section 4, simulation results are provided to show the advantages of the proposed scheme compared to the literature. Finally, concluding remarks are presented in Section 5.

2. Preliminaries and system descriptions

In EHA systems that mostly consist of a double-rod hydraulic cylinder and a proportional valve (see Figure 1), the exact mathematical model is required for a realistic analysis.

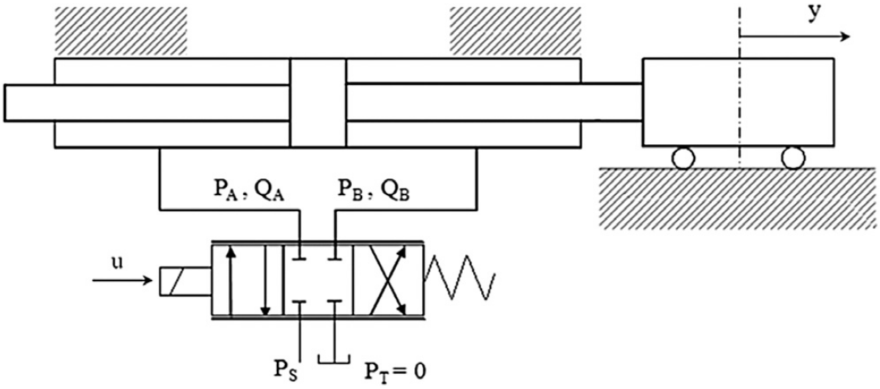

Figure 1: Graphical representation of the hydraulic actuation system [16].

In the following, mathematical model of a class of electro-hydraulic actuation system is considered [16].

$$\begin{aligned} \ddot{y} &= -\frac{b}{m}\dot{y} - \frac{F_f(\dot{y})}{m} + \frac{A_P P_L}{m} \\ \dot{P}_L &= -\frac{2\rho A_P}{V_0}\dot{y} + \frac{2\rho\Psi(v_e)\sqrt{P_S - |P_L|}}{V_0} \\ \ddot{v}_e &= -\omega_{nv}^2 v_e - 2\epsilon_v \omega_{nv}\dot{v}_e + \omega_{nv}^2(k_e u + v_{e0}) \end{aligned} \tag{1}$$

where y denotes the position of the piston, $P_L = P_A - P_B$ is the pressure due to the external load and supply, where according to Figure 1, P_A and P_B (Q_A and Q_B) are the pressures (the flows) inside the two-cylinder chambers, v_e is the spool valve displacement signal, u is the valve command, b is the viscous friction coefficient, m is the mass of the load, $F_f(\dot{y})$ is the friction force, A_P is the piston area, ρ is the effective bulk modulus, V_0 is the volume of each chamber for the piston centered position, $\Psi(v_e)$ is a gain that depends on the geometry of the adopted proportional valve, P_s and P_T are the supply and tank pressures, respectively, ω_{nv} and ϵ_v are the natural frequency and the damping ratio of the valve, respectively, k_e is the input gain, and v_{e0} represents the spool position bias. In (1), the time variable t is omitted for convenience. Note that the top, middle, and bottom equations in (1) express the piston rod dynamics, the load pressure dynamics, and the proportional valve dynamics, respectively.

In this paper, it is desired to design the valve command u required for the piston's position y and its velocity $\dot{y}$ to track the bounded time-varying reference signals $y_d(t)$ and $\dot{y}_d(t)$, respectively. Let $x = [x_1, x_2, x_3, x_4, x_5]^T$ be the state vector with $x_1 = y$, $x_2 = \dot{y}$, $x_3 = P_L$, $x_4 = v_e$, and $x_5 = \dot{v}_e$. Defining the tracking errors $\bar{e}_1 = x_1 - y_d(t)$ and $\bar{e}_2 = x_2 - \dot{y}_d(t)$, the hydraulic actuation system dynamics in (1) can be rewritten as

$$\begin{bmatrix} \dot{\bar{e}}_1 \\ \dot{\bar{e}}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} \bar{e}_2 \\ -\frac{b}{m}\bar{e}_2 + \frac{A_P}{m}x_3 - \frac{b}{m}\dot{y}_d - \ddot{y}_d \\ -\frac{2\rho A_P}{V_0}\bar{e}_2 + \frac{2\rho k_q\sqrt{P_s - |x_3|}}{V_0}x_4 - \frac{2\rho A_P}{V_0}\dot{y}_d \\ x_5 \\ -\omega_{nv}^2 x_4 - 2\epsilon_v\omega_{nv}x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_e\omega_{nv}^2 \end{bmatrix} u$$

$$+ \underbrace{\begin{bmatrix} 0 \\ -\frac{F_f(\dot{y})}{m} + \eta_y(t) \\ \frac{2\rho\left(\Psi(x_4) - k_q x_4\right)\sqrt{P_s - |x_3|}}{V_0} + \eta_{P_L}(t) \\ 0 \\ \omega_{nv}^2 v_{e0} + \eta_{v_e}(t) \end{bmatrix}}_{\Delta(t,x)} \tag{2}$$

where k_q is the nominal valve constant and $\Delta(t,x) = \overline{\Delta}(t,x) + \eta(t)$ represents the summation of uncertainties $\overline{\Delta}(t,x)$ and disturbances $\eta(t) = \left[0, \eta_y, \eta_{P_L}, 0, \eta_{v_e}\right]^T$. The valve command u should be designed in a way that $\bar{e}_1$ and $\bar{e}_2$ converge to zero in a finite time. However, due to the uncertainties and external disturbances, the exact convergence to zero in finite time is impossible. Hence, the FTUB concept is considered instead. In this regard, the following fundamental definitions and remark are used.

Definition 1 [32]. The time-varying system $\dot{z} = \chi(t, z, \delta)$ is said to be bounded in a finite time T with respect to δ where $\delta^T \delta \leq \sigma$, if for some positive-definite (PD) matrix function R and any positive constants a, b, and σ, where $0 \leq a \leq b$, the condition $z^T R z < b$ is satisfied for $t \in [0, T]$ whenever $z_0{}^T R z_0 \leq a$.

Definition 2 [32]. The system $\dot{z} = \chi(t, z, \delta)$ is said to be finite-time input-to-state stable (FT-ISS) in a finite time T with respect to δ where $\|\delta\| \leq \sigma$, if the inequality

$$\|z\| \leq \alpha(\|z_0\|, \bar{\vartheta}) + \beta(\|\delta\|) \tag{3}$$

is guaranteed for any $t \geq t_0 + T$, where $\alpha(\cdot)$ and $\beta(\cdot)$ are KL-class and K-class functions, respectively, $\bar{\vartheta}$ is a time-varying function that approaches infinity as $t \to t_0 + T$. Note that, in the absence of the disturbance δ, a FT-ISS system becomes a finite-time stable (FTS) system [36].

Remark 1. Consider the change of coordinates $\Sigma_1 \to \Sigma_2$ in form of $\Sigma_2 = C\Sigma_1$, where C is defined as a monotonically increasing function diverging asymptotically to infinity as $t \to t_0 + T$. Then, if the variable Σ_2 remains stable, which does not tend to infinity, the FTUB of the first coordinate Σ_1 as $t \to t_0 + T$ is guaranteed.

3. Finite-time tracking control design

In this section, first, the constructed dynamic (2) are partially transformed into a time-varying form in which the finite-time converge of $\bar{e}_1$ and $\bar{e}_2$ is guaranteed using Lyapunov stability analysis. Then, based on presumably known upper bound of $\Delta(t, x)$, a control law is designed using the backstepping technique to guarantee the finite-time tracking of the piston position toward the desired time-varying reference signal. Next, in order to make the assumption about $\Delta(t, x)$ more realistic for practical situations, the backstepping-based feedback control law is modified to design a disturbance observer-based state feedback control law, where the uncertainties and disturbances are estimated in a finite time.

Step 1 *(Transformation into a time-varying form)*: In order to achieve a successful tracking associated with the control objectives in the paper, P_L, v_e, and $\dot{v}_e$ must be bounded, and $\bar{e}_1$ and $\bar{e}_2$ must converge to a neighborhood of zero in a finite time. For this purpose, the dynamics in (2) are partially transformed into a time-varying form using the transformation $E_i = \mu(t,\mathscr{t})\bar{e}_i$ $(i = 1$ and $2)$, where the monotonically increasing function

$$\mu(t,\mathscr{t}) = \frac{1+e^{\frac{-t}{\mathscr{t}}}}{2e^{\frac{-t}{\mathscr{t}}}} \tag{4}$$

is defined as a time-varying function. For simplicity in the following notation, the arguments time t and the design constant $\mathscr{t}$ are omitted so that $\mu(t,\mathscr{t})$ is represented as μ. The design constant $\mathscr{t}$ is a positive parameter and is selected later for finite-time convergence. After some straightforward derivations, the time derivatives of E_i, *i.e.* $\dot{E}_i = \dot{\mu}\bar{e}_i + \mu\dot{\bar{e}}_i$, can be expressed as

$$\begin{aligned} \dot{E}_1 &= \frac{\dot{\mu}}{\mu}E_1 + E_2 \\ \dot{E}_2 &= \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m}\right)E_2 + \frac{A_P}{m}\mu x_3 - \mu\left[\ddot{y}_d + \frac{b}{m}\dot{y}_d\right] + \mu\Delta_2 \end{aligned} \tag{5}$$

where $\Delta_2 = -\frac{F_f(\dot{y})}{m} + \eta_y(t)$. According to Remark 1, if E_i dynamics become stable (not necessarily in a finite time), then the FTUB of $\bar{e}_i$ $(i = 1$ and $2)$ is guaranteed.

Step 2 *(Finite-time backstepping-based feedback control law)*: In this section, first, based on presumably known upper bounds of $\Delta(t,x)$'s, a feedback control law is designed to guarantee the finite-time tracking of the piston's position. For this purpose, a hierarchical procedure based on the backstepping approach is conducted. We start with the special case of (5), *i.e.*

$$\dot{E}_1 = \frac{\dot{\mu}}{\mu} E_1 + E_2 \tag{6}$$

which will be generalized later. Based on the expression for μ given in (4), it can be seen that $\frac{\dot{\mu}}{\mu} = \frac{1}{2t}(2 - \mu^{-1}) > 0$, where $\frac{1}{2t} < \frac{\dot{\mu}}{\mu} \leq \frac{1}{t}$. Therefore, the dynamics of E_1 in (6) are not stable when $E_2 = 0$. Hence, based on the backstepping approach [11], the dynamics of E_1 can be stabilized by setting $E_{2d} = -a_1 E_1 \triangleq \Phi_1(E_1)$, where E_{2d} denotes the designed visual input E_2, $a_1 > \frac{1}{t}$. This can be shown by selecting a Lyapunov function of the form $V_1(E_1) = 0.5\, E_1^2$ with the time derivative $\dot{V}_1 = \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2$, where $\frac{\dot{\mu}}{\mu} - a_1 < 0$. That is, the stability of E_1 and, consequently, the FTUB of $\bar{e}_1$ is guaranteed.

Next, using the backstepping approach, the change of variable $\tilde{E}_2 = E_2 - \Phi_1(E_1)$ is applied to rewrite the transformed dynamics in (5) as

$$\begin{gathered}\dot{E}_1 = \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1 + \tilde{E}_2 \\ \dot{\tilde{E}}_2 = \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m}\right) \tilde{E}_2 + \frac{A_P}{m} \mu x_3 - \mu \left[\ddot{y}_d + \frac{b}{m} \dot{y}_d\right] + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m}\right) \Phi_1 - \dot{\Phi}_1 + \mu \Delta_2\end{gathered} \tag{7}$$

where $\dot{\Phi}_1 = \frac{\partial \Phi_1}{\partial t} + \frac{\partial \Phi_1}{\partial E_1} \dot{E}_1$. It is desired to guarantee the stability of the dynamics in (7) in the presence of Δ_2, where x_3 is considered as the input. If x_3 is designed as

$$x_{3d} = \left(\frac{A_P}{m} \mu\right)^{-1} \left(-E_1 - a_2 \tilde{E}_2 + \mu \left[\ddot{y}_d + \frac{b}{m} \dot{y}_d\right] - \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m}\right) \Phi_1 + \dot{\Phi}_1 - \mu k_2 \operatorname{sgn}(\tilde{E}_2)\right) \triangleq \Phi_2(E_1, \tilde{E}_2) \tag{8}$$

where x_{3d} denotes the designed visual input x_3, $a_2 > \frac{1}{t} - \frac{b}{m}$ and $k_2 > |\Delta_2|$ is the only information needed to be known about Δ_2, then, stability can be shown using the Lyapunov approach. For this purpose, a Lyapunov function of the form $V_2(E_1, \tilde{E}_2) = V_1(E_1) + 0.5\, \tilde{E}_2^2$ with $\dot{V}_2 = \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 + \mu \tilde{E}_2 \Delta_2 - \mu k_2 |\tilde{E}_2|$ is selected. Since $\frac{\dot{\mu}}{\mu} - a_1 < 0$, $\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2 < 0$, and $k_2 > |\Delta_2|$, it can be seen that $\dot{V}_2 <$

$\left(\frac{\dot{\mu}}{\mu}-a_1\right)E_1^2+\left(\frac{\dot{\mu}}{\mu}-\frac{b}{m}-a_2\right)\tilde{E}_2^2<0$. Therefore, asymptotic stability of E_1, $\tilde{E}_2$, and E_2, and, consequently, the FTUB of $\bar{e}_1$ and $\bar{e}_2$ are guaranteed. In order to backstep, the change of variable $\tilde{x}_3=x_3-\Phi_2(E_1,\tilde{E}_2)$ is applied to rewrite the first three equations in (2) and (5) as

$$\dot{E}_1=\left(\frac{\dot{\mu}}{\mu}-a_1\right)E_1+\tilde{E}_2$$
$$\dot{\tilde{E}}_2=-E_1+\left(\frac{\dot{\mu}}{\mu}-\frac{b}{m}-a_2\right)\tilde{E}_2+\frac{A_P}{m}\tilde{x}_3-\mu\left(k_2\,\mathrm{sgn}(\tilde{E}_2)-\Delta_2\right) \tag{9}$$
$$\dot{\tilde{x}}_3=-\frac{2\rho A_P}{\mu V_0}\left(\tilde{E}_2+\Phi_1\right)-\frac{2\rho A_P}{V_0}\dot{y}_d-\dot{\Phi}_2+\frac{2\rho k_q\sqrt{P_s-|\tilde{x}_3+\Phi_2|}}{V_0}x_4+\Delta_3$$

where $\dot{\Phi}_2=\frac{\partial\Phi_2}{\partial t}+\frac{\partial\Phi_2}{\partial E_1}\dot{E}_1+\frac{\partial\Phi_2}{\partial\tilde{E}_2}\dot{\tilde{E}}_2$. To guarantee the stability of dynamics in (9) in the presence of $\Delta_3=\frac{2\rho\left(\Psi(x_4)-k_qx_4\right)\sqrt{P_s-|x_3|}}{V_0}+\eta_{P_L}(t)$, the input x_4 is designed as

$$x_{4d}=\left(\frac{2\rho k_q\sqrt{P_s-|\tilde{x}_3+\Phi_2|}}{V_0}\right)^{-1}\left(-\frac{A_P}{m}\tilde{E}_2+\frac{2\rho A_P}{V_0}\left(\tilde{E}_2+\Phi_1\right)+\frac{2\rho A_P}{V_0}\dot{y}_d+\dot{\Phi}_2-a_3\tilde{x}_3-k_3\,\mathrm{sgn}(\tilde{x}_3)\right)\triangleq\Phi_3(E_1,\tilde{E}_2,\tilde{x}_3) \tag{10}$$

where x_{4d} denotes the designed visual input x_4, $a_3>0$ and $k_3>|\Delta_3|$ is the only information required to be known about Δ_3. Then, a Lyapunov function can be selected in the form $V_3(E_1,\tilde{E}_2,\tilde{x}_3)=V_2(E_1,\tilde{E}_2)+0.5\,\tilde{x}_3^2$ satisfying $\dot{V}_3<\left(\frac{\dot{\mu}}{\mu}-a_1\right)E_1^2+\left(\frac{\dot{\mu}}{\mu}-\frac{b}{m}-a_2\right)\tilde{E}_2^2-a_3\tilde{x}_3^2<0$. Therefore, asymptotic stability of E_1, $\tilde{E}_2$, E_2, $\tilde{x}_3$, and x_3, and, consequently, the FTUB of $\bar{e}_1$ and $\bar{e}_2$ are guaranteed. Now, by applying the change of variable $\tilde{x}_4=x_4-\Phi_3(E_1,\tilde{E}_2,\tilde{x}_3)$, the first four equations in (2) and (5) can be rewritten as

$$
\begin{aligned}
\dot{E}_1 &= \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1 + \tilde{E}_2 \\
\dot{\tilde{E}}_2 &= -E_1 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2 + \frac{A_P}{m} \tilde{x}_3 - \mu\left(k_2 \operatorname{sgn}\left(\tilde{E}_2\right) - \Delta_2\right) \\
\dot{\tilde{x}}_3 &= -\frac{A_P}{m} \tilde{E}_2 - a_3 \tilde{x}_3 + \frac{2\rho k_q \sqrt{P_S - |\tilde{x}_3 + \Phi_2|}}{V_0} \tilde{x}_4 - k_3 \operatorname{sgn}(\tilde{x}_3) + \Delta_3 \\
\dot{\tilde{x}}_4 &= -\dot{\Phi}_3 + x_5
\end{aligned}
\tag{11}
$$

where $\dot{\Phi}_3 = \frac{\partial \Phi_3}{\partial \mathrm{t}} + \frac{\partial \Phi_3}{\partial E_1} \dot{E}_1 + \frac{\partial \Phi_3}{\partial \tilde{E}_2} \dot{\tilde{E}}_2 + \frac{\partial \Phi_3}{\partial \tilde{x}_3} \dot{\tilde{x}}_3$. To guarantee the stability of the dynamics in (11), the input x_5 is selected in the form of

$$
x_{5d} = \dot{\Phi}_3 - \frac{2\rho k_q \sqrt{P_S - |\tilde{x}_3 + \Phi_2|}}{V_0} \tilde{x}_3 - a_4 \tilde{x}_4 \triangleq \Phi_4\left(E_1, \tilde{E}_2, \tilde{x}_3, \tilde{x}_4\right) \tag{12}
$$

where x_{5d} denotes the designed visual input x_5, $a_4 > 0$. The Lyapunov function is selected as $V_4\left(E_1, \tilde{E}_2, \tilde{x}_3, \tilde{x}_4\right) = V_3\left(E_1, \tilde{E}_2, \tilde{x}_3\right) + 0.5\, \tilde{x}_4^2$ with $\dot{V}_4 < \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3 \tilde{x}_3^2 - a_4 \tilde{x}_4^2 < 0$. This guarantees asymptotic stability of E_1, $\tilde{E}_2$, E_2, $\tilde{x}_3$, x_3, $\tilde{x}_4$, and x_4, and, consequently, the FTUB of $\bar{e}_1$ and $\bar{e}_2$. Finally, by applying the final change of variable $\tilde{x}_5 = x_5 - \Phi_4\left(E_1, \tilde{E}_2, \tilde{x}_3, \tilde{x}_4\right)$, dynamics in (2) and (5) can be rewritten as

$$
\begin{aligned}
\dot{E}_1 &= \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1 + \tilde{E}_2 \\
\dot{\tilde{E}}_2 &= -E_1 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2 + \frac{A_P}{m} \tilde{x}_3 - \mu\left(k_2 \operatorname{sgn}\left(\tilde{E}_2\right) - \Delta_2\right) \\
\dot{\tilde{x}}_3 &= -\frac{A_P}{m} \tilde{E}_2 - a_3 \tilde{x}_3 + \frac{2\rho k_q \sqrt{P_S - |\tilde{x}_3 + \Phi_2|}}{V_0} \tilde{x}_4 - k_3 \operatorname{sgn}(\tilde{x}_3) + \Delta_3 \\
\dot{\tilde{x}}_4 &= -\frac{2\rho k_q \sqrt{P_S - |\tilde{x}_3 + \Phi_2|}}{V_0} \tilde{x}_3 - a_4 \tilde{x}_4 + \tilde{x}_5 \\
\dot{\tilde{x}}_5 &= -\omega_{nv}^2 (\tilde{x}_4 + \Phi_3) - 2\epsilon_v \omega_{nv} (\tilde{x}_5 + \Phi_4) - \dot{\Phi}_4 + k_e \omega_{nv}^2 u + \Delta_5
\end{aligned}
\tag{13}
$$

where $\dot{\Phi}_4 = \frac{\partial \Phi_4}{\partial \mathrm{t}} + \frac{\partial \Phi_4}{\partial E_1} \dot{E}_1 + \frac{\partial \Phi_4}{\partial \tilde{E}_2} \dot{\tilde{E}}_2 + \frac{\partial \Phi_4}{\partial \tilde{x}_3} \dot{\tilde{x}}_3 + \frac{\partial \Phi_4}{\partial \tilde{x}_4} \dot{\tilde{x}}_4$. It can be shown that the dynamics in (13) are asymptotically stable in the presence of $\Delta_5 = \omega_{nv}^2 v_{e0} + \eta_{v_e}(t)$ when the valve command u in that equation is selected as the state feedback control of the form

$$u = (k_e\omega_{nv}^2)^{-1}\Big(-\tilde{x}_4 + \omega_{nv}^2(\tilde{x}_4 + \Phi_3) + 2\epsilon_v\omega_{nv}(\tilde{x}_5 + \Phi_4) + \dot{\Phi}_4 - a_5\tilde{x}_5 - k_5\,\mathrm{sgn}(\tilde{x}_5)\Big), \tag{14}$$

where $a_5 > 0$ and $k_5 > |\Delta_5|$ is the only information needed to be known about Δ_5. Note that k_i ($i = 2$, 3, and 5) is not required to be small and bounded, but it needs to be known. By selecting a Lyapunov function of the form $V_5(E_1, \tilde{E}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5) = V_4(E_1, \tilde{E}_2, \tilde{x}_3, \tilde{x}_4) + 0.5\,\tilde{x}_5^2$, it can be shown that $\dot{V}_5 < \left(\frac{\dot{\mu}}{\mu} - a_1\right)E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right)\tilde{E}_2^2 - a_3\tilde{x}_3^2 - a_4\tilde{x}_4^2 - a_5\tilde{x}_5^2 < 0$. Therefore, asymptotic stability of E_1, $\tilde{E}_2$, E_2, $\tilde{x}_3, x_3$, $\tilde{x}_4$, x_4, $\tilde{x}_5$, x_5, and, consequently, the FTUB of $\bar{e}_1$ and $\bar{e}_2$ are guaranteed. The discussion above is summarized in the following lemma.

Lemma 1: Consider the system given by (2), where the uncertainties Δ_i ($i = 2$, 3, and 5) satisfy inequalities $|\Delta_2| < k_2$, $|\Delta_3| < k_3$, and $|\Delta_5| < k_5$. If the state feedback control law u is designed as in (14) with k_i sufficiently large and the positive constant gains a_1 and a_2 satisfying $a_1 > \frac{1}{t}$ and $a_2 > \frac{1}{t} - \frac{b}{m}$, then all states of the closed-loop system constructed by (2) and (14) are bounded. Furthermore, the states y and $\dot{y}$ track the time-varying reference signals y_d and $\dot{y}_d$, respectively, in a finite time.

Proof: Inspired by Remark 1, since the finite-time stability of system Σ_1 is equivalent to the Lyapunov stability of system Σ_2, based on the step 1, system (2) is written as follows:

$$\begin{bmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} \frac{\dot{\mu}}{\mu}E_1 + E_2 \\ \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m}\right)E_2 + \frac{A_P}{m}x_3 - \mu\left[\ddot{y}_d + \frac{b}{m}\dot{y}_d\right] \\ -\frac{2\rho A_P}{\mu V_0}E_2 + \frac{2\rho k_q\sqrt{P_s - |x_3|}}{V_0}x_4 - \frac{2\rho A_P}{V_0}\dot{y}_d \\ x_5 \\ -\omega_{nv}^2 x_4 - 2\epsilon_v\omega_{nv}x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_e\omega_{nv}^2 \end{bmatrix} u + \begin{bmatrix} 0 \\ \mu\Delta_2 \\ \Delta_3 \\ 0 \\ \Delta_5 \end{bmatrix} \tag{15}$$

The continuation of the proof is a direct consequence of the above formulations, where by selecting the Lyapunov's sequence as V_1, V_2, V_3, V_4, and V_5, respectively, the results of $\dot{V}_1 < 0$, $\dot{V}_2 < \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 < 0$, $\dot{V}_3 < \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3 \tilde{x}_3^2 < 0$, $\dot{V}_4 < \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3 \tilde{x}_3^2 - a_4 \tilde{x}_4^2 < 0$, and $\dot{V}_5 < \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3 \tilde{x}_3^2 - a_4 \tilde{x}_4^2 - a_5 \tilde{x}_5^2 < 0$ are obtained, respectively. Therefore, asymptotic stability of E_1, E_2, x_3, x_4, x_5, and, consequently, the FTUB of $\bar{e}_1$ and $\bar{e}_2$ are guaranteed, where the states y and $\dot{y}$ track the time-varying reference signals y_d and $\dot{y}_d$, respectively, in a finite time. This completes the proof. □

Since the upper-boundedness of the uncertainties is not always a reasonable assumption in real world, for a realistic analysis, that assumption is relaxed in Step 3 below by proposing a disturbance observer-based control scheme.

Step 3 (*Extension to observer-based control)*: The disturbance observer is proposed as

$$\hat{\Delta}_i = -\int_0^t \alpha_i \operatorname{sgn}\left(\dot{e}_i + \beta e_i^{q/p}\right) dt - \beta e_i^{q/p} \tag{16}$$

where the adaptive gains $\alpha_i(.)$ $(i = 2, 3, \text{and } 5)$ are class K functions to be designed, such that the convergence of the estimated uncertainties Δ_i to small neighborhoods of their corresponding actual values is guaranteed in a finite time. In (16), $\beta > 0$ is a design constant, $p > 0$ and $q > 0$ are odd integers satisfying the condition $p > q$. Also, in that equation, the observation errors are defined as $e_i = \hat{x}_i - x_i$, where $\hat{x}_i$'s are obtained as follow by inspired of [32]

$$\begin{aligned} \dot{\hat{x}}_2 &= -\frac{b}{m}\hat{x}_2 + \frac{A_P}{m} x_3 + \hat{\Delta}_2 \\ \dot{\hat{x}}_3 &= -\frac{2\rho A_P}{V_0}\hat{x}_2 + \frac{2\rho k_q \sqrt{P_s - |x_3|}}{V_0} x_4 + \hat{\Delta}_3 \\ \dot{\hat{x}}_5 &= -\omega_{nv}^2 x_4 - 2\epsilon_v \omega_{nv} \hat{x}_5 + k_e \omega_{nv}^2 u + \hat{\Delta}_5 \end{aligned} \tag{17}$$

where the state variables $x_1 = y$, $x_3 = P_L$, and $x_4 = v_e$, as the output of the system is measurable and available online. The discussion above is summarized in the following theorem.

Theorem 1: Consider the system in (2). Using the observers in (16) and (17), there exist suitable parameters such that the observation errors e_i $(i = 2, 3, \text{and } 5)$ are globally FTUB.

Proof. First, define a sliding variable $s_i(t) = \dot{e}_i + \beta e_i^{q/p}$ $(i = 2, 3, \text{and } 5)$. Then, by choosing the Lyapunov functions as

$$V_i = \frac{s_i^2}{2} \tag{18}$$

their time derivatives along the system's trajectories become

$$\dot{V}_i = s_i \dot{s}_i = s_i \left(\ddot{e}_i + \frac{\beta q}{p} e_i^{\frac{q}{p}-1} \dot{e}_i \right) = s_i \left(\ddot{\hat{x}}_i - \ddot{x}_i + \frac{\beta q}{p} e_i^{\frac{q}{p}-1} \dot{e}_i \right) \tag{19}$$

After substituting the expression $\frac{\beta q}{p} e_i^{\frac{q}{p}-1} \dot{e}_i = -\alpha_i \operatorname{sgn}\left(\dot{e}_i + \beta e_i^{q/p}\right) - \dot{\hat{\Delta}}_i$ from the time derivative of the disturbance observer (16) into (19) and simplification, the latter becomes

$$\dot{V}_i = s_i\left(-\alpha_i \operatorname{sgn}(s_i(t)) - \dot{\Delta}_i\right) \tag{20}$$

where based on [32], in this step it has been assumed $\ddot{x}_i = \ddot{\hat{x}}_i$. Under the Lipschitz-like condition $|\dot{\Delta}_i| < \sigma_i$ $(i = 2, 3, \text{and } 5)$,

$$\dot{V}_i \le -\alpha_i |s_i| + |\dot{\Delta}_i||s_i| \le -(\alpha_i - \sigma_i)|s_i| \tag{21}$$

Let the inverse of the function $\alpha_i(\cdot)$ be $\alpha_i^{-1}(\cdot)$. Then, for any $|s_i| > \alpha_i^{-1}(\sigma_i)$, $\alpha_i(\cdot) > \sigma_i$ and hence $\dot{V}_i < 0$. Therefore, the function $\dot{s}_i = -\alpha_i \operatorname{sgn}(s_i(t)) - \dot{\Delta}_i$ satisfies the stability criterion in Theorem 1. Therefore, s_i is bounded as $|s_i| \le \alpha_i^{-1}(\sigma_i)$ in the finite time $T_i = \frac{|s_i(0)|}{\alpha_i(|s_i(0)|)-\sigma_i}$ and the FTUB of the observation error e_i is guaranteed. This completes the proof. □

Now, we can update the state feedback control law (14) by using the estimation of Δ_i proposed by (16). After lengthy but straightforward calculations, the FT-DOTC law is obtained as

$$u = (k_e\omega_{nv}^2)^{-1}\Big(-\tilde{\mathcal{X}}_4 + \omega_{nv}^2(\tilde{\mathcal{X}}_4 + \bar{\Phi}_3) + 2\epsilon_v\omega_{nv}(\tilde{\mathcal{X}}_5 + \bar{\Phi}_4) + \dot{\bar{\Phi}}_4 - a_5\tilde{\mathcal{X}}_5 - (\hat{\Delta}_5 + \bar{\Theta}_5)\mathrm{sgn}(\tilde{\mathcal{X}}_5)\Big) \tag{22}$$

where $\bar{\Theta}_i$ $(i = 2, 3, \text{and } 5)$ is to be designed to suppress the effect of the estimation errors that behave as small uncertainties in the states. It is worth noting that, the variables x's defined in the past steps and $\mathcal{X}$'s used here do not have any fundamental differences, where the capital letter $\mathcal{X}$ has been used only for Step 3. Also $\bar{\Phi}$'s are new visual variables that are presented later. In the following theorem, it is shown that without any knowledge about the upper bounds of the uncertainties, the closed-loop dynamics obtained by (2), (16), (17), and (22) remain ultimately bounded and the output variable tracks the reference signal within a finite time.

Theorem 2. Consider the system (2), the uncertainties Δ_i $(i = 2, 3, \text{and } 5)$ satisfy the Lipschitz-like conditions $|\dot{\Delta}_2| < \sigma_2$, $|\dot{\Delta}_3| < \sigma_3$, and $|\dot{\Delta}_5| < \sigma_5$. If u is designed as the disturbance observer-based state feedback control law in (22) with $\bar{\Theta}_2 > \alpha_2^{-1}(\sigma_2)$, $\bar{\Theta}_3 > \alpha_3^{-1}(\sigma_3)$, and $\bar{\Theta}_5 > \alpha_5^{-1}(\sigma_5)$ and the positive constant gains $a_1$and a_2 are designed such that $a_1 > \frac{1}{t}$ and $a_2 > \frac{1}{t} - \frac{b}{m}$, then all states of the closed-loop system (2), (16), (17), and (22) are bounded and the variables y and $\dot{y}$ track the time-varying reference signals y_d and $\dot{y}_d$, respectively, in a finite time.

Proof. The proof is a direct consequence of Lemma 1 by replacing (23) and (24) with (8) and (10), respectively.

$$\mathcal{X}_3 = \left(\frac{A_P}{m}\right)^{-1}\left(-E_1 - a_2\tilde{E}_2 + \mu\left[\ddot{y}_d + \frac{b}{m}\dot{y}_d\right] - \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m}\right)\Phi_1 + \dot{\Phi}_1 - \mu(\hat{\Delta}_2 +\right.$$

$$\left.\bar{\Theta}_2)\,\mathrm{sgn}(\tilde{E}_2)\right) \triangleq \bar{\Phi}_2(E_1, \tilde{E}_2) \tag{23}$$

and

$$\mathcal{X}_4 = \left(\frac{2\rho k_q\sqrt{P_S - |\tilde{\mathcal{X}}_3 + \bar{\Phi}_2|}}{V_0}\right)^{-1}\left(-\frac{A_P}{m}\tilde{E}_2 + \frac{2\rho A_P}{V_0}(\tilde{E}_2 + \Phi_1) + \frac{2\rho A_P}{V_0}\dot{y}_d + \dot{\bar{\Phi}}_2 - a_3\tilde{\mathcal{X}}_3 -\right.$$

$$\left.(\hat{\Delta}_3 + \bar{\Theta}_3)\,\mathrm{sgn}(\tilde{\mathcal{X}}_3)\right) \triangleq \bar{\Phi}_3(E_1, \tilde{E}_2, \tilde{\mathcal{X}}_3) \tag{24}$$

By replacing (23) with (8), we obtain

$$\dot{V}_2 = \left(\frac{\dot{\mu}}{\mu} - a_1\right)E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right)\tilde{E}_2^2 - \mu|\tilde{E}_2|\bar{\Theta}_2 - \left(\mu|\tilde{E}_2|\hat{\Delta}_2 - \mu\tilde{E}_2\Delta_2\right) \tag{25}$$

For the convenience, depending on the term $-[\mu|\tilde{E}_2|\hat{\Delta}_2 - \mu\tilde{E}_2\Delta_2]$ two different cases may occur:

Case 1 ($-[\mu|\tilde{E}_2|\hat{\Delta}_2 - \mu\tilde{E}_2\Delta_2] \le \mu|\tilde{E}_2||\hat{\Delta}_2| - \mu|\tilde{E}_2||\Delta_2|$): In this case, $-[\mu|\tilde{E}_2|\hat{\Delta}_2 - \mu\tilde{E}_2\Delta_2]$ satisfies $-[\mu|\tilde{E}_2|\hat{\Delta}_2 - \mu\tilde{E}_2\Delta_2] \le \mu|\tilde{E}_2|[|\hat{\Delta}_2| - |\Delta_2|] \le \mu|\tilde{E}_2|\alpha_2^{-1}(\sigma_2)$. Then, $\dot{V}_2$ is achieved as,

$$\dot{V}_2 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right)E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right)\tilde{E}_2^2 + \mu|\tilde{E}_2|(\alpha_2^{-1}(\sigma_2) - \bar{\Theta}_2) \tag{26}$$

Since $\bar{\Theta}_2$ and $\alpha_2^{-1}(\sigma_2)$ are positive, if $\bar{\Theta}_2 > \alpha_2^{-1}(\sigma_2)$ such that $\alpha_2^{-1}(\sigma_2) - \bar{\Theta}_2 < 0$, then $\dot{V}_2 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right)E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right)\tilde{E}_2^2$.

Case 2 ($-[\mu|\tilde{E}_2|\hat{\Delta}_2 - \mu\tilde{E}_2\Delta_2] \le \mu|\tilde{E}_2||\Delta_2| - \mu|\tilde{E}_2||\hat{\Delta}_2|$): In this case, $-[\mu|\tilde{E}_2|\hat{\Delta}_2 - \mu\tilde{E}_2\Delta_2]$ satisfies $-[\mu|\tilde{E}_2|\hat{\Delta}_2 - \mu\tilde{E}_2\Delta_2] \le \mu|\tilde{E}_2|[|\Delta_2| - |\hat{\Delta}_2|] \le -\mu|\tilde{E}_2|\alpha_2^{-1}(\sigma_2)$, and $\dot{V}_2$ is achieved as,

$$\dot{V}_2 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right)E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right)\tilde{E}_2^2 + \mu|\tilde{E}_2|(-\alpha_2^{-1}(\sigma_2) - \bar{\Theta}_2) \tag{27}$$

where $\dot{V}_2 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2$ is guaranteed without any concern. Therefore, the condition $\bar{\Theta}_2 > \alpha_2^{-1}(\sigma_2)$ is a sufficient condition to guarantee $\dot{V}_2 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2$. Subsequently, by replacing (24) with (10), we obtain

$$\dot{V}_3 = \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 - |\tilde{\mathcal{X}}_3|\bar{\Theta}_3 - \left(|\tilde{\mathcal{X}}_3|\hat{\Delta}_3 - \tilde{\mathcal{X}}_3\Delta_3\right) \quad (28)$$

where $V_3(E_1, \tilde{E}_2, \tilde{\mathcal{X}}_3) = V_2(E_1, \tilde{E}_2) + 0.5\, \tilde{\mathcal{X}}_3^2$. Now, as in the previous step, depending on the term $-[|\tilde{\mathcal{X}}_3|\hat{\Delta}_3 - \tilde{\mathcal{X}}_3\Delta_3]$ two different cases may occur:

Case 1 ($-[|\tilde{\mathcal{X}}_3|\hat{\Delta}_3 - \tilde{\mathcal{X}}_3\Delta_3] \le |\tilde{\mathcal{X}}_3||\hat{\Delta}_3| - |\tilde{\mathcal{X}}_3||\Delta_3|$): In this case, $-[|\tilde{\mathcal{X}}_3|\hat{\Delta}_3 - \tilde{\mathcal{X}}_3\Delta_3]$ satisfies $-[|\tilde{\mathcal{X}}_3|\hat{\Delta}_3 - \tilde{\mathcal{X}}_3\Delta_3] \le |\tilde{\mathcal{X}}_3|[|\hat{\Delta}_3| - |\Delta_3|] \le |\tilde{\mathcal{X}}_3|\alpha_3^{-1}(\sigma_3)$. Then, $\dot{V}_3$ is achieved as,

$$\dot{V}_3 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 + |\tilde{\mathcal{X}}_3|(\alpha_3^{-1}(\sigma_3) - \bar{\Theta}_3) \quad (29)$$

Since $\bar{\Theta}_3$ and $\alpha_3^{-1}(\sigma_3)$ are positive, if $\bar{\Theta}_3 > \alpha_3^{-1}(\sigma_3)$ such that $\alpha_3^{-1}(\sigma_3) - \bar{\Theta}_3 < 0$, then $\dot{V}_3 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2$.

Case 2 ($-[|\tilde{\mathcal{X}}_3|\hat{\Delta}_3 - \tilde{\mathcal{X}}_3\Delta_3] \le |\tilde{\mathcal{X}}_3||\Delta_3| - |\tilde{\mathcal{X}}_3||\hat{\Delta}_3|$): In this case, $-[|\tilde{\mathcal{X}}_3|\hat{\Delta}_3 - \tilde{\mathcal{X}}_3\Delta_3]$ satisfies $-[|\tilde{\mathcal{X}}_3|\hat{\Delta}_3 - \tilde{\mathcal{X}}_3\Delta_3] \le |\tilde{\mathcal{X}}_3|[|\Delta_3| - |\hat{\Delta}_3|] \le -|\tilde{\mathcal{X}}_3|\alpha_3^{-1}(\sigma_3)$, and $\dot{V}_3$ is achieved as,

$$\dot{V}_3 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 + |\tilde{\mathcal{X}}_3|(-\alpha_3^{-1}(\sigma_3) - \bar{\Theta}_3) \quad (30)$$

where $\dot{V}_3 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2$ is guaranteed without any concern. Therefore, the condition $\bar{\Theta}_3 > \alpha_3^{-1}(\sigma_3)$ is a sufficient condition to guarantee $\dot{V}_3 \le \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2$. Finally, after replacing the FT-DOTC law (22) with the state feedback control law in (14), the updated Lyapunov function $V_5(E_1, \tilde{E}_2, \tilde{\mathcal{X}}_3, \tilde{\mathcal{X}}_4, \tilde{\mathcal{X}}_5)$ is selected such that it satisfies

$$\dot{V}_5 = \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 - a_4\tilde{\mathcal{X}}_4^2 - a_5\tilde{\mathcal{X}}_5^2 - |\tilde{\mathcal{X}}_5|\bar{\Theta}_5 - \left(|\tilde{\mathcal{X}}_5|\hat{\Delta}_5 - \tilde{\mathcal{X}}_5\Delta_5\right) \tag{31}$$

Now, as in the previous step, depending on the term $-[|\tilde{\mathcal{X}}_5|\hat{\Delta}_5 - \tilde{\mathcal{X}}_5\Delta_5]$ two different cases may occur:

Case 1 $(-[|\tilde{\mathcal{X}}_5|\hat{\Delta}_5 - \tilde{\mathcal{X}}_5\Delta_5] \leq |\tilde{\mathcal{X}}_5||\hat{\Delta}_5| - |\tilde{\mathcal{X}}_5||\Delta_5|)$**:** In this case, $-[|\tilde{\mathcal{X}}_5|\hat{\Delta}_5 - \tilde{\mathcal{X}}_5\Delta_5]$ satisfies $-[|\tilde{\mathcal{X}}_5|\hat{\Delta}_5 - \tilde{\mathcal{X}}_5\Delta_5] \leq |\tilde{\mathcal{X}}_5|[|\hat{\Delta}_5| - |\Delta_5|] \leq |\tilde{\mathcal{X}}_5|\alpha_5^{-1}(\sigma_5)$. Then, $\dot{V}_5$ is achieved as,

$$\dot{V}_5 \leq \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 - a_4\tilde{\mathcal{X}}_4^2 - a_5\tilde{\mathcal{X}}_5^2 + |\tilde{\mathcal{X}}_5|(\alpha_5^{-1}(\sigma_5) - \bar{\Theta}_5) \tag{32}$$

Since $\bar{\Theta}_5$ and $\alpha_5^{-1}(\sigma_5)$ are positive, if $\bar{\Theta}_5 > \alpha_5^{-1}(\sigma_5)$ such that $\alpha_5^{-1}(\sigma_5) - \bar{\Theta}_5 < 0$, then $\dot{V}_5 \leq \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 - a_4\tilde{\mathcal{X}}_4^2 - a_5\tilde{\mathcal{X}}_5^2$.

Case 2 $(-[|\tilde{\mathcal{X}}_5|\hat{\Delta}_5 - \tilde{\mathcal{X}}_5\Delta_5] \leq |\tilde{\mathcal{X}}_5||\hat{\Delta}_5| - |\tilde{\mathcal{X}}_5||\Delta_5|)$**:** In this case, $(-[|\tilde{\mathcal{X}}_5|\hat{\Delta}_5 - \tilde{\mathcal{X}}_5\Delta_5]$ satisfies $-[|\tilde{\mathcal{X}}_5|\hat{\Delta}_5 - \tilde{\mathcal{X}}_5\Delta_5] \leq |\tilde{\mathcal{X}}_5|[|\Delta_5| - |\hat{\Delta}_5|] \leq -|\tilde{\mathcal{X}}_5|\alpha_5^{-1}(\sigma_5)$, and $\dot{V}_5$ is achieved as,

$$\dot{V}_5 \leq \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 - a_4\tilde{\mathcal{X}}_4^2 - a_5\tilde{\mathcal{X}}_5^2 + |\tilde{\mathcal{X}}_5|(-\alpha_5^{-1}(\sigma_5) - \bar{\Theta}_5) \tag{33}$$

where $\dot{V}_5 \leq \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 - a_4\tilde{\mathcal{X}}_4^2 - a_5\tilde{\mathcal{X}}_5^2$ is guaranteed straightforward. Therefore, the condition $\bar{\Theta}_5 > \alpha_5^{-1}(\sigma_5)$ is a sufficient condition to guarantee $\dot{V}_5 \leq \left(\frac{\dot{\mu}}{\mu} - a_1\right) E_1^2 + \left(\frac{\dot{\mu}}{\mu} - \frac{b}{m} - a_2\right) \tilde{E}_2^2 - a_3\tilde{\mathcal{X}}_3^2 - a_4\tilde{\mathcal{X}}_4^2 - a_5\tilde{\mathcal{X}}_5^2$. That is, the closed-loop dynamics obtained by (2), (16), (17), and (22) remain ultimately bounded and the output variables y and $\dot{y}$ track the time-varying reference signals y_d and $\dot{y}_d$, respectively, within a finite time. This completes the proof. $\square$

3.1.Implementation issues

According to (4), the time-varying transformation function $\mu(t, \ell)$ approaches infinity when $t \to \infty$. To avoid this, that transformation function can be modified as

$$\mu(t, \ell) = \begin{cases} 1 + e^{\frac{-t}{\ell}} \Big/ 2e^{\frac{-t}{\ell}} & t \leq \delta_t \\ \mu_{Max} & t > \delta_t \end{cases} \tag{34}$$

where for a positive real constant δ_t, the constant μ_{Max} is defined as $1 + e^{\frac{-\delta_t}{\ell}} / 2e^{\frac{-\delta_t}{\ell}}$. The constant δ_t should be designed based on a trade-off between the finite-time efficiency and the control efforts. Note that the stability analyses can be modified to address the new definition for the transformation function $\mu(\cdot)$ given in (33). For this purpose, the stability analysis is performed during two different time intervals of $t \leq \delta_t$ and $t > \delta_t$. The previous analysis is valid during the first time interval. Also, since we have already proved that all the states, including the observed states, remain bounded for all time, this statement will also remain valid for the second time interval.

Furthermore, according to the definition of the sliding variable $s_i(t)$ $(i = 2, 3, \text{and } 5)$ given in the proof of Theorem 1, $\left(1 - \frac{s_i(t)}{\dot{e}_i}\right) \dot{e}_i + \beta e_i^{q/p} = 0$, where by letting $\dot{e}_i > s_i(t)$, $1 - \frac{s_i(t)}{\dot{e}_i} > 0$, and keeps the same property of finite-time stability as that in $s_i(t)$, which reversely means that the velocity of tracking error converges to the region $\dot{e}_i \leq s_i(t)$ in a finite time. By simplifying $\dot{e}_i$, it can be seen that $\dot{e}_i = \hat{\Delta}_i - \Delta_i$. Hence, combining this with $\dot{e}_i \leq s_i(t)$ and the result of Theorem 1, *i.e.* $|s_i| \leq \alpha_i^{-1}(\sigma_i)$, yields

$$|\dot{e}_i| = |\hat{\Delta}_i - \Delta_i| \leq |s_i(t)| \leq \alpha_i^{-1}(\sigma_i) \tag{35}$$

Therefore, for any desired error bound $\alpha_i^{-1}(\sigma_i)$, there exists a finite time such that $|\hat{\Delta}_i - \Delta_i| \leq \alpha_i^{-1}(\sigma_i)$ and, by tuning parameters, the size of the above region can be accurately suppressed to be arbitrarily small.

Compared to conventional methods, the proposed scheme discussed above is straightforward to implement and realize where its block diagram is shown in Figure 2.

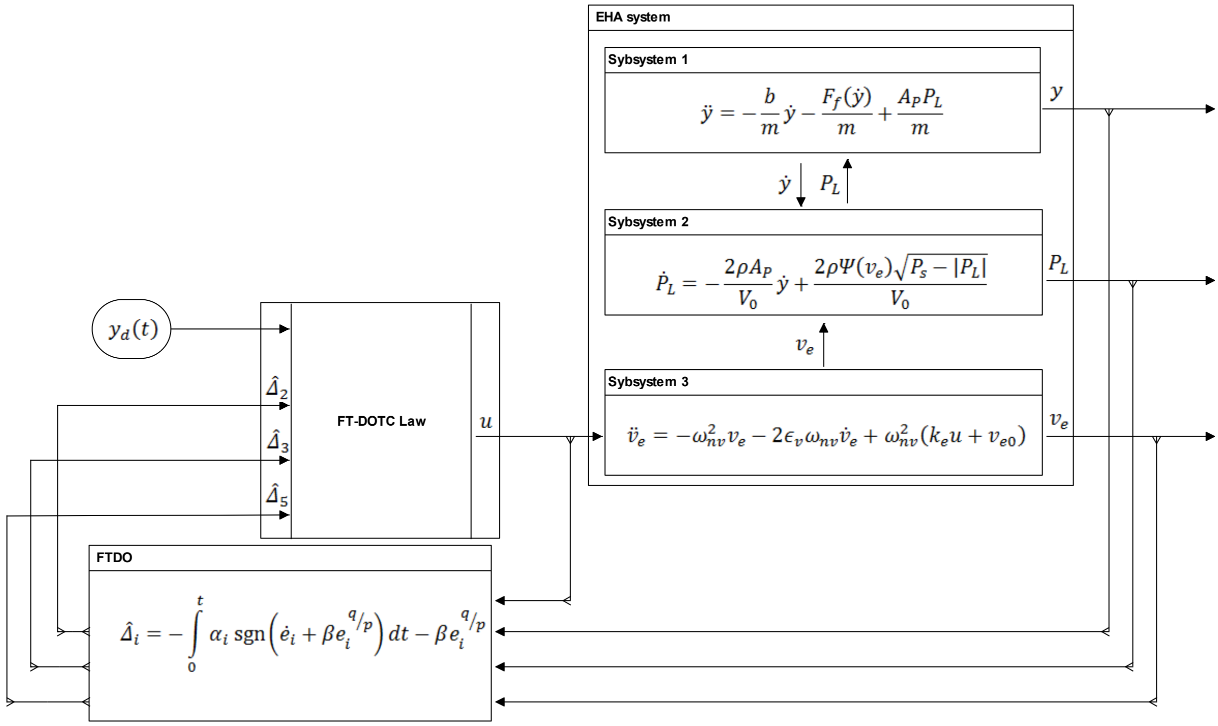

Figure 2: Block diagram of the proposed FT-DOTC law scheme for the EHA system (1).

By an appropriate selection of the observer and controller parameters such that they satisfy the conditions mentioned in the stability analyses; uncertainties can be estimated in a finite time before any divergence occurs. A recommended guideline on the selection of the parameters is provided in the following steps:

(1) Since the estimates of uncertainties obtained through the FTDO in (16) is fed into the proposed FT-DOTC law in (22), the observer in (16) should be faster than the controller in (22). Therefore, to guarantee the estimation in a finite time, β is proposed as a positive constant, where increasing its value increases the convergence time significantly but reduces the control effort considerably, and vice versa. Also, p and q are

positive odd integers, satisfying the condition $p > q$, where decreasing the value of the fraction $^{q}/_{p}$ reduces the convergence time considerably but increases the control effort significantly, and vice versa.

(2) Then, in order to suppress the effect of the estimation errors, for class K functions $\alpha_i(.)$ (the adaptive gains $\bar{\Theta}_i$ are designed as $\bar{\Theta}_2 > \alpha_2^{-1}(\sigma_2)$, $\bar{\Theta}_3 > \alpha_3^{-1}(\sigma_3)$, and $\bar{\Theta}_5 > \alpha_5^{-1}(\sigma_5)$.

(3) Next, to achieve proper tracking in a finite time, the design parameter t is selected based on the trade-off between the control effort and the convergence time. Increasing the value of t reduces the control effort, while it increases the convergence time, and vice versa.

(4) Finally, the positive constant gains a_i for $i = 1, 2, \cdots, 5$ are selected to satisfy

$$a_1 > \frac{1}{t}, \qquad a_2 > \frac{1}{t} - \frac{b}{m}, \qquad a_i > 0 \ \ (i = 3, 4, 5) \tag{36}$$

The effect of the parameters tuning on the system performance is illustrated and further discussed in the numerical simulations section.

4. Numerical simulations and discussion

In this section, numerical simulations are presented to analyze the effectiveness of the proposed FT-DOTC law in (22) and the FTDO in (16) for the nonlinear EHA system in (2). Then, further simulations are reported to validate the performance of the proposed framework in the presence of unknown measurement noise. To clearly indicate the efficacy of the proposed approach, the numerical simulation results obtained using this technique is compared with those obtained using the OFCDR method in [35]. The OFCDR method is selected as the control benchmark since the efficiency of that method compared with the PID, SMC, and RBC methods was already demonstrated in [35]. It is shown in the following that the proposed scheme is advantageous over OFCDR in terms of system performance indices, which makes it a qualified alternative approach in the observer design with noteworthy potential. In the numerical simulations below, it is assumed that initial conditions for all state variables are zero and that the parameters in the observer are the same as those in the nonlinear EHA system. Specifically, the numerical values for the parameters used are provided in Table 1 [16].

Table 1
Nominal parameters of the hydraulic actuator [16]

	Symbol	Unit	Value
Piston mass	m	kg	440
Piston area	b	m^2	0.01
Centered camera volume	V_0	m^3	0.004
Bulk modulus	ρ	Pa	$1e9$
Input gain	k_e	-	0.49
Valve natural frequency	ω_{nv}	s^{-1}	152
Valve damping coefficient	ϵ_v	-	0.92
Supply pressure	P_s	Pa	$1e7$

First, to understand the dead-zone's effect, the results of the FTDO in (16) are compared to the high gain disturbance observer in [35], as shown in Figure 3, by employing a low-frequency sine wave $u = 2\sin(\pi t)$ as input. It can be seen in Figure 3 that the observer designed based on the high gain disturbance observer approach in [35] fails to estimate the dead-zone effect and is not able to sense the fast changes in the friction force in a short time. While, in response to the considered valve command signal, one can find from Figure 3 that by employing the proposed disturbance observer, the friction force and the dead-zone effect variables remain at a small neighborhood of the real states in a finite time. It can be seen that the proposed observer can achieve higher observation accuracy and faster convergence speed, where the proposed disturbance observer (16) coupled with (17) has a smaller convergence time (less than 0.168 s, 0.1 s, and 0.0005 s, respectively) compared to the high gain disturbance observer (almost equal to 0.3 s, 1.15 $\pm$ 0.1 s, and 0.09 s, respectively) [35].

Remark 2: One of the main drawbacks of the previous disturbance observers is that the upper bound of the unknown terms is often assumed to be known, which is not feasible in practical situations. To overcome this problem, the disturbance observer (16) can deal with strong uncertainties with unknown upper bounds, where estimation with finite-time boundedness is guaranteed in a straightforward manner.

Next, the performance of the proposed scheme is evaluated in terms of finite-time tracking of the bounded time-varying reference signals $y_d(t) = \sin(t)$ and $\dot{y}_d(t) = \cos(t)$.

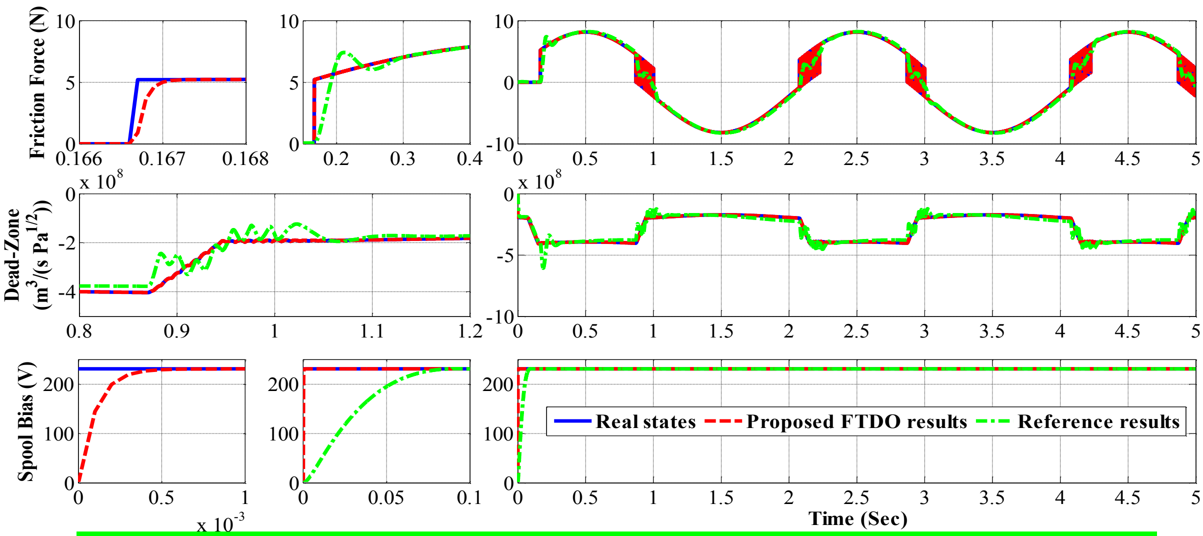

Figure 3: Simulated uncertainties and estimation results of the open-loop system (solid line for actual states (real states), dashed line for proposed FTDO results, and dash-dotted line for reference results [35]).

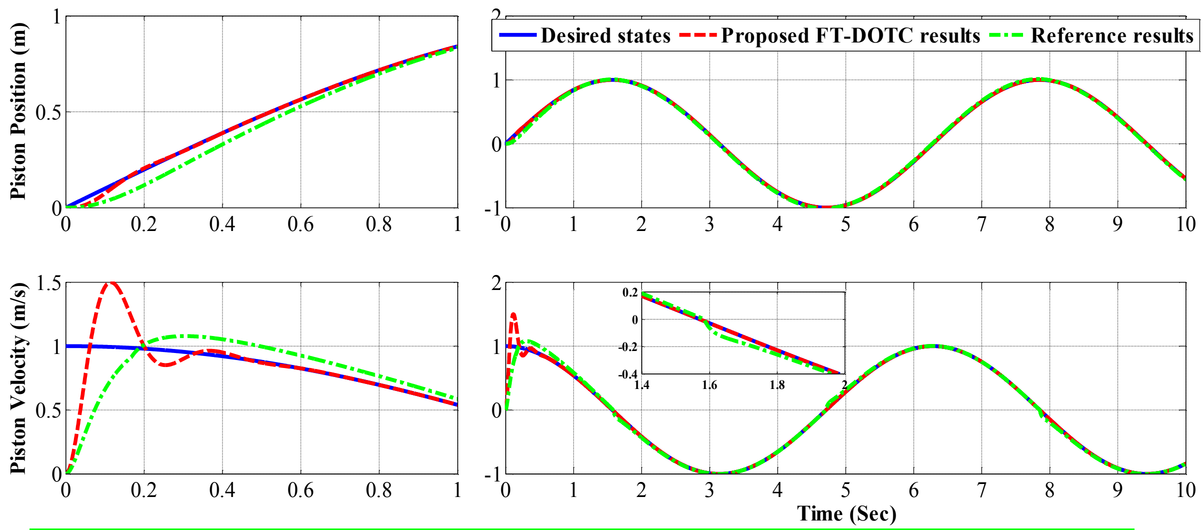

Figure 4: Reference signals (desired states) and position and velocity tracking results of the closed-loop system (solid line for desired states, dashed line for proposed FT-DOTC results, and dash-dotted line for reference results [35]).

Figure 4 illustrates the important efficiency of the proposed scheme, where the state variables x_1 and x_2 track the reference signals y_d and $\dot{y}_d$ in a shorter time and with less tracking error (0.2 s and 0.5 s, respectively) than OFCDR (1 ± 0.1 s and 1.5 ± 0.5 s, respectively). To further demonstrate the merits of the proposed scheme, the performance of studied approaches are evaluated and compared quantitatively in Table 2 by defining RMS performance index $J_{\bar{e}} = \left\| \sqrt{\frac{1}{t_s} \int_0^{t_s} \bar{e}^T \bar{e}\, dt} \right\|$ based on the infinity norm of the tracking error vector $\bar{e} = [\|\bar{e}_1\|, \|\bar{e}_2\|]^T$, where $t_s = 10$ s denotes the simulation time.

Table 2
Comparative results between RMS performance indexes.

	Tracking error vector $\bar{e} = [\|\bar{e}_1\|, \|\bar{e}_2\|]^T$	**performance index $J_{\bar{e}}$**
Proposed scheme results	$\bar{e} = \begin{bmatrix} 0.9790 \\ 454.3910 \end{bmatrix}$	$J_{\bar{e}} = 454.3920$
OFCDR results [35]	$\bar{e} = \begin{bmatrix} 36.5773 \\ 642.1908 \end{bmatrix}$	$J_{\bar{e}} = 643.2316$

According to Table 2, the velocity tracking error is much larger than the position tracking error. This can be reasonable in practice because the characteristics of the load connected to the piston change very rapidly with the changes in friction force. As can be seen from the calculated indexes, without any knowledge about the upper bounds of the uncertainties, the tracking error of the proposed scheme is almost 30% less than that obtained using the OFCDR approach in [35], where the RMS performance index $J_{\bar{e}}$ obtained by the proposed scheme has the L_2 norm 454.39205464224 and the OFCDR approach in [35] has the L_2 norm 643.2316242847. It can be concluded that the proposed scheme has a better performance in terms of tracking errors in the presence of uncertainties.

In Figure 5, the time histories of the control signals obtained using the proposed controller and OFCDR are presented. Since the tracking performance in the proposed scheme has been achieved in less finite time, so a sudden increase in the control law is produced, where it can be reasonable in practice. However, there is the problem of large overshoot compared with the OFCDR in [35], where practical constraints in terms of input saturation will be addressed in future works. In addition, as can be seen in Figure 5, the proposed FT-DOTC law (22) has a smoother behavior compared with the OFCDR control law in [35], where the valve input voltage is produced without any chattering phenomena. To show the merits of the proposed FTDO (14) in the closed-loop system obtained by (2), (16), (17), and (22), the results of the two observers are compared in Figure 6, where it is shown that the proposed observer can perceive quick changes in uncertainties accurately, while the observer [35] is not able to do so.

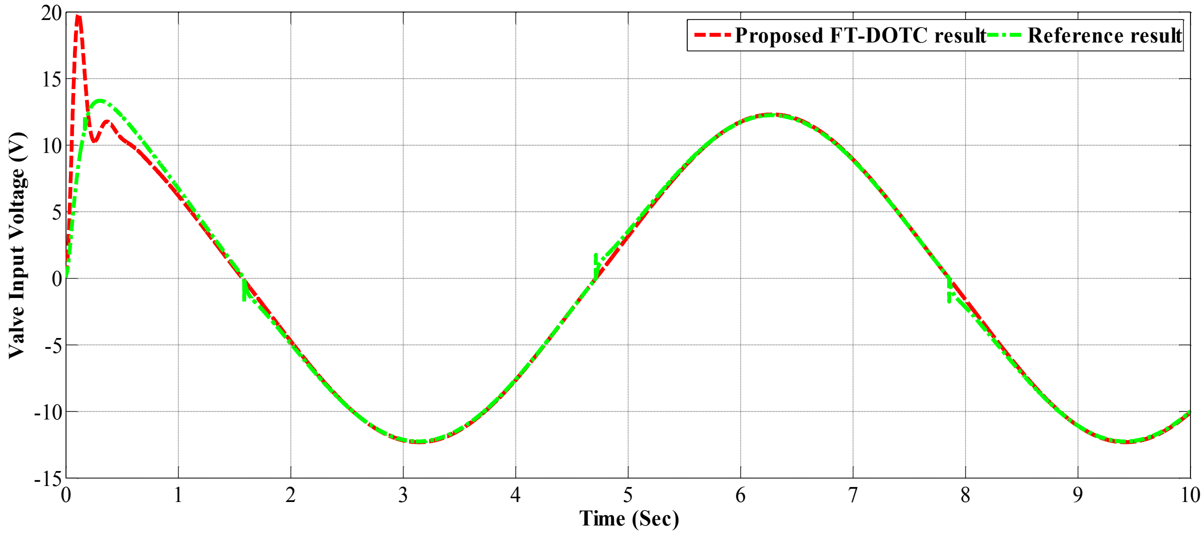

Figure 5: Valve input voltage in the closed-loop system (dashed line for proposed FT-DOTC result and dash-dotted line for reference result [35]).

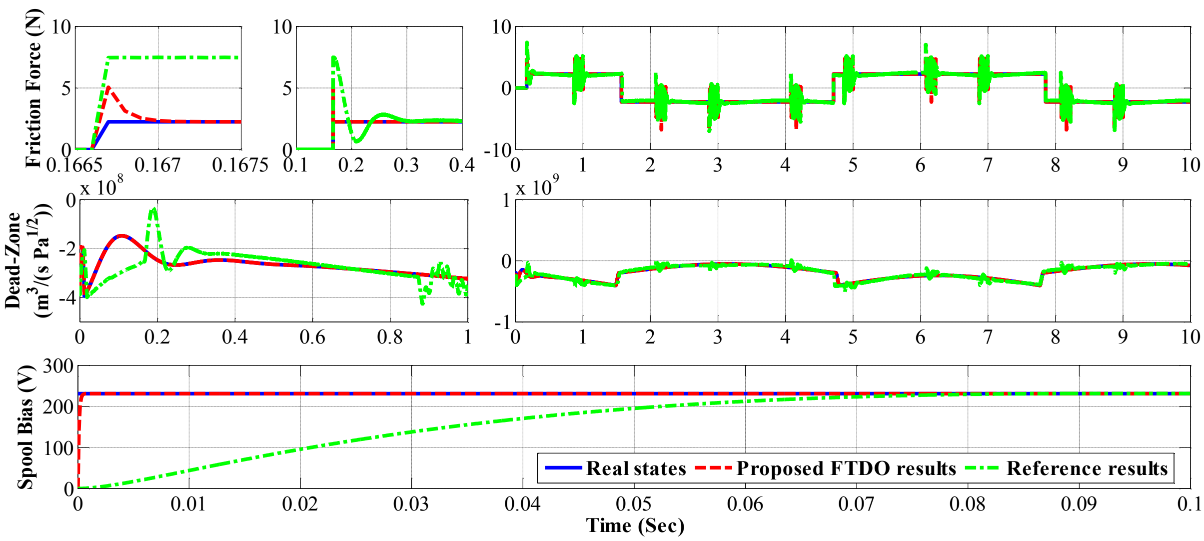

Figure 6: Simulated uncertainties and estimation results of the closed-loop system (solid line for actual states, dashed line for proposed FTDO results, and dash-dotted line for reference results [35]).

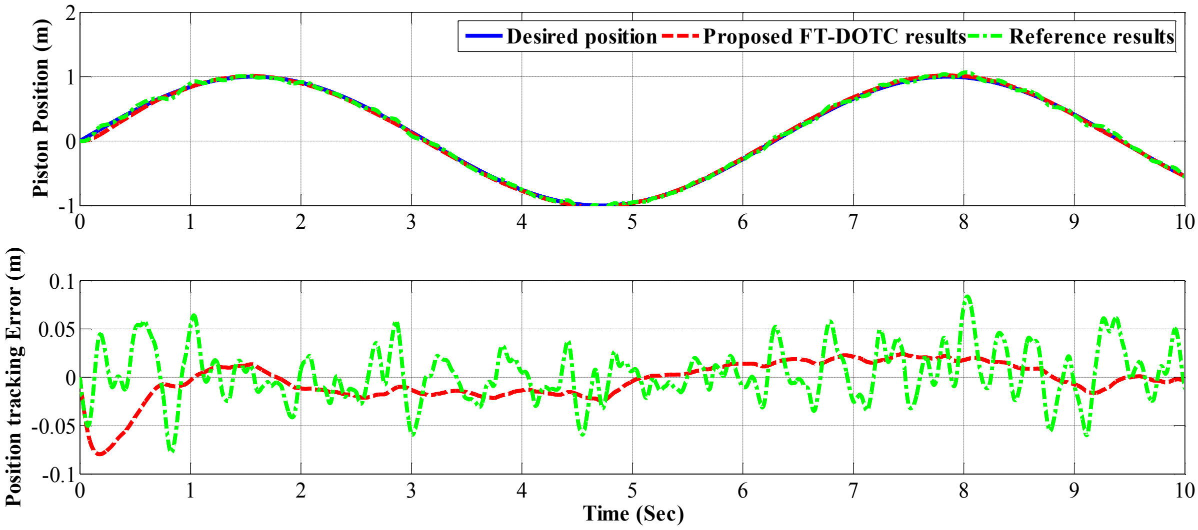

Figure 7: Reference signals and position and velocity tracking results of the closed-loop system in the presence of unknown measurement noise (solid line for desired position, dashed line for proposed FT-DOTC results, and dash-dotted line for reference results [35]).

It can be clearly seen in Figure 6 that the proposed scheme achieves high estimation performance in the closed-loop system. To evaluate the robustness of the proposed scheme in the presence of unknown measurement noise, Figure 7 is presented, where it can be seen that the tracking error obtained using the proposed approach is less than OFCDR in [35]. It is worth noting that, the considered unknown measurement noise in this paper is a Band-Limited White Noise (Refer to the Band-Limited White Noise block in the Simulink Matlab software), where according to the Figures 7 and 3, the proposed approach is completely robust in the presence of unknown measurement noise with unknown upper bound.

5. Conclusions

In this paper, a continuous disturbance observer-based backstepping finite-time tracking control has been presented for a class of electro-hydraulic actuator (EHA) systems. Without any knowledge about the upper bounds of the uncertainties, a finite-time disturbance observer has been proposed to estimate the uncertainties and disturbances, where the estimation error converges into an arbitrarily small neighborhood around zero in a finite time by tuning the design parameters. Then, a finite-time disturbance observer-based tracking control law was designed to guarantee the finite-time tracking of the piston position toward a desired time-varying reference signal. The key idea in this regard was to employ a monotonically-increasing function associated with the control objective to improve the control performance, where the tracking errors dynamics are transformed into a time-varying form and the finite-time boundedness criterion is guaranteed using Lyapunov stability analysis. In this regard, time-varying gains of the controller were designed in a straightforward manner. Finally, numerical simulations were provided to demonstrate the effectiveness of the proposed scheme for the EHA system with unknown measurement noise. The results are compared to those obtained using a state-of-the-art observer-based control strategy in the literature, where the competence of the proposed scheme is numerically proved which makes it a qualified alternative approach with noteworthy advantages. In conclusion, the following objectives are successfully achieved: (1) Without any knowledge about the upper bound of uncertainties, the proposed disturbance observer-based control law is capable of tracking the reference signal in the presence of uncertainties with unknown upper bound. (2) By introducing a time-varying conversion, finite-time tracking has been guaranteed by using Lyapunov stability analysis. In terms of more interesting topics, an optimal algorithm is suggested to be designed for choosing all the observer parameters. Moreover, output feedback control in the presence of symmetric input saturation can be considered to investigate the possibility of decreasing the number of physical sensors to control the robotic manipulators.

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