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The Global Component of Inflation Volatility[*]

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Abstract

Global developments play an important role for domestic inflation rates. Earlier literature has found that a substantial amount of the variation in a large set of national inflation rates can be explained by a single global factor. However, inflation volatility has been typically neglected, while it is clearly relevant both from a policy point of view and for structural analysis and forecasting. We study the evolution of inflation rates in several countries, using a novel model that allows for commonality in both levels and volatilities, in addition to country-specific components. We find that inflation volatility is indeed important, and a substantial fraction of it can be attributed to a global factor that is also driving inflation levels and their persistence. The extent of commonality among core inflation rates and volatilities is substantially smaller than for overall inflation, which leaves scope for national monetary policies. Finally, we show that the point and density forecasting performance of the model is good relative to standard benchmarks, which provides additional evidence on its reliability.

Keywords: Inflation, Volatility, Global factors, Large Datasets, Multivariate Autoregressive Index models, Reduced Rank Regressions, Forecasting.

J.E.L. Classification: E31, F62, C32, E37, C53.

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1 Introduction

Global developments play an important role for domestic inflation. Earlier literature has found that a substantial amount of the variation in a large set of national inflation rates can be explained by a single global factor (Ciccarelli & Mojon, 2010). This is evident in Figure 1, which reports the time series of CPI inflation rates for 20 OECD countries, over the period 1960Q1-2016Q4, together with their first principal component (PC).

The commonality documented in Figure 1, and modelled in contributions such as Ciccarelli & Mojon (2010), Mumtaz & Surico (2012), Lodge & Mikolajun (2016) refers to movements in the *conditional mean* of inflation. This paper is motivated by the observation that there is a significant degree of commonality also in the *conditional variance* of inflation rates of different countries. For example, Figure 2 plots estimated volatilities for the same 20 OECD countries (based on an autoregressive model with stochastic volatility), along with their first PC: The first principal component explains almost 60% of the variation in the volatilities.

Based on this observation, this paper introduces a new approach allowing to model commonality in both the conditional means (levels) and conditional variances (volatilities) of a cross section of macro-economic time series. The approach builds on the multivariate index (MAI) model of Reinsel (1983), and its Bayesian implementation in Carriero, Kapetanios & Marcellino (2016). A MAI model is a vector autoregression (VAR) with a particular reduced rank structure imposed on the coefficient matrices, such that each variable is driven by the lags of a limited number of linear combinations of all variables (so called Indexes), which can be considered as observable common factors.

As we document in Section 4, the simple MAI model does not offer a good representation of the data, and we extend it in two directions. First, we add time variation in the volatilities in the form of stochastic volatility. This allows to capture the empirically clear phenomenon of changing volatilities in inflation rates, which is particularly evident over the rather long sample we consider in the empirical application. Second, we introduce an individual AR component, which helps to improve the in-sample fit by capturing the country-specific dynamics. Stochastic volatility (SV) was introduced in the MAI model by Carriero, Corsello & Marcellino (2020), while Cubadda & Guardabascio (2017) allowed for the possibility of autoregressive (AR) terms to capture individual components. We combine these features into the MAI-AR-SV model, obtain an analytical representation for the indexes (observable factors) law of motion, derive a decomposition of the SV into common and individual terms, and provide the conditional posterior kernels necessary for estimation via Bayesian MCMC methods.

Importantly, the approach presented here hinges on a reduced rank VAR rather than on a factor model. This sets our approach apart from other contributions in the literature. Contributions such as Mumtaz & Surico (2012), Mumtaz & Theodoridis (2017) and Mumtaz & Musso (2021) rely on a factor model in which the unobservable factor has time varying volatility. In all these contributions the common volatility factor is merely the volatility of the common factor. Since the factor enters the conditional mean of the process, the factor volatility partly explains that of the data. However, neither the factor nor its volatility can explain the conditional variance of the shocks. Instead, in the approach presented here the observable common factor is common to both the conditional mean and the conditional variance of the model. Our methodology is also substantially different from the one in Delle Monache, Petrella & Venditti (2016), which extends the model of Stock & Watson (2007). Their model features a common permanent component with its own changing volatility estimated in a non-Bayesian setting where time variation is driven by likelihood scores, and it is applied in a multi-country inflation setting for the euro area. Another paper of interest is Gorodnichenko & Ng (2017) that allows for common but unrelated factors in the levels and volatility of the variables, assessing the effects of level and volatility shocks and their relative importance.

We work with a single index model where the index (a linear combination of all the national inflation rates) represents the global factor that drives both levels and volatilities of all national inflation rates. Inflation levels and volatilities also have an individual, country-specific, component, whose relative importance with respect to the global component is time-varying and empirically determined.

Empirically, we find that both the autoregressive component (AR) and the time varying volatility (SV) of the MAI-AR-SV model are needed to provide a proper statistical representation for the inflation rates. Moreover, the single common factor explains on average about 70% of the variability of all inflation rates. Futhermore, there is also substantial commonality in the inflation volatilities, increased in the last two decades. The average (across countries) share of stochastic volatility explained by the global component spans from 20% to 65% throughout the sample.

While various sources can be behind the global inflation factor, it turns out that since the early '90s its level and volatility are strongly correlated with those of Chinese PPI and Oil inflation. Measures of global slack seem not to have additional explanatory power, and US monetary policy shocks are basically uncorrelated with shocks to the global inflation factor. Hence, supply seems to matter more than demand to explain the global component of inflation and its volatility.

We also find that the global inflation factor is highly persistent, and this persistence is transmitted to the global component of the national inflation rates, in line with Ciccarelli & Mojon (2010). Level components explained by the common factor show a larger degree of persistence than individual components.

When we repeat the same analysis on a panel of non-Food and non-Energy inflation rates for the same set of OECD countries, using data available for the period 1979Q1-2016Q4, we find a smaller but non-negligible degree of commonality. The global core inflation factor explains roughly 25% of the variability of core CPI inflation levels. The average (across countries) share of volatility explained by the global component ranges from 10% to 20% throughout the sample, without displaying sizable variation over time as was the case for headline inflation rates. There is a substantial national component of core inflation, both in the levels and volatilities, which leaves scope for national monetary policies.

Finally, point and density forecast evaluations show that the MAI-AR-SV model has a very good out of sample performance for inflation rates, when compared with a set of multivariate and univariate competitors, and the SV specification is particularly relevant for the proper calibration of density forecasts. These results hold for both all items inflation and core inflation rates, and provide further empirical support for our proposed model.

The paper is structured as follows. Section 2 introduces the econometric model and the volatility decomposition. Section 3 discusses the choice of prior distributions and the Markov Chain Monte Carlo sampler used to draw from the posterior distribution (with additional details in the Appendix). Section 4 presents the data and the empirical results on the commonality in inflation rate levels and volatilities, and on their drivers. Section 5 assesses the point and density forecasting performance of the MAI-AR-SV inflation model. Section 6 concludes. The online Appendix provides additional details and empirical results.

2 The econometric model

2.1 The MAI-AR-SV model

We assume that the model for the n-dimensional zero mean process[1] y_t containing the inflation rates of interest is:

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + \sum_{\ell=1}^{p} A_\ell \cdot B_0 \cdot y_{t-\ell} + u_t, \tag{1}$$

where $A_\ell,\ \ell = 1, \ldots, p$ are $n \times r$ matrices, B_0 is an $r \times n$ matrix, and $\Gamma_\ell,\ \ell = 1, \ldots, q$ are n -dimensional diagonal matrices with diagonal elements $\gamma_{1,\ell}, \gamma_{2,\ell}, \ldots, \gamma_{n,\ell}$.

In this model, each of the n variables in y_t is driven by its own lags, capturing country-

[1] A non-zero mean can be easily allowed by inserting an intercept in the model.

specific features of inflation, with associated coefficients Γ_ℓ, by the lags of r common observable factors ($B_0 y_{t-\ell}$, the "indexes"), capturing global features of inflation, with associated loading matrices A_ℓ, and by error terms, u_t, whose properties are described below. With respect to an unrestricted Vector Autoregression, the model above leads to a substantial reduction in the number of parameters.[2]

The product $A_\ell B_0$ is an $n \times n$ matrix with reduced rank (r), and there is one matrix for each lag $\ell \in \{1, \ldots, p\}$. As it happens also in factor models and in cointegrated VARs, it is the case that one can rotate these matrices arbitrarily, e.g. $A_\ell B_0 = A_\ell Q' \cdot Q B_0$, where Q is an orthogonal matrix, and hence proper identification restrictions are needed to pin down only one of these possible rotations. Identification can be straightforwardly achieved for example as in Reinsel (1983) by assuming that the first r rows and columns of B_0 form an identity matrix, that is $B_0 = [I_r \;\; \widetilde{B}_0]$. We will follow a similar approach, see Section 3.1.1 for details.

In expression (1), the error term u_t is the innovation process of y_t. It is assumed to be uncorrelated over time, with multivariate Gaussian distribution $u_t \sim N(0, \Omega_t)$, where Ω_t is a time-varying variance-covariance matrix. Following Cogley & Sargent (2005) and Primiceri (2005), we operate a triangular factorization on the full matrix Ω_t, so that the errors u_t can be written as $u_t = G^{-1}\Sigma_t \varepsilon_t$, where ε_t is i.i.d. with multivariate Gaussian distribution $\varepsilon_t \sim N(0, I_n)$, G is a triangular matrix containing reduced form covariances.[3] and Σ_t is a diagonal matrix containing the stochastic volatility states. This implies the following factorization for the variance covariance matrix Ω_t:

$$\Omega_t = G^{-1}\Sigma_t \Sigma_t \left(G^{-1}\right)' \tag{2}$$

$$G = \begin{bmatrix} 1 & 0 & \ldots & \ldots & 0 \\ g_1 & 1 & \ddots & \ddots & \vdots \\ g_2 & g_3 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ g_{m-n+2} & g_{m-n+3} & \ldots & g_m & 1 \end{bmatrix}, \qquad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & \ldots & \ldots & 0 \\ 0 & \sigma_{2,t} & \ddots & \ddots & \vdots \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 0 & \sigma_{n,t} \end{bmatrix}. \tag{3}$$

The the non-zero off-diagonal elements in the matrix G are further collected in the vector $g = (g_1, g_2, ..., g_m)'$ which has dimension $m = n\,(n-1)/2$. Following Primiceri (2005), the law of motion for the time-varying (TV) standard deviations, collected in the vector

[2] In our empirical application, we have $p = q = 4$, $r = 1$ and $n = 20$, so that there are 180 parameters in the MAI-AR-SV while there would be 1600 parameters in an unrestricted VAR.

[3] The matrix G can be also made time-varying, but at the cost of a substantial increase in computational complexity when the number of variables is large.

$\sigma_t = \left[\sigma_{1,t}, \ldots, \sigma_{n,t}\right]'$, is defined in logarithms as:

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ Q_\sigma \right). \tag{4}$$

We will refer to the model in (1) with the volatility specification in (2), (3) and (4) as the MAI-AR-SV model.

2.2 An alternative representation of the MAI-AR-SV model

Let us define the observable factors driving all variables as

$$F_t \equiv B_0 \cdot Y_t, \tag{5}$$

and note that the following decomposition holds:[4]

$$I_n = \Omega_t B_0' \Xi_t^{-1} B_0 + B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp} \Omega_t^{-1}, \tag{6}$$

where $B_{0\perp}$ is the $(n-r) \times n$ orthogonal matrix of B_0 such that $B_0 B_{0\perp}' = \mathbf{0}_{r\times(n-r)}$, $\Xi_t = B_0 \Omega_t B_0'$ and $\Xi_{\perp,t} = B_{0\perp} \Omega_t^{-1} B_{0\perp}'$. Let us also define

$$G_t = B_{0\perp} \Omega_t^{-1} y_t. \tag{7}$$

where G_t is a vector containing $n - r$ variables.

Using (5)-(7), we can now write the MAI-AR-SV model in (1)-(2) as

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell [\Omega_t B_0' \Xi_t^{-1} B_0 + B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp} \Omega_t^{-1}]\ y_{t-\ell} + \sum_{\ell=1}^{p} A_\ell \cdot B_0\ y_{t-\ell} + u_t,$$

or

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} (\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + u_t. \tag{8}$$

Next, we derive the model for the factors F_t implied by the MAI-AR-SV model. Star-

[4] See Carriero et al. (2016) and the references therein for details.

ting from (8) and multiplying both sides of it either by B_0 or by $B_{0\perp}\Omega_t^{-1}$, we obtain:

$$F_t = \sum_{\ell=1}^{q} B_0\Gamma_\ell B'_{0\perp}\Xi^{-1}_{\perp,t}G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_0(\Gamma_\ell\Omega_t B'_0\Xi_t^{-1} + A_\ell)F_{t-\ell} + \omega_t, \tag{9}$$

$$G_t = \sum_{\ell=1}^{q} B_{0\perp}\Omega_t^{-1}\Gamma_\ell B'_{0\perp}\Xi^{-1}_{\perp,t}G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_{0\perp}\Omega_t^{-1}(\Gamma_\ell\Omega_t B'_0\Xi_t^{-1} + A_\ell)F_{t-\ell} + \psi_t,$$

where

$$\begin{bmatrix}\omega_t\\ \psi_t\end{bmatrix} = \begin{bmatrix}B_0 u_t\\ B_{0\perp}\Omega_t^{-1}u_t\end{bmatrix} \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \begin{bmatrix}\Xi_t & 0\\ 0 & \Xi_{\perp,t}\end{bmatrix}\right). \tag{10}$$

Note that u_t are the innovations to y_t, ω_t and ψ_t are innovations to F_t and G_t, respectively, and that they are mutually uncorrelated:

$$E(\omega_t\psi_t') = E(B_0u_tu_t'\,\Omega_t^{-1}B_{0\perp}') = B_0\Omega_t\Omega_t^{-1}B_{0\perp}' = 0. \tag{11}$$

Hence, the r observable factors F_t and the $n-r$ components G_t jointly evolve as a VAR, with block uncorrelated errors.

This shows that in this model the data are decomposed into their projections on two orthogonal subspaces, one of which (ω_t) takes into account the bulk of the commonalities, while the other (ψ_t) takes into account individual variation. In this paper we use the wording "individual variation" to denote a variation which is uncorrelated with the common variation and as such can be interpreted as variation specific to an individual variable or a subset of variables.

The model in (8)-(9) is similar to a factor augmented VAR (FAVAR) model, as for example in Bernanke et al. (2005), or Stock & Watson (2005) who also allow for variable-specific AR terms. However, it differs from a FAVAR in three important ways. First, it features stochastic volatility in the shocks driving the common as well as the individual variation, an aspect which turns out to be particularly relevant for modelling inflation. Second, while in the FAVAR model the factors are unobservable, in a MAI the factors are observable, a fact which greatly simplifies model specification and estimation. Finally, unobserved factors in a typical factor model follow a VARMA process, as emphasized by Dufour & Stevanović (2013). Instead the MAI implies a VAR process for the observable factors F_t.

2.3 Decomposing the volatilities

We have discussed above that $\omega_t = B_0u_t$ is the source of the commonalities in the levels of the series. Furthermore, in our approach, the same factors drive both the levels and the volatility of the variables.[5] Applying again the decomposition in (6) to the reduced form innovations u_t we obtain:

$$u_t = \Omega_t B_0' \Xi_t^{-1} \omega_t + B_{0\perp}' \left(B_{0\perp} \Omega_t^{-1} B_{0\perp}' \right)^{-1} \psi_t, \tag{12}$$

with $\Xi_t = B_0 \Omega_t B_0'$. Since there is no correlation between ω_t and ψ_t the epression above decomposes the total conditional volatility of the innovations into the volatility of the common component and that of the individual components:

$$\Omega_t = \underbrace{\Omega_t B_0' \Xi_t^{-1} B_0 \Omega_t}_{\text{common}} + \underbrace{B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp}}_{\text{individual}}.$$

2.4 Decomposing the levels and computing IRFs

It is interesting to decompose the inflation rates in y_t into their common and individual components, where the common component is driven by the common shocks ω_t and the individual component by the individual shocks ψ_t. The decomposition can be also used to compute impulse response functions (IRFs) to both common and individual shocks.

We cannot directly use the model in (8), as both F_t and G_t are driven by both ω_t and ψ_t. However, we can use the projection approach, proposed for example by Jordà (2005) for IRF computation, to obtain the decomposition:

$$y_t = B_1(L)\omega_t + B_2(L)\psi_t,$$

where ω_t and ψ_t are defined in (10). The common and individual components are uncorrelated at all leads and lags, due to the absence of correlation between ω_t and ψ_t. Therefore, empirically, we can obtain the common component as the fitted value in a regression of y_t on contemporaneous and lagged values of the (estimated) common shocks ω_t (and the individual component as y_t minus the estimated common component), while the IRFs to common shocks are computed from the elements of $B_1(L)$[6].

[5] It is also possible to envisage models in which specific factors drive the comovements in the volatilities. This kind of assumption is often made in the financial literature but may also be put forward in the macroeconomic literature, see for instance Gorodnichenko & Ng (2017).

[6] In our empirical application on inflation, we have a single factor ($r = 1$), which explains on average more than 70% of the cross-country inflation variability. In this case, ω_t is a scalar, which further simplifies the computation of the common component of inflation rates, and their impulse response functions to global shocks.

3 Estimation

The model is estimated using Markov Chain Monte Carlo (MCMC) techniques. In this Section we discuss, in turn, the priors on the model coefficients and the MCMC algorithm we use to obtain draws from the posterior distributions.

3.1 Specification of the prior distributions

The prior distributions are constructed in various steps, which generally require the use of a training sample $\{-T^*, \ldots, -1, 0\}$.

3.1.1 Prior on B_0

In order to identify the model, we need to restrict at least r^2 elements in the $r \times n$ matrix B_0. Given that in our case $r = 1$, then $B_0 = \begin{bmatrix} b_{0,1} & \ldots & b_{0,n} \end{bmatrix}$ is a row vector of weights, and as identifying restrictions we simply normalize to 1 the first weight $b_{0,1}$.

Prior knowledge for the unrestricted elements of B_0 is elicited with a Normal distribution. To set the prior moments for the elements of $B_0 = \begin{bmatrix} b_{0,1} & \ldots & b_{0,n} \end{bmatrix}$, we compute the largest eigenvalue score S_t from the principal component analysis on the set of variables, and then run the following univariate regressions for $k = 2, \ldots, n$:

$$S_t = b_{0,k} \cdot y_{t,k} + u_{k,t}, \quad u_{k,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_k^2\right).$$

The OLS estimates of these regressions, and their standard errors, are used to calibrate mean and variances of the prior distributions.

3.1.2 Prior on A

Defining $A \equiv \begin{bmatrix} A_1 & \ldots & A_p \end{bmatrix}$, the prior on $a = vec\left(A'\right)$ is multivariate Normal, centered on $\mathbf{0}$, and with diagonal variance V_a resembling a Minnesota prior. In particular, it holds that

$$V_a = Diag\left(\begin{bmatrix} \widehat{\sigma}_{y,1}^2 \\ \vdots \\ \widehat{\sigma}_{y,n}^2 \end{bmatrix}\right) \otimes \begin{bmatrix} \Psi_1 & 0 & \ldots & 0 \\ 0 & \Psi_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \Psi_p \end{bmatrix}, \forall \ell \quad \Psi_\ell = \frac{\lambda_a}{\ell^d} \cdot Diag\left(\begin{bmatrix} \widehat{\sigma}_{F,1}^{-2} \\ \vdots \\ \widehat{\sigma}_{F,r}^{-2} \end{bmatrix}\right),$$

where $\widehat{\sigma}_{y,j}^2$ and $\widehat{\sigma}_{F,s}^2$ are the residual variances of a univariate AR(1) for, respectively, each variable j and each factor s (computed using the prior mean of B_0). λ_a is a tightness parameter, and d is a decay parameter. We chose standard calibration borrowing from the VAR literature, i.e. $\lambda_a = 0.2$ and $d = 2$.

3.1.3 Prior on Γ_ℓ

The prior distribution on the AR coefficients in the matrices Γ_ℓ, collected in the column vector γ (see the Appendix for details), is a multivariate Normal distribution. In the spirit of a Minnesota prior, we choose an a priori unitary mean for the first lag of each variable whose dynamics resemble a random walk, and a zero mean for the higher lags. Regarding the a priori covariance matrix, we assume no correlation across coefficients of different lags and variables, and we set a prior structure for the variances which resembles the Minnesota prior, using the tightness and decay parameters:

$$\bar{\gamma} = \begin{bmatrix} \bar{\gamma}_1 \\ \bar{\gamma}_2 \\ \vdots \\ \bar{\gamma}_q \end{bmatrix} = \begin{bmatrix} \mathbf{1}_{n\times 1} \\ \mathbf{0}_{n\times 1} \\ \vdots \\ \mathbf{0}_{n\times 1} \end{bmatrix}, \qquad V_\gamma = \lambda_\gamma \cdot Diag\left(\begin{bmatrix} 1^{-d} \\ 2^{-d} \\ \vdots \\ q^{-d} \end{bmatrix} \right) \otimes I_n.$$

Considering the smaller number of elements in γ in our empirical application, we chose a less tight prior calibration than in the case of A , i.e. $d = 1$ and $\lambda_\gamma = 1$.

3.1.4 Prior on Ω_t

The prior for the elements of G in expression (2), the matrix containing the stable covariances among the MAI errors, is a multivariate Normal distribution centered at zero, with large diagonal covariance matrix.

The priors to produce inference on the elements of σ_t in (4) are set as follows. The prior for σ_0 is a multivariate Normal, centered at $\begin{bmatrix} \widehat{\sigma}^2_{y,1} & \widehat{\sigma}^2_{y,2} & \dots & \widehat{\sigma}^2_{y,n} \end{bmatrix}'$ estimated as standard deviations of univariate AR(1) residuals on each observable, with identity covariance matrix, as in Primiceri (2005). The prior distribution for the innovation covariance matrix Q_σ is calibrated as in Primiceri (2005).

3.2 Gibbs Sampler

This subsection describes each step of the Gibbs Sampler (GS) used to simulate from the joint posterior distribution of both parameters $\theta = \{\gamma, A, B_0, G, Q_\sigma\}$ and unobservable states $\{\sigma_t\}_{t=1}^T$ of the MAI-AR-SV model. Moreover, for volatility estimation the Omori et al. (2007) procedure requires drawing the indexes of Normal components of a mixture approximating a $\log \chi^2_1$ distribution labeled as $\{S_t\}_{t=1}^T$. This MCMC estimation approach is needed as the joint posterior distribution cannot be analytically determined. The algorithm innovates with respect to Carriero et al. (2020) since it introduces the step to draw the AR coefficients, and it builds upon Carriero et al. (2016) to draw the reduced rank structure of the VAR coefficients ($A \cdot B_0$), and on Primiceri (2005) and Del Negro & Primiceri (2015) to draw the time-varying volatilities.

The steps are the following:

1. Draw a history of volatilities $\{\sigma_t\}_{t=1}^T \Big| \theta, \{S_t\}_{t=1}^T,$
2. Draw $\theta, \{S_t\}_{t=1}^T \,|\, \{\sigma_t\}_{t=1}^T$. This second step is further split as follows:
   (a) Draw the elements in $\theta | \, \{\sigma_t\}_{t=1}^T$
      i. Draw the covariance of volatilities' innovations $Q_\sigma \Big| \gamma, A, B_0, G, \{\sigma_t\}_{t=1}^T$
      ii. Draw the AR coefficients $\gamma \Big| A, B_0, G, Q_\sigma, \{\sigma_t\}_{t=1}^T,$
      iii. Draw the loadings $A \Big| \gamma, B_0, G, Q_\sigma, \{\sigma_t\}_{t=1}^T,$
      iv. Draw the factor weights $B_0 \Big| \gamma, A, G, Q_\sigma, \{\sigma_t\}_{t=1}^T,$
      v. Draw the off-diagonal elements in $G \Big| \gamma, A, B_0, Q_\sigma, \{\sigma_t\}_{t=1}^T$
   (b) Draw a history of indexes of the mixture in $\{S_t\}_{t=1}^T | \theta, \{\sigma_t\}_{t=1}^T$

It is important to note that steps *a* and *b* must be performed in this order, as they deliver a draw from the joint posterior of parameters and mixture states via a conditional/marginal factorization. Note also that steps *iii* and *iv* have $y_t - \mathcal{X}_t \cdot \gamma$ as dependent variable in order to draw A and B_0, while in step *ii* we use $y_t - A \cdot Z_t$ to draw the AR coefficients, where

$$Z_t \equiv (I_p \otimes B_0) \cdot vec\left(\begin{bmatrix} y_{t-1} & \dots & y_{t-p} \end{bmatrix}\right), \quad \mathcal{X}_t = \begin{bmatrix} Diag(y_{t-1}) & \dots & Diag(y_{t-q}) \end{bmatrix}.$$

The Gibbs Sampler is described in more detail in section A of the Appendix. Section C of the Appendix shows some Convergence Diagnostics for the estimated model.

4 The Global Component of Inflation Volatility

4.1 Data

Following the literature on global inflation developments in advanced economies (e.g. Ciccarelli & Mojon, 2010 and Borio & Filardo, 2007) we collected a panel of Consumer Price

Indices for a set of 20 OECD countries[7], downloaded from the *OECD main economic indicators* database. The dataset includes 240 observations at quarterly frequency, covering the period from 1960-Q1 to 2019-Q4. We then constructed inflation rates as year on year changes of the indexes[8]. Apart from being inconsistent with the related literature that focuses on advanced economies, including emerging economies in our dataset would have substantially reduced the sample size, limiting the long-run scope of the analysis.

4.2 Model specification

As a first step in the analysis, and motivated by previous literature and preliminary evidence, we find that estimating a MAI model with a single global factor ($r = 1$) captures very well the commonality (i.e. the slow moving global comovements) of the inflation rates, as in a factor model but starting from a VAR representation that allows for more complex dynamics. Yet, estimation of a simple MAI model with constant volatilities produces residuals that appear very autocorrelated and heteroskedastic. Adding the individual AR components to the MAI model, and hence estimating the MAI-AR model, generally eliminates residual autocorrelation, but there remains evidence of heteroskedasticity (based on the correlogram of the standardized residuals and their squares reported in the Appendix). Therefore, as a final step, we add SV to capture the empirically clear phenomenon of changing conditional volatilities in inflation rates, particularly evident over the rather long sample that we are dealing with. The standardized residuals of the MAI-AR-SV model, and their squares, show no significant autocorrelation (see the Appendix), supporting the use of this specification, which in addition has the best in-sample fit among the alternatives considered. This specification process also revelead that $p = q = 6$ is the lowest lag length for, respectively, the MAI and AR components, resulting in no significant autocorrelation in the associated standardized residuals and their squares.

In order to have a broad idea of how our model compares with factor models we have

[7]USA, Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK

[8]Ciccarelli & Mojon (2010) use Year on Year changes of CPI inflation rates for the bulk of their analysis. O'Reilly & Whelan (2005) adopt the same transformation stressing that is cited in the ECB's official inflation mandate. Lodge & Mikolajun (2016) point out that using YoY changes in CPI is preferable since this transformation produces no seasonal pattern by construction. Nevertheless, we have repeated all the estimations using the Quarter on Quarter transformation, which displays more variations at higher frequencies. The evidence of commonality is similar to those obtained via the YoY transformation, even though posterior variance is generally higher. Results are available upon request.

estimated the following Dynamic Factor Model with Stochastic Volatility (DFM-SV):

$$
\begin{aligned}
y_{i,t} &= B_i \cdot F_t + e_{i,t} \\
e_{i,t} &= \sum_{\ell=1}^{p} \rho_{i,\ell} \cdot e_{i,t-\ell} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim N(0, \sigma_{i,t}) \\
\ln(\sigma_{i,t}) &= \mu_{\sigma,i} + \phi_{\sigma,i} \ln(\sigma_{i,t-1}) + q_{\sigma,i} \cdot \eta_{i,t} \\
F_t &= \mu_F + \sum_{\ell=1}^{p} \beta_\ell F_{t-\ell} + \omega_t, \quad \omega_t \sim N(0, \sigma_{F,t}) \\
\ln(\sigma_{F,t}) &= \mu_{\sigma,F} + \phi_{\sigma,F} \ln(\sigma_{i,t-1}) + q_{\sigma,F} \cdot \upsilon_t \\
&\left[\upsilon_t, \eta_{1,t}, \ldots, \eta_{n,t}\right]' \underset{iid}{\sim} N(\mathbf{0}, I_{n+1})
\end{aligned}
$$

Note that the volatilities of the factor and of the individual components have completely separate laws of motion. The model above features one global factor, therefore it is a one-factor version of the model by Mumtaz & Theodoridis (2017) (which contains other types of factors needed to fit a much more elaborate dataset). We refer to Mumtaz & Theodoridis (2017) for details about estimation.[9] The Appendix shows that the common factor and associated stochastic volatility from the DFM-SV and the MAI-AR-SV are very similar. The two models have a very similar in sample fit but the MAI-AR-SV delivers more precise estimates of the common level and volatilities, and we conjecture that this is because it does not need to filter out an unobservable factor. Moreover, the estimated MAI-AR-SV residuals, and even more their squares, show cleaner autocorrelograms than their counterparts from the DFM-SV (see the Appendix).

4.3 Common component and global shock transmission

The average explained variance of the MAI-AR-SV is about 94% (about 73% without the AR components). In particular, the MAI-AR-SV specification is able to capture both the low and the high frequency variation of each inflation series, due to the presence of both common and country-specific autoregressive components.

Figure 3 reports the inflation rates for each country along with the posterior bands and median of the estimated common global inflation factor, obtained from the MAI-AR-SV specification. The estimated global factor is similar to the first principal component (PC) of the inflation rates. The first PC is used to form the prior on the B_0 coefficients but the prior variance we set on B_0 is large enough that the detected similarity is data

[9] We have used a modified version of the code available from the authors' website, with the same lag length and similar shrinkage as in our MAI-AR-SV model.

driven rather than dictated by the prior. Also, the global factor captures very well the low-frequency variation that in the univariate context of Stock & Watson (2007) is driven by the permanent component (stochastic trend). Moreover, an average of our estimated common components is also broadly comparable to an OECD measure of global inflation.

The results so far are in line with the findings of Ciccarelli & Mojon (2010), even though their sample stops in 2008. As reported also by Ferroni & Mojon (2016), our analysis suggests strong commonality in inflation developments across OECD continues also in the more recent period, and, actually, it has been particularly high during the last financial crisis[10].

We now use a simple local projection analysis to assess how a shock to the global factor is transmitted across countries. Figure 4 reports the responses to a unitary impulse to ω_t, the innovation of the global factor's law of motion. We observe a similar hump shaped pattern in most countries. With few exceptions, on impact and in the first periods after the impulse, national inflation tends to increase, and then after some quarters it reverts back to the impact response or even to lower values in some countries. This evidence shows how a global shock to headline inflation rates induces a similar response over a set of advanced economies, reinforcing our understanding of the degree of commonality.

Finally, using the decomposition of equation (12), in which reduced form residuals of each variable are decomposed into their common and individual components, we computed the cross-country correlations among the individual errors. We found only very small and slightly negative correlations across countries (with an average of -0.05), confirming that the individual errors represent mostly country-specific dynamics.

4.4 Levels decomposition and persistence

Using the level decomposition discussed in section 2.4, we are able to decompose the observed inflation series of each country into orthogonal components driven, respectively, by common and individual shocks. Moreover, for each country we measure how much variation is explained by each component.

Figures 5 reports the level decomposition of inflation rates into common and individual components, compared with the actual series. The common components tend to explain more than 50% of almost all countries' inflation rates, and are particularly important in large economies like the US, UK, Germany and Japan.

Stock & Watson (2007) discuss the persistence of US inflation, using as a measure of persistence the largest autoregressive root of the levels' process. Inference about this measure of persistence is made possible by the Stock (1991) method, which is appropriate

[10]Using a shorter sample of inflation rates (1993-2014), Ferroni & Mojon (2016) find that the fraction of national inflation rates' variance that is explained by Global Inflation remains dominant.

when dealing with series displaying high levels of persistence. Stock & Watson (2007) do not find strong evidence of persistence changes in US inflation from the 1970s onwards, reporting the largest AR root of US CPI inflation comprised between 0.85 and 1.05 (as 90% confidence interval). O'Reilly & Whelan (2005) report little evidence of instability for inflation persistence in the Euro Area since the 1970s; they report rolling confidence intervals for the largest AR root of Euro Area CPI inflation that are centered around 0.9 across almost the entire sample.

In light of this literature, using the entire sample, we computed the 90% confidence intervals (CI) for the largest AR roots of all national CPI inflation series, of their common and individual components, and of the global factor. Figure 6 compares the CI for the largest AR root of the observed series, their components and the global factor, separately for each country. The picture clearly shows how the common global components tend to preserve the high persistence of the observed series, while the individual country-specific components display wider confidence intervals centered on slightly smaller values. The global factor shows a very narrow CI centered on 0.99.

These results are in line with what reported by Ciccarelli & Mojon (2010), who argue that "*the global component captures the most persistent and possibly nonstationary part of inflation*". Indeed, using a different methodology, they report smaller persistence for the so called "national" components; interpreting such results, they consider the global factor as an attractor and the main driver of persistence coming from the observed data. However, for this specific exercise they use annualized quarter on quarter inflation rates, which is a transformation that tends to display a smaller degree of persistence than the year on year transformation. Performing our analysis using quarter on quarter CPI changes, we measure a degree of persistence in line with Ciccarelli & Mojon (2010) for both global and national inflation components.

Finally, in order to check whether our results, and in particular the extent of commonality among inflation rates, would be different if treating the inflation rates as integrated variables, we performed the following exercise. First, we computed the cumulated first PC obtained from the first differences of inflation rates. In the Appendix it is compared with the first PC of the levels (our benchmark), and it is shown that they are very similar. Given our Bayesian context, the PC affects our results only as it determines the prior distributions of the elements of B_0. Therefore, as a second step, we used the cumulated PC to calibrate the prior distributions of the elements in B_0. The resulting first moments of the priors were very close to the benchmark ones, but the prior variances were much higher because of the smaller precision of estimation with first differenced data. Hence, we prefer to stick to the original specification in levels.

4.5 Time-varying volatility decomposition

Figure 7 reports the posterior bands of the estimated conditional inflation volatilities of all countries for the MAI-AR-SV, along with the decomposition discussed in Section 2. The estimated volatilities display a relevant degree of commonality. Indeed, the first principal component of the volatilities explains on average about 50% of their variation.

To better understand what is driving the volatilities, Figure 8 presents the contribution of the estimated common and individual components in determining volatilities. It turns out that the contribution of the common component is non trivial, reaching values above 50% for some countries and time periods, especially during the last decades, in particular during the Great Recession. In the 1970s the contribution of the global component to stochastic volatility is generally smaller than in the Great Recession, standing for a smaller degree of cross-country synchronization of changes in the unexplained residuals' size.

In this multi-country context, it is complex to understand the structural drivers of the common inflation volatility component. However, for a single country this can be done. Carriero et al. (2020), focusing on the US, find that supply shocks are particularly important, with demand shocks ranked second and monetary/financial shocks third. We will see later on that the importance of supply factors is also confirmed for other countries.

Figure 9 shows the posterior bands of the global factor volatility, that is $(\Xi_t)_{t=1}^{T}$. Global inflation volatility was moderate during the 1960s, increased dramatically during the 1970s before the sharp reduction starting in the 1980s associated with the change in monetary policy to fight inflation occurred in several countries. These results are in line with the US inflation volatility estimated by Stock & Watson (2007). Global inflation volatility has remained very low until mid 2000s, reaching a new spike during the Great Recession, before turning back to the historically low values in the last years of the sample. Time variation is significant and relatively large throughout the entire estimation sample.

4.6 Temporal Stability

Considering that large low-frequency movements characterize mostly the first part of the sample, to check for possible changes in the importance of the global component, we have re-estimated the models for the subsamples before and after 1995 (1960-1994 and 1995-2019). The Appendix contains the split-samples counterpart of level decompositions, and a table with the share of country-specific variances explained by the common component over the different samples. Even though the second subsample is shorter and does not contain the global disinflation trend (low frequency movement) of the 1980s and early 1990s, we still find an important degree of commonalities since 1995, though in general lower except for a couple of countries (Greece and Portugal), stressing that the global component has remained important in capturing common dynamics in the most recent period.

As a further stability check, we also recursively estimated the MAI-AR-SV model since 2004, by running multiple chains adding each time a new quarter to the sample (on a server, exploiting CPU parallelization). In the Appendix, we present figures with posterior medians and credible regions for the loadings in the A matrices and the factor weights in B_0. It turns out that recursive estimates are relatively stable over time, and stability is not impaired significantly when the crisis observations of 2008-2009 enter into the sample.

This stability evidence reinforces the validity of our modeling strategy, and it is in line with a recent contribution by Chan & Eisenstat (2018), that shows how multivariate modeling of macro variables does not benefit from time-varying parameters once stochastic volatility is taken into account.

4.7 Possible drivers of global inflation's level and volatility

While there can be many drivers of the global inflation factor, Figure 10 shows that after the 90's it is correlated with the Chinese PPI inflation rate and the Oil inflation rate, with correlations around 0.7 and 0.3 respectively. To compute Oil inflation, we use the WTI price ($/barrel), while Chinese PPI inflation is available only from the early 1990s.

To obtain more reliable results on the role of these two variables as drivers of global inflation, we have regressed the global factor on its own lags and on the simultaneous and lagged values of the Chinese PPI and Oil inflation rates, starting the sample in the early '90s due to data availability. The results indicate that the regressors are significant, with the share of explained variance increasing to more than 90% when Chinese PPI and Oil inflation are both included in the specification, compared to roughly 75% in the AR model, and less autocorrelated residuals. If either of these regressors is included separately and individually, the variance explained reaches about 86%, pointing still to an increased explanatory power and to their equivalence. Adding to such regressions the contemporaneous and lagged OECD global output gap does not increase the explanatory power. These findings are also qualitatively confirmed in regressions with differenced variables and suggest that supply factors are more relevant than demand ones as drivers of global inflation dynamics[11].

[11] Using quarterly data from the 1990s within an empirical Phillips-Curve estimation framework, Forbes (2018) provides evidence that four global factors (global slack, commodity prices, exchange rates and the degree of global price competition among producers) contribute to national inflation dynamics, even though quite heterogeneously across countries. Moreover, the evidence provided shows that global factors have not become as important in explaining core inflation dynamics as they have for headline CPI and that the explanatory power of different global factors is very heterogeneous across time and across countries.

To further explore some other relevant sources of global inflation fluctuations, we regressed the innovation to the common component, ω_t , on simultaneous and lagged values of two estimated exogenous innovations to the US economy: the Gertler & Karadi (2015) measure of monetary policy shocks and the Beaudry & Portier (2006) measure of TFP news shock[12]. The evidence shows that such important exogenous shocks relative to the US economy, which typically explain a large share of business cycle fluctuations according to Ramey (2016), have almost no explanatory power for global inflation innovations. Miranda-Agrippino & Rey (2020) also find that US monetary policy shocks explain a very small share of inflation variance.

Next, to understand which global forces may correlate with global CPI inflation volatility, we estimated stochastic volatilities from univariate AR-SV models for Oil inflation, the Chinese PPI inflation and the global output gap. A comparison of the estimated (median) volatilities is reported in Figure 11. From visual inspection, a positive co-movement between Oil and global CPI inflation volatility stands out, showing a correlation around 0.5 from the early 1990s. Also the Chinese PPI inflation volatility displays a positive correlation with that of global CPI: the correlation is around 0.8 from the early 1990s. The volatility of the output gap is instead much lower, except for the recessionary period.

To provide additional evidence, we have regressed the (changes in the) volatility of the global inflation factor on its lags and on the contemporaneous and lagged (changes in the) volatility of Oil, Chinese PPI and output gap, either separately or jointly. The largest explanatory power is achieved by the volatility of the Chinese PPI, with an R^2 of about 0.74 with respect to about 0.3 for the AR model. Hence, overall, we find evidence for the volatility of Chinese PPI as a driver of the volatility of global inflation, with the Global Output Gap as second best.

So far we have analyzed separately the drivers of level and volatility of global inflation. Joint evaluation leads to the same conclusions. In particular we have conducted a joint analysis by modelling the global inflation factor, Oil inflation, Chinese PPI inflation and global output gap with a VAR model with stochastic volatility. The estimation results confirm the relevance of lags of Oil and Chinese PPI in the equation for the global inflation factor, not of global output gap. Moreover, the first principal component of the four estimated stochastic volatilities explains about 90% of their movements since the early 1990s, even though the stochastic volatility of the global output gap has almost zero weight.

These findings are in line with our results.

[12]Ramey (2016) reviews and discusses several shocks driving US economic fluctuations. The Gertler & Karadi (2015) monetary policy shocks are identified at high-frequency using federal funds futures, and then aggregated at lower frequencies. The Beaudry & Portier (2006) series of news shock to productivity is identified via short and long run restrictions using data on stock prices.

4.8 Commonality in core inflations

In light of the correlation (both in levels and volatilities) between the global component of headline CPI inflation and Oil, it is important to detect how much core components of the CPIs remain correlated across countries. To this end, the same exercises of this section have been performed using the non-Food and non-Energy Consumer Prices Indices for the same set of countries, downloaded from the *OECD main economic indicators* database. These data are available only from the late '70s onwards.

Non-Food and non-Energy inflation tends to display a lower degree of commonality, already from a quick graphical inspection[13]. Results of the decomposition are collected in Appendix. They indicate a smaller importance of the common component both in volatilities and in levels: the global core inflation factor explains roughly 40% of the variability of core CPI inflation levels, while the average (across countries) share of stochastic volatility explained by the global component spans from 10% to 20% throughout the sample[14]. The fact that core inflation dynamics remain mostly heterogenous across countries, leaves ample scope for domestic monetary policies.

5 Forecasting Inflation with the MAI-AR-SV model

To provide further evidence on the usefulness of the MAI-AR-SV as a model for multi-country inflation, we now evaluate its out of sample properties, also in comparison with a set of standard competitors.

Using the same inflation series employed in the structural analysis, several models are recursively estimated and used to produce 1- up to 8-quarter-ahead forecasts over a sample going from 1990Q1 to 2019Q4. The models under evaluation are: AR, AR-SV, VAR, VAR-SV, MAI-AR, MAI-AR-SV, DFM-SV.

All models are estimated using Bayesian techniques. AR and VAR priors are constructed using the standard Litterman (1986) a priori assumption of univariate random walk processes. The SV prior in all models except the DFM-SV is calibrated as in Pri-

[13]The lower degree of commonality may also stand for a low degree of synchronization between national core inflation rates. Indeed, less volatile components of the CPI may be less responsive to global changes in goods' prices, and most importantly their time response can be heterogenous across countries. This phenomenon is less evident in headline inflation rates since the most volatile components of prices are more synchronous over the world and determine a non-trivial amount of variation in goods' prices.

[14]To understand whether the different results we got for headline and core inflation rates could be due to the different samples, we have also repeated the analysis for headline inflation using the same sample as for core (i.e., from 1979 onwards). The results are qualitatively similar to what illustrated in the previous subsections. There is a decline in the fraction of variance explained by the global inflation factor to around 50%, but this value is still about two times larger than that for the core inflation rates. The decline in the explanatory power of the global inflation factor over the shorter sample can be attributed to the global relevance of oil price fluctuations in the '70s.

miceri (2005). The MAI priors are specified as in Section 3. The priors for the DFM-SV are calibrated mostly as in Mumtaz & Theodoridis (2017), but they have been adjusted for the factor loadings and the factor's law of motion in order to make them comparable to the MAI models.

Diagnostics are then computed for both point and density forecasting, following the evaluation framework of Clark & Ravazzolo (2015). Specifically, to evaluate the accuracy in terms of point forecasting, we compute the forecasts posterior medians for all vintages, models, variables and horizons. Then, we compute the Root Mean Squared Forecast Error (RMSFE) for each model, variable and horizon, using the variation across vintages. Hence, for each variable $j \in \{1, \ldots, n\}$, each horizon $h \in \{1, \ldots, H\}$ and each model $m \in \{1, \ldots, M\}$ we compute:

$$RMSE_{j,h}^{m} = \sqrt{\frac{1}{T^*} \sum_{t=T+1}^{T+T^*} \left(y_{j,t+h} - \widehat{y}_{j,t+h}^{m}\right)^2},$$

where $\widehat{y}_{j,t+h}^{m}$ is the median of the posterior distribution $\left(\widehat{y}_{j,t+h}^{m,i}\right)_{i=1}^{L_c}$ (L_c is the length of the discretized posterior distribution). To test for significance of the squared forecast errors differences across models, we compute the Diebold & Mariano (1995) t-tests for equality of the average loss [15].

To evaluate models in terms of density forecasting, we use two measures of accuracy: the average log-predictive score and the average Continuous Ranked Probability Score (CRPS). Even in this case, to test for significantly different performances we employ the Diebold and Mariano test, following Clark & Ravazzolo (2015).

Log Predictive Scores are obtained via non-parametric kernel smoothing density estimators. Adopting a normal kernel $\mathcal{K}_{\mathcal{N}}(\cdot)$ and following an optimal selection strategy of the bandwith parameter $\widehat{\mathcal{H}}$, we can compute for each variable, model, horizon and vintage the empirical density evaluted at the actual observation $y_{j,t+h}$, that is:

$$\widehat{f}_m\left(y_{j,t+h}, \widehat{\mathcal{H}}\right) = \frac{1}{\widehat{\mathcal{H}} \cdot L_c} \sum_{i=1}^{L_c} \mathcal{K}_{\mathcal{N}}\left(\frac{y_{j,t+h} - \widehat{y}_{j,t+h}^{m,i}}{\widehat{\mathcal{H}}}\right).$$

Then, applying logarithms and computing the average across forecasting vintages yields

[15] Monte Carlo evidence in Clark & McCracken (2011) and Clark & McCracken (2015) indicates that, with nested models, the Diebold-Mariano test compared against normal critical values can be viewed as a somewhat conservative (conservative in the sense of tending to have size modestly below nominal size) test for equal accuracy in the finite sample.

the average log score for each variable, model and horizon:

$$\overline{logScore}_{j,h}^{m} = \frac{1}{T^*} \sum_{t=T+1}^{T+T^*} \log \widehat{f}_m \left(y_{j,t+h}, \widehat{\mathcal{H}} \right).$$

To compute the average CRPS, following Clark & Ravazzolo (2015), we first compute the CRPS per each variable, model, horizon and vintage, making use of the actual observations, the posterior distribution $\left(\widehat{y}_{j,t+h}^{m,i}\right)_{i=1}^{L_c}$ and a random permutation of the latter $\left(\widehat{y}_{j,t+h}^{m,i'(i)}\right)_{i=1}^{L_c}$ where $i' : \{1, \dots, L_c\} \to \{1, \dots, L_c\}$ is randomly drawn without replacement. Lastly, we simply compute the average across time vintages:

$$CRPS_{j,t+h}^{m} = \frac{1}{L_c} \sum_{i=1}^{L_c} \left| \widehat{y}_{j,t+h}^{m,i} - y_{j,t+h} \right| - \frac{1}{2 \cdot L_c} \sum_{i=1}^{L_c} \left| \widehat{y}_{j,t+h}^{m,i} - \widehat{y}_{j,t+h}^{m,i'(i)} \right|,$$

$$\overline{CRPS}_{j,h}^{m} = \frac{1}{T^*} \sum_{t=T+1}^{T+T^*} CRPS_{j,t+h}^{m}.$$

Figure 12 portrays the relative performance of the competing set of models against the benchmark MAI-AR-SV model, for each country and two selected horizons. Models' point forecasting performance is reported as a ratio between their own Root Mean Squared Errors and the benchmark's, so that values larger than one imply that the MAI-AR-SV produces more accurate point forecasts. The MAI-AR-SV model improves significantly upon its counterparts on most variables, especially at short horizons, even though in a smaller number of cases this is reversed. The AR-SV and the DFM-SV show competitive point forecasting performance, especially at longer horizons. The highly parametrized VARs generally achieve a lower degree of point forecasting accuracy than the benchmark[16].

Moving to density forecast evaluation, Figure 13 reports the relative average log predictive scores for the chosen set of models and horizons. Alternative models' performance is reported in terms of log-scores differences with the benchmark MAI-AR-SV, so that negative values favor the MAI-AR-SV. The benchmark model generally improves upon its competitors: the difference is negative and significant in most cases. Also in this case, the AR-SV and the DFM-SV remain competitive alternatives, especially at longer horizons. Figure 14 shows the CRPS reported comparatively as a ratio, where values greater than one indicate a worse density forecasting performance with respect to the MAI-AR-SV. Results are in line with the log-scores, with the benchmark model improving significantly upon most of its competitors[17].

To conclude, our evidence shows that the MAI-AR-SV is a good forecasting model

[16]Tables 1a and 1b in the online appendix report detailed results.

[17]More detailed forecasting results are reported in Appendix B.

for inflation rates. The introduction of SV is particularly relevant to improve density forecasts. This evidence is in line with findings reported by Clark & Ravazzolo (2015) and D'Agostino et al. (2013). Moreover, the introduction of the MAI component helps increasing the forecasting power. Indeed, the benchmark model attains a higher degree of forecasting accuracy with respect to the standard unrestricted VAR estimated using a Minnesota Prior as shrinkage device. The DFM-SV with a single unobservable factor proves to be a valid alternative, particularly at longer horizons. Adding a second unobservable factor deteriorates considerably the performance of the DFM-SV.

6 Conclusions

Global developments play an important role in the determination of inflation rates, and indeed earlier literature has found that a substantial amount of the variation in a large set of national inflation rates can be explained by a single global factor. This literature has typically neglected inflation (conditional) volatility, while volatility is clearly relevant both from a policy point of view and for structural analysis and forecasting.

In this paper we study the evolution of inflation rates in many countries, using a novel model that allows for commonality in both levels and volatilities, in addition to country-specific components. We find that allowing for inflation volatility is indeed important, and a large fraction of it can be attributed to a global factor that is also driving the inflation levels.

While other sources can be behind this global factor, it turns out that since the early '90s it is strongly correlated with the Chinese PPI and to a lesser extent with Oil prices. Moreover, also the global factor stochastic volatility is highly correlated with that of Chinese PPI.

Repeating the same analysis on core inflation rates for the same set of OECD countries, the model finds a smaller but non-negligible degree of commonality. The substantial national component of core inflation level and volatility leaves ample scope for domestic monetary policies.

The MAI-AR-SV shows also very good out of sample properties, achieving comparable or better forecasting performance for inflation when compared with a set of prominent alternative models, especially in terms of density forecasting.

Finally, as we have seen that the MAI-AR-SV model can be reparametrized as an (observable) factor augmented VAR with stochastic volatility, it has a wide range of applicability in empirical macroeconomics.

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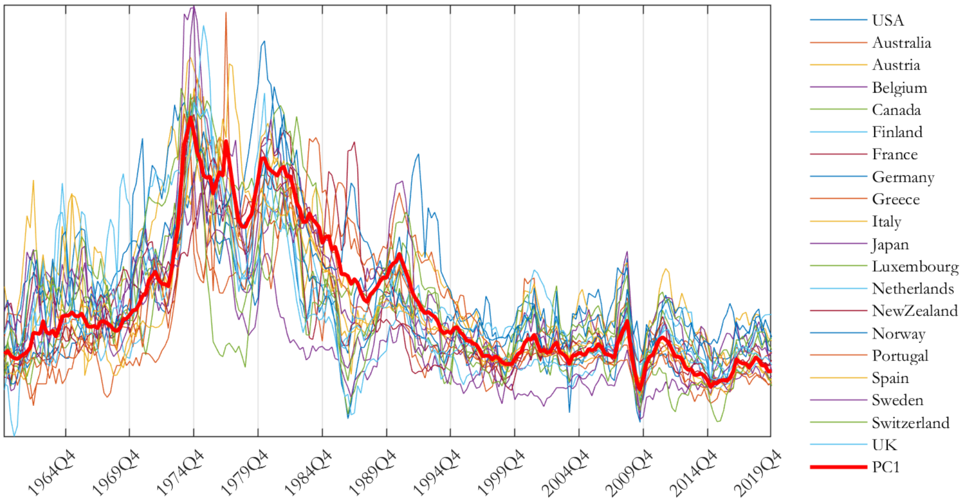

Figure 1: Inflation rates and their first principal component (thick red line)

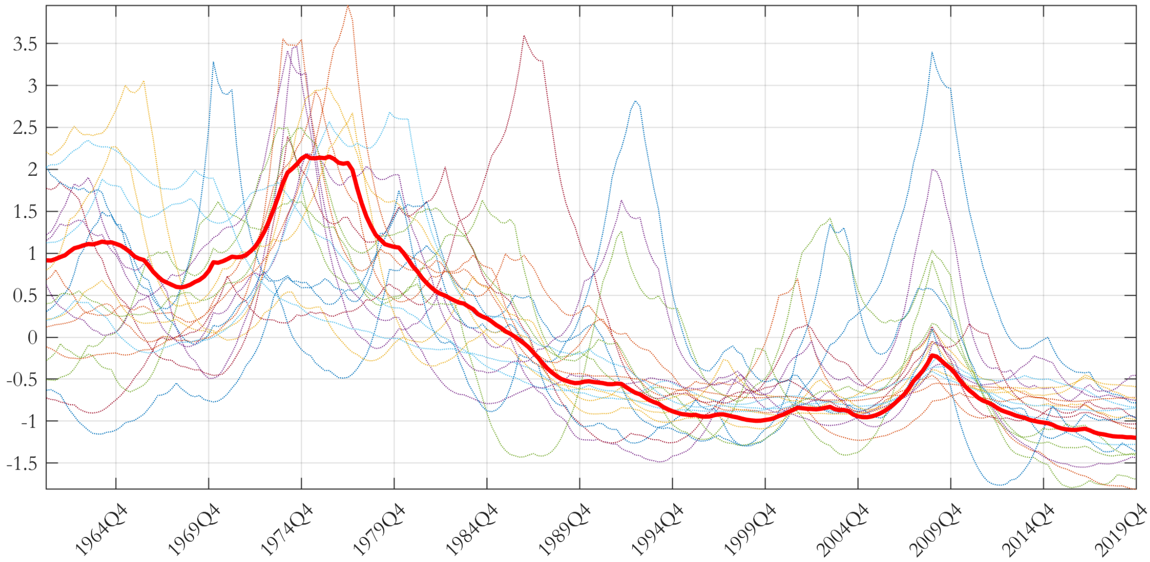

Figure 2: CPI inflation rates SVs from univariate AR-SV, and their first PC

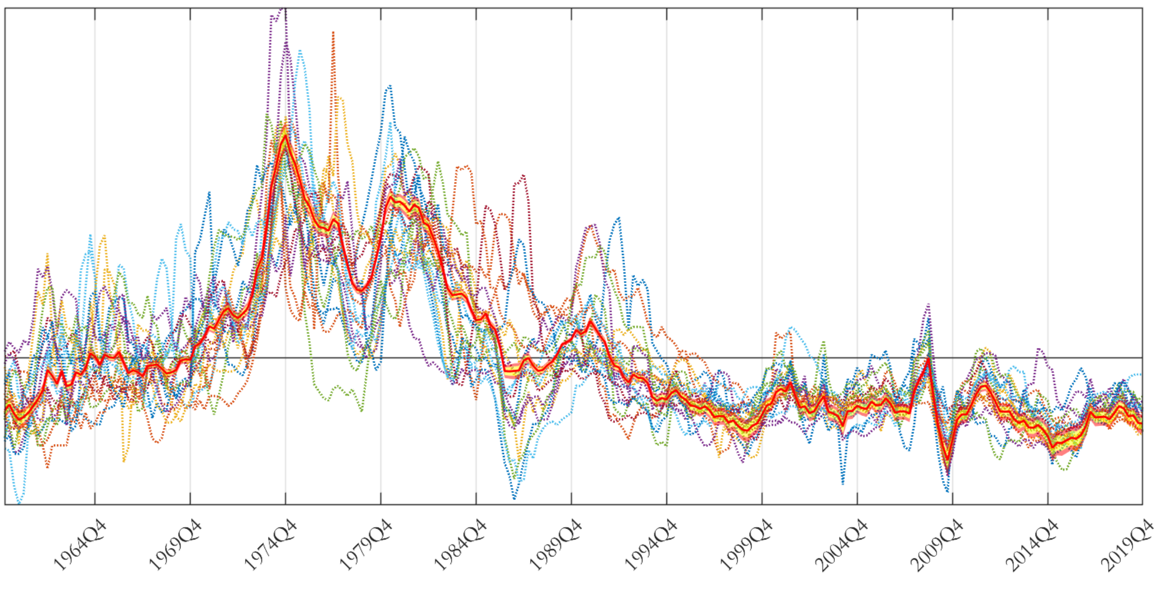

Figure 3: MAI-AR-SV estimated common factor (with posterior bands) Vs Data

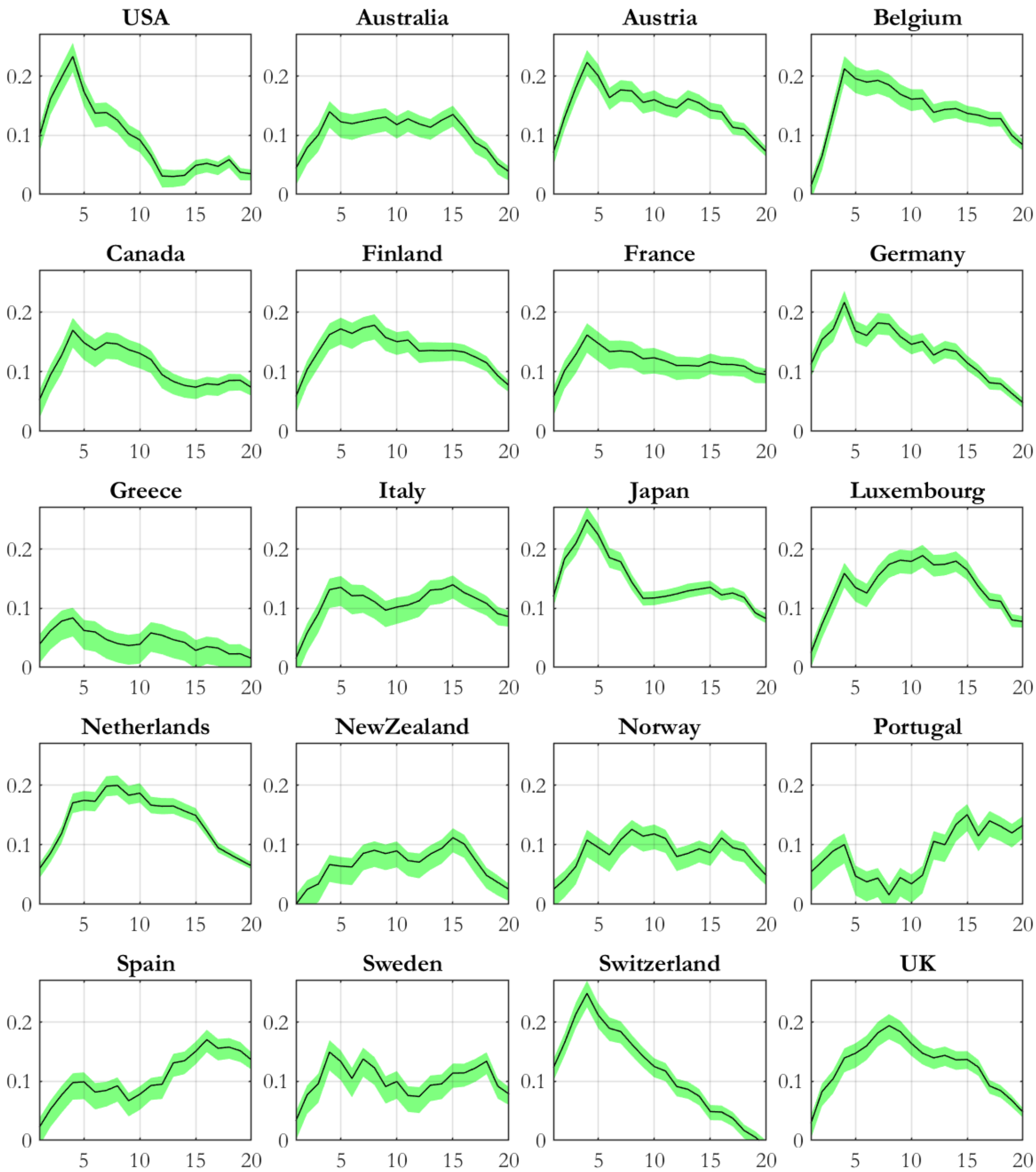

Figure 4: Responses of country inflation rates to a unitary impulse of the global inflation factor

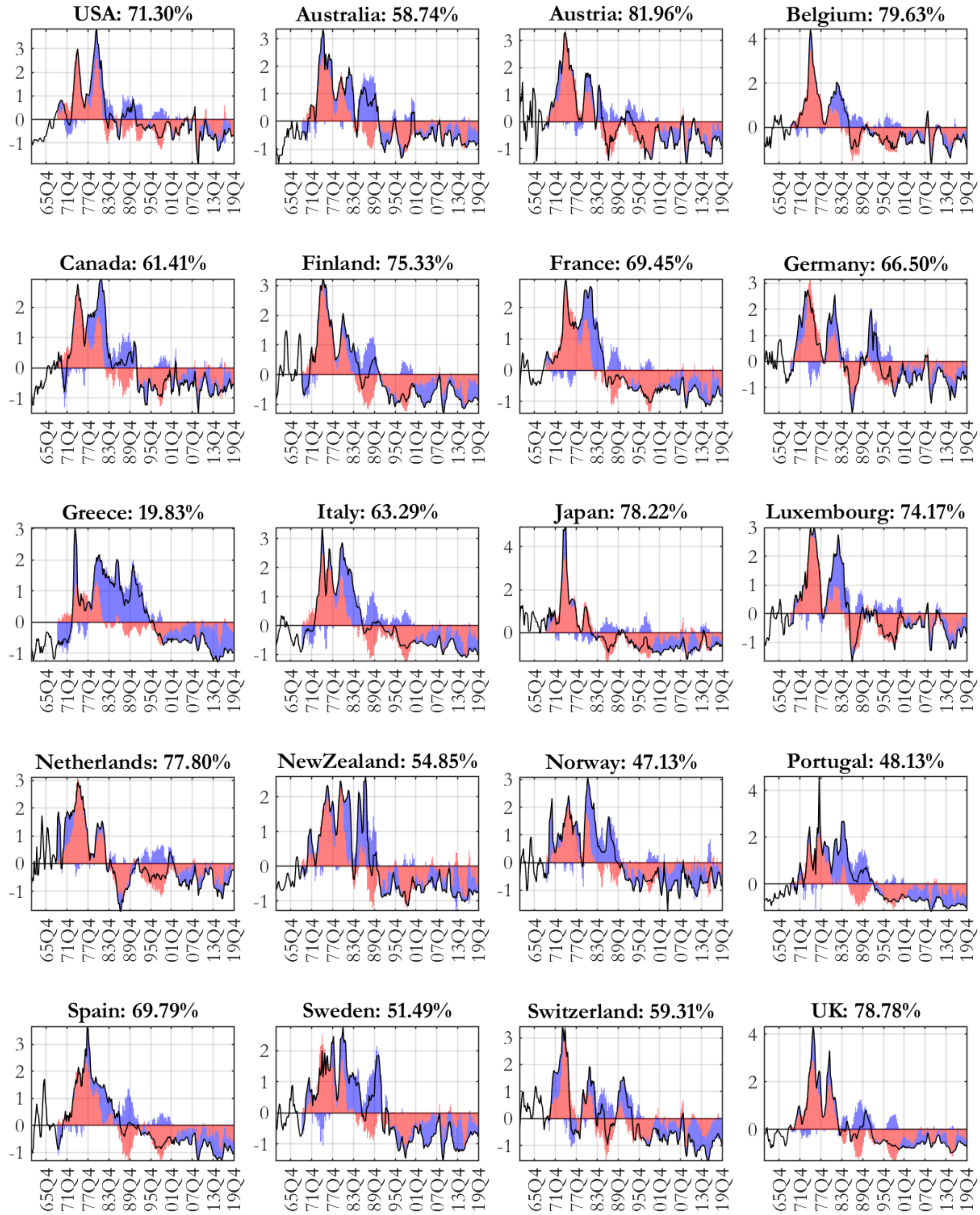

Figure 5: MAI-AR-SV, Inflation levels decomposition into common (red) and individual (blue) components

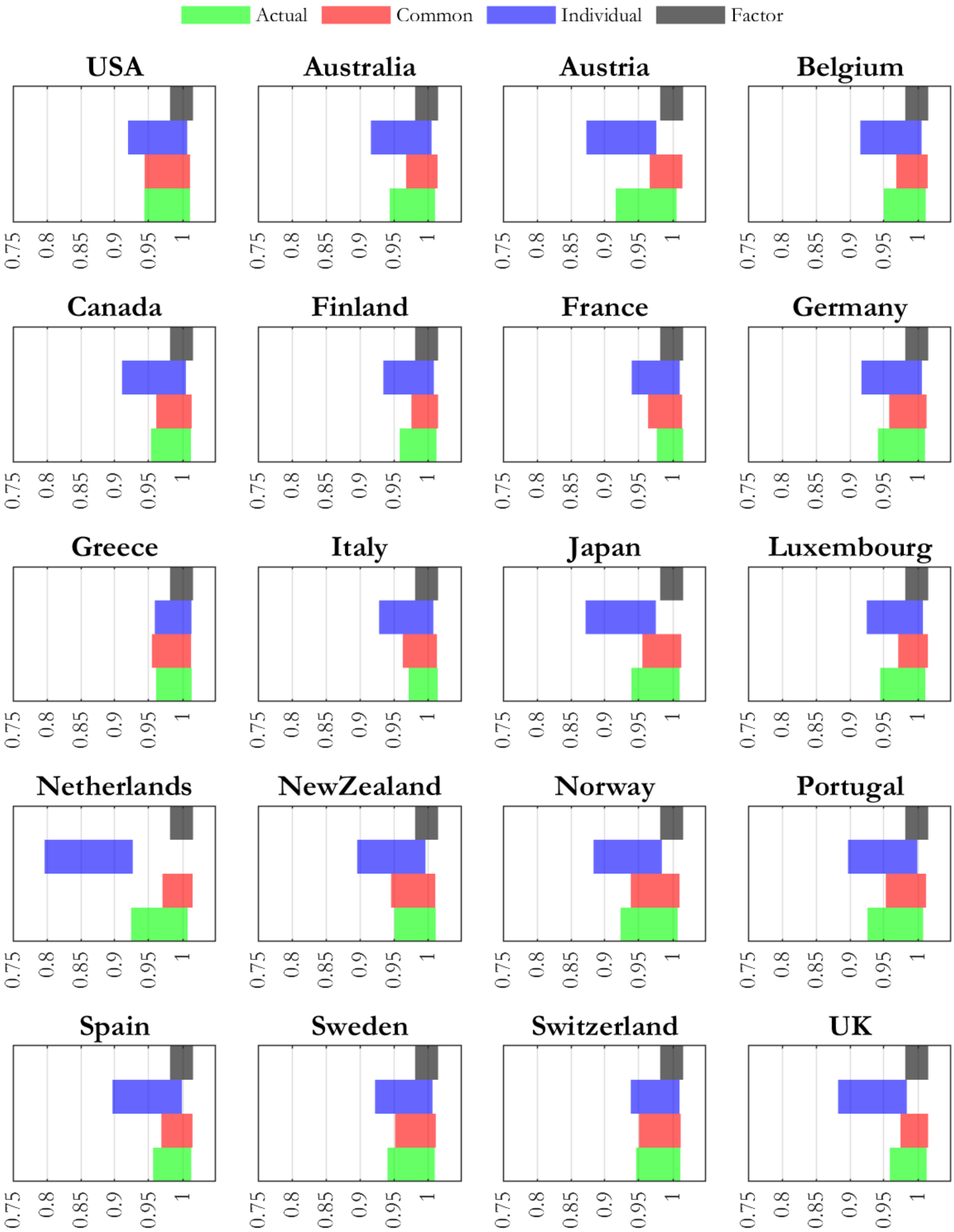

Figure 6: Largest Autoregressive Root (90% confidence intervals) CPI inflation levels, components, and global factor.

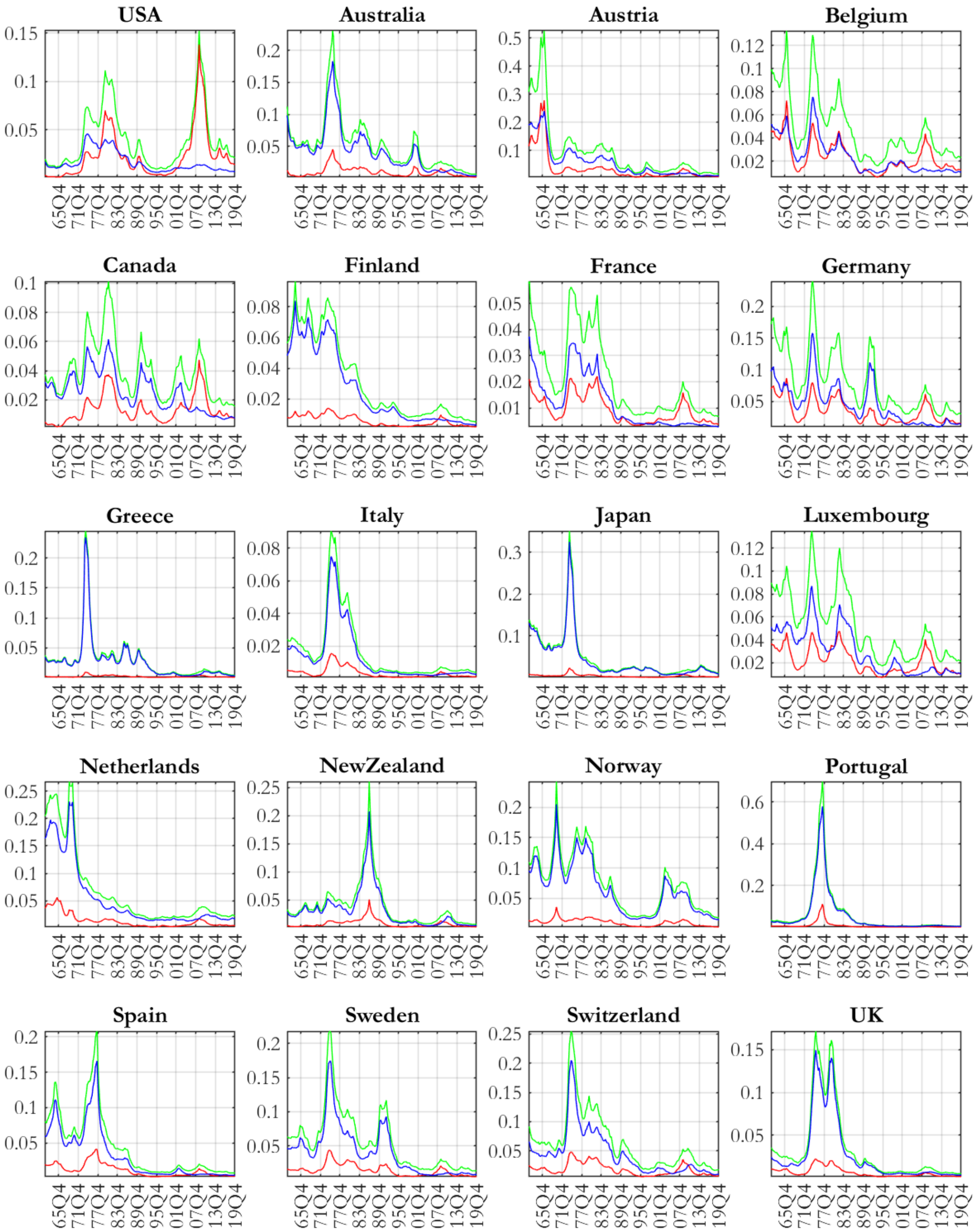

Figure 7: MAI-AR-SV, Residuals' Volatility, TV decomposition, Common (red), individual (blue), total (green)

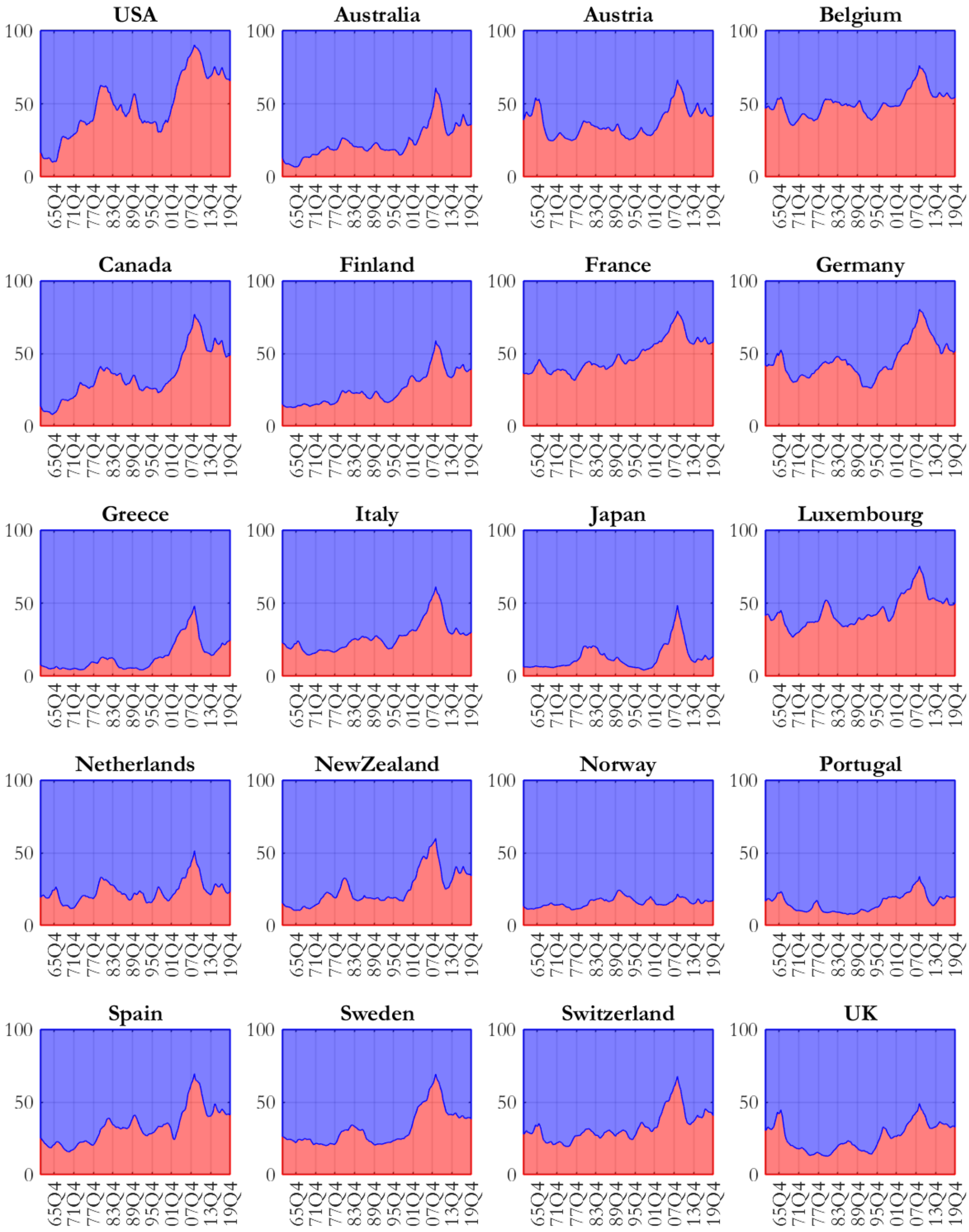

Figure 8: MAI-AR-SV, Residuals' Volatility, TV decomposition shares (%), Common (red), individual (blue)

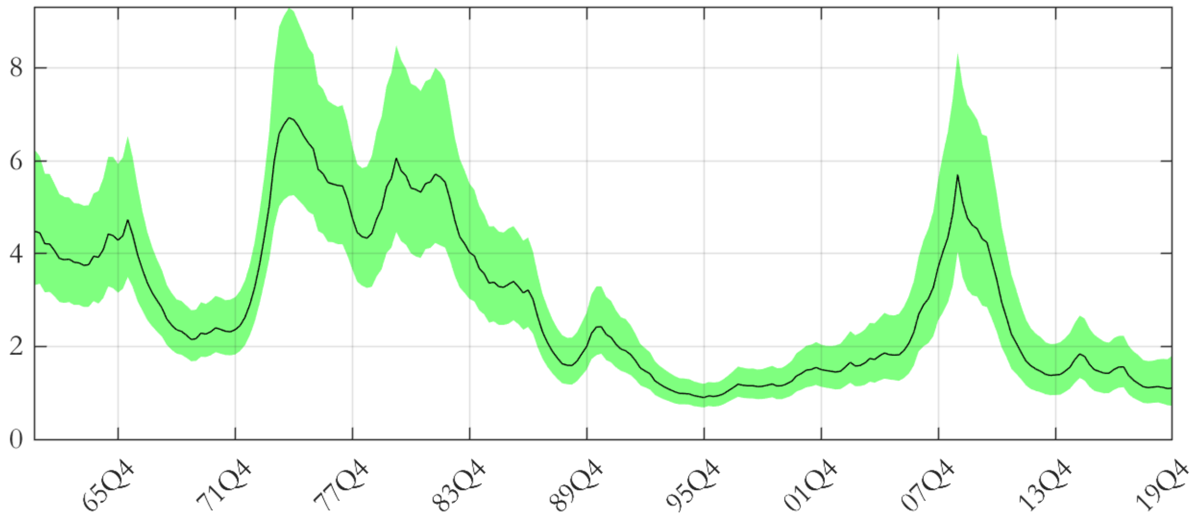

Figure 9: MAI-AR-SV, Global Factor Volatility, Posterior bands.

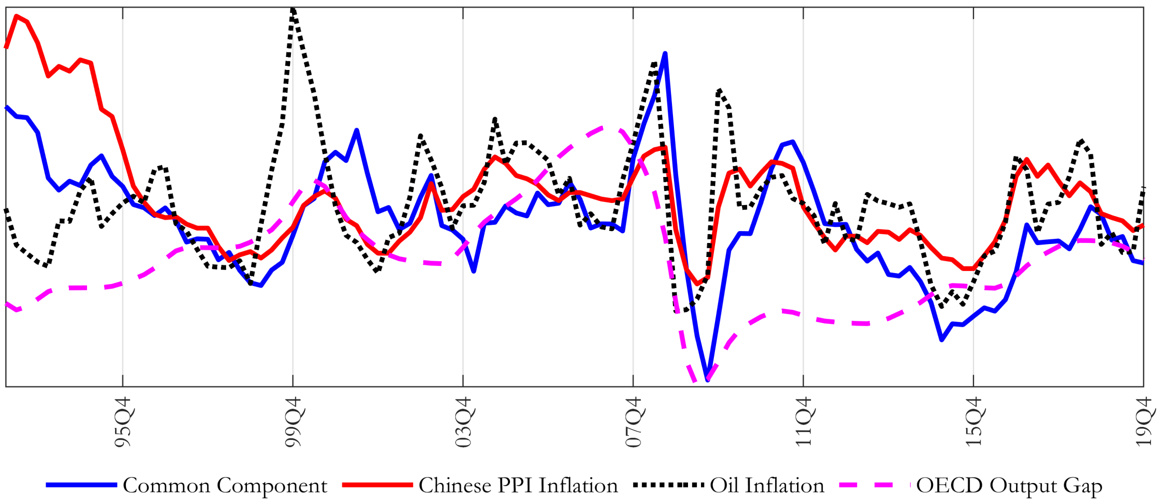

Figure 10: Comparing MAI component with Chinese PPI, Oil Inflation and Global Output Gap

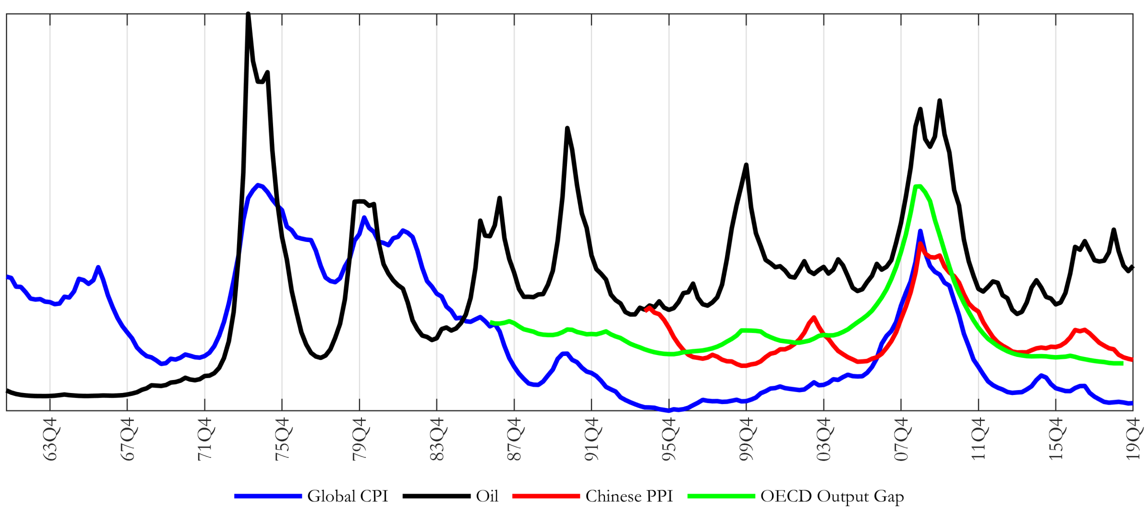

Figure 11: Median Volatilities of Global Factor (MAI-AR-SV), Oil inflation (AR-SV), Chinese PPI (AR-SV) and Global Output Gap (AR-SV)

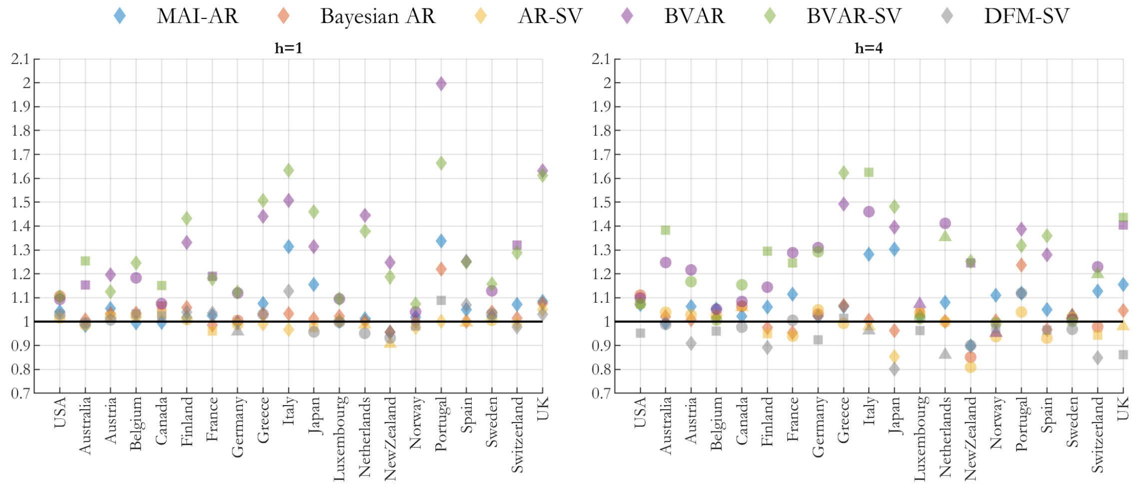

Figure 12: Relative Root Mean Squared Forecast Errors (ratios with MAI-AR-SV)

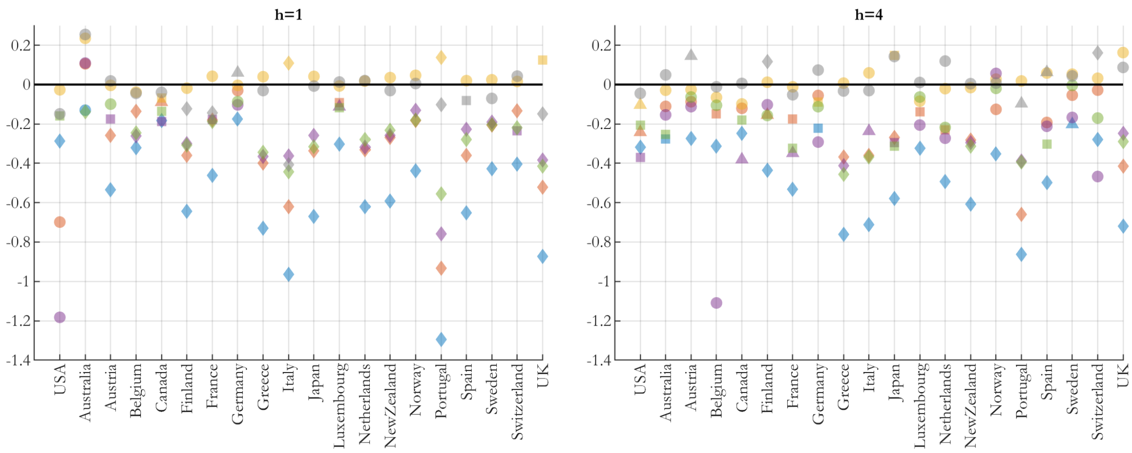

Figure 13: Relative Log Predictive Scores (differences with MAI-AR-SV)

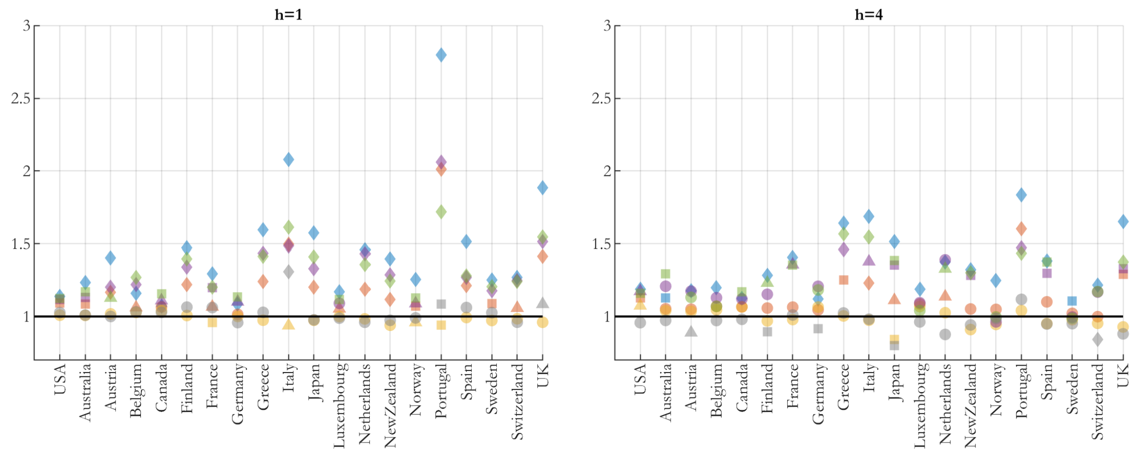

Figure 14: Relative Continuous Rank Probability Scores (ratios with MAI-AR-SV)

The filled marker is of different shape when the criterion is significantly difference between models, according to the Diebold Mariano t- statistic.

Not significant | Significant at 10% | Significant at 5% | Significant at 1%