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Managing the tragedy of commons and polluting emissions: a unified view*

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Abstract

We address the issue of regulating both polluting emissions through a generic tax and access to a common resource pool in a dynamic oligopoly game. Our analysis shows that once industry structure is regulated so as to induce the industry to harvest the resource in correspondence of the maximum sustainable yield, social welfare is either independent or decreasing in the tax if firms do not invest in abatement technologies, while, if they do, the policy maker may increase the tax to foster both individual and aggregate green research and development to attain abatement technologies, ideally up to the level at which emissions and the associated environmental damage are nil. This also allows us to detect the arising of the win-win solution associated to the strong form of the Porter hypothesis. We extend the analysis to encompass product differentiation and monopolistic competition, to show that qualitatively analogous conclusions obtain.

Keywords: OR in environment and climate change; sustainability; differential games; green research and development; Porter hypothesis

1 Introduction

The analysis of natural resource extraction is, most frequently, disjoint from the analysis of global warming as a result of anthropic activities. Yet, there obviously exists an intimate connection between the two, irrespective of whether the resources at stake are renewable ones or not, as producing

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final consumption goods involves both, and production and consumption jointly contribute to the accumulation of polluting emissions. Indeed, there exists a relatively small literature including a few notable contributions which have proposed a joint analysis of renewable resource and pollution dynamics in optimal control models sometimes accomodating also endogenous growth (see Tahvonen, 1991; Tahvonen and Kuuluvainen, 1991, 1993; Ayong Le Kama, 2001; and Wirl, 2004). All of these focus on the assumptions of (i) a representative consumer either autonomous or flanked by a policy maker, and (ii) a perfectly competitive economic system

However, the global economy is supplied by industrial sectors populated by large firms endowed with non-negligible degrees of market power. Hence, a dynamic game encompassing the dynamics of pollution and that of natural resources facing the impact of firms exploiting them to supply the global market should be part of the research agenda, and possibly at the top of it. On the basis of these considerations, we propose a comprehensive view of the impact of an oligopolistic industry on a renewable resource used to produce consumption goods implying polluting emissions, complemented by a policy analysis.

To this purpose, we will illustrate a differential oligopoly game in which the linear dynamics of polluting emissions¹ combines with the logistic growth of a renewable resource as in the VLV model (Verhulst, 1838; Lotka, 1925; Volterra, 1931). In this respect, it is worth adding that several recent developments of the debate about the economic exploitation of renewables have relied on either a piecewise linear ('tent-shaped') or simply a linear approximation of the logistic growth function appearing in the VLV setup.² These approaches, which have delivered a large spectrum of valuable results, among which a detailed analysis of the continuum of feedback strategies and the arising of voracity, either disregard the maximum sustainable yield altogether, because it does not exist if the population dynamics is linear, or transform it in a non-differentiable point in the piecewise linear case.

This paper builds upon the extension of the VLV model investigated in Lambertini and Leitmann (2019). There, it is illustrated that, the nonlinear dynamics of the renewable resource notwithstanding, the differential oligopoly game of resource extraction yields a subgame perfect (or strongly time consistent) equilibrium under open-loop information, in correspondence of which the policy maker may regulate access to the common pool resource in such a way to induce the industry to locate itself arbitrarily close to the maximum sustainable yield, ensuring thus a sustainable exploitation path in the long run. In this respect, our analysis is connected with the debate about the optimal number of

¹This the same appearing, e.g., in Benckroun and Long (1998) and uncountably many others. For an overview of the related literature, see Lambertini (2013, ch. 8, and 2017, ch. 7).

²Models of the first type appear in Benckroun (2003, 2008), Colombo and Labrecciosa (2013a, 2015) and Benckroun and Long (2016), while some of those belonging to the second can be found in Fujiwara (2008), Colombo and Labrecciosa (2013b) and Lambertini and Mantovani (2014, 2016).

firms in the commons (see, *inter alia*, Cornes *et al.*, 1986; Mason *et al.*, 1988; and Mason and Polasky, 1997). This result is also shown to hold in the perfectly competitive case, where the exogenous market price delivered by an infinitely elastic demand becomes an additional regulatory tool in the hands of the public authority in order to achieve the desired outcome.

Here, we expand the model to account for the environmental impact of production and/or consumption of the related final good, and the analysis of the equilibrium welfare involving profits, consumer surplus and the environmental balance consisting of the residual resource stock and the convex environmental damage caused by polluting emissions. To complete the picture, we admit the presence of an emission tax and consider the possibility for firms to invest in green research and development (R&D henceforth) efforts for emission abatement.

A few remarks are in order concerning the use of the emission tax by the regulator. As is well known, under imperfect competition the efficient tax does not induce firms to exactly internalise the marginal environmental damage, due to the presence of extraprofits and consumer surplus. That is, any demand function less than perfectly elastic poses a serious problem to the authority because maximising welfare is not equivalent to fully internalising the externality. Indeed, Buchanan (1969) and Barnett (1980) have shown that the welfare-maximising tax falls short of the marginal environmental damage in monopoly, and the same happens, in general, under oligopolistic behaviour (see Simpson, 1995; and Katsoulacos and Xepapadeas, 1995), with few exceptions emerging under cost asymmetries across firms and in correspondence of the limit properties of the free entry equilibrium. The fact is, in brief, that using emission taxation to maximise welfare leads, more often than not, to a compromise driven by the well known tradeoff between consumer and producer surplus on one side and the environmental balance on the other.

Our approach offers a new perspective about this long standing issue, as, instead, we discover that regulating entry to drive industry harvest in correspondence of the maximum sustainable yield partially or completely sterilises tax policy (if firms only react through output) or allows the authority to use the tax in order to increase (indeed, maximise) the greenness of technology (if firms implement R&D project under the stimulus provided by the emission tax itself). The intuition behind this result is that regulating access to the commons in order to achieve harvesting at the maximum sustainable yield amounts to regulating individual and total output, thereby disconnecting the tax from the remaining part of the environmental balance, namely, the damage caused by emissions. All of this is proved keeping the model as general as possible, and specifying only the essential qualitative properties of the building blocks, such as market demand, production costs, a convex damage function, and the system of differential equations governing the renewable resource and firms' emissions.

Then, we revert to a fully specified setup through which these results are generated by augmenting

in several directions the basic model, which initially accounts for a Cournot industry supplying a homogeneous good produced at constant returns to scale through the extraction of a renewable resource. The basic layout indeed implies that regulating access to attain harvesting at the maximum sustainable yield fully sterilises the emission tax, whose only effect is purely redistributive. As soon as one admits the presence of (i) decreasing returns, (ii) product differentiation and (iii) R&D for emission abatement, the scenario engendered by the model changes significantly. In particular, adding R&D investments to the initial version of the model suffices to yield the most relevant insight this paper offers, since in this case aggregate R&D efforts increase monotonically in the tax and the regulator may use it to decrease both emissions and the resulting environmental damage. Furthermore, (i) there exists a unique emission tax at which emissions and the damage are nil and still profits and consumer surplus are strictly positive; and (ii) this tax rate exceeds the welfare-maximising tax.

The resulting tradeoff posed to the policy maker prompts a verification of the Porter hypothesis (Porter, 1991; Porter and van der Linde, 1995)³ in its strong form, according to which firms subject to environmental regulation react by going green, which delivers a win-win solution whereby industry profits go up together with social welfare. Indeed, we prove this holds true by comparing the equilibrium profit and welfare levels across the first two settings just mentioned. The tradeoff between welfare and greenness disappears once R&D investments are just high enough to reduce emissions to zero, because emission taxation disappears as well.

Analogous considerations also apply in correspondence of the equilibrium outcome of the differential game associated with the full model in which firms offer differentiated varieties and operate under decreasing returns to scale, as well as in the alternative scenario in which, all else equal, we assume the presence of monopolistic competition.

The remainder of the paper is structured as follows. Section 2 illustrates the layout of the model in its general version, to derive the main results independently of specific assumptions concerning demand, cost and damage functions, and also discusses the implications of the Ramsey rule, which disappears as soon as one focusses upon the degenerate feedback solution under open-loop information. The specialised oligopoly game with product homogeneity and constant returns to scale is in section 3, while further extensions to decreasing returns, product differentiation and monopolistic competition are in section 4. Section 5 contains a few concluding remarks and briefly indicates avenues for future research.

³The stream of theoretical and empirical research on the Porter hypothesis is too large to be accounted for comprehensively here (for exhaustive overviews, see Lanoie *et al.* 2011, Ambec *et al.*, 2013, and Lambertini, 2017). To the best of our knowledge, the only dynamic model in this vein is that of Xepapadeas and de Zeeuw (1999), where firms change their capital stocks under the stimulus exerted by emission taxation.

2 Resource extraction and polluting emissions

Consider a market existing over continuous time $t \in [0, \infty)$, being supplied by $n \geq 1$ identical firms exploiting a renewable resource $X(t)$ to produce a homogeneous final good sold to consumers. The dynamics of the natural resource is as in the VLV model,

$$\dot{X}(t) = \delta X(t) [1 - \beta X(t)] - \gamma Q(t) \quad (1)$$

in which β , γ and δ are positive constants, and $Q(t) = \sum_{i=1}^n q_i(t)$ is the sum of the n firms' individual harvest at any time t . Through an appropriate choice of measure, $q_i(t)$ and $Q(t)$ are also the instantaneous individual and industry output levels.

Production and/or consumption are responsible of polluting emissions, whose total amount at any instant is $S(t) = \sum_{i=1}^n s_i(t)$, $s_i(t)$ being the emission level imputed to $q_i(t)$, in such a way that we have n additional state equations of the following form:

$$\dot{s}_i(t) = \nu q_i(t) - \eta s_i(t) \quad (2)$$

where ν and η are positive constants. Before proceeding any further, a few words are appropriate to justify this choice, which excludes the double commons case. Indeed, we could have modelled the latter by stipulating the existence of a single state $S(t)$ being determined by the production/consumption of industry output $Q(t)$, in such a way that firms should account for a single state equation $\dot{S}(t) = \nu \sum_{i=1}^n q_i(t) - \eta S(t)$. Accordingly, firms would then be subject to a taxation levied on $S(t)$. Indeed, this choice would replicate the qualitative nature of our main results, with a *caveat* related to the role of green R&D, which would appear in the pollution dynamics as a public good, since the contribution of any single firm in terms of emission abatement would alleviate the tax burden borne by all firms alike. This would thus give rise to a scenario which would not be adherent to casual observation of firms' behaviour. A brief sketch of this alternative formulation of the problem, confined to the linear Cournot model, is in the Appendix.

Back to our model, the first scenario envisages Cournot competition with homogeneous goods. Accordingly, we assume that, at any t , firms face a single inverse demand function $p(Q(t))$, with $\partial p(Q(t))/\partial Q(t) < 0$ and $\partial^2 p(Q(t))/\partial Q^2(t) \geq 0$ (the latter condition ensures concavity of instantaneous profits, see Dixit, 1986). Moreover, firms share the same technology summarised by a cost function $C_i(q_i(t))$, with $\partial C_i(q_i(t))/\partial q_i(t) > 0$ and $\partial^2 C_i(q_i(t))/\partial q_i^2(t) \geq 0$, i.e., marginal cost is non-decreasing. Then, the regulator imposes a tax levied on each individual firm's emissions, in such a way that the instantaneous profit function of firm i is $\pi_i(t) = p(Q(t)) q_i(t) - C_i(q_i(t)) - \tau s_i(t)$.

Firm i has choose harvest $q_i(t)$ so as to maximise the discounted profit flow $\Pi_i(t) = \int_0^\infty \pi_i(t) e^{-\rho t} dt$ under the constraint posed by (1). The Hamiltonian function of firm i is therefore

$$\mathcal{H}_i(t) = p(Q(t)) q_i(t) - C_i(q_i(t)) - \tau s_i(t) + \lambda_{ii}(t) [\nu q_i(t) - \eta s_i(t)] \quad (3)$$

$$+ \sum_{j \neq i} \lambda_{ij}(t) [\nu q_j(t) - \eta s_j(t)] + \mu_i(t) [\delta X(t) (1 - \beta X(t)) - \gamma Q(t)]$$

to be maximised w.r.t. $q_i(t)$, the initial conditions being $X_0 = X(0) > 0$ and $s_{i0} = s_i(0) \geq 0$ for all $i = 1, 2, \dots, n$.

Suppose firms operate under open-loop information. The first order condition (FOC) taken w.r.t. q_i is (henceforth, we omit the explicit indication of the time argument):

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = \frac{\partial \pi_i}{\partial q_i} + \lambda_{ii} \nu - \mu_i \gamma = 0. \quad (4)$$

To ease the exposition of the ensuing discussion, we may reformulate the profit function as $\pi_i = \tilde{\pi}_i - \tau s_i$, where $\tilde{\pi}_i = p(Q) q_i - C_i(q_i)$ is the instantaneous profit function in the absence of taxation. This allows us to rewrite the above FOC as follows:

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = \frac{\partial \tilde{\pi}_i}{\partial q_i} + \lambda_{ii} \nu - \mu_i \gamma = 0 \quad (5)$$

since only gross profits are involved in the choice of q_i .

The associated system of the $n + 1$ costate equations is

$$\dot{\lambda}_{ii} = (\eta + \rho) \lambda_{ii} + \tau \quad (6)$$

$$\dot{\lambda}_{ij} = (\eta + \rho) \lambda_{ij}, \quad \forall j \neq i \quad (7)$$

$$\dot{\mu}_i = [\delta (2\beta X - 1) + \rho] \mu_i \quad (8)$$

This implies that the costate dynamics associated with λ_{ij} and μ_i is described by a differential equation in separable variables, admitting the solutions $\lambda_{ij} = 0$ and $\mu_i = 0$ at all times. This is not true for λ_{ii} except in the special case in which $\tau = 0$, i.e., if emission taxation is permanently absent. However, the game is linear in the vector $s = (s_1, s_2, \dots, s_n)$ of emissions, and state-redundant w.r.t. the resource stock X . These two properties imply that the open-loop solution of the game is indeed subgame perfect (or, strongly time consistent), although the structure of the game is not linear in one of the state variables (cf. Dockner *et al.*, 2000).⁴

Before proceeding with the discussion of the solution of the oligopoly game, it is useful to define the welfare function, as follows:⁵

$$SW = \sum_{i=1}^n \pi_i + CS + TI - D + X \quad (9)$$

⁴This property of the VLV model was originally pointed out by Goh *et al.* (1974). See also Leitmann (1973).

⁵This definition of the welfare function accounts for the combination of industry profits, consumer surplus and the *environmental balance* $X - D$. This formulation captures the nature of the problem posed to the policy maker, which may face a tradeoff between the traditional components of welfare and the environmental ones. The expression in (9) appears in too many contributions to list them all here, but see, *inter alia*, Lambertini (2013).

in which $CS = \int_0^{Q^{CN}} p(Q) dQ$ is consumer surplus, amounting to the integral of the demand function up to the Cournot-Nash industry output Q^{CN} ; $TI = \tau S = \tau \sum_{i=1}^n s_i$ is the income generated by emission taxation, which is redistributed to society (for example, in the form of infrastructures and public services); and $D = \zeta S^2$ is the environmental damage, convex in aggregate emissions. The definition of SW in (9) applies at all times, as well as in steady state.

2.1 The Ramsey rule and its implications

We set out to solving the game supposing $\mu_i \neq 0$. This case generates a Ramsey rule governing the economic exploitation of the natural resource, as is well known since Gordon (1954), Smith (1969), Clark (1975), Peterson and Fisher (1977), Berck and Perloff (1984), Clark (1990) and many others. The main difference between these contributions and ours is that we specify the harvesting strategy as a quantity while in the aforementioned references the harvest is defined as a production function in terms of the stock and a labour input, which typically implies multiple equilibria at the intersection between the resulting harvest curve and the concave locus defining the reproduction rate of the stock.⁶

To obtain the Ramsey rule, we may manipulate (4) and (6-8). From (4), we have

$$\dot{\mu}_i = \frac{\dot{\pi}_i}{\gamma} \quad (10)$$

where $\pi'_i = \partial \pi_i / \partial q_i$; $\dot{\pi}_i = \partial \pi'_i / \partial t$ and $\mu_i = \pi'_i + \lambda_{ii} \nu$. Solving (6) we obtain $\lambda_{ii} = \nu \tau / (\delta + \rho)$, with the integration constant set to zero. Then, noting that the r.h.s. of (8) and (10) must coincide, and substituting λ_{ii} with the above expression, delivers the following:

$$\dot{\pi}_i = \frac{[\pi'_i (\delta + \rho) + \nu \tau] [\delta (2\beta X - 1) + \rho]}{\delta + \rho} \quad (11)$$

which substitutes the usual control equation that would describe the dynamics of q_i . The stationarity condition $\dot{\pi}_i = 0$ is satisfied by the solutions of the system

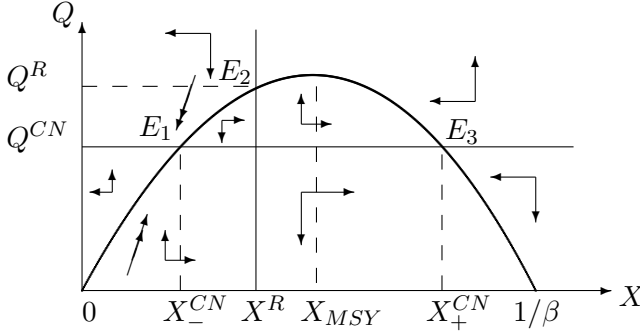
$$\begin{aligned} \pi'_i (\delta + \rho) + \nu \tau &= 0 \\ \delta (2\beta X - 1) + \rho &= 0 \end{aligned} \quad (12)$$

While the first equation yields the market-driven output level(s), the second yields the Ramsey solution at $X^R = (\delta - \rho) / (2\beta\delta)$ which depends on discounting and the parameters characterising the VLV equation. X^R is the same as in Lambertini and Leitmann (2019, p. 2), and is admissible iff $\delta > \rho$. In such a case, the existence of this solution dictated by the Ramsey rule prevents the industry from

⁶An alternative but largely analogous approach underpins the poach-and-trade model in Damania and Bulte (2007), where agents solve a static game delivering the optimal strategies which, unlike what happens in the present model, are determined by a Schaefer production function.

harvesting at the maximum sustainable yield (*MSY*), $X_{MSY} = 1/(2\beta)$, since $X_{MSY} > X^R$ for all $\rho > 0$.

Figure 1 The phase diagram under Cournot competition, $\delta > \rho$

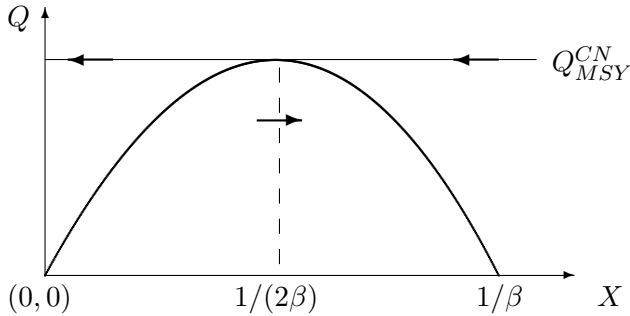


The corresponding phase diagram in the space (X, q) is illustrated in Figure 1, which replicates the analogous graph in Lambertini and Leitmann (2019, p. 3). The arrows identify the saddle path to the stable steady state in E_1 , to the left of the Ramsey solution along the vertical line at X^R . Hence, we may turn our attention to the scenario in which, the Ramsey rule being absent, firms may indeed play or be induced to play strategies achieving collective harvesting at the *MSY*.

2.2 The degenerate feedback solution

We are now in a position to outline the equilibrium of the game engendered by posing $\lambda_{ij} = 0$ and $\mu_i = 0$ at all times and to infer a key property of the model. Recalling the main result in Lambertini and Leitmann (2019), concerning the possibility for the authority to regulate access to the commons to force the industry to harvest in correspondence of the *MSY*, we may conclude that (i) if indeed the authority grants access to a specific number of firms, $n_{MSY}(\tau)$, accounting for the presence of the emission tax, then the industry output $Q_{MSY}^{CN}(n_{MSY}(\tau))$ is indeed ‘fixed’ to ensure the attainment of the *MSY* at any time, as in Figure 2 (which corresponds to Figure 3 in Lambertini and Leitmann, 2019, p. 4), where the arrows illustrate the fact that the tangency solution at the *MSY* is semi-stable.

Figure 2 Harvesting at the *MSY*



Firms solve (5) to find the optimal individual quantity and then also its control equation. Moreover,

again from (5), we have the expression of the shadow value attached by firm i to its own emissions, $\lambda_{ii} = \frac{1}{\nu} \frac{\partial \tilde{\pi}_i}{\partial q_i}$, which can be differentiated w.r.t. time to obtain $\dot{\lambda}_{ii} = \frac{d\tilde{\pi}'/dt}{\nu}$, where $d\tilde{\pi}'/dt \equiv d(\partial \tilde{\pi}_i / \partial q_i) / dt$. The r.h.s. of the above expression must be equal to the r.h.s. of (6), and consequently the expression of the optimal shadow value is $\lambda_{ii} = (d\tilde{\pi}'/dt + \nu\tau) / [\nu(\eta + \rho)]$, which in turn implies that increasing the tax pressure will decrease individual and industry optimal outputs, respectively defined as $q^{CN}(n, \tau)$ and $Q^{CN}(n, \tau)$ at any time, and of course in steady state.

Now the matter is that the regulator wants firms to harvest at the MSY at all times to preserve the biological stock forever, and therefore we may plug $X = 1/(2\beta)$ into (1) and then impose $n = n_{MSY}(\tau)$ in such a way that $Q_{MSY}^{CN} = \delta/(4\beta\gamma)$. Here it is worth noting that this number of firms will necessarily be increasing in τ , i.e., $\partial n_{MSY}(\tau)/\partial\tau > 0$ in order to keep Q_{MSY}^{CN} constant, since, as noted above, $q^{CN}(n, \tau)$ and $Q^{CN}(n, \tau)$ are necessarily decreasing in τ .

The discussion carried out thus far also implies that consumer surplus $C_{MSY}^{CN} = (Q_{MSY}^{CN})^2/2$ and the steady state amount of emissions $S_{MSY}^{CN} = \nu Q_{MSY}^{CN}/\eta$ are independent of the emission tax, and so is the environmental damage $D_{MSY}^{CN} = \delta^2\nu^2\zeta/(16\beta^2\gamma^2\eta^2)$ as well.

The only component of welfare being affected by the tax is the aggregate profit of the industry, because individual profits are $\pi_{MSY}^{CN} = p(Q_{MSY}^{CN}) \cdot q_{MSY}^{CN} - C(q_{MSY}^{CN}) - \tau\nu q_{MSY}^{CN}/\eta$ and their aggregate value is

$$\Pi_{MSY}^{CN} = n_{MSY}(\tau) \cdot \pi_{MSY}^{CN} = p(Q_{MSY}^{CN}) \cdot Q_{MSY}^{CN} - n_{MSY}(\tau) \cdot C(q_{MSY}^{CN}) - \frac{\tau\nu Q_{MSY}^{CN}}{\eta} \quad (13)$$

Note that aggregate revenues are unaffected by τ , which appears only in the total cost function either explicitly (in the tax burden) or implicitly (in $n_{MSY}(\tau)$ and q_{MSY}^{CN} , which contains the latter). The partial derivative of (13) is

$$\frac{\partial \Pi_{MSY}^{CN}}{\partial \tau} = - \frac{\eta [C_i(q_{MSY}^{CN}) + n_{MSY}(\tau) \cdot C'(q_{MSY}^{CN})] n'_{MSY}(\tau) + \nu Q_{MSY}^{CN}}{\eta} \quad (14)$$

where

$$\begin{aligned} C'(q_{MSY}^{CN}) &\equiv \frac{\partial C(q_{MSY}^{CN})}{\partial \tau} = \frac{\partial C(q^{CN})}{\partial q} \cdot \frac{\partial q^{CN}}{\partial \tau} \Big|_{n=n_{MSY}(\tau)} < 0 \\ n'_{MSY}(\tau) &\equiv \frac{\partial n_{MSY}(\tau)}{\partial \tau} > 0 \end{aligned} \quad (15)$$

Now, if production takes place at constant marginal cost, then $n_{MSY}(\tau) \cdot C(q_{MSY}^{CN}) = C(Q_{MSY}^{CN})$, whereby $C'(Q_{MSY}^{CN}) = 0$ because Q_{MSY}^{CN} is independent of τ . On the other hand, social welfare will be independent of τ because of the redistribution of the tax income to consumers. Hence, we may claim

Proposition 1 *If firms sell a homogeneous good and productive technology is characterised by constant returns to scale, provided access to the commons is regulated to ensure industry harvesting at the MSY , equilibrium welfare will be independent of the emission tax.*

This also implies the following ancillary result:

Corollary 2 *Under product homogeneity and regulated access to the commons at $n_{MSY}(\tau)$, irrespective of the nature of returns to scale in production, the emission tax will have no impact on the size of the environmental damage. Consequently, the public authority may modify tax pressure for purely redistributive purposes.*

This amounts to saying that once industry structure has been "frozen" to guarantee the survival of the biological resource in the long run, the primary effect of the emission tax, originally envisaged to be an instrument to reduce the environmental impact of production and/or consumption, has been entirely sterilised.

This remains largely true (although not 100% so) if we relax the admittedly restrictive assumption concerning product homogeneity to adopt the more realistic view that each firm sells a differentiated variety, in such a way that firm i faces a product-specific inverse demand function $p_i(q_i, Q_j)$, in which $Q_j = \sum_{j \neq i} q_j$. This assumption, in particular, implies that individual and industry equilibrium profits will indeed depend on τ even under constant returns to scale, as can be ascertained using the well known model in Ottaviano *et al.* (2002), which encompasses the earlier approach due to Singh and Vives (1984).

The specific reason is that equilibrium prices will incorporate the n firms' outputs in an asymmetric way, due to the presence of even the smallest degree of product differentiation. However, it remains true that fixing n and consequently Q to induce the industry to extract the resource from its natural habitat in correspondence of the MSY opens up a novel and interesting perspective on the role of emission taxation, showing that (i) it can be finely tuned to drive emissions and the related environmental damage to zero, although (ii) the tax level achieving this result is systematically higher than that maximising total welfare, precisely because under product differentiation - and irrespective of the nature of the market regime - the environmental damage is affected by the tax, which partially restores the traditional tradeoff between profits and consumer surplus on one side and the environmental balance on the other (still, were product differentiation *sufficiently small*, this consequence would be *almost negligible*).

A somewhat different story can be told if one admits the possibility for firms to react to emission taxation by investing in abatement technologies, with the twofold objective of cleaning their technology and softening the impact of the emission tax on their profits.

To investigate this scenario, it suffices to modify the above model to include investments in abatement technologies. The simplest way to do so is to assume perfect patent protection, i.e., no spillovers or information leakages across firms. Firm i 's profit function becomes $\pi_i(t) = p(Q(t))q_i(t) -$

$C_i(q_i(t)) - \tau s_i(t) - bk_i^2(t)$, in which $bk_i^2(t)/2$ is the instantaneous cost associated with an abatement effort $k_i(t) > 0$, this becoming the second control in the hands of each firm. This R&D effort enters the state equation of individual emissions in the following way:

$$\dot{s}_i(t) = \nu q_i(t) - \eta s_i(t) - \phi k_i(t) \quad (16)$$

where ϕ is a positive constant. In this respect, it is worth noting that we identify the present layout of R&D as being for abatement technologies in the sense that it leaves unaltered parameter ν , which measures the number of CO_2 -equivalent molecules emitted by production/consumption. We may label this technology as an end-of-pipe device designed to capture carbon before its release in the atmosphere. Conversely, if $k_i(t)$ were to modify ν , the above state equation would write as $\dot{s}_i(t) = \nu_i(t) q_i(t) - \eta s_i(t) - \phi k_i(t)$ and it would be necessarily accompanied by an additional state equation like $\dot{\nu}_i(t) = \nu_i(t) [1 - \phi k_i(t)]$, so that as long as $k_i(t) < 1/\phi$, $\nu_i(t)$ increases, and conversely. However, the multiplicative effect appearing in the latter equation would prevent the attainment of a degenerate feedback equilibrium under open-loop information (for more on this modelling strategy and its implications, see Dragone *et al.*, 2013).

The remaining building blocks of the model are unmodified, so that the Hamiltonian of firm i is

$$\begin{aligned} \mathcal{H}_i(t) = & p(Q(t)) q_i(t) - C_i(q_i(t)) - \tau s_i(t) - bk_i^2(t) + \\ & \lambda_{ii}(t) [\nu q_i(t) - \eta s_i(t) - \phi k_i(t)] + \sum_{j \neq i} \lambda_{ij}(t) [\nu q_j(t) - \eta s_j(t) - \phi k_j(t)] \\ & + \mu_i(t) [\delta X(t) (1 - \beta X(t)) - \gamma Q(t)] \end{aligned} \quad (17)$$

Given the additive separability of the model w.r.t. controls, the FOC on $q_i(t)$ and the dynamics of $\lambda_{ii}(t)$ are the same as above, while we have an additional FOC taken on $k_i(t)$ (once again, we omit the time argument):

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = -2bk_i(t) - \lambda_{ii}\phi = 0 \quad (18)$$

Additionally, it remains true that the game is state-redundant and therefore $\lambda_{ij} = 0$ and $\mu_i = 0$ at all times.

Since λ_{ii} appears in both FOCs, we have $\lambda_{ii} = (d\tilde{\pi}'/dt + \nu\tau) / [\nu(\eta + \rho)] - 2bk_i/\phi$. The first option can be used to treat the dynamics of q_i , while the second is useful to characterise the dynamics of emission abatement:⁷

$$\dot{k}_i = -\frac{\lambda_{ii}\phi}{b} = \frac{b(\eta + \rho)k_{ii} - \phi\tau}{b} \quad (19)$$

⁷As will become evident in the ensuing examples, the two control equations will not, in general, be linearly independent, because of the presence of the adjoint variable in both FOCs. This is surely the case with a linear market demand for a homogeneous good: in this case, the FOC w.r.t. $q_i(t)$ can be solved to find the optimal individual output.

solving which at any time t we obtain the optimal symmetric abatement effort of the generic firm, $k^*(\tau) = \phi\tau / [2b(\eta + \rho)] + e^{t(\eta+\rho)}\mathbb{C}$, where the integration constraint \mathbb{C} can be set equal to zero. Since the above expression is independent of n , which - as we know from the above discussion - has been in the meantime set equal to $n_{MSY}(\tau)$ by the regulator, the industry-wide investment in abatement technologies is $K^*(\tau) = n_{MSY}(\tau) \cdot k^*(\tau)$, and

$$\frac{\partial K^*(\tau)}{\partial \tau} = \frac{\partial n_{MSY}(\tau)}{\partial \tau} \cdot k^*(\tau) + n_{MSY}(\tau) \cdot \frac{\partial k^*(\tau)}{\partial \tau} > 0 \quad (20)$$

since $\partial n_{MSY}(\tau) / \partial \tau > 0$ and $\partial k^*(\tau) / \partial \tau > 0$.

All else equal, this very fact modifies the picture in one very relevant respect, namely, the regulator's attitude about the proper use of the tax. While in absence of R&D activities on the part of firms the public authority could only think of the emission tax as a redistributive tool, now it sees this environmental policy for what it should indeed be, namely, a tool for triggering and boosting green R&D.

In particular, this exercise has shown the arising of an Arrovian result (Arrow, 1962),⁸ according to which increasing the tax drives a monotone increase in the aggregate abatement effort carried out by the entire industry:

Proposition 3 *If firms react to emission taxation by implementing R&D projects for abatement technology under full patent protection, any tax increase causes individual and aggregate R&D to increase monotonically since $n_{MSY}(\tau)$ is itself increasing in the tax rate to guarantee the attainment of the MSY.*

Of course, the non-negativity of profits poses an upper bound to the tax increase and therefore also to the expansion of R&D investments, but in line of principle the policy maker can push the tax up to a level at which equilibrium individual profits are positive although arbitrarily small. Indeed, the following example, based upon a linear market demand, will show that the public authority may actually drive the whole industry to a green production pattern. In doing so, we will also assess the relative size of such tax and that associated with welfare maximisation, to illustrate that a government may systematically face a tradeoff between achieving the highest possible degree of greenness and welfare maximisation, and nonetheless this conundrum can be solved through the arising of a win-win

⁸Arrow (1962) proposed the view, holding that increasing the intensity of competition (or industry fragmentation) would bring to bear higher innovation incentives, in contrast with Schumpeter's (1942) position according to which innovation incentives would be monotonically increasing in market power and concentration. The lively debate discussing these opposite views in the theory of industrial organization can be appreciated by reading Tirole (1988) and Reinganum (1989), *inter alia*. An analogous discussion about green R&D exists in environmental economics as well (see, for example, Feichtinger *et al.*, 2016, and Lambertini *et al.*, 2017).

solution vindicating the Porter hypothesis. This happens because once technology is green, taxation becomes ineffective and therefore the aforementioned tradeoff vanishes.

Moreover, given the additively separable structure of the model, the presence of product differentiation and/or decreasing returns to scale in production do not affect the above claims. Hence, we have found a perspective that allows a regulator to get two eggs in one basket: first appropriately define n as a function of τ in such a way to preserve the renewable resource forever, and then exploit the monotonicity of $n_{MSY}(\tau)$ in τ to spur firms to clean up their technology.

To illustrate the above results, we may resort to different specialised models based on plausible sets of specific functional forms of demand and cost functions, with and without R&D investments in abatement technologies.

3 Linear demand and constant returns to scale

Let the n Cournot firms supply a homogeneous good whose market demand function is linear, $p(t) = a - Q(t)$, where a is a positive constant. All firms use the same technology at constant returns to scale, whereby the individual instantaneous cost function is $C_i(q_i(t)) = cq_i(t)$, where the time-invariant marginal cost $c \in (0, a)$. To ease the ensuing exposition, we may define $A \equiv a - c$. Each individual firm's emissions $s_i(t)$ are being taxed at all times at the rate $\tau > 0$. Consequently, firm i 's instantaneous profit function can be written as $\pi_i(t) = [A - q_i(t) - Q_{-i}(t)]q_i(t) - \tau s_i(t)$, in which $Q_{-i}(t) \equiv \sum_{j \neq i} q_j(t)$. We shall start by illustrating the game in which firms do not invest in abatement technologies.

In absence of green R&D, the problem of firm i is

$$\begin{aligned} \max_{q_i(t)} \int_0^\infty e^{-\rho t} \pi_i(t) dt, \\ \dot{s}_i(t) &= \nu q_i(t) - \eta s_i(t), \quad s_i(0) = s_{i0}, \quad i = 1, 2, \dots, n \\ \dot{X}(t) &= \delta X(t) [1 - \beta X(t)] - \gamma Q(t), \quad X(0) = X_0. \end{aligned} \tag{21}$$

which implies that firm i must choose its output level to maximise the following Hamiltonian:⁹

$$\mathcal{H}_i = \pi_i + \lambda_{ii}(\nu q_i - \eta s_i) + \sum_{j \neq i} \lambda_{ij}(\nu q_j - \eta s_j) + \mu_i [\delta X (1 - \beta X)] \tag{22}$$

in which $\{\lambda_{ii}, \lambda_{ij}, \mu_{ii}\}$ is the vector of the $n+1$ costate variables. The differential game is solved under open-loop information, with firms playing simultaneously and non-cooperatively at all times, which amounts to saying that we are about to characterise the Cournot-Nash open-loop equilibrium and its implications.

⁹Henceforth we shall omit the explicit indication of the time argument for the sake of brevity.

The first order condition (FOC) on individual output is

$$A - 2q_i - \sum_{j \neq i} q_j + \lambda_{ii}\nu - \mu_i\gamma = 0 \quad (23)$$

which yields

$$\lambda_{ii} = \frac{2q_i + \sum_{j \neq i} q_j + \mu_i\gamma - A}{\nu} \quad (24)$$

and it is accompanied by the following costate equations:

$$\begin{aligned} \dot{\lambda}_{ii} &= (\eta + \rho)\lambda_{ii} + \tau \\ \dot{\lambda}_{ij} &= (\eta + \rho)\lambda_{ij} \\ \dot{\mu}_i &= [\delta(2X\beta - 1) + \rho]\mu \end{aligned} \quad (25)$$

and the transversality conditions $\lim_{t \rightarrow \infty} \lambda_{ii}s_i = 0$, $\lim_{t \rightarrow \infty} \lambda_{ij}s_j = 0$ and $\lim_{t \rightarrow \infty} \mu_i X_i = 0$.

Now note that (25) imply that solutions $\lambda_{ij} = \mu_i = 0$ are admissible at all times. Using these solutions and imposing symmetry across output levels and λ_{ii} 's, the solution of (23) at a generic instant is $q^{CN} = (A + \lambda\nu)/(n + 1)$, which, in turn, delivers the control equation $\dot{q} = \dot{\lambda}\nu/(n + 1)$. This, on the basis of (24-25), becomes

$$\dot{q} = \frac{(\eta + \rho)[(n + 1)q - A] + \tau\nu}{n + 1} \quad (26)$$

The steady state values of the $n + 1$ states and the single control are

$$\begin{aligned} \hat{s} &= \frac{\nu[A(\eta + \rho) - \nu\tau]}{(n + 1)\eta(\eta + \rho)} = \frac{\nu}{\eta}\hat{q} \\ \hat{X}_{\pm} &= \frac{(n + 1)\delta(\eta + \rho) \pm \sqrt{\Theta}}{2(n + 1)\beta\delta(\eta + \rho)} \\ \hat{q} &= \frac{A(\eta + \rho) - \nu\tau}{(n + 1)(\eta + \rho)} \end{aligned} \quad (27)$$

where $\Theta \equiv (n + 1)^2\delta^2(\eta + \rho)^2 + 4(n + 1)\delta(\eta + \rho)n\beta\gamma[\nu\tau - A(\eta + \rho)]$, and $\hat{q} > 0$ implies $\hat{X}_+ \geq \hat{X}_- > 0$.

Standard stability analysis (assuming $\lambda_{ij} = 0$ and $\mu_i = 0$ for all t) of the equilibrium implies two strictly positive and one negative eigenvalue for both steady state values of X . The fourth eigenvalue is negative for \hat{X}_+ and positive for \hat{X}_- . Both equilibria are saddle points but only \hat{X}_+ is a candidate for an optimal solution.

Regulating access to the commons to ensure $\hat{X}_+ = X_{MSY} = 1/(2\beta)$ requires imposing

$$n = n_{MSY} \equiv \frac{\delta(\eta + \rho)}{4\beta\gamma[A(\eta + \rho) - \nu\tau] - \delta(\eta + \rho)} \quad (28)$$

This implies the following:

Proposition 4 *Harvesting at the MSY requires*

$$n = n_{MSY} \equiv \frac{\delta(\eta + \rho)}{4\beta\gamma [A(\eta + \rho) - \nu\tau] - \delta(\eta + \rho)}$$

with n_{MSY} being positive and at least equal to one for all $A \in (A_{min}, A_{max})$, where

$$\begin{aligned} A_{min} &\equiv \frac{4\beta\gamma\nu\tau + \delta(\eta + \rho)}{4\beta\gamma(\eta + \rho)} \\ A_{max} &\equiv \frac{2\beta\gamma\nu\tau + \delta(\eta + \rho)}{2\beta\gamma(\eta + \rho)} \end{aligned}$$

From

$$\frac{\partial n_{MSY}}{\partial \tau} = \frac{4\beta\delta\gamma\nu(\eta + \rho)}{[4\beta\gamma(A(\eta + \rho) - \nu\tau) - \delta(\eta + \rho)]^2} > 0 \quad (29)$$

we see that increasing taxation must go along with expanding access to the commons in order for the industry to keep harvesting the resource at the MSY, to compensate the negative effect of emission taxation on the steady state value of production \hat{q} :

$$\frac{\partial \hat{q}}{\partial \tau} = -\frac{\nu}{(n+1)(\eta + \rho)} \quad (30)$$

Now we may look at the equilibrium level of the environmental damage, which, for a generic industry structure, amounts to

$$\hat{D}(n) = \frac{n^2\nu^2\zeta [A(\eta + \rho) - \nu\tau]^2}{(n+1)^2\eta^2(\eta + \rho)^2} \quad (31)$$

However, once $n = n_{MSY}$, this simplifies as $\hat{D}(n_{MSY}) = \delta^2\nu^2\zeta / (16\beta^2\gamma^2\eta^2)$, i.e., it becomes independent of the emission tax, and the same of course applies to the equilibrium level of aggregate emissions. Moreover, the same holds for aggregate output, which corresponds to $\hat{Q}(n_{MSY}) = \delta / (4\beta\gamma)$, and consequently also for consumer surplus, $\widehat{CS}(n_{MSY}) = \delta^2 / (32\beta^2\gamma^2)$.

Therefore, we may claim:

Proposition 5 *Once the policy maker pursues the attainment of harvesting at the MSY, the environmental balance $X_{MSY} - \hat{D}(n_{MSY})$ is independent of industry structure.*

This of course is not true for profits, $n_{MSY} \cdot \hat{\pi}(n_{MSY}) = \delta [4\beta\gamma\eta(A - \tau) - \delta\eta] / (16\beta^2\gamma^2\eta)$, due to the presence of the emission tax. However, since the income generated by emission taxation is being redistributed, the equilibrium level of social welfare is indeed independent of the tax:

$$\widehat{SW}(n_{MSY}) = \frac{16\beta\gamma^2\eta^2 + \delta [(8A\beta\gamma - \delta)\eta^2 - 2\delta\nu^2\zeta]}{32\beta^2\gamma^2\eta^2} \quad (32)$$

Note that $\widehat{SW}(n_{MSY})$ may take a negative value. A sufficient condition for this not to happen is $A \geq \hat{A}(n_{MSY}) \equiv \delta(\eta^2 + 2\nu^2\zeta) / (8\beta\gamma\eta^2)$. Hence, the detailed exposition of the model with linear demand, product homogeneity and constant returns to scale confirms the contents of Proposition 1 and Corollary 2, whereby we may reformulate those conclusions as follows:

Proposition 6 *If firms sell a homogeneous good with a common technology characterised by a constant marginal cost, limiting access at n_{MSY} sterilises the emission tax, which has no impact on the environmental damage and can be exclusively used to redistribute surplus between firms and consumers.*

3.1 Abatement technology

We are now in a position to expand the above model in order for it to accommodate firms' R&D efforts in abatement technologies, spurred by emission taxation. In this setting, each firm controls two variables, output and the R&D effort, the latter involving a convex instantaneous cost and entering the kinematics of s_i , in such a way that the key elements of the model can be described as follows. Instantaneous individual profits are $\pi_i(t) = [A - Q(t)] q_i(t) - \tau s_i(t) - bk_i^2(t)$, and the relevant state equation of individual emissions becomes (16), which is based on the assumption that the individual firm receives no spillover from any of the rivals.

Hence, firm i 's Hamiltonian is

$$\begin{aligned} \mathcal{H}_i(t) = & [A - Q(t)] q_i(t) - \tau s_i(t) - bk_i^2(t) + \lambda_{ii}(t) [\nu q_i(t) - \eta s_i(t) - \phi k_i(t)] + \\ & \sum_{j \neq i} \lambda_{ij}(t) [\nu q_j(t) - \eta s_j(t) - \phi k_j(t)] + \mu_i(t) [\delta X(t) (1 - \beta X(t)) - \gamma Q(t)] \end{aligned} \quad (33)$$

and therefore (23) and (25) still apply, accompanied by the FOC taken w.r.t. the abatement effort, which is (18). Our purpose here is to illustrate the arising of Proposition 3.

After posing $\mu_i = 0$ and $\lambda_{ij} = 0$ for all $j \neq i$ at all times, and imposing symmetry, we may solve (18) to obtain $k = -\phi\lambda/(2b)$ and thus also $\dot{k} = -\phi\dot{\lambda}/(2b)$, in such a way that the two control equations are

$$\dot{k} = -\frac{\phi[\lambda(\eta + \rho) + \tau]}{2b}; \quad \dot{q} = \frac{\nu[\lambda(\eta + \rho) + \tau]}{n + 1} \quad (34)$$

which are not linearly independent. Consequently, we shall use the FOC (23) to identify the Cournot output, and (18) to identify the expression of the shadow price, $\lambda = -2bk/\phi$. Doing so, we obtain the following system:

$$\begin{aligned} \dot{k} &= k(\eta + \rho) - \frac{\phi\tau}{2b} \\ A - (n + 1)q - \frac{2b\nu k}{\phi} &= 0 \\ \dot{s} &= \nu q - \eta s - \phi k \\ \dot{X} &= \delta X (1 - \beta X) - \gamma(n - 1)q \end{aligned} \quad (35)$$

which, imposing stationarity, delivers

$$\begin{aligned} \hat{k} &= \frac{\phi\tau}{2b(\eta + \rho)}; \quad \hat{q} = \frac{A(\eta + \rho) - \nu\tau}{(n + 1)(\eta + \rho)} \\ \hat{s} &= \frac{\nu\hat{q} - \phi\hat{k}}{\eta}; \quad \hat{X}_{\pm} = \frac{\delta \pm \sqrt{\Psi}}{2\beta\delta} \end{aligned} \quad (36)$$

where

$$\Psi \equiv \frac{\delta [(\eta + \rho) (\delta + n (\delta - 4A\beta\gamma)) + 4n\beta\gamma\nu\tau]}{(n + 1) (\eta + \rho)} \quad (37)$$

At this point we may follow the same procedure as in the previous setting, by noting that the stable solution for the resource stock is necessarily \widehat{X}_+ , and impose $\widehat{X}_+ = 1/(2\beta)$ to find

$$n_{MSY}(\tau) = \frac{\delta (\eta + \rho)}{(\eta + \rho) (4A\beta\gamma - \delta) - 4\beta\gamma\nu\tau} \quad (38)$$

which is positive and at least equal to one for all

$$A \in \left(\frac{\delta (\eta + \rho) + 4\beta\gamma\nu\tau}{4\beta\gamma (\eta + \rho)}, \frac{\delta (\eta + \rho) + 2\beta\gamma\nu\tau}{2\beta\gamma (\eta + \rho)} \right] \quad (39)$$

At $n = n_{MSY}(\tau)$, once again $\widehat{Q}(n_{MSY}(\tau)) = \delta/(4\beta\gamma)$, and therefore consumer surplus $\widehat{CS}(n_{MSY}(\tau)) = \delta^2/(32\beta^2\gamma^2)$.

The corresponding environmental damage is

$$\widehat{D}(n_{MSY}(\tau)) = \frac{\zeta [b\delta\nu ((4A\beta\gamma - \delta) (\eta + \rho) - 4\beta\gamma\nu\tau) - 2\beta\gamma\delta\phi^2\tau]^2}{16b^2\beta^2\gamma^2\eta^2 [(4A\beta\gamma - \delta) (\eta + \rho) - 4\beta\gamma\nu\tau]^2} \quad (40)$$

and this expression becomes nil at

$$\tau^* = \frac{b\nu (4A\beta\gamma - \delta) (\eta + \rho)}{2\beta\gamma (2b\nu^2 + \phi^2)} > 0 \quad (41)$$

which obviously also solves $\partial\widehat{D}(n_{MSY}(\tau))/\partial\tau = 0$. Of course, the adoption of τ^* also implies $\widehat{s}(n_{MSY}) = 0$. This proves the following:

Proposition 7 *If firms undertake R&D projects for abatement technologies under full patent protection, and the regulator imposes $n = n_{MSY}(\tau)$, there exists a unique emission tax rate τ^* at which individual and aggregate emissions drop to zero.*

As for aggregate R&D, $\widehat{K}(n_{MSY}(\tau)) = \delta\phi\tau/[2b((4A\beta\gamma - \delta)(\eta + \rho) - 4\beta\gamma\nu\tau)]$, we have that $\partial\widehat{K}(n_{MSY}(\tau))/\partial\tau > 0$ always, which proves the following

Corollary 8 *If firms undertake R&D projects for abatement technologies under full patent protection, and the regulator imposes $n = n_{MSY}(\tau)$, then aggregate R&D investment is monotonically increasing in the emission tax rate.*

Finally, simplifying the expressions of individual output, profits and social welfare, we obtain

$$\begin{aligned} \widehat{q}(n_{MSY}(\tau), \tau^*) &= \frac{(4A\beta\gamma - \delta) \phi^2}{4\beta\gamma (2b\nu^2 + \phi^2)} \\ \widehat{\pi}(n_{MSY}(\tau), \tau^*) &= \frac{(4A\beta\gamma - \delta)^2 \phi^2 (b\nu^2 + \phi^2)}{16\beta^2\gamma^2 (2b\nu^2 + \phi^2)^2} \\ \widehat{SW}(n_{MSY}(\tau), \tau^*) &= \frac{8b\beta\gamma\nu^2 [(a - c) \delta + 4\gamma] + [8\beta\gamma (A\delta + 2\gamma) - \delta^2] \phi^2}{32\beta^2\gamma^2 (2b\nu^2 + \phi^2)^2} \end{aligned} \quad (42)$$

where $\hat{q}(n_{MSY}(\tau), \tau^*)$ and $\hat{\pi}(n_{MSY}(\tau), \tau^*)$ are positive in view of the constraint ensuring the positivity of the output level at any instant. Consequently, in this setting the regulator need not take into account the non-negativity of either outputs or profits when looking for the tax rate driving the whole industry along a sustainable path. However, once again the equilibrium welfare level may take a negative value. A sufficient condition for this not to happen is $A \geq \hat{A}(n_{MSY}(\tau), \tau^*) \equiv (\delta^2 - 16\beta\gamma^2) / (8\beta\gamma\delta)$, and it is easily checked that $\hat{A}(n_{MSY}(\tau), \tau^*) < \hat{A}(n_{MSY})$ always.

Additionally, we may briefly dwell upon the possibility for the public authority to implement the welfare-maximising tax rate. In this respect, it is easily checked that, in correspondence of $\tau = \tau^*$, the partial derivative of $\widehat{SW}(n_{MSY}(\tau))$ w.r.t. to τ simplifies to

$$\left. \frac{\partial \widehat{SW}(n_{MSY}(\tau))}{\partial \tau} \right|_{\tau=\tau^*} = -\frac{\delta\nu(b\nu^2 + \phi^2)}{4\beta\gamma(\eta + \rho)\phi^2} < 0 \quad (43)$$

which reveals that the welfare-maximising tax falls short of the tax rate ensuring the elimination of polluting emissions. This, intuitively, is imputable to the presence of τ in the expression of equilibrium profits, and is a potential source of conflicts inside a government. The same problem reproduces itself in the remainder of the paper, with the equivalent of (43) holding systematically across market regimes.

Needless to say, the result spelled out in Proposition 7 should be considered with some caution, as one cannot expect emissions to literally drop to zero. However, this setup says that there exists a scenario in which regulating access and emissions may create a synergy that has remained overlooked thus far, capable of creating a sustainable path accounting for two key dimensions, i.e., the preservation of natural resources and a significant reduction of polluting emissions. This prompts for a check of the Porter hypothesis in its strong form.

3.2 A validation of the Porter hypothesis

Now we are in a position to comparatively assess equilibrium profits and welfare levels engendered by the two scenarios analysed thus far. This exercise is carried out by posing $\tau = \tau^*$ in $\hat{\pi}(n_{MSY})$, so as to verify whether the tax rate driving to zero emissions in the game including investment in abatement technologies is conducive to the win-win solution. As for profits, we have

$$\hat{\pi}(n_{MSY}(\tau), \tau^*) - \hat{\pi}(n_{MSY}) = \frac{b(4A\beta\gamma - \delta^2)(\eta + 2\rho)\phi^2\nu^2}{16\beta^2\gamma^2\eta(2b\nu^2 - \phi^2)} > 0 \quad (44)$$

always, while the comparison of welfare levels yields

$$\widehat{SW}(n_{MSY}(\tau), \tau^*) - \widehat{SW}(n_{MSY}) = \frac{[(b(\eta^2 + 2\nu^2\zeta) + \phi^2\zeta)\delta - 4Ab\beta\gamma\eta^2]\delta\nu^2}{16\beta^2\gamma^2\eta^2(2b\nu^2 - \phi^2)} \quad (45)$$

which is positive for all $A \in (0, A_{PH})$, $A_{PH} \equiv [b(\eta^2 + 2\nu^2\zeta) + \phi^2\zeta]\delta / (4b\beta\gamma\eta^2)$. As a last step, it is easily checked that $A_{PH} > \hat{A}(n_{MSY}) > \hat{A}(n_{MSY}(\tau), \tau^*)$. Accordingly, we may formulate the following

Proposition 9 *If $A \in (\widehat{A}(n_{MSY}), A_{PH})$, the adoption of τ^* delivers the win-win solution.*

It is worth stressing that this result is a consequence of the parallel policy regulating access to the common pool, which sterilises the effects of emission taxation when firms react to it by modifying outputs only. What at first sight may appear as a simple redistributive mechanism confined to a special case turns out in fact to open the path to the vindication of the Porter hypothesis in its strong form.

4 Extensions: decreasing returns, product differentiation and monopolistic competition

Here we extend the model to encompass convex production costs, the representative consumer's preference for variety and monopolistically competitive behaviour on the part of firms, each of which is supplying a single variety. The first variation entails that firm i 's instantaneous cost function becomes $C_i(q_i(t)) = cq_i(t) + dq_i^2(t)$, with d being a positive constant. The second and third extensions require a new definition of the demand structure engendered by consumers' taste for variety, which makes it possible to characterise both oligopolistic and monopolistically competitive equilibria. To this aim, we adopt the same utility function as in Ottaviano *et al.* (2002) and the subsequent literature.¹⁰ The preferences of the representative consumer at any instant t are described by the following utility function:

$$U = a \sum_{i=1}^n q_i(t) - \frac{\xi - \sigma}{2} \sum_{i=1}^n q_i^2(t) - \frac{\sigma}{2} \left(\sum_{i=1}^n q_i(t) \right)^2 + q_0(t) \quad (46)$$

where $q_0(t)$ is the numeraire good, while ξ and σ are constants, with $\xi > 0$ and $\sigma \in [-\xi, \xi]$, in such a way that if $\sigma < 0$, consumers want to buy complements, if $\sigma > 0$, consumers want to buy substitutes, and $\sigma = 0$ each good is sold on a different market (and each firm is a pure monopolist). Hence, any $\sigma \in (0, \xi)$ provides an inverse measure of substitutability between any two imperfectly substitutable varieties. The homogeneous good case arises if $\sigma = \xi$.

The individual inverse demand function resulting from the constrained maximization of (46) is

$$p_i(t) = a - (\xi - \sigma) q_i(t) - \sigma \sum_{i=1}^n q_i(t) \quad (47)$$

if firms are Cournot oligopolists, and

$$p_i(t) = a - (\xi - \sigma) q_i(t) - \sigma Q(t) \quad (48)$$

¹⁰In particular, the detailed illustration of the monopolistically competitive equilibrium emerging from (46) can be found in Cellini *et al.* (2004, 2020).

if they are monopolistically competitive. In the above demand functions, the appearance of aggregate output $Q(t)$ indicates that under monopolistic competition each individual firm is unaware of its role in the aggregate behaviour of the industry. Note that if $\sigma = \zeta = 1$, we are back to the homogeneous good case investigated initially.

In both scenarios, the instantaneous profit function of firm i is

$$\pi_i(t) = [p_i(t) - c - dq_i(t)] q_i(t) - \tau s_i(t) \quad (49)$$

if firms do not invest in emission abatement, and

$$\pi_i(t) = [p_i(t) - c - dq_i(t)] q_i(t) - \tau s_i(t) - bk_i^2(t) \quad (50)$$

if they do, and $p_i(t)$ must be replaced with the appropriate version of the inverse demand function depending on the nature of market competition. As in section 3, to save upon notation we shall define $A \equiv a - c$. We set out to illustrate the first case, under the assumption that firms are Cournot oligopolists.

4.1 The oligopoly game

If Cournot firms only choose output levels, the relevant Hamiltonian of the generic firm is

$$\begin{aligned} \mathcal{H}_i(t) = & \left[a - (\xi - \sigma) q_i(t) - \sigma \sum_{i=1}^n q_i(t) - c - dq_i(t) \right] q_i(t) - \tau s_i(t) + \\ & \lambda_{ii}(t) [\nu q_i(t) - \eta s_i(t) - \phi k_i(t)] + \sum_{j \neq i} \lambda_{ij}(t) [\nu q_j(t) - \eta s_j(t) - \phi k_j(t)] + \\ & \mu_i(t) [\delta X(t) (1 - \beta X(t)) - \gamma Q(t)] \end{aligned} \quad (51)$$

The FOC on q_i is

$$A - 2(d + \xi) q_i - \sigma \sum_{j \neq i} q_j + \lambda_{ii} \nu - \mu_i \gamma = 0 \quad (52)$$

which yields $\lambda_{ii} = [2(d + \xi) q_i + \sigma \sum_{j \neq i} q_j + \mu_i \gamma - A] / \nu$, while costate equations are the same as in (25); analogously, transversality conditions coincide with those appearing below (25). The game preserves the characteristics outlined above, in particular it remains state-redundant w.r.t. X and, being linear in polluting emissions, its open-loop solution is strongly time consistent. Hence, again from the FOC, after imposing symmetry across controls and posing $\lambda_{ij} = \mu_i = 0$, we have

$$q^{CN} = \frac{A + \lambda \nu}{2(d + \xi) + \sigma(n + 1)} \Rightarrow \dot{q} = \frac{\dot{\lambda} \nu}{2(d + \xi) + \sigma(n + 1)} \quad (53)$$

which can be explicitly written as follows:

$$\dot{q}^{CN} = \frac{(\eta + \rho) [(2(d + \xi) + \sigma(n + 1)) q - A] + \tau \nu}{n + 1} \quad (54)$$

which, together with the $n + 1$ state equations (1-2) forms the state-control system of the present formulation of the game.

The coordinates of the steady state points are (stability can be verified by proceeding as in section 3)

$$\begin{aligned}\widehat{s}^{CN} &= \frac{\nu [A(\eta + \rho) - \nu\tau]}{\eta(\eta + \rho) [2(d + \xi) + \sigma(n + 1)]} = \frac{\nu}{\eta} \widehat{q} \\ \widehat{X}_{\pm}^{CN} &= \frac{\delta(\eta + \rho) (2(d + \xi) + \sigma(n + 1)) \pm \sqrt{\Phi}}{2\beta\delta(\eta + \rho) [2(d + \xi) + \sigma(n + 1)]} \\ \widehat{q}^{CN} &= \frac{A(\eta + \rho) - \nu\tau}{(\eta + \rho) [2(d + \xi) + \sigma(n + 1)]}\end{aligned}\quad (55)$$

where

$$\begin{aligned}\Phi &\equiv \delta(\eta + \rho) [2(d + \xi) + \sigma(n + 1)] \times \\ &[(\eta + \rho) (\delta (2(d + \xi) + \sigma(n + 1)) - 4An\beta\gamma) + 4n\beta\gamma\nu\tau]\end{aligned}\quad (56)$$

which obviously coincide with Θ iff $d = 0$ and $\sigma = \xi = 1$. If this is not true, then imposing harvesting at the *MSY* implies that the resulting expression identifying the number of firms being granted access to the common pool will contain demand parameters (as well as τ , of course), and this will necessarily affect the individual firm's equilibrium price in such a way that the sum of the traditional components of welfare (profits and consumer surplus) will not be independent of the emission tax. The number of firms ensuring the industry harvest at the maximum sustainable yield is

$$n_{MSY}^{CN} = \frac{\delta(\eta + \rho) [2(d + \xi) - \sigma]}{(\eta + \rho) (4A\beta\gamma - \delta\sigma) - 4\beta\gamma\nu\tau}\quad (57)$$

which, just like (28), is monotonically increasing in the tax rate τ . Intuitively, it is also increasing in d , as this parameter determines the slope of the convex component of production costs. It can be easily checked that n_{MSY}^{CN} is positive and at least equal to one for all

$$A \in \left(\frac{\delta(\eta + \rho)\sigma + 4\beta\gamma\nu\tau}{4\beta\gamma(\eta + \rho)}, \frac{(d + \xi)\delta(\eta + \rho) + 2\beta\gamma\nu\tau}{2\beta\gamma(\eta + \rho)} \right)\quad (58)$$

Now, if $n = n_{MSY}^{CN}$ and therefore $\widehat{X} = 1/(2\beta)$, the remaining equilibrium expressions of the relevant magnitudes look as follows:

$$\widehat{Q}^{CN}(n_{MSY}(\tau)) = \frac{\delta}{4\beta\delta}; \widehat{D}^{CN}(n_{MSY}(\tau)) = \frac{\delta^2\nu^2\zeta}{16\beta^2\gamma^2\eta^2}\quad (59)$$

i.e., the same as in section 3, while¹¹

$$\begin{aligned}n_{MSY}^{CN}\widehat{\pi}^{CN}(n_{MSY}(\tau)) + \widehat{CS}^{CN}(n_{MSY}(\tau)) + n_{MSY}^{CN}\tau\widehat{s}^{CN}(n_{MSY}(\tau)) = \\ \frac{\delta [4A\beta\gamma(\eta + \rho) (2d + 3\xi - \sigma) - \delta\xi\sigma(\eta + \rho) + 4\beta\gamma\nu(2d + \xi - \sigma)\tau]}{32\beta^2\gamma^2(\eta + \rho) [2(d + \xi) - \sigma]}\end{aligned}\quad (60)$$

¹¹In this case, due to the presence of imperfect product substitutability, consumer surplus is defined as $CS = [\sum_{i=1}^n (a - p_i) q_i] / 2$ (cf. Ottaviano *et al.*, 2002; Cellini *et al.*, 2004, *inter alia*).

clearly illustrates that τ plays a purely redistributive role, and the traditional part of social welfare is independent of emission taxation at equilibrium iff $d = 0$ and $\xi = \sigma$. All of this can be summarised in the following:

Proposition 10 *If the differential game takes place under product differentiation and decreasing returns to scale in production, regulating access to the commons at n_{MSY}^{CN} makes environmental damage independent of the emission tax. Hence, the latter has at most a purely redistributive effect on profits and consumer surplus.*

As a last remark, it is worth noting the expression in (60) can be obtained in two seemingly different but substantively equivalent ways. The first consists in replacing individual output with

$$\widehat{q}^{CN}(n_{MSY}^{CN}(\tau)) = \frac{(4A\beta\gamma - \delta\sigma)(\eta + \rho) - 4\beta\gamma\nu\tau}{4\beta\gamma(\eta + \rho)[2(d + \xi) - \sigma]} \quad (61)$$

while the second consists in writing $\widehat{CS}^{CN}(n_{MSY}^{CN}(\tau)) = \left(\widehat{Q}^{CN}(n_{MSY}^{CN}(\tau))\right)^2/2$ and then writing the equilibrium expression of individual profits and emissions as

$$\begin{aligned} \widehat{\pi}^{CN}(n_{MSY}(\tau)) &= \left[A - \frac{(\xi + (n_{MSY}^{CN}(\tau) - 1)\sigma - d)\widehat{Q}^{CN}(n_{MSY}^{CN}(\tau))}{n_{MSY}^{CN}(\tau)} \right] \frac{\widehat{Q}^{CN}(n_{MSY}^{CN}(\tau))}{n_{MSY}^{CN}(\tau)} \\ &\quad - \tau\widehat{s}(n_{MSY}^{CN}(\tau)) \\ \widehat{s}^{CN}(n_{MSY}^{CN}(\tau)) &= \frac{\nu\widehat{Q}^{CN}(n_{MSY}^{CN}(\tau))}{\eta n_{MSY}^{CN}(\tau)} \end{aligned} \quad (62)$$

Then it is easily checked that both procedures yield (60).

4.2 Abatement technology with product differentiation

As in section 3.1, the presence of investments in emission abatement implies that each firm has two controls. The Hamiltonian function becomes

$$\begin{aligned} \mathcal{H}_i(t) &= \left[a - (\xi - \sigma)q_i(t) - \sigma \sum_{i=1}^n q_i(t) - c - dq_i(t) \right] q_i(t) - \tau s_i(t) - bk_i^2(t) + \\ &\quad \lambda_{ii}(t)[\nu q_i(t) - \eta s_i(t) - \phi k_i(t)] + \sum_{j \neq i} \lambda_{ij}(t)[\nu q_j(t) - \eta s_j(t) - \phi k_j(t)] \\ &\quad + \mu_i(t)[\delta X(t)(1 - \beta X(t)) - \gamma Q(t)] \end{aligned} \quad (63)$$

and given additive separability, the FOCs taken w.r.t. $k_i(t)$ and $q_i(t)$ coincide with (18) and (52), respectively. Once again the two control equations being not linearly independent, we must solve the system

$$\begin{aligned} \dot{k} &= k(\eta + \rho) - \frac{\phi\tau}{2b} \\ A - [2(d + \xi) + \sigma(n - 1)]q - \frac{2b\nu k}{\phi} &= 0 \\ \dot{s} &= \nu q - \eta s - \phi k \\ \dot{X} &= \delta X(1 - \beta X) - \gamma(n - 1)q \end{aligned} \quad (64)$$

made up by the control equation of the abatement effort, the FOC on the output level, and the state equations. Solving it, we obtain

$$\begin{aligned}\widehat{k}^{CN} &= \frac{\phi\tau}{2b(\eta + \rho)}; \widehat{q}^{CN} = \frac{A(\eta + \rho) - \nu\tau}{(\eta + \rho)[2(d + \xi) + \sigma(n - 1)]} \\ \widehat{s}^{CN} &= \frac{\nu\widehat{q}^{CN} - \phi\widehat{k}^{CN}}{\eta}; \widehat{X}_{\pm}^{CN} = \frac{\delta \pm \sqrt{\Lambda}}{2\beta\delta}\end{aligned}\quad (65)$$

where \widehat{X}_{+}^{CN} is stable, and

$$\Lambda \equiv \delta \left[\delta - \frac{4n\beta\gamma(A(\eta + \rho) - \nu\tau)}{(\eta + \rho)[2(d + \xi) + \sigma(n - 1)]} \right] \quad (66)$$

which coincides with (37) iff $d = 0$ and $\xi = \sigma = 1$.

The industry structure at which harvest takes place in correspondence of the MSY is $n_{MSY}^{CN}(\tau)$, i.e., the same as in (57). If $n = n_{MSY}^{CN}(\tau)$, the total abatement effort at the industry level is

$$\widehat{K}^{CN}(n_{MSY}^{CN}(\tau)) = \frac{\delta\phi[2(d + \xi) - \sigma]\tau}{2b[(4A\beta\gamma - \delta\sigma)(\eta + \rho) - 4\beta\gamma\nu\tau]} \quad (67)$$

which is monotonically increasing in τ in the admissible parameter range.

Moreover, as in section 3.1, once again, the equilibrium amount of the environmental damage is not independent of the emission tax,

$$\widehat{D}^{CN}(n_{MSY}(\tau)) = \frac{\zeta [b\delta\nu((4A\beta\gamma - \delta\sigma)(\eta + \rho) - 4\beta\gamma\nu\tau) - 2\beta\gamma\delta(2(d + \xi) - \sigma)\phi^2\tau]^2}{16b^2\beta^2\gamma^2\eta^2[(4A\beta\gamma - \delta\sigma)(\eta + \rho) - 4\beta\gamma\nu\tau]^2} \quad (68)$$

which is equal to zero at

$$\tau^{CN} = \frac{b\nu(4A\beta\gamma - \delta\sigma)(\eta + \rho)}{2\beta\gamma[2b\nu^2 + (2(d + \xi) - \sigma)\phi^2]} > 0 \quad (69)$$

and of course the latter also solves $\partial\widehat{D}^{CN}(n_{MSY}^{CN}(\tau))/\partial\tau = 0$. Intuitively, the above expression coincides with (41) for $d = 0$ and $\xi = \sigma = 1$. There remains to stress that, if $\tau = \tau^{CN}$, the expression of the aggregate abatement effort, $\widehat{K}^{CN}(n_{MSY}^{CN}(\tau^{CN})) = \delta\nu/(4\beta\gamma\phi)$ is independent of product differentiation and the shape of production costs (and therefore it applies as well to the simplified model exposed in section 3.1). To sum up:

Proposition 11 *If the differential game takes place under product differentiation and decreasing returns to scale in production, regulating access to the commons at n_{MSY} allows the regulator to use the tax to boost firms' abatement effort in order to minimise individual and aggregate emissions as well as the resulting environmental damage.*

4.3 Monopolistic competition

The last scenario we want to illustrate is that in which firms behave as monopolistically competitive agents, and the relevant specification of the individual demand function is (48). To begin with, we

look at the case in which firms control output levels only. Each firm i maximises its discounted flow of profits, whose instantaneous expression is (49), under the set of constraints

$$\begin{aligned}\dot{s}_i(t) &= \nu q_i(t) - \eta s_i(t), \quad s_i(0) = s_{i0}, \quad i = 1, 2, \dots, n \\ \dot{X}(t) &= \delta X(t) [1 - \beta X(t)] - \gamma Q(t), \quad X(0) = X_0\end{aligned}\tag{70}$$

and becomes aware of the fact that $Q(t) = q_i(t) + \sum_{j \neq i} q_j(t)$ only *after* taking FOCs.

The relevant Hamiltonian is

$$\begin{aligned}\mathcal{H}_i(t) &= [A - (\xi - \sigma) q_i(t) - \sigma Q(t) - dq_i(t)] q_i(t) - \tau s_i(t) + \\ &\lambda_{ii}(t) [\nu q_i(t) - \eta s_i(t) - \phi k_i(t)] + \sum_{j \neq i} \lambda_{ij}(t) [\nu q_j(t) - \eta s_j(t) - \phi k_j(t)] \\ &+ \mu_i(t) [\delta X(t) (1 - \beta X(t)) - \gamma Q(t)]\end{aligned}\tag{71}$$

and therefore the FOC is

$$A - 2(d + \xi - \sigma) q_i - \sigma Q + \lambda_{ii} \nu = 0\tag{72}$$

while costate equations are the same as above, and we may set $\lambda_{ij} = \mu_i = 0$ at all times, for all $j \neq i$. Now, posing $\lambda_{ii} = \lambda$, $q_i = q$ and $Q = nq$, from (72) we obtain $q^{mc} = (A + \lambda \nu) / [2(d + \xi) + \sigma(n - 2)]$, where superscript *mc* mnemonics for monopolistic competition. Therefore, the control equation is

$$\dot{q}^{mc} = \frac{\lambda \nu}{2(d + \xi) + \sigma(n - 2)} = \frac{[\lambda(\eta + \rho) + \tau] \nu}{2(d + \xi) + \sigma(n - 2)}\tag{73}$$

which, given that $\lambda = [(2(d + \xi) + \sigma(n - 2))q - A] / \nu$, can be rewritten in its final form as

$$\dot{q}^{mc} = \frac{(\eta + \rho) [(2(d + \xi) + \sigma(n - 2))q - A] + \nu \tau}{2(d + \xi) + \sigma(n - 2)}\tag{74}$$

The stable steady state has the following coordinates:

$$\begin{aligned}\hat{s}^{mc} &= \frac{\nu [A(\eta + \rho) - \nu \tau]}{\eta(\eta + \rho) [2(d + \xi) + \sigma(n - 2)]} = \frac{\nu}{\eta} \hat{q}^{mc} \\ \hat{X}_+^{mc} &= \frac{\delta(\eta + \rho) (2(d + \xi) + \sigma(n - 2)) + \sqrt{\Xi}}{2\beta\delta(\eta + \rho) [2(d + \xi) + \sigma(n - 2)]} \\ \hat{q}^{mc} &= \frac{A(\eta + \rho) - \nu \tau}{(\eta + \rho) [2(d + \xi) + \sigma(n - 2)]}\end{aligned}\tag{75}$$

where

$$\begin{aligned}\Xi &\equiv \delta(\eta + \rho) [2(d + \xi) + \sigma(n - 2)] \times \\ &[\delta(\eta + \rho) (2(d + \xi) + \sigma(n - 2)) - 4n\beta\gamma (A(\eta + \rho) - \nu \tau)]\end{aligned}\tag{76}$$

and the number of firms ensuring that the harvest is carried out at the *MSY* is

$$n_{MSY}^{mc}(\tau) = \frac{2\delta(\eta + \rho)(d + \xi - \sigma)}{(\eta + \rho)(4A\beta\gamma - \delta\sigma) - 4\beta\gamma\nu\tau}\tag{77}$$

which, unsurprisingly, entails that the resulting environmental industry output and environmental damage coincide with (59). This, together with the parallel result that $\hat{X} = 1/(2\beta)$, immediately implies the following

Proposition 12 *As far as the environmental balance is concerned, once access to the commons is regulated to ensure harvesting at the MSY , monopolistic competition is observationally equivalent to the differentiated oligopoly, all else equal.*

Of course the same does not hold when it comes to the output and price levels of each single product variety. To see this, it is sufficient to compare the expression of the equilibrium individual output under monopolistic competition,

$$\widehat{q}^{mc}(n_{MSY}^{mc}(\tau)) = \frac{(4A\beta\gamma - \delta\sigma)(\eta + \rho) - 4\beta\gamma\nu\tau}{8\beta\gamma(\eta + \rho)(d + \xi - \sigma)} \quad (78)$$

with (61), to verify that

$$\widehat{q}^{mc}(n_{MSY}^{mc}(\tau)) - \widehat{q}^{CN}(n_{MSY}^{CN}(\tau)) = \frac{\sigma[(4A\beta\gamma - \delta\sigma)(\eta + \rho) - 4\beta\gamma\nu\tau]}{8\beta\gamma(\eta + \rho)(d + \xi - \sigma)[2(d + \xi) - \sigma]} \quad (79)$$

is strictly positive in the parameter range wherein both $\widehat{q}^{mc}(n_{MSY}^{mc}(\tau))$ and $\widehat{q}^{CN}(n_{MSY}^{CN}(\tau))$ are positive. Then, necessarily, $n_{MSY}^{mc}(\tau) < n_{MSY}^{CN}(\tau)$, simply because, given $\widehat{Q}^{cm}(n_{MSY}^{cm}(\tau)) = \widehat{Q}^{CN}(n_{MSY}^{CN}(\tau))$, each monopolistically competitive firm produces more than its oligopolistic counterpart. The last related remark concerns the associated equilibrium prices, whose expressions can be calculated on the basis of

$$\widehat{p}^J(n_{MSY}^J(\tau)) = a - [\xi + \sigma(n_{MSY}^J(\tau) - 1)]\widehat{q}^J(n_{MSY}^J(\tau)), \quad J = CN, mc \quad (80)$$

Their difference is

$$\begin{aligned} \widehat{p}^{CN}(n_{MSY}^{CN}(\tau)) - \widehat{p}^{mc}(n_{MSY}^{mc}(\tau)) &= \frac{(\xi - \sigma)\sigma[(4A\beta\gamma - \delta\sigma)(\eta + \rho) - 4\beta\gamma\nu\tau]}{8\beta\gamma(\eta + \rho)(d + \xi - \sigma)[2(d + \xi) - \sigma]} \\ &= (\xi - \sigma)[\widehat{q}^{mc}(n_{MSY}^{mc}(\tau)) - \widehat{q}^{CN}(n_{MSY}^{CN}(\tau))] > 0, \end{aligned} \quad (81)$$

and the reason can be found in the different interplay between individual output and the number of firms being granted access to the commons in the two regimes, as it appears from (80): the ranking of prices across regimes tells that the individual output restriction due to oligopolistic behaviour matters more than the increase in the number of firms as compared to monopolistic competition.

It is also worth adding that, almost paradoxically, the fact that $n_{MSY}^{mc}(\tau) < n_{MSY}^{CN}(\tau)$ implies that consumers are offered a wider range of varieties in oligopoly than in monopolistic competition, and at higher market prices - as opposed to the traditional wisdom we have inherited from Helpman and Krugman (1985, 1989) and the subsequent literature - precisely because access to the commons is regulated so as to spur the industry to harvest at the MSY .

4.4 Abatement technology under monopolistic competition

Here the monopolistic competition model is modify to include R&D for emission abatement, in the same form as in section 3.1. The Hamiltonian of firm i becomes

$$\begin{aligned} \mathcal{H}_i(t) = & [A - (\xi - \sigma) q_i(t) - \sigma Q(t) - dq_i(t)] q_i(t) - \tau s_i(t) - bk_i^2(t) + \\ & \lambda_{ii}(t) [\nu q_i(t) - \eta s_i(t) - \phi k_i(t)] + \sum_{j \neq i} \lambda_{ij}(t) [\nu q_j(t) - \eta s_j(t) - \phi k_j(t)] \\ & + \mu_i(t) [\delta X(t) (1 - \beta X(t)) - \gamma Q(t)] \end{aligned} \quad (82)$$

accompanied by the usual set of initial and transversality conditions.

We may skip the detailed exposition of FOCs and costate equations, to focus our attention to the essential elements. As in the oligopoly game with R&D for emission abatement, also here the control equations are not linearly independent, which brings us to solve a system made up by three differential equations and the FOC on output:

$$\begin{aligned} \dot{k} &= k(\eta + \rho) - \frac{\phi\tau}{2b} \\ A - [2(d + \xi) + \sigma(n - 2)]q - \frac{2b\nu k}{\phi} &= 0 \\ \dot{s} &= \nu q - \eta s - \phi k \\ \dot{X} &= \delta X(1 - \beta X) - \gamma(n - 1)q \end{aligned} \quad (83)$$

which delivers the following stable steady state equilibrium:

$$\begin{aligned} \hat{s}^{mc} = \frac{\nu \hat{q}^{mc} - \phi \hat{k}^{mc}}{\eta}; \hat{X}_+^{mc} &= \frac{\delta + \sqrt{\delta \left(\delta - \frac{4n\beta\gamma [A(\eta + \rho) - \nu\tau]}{(\eta + \rho) [2(d + \xi) + \sigma(n - 2)]} \right)}}{2\beta\delta} \\ \hat{q}^{mc} = \frac{A\phi - 2b\nu \hat{k}^{mc}}{\phi [2(d + \xi) + \sigma(n - 2)]}; \hat{k}^{mc} &= \frac{\phi\tau}{2b(\eta + \rho)} \end{aligned} \quad (84)$$

Now note that

Remark 13 $\hat{k}^{mc} = \hat{k}^{CN}$ if τ is given.

This of course (i) is due to the additive separability of the model, which imply that, all else equal, the individual firm exerts exactly the same effort irrespective of the nature of market competition; but (ii) as we are about to see, the equilibrium level of the emission tax, as well as the optimal industry structure, cannot be invariant in the market regime.

The number of firms needed for the industry harvest to take place at the MSY is $n_{MSY}^{mc}(\tau)$, the same as in (77), for any generic tax rate. Moreover, we also now that $n_{MSY}^{CN}(\tau) > n_{MSY}^{mc}(\tau)$, which has a straightforward implication, namely, that Cournot firms, outnumbering they monopolistically competitive counterparts, will outperform them as far as the aggregate R&D effort for abatement technologies is concerned. This claim can be formulated as follows:

Proposition 14 For any given tax rate τ , since $n_{MSY}^{CN}(\tau) > n_{MSY}^{mc}(\tau)$, then $\widehat{K}^{CN}(n_{MSY}^{CN}(\tau)) > \widehat{K}^{mc}(n_{MSY}^{mc}(\tau))$.

To complete the picture, we may note that the claims contained in Propositions 10-11 can be replicated for monopolistic competition as well. Moreover, $Q = \widehat{Q}^{cm}(n_{MSY}^{cm}(\tau))$ and the steady state equilibrium level of the environmental damage

$$\widehat{D}^{cm}(n_{MSY}^{cm}(\tau)) = \frac{\zeta [b\delta\nu((\eta + \rho)(4A\beta\gamma - \delta\rho) - 4\beta\gamma\nu\tau) + 4\beta\gamma\delta(d + \xi - \sigma)\phi^2\tau]^2}{16b^2\beta^2\gamma^2\eta^2[(\eta + \rho)(4A\beta\gamma - \delta\rho) - 4\beta\gamma\nu\tau]^2} \quad (85)$$

takes its minimum (equal to zero) at

$$\tau^{mc} = \frac{b\nu(\eta + \rho)(4A\beta\gamma - \delta\rho)}{4\beta\gamma[b\nu^2 + (d + \xi - \sigma)\phi^2]} > 0 \quad (86)$$

and comparing (86) with (69) we obtain

$$\tau^{mc} - \tau^{CN} = \frac{b\nu(\eta + \rho)\sigma(4A\beta\gamma - \delta\rho)\phi^2}{4\beta\gamma[b\nu^2 + (d + \xi - \sigma)\phi^2][2b\nu^2 + (2(d + \xi) - \sigma)\phi^2]} > 0 \quad (87)$$

which proves

Corollary 15 If $n = n_{MSY}^J(\tau)$, $J = CN, cm$, then $\tau^{mc} > \tau^{CN}$ in the whole parameter range in which equilibrium magnitudes are positive in both market regimes.

Intuitively, this is due to the different number of firms supplying the market in the two cases. Once $\widehat{K}^J(n_{MSY}^J(\tau))$ has been simplified by using the appropriate expression for τ^J , we may draw the obvious that aggregate (but not individual) abatement efforts coincide across regimes at $\widehat{K}^{CN}(n_{MSY}^{CN}(\tau^{CN})) = \widehat{K}^{mc}(n_{MSY}^{mc}(\tau)) = \delta\nu/4\beta\gamma\phi$, which allows us to formulate our last claim:

Corollary 16 If $n = n_{MSY}^J(\tau)$ and $\tau = \tau^J$, $J = CN, cm$, then Cournot oligopoly and monopolistic competition are observationally equivalent in terms of aggregate $R\&D$ investment for emission abatement.

5 Concluding remarks

Very often, and for a long time, resource exploitation and the environmental damage brought about by polluting emissions have been treated separately. This is reasonable in view of the analytical tractability of both issues when taken in isolation. However, it is intuitive that they are intimately connected, as the global economic system extracts natural resources to supply markets around the globe with consumption goods, and this involves a growing volume of CO_2 -equivalent emissions which can be imputed to either production or consumption or, most commonly, both. This very fact prompts

for comprehensive models investigating both sides of the coin together, and assessing the role and scope of regulation in a full-fledged setup.

With this in mind, we have proposed a differential game in oligopoly or monopolistic competition which, relying on the arising of a strongly time consistent equilibrium under open-loop rules, reveals that regulating access to the commons so as to ensure the long-run sustainability of renewable resource extraction implies that emission taxation may only determine a welfare-neutral surplus transfer between consumers and producers, or may be redirected to increase investments in green technologies as much as possible. In line of principle, so much so that it might also reduce emissions and the related environmental damage to zero. In this case, we have also found out a confirmation of the arising of a win-win solution confirming the Porter hypothesis, under a plausible conditions on parameters.

Our analysis has left intentionally aside the identification of the welfare-maximising tax (which, however, in the basic setup does not exist) and its relation with the marginal environmental damage, precisely because it can be used to drive emissions very close to zero in terms of the theoretical model and, more realistically, below a given threshold compatible with the objectives of the Paris Agreement and the explicit indications by IPCC from COP24 onwards. Moreover, this first attempt at modelling the dynamics of resource extraction and greenhouse gases together seems promising in terms of the possibility of investigating the analogous problem related to the exploitation of non-renewables - in particular, fossil fuels - and its cumulative impact on climate. In this respect, this contribution shares several features with a parallel discussion taking place in the area of industrial organization, very well accounted for in Asker (2020).

Appendix

The easiest way of approaching the double commons case consists in substituting the set of n state equations (2) with $\dot{S}(t) = \nu \sum_{i=1}^n q_i(t) - \eta S(t)$ to reconstruct the analysis appearing in section 3, on the basis of the Hamiltonian function of the single firm now being written as

$$\mathcal{H}_i = \pi_i + \lambda_i \left(\nu \sum_{i=1}^n q_i - \eta S \right) + \mu_i [\delta X (1 - \beta X)] \quad (88)$$

with $\pi_i = (A - q_i - Q_{-i}) q_i - \tau S$. From the FOC

$$A - 2q_i - \sum_{j \neq i} q_j + \lambda_i \nu - \mu_i \gamma = 0 \quad (89)$$

and the set of costate equations

$$\begin{aligned} \dot{\lambda}_i &= (\eta + \rho) \lambda_i + \tau \\ \dot{\mu}_i &= [\delta (2X\beta - 1) + \rho] \mu \end{aligned} \quad (90)$$

we obtain $q^{CN} = (A + \lambda\nu) / (n + 1)$, $\lambda_i = [(n + 1)q^{CN} - A] / \nu$ and $\mu_i = 0$ for all $i = 1, 2, \dots, t$ at all times. Then, the control equation $\dot{q} = \lambda\nu / (n + 1)$ can be written explicitly to verify that it coincides with (26). Hence, the coordinates of the steady state are

$$\begin{aligned}\widehat{S} &= \frac{n\nu [A(\eta + \rho) - \nu\tau]}{(n + 1)\eta(\eta + \rho)} = \frac{n\nu}{\eta}\widehat{q} \\ \widehat{X}_{\pm} &= \frac{(n + 1)\delta(\eta + \rho) \pm \sqrt{\Upsilon}}{2(n + 1)\beta\delta(\eta + \rho)}; \widehat{q} = \frac{A(\eta + \rho) - \nu\tau}{(n + 1)(\eta + \rho)}\end{aligned}\quad (91)$$

where $\Upsilon \equiv (n + 1)\delta(\eta + \rho) [(n + 1)\delta(\eta + \rho) - 4n\beta\gamma(A(\eta + \rho) - \nu\tau)]$. Again, $\widehat{q} > 0$ implies $\widehat{X}_+ \geq \widehat{X}_- > 0$. For obvious reasons, the number of firms n_{MSY} delivering $\widehat{X}_+ = X_{MSY} = 1/(2\beta)$ is (28), and once we substitute it into the relevant equilibrium magnitudes we obtain $\widehat{D}(n_{MSY}) = \delta^2\nu^2\zeta / (16\beta^2\gamma^2\eta^2)$, $\widehat{Q}(n_{MSY}) = \delta / (4\beta\gamma)$ and $\widehat{CS}(n_{MSY}) = \delta^2 / (32\beta^2\gamma^2)$, while individual profits differ from those appearing at the steady state of the model in section 3 because each firm is taxed on the basis of industry-wide emissions.

Now, should firms activate R&D projects to abate polluting emissions, one ought to rewrite the state equation as $\dot{S}(t) = \nu \sum_{i=1}^n q_i(t) - \eta S(t) - \phi \sum_{i=1}^n k_i(t)$, relying on the disputable assumption that all of them do invest, but this would entail describing a situation in which any individual effort would amount to delivering a public good and therefore could easily trigger a free-riding problem as every single firm could internalise the benefits generated by any abatement effort carried out by rivals.

References

- 1] Ambec, S., M. Cohen, S. Elgie and P. Lanoie (2013), “The Porter Hypothesis at 20: Can Environmental Regulation Enhance Innovation and Competitiveness?”, *Review of Environmental Economics and Policy*, 7, 2-22.
- 2] Arrow, K.J. (1962), “Economic Welfare and the Allocation of Resources for Invention”, in R. Nelson (ed.), *The Rate and Direction of Industrial Activity*, Princeton, NJ, Princeton University Press.
- 3] Asker, J. (2020), “Cartels and Welfare”, Plenary Lecture, EARIE 2020, available online at <https://www.youtube.com/watch?v=6HSc7ImHppg&t=1865s>
- 4] Ayong Le Kama, A.D. (2001), “Sustainable Growth, Renewable Resources and Pollution”, *Journal of Economic Dynamics and Control*, 25, 1911-18.
- 5] Barnett, A. (1980), “The Pigouvian Tax Rule under Monopoly”, *American Economic Review*, 70, 1037-41.
- 6] Benckroun, H. (2003), “Unilateral Production Restrictions in a Dynamic Duopoly”, *Journal of Economic Theory*, 111, 214-39.
- 7] Benckroun, H. (2008). “Comparative Dynamics in a Productive Asset Oligopoly”, *Journal of Economic Theory*, 138, 237-61.
- 8] Benckroun, H. and N.V. Long (1998), “Efficiency Inducing Taxation for Polluting Oligopolists”, *Journal of Public Economics*, 70, 325-42.

- 9] Benchekroun, H. and N.V. Long (2016), "Status Concern and the Exploitation of Common Pool Renewable Resources", *Ecological Economics*, 125, 70-82.
- 10] Berck, P. and J.M. Perloff (1984), "An Open-Access Fishery with Rational Expectations", *Econometrica*, 52, 489-506.
- 11] Buchanan, J.M. (1969), "External Diseconomies, Corrective Taxes, and Market Structure", *American Economic Review*, 59, 174-77.
- 12] Cellini, R., L. Lambertini and G. Ottaviano (2004), "Welfare in a Differentiated Oligopoly with Free Entry: A Cautionary Note", *Research in Economics*, 58, 125-33.
- 13] Cellini, R., L. Lambertini and G.I.P. Ottaviano (2020), "Strategic Inattention, Delegation and Endogenous Market Structure", *European Economic Review*, 121, article 103324, 1-10.
- 14] Clark, C.W. (1990), *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*, New York, Wiley.
- 15] Clark, C.W. and G.R. Munro (1975), "The Economics of Fishing and Modern Capital Theory", *Journal of Environmental Economics and Management*, 2, 92-106.
- 16] Colombo, L. and P. Labrecciosa (2013a), "Oligopoly Exploitation of a Private Property Productive Asset", *Journal of Economic Dynamics and Control*, 37, 838-53.
- 17] Colombo, L. and P. Labrecciosa (2013b), "On the Convergence to the Cournot Equilibrium in a Productive Asset Oligopoly", *Journal of Mathematical Economics*, 49, 441-45.
- 18] Colombo, L. and P. Labrecciosa (2015), "On the Markovian Efficiency of Bertrand and Cournot Equilibria", *Journal of Economic Theory*, 155, 332-58.
- 19] Cornes, R., C. Mason and T. Sandler (1986), "The Commons and the Optimal Number of Firms", *Quarterly Journal of Economics*, 101, 641-46.
- 20] Damania, R. and E.H. Bulte (2007), "The Economics of Wildlife Farming and Endangered Species Conservation", *Ecological Economics*, 62, 461-72.
- 21] Dockner, E.J., Jørgensen, S., Long, N.V. and Sorger, G. (2000). *Differential Games in Economics and Management Science*, Cambridge, Cambridge University Press.
- 22] Dragone, D., L. Lambertini and A. Palestini (2013), "The Incentive to Invest in Environmental-Friendly Technologies: Dynamics Makes a Difference", in J. Crespo Cuaresma, T. Palokangas and A. Tarasjev (eds), *Green Growth and Sustainable Development*, Heidelberg, Springer, 165-88.
- 23] Feichtinger, G., L. Lambertini, G. Leitmann and S. Wrzaczek (2016), "R&D for Green Technologies in a Dynamic Oligopoly: Schumpeter, Arrow and Inverted U's", *European Journal of Operational Research*, 249, 1131-38.
- 24] Fujiwara, K. (2008), "Duopoly Can Be more Anti-competitive than Monopoly", *Economics Letters*, 101, 217-19.

- 25] Goh, B., G. Leitmann and T. Vincent (1974), "Optimal Control of the Prey-Predator System", *Journal of Mathematical Biosciences*, 19, 263-86.
- 26] Gordon, H.S. (1954), "The Economic Theory of a Common-Property Resource: The Fishery", *Journal of Political Economy*, 62, 124-42.
- 27] Helpman, E. and P. Krugman (1985), *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy*, Cambridge, MA, MIT Press.
- 28] Helpman, E. and P. Krugman (1989), *Trade Policy and Market Structure*, Cambridge, MA, MIT Press.
- 29] Katsoulacos, Y. and A. Xepapadeas (1995), "Environmental Policy under Oligopoly with Endogenous Market Structure", *Scandinavian Journal of Economics*, 97, 411-20.
- 30] Lambertini, L. (2013), *Oligopoly, the Environment and Natural Resources*, London, Routledge.
- 31] Lambertini, L. (2017), "Green Innovation and Market Power", *Annual Review of Resource Economics*, 9, 231-52.
- 32] Lambertini, L. (2018), *Differential Games in Industrial Economics*, Cambridge, Cambridge University Press.
- 33] Lambertini, L. and G. Leitmann (2019), "On the Attainment of the Maximum Sustainable Yield in the Verhulst-Lotka-Volterra Model", *Automatica*, 110, article 108555, 1-5.
- 34] Lambertini, L. and A. Mantovani (2014), "Feedback Equilibria in a Dynamic Renewable Resource Oligopoly: Pre-emption, Voracity and Exhaustion", *Journal of Economic Dynamics and Control*, 47, 115-22.
- 35] Lambertini, L. and A. Mantovani (2016), "On the (In)stability of Nonlinear Feedback Solutions in a Dynamic Duopoly with Renewable Resource Exploitation", *Economics Letters*, 143, 9-12.
- 36] Lambertini, L. J. Poyago-Theotoky and A. Tampieri (2017), "Cournot Competition and "Green" Innovation: An Inverted-U Relationship", *Energy Economics*, 68, 116-23.
- 37] Lanoie, P., J. Laurent-Lucchetti, N. Johnstone and S. Ambec (2011), "Environmental Policy, Innovation and Performance: New Insights on the Porter Hypothesis", *Journal of Economics and Management Strategy*, 20, 803-42.
- 38] Leitmann, G. (1973), "A Minimum Principle for a Population Equation", *Journal of Optimization Theory and Applications*, 9, 155-56.
- 39] Lotka, A.J. (1925), *Elements of Physical Biology*, Philadelphia, Williams and Wilkins.
- 40] Mason, C. and S. Polasky (1997), "The Optimal Number of Firms in the Commons: A Dynamic Approach", *Canadian Journal of Economics*, 30, 1143-60.
- 41] Mason, C., T. Sandler and R. Cornes (1988), "Expectations, the Commons, and Optimal Group Size", *Journal of Environmental Economics and Management*, 15, 99-110.
- 42] Ottaviano, G.I.P., T. Tabuchi and J.-F. Thisse (2002), "Agglomeration and Trade Revisited", *International Economic Review*, 43, 409-35.
- 43] Peterson, F.M. and A.C. Fisher (1977), "The Exploitation of Extractive Resources: A Survey", *Economic*

Journal, 87, 681-721.

- 44] Porter, M.E. (1991), "America's Green Strategy", *Scientific American*, 264, 168.
- 45] Porter, M.E. and C. van der Linde (1995), "Toward a New Conception of the Environment-Competitiveness Relationship", *Journal of Economic Perspectives*, 9, 97-118.
- 46] Reinganum, J. (1989), "The Timing of Innovation: Research, Development and Diffusion", in R. Schmalensee and R. Willig (eds.), *Handbook of Industrial Organization*, Amsterdam, North-Holland, Vol. 1.
- 47] Schumpeter, J. A. (1942), *Capitalism, Socialism and Democracy*, New York: Harper.
- 48] Simpson, R.D. (1995), "Optimal Pollution Taxation in a Cournot Duopoly", *Environmental and Resource Economics*, 6, 359-69.
- 49] Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Differentiated Duopoly", *RAND Journal of Economics*, 15, 546-54.
- 50] Smith, V. (1969), "On Models of Commercial Fishing", *Journal of Political Economy*, 77, 181-98.
- 51] Tahvonen, O. (1991), "On the Dynamics of Renewable Resource Harvesting and Pollution Control", *Environmental and Resource Economics*, 1, 97-117.
- 52] Tahvonen, O. and J. Kuuluvainen (1991), "Optimal Growth with Renewable Resources and Pollution", *European Economic Review*, 35, 650-61.
- 53] Tahvonen, O. and J. Kuuluvainen (1993), "Economic Growth, Pollution and Renewable Resources", *Journal of Environmental Economics and Management*, 24, 101-18.
- 54] Tirole, J. (1988), *The Theory of Industrial Organization*, Cambridge, MA, MIT Press.
- 55] Verhulst, P.H. (1838), "Notice sur la loi que la population poursuit dans son accroissement", *Correspondance mathématique et physique*, 10, 113-21.
- 56] Volterra, V. (1931), "Variations and Fluctuations of the Number of Individuals in Animal Species Living Together", in R.N. Chapman (ed.), *Animal Ecology*, New York, McGraw-Hill.
- 57] Wirl, F. (2004), "Sustainable Growth, Renewable Resources and Pollution: Thresholds and Cycles", *Journal of Economic Dynamics and Control*, 28, 1149-57.
- 58] Xepapadeas, A. and A. de Zeeuw (1999), "Environmental Policy and Competitiveness: The Porter Hypothesis and the Composition of Capital", *Journal of Environmental Economics and Management*, 37, 165-82.