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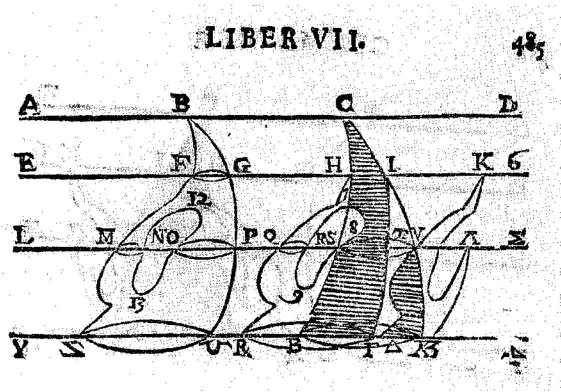

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Contents

What's the Matter with the Deductive Nomological Model?

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Abstract. The philosophical discussion about mathematical explanation has inherited the very same sense of dissatisfaction that philosophers of science expressed, in the context of scientific explanation, towards the famous deductive-nomological model. This model is regarded as unable to cover cases of mathematical explanations and, furthermore, it is largely ignored in the relevant literature. Surprisingly enough, the reasons for this ostracism are not sufficiently manifest among philosophers of mathematics. In this paper, I consider a possible extension of the deductive nomological model to the case of mathematical explanations in science and I claim that there are at least two good reasons to judge the deductive-nomological picture of explanation as inadequate in that context: it cannot deal with mathematical operations or procedures which play a key role in explanatory practices but which do not come under the form of statements; it is not a sufficiently good indicator of the intuitions coming from the scientific practice, thus imposing a picture of explanation which is not authentic.

1. Introduction

The expression 'mathematical explanation' is generally used to indicate two distinct classes, or senses, of explanation: mathematical explanations in empirical sciences (MES) and mathematical explanations in mathematics (MEM). In both these explanations mathematics is regarded as playing an essential role in the explanation provided, i.e. mathematics unveils the reason *why* a particular state of affairs is true, althought MES and MEM denote different things: the former are explanations in empirical sciences that make use of mathematics, whereas MEM refer to explanatory practices that take place within the realm of mathematics itself.

The existence of mathematical explanations, both in the sense of MES and of MEM, is now largely recognized in the literature [**17**]. There is no consensus, however, on how they work and whether they can be captured through a model. A possible strategy in the investigation of these explanations has been to assess a model of scientific explanation, such as Kitcher's unification model or Van Fraassen's pragmatic account, on a case of mathematical explanation (MES or MEM) which has been recognized as genuinely explanatory by the scientists in their practice [**10, 21**]. It is therefore natural to ask what is the result of such an assessment in the case

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of the well-known deductive-nomological (D-N) model of scientific explanation advanced by Carl Hempel and Paul Oppenheim in their famous essay "Studies in the Logic of Explanation" [**12**].

An examination of the contemporary studies on explanation shows that the philosophical discussion about mathematical explanation has inherited the sense of dissatisfaction that philosophers of science expressed, in the context of scientific explanation, towards the D-N model. With the only exception of a few papers, in fact, the –exponentially growing up– literature on mathematical explanation either rules out the possibility to use such a model for cases of MES and MEM, or simply ignores it, thus contributing to sanction its inefficacy (Cf. [**1**] and [**7**]). The consequence of this attitude is that any potential amendment to the model is cast aside from the beginning. Nevertheless, the reasons for this profound skepticism are not clear enough and they remains rather unexplored. As it was said above, other models of scientific explanation have been thought to extend to mathematical explanation and they have been subjected to an accurate analysis. These assessments have pointed to some problems of the models and therefore they disclosed the reasons why these models (or their extensions) are not good candidate to cover mathematical explanation in the classes MES and MEM. More importantly, they have permitted to progress in the philosophical analysis of scientific and mathematical explanation. On the other hand, there is no trace of such an evaluation for the case of the D-N model in the context of mathematical explanation. Why is such a a classical theory of explanation in trouble when faced with a case of mathematical explanation? Surprisingly, although philosophers manifest a negative attitude towards the use of the model in the context of mathematical explanation, the literature has remained rather silent in this regard.

One reason for this philosophical peacefulness may be that there is an extraordinarily obvious reason for it. But what exactly is such an obvious reason? The D-N model was introduced as model of scientific explanation by Carl Hempel and Paul Oppenheim in 1948. These authors conceived explanation as a mere logical deduction (the explanandum is deduced from laws of nature and initial conditions) whose constituents have to satisfy certain logical and empirical conditions of adequacy:

- Logical conditions of adequacy:
  - R_1 The explanandum must be a logical consequence of the explanans.
  - R_2 The explanans must contain general laws, and these must actually be required for the derivation of the explanandum and use no accidental generalizations.
  - R_3 The explanans must have empirical content: that is, it must capable, at least in principle, of test by experiment and observation.
- Empirical condition of adequacy:
  - R_4 The sentences constituting the explanans must be true.

On this account, it seems therefore meaningless to speak of 'mathematical explanation'. And this simply because according to this picture of explanation what is primary to explanation are laws of nature and initial (empirical) conditions, whereas mathematics has no explanatory import. Of course, this does not mean that Hempel and Oppenheim denied that mathematics contributes to explanation.

Mathematics helps in formulating laws of nature, which are the central ingredient of the D-N model, and therefore it has an essential role in explanation. However, the D-N model mirrors the idea that the explanatory power is not conveyed by the mathematics involved, thus ruling out the possibility of any genuine mathematical explanation. This would be the obvious reason why the D-N model is banned from the debate on mathematical explanation (Cf. [**7**, p. 172]).

Although this 'obvious reason' is perfectly reasonable (and, I think, unquestionable), this point remains implicit in much of the debate on mathematical explanation. Besides, it does not exclude that the D-N model may be modified and improved in order to capture genuine cases of MES or MEM. Whether a model of scientific explanation is dismissed as good candidate in the case of mathematical explanation, the reason for this choice should be made clear in all its aspects. This was the case for Kitcher's unification model or Van Fraassen's pragmatic approach evaluated in the context of mathematical explanation, however this is not what happens for the D-N model. Nevertheless, the dissatisfaction with respect to this model, or better towards such a picture of explanation, remains.

The main purpose of this paper is to make this dissatisfaction more explicit and, consequently, more satisfactory. I will suggest that the D-N model cannot be extended, at least if we retain the original conception of explanation put forward by Hempel and Oppenheim, to cover cases of mathematical explanation. And this for two (related) difficulties that the D-N picture of explanation faces in the context of mathematical explanations in science and in mathematics: (*a*) it cannot deal with mathematical operations or procedures which do not come under the form of statements; (*b*) it is not a sufficiently good indicator of the intuitions coming from the scientific practice, thus imposing a picture of explanation which is not authentic (at least if we have faith in the intuitions of our scientists and mathematicians). In the following section I will consider a possible extension of the D-N account for mathematical explanations. My discussion will be based on a paper by Alan Baker, in which such an idea is sketched (i.e. the idea that the D-N model can extend to mathematical explanations if properly modified). In section 3 I will point to the limitations that the extended version of the D-N model has in the context of mathematical explanation. The last section of the paper contains my conclusions.

2. Extending the D-N model to Mathematical Explanations

A number of classical counterexamples undermine the claim that the D-N model provides sufficient conditions for successful scientific explanation[1]. These counterexamples are implicitly based on the following intuition: what is taken to be genuinely explanatory is what is taken to be explanatory in science, and more precisely what is taken to be a bona fide explanation by the working scientists. Roughly, the authority in declaring something an explanation is the authority of practicing scientists. For instance, one deduction in the famous asymmetry problem is regarded as explanatory in science and within a scientific context, whereas the other is not[2]. This provides the philosopher of science with some evidence that he is confronted with a genuine explanation. A model of explanation must, whether correct, reflect this intuition. In fact, this is why such counterexamples constitute a real problem

[1] The *loci classici* for these counterexamples are [**23**], [**6**] [**19**].

[2] Here I assume the reader is familiar with the classical problem of asymmetry, first put forward by Sylvain Bromberger [**6**].

for the D-N model.

Whether we come to mathematical explanation, and we examine how the the D-N model is discussed in that context, we find the same sense of dissatisfaction that philosophers feel towards the model in the case of scientific explanation. When an empirical phenomenon or a mathematical fact is accounted for through mathematics, and the mathematical machinery is recognized by scientists as yielding explanatory power, philosophers of science want their model of explanation to recognize this case as a bona fide explanation[3]. This would permit them to mirror the intuitions of scientists and account for the evidence. However, it is simple to see why the D-N model fails to detect as explanatory such cases of mathematical explanation, thus betraying their expectations. According to condition R_3, the explanans must have empirical content. Thus mathematical explanans are automatically ruled out as genuine explanations by the D-N model. But, once again, to accept this conclusion would be quite unfavorable from a philosophical standpoint. Indeed, it would force us to deny that we are confronted with a genuine explanation and that mathematics plays any explanatory role – regardless whether the case is taken to be a genuine explanation by working scientists and mathematics is considered to play an explanatory role.

In short, to accept the D-N model in its original form simply means to deny that mathematical explanations (MES and MEM) exist. Nevertheless, the intuitions from the scientific practice seem to suggest that this is not true. And, after all, accounting for these intuitions through some conceptual framework is what philosophy of science is supposed to do, or at least one of its main tasks. There is, however, one question that we still want answered and that has not been addressed yet, namely: What about a possible implementation of the D-N model in the context of mathematical explanation? In the following paragraphs of this section I am going to consider this question.

In his paper "Are there genuine mathematical explanations of physical phenomena?" [**1**], Alan Baker presents a case of MES from evolutionary biology. The specific biological phenomenon concerns the life-cycle of an insect called *periodical cicada*. Baker observes how biologists offer an explanation of the prime-numbered-year cicada life-cycle in terms of specific ecological facts, general biological laws and a number theoretic result: "prime periods minimize intersections compared to non-prime periods"[4]. The explanations given by the practice biologists use this mathematical result as an essential element in giving their explanation and mathematics results therefore essential to the structure of the general explanation. According to Baker, the cicada case represents a genuine MES.

In the third part of his paper, Baker consider as possible candidates to cover his example of MES three accounts of scientific explanation: the causal model, the D-N model and Van Fraassen's pragmatic model. The causal account is soon rejected as a possible account for MES because of its incompatibility with any genuine mathematical explanation. On the other hand, Baker claims that the D-N account and the pragmatic account "both support the claim that the cicada case study is an example of a genuinely explanatory application of mathematics to science" [**1**, p. 235]. Although he does not further substantiate this claim, I take it as

[3]There are, of course, exceptions. For instance, Juha Saatsi denies that intuitions from the practice of scientists provide such evidences [**18**, p. 153].

[4]The number theoretic result is actually a consequence of two lemmas. See [**1**, p. 232].

starting point for a discussion of an extension of the D-N model to mathematical explanations. In passing, let me observe that it seems quite surprising that here Baker suggests to rehabilitate, for a case of mathematical explanation, a model of scientific explanation which was addressed to explanations and predictions taking the form of logical derivations from *observational* statements. However, our stupefaction does not exclude that such a move might reinforce previous criticisms, or even have positive repercussions on the study of mathematical explanations.

In what sense is it possible for the D-N model to cover the cicada explanation? Baker remarks that the explanation-schema employed by the biologists has a layout similar to the inferential layout proposed by the D-N model. However, the premise which contains the statement "prime periods minimize intersection" in the cicada's explanation scheme refers to a mathematical theorem, which does not have empirical content and does not represent a law of nature, thus violating conditions R_2 and R_3. To circumvent this difficulty, he suggests that the D-N model would need an extension based on the "broadening of the category of laws of nature to include mathematical theorems and principles" [**1**, p. 235]. Although Baker does not push further this idea, if we follow his suggestion condition R_2 of the original D-N model would assume the following form:

$R_2{}^*$ The explanans must contain general laws, which include mathematical theorems (i.e. analytic truths), and these must actually be required for the derivation of the explanandum.

Furthermore, also the logical condition of adequacy R_3 should be modified in order for the model to admit mathematical explanations:

$R_3{}^*$ The explanans must have empirical or mathematical content.

What about conditions R_1 and R_4? When we come to the *empirical* condition of adequacy R_4, it might be thought that this condition should be left out in the case in which the explanans contain mathematical statements. However, observe that it poses no problem once we adopt a view on truth-values of mathematical statements that appeals to the standards of mathematics itself. If we adopt such a point of view, known as *mathematical naturalism* [**15**], when we defend the truth of a mathematical claim we do not need to appeal to standards outside of mathematics. Rather, we assume that mathematics provide their own sui generis standards of justification, in the same way that physics does. On this account, the singular sentence 'Mt. Everest is snowcapped' is as true as the mathematical statement '$7 + 5 = 12$'. Of course, to preserve R_4 with the name of 'empirical condition of adequacy' would be misleading in the context of our extension to mathematical explanations. This is why I will refer to this condition simply as R_4* ('The explanans must be true'). Finally, let's leave R_1 unchanged and, for convenience sake, name it R_1* ('The explanandum must be a logical consequence of the explanans').

Call D-N*, or D-N Extended, the resulting account of explanation, whose criteria of explanation are fixed by conditions R_1* ... R_4*. On the D-N* account, a phenomenon or a mathematical fact is said to be explained when it is possible to deduce a statement (the *explanandum* E*) from some statements (the *explanans*) according to the conditions of adequacy R_1* ... R_4*. Of course, with respect to the D-N model, here the explanandum can be a statement describing an empirical phenomenon *or* a mathematical statement such as, for instance, ' For every real

$x > 0$ and every integer $n > 0$ there is one and only one positive real y such that $y^n = x$ '. The two situations correspond to a case of MES and MEM, respectively.

The important aspect of this extension is that, although some amendments have been introduced to cover mathematical explanations, the original and fundamental intuition behind the D-N model has been preserved: to have a genuine explanation is to have a sound deductive argument which makes use of at least one lawful connection. As specified by R_2*, the law-like generalizations considered here include mathematical theorems.

3. What's the Matter?

In the previous section I pursued further Alan Baker's suggestion and I showed how the D-N model could be modified to handle mathematical explanations inside its structure. I called D-N* this extension, which has not been explored yet. To extend the basic intuition of the deductive-nomological model has required some changes to the criteria for sound explanation, however it has preserved the basic intuition of the original account proposed by Hempel and Oppenheim. In the present section I shall offer two examples (of MES and MEM, respectively) that are intended to show that (and why) the D-N* account lacks the resources to correctly account for mathematical explanations within its deductive structure. The reason for its failure will serve as a reinforcement of the dissatisfaction seen in the first section.

It is now time to check the effectiveness of the D-N* model on cases which have been recognized as bona fide mathematical explanations. I will consider two examples of mathematical explanation, one for MES and one for MEM in turn. What I am going to show is that a quick assessment of the model on these examples highlights two (related) difficulties that the Hempelian picture of explanation faces in the context of mathematical explanations in science and in mathematics: (*a*) it cannot deal with mathematical operations or procedures which do not come under the form of statements but which are regarded as playing an explanatory role; (*b*) it is not a sufficiently good indicator of the intuitions coming from the scientific practice, thus imposing a picture of explanation which is not authentic. The two difficulties disclose two general problems: the D-N* model does not provide necessary conditions for mathematical explanation, i.e. there are cases which are regarded as bona fide mathematical explanations but which do not qualify as explanations according to the D-N* model; the D-N* model does not provide sufficient conditions for mathematical explanation, i.e. there are sets of statements that qualify as explanations according to the D-N model yet the scientists do not normally think of them as explanatory.

First of all, what about Baker's case of cicadas? Is the D-N* model able to recognize this case as a bona fide mathematical explanation? In cases of MES such as that of cicadas the explanandum, i.e. the sentence which describes the phenomenon to be explained, is a logical consequence of the explanans (via a 5-steps deductive argument analyzed by Baker). Thus criterion R_1* is satisfied. In addition, it is straightforward to see that R_2*, R_3* and R_4* are fulfilled too, thus making possible for the D-N* to consider the cicada example as a bona fide mathematical explanation. After all, we have built our extension of the original D-N model upon Baker's considerations concerning that particular case of MES. However, not all MES exhibit the structure of the cicada explanation. There are MES which are recognized as such in scientific practice and in which the mathematical component of the explanation does not come in the form of a theorem, i.e. a mathematical statement (as required by R_2*). For instance, in their paper "The Explanatory Power of Phase Spaces" [**14**], Aidan Lyon and Mark Colyvan consider such an example of MES. They take into account a particular physical systems called 'Hénon-Heiles system'. This systems is formed by a particle moving in the bidimensional potential $U(q_x, q_y) = \frac{1}{2}(q_x^2 + q_y^2) + q_x q_y^2 - \frac{1}{2}q_y^3$. We want to explain the behaviour (regular or not) of the system for different energies. Let's therefore take the sentence describing the (regular or chaotic) motion of the system as our explanandum. Now, it turns out that there are two mathematical routes to study the behaviour of the system. We can study the system through the Lagrangian analysis, or we can adopt the Hamiltonian formulation which comes with a particular mathematical structure called 'phase space'. The Lagrangian formulation is obtained by introducing the Lagrangian function $L = T - U$, where T is the kinetic energy of the system, and successively obtaining the equations of the motion from the so called *Lagrange's equations*. In this formulation, a system with n degrees of freedom possesses n (second-order) differential equations of motion, while the state of the system is represented by a point in an n-dimensional *configuration space* whose coordinates q_i are called 'generalized coordinates'. The Hamiltonian formulation, on the other hand, is "based on a fundamentally different picture" [**9**, p. 335] and permits to describe the motion in terms of *first-order* equations of motion, known as *Hamilton's canonical equations of motion*. These equations describe the behavior of the system point in a particular space, the phase space, which has $2n$-dimensions and whose coordinates are the $2n$ independent variables which appear in the canonical equations of motion. In other words, in the Hamiltonian formulation of mechanics the dynamics of our Hénon-Heiles system is defined by the evolution of points ('trajectories') in the phase space[5].

The fact that the Hénon-Heiles system exhibits regular or chaotic motion is deduced visually from a representation in the phase space. By considering the total energy of the system E constant, we lower the dimensionality of the phase space by one. Next we take a 2-dimensional cross section of this hypersurface in the phase space and we map the intersections of the trajectories with the plane by using a function called *Poincaré map*. Finally, we look at the dots made by the solutions (orbits of the system) on the Poincaré section and we can visually grasp qualitative informations about the dynamics of the system at that particular energy. Thus the phase space, with its mathematical apparatus, is regarded to have an explanatory role:

> The explanatory power is in the structure of the phase space and the Poincaré map. So it seems that this is a case where using the phase space is essential to our understanding and ability to explain certain features of the world [**14**, p. 14]

Now, it is important to note again that the Hamiltonian procedure involving phase space is not the only alternative for the study of the system. However, to analyze the system via the Lagrangian route seems not to convey the sense of explanatoriness that we obtain from the use of the phase space theory in the Hamiltonian formalism:

[5]Of course, there are technicalities I am glossing over here for the purposes of exposition. See [**14**] for the full treatment.

> [...] although there is a Lagrangian formulation of the theory in question that does not employ phase spaces, the cost of adopting such an approach is a loss of explanatory power [**14**, p. 2]

From Lyon and Colyvan's example two important points emerge: first, even though mathematics comes as an essential ingredient, it is not a particular theorem (i.e. a mathematical law) which participates in the explanation; second, although two mathematical procedures are acceptable as to study the physical phenomenon (regular or chaotic motion of the particle moving in the potential), only one of them carries explanatory power. Consequently, in the context of this example, the D-N* is confronted with the following problem: the model cannot deal with mathematical operations or procedures (such as the use of the Hamiltonian formalism including phase space and Poincaré map) which do not come under the form of statements, and therefore it does not recognize the explanation as genuine[6]. What is more, even if we would have such mathematical procedures under the form of statements, the D-N* model would lack in resources to discriminate between the explanatory mathematical procedure and the non-explanatory one. In fact, these procedures (the Lagrangian and the Hamiltonian) are both formally correct from a mathematical point of view, and therefore the D-N* would consider both equally explanatory on the basis of criterion R_1* (both are good ingredients of the logical deduction). But, again, note that scientists consider as bona fide mathematical explanation only one mathematical procedure, that which uses the Hamiltonian formalism. As a consequence, to regard the Lagrangian treatment of the Hénon-Heiles system as equally explanatory, as the D-N* model suggests, would go against the opinion of scientists and would not do justice to their intuitions.

In the previous lines I showed that the D-N* model does not recognize as explanatory the example of MES in question because of difficulty (*a*). Furthermore, I claimed that even if the D-N* model would be capable of bypassing difficulty (*a*), it would suggest a picture of explanation which does not fit with the intuitions coming from the scientific practice. In other words, difficulty (*b*) remains. The second example I want to consider is a case of mathematical explanation, and in particular a case of MEM, where the difficulty (*b*) is even more pronounced. To anticipate the point, the D-N* model identifies every formal proof in mathematics as a genuine explanation. Every formal proof, in fact, inevitably follows a logical deductive schema in which the basic concept is that of a statement being a logical consequence of some other statements. Moreover, when we consider such a proof conditions R_2*, R_3* and R_4* are fulfilled as well. But, once again, does the D-N* model accurately mirror the intuitions coming from the practicing mathematicians? It seems that this is not the case, and this exactly because mathematicians do not consider *every* correct formal proof as an explanation of a mathematical result. There are several examples of MEM which have been discussed in the literature and which might be used here to illustrate this point ([**17**] provides a survey of various cases). Nevertheless, for the purposes of this paper it will be sufficient to consider just one case.

In his paper "Explanatory Unification and the Causal Structure of the World", Philip Kitcher points out that explanatory asymmetries arise also in the domain of mathematics and are not a privilege of the causal debate on explanation [**13**, p.

[6] Note the difference between Lyon and Colyvan's example and the example chosen by Baker, where the mathematics involved was expressed by a single theorem.

425][7]. To illustrate his point he considers the proof of a property of finite groups by means of one specific axiomatization of the theory of finite groups. In that case, one particular axiomatization containing the existence of the inverse and idempotent elements is preferred by the mathematicians in order to 'explain' why finite groups satisfy the division property. On the other hand, the reverse derivation, i.e. the derivation of the existence of an idempotent element and of inverses from the division property, is regarded as a less natural and non-explanatory derivation (though formally valid). The former derivation is regarded by the mathematicians as explanatory on the basis of its capacity of providing more general results, whereas the other derivation does not provide such possibility:

> It is not hard to see a reason for the distinguishing of the derivations: the preferred derivation can be generalized to achieve more wide-ranging results [...] the explanatory derivation is similar to derivations we could provide for a more general result; the nonexplanatory derivation cannot be generalized, it applies only to the local case [**13**, p. 425]

We want to analyse now the situation through the lenses of the D-N* model. What we discover pretty soon is that, analogously to what happens when the D-N is confronted with cases of explanatory asymmetries in the empirical sciences, such as in the classical asymmetry example of the flagpole and the shadow put forward by Bromberger [**6**], the D-N* model lacks resources to discriminate between the two (formally valid) proofs and pick out the bona-fide mathematical explanation. Indeed, according to the D-N* model, both the mathematical deductions meet criteria R_1* ... R_4*, and are therefore recognized as genuine mathematical explanations. Certainly proving theorems is the canonical means of obtaining knowledge of mathematical facts. However, it seems that if we limit our picture of the cognitive activity informing mathematical reasoning to include just deductively valid arguments from indubitable premises, then we will not have any access to the sorts of reasons that we are interested in here. This is why the D-N* model, which is based on such a picture, does not discriminated between the explanatory derivation and the non-explanatory one in the example put forward by Kitcher.

The general moral of the previous lines is that the D-N* model does not accurately mirror important aspects of scientific practice and, furthermore, it does impose a picture of explanation which is not authentic. By adopting the D-N* we are left with a schema which is nothing more than a purely logical deduction. Consequently, explaining an outcome E (a statement describing an empirical phenomenon or a mathematical statement) is just a matter of showing that it is expectable on the basis of lawful connections (where laws also include mathematical theorems). Although this would be at odds with the evidence coming from the scientific practice, as the two previous examples show, it might be thought that this observation cannot be used to criticize the D-N*. In fact, once we adopt the Hempelian perspective that explanation *is* logical deduction, we do not need to resort to such 'evidences' from scientific practice. It should be noted, however, that the import of such evidences is considered by many philosophers as of primary importance for the philosophical analysis of the notion of explanation in science and

[7]In passing, let me note that this is the first time, at least to my knowledge, that such an analogy, i.e. the analogy between explanatory asymmetries in empirical science and explanatory asymmetries in mathematics, is brought to light.

mathematics [**16**]. And, once again, it seems very natural to think that accounting for such evidences is one of the main task of philosophy of science.

4. Conclusions

Despite the great interest in the linkage between scientific explanation and mathematical explanation, an extensive discussion of models of scientific explanation in the context of mathematical explanation has not been offered and work is just beginning. In this paper I showed how the D-N model of scientific explanation can be thought to extend to mathematical explanation when the conditions of adequacy on which it depends are properly modified. My analysis was intended to clarify the reasons why the picture of explanation which comes with the D-N model is insufficient to cover mathematical explanations. In fact, these reasons are not explicitly stated in the literature on explanation, although it is often assumed that the D-N model cannot cover mathematical explanations.

It was not my intention here to discuss every possible way to extend the D-N model to mathematical explanation. I have picked out only one possible extension, which I named D-N*. By pointing to difficulties (a) and (b), I showed how the D-N* suffers problems of necessity and sufficiency: there exist mathematical explanations that don't fit the model but are regarded as bona-fide explanations by the scientists; there are mathematical explanations that fit the model but that are not considered as genuine explanations in the scientific practice. Moreover, I put forward a diagnosis of such difficulties. The failure of the D-N* model, as in the case of the original model, has do to with the inability of logic alone to mirror particular non-logical features of the case studies which carry explanatory import. In the two examples considered in the previous section these non-logical features are the capacity mathematics has of making possible to infer visually some information on a diagram and its capacity of providing more general results. Making logical deduction the hallmark of explanation preserves the basic intuition of the D-N model but amounts to the imposition of a defining characteristic feature on what ought to be counted as 'mathematical explanation'. The resulting picture of explanation is incomplete and not satisfactory. And my general feeling is that every effort to extend the D-N model picture of explanation to MES and MEM would face impediments similar to those highlighted here and would therefore be unsuccessful.

Of course, my dissatisfaction with the D-N and the D-N*, or better with the specific picture of explanation which stands behind these models, is based on the following assumption: the intuitions coming from the scientific practice, although not providing normative and rigid standards of explanation, supply philosophers with some guidelines in the study of explanation. And these intuitions should be mirrored by our philosophical accounts of explanation. Nevertheless, this is something which I regard as natural (and widely accepted) not only in the context of explanation, but more generally in the context of philosophy of science. I would be happy about the following situation: the D-N* model, or any other account of explanation, identifies a mathematical explanation as genuine and that particular explanation is recognized as a bona fide explanation in the scientific practice as well. This situation would be, I think, an indicator of the fruitfulness of the philosophical investigation which lies behind that account of explanation. On the other hand, I regard neither reasonable nor philosophically fructuous that a philosophical model of explanation does impose on the scientists a normative criterion of explanation that the scientists would not accept.

As a whole, my results confirm the dissatisfaction which philosophers of science feel towards the use of the D-N model in the context of mathematical explanation. Furthermore, they highlight the reasons why the picture of explanation which stands behind that account is problematic, thus making our sense of dissatisfaction more satisfactory.

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