Trajectory Tracking for Discrete-Time Port-Hamiltonian Systems

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***Abstract*—This letter presents a regulator for nonlinear, discrete-time port-Hamiltonian systems that lets the state track a reference signal. Similarly to continuous-time approaches, the synthesis is based on the mapping via state-feedback of the open-loop error system to a target one in port-Hamiltonian form, and with an asymptotically stable origin that corresponds to the perfect tracking condition. The procedure is formally described by a matching equation that, in continuous-time, turns out to be a nonlinear partial differential equation (PDE). This is not the case for sampled-data systems, so an algebraic approach is proposed. The solution is employed to construct a dynamical regulator that performs an "approximated" mapping. The stability analysis relies on Lyapunov arguments.**

***Index Terms*—Discrete-time systems, port-hamiltonian systems, sampled-data control.**

I. Introduction

PORT-HAMILTONIAN systems have been introduced about thirty years ago to model continuous-time, lumped-parameter physical systems, [1]. The discrete-time extension is a popular framework for the geometric integration of ordinary differential equations (ODE) that preserves either the *structure* or the *energy* of continuous-time systems. So, two categories can be found in literature, i.e., the geometric and the energetic integrators. In the autonomous case, the number of contributions is large, and most of the results can be found in textbooks, [2]. As far as non-autonomous, discrete-time port-Hamiltonian systems are concerned, a definition on discrete manifolds can be found in [3]. When the state space is a vector space, sampled-data (port-)Hamiltonian systems based on symplectic integrators have been employed, e.g., in [4]–[8].

The discrete-time systems studied here belong instead to the family of energy-preserving integrators, [9], [10]. As in [11]–[14], their dynamics depend on the discrete gradient of the Hamiltonian function. The advantage is that an energy-balance relation is easily obtained, thus making the stability analysis based on Lyapunov arguments simpler. The side-effect is that the state equation is implicit. Not only continuous-time *physical* systems but also systems that naturally evolve in discrete-time, such as numerical algorithms for convex or non-convex optimization problems [15], fit into this framework. This letter deals with control design, and its contribution is a regulator that lets a nonlinear, discrete-time, port-Hamiltonian system track a reference trajectory.

The regulator implementations are digital, so the development of synthesis methodologies in discrete-time and inspired by energy-based design paradigms is relevant not only from a theoretical point of view. Since the regulator is coupled with continuous-time dynamics, the idea has been to work with models that belong to the family of energy-preserving integrators and rely on the *energy* as the "lingua franca" to study the closed-loop performances. This approach does not require that the coupling between plant and digital controller is passive despite the presence of the sample-and-hold block, [16], [17]. This problem is beyond the scopes of this letter, and so is not tackled here. The state-feedback action is designed so that the open-loop dynamics is mapped to a time-varying error system in a port-Hamiltonian form that generalises [13]. Such a target system has desired structure, energy function, and stability properties.

The synthesis procedure is similar to [18], where canonical transformations were employed, and [19] within the Interconnection and Damping Assignment Passivity-based Control (IDA-PBC) framework, both in the continuous-time case. An extension to sampled-data systems has been proposed in [11], [20], [21]. In [21], the focus is on the design of energy-based regulators to stabilize on constant references mechanical systems modeled by symplectic, discrete-time port-Hamiltonian systems. Here, instead, the energy-preserving, nonlinear, time-varying case is tackled together with the implicit formulation of the state equation.

The mapping between open-loop and desired error systems requires solving a *matching equation*. In continuous-time, this equation is a nonlinear partial differential equation (PDE), while in the digital case it is algebraic in the discrete gradient. Only in particular cases, it is possible to transform the latter into a PDE that is instrumental to get a solution in discrete-time. To overcome such a limitation, the approach proposed in [22] has been extended to deal with time-varying *and* sample-data systems to obtain an algebraic solution to the matching equation. Note that the corresponding stabilizing laws are not necessarily based on the discrete gradient of an energy function. To conclude, the design methodology

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is illustrated with an example in which the plant is a two degrees-of-freedom, fully-actuated, planar manipulator.

Notation: For a matrix $M \in \mathbb{R}^{n\times m}$, $M^\perp$ and M^+ are the full-rank left annihilator and pseudo-inverse, respectively. If $n = m$, $\operatorname{sym} M = \frac{1}{2}(M + M^{\mathrm{T}})$ is the symmetric part. Given the sampled variable $x_k \in \mathbb{R}^n$, the pair (x_k, x_{k+1}) is denoted by $\hat{x}_k \in \mathbb{R}^{2n}$. Finally, the state of the error system is z_k and the quantities related to its evolution are equipped with a $'$.

II. Discrete-Time Port-Hamiltonian Systems

In this section, the discrete-time formulation of the following nonlinear, continuous-time, time-varying port-Hamiltonian system is presented, [1], [18]:

$$\begin{cases} \dot{x}(t) = F(x(t), t)\nabla H(x(t), t) + G(x(t), t)u(t) \\ y(t) = G^{\mathrm{T}}(x(t), t)\nabla H(x(t), t) \\ x(0) = x_0. \end{cases} \tag{1}$$

In (1), $t \in \mathbb{R}_{\geq 0}$ and $x(t) \in \mathbb{R}^n$ are the time and state variable, $u(t), y(t) \in \mathbb{R}^m$ the input and output, $F(x, t) \in \mathbb{R}^{n\times n}$ is such that $\operatorname{sym} F(x, t) \leq 0$ for all x and t, $G(x, t) \in \mathbb{R}^{n\times m}$ is full-rank, and $H : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ is the Hamiltonian (energy) function. Along system's trajectories we have that

$$\begin{aligned} \dot{H}(x(t), t) = {} & \nabla^{\mathrm{T}} H(x(t), t)F(x(t), t)\nabla H(x(t), t) \\ & + y^{\mathrm{T}}(t)u(t) + \partial_t H(x(t), t), \end{aligned} \tag{2}$$

where ∂_t denotes the partial derivative with respect to time. Consequently, if $H(x, t)$ is lower-bounded and $\partial_t H(x, t) \leq 0$, (1) is passive and $H(x, t)$ is the storage function.

To get a discrete-time formulation of (1), the time derivative of the state is approximated by the finite difference as

$$\dot{x}(t_k) \simeq \tfrac{1}{\tau}(x_{k+1} - x_k) \tag{3}$$

being $t_k = k\tau$, with $\tau > 0$ and $k \in \mathbb{N}$, the time samples and $x_k = x(t_k)$. Besides, a discrete approximation of the gradient operator is necessary, see, e.g., [9, Definition 3.1].

Definition 1: Let $\Phi : \mathbb{R}^p \to \mathbb{R}$ be a continuously differentiable function. A *discrete gradient* $\bar{\nabla}\Phi : \mathbb{R}^p \times \mathbb{R}^p \to \mathbb{R}^p$ is a continuous map such that for all $\varphi, \varphi_+ \in \mathbb{R}^p$ we have

$$\begin{aligned} (\varphi_+ - \varphi)^{\mathrm{T}} \bar{\nabla}\Phi(\varphi, \varphi_+) &= \Phi(\varphi_+) - \Phi(\varphi), \\ \lim_{\varphi_+ \to \varphi} \bar{\nabla}\Phi(\varphi, \varphi_+) &= \nabla\Phi(\varphi). \end{aligned} \tag{4}$$

Typical examples are the mean value [23, Th. 2.1], or the Gonzalez discrete gradients, [9, Proposition 3.1]. If we have a quadratic function $\Phi(\varphi) = \varphi^{\mathrm{T}} Q\varphi$ with $Q = Q^{\mathrm{T}}$, it is easy to check that $\bar{\nabla}\Phi(\varphi, \varphi_+) = Q(\varphi + \varphi_+)$.

Based on (1), (3) and Definition 1, the discrete-time formulation of a port-Hamiltonian dynamics is:

$$\begin{cases} x_{k+1} = x_k + \tau\bar{F}(\hat{x}_k, \hat{t}_k)\bar{\nabla}H(\hat{x}_k, \hat{t}_k) + \tau\bar{G}(\hat{x}_k, \hat{t}_k)u_k \\ y_k = \bar{G}^{\mathrm{T}}(\hat{x}_k, \hat{t}_k)\bar{\nabla}H(\hat{x}_k, \hat{t}_k) \end{cases} \tag{5}$$

in which $u_k = u(k\tau)$, with $k \in \mathbb{N}$. The matrix-valued functions $\bar{F} : \mathbb{R}^{2n} \times \mathbb{R}^2 \to \mathbb{R}^{n\times n}$ and $\bar{G} : \mathbb{R}^{2n} \times \mathbb{R}^2 \to \mathbb{R}^{n\times m}$ are discrete approximations of F and G, respectively, i.e., for all $x, x_+ \in \mathbb{R}^n$ and $t, t_+ \in \mathbb{R}$ we have that

$$\begin{aligned} \bar{F}(x, x_+, t, t_+) + \bar{F}^{\mathrm{T}}(x, x_+, t, t_+) &\leq 0 \\ \bar{F}(x, x, t, t) &= F(x, t) \\ \bar{G}(x, x, t, t) &= G(x, t). \end{aligned}$$

Admissible choices for $\bar{F}$ are $F(\frac{1}{2}(x + x_+), \frac{1}{2}(t + t_+))$ or $\frac{1}{2}F(x, t) + \frac{1}{2}F(x_+, t_+)$, and similarly for $\bar{G}$. From (4), we get the discrete-time counterpart of (2), i.e.:

$$\begin{aligned} \tfrac{1}{\tau}\big[H(x_{k+1}, t_{k+1}) - H(x_k, t_k)\big] & \\ = \bar{\nabla}^{\mathrm{T}} H(\hat{x}_k, \hat{t}_k)\bar{F}(\hat{x}_k, \hat{t}_k)\bar{\nabla}H(\hat{x}_k, \hat{t}_k) & \\ + y_k^{\mathrm{T}} u_k + \bar{\partial}_t H(\hat{x}_k, \hat{t}_k), & \end{aligned} \tag{6}$$

where with some abuse in notation $\bar{\partial}_t$ denotes the discrete partial derivative.

Remark 1: The drawback of the discrete-time system (5) is that the dynamics is in implicit form. This is the price to pay to have "for free" the energy-balance relation (6), which is the starting point for control design and stability analysis. Such an implicit equation can be made explicit, i.e., solved for x_{k+1}, in the linear case, because H is quadratic and F and G in (1) are constant. This approach has been followed in [11], [20]. In the nonlinear case, however, it is necessary to keep such an implicit formulation. Based on passivity arguments and inspired by [15], under mild conditions on the structure of the system and on the discrete gradient, it is possible to show that trajectories exist independently from the sampling time. This is the same rationale pursued in [13] where a similar result has been proved under the hypothesis that the sampling time is sufficiently small. In this letter we assume that the dynamical equations are well-posed, so the next state can be always (at least numerically) computed.

III. Trajectory Tracking: Problem Formulation

The goal is to develop a controller that lets the discrete-time system (5) track a feasible trajectory $x_k^\star \in \mathbb{R}^n$. If (5) is the sampled version of (1) and $x^\star(t)$ the desired trajectory, then $x_k^\star = x^\star(t_k)$, with $k \in \mathbb{N}$. Inspired by [18], the idea is to obtain an error system for (5) in port-Hamiltonian form via state-feedback, and for which the origin is either the minimum of the Hamiltonian function and the configuration that guarantees perfect tracking. So, the origin has also to be an asymptotically stable equilibrium. Let us denote by

$$z_k = \Psi(x_k, t_k), \quad k \in \mathbb{N} \tag{7}$$

the error variable, in which $\Psi(x, t)$ is a change of coordinates such that $\Psi(x_k, t_k) = 0$ if and only if $x_k = x_k^\star$. From (7) and Definition 1, we get that

$$z_{k+1} - z_k = \bar{\nabla}^{\mathrm{T}}\Psi(\hat{x}_k, \hat{t}_k)(x_{k+1} - x_k) + \tau\bar{\partial}_t\Psi(\hat{x}_k, \hat{t}_k) \tag{8}$$

since $t_{k+1} - t_k = \tau$ and with the x_k dynamics given in (5). The conditions that a state-feedback control action

$$u_k = \beta(\hat{x}_k, \hat{t}_k) + u_k', \quad k \in \mathbb{N} \tag{9}$$

has to obey so that the z_k dynamics is in port-Hamiltonian form are summarized in the next proposition. In (9), $u_k' \in \mathbb{R}^m$ is an auxiliary control input, specified later.

Proposition 1: Let us consider (5) and the two functions $H_a : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ and $\bar{F}_a : \mathbb{R}^{2n} \times \mathbb{R}^2 \to \mathbb{R}^{n\times n}$ such that $H_d = H + H$ and $\bar{F}_d = \bar{F} + \bar{F}_a$ satisfy the same conditions for H and $\bar{F}$ in (5). The state-feedback law $\beta(\hat{x}_k, \hat{t}_k)$ in (9) leads to a discrete-time port-Hamiltonian dynamics for z_k introduced in (7) if and only if

$$\bar{\nabla}^{\mathrm{T}}\Psi(\hat{x}_k,\hat{t}_k)\big[\bar{F}(\hat{x}_k,\hat{t}_k)\bar{\nabla}H_a(\hat{x}_k,\hat{t}_k)+\bar{F}_a(\hat{x}_k,\hat{t}_k)\bar{\nabla}H_d(\hat{x}_k,\hat{t}_k) - \bar{G}(\hat{x}_k,\hat{t}_k)\beta(\hat{x}_k,\hat{t}_k)\big]-\bar{\partial}_t\Psi(\hat{x}_k,\hat{t}_k)=0. \quad (10)$$

Proof: From (5), (8) and (9), we get that

$$z_{k+1}-z_k=\tau\bar{\nabla}^{\mathrm{T}}\Psi\big[\bar{F}\bar{\nabla}H+\bar{G}\big(\beta+u_k'\big)\big]+\tau\bar{\partial}_t\Psi, \quad (11)$$

which leads to $z_{k+1}-z_k=\tau\bar{\nabla}^{\mathrm{T}}\Psi\bar{F}_d\bar{\nabla}H_d+\tau\bar{\nabla}^{\mathrm{T}}\Psi\bar{G}u_k'$ when combined with (10). Note that, in the previous expressions, the dependence on $(\hat{x}_k,\hat{t}_k)$ has been omitted. Now, with (7) in mind, if

$$\begin{aligned} H_d'(z_k,t_k)&=H_d(x_k,t_k)\\ \bar{F}_d'(\hat{z}_k,\hat{t}_k)&=\bar{\nabla}^{\mathrm{T}}\Psi(\hat{x}_k,\hat{t}_k)\bar{F}_d(\hat{x}_k,\hat{t}_k)\bar{\nabla}^{\mathrm{T}}\Psi(\hat{x}_k,\hat{t}_k)\\ \bar{G}'(\hat{z}_k,\hat{t}_k)&=\bar{\nabla}^{\mathrm{T}}\Psi(\hat{x}_k,\hat{t}_k)\bar{G}(\hat{x}_k,\hat{t}_k), \end{aligned} \quad (12)$$

we get that

$$z_{k+1}=z_k+\tau\bar{F}_d'(\hat{z}_k,\hat{t}_k)\bar{\nabla}H_d'(\hat{z}_k,\hat{t}_k)+\tau\bar{G}'(\hat{z}_k,\hat{t}_k)u_k', \quad (13)$$

where $\hat{z}_k=(z_k,z_{k+1})\in\mathbb{R}^{2n}$ and since $\bar{\nabla}H_d=\bar{\nabla}^{\mathrm{T}}\Psi\bar{\nabla}H_d'$. Due to the fact that $F_d+F_d^{\mathrm{T}}\leq 0$, the same property is valid for F_d', and so (13) is in port-Hamiltonian form. ■

As in (5), to get a balance relation similar to (6), the output of (13) is defined as $y_k'=\bar{G}'^{\mathrm{T}}(\hat{z}_k,\hat{t}_k)\bar{\nabla}H_d'(\hat{z}_k,\hat{t}_k)$. Then, the design of a controller that lets (5) track the trajectory $x_k^\star$ reduces to solve the matching equation (10) with some function $H_d'(z_k,t_k)$ that has a (local) minimum at the origin. The convergence rate is improved by acting on the (u_k',y_k') port by imposing that

$$u_k'=-K_d(z_k,t_k)y_k', \quad (14)$$

with $K_d:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^{m\times m}$ and such that $K_d=K_d^{\mathrm{T}}\geq 0$. In the z_k coordinates, (10) becomes

$$\bar{F}'(\hat{z}_k,\hat{t}_k)\bar{\nabla}H_a'(\hat{z}_k,\hat{t}_k)+\bar{F}_a'(\hat{z}_k,\hat{t}_k)\bar{\nabla}H_d'(\hat{z}_k,\hat{t}_k)= =\bar{G}'(\hat{z}_k,\hat{t}_k)\beta'(\hat{z}_k,\hat{t}_k)+\bar{\partial}_t\Psi'(\hat{z}_k,\hat{t}_k) \quad (15)$$

where H_a', $\bar{F}'$, $\bar{F}_a'$, $\bar{\partial}_t\Psi'$, and β' are in the coordinates of the error system. Relation (15) can be re-written as

$$\bar{G}'^{\perp}(\hat{z}_k,\hat{t}_k)\big[\bar{F}'(\hat{z}_k,\hat{t}_k)\bar{\nabla}H_a'(\hat{z}_k,\hat{t}_k) + \bar{F}_a'(\hat{z}_k,\hat{t}_k)\bar{\nabla}H_d'(\hat{z}_k,\hat{t}_k)-\bar{\partial}_t\Psi'(\hat{z}_k,\hat{t}_k)\big]=0 \quad (16)$$

and

$$\beta'(\hat{z}_k,\hat{t}_k)=\bar{G}'^{+}(\hat{z}_k,\hat{t}_k)\big[\bar{F}'(\hat{z}_k,\hat{t}_k)\bar{\nabla}H_a'(\hat{z}_k,\hat{t}_k)+\bar{F}_a'(\hat{z}_k,\hat{t}_k) \times\bar{\nabla}H_d'(\hat{z}_k,\hat{t}_k)-\bar{\partial}_t\Psi'(\hat{z}_k,\hat{t}_k)\big]. \quad (17)$$

If a solution for (16) can be computed, (17) provides the control action that maps (5) into the target error system (13). The obtained solution has to guarantee that the zero equilibrium is (locally) asymptotically stable to achieve perfect tracking of the reference $x_k^\star$.

IV. Algebraic Solution of the Matching Equation

The aim of this section is to develop an algebraic procedure that allows us to solve the matching condition (16). To achieve this, let us introduce the function $K_a':\mathbb{R}^{2n}\times\mathbb{R}^2\to\mathbb{R}^n$ and re-write (16) as

$$\bar{G}'^{\perp}(\hat{z}_k,\hat{t}_k)\big[\bar{F}_d'(\hat{z}_k,\hat{t}_k)K_a'(\hat{z}_k,\hat{t}_k) + \bar{F}_a'(\hat{z}_k,\hat{t}_k)\bar{\nabla}H'(\hat{z}_k,\hat{t}_k)-\bar{\partial}_t\Psi'(\hat{z}_k,\hat{t}_k)\big]=0. \quad (18)$$

These two equations are equivalent if and only if K_a' solution of (17) is also the discrete gradient of a real-valued function, namely H_a'. In the continuous-time case, such a requirement can be met by solving the PDE associated to the matching condition. This is not the case in the discrete-time setting. For this reason, inspired by [22], an algebraic solution of the matching equation (18) is now illustrated.

The Taylor expansion of $H'(z,t)$ around $z=0$ is

$$H'(z,t)=H_0(t)+H_1^{\mathrm{T}}(t)z+\tfrac{1}{2}z^{\mathrm{T}}H_2(t)z+h(z,t) \quad (19)$$

where $H_0\in\mathbb{R}$, $H_1\in\mathbb{R}^n$, and $H_2=H_2^{\mathrm{T}}\in\mathbb{R}^{n\times n}$, while $h:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}$ is the remainder and such that $\lim_{z\to 0}\frac{h(z,t)}{\|z\|^2}=0$. Given $z,z_+\in\mathbb{R}^n$ and $t,t_+\in\mathbb{R}$, it is assumed that

$$\bar{\nabla}h(z,z_+,t,t_+)=\tfrac{1}{2}\big[h_0(z,z_+,t,t_+)z+h_0(z_+,z,t_+,t)z_+\big],$$

where $h_0:\mathbb{R}^{2n}\times\mathbb{R}^2\to\mathbb{R}^{n\times n}$ is such that $h_0(0,0,t,t_+)=0$. This hypothesis is in line with the fact that $\bar{\nabla}\Phi(\varphi,\varphi_+)=\bar{\nabla}\Phi(\varphi_+,\varphi)$, see Definition 1. Besides, it can be checked that such a property holds for polynomial functions. Then, the discrete gradient of (19) is

$$\bar{\nabla}H'(z,z_+,t,t_+)=\bar{H}_1(t,t_+)+\tfrac{1}{2}\bar{H}_2(t,t_+)(z+z_+) + \bar{\nabla}h(z,z_+,t,t_+), \quad (20)$$

where $\bar{H}_i(t,t_+)=\frac{1}{2}[H_i(t)+H_i(t_+)]$, $i=1,2$. With the procedure presented in [22] in mind, the function $K_a':\mathbb{R}^{2n}\times\mathbb{R}^2\to\mathbb{R}^n$ is defined as

$$K_a'(z,z_+,t,t_+)=\tfrac{1}{2}\bar{P}_1(z,z_+,t,t_+)(z+z_+) - \bar{H}_1(t,t_+)+\bar{P}_0(z,z_+,t,t_+). \quad (21)$$

Suppose that $\bar{P}_1(z,z_+,t,t_+)=\frac{1}{2}[P_1(z,t)+P_1(z_+,t_+)]$, being $P_1:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^{n\times n}$ a function such that $P_1(z,t)=P_1^{\mathrm{T}}(z,t)$ for all $z\in\mathbb{R}^n$ and $t\in\mathbb{R}$, and that

$$\bar{P}_1=-\bar{F}_d'^{+}\big[\bar{G}'\Lambda_1+\bar{F}_a'\big(\bar{H}_2+h_0\big)\big] \quad (22)$$

for a function $\Lambda_1:\mathbb{R}^{2n}\times\mathbb{R}^2\to\mathbb{R}^{m\times n}$. In (22), the dependence on (z,z_+,t,t_+) has been omitted to simplify the notation. As before, $\bar{P}_0(z,z_+,t,t_+)=\frac{1}{2}[P_0(z,t)+P_0(z_+,t_+)]$, where $P_0:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^n$ is a function such that

$$\bar{P}_0=\bar{F}_d'^{+}\big[\bar{G}'\Lambda_0+\big(\bar{F}'\bar{H}_1+\bar{\partial}_t\Psi'\big)\big] \quad (23)$$

for a $\Lambda_0:\mathbb{R}^{2n}\times\mathbb{R}^2\to\mathbb{R}^{m\times n}$. Note that if in (21) we set $z=z_k$, $z_+=z_{k+1}$, $t=t_k$ and $t_+=t_{k+1}$, then $K_a'(\hat{z}_k,\hat{t}_k)$ satisfies the matching condition (18).

For any $R=R^{\mathrm{T}}>0$, let

$$H_d'(z,w,t)=H'(z,t)+H_a'(z,w,t) \quad (24)$$

with

$$H'_a(z,w,t) = -H_1^{\mathrm{T}}(t)z + P_0^{\mathrm{T}}(w,t)z + \tfrac{1}{2}z^{\mathrm{T}}P_1(w,t)z + \tfrac{1}{2}(z-w)^{\mathrm{T}}R(z-w)$$

be a desired Hamiltonian function defined on $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \ni (z,w,t)$ and for which the origin is a (local) minimum if for all t and $\kappa > 0$ sufficiently large, [22]:

$$P_0(0,t) = 0 \quad P_1(0,t) + H_2(t) > \kappa I_n. \tag{25}$$

The idea is to design the w_k dynamics so that the control input for (5) is given by (9) and (17), but with $\bar{\nabla}H'_a$ replaced by the K'_a defined in (21), and that (z_k, w_k) goes to $(0,0)$. This problem is tackled in the next section.

V. Control Design and Stability Analysis

The goal is to have a closed-loop system with state variable (z_k, w_k) whose evolution depends on the discrete gradient of the energy function (24). So, the first step consists in computing $\bar{\nabla}H'_a$. With some abuse in notation, we get that

$$\bar{\nabla}H'_a(\hat{z}_k,\hat{w}_k,\hat{t}_k) = \begin{pmatrix}\bar{\nabla}_z H'_a(\hat{z}_k,\hat{w}_k,\hat{t}_k)\\ \bar{\nabla}_w H'_a(\hat{z}_k,\hat{w}_k,\hat{t}_k)\end{pmatrix}$$

where

$$\begin{aligned}\bar{\nabla}_z H'_a(\hat{z}_k,\hat{w}_k,\hat{t}_k) &= -\bar{H}_1(\hat{t}_k) + \bar{P}_0(\hat{w}_k,\hat{t}_k) + \tfrac{1}{2}\bar{P}_1(\hat{w}_k,\hat{t}_k)\\ &\quad\times (z_k + z_{k+1}) + \tfrac{1}{2}R(z_k + z_{k+1} - w_k - w_{k+1})\\ \bar{\nabla}_w H'_a(\hat{z}_k,\hat{w}_k,\hat{t}_k) &= \tfrac{1}{2}\Theta(\hat{z}_k,\hat{w}_k,\hat{t}_k)\hat{z}_k\\ &\quad - \tfrac{1}{2}R(z_k + z_{k+1} - w_k - w_{k+1})\end{aligned} \tag{26}$$

in which $\Theta(\hat{z}_k,\hat{w}_k,\hat{t}_k)$ takes into account the discrete gradients with respect to w of the terms in H'_a that depend on P_0 and P_1. If there exists a function $\Theta_1(\hat{z}_k,\hat{w}_k,\hat{t}_k)$ such that

$$\begin{aligned}&\left[\bar{P}_1(\hat{w}_k,\hat{t}_k) - \bar{P}_1(\hat{z}_k,\hat{t}_k)\right](z_k + z_{k+1})\\ &\quad = \Theta_1(\hat{z}_k,\hat{w}_k,\hat{t}_k)(z_k + z_{k+1} - w_k - w_{k+1})\end{aligned}$$

and a function $\Theta_0(\hat{z}_k,\hat{w}_k,\hat{t}_k)$ such that

$$\bar{P}_0(\hat{w}_k,\hat{t}_k) - \bar{P}_0(\hat{z}_k,\hat{t}_k) = \Theta_0(\hat{z}_k,\hat{w}_k,\hat{t}_k)(z_k + z_{k+1} - w_k - w_{k+1})$$

then the first relation in (26) can be rewritten as

$$\begin{aligned}\bar{\nabla}_z H'_a(\hat{z}_k,\hat{w}_k,\hat{t}_k) &= -\bar{H}_1(\hat{t}_k) + \bar{P}_0(\hat{z}_k,\hat{t}_k)\\ &\quad + \tfrac{1}{2}\bar{P}_1(\hat{z}_k,\hat{t}_k)(z_k + z_{k+1}) + \tfrac{1}{2}\left[R - \Theta_{01}(\hat{z}_k,\hat{w}_k,\hat{t}_k)\right]\\ &\quad\times (z_k + z_{k+1} - w_k - w_{k+1})\end{aligned} \tag{27}$$

with $\Theta_{01} = \Theta_0 + \Theta_1$. From (17) and (21), we obtain that

$$\begin{aligned}&\bar{G}'(\hat{z}_k,\hat{t}_k)\beta'(\hat{z}_k,\hat{t}_k) + \bar{\partial}_t\Psi'(\hat{z}_k,\hat{t}_k)\\ &\quad = \bar{F}'_d(\hat{z}_k,\hat{t}_k)\left[\tfrac{1}{2}\bar{P}_1(\hat{z}_k,\hat{t}_k)(z_k + z_{k+1}) - \bar{H}_1(\hat{t}_k) + \bar{P}_0(\hat{z}_k,\hat{t}_k)\right]\\ &\qquad + \bar{F}'_a(\hat{z}_k,\hat{t}_k)\bar{\nabla}H'(\hat{z}_k,\hat{t}_k).\end{aligned} \tag{28}$$

Consequently, with an eye on (27), for the extended system

$$\begin{cases} z_{k+1} = z_k + \tau\bar{F}'_d(\hat{z}_k,\hat{t}_k)\bar{\nabla}_z H'_d(\hat{z}_k,\hat{w}_k,\hat{t}_k)\\ \quad + \tau\bar{G}'(\hat{z}_k,\hat{t}_k)u'_k - \tfrac{1}{2}\tau\bar{F}'_d\left[R - \Theta_{01}(\hat{z}_k,\hat{w}_k,\hat{t}_k)\right]\\ \quad\cdot (z_k + z_{k+1} - w_k - w_{k+1})\\ w_{k+1} = w_k - \tau K_w\bar{\nabla}_w H'_d(\hat{z}_k,\hat{w}_k,\hat{t}_k)\end{cases} \tag{29}$$

the z_k dynamics is (11) with the control input that appears in (28). The additional input u'_k is instrumental to increase the convergence rate. As far as the w_k dynamics is concerned, $K_w = K_w^{\mathrm{T}} > 0$ is selected so that w_k asymptotically tends to z_k. After few passages, for (29) we get that

$$\begin{aligned}\tfrac{1}{\tau}\Delta_k H'_d &= \bar{\nabla}_z^{\mathrm{T}} H'_d \bar{F}'_d \bar{\nabla}_z H'_d - \bar{\nabla}_w^{\mathrm{T}} H'_d K_w \bar{\nabla}_w H'_d\\ &\quad + \bar{\nabla}_z^{\mathrm{T}} H'_d \bar{G}' u'_k - \tfrac{1}{2}\bar{\nabla}_z^{\mathrm{T}} H'_d \bar{F}'_d[R - \Theta_{01}]\\ &\quad\cdot (z_k + z_{k+1} - w_k - w_{k+1}) + \bar{\partial}_t H'_d\end{aligned} \tag{30}$$

where $\Delta_k H'_d = H'_d(z_{k+1}, w_{k+1}, t_{k+1}) - H'_d(z_k, w_k, t_k)$, and the dependence on $\hat{z}_k$, $\hat{w}_k$ and $\hat{t}_k$ has been omitted. From such a relation, the dual output for (29) is defined as

$$y'_k = \bar{G}'^{\mathrm{T}}(\hat{z}_k,\hat{t}_k)\bar{\nabla}_z H'_d(\hat{z}_k,\hat{w}_k,\hat{t}_k), \tag{31}$$

and additional dissipation is introduced if (14) holds.

In a similar way as in [22], the stability result relies on a Lyapunov analysis based on the energy-balance relation (30), which is now re-written to make such a final step as much simpler as possible. The first condition in (15) implies that P_0 is vanishing in case of perfect tracking. This means that there exists $\tilde{P}_0 : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n\times n}$ such that $P_0(z,t) = \tilde{P}_0(z,t)z$ and $\lim_{z\to 0}\tilde{P}_0(z,t)z = 0$ for all t. If

$$\xi_k := \begin{pmatrix}\hat{z}_k\\ z_k + z_{k+1} - w_k - w_{k+1}\end{pmatrix} \in \mathbb{R}^{3n}$$

then from (20) and (27) we get that

$$\bar{\nabla}_z H'_d(\hat{z}_k,\hat{w}_k,\hat{t}_k) = \Gamma_z(\hat{z}_k,\hat{w}_k,\hat{t}_k)\xi_k \tag{32}$$

where $\Gamma_z = (\Gamma_{z1},\ \Gamma_{z2},\ \Gamma_{z3}) \in \mathbb{R}^{n\times 3n}$, with

$$\begin{aligned}\Gamma_{z1}(\hat{z}_k,\hat{w}_k,\hat{t}_k) &= \tfrac{1}{2}\big[\bar{H}_2(\hat{t}_k) + \tilde{P}_0(z_k,t_k) + \bar{P}_1(\hat{z}_k,\hat{t}_k)\\ &\quad + h_0(z_k, z_{k+1}, t_k, t_{k+1})\big]\\ \Gamma_{z2}(\hat{z}_k,\hat{w}_k,\hat{t}_k) &= \tfrac{1}{2}\big[\bar{H}_2(\hat{t}_k) + \tilde{P}_0(z_{k+1},t_{k+1}) + \bar{P}_1(\hat{z}_k,\hat{t}_k)\\ &\quad + h_0(z_{k+1}, z_k, t_{k+1}, t_k)\big]\\ \Gamma_{z3}(\hat{z}_k,\hat{w}_k,\hat{t}_k) &= \tfrac{1}{2}\big[R - \Theta_{01}(\hat{z}_k,\hat{w}_k,\hat{t}_k)\big]\end{aligned}$$

From the second relation in (26) we get that

$$\bar{\nabla}_w H'_d(\hat{z}_k,\hat{w}_k,\hat{t}_k) = \Gamma_w(\hat{z}_k,\hat{w}_k,\hat{t}_k)\xi_k, \tag{33}$$

where

$$\Gamma_w(\hat{z}_k,\hat{w}_k,\hat{t}_k) = \tfrac{1}{2}\big(\Theta(\hat{z}_k,\hat{w}_k,\hat{t}_k) \quad -R\big) \in \mathbb{R}^{n\times 3n}. \tag{34}$$

Because of (32), the second term in (30) is now re-written as $\xi_k^{\mathrm{T}}\,\Xi(\hat{z}_k,\hat{w}_k,\hat{t}_k)\xi_k$, with $\Xi = \frac{1}{2}(0_{3n\times 2n},\ \Gamma_z^{\mathrm{T}}\bar{F}'_d[R - \Theta_{01}])$, so that the left side of (30) becomes

$$-\xi_k^{\mathrm{T}}\big[-\Gamma_z^{\mathrm{T}}\bar{F}'_d\Gamma_z + \Xi\big]\xi_k - \xi_k^{\mathrm{T}}K_w\xi_k + y_k'^{\mathrm{T}}u'_k, \tag{35}$$

where (31) and (33) have been taken into account.

Proposition 2: Let us consider the discrete-time system (29) with "energy" function $H'_d(z,w,t)$ and output y'_k defined in (24) and (31), respectively. Assume that (25) holds true that u'_k is given as in (14). Let

$$\begin{aligned}\mathbb{H} &= \Gamma_z^{\mathrm{T}}\big[-\mathrm{sym}\,\bar{F}'_d + \bar{G}'K_d\bar{G}'^{\mathrm{T}}\big]\Gamma_z + \mathrm{sym}\,\Xi\\ \mathbb{Z} &= \begin{pmatrix} I\\ R^{-1}\Theta\end{pmatrix}\end{aligned} \tag{36}$$

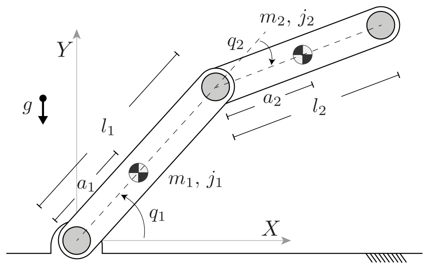

Fig. 1. The two degrees-of-freedom planar manipulator.

with $\mathbb{H}$ and $\mathbb{Z}$ functions of $(\hat{z}_k, \hat{w}_k, \hat{t}_k)$. If there exist $\bar{H}_d > 0$ and $K_d = K_d^{\mathrm{T}} \geq 0$ such that for all $\hat{z}_k$, $\hat{w}_k$ and $\hat{t}_k$ for which $H'_d(z_k, w_k, t_k) \leq \bar{H}_d$ and $H'_d(z_{k+1}, w_{k+1}, t_{k+1}) \leq \bar{H}_d$, we have that

$$\begin{gathered} \mathbb{Z}^{\mathrm{T}}\mathbb{H}\mathbb{Z} \geq 0 \\ \operatorname{Ker} \mathbb{Z}^{\mathrm{T}}\mathbb{H}\mathbb{Z} = \operatorname{Ker} \mathbb{Z}^{\mathrm{T}}\mathbb{H}^2\mathbb{Z} \end{gathered} \tag{37}$$

and $\bar{\partial}\Psi'(\hat{z}_k, \hat{w}_k, \hat{t}_k) \leq 0$ along the trajectories of (29), there exists $\kappa_w \geq 0$ so that, if $K_w \geq \kappa_w I$, then $(0, 0)$ is a locally stable equilibrium. Besides, if (29) with output (31) is zero-state detectable, then such an equilibrium is locally asymptotically stable.

Proof: This result is a slight modification of [22, Proposition 2], and so the proof is based on similar arguments. Note that $\operatorname{Im} \mathbb{Z} = \operatorname{Ker} \Gamma_w$, with Γ_w defined in (34). From [24, Th. 4.2], we have that if (37) are met, then there exists a positive κ_w such that for all $\kappa \geq \kappa_w$, we have that $\mathbb{H} + \kappa\Gamma_w^{\mathrm{T}}\Gamma_w \geq 0$. For the autonomous system resulting from (29) in which u'_k is given as in (14) with y'_k defined in (31), because of (36) and once $K_w \geq \kappa_w I$, the balance relation (30) can be compactly written as

$$\tfrac{1}{\tau}\Delta_k H'_d = -\xi_k^{\mathrm{T}}\left[\mathbb{H} + \Gamma_w^{\mathrm{T}} K_w \Gamma_w\right]\xi_k \leq 0 \tag{38}$$

where (35) and the fact that $\bar{\partial}\Psi' \leq 0$ have been taken into account. Relation (38) is satisfied in a neighborhood of the origin of (29), and this implies that such an equilibrium is locally stable. As in [18, Th. 1], asymptotic stability follows from the fact that (29) with output mapping (31) is zero-state detectable, once the feedback gain K_d in (14) is replaced by $K_d + \varepsilon_d I_m$, with $\varepsilon_d > 0$. ■

VI. Numerical Example

The design methodology is applied to a the two degrees-of-freedom planar manipulator of Fig. 1. This is the same example treated in [21] but within the symplectic integrators framework and in case of regulation on constant references.

The Hamiltonian is $H(q, p) = \frac{1}{2}p^{\mathrm{T}}M^{-1}(q)p + V(q)$, where $q = (q_1, q_2) \in \mathbb{R}^2$ are the generalized coordinates,

$$M(q) = \begin{pmatrix} M_{11}(q) & M_{12}(q) \\ M_{12}(q) & M_{22} \end{pmatrix}$$

is the inertia matrix in which $M_{22} = j_2 + m_2 a_2^2$, and

$$\begin{aligned} M_{11}(q) &= j_1 + j_2 + m_1 a_1^2 + m_2\left(l_2^2 + a_2^2 + 2a_2 l_1 \cos q_2\right) \\ M_{12}(q) &= j_2 + m_2\left(a_2^2 + a_2 l_1 \cos q_2\right), \end{aligned}$$

$p(t) = M(q(t))\dot{q}(t) \in \mathbb{R}^2$ are the generalized momenta, and $V(q) = m_1 g a_1 \sin q_1 + m_2 g(l_1 \sin q_1 + a_2 \sin(q_1 + q_2))$ is the potential energy, with g the gravity acceleration. The continuous-time model is (1), with $x = (q, p) \in \mathbb{R}^4$, and

$$F = \begin{pmatrix} 0 & I_2 \\ -I_2 & -b\,I_2 \end{pmatrix} \quad G = \begin{pmatrix} 0 \\ I_2 \end{pmatrix}$$

being $b \geq 0$ a coefficient that takes into account the viscous friction. The inputs $u(t) \in \mathbb{R}^2$ are the torques applied at each joint. The parameters are reported in Table I. The goal is to let the robot track a reference trajectory $q^\star(t)$ or, equivalently, that $x(t)$ converges to $x^\star(t) = (q^\star(t), p^\star(t))$, with $p^\star(t) = M(q^\star(t))\dot{q}^\star(t)$. The reference path in the workspace (X, Y) is depicted in Fig. 2 (red line).

TABLE I
Parameters of the Planar Manipulator of Fig. 1

parameter	value	parameter	value
$m_1 = m_2$	1	$j_1 = j_2$	0.1
$l_1 = l_2$	1	$a_1 = a_2$	0.5
b	0.05	τ	0.1
(q_0, p_0)	$\left(\frac{\pi}{3}, \frac{\pi}{3}, 0, 0\right)$		

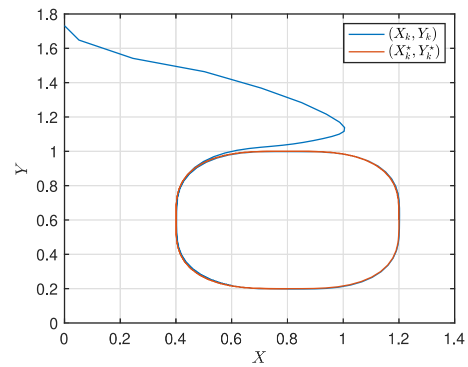

Fig. 2. Reference path and closed-loop trajectory in the workspace.

In the continuous-time case, the tracking problem can be solved as illustrated in [18, Remark 4]. Besides, the matching equation (16) allows for a perfect cancelation of the potential energy $V(q)$. However, in these simulations a steady-state compensation has been performed to precisely follow the procedure of Section IV. First of all, the coordinate transformation (7) is defined as $z_k = (\bar{q}_k, \bar{p}_k)$, with $\bar{q}_k = q_k - q_k^\star$ and $\bar{p}_k = p_k - p_k^\star$, which implies that, in (19), we have that

$$H_1 = \begin{pmatrix} \nabla V(q^\star) + \frac{1}{2}\nabla_{\bar{q}}[p^{\star\mathrm{T}} M^{-1}(\bar{q} + q^\star)p^\star]_{\bar{q}=0} \\ M^{-1}(q^\star)p^\star \end{pmatrix}.$$

In the design procedure, it is not necessary to modify the system structure, so $F'_a = 0$. A K'_a that satisfies the matching condition (18) is with $P_1(\bar{q}, \bar{p}) = \operatorname{diag}(P_{11}(\bar{q}, \bar{p}), 0_2)$ in (22), with $P_{11}(\bar{q}, \bar{p}) \in \mathbb{R}^{2\times 2}$, and $P_0 = 0$ in (21). The idea is to have a gain that increases with a weighted norm of the position error in a controller that is not realisable thanks to the discrete gradient of a scalar function. So, in (25) we let $P_{11}(\bar{q}) = \operatorname{diag}(p_1(\bar{q}), p_2(\bar{q}))$, with

$$\begin{aligned} p_1(\bar{q}) &= k_0 + k_1\left[\alpha_1\bar{q}_1^2 + (1 - \alpha_1)\bar{q}_2^2\right] \\ p_2(\bar{q}) &= k_0 + k_1\left[(1 - \alpha_2)\bar{q}_1^2 + \alpha_2\bar{q}_2^2\right] \end{aligned} \tag{39}$$

being k_i a positive scalar and $\alpha_i \in [0, 1]$, $i = 1, 2$.

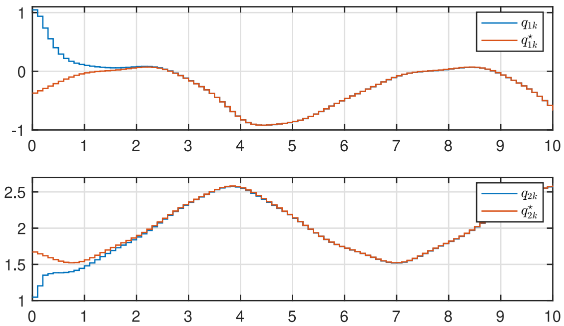

Fig. 3. Reference trajectory and closed-loop response in the joint space.

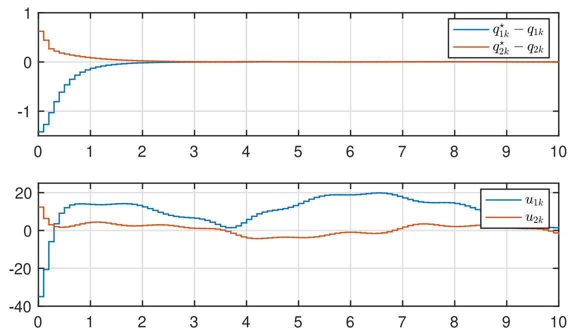

Fig. 4. Tracking error and applied torques at each joint.

The trajectories of the closed-loop system in the work and joint spaces are reported in Fig. 2 and 3, respectively. On the other hand, the tracking error in the joint space and the control inputs are in Fig. 4. As far as the controller parameters are concerned, in the desired Hamiltonian function (24) we have that $R = 20\,I_4$, and that $k_i = 15$ and $\alpha_i = 0.8$, $i = 1, 2$, in (39). Note that in (24), $w \in \mathbb{R}^4$. The w_k dynamics is defined as in (29), with $w_0 = \frac{1}{2} z_0$ and $K_w = 25\,I_4$, while the damping injection gain in (14) is $K_d = 10\,I_2$. Finally, the mean-value discrete gradient has been employed, but similar results have been also achieved with the Gonzalez one.

VII. Conclusion and Future Activities

In this letter, a framework for the design of passivity-based control laws to let nonlinear, sample-data port-Hamiltonian systems track a reference trajectory has been proposed. The synthesis procedure requires solving a matching equation. Inspired by a similar result developed in the continuous-time setting, an algebraic solution is computed. So, we take advantage of the algebraic nature of such a matching equation, and we can obtain control laws that do not necessarily depend on the gradient of an energy function. Regarding the future research activities, since this design technique is model-based, one of the topics is to investigate its robustness against parametric uncertainties, as long as the performances that are obtained when the controller acts on continuous-time dynamics. A second topic is to pair such a design technique with optimization schemes. The idea is to rely on such tools to select the controller parameters or even its structure.

References

[1] A. van der Schaft, *L_2-Gain and Passivity Techniques in Nonlinear Control* (Communication and Control Engineering). Cham, Switzerland: Springer Int., 2017.

[2] K. Feng and M. Qin, *Symplectic Geometric Algorithms for Hamiltonian Systems*. Berlin, Germany: Springer-Verlag, 2010.

[3] V. Talasila, J. Clemente-Gallardo, and A. van der Schaft, "Discrete port-Hamiltonian systems," *Syst. Control Lett.*, vol. 55, no. 6, pp. 478–486, 2006.

[4] R. Ruth, "A canonical integration technique," *IEEE Trans. Nucl. Sci.*, vol. NS-30, no. 4, pp. 2669–2671, Apr. 1983.

[5] O. Gonzalez and J. Simo, "On the stability of symplectic and energy-momentum algorithms for non-linear hamiltonian systems with symmetry," *Comput. Methods Appl. Mech. Eng.*, vol. 134, nos. 3–4, pp. 197–222, 1996.

[6] J. Marsden and M. West, "Discrete mechanics and variational integrators," *Acta Numerica*, vol. 10, pp. 357–514, Jun. 2001.

[7] S. Dai and X. Koutsoukos, "Safety analysis of integrated adaptive cruise control and lane keeping control using discrete-time models of port-Hamiltonian systems," in *Proc. Amer. Control Conf. (ACC)*, Jun. 2017, pp. 2980–2985.

[8] P. Kotyczka and L. Lefèvre, "Discrete-time port-Hamiltonian systems: A definition based on symplectic integration," *Syst. Control Lett.*, vol. 113, Nov. 2019, Art. no. 104530.

[9] O. Gonzalez, "Time integration and discrete Hamiltonian systems," *J. Nonlinear Sci.*, vol. 6, pp. 449–467, Mar. 1996.

[10] G. Quispel and G. Turner, "Discrete gradient methods for solving ODEs numerically while preserving a first integral," *J. Phys. A Math. Gen.*, vol. 29, no. 13, p. L341, 1996.

[11] L. Gören-Sümer and Y. Yalçin, "Gradient based discrete-time modeling and control of hamiltonian systems," in *Proc. 17th IFAC World Congr.*, Seoul, South Korea, Nov. 2008, pp. 212–217.

[12] S. Monaco, D. Normand-Cyrot, and F. Tiefensee, "Nonlinear port controlled hamiltonian systems under sampling," in *Proc. 28th Chin. Control Conf. Decis. Control (CDC/CCC)*, Dec. 2009, pp. 1782–1787.

[13] S. Aoues, M. Di Loreto, D. Eberard, and W. Marquis-Favre, "Hamiltonian systems discrete-time approximation: Losslessness, passivity and composability," *Syst. Control Lett.*, vol. 110, pp. 9–14, Dec. 2017.

[14] A. Moreschini, M. Mattioni, S. Monaco, and D. Normand-Cyrot, "Discrete port-controlled hamiltonian dynamics and average passivation," in *Proc. IEEE 58th Annu. Conf. Decis. Control (CDC)*, Nice, France, Mar. 2019, pp. 1430–1435.

[15] M. Ehrhardt, E. Riis, T. Ringholm, and C.-B. Schönlieb, "A geometric integration approach to smooth optimisation: Foundations of the discrete gradient method," 2020, *arXiv:1805.06444*.

[16] S. Stramigioli, C. Secchi, A. van der Schaft, and C. Fantuzzi, "Sampled data systems passivity and discrete port-Hamiltonian systems," *IEEE Trans. Robot.*, vol. 21, no. 4, pp. 574–587, Aug. 2005.

[17] R. Costa-Castello and E. Fossas, "On preserving passivity in sampled-data linear systems," in *Proc. Amer. Control Conf.*, Minneapolis, MN, USA, Jun. 2006, pp. 4373–4378.

[18] K. Fujimoto, K. Sakurama, and T. Sugie, "Trajectory tracking control of port-controlled hamiltonian systems via generalized canonical transformations," *Automatica*, vol. 39, no. 12, pp. 2059–2069, Dec. 2003.

[19] R. Ortega and E. Garcia-Canseco, "Interconnection and damping assignment passivity-based control: A survey," *Eur. J. Control*, vol. 10, no. 5, pp. 432–450, 2004.

[20] A. Moreschini, M. Mattioni, S. Monaco, and D. Normand-Cyrot, "Stabilization of discrete port-Hamiltonian dynamics via interconnection and damping assignment," *IEEE Control Syst. Lett.*, vol. 5, no. 1, pp. 103–108, Jan. 2021.

[21] P. Kotyczka and T. Thoma, "Symplectic discrete-time energy-based control for nonlinear mechanical systems," *Automatica*, vol. 133, Nov. 2021, Art. no. 109842.

[22] K. Nunna, M. Sassano, and A. Astolfi, "Constructive interconnection and assignment for port-controlled hamiltonian systems," *IEEE Trans. Autom. Control*, vol. 60, no. 9, pp. 2350–2361, Sep. 2015.

[23] A. Harten, P. Lax, and B. Van Leer, "On upstream Differencing and Godunov-type schemes for hyperbolic conservation laws," *SIAM Rev.*, vol. 25, no. 1, pp. 35–61, 1983.

[24] K. Anstreicher and M. Wright, "A note on the augmented hessian when the reduced Hessian is Semidefinite," *SIAM J. Control Optim.*, vol. 11, no. 1, pp. 243–253, 2000.

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