



ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

ARCHIVIO ISTITUZIONALE
DELLA RICERCA

Alma Mater Studiorum Università di Bologna Archivio istituzionale della ricerca

The probability of multidimensional poverty: A new approach and an empirical application to EU-SILC data

This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

Published Version:

Paolo Liberati, Giuliano Resce, Francesca Tosi (2023). The probability of multidimensional poverty: A new approach and an empirical application to EU-SILC data. *THE REVIEW OF INCOME AND WEALTH*, 69(3), 668-700 [10.1111/roiw.12598].

Availability:

This version is available at: <https://hdl.handle.net/11585/888405> since: 2023-11-21

Published:

DOI: <http://doi.org/10.1111/roiw.12598>

Terms of use:

Some rights reserved. The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

This item was downloaded from IRIS Università di Bologna (<https://cris.unibo.it/>).
When citing, please refer to the published version.

(Article begins on next page)

This is the final peer-reviewed accepted manuscript of:

Liberati, P., Resce, G., & Tosi, F. (2023). The probability of multidimensional poverty: A new approach and an empirical application to EU-SILC data. *Review of Income and Wealth*, 69(3), 668-700.

The final published version is available online at:

<https://doi.org/10.1111/roiw.12598>

Terms of use:

Some rights reserved. The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

This item was downloaded from IRIS Università di Bologna (<https://cris.unibo.it/>)

When citing, please refer to the published version.

The probability of multidimensional poverty:

A new approach and an empirical application to EU-SILC data

Paolo Liberati, Roma Tre University

Giuliano Resce, University of Molise

Francesca Tosi, Alma Mater Studiorum – University of Bologna

Abstract This paper proposes a novel method to analyse multidimensional poverty by using a large set of feasible weights to summarise the information about the poor, which allows to remain agnostic about the relative importance given to different poverty dimensions. This method allows to calculate the individual probability of being poor in a multidimensional perspective. The distribution of individual probabilities can then be combined with Generalised Lorenz dominance techniques to derive unanimous consent for a wide class of social welfare functions with a minimum load of value judgments. The innovations proposed here allow to move from a dual definition of poverty, where poor and non-poor individuals are classified in a mutually exclusive context, to a continuous measure of deprivation capturing both the extensive and intensive margin of multidimensional poverty. The empirical application of the method consists of measuring multidimensional poverty in ten selected countries using four waves of EU-SILC data (2008-2014).

Keywords: Multidimensional poverty; Dominance; Europe.

JEL classification: I3; H00; D63; C43

1. Introduction

There is widespread agreement on the need to conceptualise poverty as a multidimensional phenomenon. Low consumption or income is surely at the heart of the notion of poverty but several other domains are systematically concerned by inadequate living standards (Ferreira and Lugo, 2013). Since the pioneering works of Tsui (2002) and Bourguignon and Chakravarty (2003), a wealth of approaches were developed to measure deprivation in multiple dimensions (see among others Alkire and Foster, 2011; Chakravarty *et al.*, 1998; Cheli and Lemmi, 1995; Chiappero-Martinetti, 1994; Deutsch and Silber, 2005; Maasoumi and Lugo, 2008).

However, multidimensional poverty measures are far from being universally welcomed. To begin with, which dimensions matter and who should be selecting them are questions that repeatedly raise issues of ethics and legitimacy.¹ Retrieving information on shared societal values and priorities is not straightforward, especially when the analysis is carried out at international or even at the global level (Alkire, 2007). Selecting deprivation indicators and poverty thresholds – both within and across indicators – requires further sensitive decisions, although they end up being data-driven in most cases, especially when the poverty analysis is based on the counting of deprivations framework (Alkire *et al.*, 2015).

Relative weights attached to attributes of different nature are also a matter of concern. In the income-centred framework, prices are commonly used to aggregate components of consumption expenditure (or the incomes used to finance such consumption). They are then used to compose an index of aggregate consumption to be compared with an aggregate poverty line defined in the same space. Even though there exist different reasons why prices might not be ideal welfare weights (Ferreira and Lugo, 2013), they provide a clear understanding of the effects of the weighting scheme (Maasoumi and Lugo, 2008), as they explicitly address the trade-offs between different goods and services, or the rate at which consumers are willing to trade one unit of an expenditure component for another (i.e. the marginal rate of substitution (MRS) between two goods).²

¹ See for instance the Sustainable Development Goals experience (Fukuda-Parr, 2016).

² In this case, the MRS between two dimensions j_1 and j_2 can be defined as the amount of dimension 2 an individual is willing to give up for an extra unit of dimension 1, while maintaining

Similarly, in a multidimensional setting relative weights play the central role of determining trade-offs between dimensions. They reflect value judgments and possibly the very structure of social preferences. For these reasons, the setting of a weighting system is inevitably affected by the formulation of strong normative assumptions and ethical considerations on what a ‘good life’ is, and it should be made as explicitly as possible.

The literature provides an array of methods to set relative weights in a multidimensional context (Decancq and Lugo, 2013), although in practice equal weights are often assumed among dimensions, i.e. equally important from an ethical point of view, as in the case of the Human Development Index (UNDP, 1990). Moreover, quantifying how many units of, say, education an individual would give up to compensate an extra year of life is a rather complicated task. In the first place, such an evaluation would require an amount of information that might be uneasy or even impossible to retrieve. Second, the MRS between any two dimensions could vary from an individual to another based on the actual levels of the achievements. This has relevant implications, as assuming one specific vector of weights may heavily affect interpersonal comparisons (Foster *et al.*, 2013) and social outcomes, leading to ambiguous results.

In this paper, we choose to focus on the specific issue of weights with the aim of minimising the degree of arbitrariness that is often embodied in their use. Our aim is not that of defining an alternative poverty index, but rather that of estimating the individual probabilities of being multidimensionally poor and the average probability of experiencing multidimensional poverty after testing it for a large set of vectors of weights. In this perspective, our method is a complementary way to understand the characteristics of multidimensional poverty.

To show the relevance of our approach, we compare selected European Union (EU) countries by estimating the probability to be multidimensional poor for a wide set of feasible vectors of weights. To this purpose, we address the issue of weighting indicators by applying Stochastic Multicriteria Acceptability Analysis (SMAA) (Lahdelma and Salminen, 2001), which allows to embody unknown preferences on the weights assigned to each poverty indicator or dimension. Such method was previously used in economics

the same level of well-being: $MRS_{j_1, j_2} = \frac{\partial I(x)}{\partial x_{j_1}} / \frac{\partial I(x)}{\partial x_{j_2}}$, where $I(x)$ is the well-being index and x is the vector of achievements for all j dimensions.

to empirically investigate well-being at the country level both in Italy and the US, by analysing regional data (Greco et al. 2018; Lagravinese et al., 2019, 2020). In this article, we show that the same approach can be adapted to multidimensional poverty analysis, and we apply it for the first time to individual-level data, specifically to Eurostat's Survey on Income and Living Conditions (EU-SILC). We also show that the proposed method can be used to robustly assess if a specific population sub-group in a given country is more likely to be considered multidimensionally deprived than the same sub-group in other countries.

The article is organised as follows: Section 2 illustrates the logic behind counting approaches to multidimensional poverty measurement and discusses the issue of relative weights and the most popular methods to assign them. Section 3 introduces a new method for measuring multidimensional poverty and Section 4 presents its empirical application to the EU-SILC data, discussing the results from both a cross-country and a diachronic perspective. Finally, in Section 5 the Generalised Lorenz dominance technique is used to perform pairwise country comparisons of the distribution of probabilities to rank them from a social perspective with the minimum load of value judgments. Section 6 concludes.

2. Assessing multidimensional poverty

2.1 Counting methods for identifying the multidimensionally poor

Poverty measurement implies the accomplishment of two fundamental tasks: the first is to identify the poor among the total population; the second is to aggregate the information on the poor to give an aggregate measurement of poverty, either through a poverty index (Sen, 1976) or by using dominance ordering (see Deaton, 1997).

When performed in a multidimensional setting, the identification step requires to make several choices, including defining suitable dimensions and indicators, setting the corresponding poverty cut-offs, and defining a system of weights to be applied to each dimension or indicator. If multidimensional poverty is conceived as the occurrence of multiple *simultaneous* deprivations, one could study their cumulative distribution across

the population by looking at the multiple achievements in which individuals fall short at once.

Empirically, this method translates into aggregating the information about the poor first across dimensions, and then – to obtain a single population-wide multidimensional poverty measure – across individuals, as for instance in the case of the Global MPI (Alkire and Santos, 2014). In this perspective, two specific problems may arise: the first is related to imposing a restricted choice of the usable data, which must come from the same source as of the studied population and which might not cover all the domains that could be of interest for a multidimensional poverty analysis. When such data are available, the cumulative distribution of deprivations can be assessed by simply counting the dimensions in which individuals are deprived and by assigning scores correspondingly (Atkinson, 2003), which requires to define a set of relevant indicators and their corresponding cut-offs, and then to create binary deprivation scores for each observational unit. To that aim, although not explicitly promoted by any specific development or welfare economics theory, the counting approach has become widespread, especially in the empirical literature (see Nolan and Whelan, 1996, 2011). Starting from the resulting deprivation matrix, a second and even more relevant issue remains, i.e. that of assigning a weight to each poverty dimension. Then, the weighted sum of deprivations is computed to produce an overall poverty score and, finally, to identify the multidimensionally poor by setting a poverty cut-off. Such a framework underpins a much-used approach to multidimensional poverty measurement, exemplified by the Alkire-Foster method (Alkire and Foster, 2011), also known as the *dual cut-off* approach.

It is on this second specific issue that the paper will focus, which means that not all the shortcomings of a multidimensional approach to poverty measurement will be addressed in this context. Yet – as a first step in the analysis of multidimensional poverty – the herein proposed method for minimising arbitrariness in choosing weights will contribute to significantly reduce the overall degree of arbitrariness in its use.

2.2 *The issue of relative weights*

One relevant critique to multidimensional poverty indices concerns how to set the weights attached to attributes of different nature, as they implicitly reveal how small changes in the achievements of different well-being dimensions can or cannot compensate each other. Decancq and Lugo (2013) categorize existing weighting methods into three main classes: data-driven, normative, and hybrid.

Data-driven approaches – like frequency-based weights, statistical weights, and most-favourable weights – are a function of the distribution of the achievements in the society and are not based on value judgements about trade-offs between different life domains. Frequency-based weights often assign an inverse relation between the frequency of deprivation in a dimension and the weight of that dimension (e.g., Deutsch and Silber, 2005), motivated by the idea that less frequent deprivations should have a higher weight because individuals would attach a higher importance to shortfalls in dimensions where the majority in their society do not fall short (Desai and Shah, 1988). Statistical weights (Krishnakumar and Nadar, 2008), on the other hand, are often classified into two broad sets: multivariate statistical methods – of which the most commonly used one is the Principal Component Analysis (Klasen, 2000; Noorbakhsh, 1998) – and explanatory models based on the idea of the latent variable, like Factor Analysis (Noble et al., 2006), the Rasch model (Fusco and Dickens, 2008), multiple indicator and multiple causes models (MIMIC) (Di Tommaso, 2007), and structural equation models (Krishnakumar, 2007; Krishnakumar and Ballon, 2008). Finally, the most-favourable weights technique, widely used to set weights in well-being indices (see Despotis, 2005a, 2005b; Mahlberg and Obersteiner, 2001; Zaim et al., 2001), is a particular case of the data envelopment analysis proposed by Melyn and Moesen (1991) and considers weights as individual-specific and endogenously determined – i.e., the highest relative weights are given to dimensions in which the person performs best.

Normative approaches depend instead on value judgements about the MRSs. Weights can either be set in an equal or unequal way, although they are always assigned according to considerations about specific trade-offs among dimensions. The degree of arbitrariness involved in this assignment could be overcome by following an ‘expert opinion

approach’, that is by letting well-informed persons decide which weighting scheme to adopt (see e.g., Chiappero-Martinetti and von Jacobi, 2012). The latter method includes the Budget Allocation Technique (Moldan and Billharz, 1997; Chowdury and Squire, 2006; Mascherini and Hoskins, 2008), where experts are asked to distribute a budget of points to the different attributes, and the Analytic Hierarchy Process (Saaty, 1987), which compares dimensions pairwise and assigns for each round a score of importance. However, to some extent, the arbitrariness of this approach is only raised at an upper level, considering the opinion of experts, but leaving unsolved how experts can be selected.

Lastly, *hybrid* approaches – like stated preference weights (Mack and Lansley, 1985; Halleröd, 1995a, 1995b; de Kruijk and Rutten, 2007; Guio et al., 2016; Bossert et al., 2013) and hedonic weights (Schokkaert, 2007; Ferrer-i-Carbonell and Freijters, 2004; Nardo et al., 2008; Fleurbaey, 2009) – are a mix of the former two.

As we will see in the next section, this paper tries to overcome the arbitrary choice involved in the weighting step by introducing a new method to measure multidimensional poverty. It is worth stressing that the aim will not be that of defining a specific poverty index, but rather that of estimating the individual probabilities of being multidimensionally poor and the average probability of experiencing multidimensional poverty after testing it for a large set of vectors of weights.

3. Measuring the probability of multidimensional poverty

Consider poverty in d dimensions across a population of n individuals. In each dimension, there is a variable number of indicators, for a total number of indicators p . Let $\mathbf{y} = [y_{kj}]$ denote the $n \times p$ matrix of achievements. Each row of the matrix $y_k = (y_{k1}, \dots, y_{kj}, \dots, y_{kp})$ gives the achievements of the k -th individual in each indicator. Each column of the matrix $y_j = (y_{1j}, \dots, y_{kj}, \dots, y_{nj})$ gives the distribution of achievements in the j -th indicator across individuals.

For each indicator, a cut-off z is defined. This gives rise to a vector of cut-offs $z = (z_1, \dots, z_j, \dots, z_p)$. In our analysis, the cut-offs are related to the presence of a given qualitative characteristic, except for monetary income. Thus, starting from the matrix \mathbf{y}

of achievements, one can get a matrix of deprivations $\mathbf{g} = [g_{kj}]$, where the generic element $g_{kj} = 1$ when $y_{kj} = z_j$ and $g_{kj} = 0$ when $y_{kj} \neq z_j$. For monetary income and ordinal variables where the outcomes are ordered from the worst to the best, one has $g_{kj} = 1$ when $y_{kj} \leq z_j$ and $g_{kj} = 0$ when $y_{kj} > z_j$. Each row of the matrix $g_k = (g_{k1}, \dots, g_{kj}, \dots, g_{kp})$ shows in which indicator the k -th individual is deprived. Each column of the matrix $g_j = (g_{1j}, \dots, g_{kj}, \dots, g_{nj})$ gives the number of individuals who are deprived in the j -th indicator.

From the matrix of deprivation, one can build a vector of, say, *deprivation scores* by calculating – for each individual – the weighted sum of deprivations. Let $C = (C_1, \dots, C_k, \dots, C_n)$ denote the vector of deprivation scores and $v = (v_1, \dots, v_j, \dots, v_p)$ the vector of weights, where $\sum_{j=1}^p v_j = 1$ and $0 \leq v_j \leq 1$, for $j = 1, \dots, p$. Thus, a unique set of weights would be required to calculate C , and this choice has some arbitrary content. By maintaining this assumption for the moment, it will be that $C_k = \sum_{j=1}^p g_{kj}v_j$, where either $g_{kj}v_j = v_j$ when the individual is deprived in the j -th indicator or $g_{kj}v_j = 0$ when the individual is not deprived in the j -th indicator. Thus, for an individual who is not deprived in any indicator, $C_k = 0$, while for an individual that is deprived in all indicators $C_k = \sum_{j=1}^p v_j = 1$. In the general case, $0 \leq C_k \leq 1$.

At this stage, the standard approach to counting and multidimensional poverty measurement also known as the *dual cut-off* method (Alkire and Foster, 2011) requires to define a further cut-off f , where $0 < f \leq 1$, to identify individuals who are multidimensionally poor, i.e. those for which $C_k \geq f$. This means that the deprivations of those individuals for which $C_k < f$ will be disregarded. Obviously, this choice has also an arbitrary content, as it introduces a discrete *absolute* cut-off to identify the poor.

Our approach removes both the need to make recourse to a unique set of weights to calculate the individual level of deprivation and the need to identify an absolute cut-off of the weighted sum of deprivations to identify the poor while providing a new method to deal with multidimensional poverty.³ With regard to the first issue, instead of setting a

³ At this stage, the proposed method is still fully consistent with the standard framework for multidimensional poverty measurement (Alkire and Foster, 2011) as it allows to proceed on to compute all partial indices needed to build the poverty measure M_0 derived through the Alkire-Foster dual cut-off method.

unique vector of weights v , we use a large set of randomized vectors of weights from a uniform distribution which allows to have an approximation of the whole space of feasible weights (see Tervonen and Ladhelma, 2007). Thus, we will have a matrix $\mathbf{v} = [v_{sj}]$ of size $m \times p$, where m is the number of vectors used. Each row of this matrix $v_s = (v_{s1}, \dots, v_{sj}, \dots, v_{sp})$ gives the specific vector of weights by which the indicators are weighted. Each column of the matrix $v_j = (v_{1j}, \dots, v_{sj}, \dots, v_{mj})$ gives instead the set of different weights by which each indicator is weighted in each replication.

It is worth noting that by applying a uniform distribution we are in fact assuming an unrestricted domain of the vectors of weights. Even though the fact that a poverty dimension is included in a multidimensional list would imply that a non-zero weight should be assigned to it, and that not all weighting schemes may be equally likely in a given society, we prefer to be neutral with respect to the distribution of social values in the population. This is done by admitting all possible views including the possibility that – even among collectively shared dimensions – different political and social attitudes might lead to totally neglect some dimensions of poverty, implicitly giving them a zero weight.

By this way, we avoid restricting the domain of admissible preferences to judge about multidimensional poverty, obtaining results in a “no matter one’s view” environment. It is worth stressing that by dealing with weights we are not removing all element of arbitrariness that characterise the analysis of multidimensional poverty; yet, we contribute to remove a fundamental element of arbitrariness, consisting in the implicit (and hidden) social preferences that are conveyed by the choice of a unique set of weights.⁴

At this stage, for any vector of weights, one has the corresponding vector $C_s = (C_{1s}, \dots, C_{ks}, \dots, C_{ns})$, i.e., the deprivation score of each individual measured by the use of the vector v_s . The final outcome is a matrix $\mathbf{h} = [h_{ij}]$ where each row $h_k = (h_{k1}, \dots, h_{ks}, \dots, h_{km})$ gives m deprivation scores for the k -th individual depending on the changes in weighting vectors, while each column $h_s = (h_{1s}, \dots, h_{ks}, \dots, h_{ns})$ gives the distribution of the deprivation scores across individuals for each weighting vector s .

⁴ Considering the economic applications of SMAA, uniform distribution of weights has been used, among others, by Greco et al. (2018); Coco et al. (2020); Resce and Shiltz (2021).

According to this framework, a new indicator of poverty at an individual level is introduced, which is based on a ranking function. We first define:

$$(1) \quad r(k, v_s) = 1 + \sum_{i \neq k} \rho[C(i, v_s) > C(k, v_s)] \quad \text{for } s = 1, \dots, m$$

where r defines the rank, $\rho = 1$ when the condition in square brackets is true and $\rho = 0$ when the same condition is false. Thus, the rank of individual k , for each vector of weights v_s , is one plus how many times the weighted average of multidimensional poverty of k ($C(k, v_s)$) is below the weighted average of multidimensional poverty of the other individuals ($C(i, v_s)$).

Thus, the value assumed by the variable $r(k, v_s)$ is one plus the number of individuals that are more multidimensionally poor than the individual k . Therefore, the higher the value of $r(k, v_s)$, the lower the poverty of the individual k . Note that $\min(r(k, v_s)) = 1$ when $C(k, v_s)$ is always above $C(i, v_s)$, i.e. the k -th individual is more multidimensionally poor than any other individual. On the other side, $\max(r(k, v_s)) = n$ where the k -th individual is less multidimensionally poor than any other individual. The final outcome of this step is a matrix of ranks $\mathbf{r} = [r_{ks}]$, in which each row $r_k = (r_{k1}, \dots, r_{ks}, \dots, r_{km})$ is – for the k -th individual – the ranks occupied for the m possible set of weights considered, while each column $r_s = (r_{1s}, \dots, r_{ks}, \dots, r_{ns})$ is the distribution of ranks across individuals for a specific vector of weights.

As a second step, we compute the total number of the vectors of weights V_r for which the k -th individual assumes precisely rank r :

$$(2) \quad V_{kr} = \sum_s \phi[r_{ks} = r]$$

in which $\phi = 1$ anytime the use of the vector v_s gives $r_{ks} = r$, and $\phi = 0$ otherwise. Note that, by construction, in the case where two or more individuals have the same achievement, they also have the same rank. The main outcome of our analysis is a measure of the deprivation score of each individual given by the probability of occupying a given rank r in the distribution:

$$(3) \quad b_{kr} = \frac{V_{kr}}{m}$$

where b_k^r is the probability that the k -th individual gets the r -th position in the ranking, given by the ratio of the number of weighting vector for which the individual assumes rank r and the total number of vectors m . It is worth clarifying at this stage that the probability of equation (3) should not be understood as a measure of poverty in the traditional sense, as it collapses into a single probability the outcomes that can be obtained when applying different set of weights to a given set of poverty indicators. In this perspective, the probability is neither an absolute nor a relative measure, as it can embody both absolute and relative indicators of poverty giving information on how likely is that different set of weights may define any individual as multidimensional poor.

The final outcome is a matrix of probabilities $\mathbf{b} = [b_{kr}]$ in which each row $b_k = (b_{k1}, \dots, b_{kr}, \dots, b_{kn})$ gives – for the k -th individual – the set of probabilities of occupying a rank from 1 to n , i.e. to be multidimensional poor with either a higher or lower probability with respect to other individuals. Each column $b_r = (b_{1r}, \dots, b_{kr}, \dots, b_{nr})$, instead, gives the probabilities of all individuals of occupying a specific rank r .⁵

At this stage, the problem faced by the standard approach of setting a further cut-off f to compute a synthetic measure of multidimensional poverty would translate into defining until which rank l the analyst wants to consider the overall probability of being multidimensionally poor. It is worth noting that in this case the choice is about a threshold represented by a rank, and not about the number of deprivations above which the individual can be considered multidimensionally poor. This means that the use of the rank l is qualitatively different from the dual cut-off approach; indeed, while in our approach the individual probabilities of occupying any given rank are independent of the choice of l , in the dual cut-off approach falling into multidimensional poverty depends on the cut-off setting the number of deprivations.

To better explain this point, it is worth noting that the threshold l simply corresponds to the preferred percentile of the distribution of probabilities one wants to focus on. It is worth recalling that $(r(k, v_s)) = 1$ indicates the position occupied by the poorest individual (equation (1)). Thus, a given threshold l simply corresponds to a subset of n

⁵ A simple example of the way in which the method works is provided in Annex 1.

possible ranks, i.e. $l = \gamma n$, where $0 < \gamma \leq 1$ is the fractional rank of the total population included in the analysis.⁶

Once the threshold l has been chosen, equation (3) allows to measure the individual cumulative probability of being below the rank l , which – for the k -th individual – is given by the sum of the probabilities of occupying any rank from 1 to l . This probability is given by:

$$(4) \quad b_k^l = \sum_{r=1}^l b_{kr} \leq 1$$

To some extent, it must be recognised that the choice of γ involves some degree of arbitrariness, and this may be interpreted as a shortcoming of the approach; yet, it is worth stressing that this choice does not affect the estimated individual probabilities of occupying any given rank (equation (3)), but only the cumulative individual probabilities of being below a given rank (equation (4)). As it can be observed, the individual cumulative probabilities of being multidimensionally poor below a given rank increases when moving γ toward 1, i.e. when considering increasing fractions of the ranking distribution. Thus, the threshold-percentile γ does not correspond to a given (weighted) sum of deprivations above which the individual can be considered multidimensionally poor as in standard poverty analysis, but rather to the subset of the distribution the analyst may consider meaningful to focus on in order to estimate the overall probability of being multidimensionally poor. It follows that, unlike any poverty cut-off, γ does not bear a normative weight and can be modified according to the aim of the analysis. It follows, in our view, that the degree of arbitrariness involved in the choice of the threshold is less conditioning than that involved in parameters that, when changed, may alter the estimated amount of poverty, as in the case of changing a poverty line in the traditional context of poverty measurement.⁷

⁶ For example, if the total population is $n = 100$, one may be interested to investigate the probability of any individual of being multidimensional poor until rank 20. In symbols, this means that $l = 20$ and the fractional rank $\gamma = 0.2$ (or 20% of the population).

⁷ It is worth stressing that under no circumstances the choice of γ should be considered prescriptive in a methodological sense; in our perspective, it neither embodies a policy advice nor it represents a constraint. Instead, it highlights the flexibility of our approach and emphasizes its distance, rather than resemblance, to any approach based on the setting of a poverty cut-off (including, but not limited to, the dual cut-off method by Alkire and Foster, 2011).

Thus, unlike in standard approaches, according to our method there is no need to define a censored matrix including the individuals who have been identified as poor while at the same time excluding the deprivations of the non-poor according to the absolute cut-off chosen. Rather, we work with a reduced matrix with $l < n$ columns, where l identifies the rank associated to a specific subset of the population the analyst wants to focus on.

By applying equation (4), the outcome is a vector $B^l = (b_1^l, \dots, b_k^l, \dots, b_n^l)$ that for each individual gives the cumulative probability of having a rank below l . It is worth recalling once again that the individual probabilities of occupying any given rank are independent of l , while the individual *cumulative* probabilities of occupying a rank below l are not, as they obviously depend on l , with the trivial outcome that the cumulative probability of occupying a rank below n is equal to 1. According to this framework, by definition, the following will be true:

$$(5) \quad \frac{1}{n} \sum_{k=1}^n b_k^l = \gamma$$

i.e. the overall average probability of being below l is equal to the fractional rank.⁸ Equation (5) simply states that the overall probability of occupying a rank below γ , i.e. below a given fraction of the population, is equal to γ . This is implicit in our approach, as we are measuring the probability of any individual of occupying any given rank; thus, the sum of the probabilities of all individuals of being below γ , is equal to γ . Of course, when splitting individuals by groups – as in the case of our empirical application – or over time, the average probabilities among groups or across time may differ, allowing to assess multidimensional poverty either cross-sectionally or dynamically (or both). In principle, the calculation of individual probabilities could potentially be extended to identify a range of continuous ranks, i.e., a continuous probability of being

⁸ For instance, consider the case of two individuals A and B and two dimensions, where A is deprived in both dimensions while B is not. For $\gamma = 1$ they both have $b_k^l = 1$, thus $\frac{1}{n} \sum_{k=1}^n b_k^l = 1$. For $\gamma = 0.5$, A has $b_k^l = 1$, B has $b_k^l = 0$, by which $\frac{1}{n} \sum_{k=1}^n b_k^l = 0.5$. For $\gamma = 0$, they both have $b_k^l = 0$, and $\frac{1}{n} \sum_{k=1}^n b_k^l = 0$.

multidimensionally poor, as opposed to a dichotomous measure distinguishing the poor from the non-poor.⁹

Starting from the estimated individual cumulative probabilities, one can obtain average probabilities of multidimensional poverty, which is our way of reducing the distribution of individual probabilities to a synthetic and comparable parameter. In particular, one may study the average probability of multidimensional poverty of one specific group over time or the same average can be used for comparisons across different groups, either in a single point in time or diachronically, as in proper panel analyses.

As an empirical application of the proposed approach, in this paper average probabilities are calculated considering groups of individuals from different countries. The average probability of individuals belonging to group q to be below rank l can be easily obtained by calculating:

$$(6) \quad b_q = \frac{1}{n_q} \sum_{k=1}^{n_q} b_{kq}^l$$

where n_q is the population of group q .¹⁰ In this case, one can compare the average probability of each group to have individuals below rank l . Just to make the point clearer, if in a group all individuals occupy a rank greater than l , then $b_{kq}^l = 0$ for all k , and thus $b_q = 0$. On the other side, if in a group all individuals have a rank lower than l , then $b_{kq}^l = 1$ for all k , and thus $b_q = 1$. In general, different groups will have different average probabilities and the size of these probabilities will help to compare the distribution of multidimensional poverty across groups.

Furthermore, the contribution of each group to the average probability of equation (5) can be obtained by the following:

$$(7) \quad b_q^y = \frac{1}{n} \sum_{k=1}^{n_q} b_{kq}^l$$

⁹ To this regard, the proposed approach provides a basis to combine two approaches of multidimensional poverty measurement: that of counting methods and the fuzzy sets approach (Cerioli and Zani, 1990; Cheli and Lemmi, 1995; Chiappero-Martinetti, 1994). We thank an anonymous referee for suggesting such an analogy, which would deserve further theoretical analysis that goes beyond the scope of the present paper.

¹⁰ Equation (6) is given for a cross-section analysis that is repeated over time. But the same equation could be used for one country over time if q is interpreted as the time parameter.

where now the contribution of each group q to the average probability is measured over the total population n instead than n_q . By this way, one can say that below the fractional rank γ , for example, a fraction $\alpha\gamma$ comes from group A and a fraction $\beta\gamma$ comes from group B , with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$. It follows from (7) that $\sum_q b_q^\gamma = \gamma$. It is worth noting that, while the value b_q^γ depends also on the size of the groups, b_q is independent from it. In the special case where (6) is equal for all groups, the relative contribution of each group to (7) would only reflect the population size n_q .

At this stage, it is worth observing how the average probability of equation (6) behaves with regard to the axiomatic structure. In particular, it satisfies the following:

a) *Symmetry*: the average probability is not affected by the exchange of deprivation scores among individuals;

b) *Population-replication invariance*: when the existing population is replicated, the possible ranks go from 1 to λn , where λ is the number of times the population is replicated (e.g. for doubling $\lambda=2$). By maintaining the same fractional rank γ , equation (6) would give the same outcome. However, it is worth noting that the same γ is not the same absolute rank l ; in the replicated population, indeed, it will be that $l^* = \gamma(\lambda n) > l$;

c) *Poverty focus*: the individual probabilities of occupying any given rank do not depend on l . However, when changing the rank l , the cumulative individual probabilities will change, and the average probability will also change. In the same vein, if the probability of an individual of occupying ranks above l will change, the average probability of being below l will not change;

d) *Monotonicity*: the average probability decreases when the probabilities of occupying ranks below l decrease; it increases when the same probabilities increase. Furthermore, if the achievements of any individual were to improve in a way that the individual is not anymore deprived in either dimension, their probability of being multidimensional poor will decrease, which means that the average probability also decreases. Obviously, if the achievements of any individual will improve without moving them out of deprivation in that specific dimension, the probability of being multidimensional poor will not change. To some extent, monotonicity does not occur if all individuals were to exit the same deprivation at once, as it would be equivalent to measure the probability of multidimensional poverty on a narrower set of dimensions.

e) *Distribution sensitivity*: in general, this axiom would require that a measure of poverty decreases more when an improvement occurs among the poorest of the poor population. As calculated in equation (6), our measure does not satisfy this axiom, as the same reduction of the probability gives the same outcome regardless of which individuals below the fractional rank are involved in this reduction. This feature, however, is shared by common poverty measures, as the standard poverty gap, and it is not necessary a shortcoming of our approach. To satisfy this axiom, our measure could be transformed as the standard poverty gap is transformed in the Foster-Greer-Thorbecke measures of poverty; for example, by raising equation (3) at a power $\varepsilon > 1$, more relevance will be given to the highest individual probabilities of being poor when calculating the average probability.

f) *Additive decomposability by subgroups*: this is implicit in equation (7).

From a computational perspective, in this paper we select $m = 10,000$ since, according to Tervonen and Ladhelma (2007), $m \geq 9,604$ allows to consider a large set of feasible weights for estimating b_{kr} , with an error limit ≤ 0.01 that can be accomplished with a confidence interval $\geq 95\%$.¹¹

In this approach, as said above, any of these vectors of weights is considered equally important irrespective of how it deviates from the popular benchmark of equal weights, usually adopted to reflect either the lack of information about individual preferences or an agnostic attitude of the analyst (Aaberge and Brandolini, 2015). Thus, one might get empirical results that are affected by a subfamily of controversial weight vectors that some groups or individuals could not agree upon. For instance, the space of feasible weights will certainly include a number of vectors of weights in which only one indicator has a weight equal to 1, and all of the remaining indicators have a zero-weight, a feature that some might not deem appropriate to assess multidimensional poverty (Marlier and Atkinson, 2010). Yet, such a situation recalls one of the most used criteria to identify the

¹¹ As the Monte Carlo simulation should have about 10,000 repetitions to be robust (Tervonen and Ladhelma 2007), it is worth noting that the method proposed in this paper is highly demanding from a computational perspective, as the model here used has a memory complexity of 68,093 Mb. With 12 indicators it would reach a memory complexity of 359,328 Mb. The complexity of the algorithm is estimated using the GuessCompX R package developed by Agenis-Nevers et al. (2019). To some extent, the growth of complexity may be thought of as the price of having more general conclusions.

poor in a multidimensional framework, called the *union* method of identification (Atkinson, 2003), according to which it is sufficient to experience deprivation in just one indicator among many to be identified as multidimensionally poor. The union method is often contrasted with the *intersection* method (Atkinson *et al.*, 2002; Duclos *et al.*, 2006), which identifies the poor as those individuals whose achievements fall short in all the considered indicators. This latter approach has also serious ethical implications, as it identifies as non-poor individuals who might suffer from extensive deprivation in multiple indicators, though not in all of them. By the same token, any intermediate approach could be easily seen just as arbitrary and not yielding to robust empirical results. In conclusion, there is no reason for a union-like weighting scheme to be judged unethical compared to the others. The representation of all possible individual preferences and shared social values that is embedded in the large set of feasible weights used in this paper is precisely the strength of our approach, rather than its weakness.

From an operational point of view, it is worth recalling that our analysis is performed considering a variable number of indicators into $d = 3$ dimensions (decent work, income, human development) in which there is a variable number of indicators. In order to avoid that the relative importance of each dimension in the calculation of C depends on the number of indicators available for each dimension, the randomized set of weights is here obtained by implementing a two-step procedure: first we bound the generation of weights such that $\sum_{j=1}^{p_1} v_{sj} \leq 1/3$; $\sum_{j=1}^{p_2} v_{sj} \leq 1/3$; $\sum_{j=1}^{p_3} v_{sj} \leq 1/3$, where p_1 , p_2 , and p_3 are the total numbers of indicators in each dimension d and $p_1 + p_2 + p_3 = p$. Since the number of indicators varies across dimensions (2, 1, and 9, respectively), we will have that each $v_{sj} \leq 1/6$ in d_1 , each $v_{sj} \leq 1/3$ in d_2 , and each $v_{sj} \leq 1/27$ in d_3 in accordance with the nested weighting principle (Alkire and Santos, 2014). As a second step, we normalized all weights to have $\sum_{j=1}^p v_{sj} = 1$ in all cases. Equations (6) and (7) will be computed with q indicating countries.¹²

It is worth noting that the choice of the previous bounding is a mere operational choice that is mostly data-driven, as we work with a different number of indicators within each dimension. Without such an upper bound, it would follow that the dimensions containing a greater number of indicators would end up weighing more than the rest, by this way

¹² The assignment of weights in the first step could also be avoided and replaced using different vectors of weights as for the calculation of individual probabilities.

affecting the results of the analysis. While the choice of imposing an upper bound to the weighting scheme can bear consequences in terms of results, we want to stress that the method *per se* does not require to impose this or any other restriction in terms of the weighting structure – hence, the generality of the proposed method is preserved also in the presence of bounds. Furthermore, it is worth recalling that even by operating such an initial restriction in terms of weighting structure it is still possible to have one or more dimensions valued as zero – i.e. the upper bound works as a constraint in the first step, but its effect can still be offset through the final normalization.

4. Empirical application

4.1. Data

In the European context, the most suitable statistical source to investigate multidimensional poverty while aggregating the information about the poor first across dimensions and then across individuals is represented by the European Union Statistics on Income and Living Conditions (EU-SILC). EU-SILC was launched in 2003 on the basis of an agreement between Eurostat and a number of Member States with the aim of providing timely and comparable annual data on variables such as income, social exclusion, material deprivation, health, education and labour at both household and individual level.

Thus, the database is wide enough to assess deprivations over multiple facets of life. Moreover, since 2010 it is used for monitoring poverty and social exclusion in the EU in accordance with the Europe 2020 Strategy, and it has been used by previous literature in some empirical studies (see Biegert, Ebbinghaus, 2020; Paulus, Tasseva, 2020; Pohlig 2021). For all these reasons, it appears to be an appropriate and sound basis of information to measure multidimensional poverty in the EU.

4.2. *Choice of dimensions and indicators*

To produce reliable statistics, procedures for selecting life domains in a multidimensional setting should also minimize the degree of arbitrariness. However, retrieving information on shared societal values can be troublesome, especially when the evaluation concerns different countries or supranational entities like the EU.

For the empirical application, we select three dimensions of well-being to evaluate in what spheres of life individuals fall short: (i) Decent work, (ii) Income, and (iii) Human development. Such dimensions reflect three important domains covered by European social policies (Atkinson et al., 2002). Moreover, they represent publicly agreed values by the European people, as testified by written public documents about common values and social development objectives, like the Charter of Fundamental Rights of the EU (European Parliament, Council of the European Union and European Commission, 2000). Such agreement was established through some consensus-building process at one point in time and remained relatively stable thereafter.¹³ Finally, suitable indicators for all these dimensions can be easily found in the EU-SILC. Although relying on existing data might not always be the most convenient strategy to select dimensions, it is usually sufficient to carry out a methodological exercise (see e.g., Bourguignon and Chakravarty, 2003 as cited by Alkire, 2007), or to test a newly proposed measurement method, as in the case of this paper.

The *Decent work* concept refers to the right to employment opportunities for productive work and the possibility to deliver a fair income in conditions of freedom, equity, security and human dignity (ILO, 1999). From the available data in the EU-SILC, we select two relevant indicators: *Activity status* and *Low work intensity*, respectively accounting for employment conditions and (quasi-) joblessness – that is, living in households where working age members worked less than 20% of their total potential during the previous 12 months – as conceived by Eurostat as part of the composite indicator At Risk of Poverty and Social Exclusion rate (AROPE).

As regards to the *Income* dimension, different indicators in the EU-SILC allow to capture the level of social protection offered to European citizens, e.g., through the

¹³ National Constitutions and laws have been exploited to retrieve information on publicly agreed values to be used in multidimensional poverty assessments – see for instance the National Council for Evaluation of Social Development Policy experience in Mexico (CONEVAL, 2010) and some scholarly initiatives (Burchi et al., 2014).

variables Family/Children related allowances, Social exclusion not elsewhere classified, and Housing allowances. Because all the policies just mentioned sustain people's standard of living by integrating their income through the channel of monetary transfers, it appears reasonable to choose an income poverty indicator as a general proxy for this dimension. The variable *Monetary poverty* (after transfers) is thus used to account for deprivations in the Income domain.

Finally, *Human development* includes all those conditions that protect the right to the human flourishing of individuals in a just and protected environment– e.g., through the right of education and the protection of human health and the environment. Different indicators from EU-SILC can be used to construct deprivation indicators in this dimension: some of them relate to human health, while some others refer to the educational attainment or to the quality of the living environment. The nine selected variables and the corresponding modalities are outlined, along with all other chosen indicators, in Table 1.

(Table 1)

4.3. The probability of being multidimensionally poor

For the empirical analysis, we selected four waves of the EU-SILC (2008, 2010, 2012, 2014) including 10 countries: Austria (AT), Belgium (BE), France (FR), Germany (DE), Greece (EL), Italy (IT), Luxemburg (LU), Portugal (PT), Spain (ES), and the United Kingdom (UK). The whole sample size is of 182,912 individual observations in 2014; 181,864 in 2012; 178,914 in 2010; and 176,518 in 2008. The average sample size by country is 11,065 individuals in Austria, 10,793 in Belgium, 22,306 in Germany, 14,177 in Greece, 27,127 in Spain, 20,618 in France, 39,520 in Italy, 9,429 in Luxemburg, 11,096 in Portugal, and 13,919 in the UK. Yearly sample sizes for each country are reported in Table 2, which shows the descriptive statistics by country and year of the individual probabilities of being below the fractional rank $\gamma=0.2$ – i.e. of being among the poorest 20% of the overall population.

In six out of nine countries, the median probability of being multidimensionally poor (column *D*) is equal to zero; exceptions are Greece, Spain, Portugal, and Italy, even

though for not all years considered. This evidence is consistent both with the endeavour to provide a robust estimation of multidimensional poverty in the selected European countries, where living conditions are on average among the highest in the world, and with the indicators chosen for the analysis, that aim at reflecting situations of acute poverty. Obviously, where the median probability is not zero, the percentage of individuals involved in multidimensional poverty is above 50% (column *F*), with the peaks being in Greece and Portugal.

The calculation of equation (6) is reported in column *A*. There are two analytical perspectives that can be exploited to interpret the results. The first one is to observe what happens *within* a given country over time. To this purpose, one can distinguish those countries where the average probability of multidimensional poverty has reduced (AT, DE, FR, LU) from those countries where it has increased (EL, ES, PT). Finally, there are also countries where the same probability has changed across the years without a clear pattern (BE, IT, UK). The second perspective is to compare the levels of the probabilities across countries.

In Austria, Germany, France, and Luxembourg, individuals have the lowest probability of being below the fractional rank $\gamma=0.2$. The highest average probability is instead recorded in Greece, Spain, and Portugal, with Belgium, Italy, and UK positioned halfway between these two extremes. From this outcome, the geographical distribution of the probability of being multidimensionally poor appears rather clear, opposing countries of the continental Europe on the one hand and countries of the Mediterranean area on the other.

(Table 2)

The interpretation of these results becomes even clearer by observing the highly skewed shape of the probability distributions for each country shown in the box plots (Figure 1). Due to the large outliers, country mean probabilities lie outside the interquartile range in most cases. However, for some Southern European countries – Greece, Spain, and Portugal – probability distributions are extremely sparse: even though country means are included in the interquartile range, extreme values attain the value of

1, as visually described by the overlapping of the maximum of the box plot and the upper bound of the probability distribution.

That means that, in these countries, there are some individuals who have a 100% probability of being among the poorest 20% of the population regardless of the weighting scheme applied to the set of multidimensional poverty indicators. Belgium and Italy also feature quite sparse distributions, with an average maximum probability exceeding 50% (Belgium in 2010 and 2012) and 90% (Italy in 2012) of being among the poorest 20%. Conversely, in Austria, France, Germany, Luxembourg and the UK, probability distributions are narrower and close to zero, suggesting a greater robustness of the individual probabilities to changes of the weighting scheme attached to different poverty dimensions.

(Figure 1)

It is also worth noting that – in some cases – yearly changes appear to be more meaningful when even small variations of the means are associated to a substantial increase (or decrease) of the interquartile range. This is the case of Belgium, where the probability of being poor durably increases after 2008 due to the sparsity of individual probabilities in the range between 0 and p_{25} . Greece and Italy, on the other hand, show a larger variability in the probability of falling into the poorest 20% in 2012 compared to the previous years, while Portugal see its probability distribution becoming even sparser in 2014.

In all these cases, the discontinuity is also driven by an enlargement of the proportion of individuals who have non-zero probability to be in the lowest quintile of the distribution: it increases by almost 7 percentage points in Greece and by 4 percentage points in Italy between 2010 and 2012; and it grows by 62 to 65% in Portugal between 2012 and 2014 (see column *F*, Table 2).

5. Dominance conditions

5.1 Extending dominance criteria to the probabilities of being non-poor

In this section, a step further is done to investigate multidimensional poverty by considering the whole distribution of probabilities below rank l by country. This means using all the information provided by vector B^l , without collapsing the information into the calculation of equation (6). To build this process, it is convenient to order individuals in each country from the lowest to the highest probability of being *non-poor*, which means to use the complement of b_k^l as an indicator of the position in the poverty distribution.¹⁴ This means that instead of using b_k^l , in this section individuals will be ordered by $x_k^l = 1 - b_k^l$, where x_k^l is the *probability of being non-poor*. By this way, the multidimensionally poorest individuals will have $x_k^l = 0$, while individuals that are not multidimensionally poor will have $x_k^l = 1$, i.e. the certainty of being non-poor. In other terms, by ordering individuals according to x_k^l , we build a distribution of probabilities of being non-poor, which can be used to investigate second-order dominance (SD) conditions through generalised Lorenz curves.

The advantage of using SD is twofold, and it may be linked to the analysis of multidimensional poverty. First, SD allows to formulate social norms according to which a given outcome may be socially preferred for a wide class of social value judgements.¹⁵ The basis of this approach can be traced back to Atkinson (1970), who was the first to give terms under which a Lorenz inequality comparison has normative significance. As shown by Lambert (1993), the Atkinson theorem tells us that for increasing and strictly concave utility functions, the distribution with the dominating Lorenz curves is preferred. When the distributions have different means, the result is extended to generalised Lorenz dominance, given by the product of Lorenz curves and the mean of the distributions.

In the specific case, conditions will be derived under which a distribution of probabilities in a given country would be socially preferred to an alternative distribution of probabilities in another country under mild conditions. Second, SD conditions possibly obtained at a given fractional rank γ would hold for all fractional ranks below γ ; thus,

¹⁴ This process is analogous to the ranking of individuals from the poorest to the richest in terms of incomes when analysing a typical income distribution.

¹⁵ See for all Lambert (1993) and Deaton (1997).

this outcome would avoid replicating the analysis to understand whether the outcome is robust to alternative hypotheses about the value of γ . In other terms, if the dominance condition holds from 0 to γ , it will hold for any fractional rank $\theta < \gamma$, which is a more general result than computing equation (6) and (7) for a *given* fractional rank. Furthermore, using SD on the overall distribution of probabilities would make irrelevant the choice of γ , in the same way as SD makes irrelevant the choice of the poverty line (Deaton, 1997).

For our purposes, to investigate multidimensional poverty, we still focus on the probability of being non-poor truncated at $\gamma=0.2$. The normative properties of this distribution can be analysed by using the Generalised Lorenz (GL) dominance of the probabilities x_k^l . As in the standard theory of Lorenz dominance, if the GL curve of the cumulated x_k^l in country *A* dominates the GL curve of x_k^l in country *B*, it would mean that in country *A* there will be a lower cumulated probability of having individuals below the fractional rank γ (and thus below any fractional rank $\theta < \gamma$).

This outcome, by analogy with the theory of dominance, can be linked to a social welfare function truncated at $\gamma=0.2$. To this purpose, one can define a class of social norms $W(x)$ – depending on the vector of probabilities of being non-poor – that satisfies $W'(x) > 0$ and $W''(x) < 0$, i.e. an increasing and concave social welfare function. These two conditions only require, respectively, that the social preference increases both when the probability of being non-poor increases and after a “transfer” of the probability of being non-poor from a higher to a lower probability.¹⁶

The case of GL dominance, however, may not occur; rather, GL curves may cross. When this happens, unanimous conclusions about a social preference are prevented. Yet, some conclusions may be achieved with the additional requirement that $W'''(x) > 0$. This feature corresponds to the principle of diminishing transfer, which embodies – in the specific case – the assumption that an increase of the probability of the poorest of being less poor increases the social preference more than an increase of the same probability of the least poor.

¹⁶ This second condition is simply a restatement of the principle of transfers that holds when income is the argument of a social welfare function, and that fundamentally embodies aversion to inequality. In the specific case, it can be translated into a generic social norm of aversion to poverty.

Intuitively, and considering two individuals j and k for which $x_j^l < x_k^l$, this condition means that an increase of x_j^l will be more socially preferred than an equivalent increase of x_k^l . It also implies that the focus is now shifted to dominance in the part of the distribution of probabilities before the crossing, and that unanimous social preferences can be drawn only until that point. This means that a social preference is potentially restricted to a narrower class of social welfare functions; indeed, unanimous conclusions about social welfare cannot be drawn on the whole distribution, as the outcome might change when changing the poverty line (Lambert, 1993).

To this regard, assume two countries s and r ; if $GL_s >_A GL_r$, where the symbol $>_A$ means that the distribution of country s intersects the distribution of country r from above until a given point, the distribution s will be socially preferred if the two following conditions are met (mean-variance condition):

$$(17) \quad \mu_s < \mu_r$$

$$(18) \quad \sigma_s^2 < \sigma_r^2 - (\mu_r - \mu_s)(2t - \mu_r - \mu_s)$$

where t is the maximum probability of being non-poor, which is equal to 1. Condition (17) simply states that the mean of the distribution s (μ_s) must be lower than the mean of the distribution r (μ_r). Condition (18) requires that the variance of the distribution s (σ_s^2) must be *sufficiently* lower than the variance of the distribution r (σ_r^2). It is also worth noting that if the mean level of the two distributions were equal, the only relevant condition would be $\sigma_s^2 < \sigma_r^2$, i.e. that the variance of s is lower than the variance of r .

When either of the two conditions does not hold, no general conclusions in terms of social preference would be possible. Contrarily, when both hold one can go a step further to measure the robustness of the social ranking to the degree of poverty aversion. This can be done by calculating a lower limit of that aversion below which social unanimous prescriptions obtained by GL no longer hold. This lower bound is given by:

$$(19) \quad Low = \frac{t(\mu_r - \mu_s)}{(\sigma_r^2 - \sigma_s^2) - (\mu_r - \mu_s)(2t - \mu_r - \mu_s)}$$

The calculation of *Low* is potentially important to understand the robustness of dominance in terms of consensus across different decision makers with different (and unknown) degrees of poverty aversion. To this purpose, the larger the difference $[(\sigma_r^2 - \sigma_s^2) - (\mu_r - \mu_s)(2t - \mu_r - \mu_s)]$, the nearer to zero will be the lower bound of equation (19). Since $Low = 0$ would connote poverty neutrality, the larger the gap, the greater is the class of $W(x)$ for which the result will hold.

This method represents a novel approach to combine poverty analysis and dominance criteria, as it combines a weight-free method of estimating the probabilities of being non-poor with a value-free way of determining social preferences that directly connect poverty levels and their distributions until a given fractional rank. It is worth noting that dominance over the probability distribution does not imply dominance across any dimension composing the probability. In our approach, SD is used to translate a problem of multidimensional poverty into a unidimensional space through the use of the cumulated probability. Thus, in a comparative perspective, our conclusions must be interpreted as indicating those cases where there is always less cumulated probability of multidimensional poverty.

Except for recent contributions by Aaberge *et al.* (2019) and Azpitarte *et al.* (2020), this is also one of the first attempt to apply dominance criteria to the issue of multidimensional poverty. Our contribution, however, differs from that by Aaberge *et al.* (2019), as in that case the analysis is based on a deprivation count distribution where no attempt is made to aggregate the count into a synthetic multidimensional poverty index at individual level. In our analysis, instead, the deprivation count distribution is the baseline to calculate the probability of each individual to be below a given fractional rank. This difference allows us to apply dominance criteria directly considering the whole distribution of probabilities obtained by aggregating the dimensions of poverty; while in Aaberge *et al.* (2019), the dominance is *sequentially* applied (either downward or upward) by progressively adding fractions of populations with a different number of deprivations.

This same difference applies with respect to Azpitarte *et al.* (2018), where necessary and sufficient conditions of dominance for classes of counting poverty measures are derived, still within a logic of sequential dominance, i.e. involving all potential poverty sets expressed as a union of subsets of multidimensional poverty. To some extent, as reported by the authors, this may involve a greater number of statistics, as the number of

elements in the sets of multidimensionality increases exponentially with the number of dimensions involved in poverty comparisons.¹⁷ Furthermore, when dominance conditions are not verified, the main conclusion is that multidimensional poverty may be sensitive to the choice of dimensional weights and poverty cut-offs, which is only a partial response to the issue of multidimensional poverty.

In our paper, instead, we elaborate a different process, as the issue of dimensional weights is condensed in a particular poverty measure, i.e. the probability of being below a given fractional rank after experimenting with a large set of feasible weights. Thus, by construction, these individual probabilities take into account the issue of multiple vectors of weights at the stage of building the index, while at the same time providing a distribution of poverty levels that can be directly dealt with the standard dominance theory. The next section illustrates the empirical outcome.

5.2 GL dominance and GL crossings in some European countries

The outcome of the GL dominance is reported in Table 3 for all years. Each panel can be easily read by rows. For example, in 2008, Austrian individuals have always a lower probability (“Lower”) than individuals from other countries of being below the fractional rank $\gamma=0.2$, except for France. For Italian individuals, instead, this probability is lower only compared to Portugal, while crossings occur with Belgium, Greece, Spain, and Luxembourg. At the same time, individuals from Greece and Portugal have the highest probability of being below the fractional rank, as “Lower” does not appear in any comparison.

The analysis is replicated in each year and gives evidence of the changes occurred in the ranking of probabilities among countries. In the panel of year 2014, changes with respect to 2008 are highlighted. Changes occur in each country, with a slight improvement of the relative position only in Italy, Luxembourg, and Portugal. A slightly worse comparative outcome can instead be traced in Austria, Belgium, Germany, Greece, Spain, and France. Finally, in the UK, a relative improvement occurs with respect to Austria, while the relative position worsens with respect to Luxembourg.

¹⁷ The authors report that with five dimensions, the number of statistics involved in dominance conditions would be more than 7,000.

In terms of social preferences, the conclusions are readily obtained. By considering the last year of the analysis, 2014, since “Lower” corresponds to all cases where the GL curve of the probabilities of being *non-poor* in the country in row dominates the GL curve of the same probabilities in the country in column, the social preference as measured by any member of the class $W = \{W: W'(x) > 0; W''(x) < 0\}$ is always for the distribution of probabilities in the country in row. It is worth noting that the dominance also implies that the social preference will be higher for any specific fractional rank $\theta < \gamma$. The opposite holds in the case where the matrix is filled by “Higher”.

Uncertain outcomes, instead, occur when GL curves cross (“Crossing”). To solve this uncertainty, we first identify the comparisons between countries where the dominance occurs in the lowest part of the distribution (i.e. before the intersection, from above). This happens in the following cases: $GL_{IT} >_A GL_{BE}$; $GL_{IT} >_A GL_{DE}$; $GL_{IT} >_A GL_{FR}$; $GL_{LU} >_A GL_{AT}$; $GL_{LU} >_A GL_{DE}$; $GL_{LU} >_A GL_{FR}$; $GL_{PT} >_A GL_{BE}$; $GL_{PT} >_A GL_{DE}$; $GL_{PT} >_A GL_{EL}$; $GL_{PT} >_A GL_{ES}$; $GL_{UK} >_A GL_{AT}$; $GL_{UK} >_A GL_{BE}$; $GL_{UK} >_A GL_{DE}$; $GL_{UK} >_A GL_{FR}$.

In all comparisons, both conditions (17) and (18) are satisfied, which means that the dominating distribution is socially preferred for any member of the restricted class $W = \{W: W'(x) > 0; W''(x) < 0; W'''(x) > 0\}$. More importantly, as shown in Table 4, the values of *Low*, as in equation (19), are calculated. For example, the dominance of Italy over France will embody a social preference for degrees of poverty aversion higher than 0.779. As can be easily seen, some crossings correspond to a higher social preference only for degrees of poverty aversion greater than 1, as in the cases of Portugal vs. Greece, Portugal vs. Spain, and Greece vs. Spain.

It is worth stressing, at this point, that this outcome is particularly important in the analysis of poverty, as it allows a double stronger conclusion with respect to the existing literature. The first derives from the fact that the probabilities of being *multidimensionally* poor are estimated without making recourse to a specific set of weights; the second derives from the fact that social welfare implications are derived from dominance theory by making use of mild assumptions about social norms. In other terms, the comparison of the probabilities of poverty among countries that is here obtained is loaded by the minimum set of arbitrary choices, in terms both of weighting the various dimensions of poverty and of linking individual probabilities to social preferences.

(Table 3)

(Table 4)

6. Conclusions

This paper proposes a novel method to analyse multidimensional poverty: instead of relying on one specific set of weights to calculate the deprivation scores, a large set of feasible (positive) weights is used to summarise the information about the poor in a distribution of probabilities of being multidimensionally poor. This method allows to remain agnostic about the importance given to the different dimensions by producing indexes that capture the individual probability of being multidimensionally poor regardless of the weighting scheme applied.

The concept of individual probability allows to move from a dual definition of poverty, where poor and non-poor individuals are classified in a mutually exclusive context, to a continuous measure of deprivation capturing both the extensive and the intensive margin of multidimensional poverty. Individual probabilities can then be combined with the generalised Lorenz dominance techniques to derive socially preferred distributions with the minimum load of value judgments.

This novel method has been used for measuring multidimensional poverty in ten selected countries (Austria, Belgium, France, Germany, Greece, Italy, Luxemburg, Portugal, Spain, and United Kingdom) using data from four waves of EU-SILC (2008, 2010, 2012, 2014). Results show that the probability distributions of being among the poorest 20% have median zero for all countries with the exceptions of Greece, Spain, Portugal, and Italy. At the same time, in Greece, Spain, and Portugal, there is a significant number of individuals who have 100% probability of being among the poorest 20% of the population regardless of the weighting scheme applied to the set of multidimensional deprivation indicators. On the contrary, in Austria, France, Germany, Luxembourg, and the UK the probability distributions of being among the poorest 20% are smaller, and the presence of large outliers means that the weights attached to the deprivation indicators can significantly change the probability of being considered multidimensionally poor.

Evidence from the pairwise GL dominance analysis shows that in 2008 Austrian individuals have always a lower probability of being among the poorest 20% of the population than any other country, except for France. Conversely, in the same year, the distribution of probabilities in Greece and Portugal never dominates other countries. Overall, the method we propose and apply in this paper sheds new light on multidimensional poverty as a concept and is able to provide new tools for multidimensional poverty assessment that are applicable in any setting where either count or continuous data are available.

From a policy perspective, the information conveyed by our approach can facilitate the operationalization of multidimensional assessments in public policy. To begin with, the observation of the probability distribution below γ may indicate the potential urgency of policy action to fight multidimensional poverty. When individual probabilities of being below γ are high, it implies that the risk of being multidimensionally poor is high as well, regardless the political and social attitudes that might be embodied in the vector of weights – i.e., that specific probability is robust to virtually any set of admissible preferences. Conversely, the opposite holds true when the estimated probabilities are low.

Our approach can also positively contribute to inform the development of appropriate policy tools to fight poverty, like income support schemes. Such policies are often challenged by the intrinsic multidimensionality of poverty, especially in the targeting phase, where the poorest individuals need to be correctly identified. Standard approaches to poverty measurement, often based on the setting of a poverty cut-off, typically require ranking potential recipients of anti-poverty interventions according to their level of need, aggregating multiple dimensions into a summary (scalar) index and then dichotomizing the examined population into poor and non-poor.

In contrast, SMAA can be used to estimate how different weighting scheme would affect the targeting step of an income support policy, at the same time ensuring that beneficiaries are selected with the minimum degree of arbitrariness in the choice of weights. More generally, moving from a dual to a continuous targeting based on the intensive margins of multidimensional poverty (i.e., the probability to be poor) allows to go beyond the ‘cut-off problem’, which is particularly meaningful when, for instance, decisions on the volume of individual cash transfer are made. In theory, as the probability defines multidimensional poverty in the continuum, the same probability can be adopted

as a criterion for cash transfers – i.e., greater multidimensional poverty requires more significant cash transfers and decisions on the volume of cash transfer requires a continuous measure of poverty.

References

- Aaberge, R., & Brandolini, A. (2015). Multidimensional Poverty and Inequality. In Atkinson, A. B. and Bourguignon, F. (eds.) (2015) *Handbook of Income Distribution Vol.2*. 1st Edition. North-Holland.
- Aaberge, R., Peluso, E., & Sigstad, H. (2019). The dual approach for measuring multidimensional deprivation: Theory and empirical evidence, *Journal of Public Economics*, 177, Article 104036.
- Agenis-Nevers, M., Bokde, N. D., Yaseen, Z. M., & Shende, M. (2019). GuessCompX: An empirical complexity estimation in *R*. *arXiv*.
- Alkire, S. & Foster, J. E. (2011). Counting and Multidimensional Poverty Measurement, *Journal of Public Economics*. 95(7–8), 476–487.
- Alkire, S., & Santos, M. E. (2014). Measuring acute poverty in the developing world: Robustness and scope of the multidimensional poverty index. *World Development*, 59, 251-274.
- Alkire, S. (2007). Choosing Dimensions: the Capability Approach and Multidimensional Poverty. In N. Kakwani & J. Silber (eds.), *The Many Dimensions of Poverty*. New York: Palgrave Macmillan.
- Alkire, S., & Foster, J. E. (2011). Counting and Multidimensional Poverty Measurement. *Journal of Public Economics*, 95(7–8), 476–487.
- Alkire, S., Foster, J. E., Seth, S., Santos, M. E., Roche, J. M., & Ballon, P. (2015). *Multidimensional Poverty Measurement and Analysis*. Oxford: Oxford University Press.
- Anand, S., & Sen, A. (1997). Concepts of Human Development and Poverty! A Multidimensional Perspective. United Nations Development Programme, Poverty and human development: Human development papers, 1-20.
- Angilella, S., Corrente, S., Greco, S., & Słowiński, R. (2016). Robust Ordinal Regression and Stochastic Multiobjective Acceptability Analysis in multiple criteria hierarchy process for the Choquet integral preference model. *Omega*, 63, 154-169.
- Atkinson A.B. (1970). On the Measurement of Inequality. *Journal of Economic Theory*, 2, 244-263.

- Atkinson, A. B. (2003). Multidimensional deprivation: contrasting social welfare and counting approaches. *The Journal of Economic Inequality*, 1(1), 51-65.
- Atkinson, A. B., Cantillon, B., Marlier, E., & Nolan, B. (2002). *Social indicators: the EU and social inclusion*. Oxford: Oxford University Press.
- Azpitarte, F., Gallegos, J., & Yalonetzky, G. (2020). On the robustness of multidimensional counting poverty orderings. *The Journal of Economic Inequality*, 18(3), 339-364.
- Biegert, T., & Ebbinghaus, B. (2020). Accumulation or absorption? Changing disparities of household non-employment in Europe during the Great Recession. *Socio-Economic Review*, mwaa003. <https://doi.org/10.1093/ser/mwaa003>
- Bossert, W., Chakravarty, S. R., & D'Ambrosio, C. (2013). Multidimensional poverty and material deprivation with discrete data. *Review of Income and Wealth*, 59(1), 29-43.
- Bourguignon, F., & Chakravarty, S. R. (2003). The measurement of multidimensional poverty. *Journal of Economic Inequality*, 1(1), 25-49.
- Burchi, F., De Muro, P., & Kollar, E. (2014). Which dimensions should matter for capabilities? A constitutional approach. *Ethics and Social Welfare*, 8(3), 233-247.
- Cerioli, A., and S. Zani (1990). A Fuzzy Approach to the Measurement of Poverty. In C. Dagum & M. Zenga (eds.), *Income and Wealth Distribution, Inequality and Poverty, Studies in Contemporary Economics*, 272-284. Springer Verlag Berlin.
- Chakravarty, S. R., Mukherjee, D., & Ranade, R. R. (1998). On the family of subgroup and factor decomposable measures of multidimensional poverty. *Research on Economic Inequality*, 8, 175-194.
- Cheli, B., & Lemmi, A. (1995). "Totally" fuzzy and relative approach to the multidimensional analysis of poverty. *Economic Notes*, 24(1), 115-134.
- Chiappero-Martinetti, E. (1994). A New Approach to Evaluation of Well-being and Poverty by Fuzzy Set Theory. *Giornale Degli Economisti e Annali Di Economia*, 53(7/9), 367-388.
- Chiappero-Martinetti, E., & von Jacobi, N. (2012). Light and shade of multidimensional indexes: how methodological choices impact on empirical results. In F. Maggino & G. Nuvolati (eds.), *Quality of life in Italy: Research and Reflections* (69-103).

Springer: Dordrecht.

- Chowdhury, S., & Squire, L. (2006). Setting weights for aggregate indices: An application to the Commitment to Development Index and Human Development Index. *Journal of Development Studies*, 42(5), 761–771.
- Coco, G., Lagravinese, R., & Resce, G. (2020). Beyond the weights: a multicriteria approach to evaluate inequality in education. *The Journal of Economic Inequality*, 18(4), 469-489.
- CONEVAL (2010). Methodology for Multidimensional Poverty Measurement in Mexico, Consejo Nacional de Evaluación de la Política de Desarrollo Social (CONEVAL).
- Deaton, A. (1997). *The analysis of household surveys: a microeconomic approach to development policy*. The World Bank.
- de Kruijk, H., & Rutten, M. (2007). Weighting dimensions of poverty based on people's priorities: Constructing a composite poverty index for the Maldives, Q-Squared Working Paper 35, Centre for International Studies, University of Toronto.
- De Matteis, D., Ishizaka, A., & Resce, G. (2019). The 'postcode lottery' of the Italian public health bill analysed with the hierarchy Stochastic Multiobjective Acceptability Analysis. *Socio-Economic Planning Sciences*, 68, Article: 100603.
- Decancq, K. & Lugo, M. A. (2013). Weights in multidimensional indices of wellbeing: An Overview. *Econometric Reviews*, 32(1):7–34.
- Decancq, K., & Schokkaert, E. (2016). Beyond GDP: Using equivalent incomes to measure well-being in Europe. *Social indicators research*, 126(1), 21-55.
- Desai, M., & Shah, A. (1988). An econometric approach to the measurement of poverty, *Oxford Economic Papers* 40(3), 505-522.
- Despotis, D.K. (2005a). Measuring human development via data envelopment analysis: The case of Asia and the Pacific, *Omega*, 33(5), 385-390.
- Despotis, D.K. (2005b). A reassessment of the Human Development Index via data envelopment analysis, *Journal of the Operational Research Society*, 56(8), 969-980.
- Deutsch, J., & Silber, J. (2005). Measuring multidimensional poverty: an empirical comparison of various approaches. *Review of Economics and Statistics*, 51(1).
- Di Tommaso, M. L. (2007). Children capabilities: A structural equation model for India.

- Journal of Socio-Economics*, 36(3), 436–450.
<https://doi.org/10.1016/j.socec.2006.12.006>
- Duclos, J.-Y., Sahn, D. E. and Younger, S. D. (2006). Robust multidimensional poverty comparisons, *Economic Journal*, 116(514), 943–968.
- European Parliament, Council of the European Union, & European Commission. Charter of Fundamental Rights of the European Union, Official Journal of the European Communities § (2000).
- Ferreira, F. H. G., & Lugo, M. A. (2013). Multidimensional poverty analysis: Looking for a middle ground. *World Bank Research Observer*, 28(2), 220–235.
- Ferrer-i-Carbonell, A., & Frijters, P. (2004). How important is methodology for the estimates of the determinants of happiness?, *The Economic Journal*, 114(497), 641-659.
- Fleurbaey, M. (2009). Beyond GDP: the quest for a measure of social welfare, *Journal of Economic Literature*, 47(4), 1029-1075.
- Foster, J. E., McGillivray, M., & Seth, S. (2013). Composite Indices: Rank Robustness, Statistical Association, and Redundancy. *Econometric Reviews*, 32(1), 37–41.
- Fusco, A., and P. Dickens (2008). The Rasch model and multidimensional poverty measurement. In N. Kakwani & J. Silber (eds.), *Quantitative Approaches to Multidimensional Poverty Measurement*. New York: Palgrave Macmillan.
- Fukuda-Parr, S. (2016). From the Millennium Development Goals to the Sustainable Development Goals: shifts in purpose, concept, and politics of global goal setting for development. *Gender and Development*, 24(1), 43–52.
- Greco, S., Ishizaka, A., Matarazzo, B., & Torrisci, G. (2018). Stochastic multi-attribute acceptability analysis (SMAA): An application to the ranking of Italian regions, *Regional Studies*, 52(4), 585-600.
- Greco, S., Ishizaka, A., Tasiou, M., & Torrisci, G. (2019). On the methodological framework of composite indices: A review of the issues of weighting, aggregation, and robustness. *Social indicators research*, 141(1), 61-94.
- Guio, A. C., Marlier, E., Gordon, D., Fahmy, E., Nandy, S., & Pomati, M. (2016). Improving the measurement of material deprivation at the European Union level. *Journal of European Social Policy*, 26(3), 219–333.
<https://doi.org/10.1177/0958928716642947>

- Halleröd, B. (1995a). Making ends meet: perceptions of poverty in Sweden. *Scandinavian Journal of Social Welfare*, 4(3), 174-189.
- Halleröd, B. (1995). The truly poor: direct and indirect consensual measurement of poverty in Sweden. *Journal of European Social Policy*, 5(2), 111-129.
- ILO (1999). Report of the Director-General: Decent Work. Report presented at the International Labour Conference, June, 87th Session, Geneva (<http://www.ilo.org/public/english/standards/relm/ilc/ilc87/rep-i.htm>).
- Klasen, S. (2000). Measuring poverty and deprivation in South Africa, *Review of Income and Wealth*, 46(1), 33-58.
- Krishnakumar, J. (2007). Going beyond functionings to capabilities: An econometric model to explain and estimate capabilities, *Journal of Human Development*, 7(1), 39–63.
- Krishnakumar, J., & Ballon, P. (2008). Estimating basic capabilities: A structural equation model approach applied to Bolivian data, *World Development* 36(6), 992-1010.
- Krishnakumar, J., & Nadar, A. (2008). On exact statistical properties of multidimensional indices based on principal components, factor analysis, MIMIC and structural equation models, *Social Indicators Research*, 86(3), 481-496.
- Lagravinese, R., Liberati, P., & Resce, G. (2019). Exploring health outcomes by stochastic multicriteria acceptability analysis: An application to Italian regions. *European Journal of Operational Research*, 274(3), 1168-1179.
- Lagravinese, R., Liberati, P., & Resce, G. (2020). Measuring Health Inequality in US: a composite index approach. *Social Indicators Research*, 147(3), 921-946.
- Lahdelma R., Salminen P. (2001). SMAA-2: Stochastic multicriteria acceptability analysis for group decision making. *Operations Research*, 49(3), 444-454.
- Lambert, P.J. (1993). *The distribution and redistribution of income*, Manchester and New York, Manchester University Press
- Liberati, P. (2015). The World Distribution of Income and its Inequality, 1970–2009. *Review of Income and Wealth*, 61(2), 248-273.
- Marlier, E., Atkinson, A. B. (2010). Indicators of poverty and social exclusion in a global context, *Journal of Policy Analysis and Management*, 29(2), 285–304.

- Maasoumi, E., & Lugo, M. A. (2008). The Information Basis of Multivariate Poverty. In *Quantitative Approaches to Multidimensional Poverty Measurement*. London: Palgrave Macmillan, 1-29.
- Mack, J., & Lansley, S. (1985). *Poor Britain*. London: Allen & Unwin.
- Mahlberg, B., & Obersteiner, M. (2001). Remeasuring the HDI by Data Envelopment Analysis (December 18, 2001). <http://dx.doi.org/10.2139/ssrn.1999372>
- Mascherini, M., & Hoskins, B. (2008). *Retrieving expert opinion on weights for the Active Citizenship Composite Indicator*. European Commission, Joint Research Centre, Institute for the Protection and Security of the Citizen.
- Melyn, W., & Moesen, W. (1991). *Towards a synthetic indicator of macroeconomic performance: Unequal weighting when limited information is available*. Katholieke Universiteit Leuven, Public Economic Working Paper 17.
- Moldan, B., & Billharz, S. (1997). *Indicators of Sustainable Development*. Chichester: John Wiley.
- Mikulić, J., Kožić, I., & Krešić, D. (2015). Weighting indicators of tourism sustainability: A critical note. *Ecological Indicators*, 48, 312–314.
- Nardo, M., Saisana, M., Saltelli, A., Tarantola, S., Hoffman, A., & Giovannini, E. (2008). *Handbook on constructing composite indicators*. Paris, OECD-JRC.
- Noble, M., Wright, G., Smith, G., & Dibben, C. (2006). Measuring multiple deprivation at the small-area level, *Environment and Planning A*, 38(1), 169-185.
- Nolan, B., & Whelan, C. (1996). *Resources, Deprivation and Poverty*. Oxford: Oxford University Press.
- Nolan, B., & Whelan, C. (2011). *Poverty and Deprivation in Europe*. Oxford: Oxford University Press.
- Noorbakhsh, F. (1998). The Human Development Index: Some technical issues and alternative indices, *Journal of International Development*, 10(5), 589-605.
- Paruolo, P., Saisana, M., & Saltelli, A. (2013). Ratings and rankings: voodoo or science? *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 176(3): 609–634.
- Paulus, A., & Tasseva, I. V. (2020). Europe Through the Crisis: Discretionary Policy Changes and Automatic Stabilizers. *Oxford Bulletin of Economics and Statistics*, 82(4), 864-888.

- Pohlig, M. (2021). Unemployment sequences and the risk of poverty: from counting duration to contextualizing sequences. *Socio-Economic Review*, 19(1), 273-305.
- Pyatt, G. (1976). On the interpretation and disaggregation of Gini coefficients, *The Economic Journal*, 86(342), 243-255.
- Ravallion, M. (2011). On multidimensional indices of poverty. *Journal of Economic Inequality*, 9(2), 235–248.
- Ray, A. K. (2008). Measurement of social development: an international comparison. *Social Indicators Research*, 86(1), 1–46.
- Resce, G., & Schiltz, F. (2021). Sustainable development in Europe: A multicriteria decision analysis. *Review of Income and Wealth*, 67(2), 509-529.
- Robeyns, I. (2003). Sen's capability approach and gender inequality: selecting relevant capabilities. *Feminist economics*, 9(2-3), 61-92.
- Saaty, R.W. (1987). The analytic hierarchy process- what it is and how it is used, *Mathematical Modelling*, 9(3-5), 161-176.
- Schokkaert, E. (2007). Capabilities and satisfaction with life, *Journal of Human Development*, 8(5), 415- 430.
- Schultz, H. (1935). Interrelations of Demand, Price, and Income. *Journal of Political Economy*, 43(4), 433–481.
- Sen, A.K. (1976). Poverty: An ordinal approach to measurement, *Econometrica*, 44(2), 219- 231.
- Tervonen, T., & Lahdelma, R. (2007). Implementing stochastic multicriteria acceptability analysis. *European Journal of Operational Research*, 178(2), 500-513.
- Tsui, K. Y. (2002). Multidimensional poverty indices. *Social Choice and Welfare*, 19(1), 69–93.
- UNDP. (1990). *Human Development Report 1990. Concept and measurement of human development*. 0-19-506481-X.
- Yitzhaki, S., & Lerman, R. I. (1991). Income stratification and income inequality. *Review of income and wealth*, 37(3), 313-329.
- Yitzhaki, S. (1994). Economic distance and overlapping of distributions, *Journal of Econometrics*, 61(1), 147-159.

Zaim, O., Fare, R., & Grosskopf, S. (2001). An economic approach to achievement and improvement indexes, *Social Indicators Research*, 56(1), 91-118.

Annex 1 – An example of the method

Here we provide a toy example to describe the different steps illustrated in section 3. Consider four individuals (A, B, C, and D) and three deprivation dimensions (X, Y, and Z). Let us suppose that A is only deprived in the dimension X, B is deprived in both X and Y, C is only deprived in Z, and D does not have any deprivation, yielding the following matrix of deprivations \mathbf{g} :

$$\mathbf{g} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Let us now consider a matrix of weights \mathbf{v} where each row represents a different set of weights ($m = 3$):

$$\mathbf{v} = \begin{bmatrix} 0.39 & 0.27 & 0.34 \\ 0.18 & 0.33 & 0.49 \\ 0.61 & 0.10 & 0.29 \end{bmatrix}$$

Each weighting scheme in \mathbf{v} sums up to 1 as, by construction, weights are normalised so that $\sum_{j=1}^p v_{sj} = 1$. In the example, weight sets are quite heterogeneous – the first set of weights is quite balanced (i.e. all deprivation dimensions are given a weight close to 0.3) while the last set is the most imbalanced one as it assigns a 0.61 importance to dimension X and only 0.10 to dimension Y.

For each set of weights (rows) in \mathbf{v} we can estimate a different deprivation score for each individual and report them in the matrix \mathbf{h} :

$$\mathbf{h} = \begin{bmatrix} 0.39 & 0.18 & 0.61 \\ 0.66 & 0.51 & 0.71 \\ 0.34 & 0.49 & 0.29 \\ 0.00 & 0.00 & 0.00 \end{bmatrix}$$

Each row contains three different deprivation scores by individual. Also in this case a certain heterogeneity can be noted: deprivation scores for individuals A, B, and C strongly depend on the set of weights used, while deprivation scores for individual D do not as they have no deprivations. From \mathbf{h} we can obtain a matrix of ranks \mathbf{r} :

$$\mathbf{r} = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

Each row contains three ranks achieved by each individual in the three repetitions. It can be observed that individual B is always the first, individual D is always the last, while the rankings of individuals A and C depend on the set of weights used. By dividing the number of times each individual occupies each ranking by the number of repetitions (3), we obtain the matrix of estimated probabilities \mathbf{b} :

$$\mathbf{b} = \begin{bmatrix} 0.00 & 0.67 & 0.33 & 0.00 \\ 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.33 & 0.67 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}$$

Each row of \mathbf{b} contains the estimated probability that each individual has to reach each ranking (from 1 to 4 by column). At this stage, to have a synthetic measure of poverty, the analyst can define until which rank l they want to consider the probabilities of being multidimensionally poor. For example, one can use $l = 1$ and have an individual probability for A=0, for B=1, for C=0, and for D=0. Using $l = 2$ one has the individual probability for A=0.67, for B=1, for C=0.33, and for D=0. Using $l = 3$ one has an individual probability for A=1 (0.67+0.33), for B=1, for C=1 (0.33+0.67), and for D=0. Obviously, the probabilities will be equal to 1 for all individuals if $l = 4$.

Table 1 – Identification strategy for an empirical multidimensional poverty assessment

DIMENSIONS	INDICATORS	VARIABLES	CUT-OFFS
DECENT WORK	Unemployment	Activity status (PX050)	2=Employee 3=Employed persons except employees 4=Other employed 5=Unemployed 6=Retired 7=Inactive 8=Other
	Low work intensity	Low work intensity (RX050)	0=No low work intensity 1=Low work intensity 2=Not applicable
INCOME	Income poverty	Monetary poverty (HX080)	0=when HX090 >= 60% of median HX090 1=when HX090 < 60% of median HX090
HUMAN DEVELOPMENT	Low educational attainment	Highest ISCED level attained (PE040)	0=Pre-primary education 1=Primary education 2=Lower secondary education 3=Upper secondary education 4=Post-secondary education 5=First stage of tertiary education (not leading directly to an advanced research qualification) 6=Second stage of tertiary education (leading to an advanced research qualification)
	Bad self-reported health	General health (PH010)	1=Very good 2=Good 3=Fair 4=Bad 5=Very Bad
	Chronic illness	Suffers from chronic illness or condition (PH020)	1=Yes 2=No
	Unmet medical needs	Unmet medical need for medical examination or treatment (PH040)	1=Yes, there was at least one occasion when the person really needed examination or treatment but did not 2=No, there was no occasion when the person really needed examination or treatment but did not
		+ Main reason for unmet medical need (PH050)	1=Could not afford to (too expensive) 2=Waiting list 3=Could not take time because of work, care for children or for others

		4=Too far to travel/no means of transportation
		5=Fear of doctor/hospital examination/treatment
		6=Wanted to wait and see if problem got better on its own
		7=Did not know any good doctor or specialist
		8=Other
Poor quality of dwelling	Leaking roof, damp walls/floor/foundation or rot in window frames/floor (HH040)	1=Yes 2=No
Inadequate sanitation facilities	Bath/shower in dwelling (HH080/HH081)	1=Yes, for sole use of the household 2=Yes, shared 3=No
	+ Indoor flushing toilet for sole use of the household (HH090/HH091)	1=Yes, for sole use of the household 2=Yes, shared 3=No
Noise	Noise from the neighbours or from the street (HS170)	1=Yes 2=No
Pollution	Pollution, grime or other environmental problems (HS180)	1=Yes 2=No
Crime	Crime, violence or vandalism in the area (HS190)	1=Yes 2=No

Source: Authors' elaborations

Note: Modalities indicating deprivation in each specific indicator are highlighted in bold.

Table 2 – Probabilities of being among the poorest 20% of the population by country and year

Year	Obs	mean	sd	p25	p50	p75	% population	Average number of deprivations
Austria								
2008	10846	0.157	0.32	0	0	0.011	40.4	7
2010	11389	0.164	0.32	0	0	0.071	39.7	7
2012	11376	0.153	0.32	0	0	0.009	40.3	6
2014	10651	0.135	0.30	0	0	0.007	37.3	6
Belgium								
2008	10073	0.191	0.35	0	0	0.083	46.3	6
2010	11331	0.204	0.36	0	0	0.261	44.3	6
2012	10534	0.207	0.36	0	0	0.242	46.7	7
2014	11236	0.186	0.35	0	0	0.184	44.0	7
Germany								
2008	22834	0.161	0.32	0	0	0.076	44.8	6
2010	22542	0.160	0.32	0	0	0.071	42.9	6
2012	22388	0.155	0.32	0	0	0.065	43.4	6
2014	21462	0.158	0.32	0	0	0.056	42.4	7
Greece								
2008	13486	0.251	0.38	0	0.006	0.544	56.7	7
2010	14178	0.266	0.38	0	0.006	0.568	57.0	6
2012	11277	0.310	0.40	0	0.009	0.764	63.7	6
2014	17768	0.262	0.38	0	0.005	0.532	58.7	6
Spain								
2008	27784	0.230	0.37	0	0.005	0.424	53.6	6
2010	28439	0.241	0.37	0	0.004	0.543	52.3	6
2012	26237	0.243	0.38	0	0	0.532	50.8	6
2014	26049	0.251	0.38	0	0.003	0.521	52.5	6
France								
2008	19493	0.162	0.32	0	0	0.078	46.7	7
2010	20412	0.158	0.32	0	0	0.074	45.2	7
2012	21908	0.155	0.32	0	0	0.065	44.6	6
2014	20659	0.146	0.31	0	0	0.055	41.3	7
Italy								
2008	42532	0.218	0.36	0	0.004	0.273	52.6	7
2010	38999	0.207	0.35	0	0	0.266	48.7	7
2012	37944	0.217	0.36	0	0	0.395	52.2	6
2014	38604	0.212	0.34	0	0.004	0.310	53.1	7
Luxembourg								
2008	7486	0.213	0.36	0	0	0.259	47.9	7
2010	9996	0.187	0.34	0	0	0.079	43.1	6
2012	12343	0.167	0.32	0	0	0.074	43.4	6
2014	7891	0.160	0.31	0	0	0.062	42.0	6
Portugal								
2008	8505	0.236	0.37	0	0.006	0.413	58.9	7
2010	9757	0.249	0.37	0	0.007	0.543	58.3	7
2012	11544	0.256	0.38	0	0.007	0.490	62.1	6
2014	14579	0.269	0.37	0	0.007	0.683	65.0	7
UK								
2008	13479	0.195	0.35	0	0	0.080	49.4	6
2010	11871	0.188	0.35	0	0	0.075	43.1	6
2012	16313	0.182	0.34	0	0	0.071	44.6	6
2014	14013	0.206	0.34	0	0	0.310	47.6	7

Source: Authors' elaborations on EU-SILC data (2008–2014)

Table 3 – GL dominance of the probability of being among the poorest 20%

2008	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT		Lower	Lower	Lower	Lower	Crossing	Lower	Lower	Lower	Lower
BE	Higher		Higher	Crossing	Crossing	Higher	Crossing	Crossing	Lower	Crossing
DE	Higher	Lower		Crossing	Crossing	Crossing	Lower	Crossing	Lower	Crossing
EL	Higher	Crossing	Crossing		Higher	Higher	Crossing	Higher	Crossing	Higher
ES	Higher	Crossing	Crossing	Lower		Higher	Crossing	Crossing	Crossing	Higher
FR	Crossing	Lower	Crossing	Lower	Lower		Lower	Lower	Lower	Crossing
IT	Higher	Crossing	Higher	Crossing	Crossing	Higher		Crossing	Lower	Higher
LU	Higher	Crossing	Crossing	Lower	Crossing	Higher	Crossing		Lower	Higher
PT	Higher	Higher	Higher	Crossing	Crossing	Higher	Higher	Higher		Higher
UK	Higher	Crossing	Crossing	Lower	Lower	Crossing	Lower	Lower	Lower	
2010	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT		Lower	Crossing	Lower	Lower	Crossing	Lower	Lower	Lower	Lower
BE	Higher		Higher	Crossing	Crossing	Higher	Crossing	Higher	Lower	Higher
DE	Crossing	Lower		Crossing	Lower	Higher	Crossing	Crossing	Lower	Crossing
EL	Higher	Crossing	Crossing		Crossing	Higher	Higher	Higher	Crossing	Higher
ES	Higher	Crossing	Higher	Crossing		Higher	Higher	Higher	Lower	Higher
FR	Crossing	Lower	Lower	Lower	Lower		Lower	Lower	Lower	Lower
IT	Higher	Crossing	Crossing	Lower	Lower	Higher		Higher	Lower	Higher
LU	Higher	Lower	Crossing	Lower	Lower	Higher	Lower		Lower	Crossing
PT	Higher	Higher	Higher	Crossing	Higher	Higher	Higher	Higher		Higher
UK	Higher	Lower	Crossing	Lower	Lower	Higher	Lower	Crossing	Lower	
2012	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT		Lower	Crossing	Lower	Lower	Lower	Lower	Lower	Lower	Lower
BE	Higher		Higher	Lower	Lower	Higher	Crossing	Higher	Lower	Higher
DE	Crossing	Lower		Lower	Lower	Crossing	Lower	Crossing	Lower	Crossing
EL	Higher	Higher	Higher		Higher	Higher	Higher	Higher	Higher	Higher
ES	Higher	Higher	Higher	Lower		Higher	Higher	Higher	Lower	Higher
FR	Higher	Lower	Crossing	Lower	Lower		Lower	Crossing	Lower	Lower
IT	Higher	Crossing	Higher	Lower	Lower	Higher		Higher	Lower	Higher
LU	Higher	Lower	Crossing	Lower	Lower	Crossing	Lower		Lower	Lower
PT	Higher	Higher	Higher	Lower	Higher	Higher	Higher	Higher		Higher
UK	Higher	Lower	Crossing	Lower	Lower	Higher	Lower	Higher	Lower	
2014	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT		Lower	Lower	Lower	Lower	Lower	Lower	Crossing	Lower	Crossing
BE	Higher		Higher	Crossing	Lower	Higher	Crossing	Higher	Crossing	Crossing
DE	Higher	Lower		Lower	Lower	Higher	Crossing	Crossing	Crossing	Crossing
EL	Higher	Crossing	Higher		Crossing	Higher	Higher	Higher	Crossing	Higher
ES	Higher	Higher	Higher	Crossing		Higher	Higher	Higher	Crossing	Higher
FR	Higher	Lower	Lower	Lower	Lower		Crossing	Crossing	Lower	Crossing
IT	Higher	Crossing	Crossing	Lower	Lower	Crossing		Higher	Lower	Higher
LU	Crossing	Lower	Crossing	Lower	Lower	Crossing	Lower		Lower	Lower
PT	Higher	Crossing	Crossing	Crossing	Crossing	Higher	Higher	Higher		Higher
UK	Crossing	Crossing	Crossing	Lower	Lower	Crossing	Lower	Higher	Lower	

Source: Authors' elaborations on EU-SILC data (2008–2014)

Note: The tables show pairwise comparisons of the GL dominance and should be read by row: individuals in countries by row can either have a “Higher”, “Crossing”, or “Lower” average probability of being below the fractional rank $\gamma = 0.2$ than individuals from each country by column

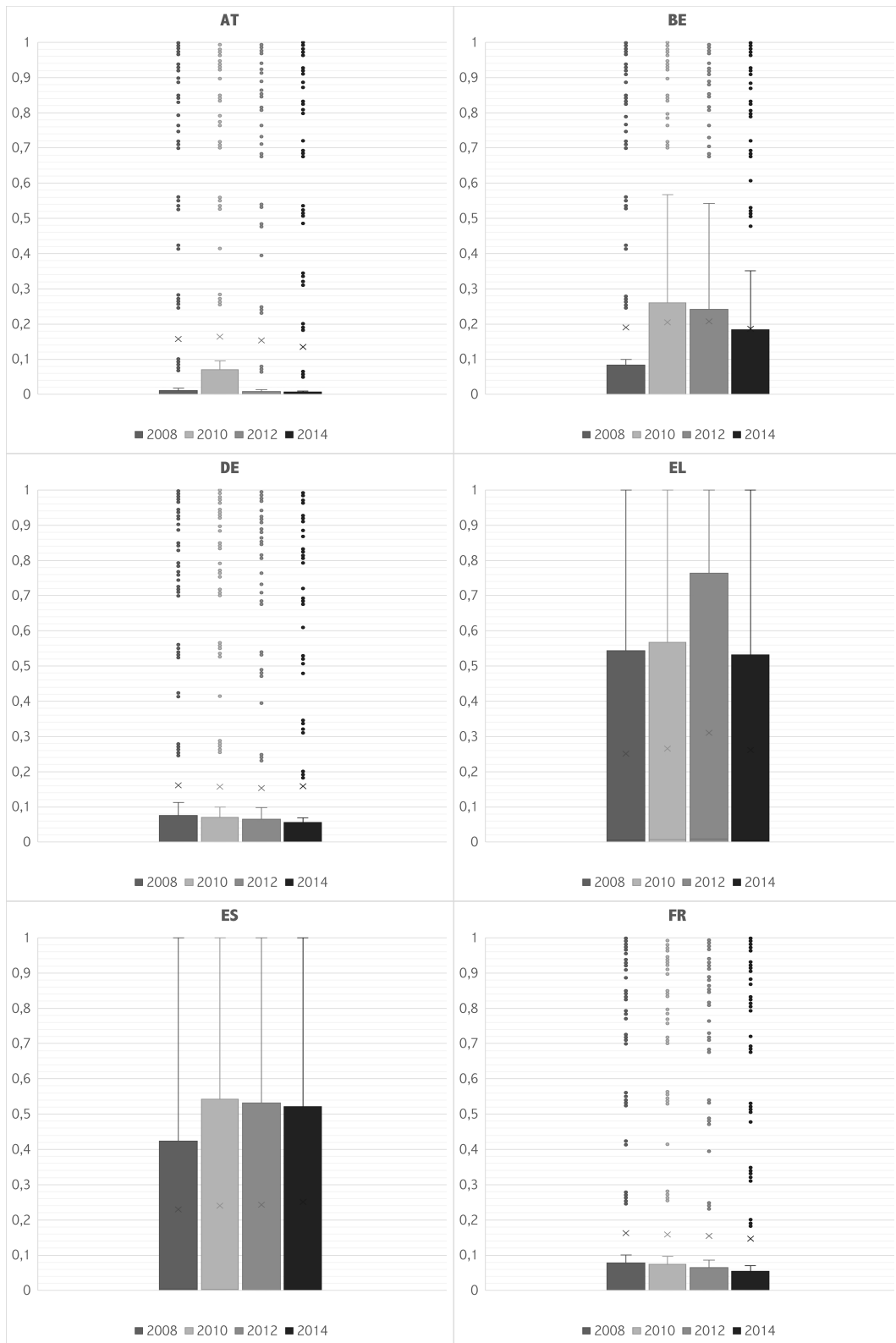
Table 4 – The lower bound of inequality aversion

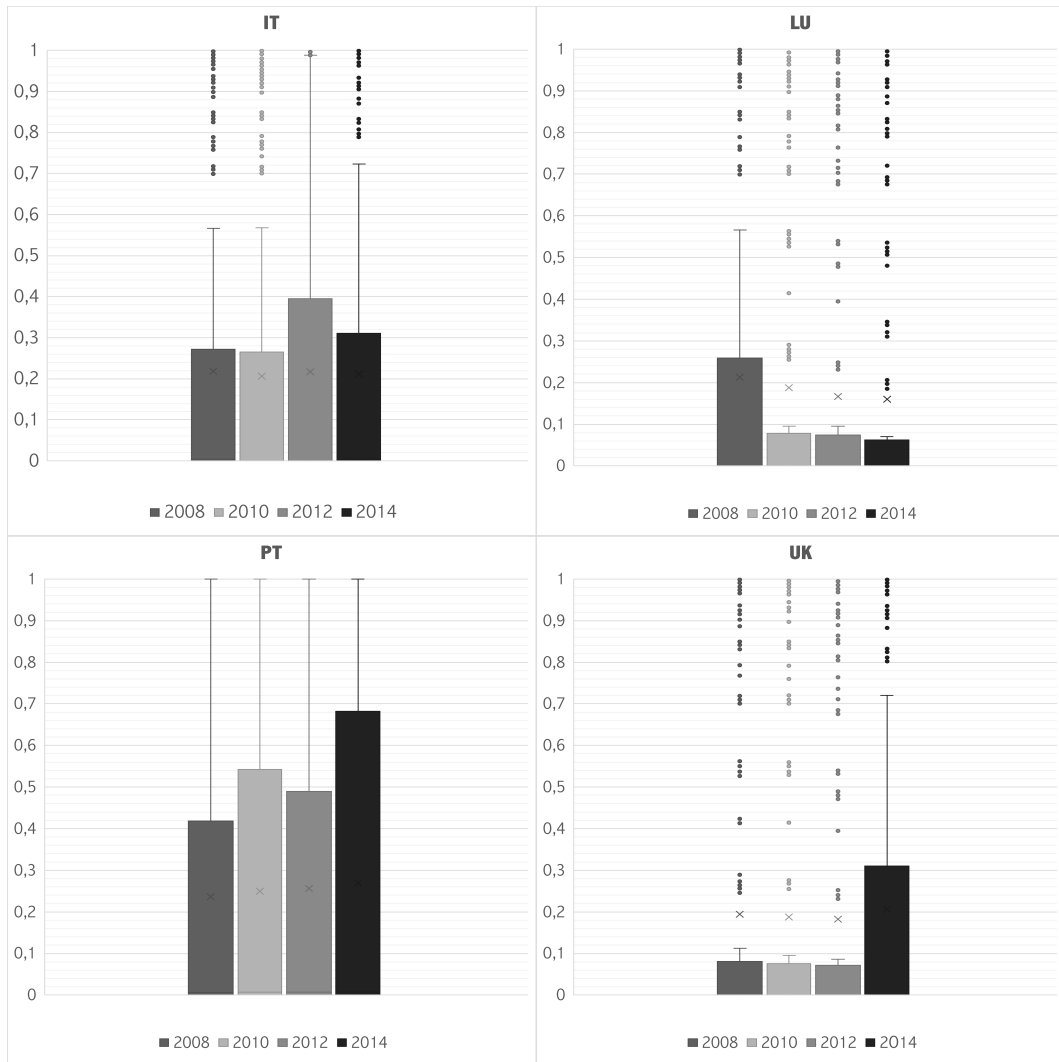
2014	AT	BE	DE	EL	ES	FR	IT	LU	PT	UK
AT										
BE										
DE										
EL					1,026					
ES										
FR										
IT		0,830	0,793			0,779				
LU	0,709		0,733			0,721				
PT		0,919	0,873	1,066	1,041					
UK	0,758	0,822	0,786			0,772				

Source: Authors' elaborations on EU-SILC data (2008–2014)

Note: The table shows pairwise comparisons of Low calculated as per equation (19) and should be read by row: when present, values in cells measure Low of countries by row compared with that of each country by column.

Figure 1 – Distributions of the probability of being among the poorest 20% of the population by country and year (box plots)





Source: Authors' elaborations on EU-SILC data (2008–2014)

Note: The boxplots show the distribution (median, 25th Percentile, 75th Percentile, and the interquartile range - IQR) of individual probabilities of being among the poorest 20% of the population by country and year. The points indicate outliers, i.e. values that fall out of the IQR.