New Multimedia Technologies as Tools for a Modern Approach to Scientific Communication and Teaching of Mathematical Sciences

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New Multimedia Technologies as Tools for a Modern Approach to Scientific Communication and Teaching of Mathematical Sciences

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Abstract This paper describes various technological tools and e-learning resources used by the authors in their work as university teachers and communicators of mathematics to a general public. As pointed out by recent developments in mathematics education research, a technology enhanced teaching may have significant potentialities at tertiary level: e-learning is an essential support to overcome some logistic obstacles such as the large number of students per teacher, the small number of hours of lesson available, the heterogeneity of the freshmen's mathematical background. Moreover, a fine design of digital environments can foster an engaging, inclusive, flexible and meaningful learning of mathematical topics. In this paper we give a perspective on some educational experiments carried out with engineering students of Università Politecnica delle Marche through specific functionalities of the Moodle platform (quiz, forum, workshop) and the dynamic geometry software Geogebra. We also report shortly on our approach to the use of new technological devices in the dissemination of scientific knowledge to a broader audience.

1 Introduction

In the last few years, research in mathematics education ([2], [4], [5]) highlighted the potentialities of e-learning environments and digital resources in supporting the teaching/learning processes at the university level: for example, they are an essential tool to face the large number of students per teacher and the small number of hours per course, to reduce the heterogeneity of freshmen background and to overcome the difficulty due to the impossibility of a close relationship between lecturer and learner.

The opportunities offered by e-learning are crucial both for students and teachers: the former have the possibility to visualize some abstract mathematical concepts and

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explore them from different representations, to check their own preparation without fear of judgment, to access different kinds of resources whenever and wherever they want. The latter can reduce their work by a suitable use of digital resources: through these tools they can keep trace of the students work, progresses and mistakes, use platforms to share clarifications to some frequent doubts rather than repeat many times the same explanations, gather some information about the cognitive styles and aptitudes of their students, which is impossible at university level without the use of technology.

In our teaching experience with engineering students, we often observe the difficulty they encounter in grasping some mathematical ideas beyond a formal and mnemonic level. In order to foster a meaningful, engaging and aware learning, we design digital resources and online environments as a teaching support. For example, through the geometry package Geogebra (which is an application for learning and teaching mathematics from primary school to university level), we implement applets with various functions: some of them allow the students to explore and visualize the mathematical concepts through animations, some explain ideas or connections, some others are devoted to self-assessment. They are collected and avaliable on the web site “Matematica per immagini e animazioni” (http://math-diism.univpm.it/progetto/).

Moreover, we provide educational activities for engineering students of Università Politecnica delle Marche by using the software Geogebra and some specific Moodle features. Some of them allow assessement and evaluation (quizzes), other ones foster the sharing of comments (forum and chat), other ones support role-play (workshop). Finally, we refer about our activities of communication of mathematics to a broad audience and education to teachers.

2 Chats

As a part of the e-learning Teaching Enhanced activity addressed to first-years students in Mechanical Engeneering, we used chats in order to increase students’ skills to deal with complex numbers. Indeed, though complex numbers are explained during the preparatory course and are a prerequisite for the Geometry course, they are often a trouble for freshmen. The chat tool is well-suited to overcome this difficulty: the supervisor posts a list of exercises on the topic she/he lectured on and the members of the audience post their answers, which are corrected by the tutor with comments and remarks and are also answered by other students creating a debate on the different strategies of approach to the solution.

The advantages of this instrument consist mainly in its flexibility and its usefulness to engross students’ attention. As for the first, the instructor can follow the learning path of the public and adapt the following steps to its reactions; in addition, the members of the learning community can work on the assignments according to the timetable they prefer. The solicitation of the chat tool is due to the exchange among students and between students and teachers: when students post their solutions, they see the comments of both the instructor and their peers, so that they have a feedback on the importance and quality of their effort.

3 Role-play and Workshop

Another appealing activity is role-play, in which the student acts as a teacher and is asked to evaluate the work of other students. This activity seems to induce students in facing learning of mathematical topics in a critical and meaningful way [1].

The setting of our experimental e-learning activity were second-year classes on Algebra and Logic of Computer and Automation Engeneering. We used the Workshop tool provided by the Moodle platform. The activity was on a voluntary basis, involving students willing to have a more interactive and engaging use of the e-learning platform.

Our experiment consisted of two parts. In the first part, the teacher proposed a list of exercises on the topics of her/his class and all the students were asked to solve the exercises and submit their own solutions. In the second part, every student was asked to assess the works of three other students, assigned automatically by the system. In order to perform the assessment task, the students used a structured assessment form provided by the teacher. The assessment form was very detailed and, for every item of the list of exercises, included a mark range and some questions about the correct solution. At the end, the students received two marks, one for their own work and another for their peer assessments of other students' work.

The students feedback on this activity was very positive: in particular, they appreciated the possibility to compare different works and often discovered that they could use more than one strategy to solve a problem. Moreover, when students act as teachers in the assessment procedure, they are induced to pay attention to the formal language and to the presentation of the solutions. Indeed, one of the main difficulties for students is how to formally express and correctly write the arguments they use in the solutions of the exercises. At the end of the activity, the files produced by the students were revised by the teacher, and some particularly well-written solution was shown to all the students in the class. Also at this final step, more mistakes and doubts of the students were clarified by the teacher. This activity is also well-suited to overcome the problems of large classes.

4 Interactive quiz: an effective tool for self-correction

One of the main difficulties in Mathematics learning consists in the correctness check of the final result and the execution of a problem or an exercise. In order to overcome this problem one can arrange interactive quiz containing not only the final result, but also the correct execution and the possibility of driving the student step by step. Moodle platform allow the use of many possible types of questions that can be used for an interactive quiz, both with open reply and with closed reply, such as multiple choice (with one or more correct answers), true/false, matching, etc. The quiz can be arranged both with a general feedback explaining the correct execution, and with a specific feedback regarding the wrong answers. For instance, in the multiple reply type questions, the teacher can put, among the wrong alternatives, the answers that usually are obtained owing to typical executions mistakes. In this way, if the student chooses such wrong answers, he receives a specific feedback explaining him what kind of mistake he made, together with an advice about the correct way.

The student can be also allowed to do more attempts, before to discover the correct execution. Each attempt, if wrong, is by indications on to the type of mistake and increasingly clear suggestions about the right way. Moreover, if a type of exercise results to be particularly hard for a student, he can repeat the same kind of question many times with different data, till to reach a good level of understanding.

Finally, the use of interactive quiz is an efficient tool for the teacher too, in order to have a clear real-time feedback on the progress of the class of students about any specific subject, since the platform furnishes a detailed report on any single question of the quiz. So, the teacher can decide to come back to a particular topic if the report of the quiz indicates a high rate of wrong answers.

5 Online resources through Geogebra to foster students' awareness in learning

Here we describe some of the educational paths which we have designed and organized in "routes of online activities"(RA) ([3]). These activities are implemented by using the software Geogebra and submitted to students through the university Moodle platform. The mathematical topic addressed by the RA is the theory of multiple integrals. In particular, we have concentrated on two specific points, which arise in the resolution of a double integral and which are frequent sources of difficulties: *(i)* the conversion between graphical and analytical representations of a subset of the plane; *(ii)* the description of a planar region as a normal domain, together with the choice of the most convenient description for the calculation of a specific integral. The RA include exploratory activities, some tasks requiring short open-ended (numerical or symbolic) answers, and one final open-ended problem. In all tasks, a planar region is given graphically and its analytical representation or its description as normal domain in Cartesian or polar coordinates is asked for. The final problem provides the analytical representation of a planar region and requires the calculation of its area and the justification of the adopted method. The RA are engineered so that the tasks have an increasing level of difficulty, from the first activities up to the final problem; they require not only a deep understanding of the involved techniques of representation, but also the metacognitive ability of evaluating different ways to obtain the solution. All the tasks envisage the possibility of asking for some hints. For each answer, request for a hint, or action by the student, the program returns a feedback, which is strongly dependent on what the student has done. When a task is correctly performed, the access to the next one becomes available.

As an example, we present a task concerning the conversion between different representations of a set. The region to be described is a half-disk (see Figure 1). In the first part of the task, the student is required to choose the correct analytic descriptions of the given subset among four options (with the possibility of asking for hints, which would provide the equations of the curves which delimit the region). In the second part, the student should describe the same subset in polar coordinates (ρ,θ) by inserting the minimum and maximum values for the parameters ρ and θ in appropriate input fields. A brief summary of the transformation between polar and Cartesian coordinates is avaliable. An immediate feedback is given both for the first and for the second part, and it concerns not only the correctness of the answer, but also the processing of the task ([8]). For example, if the answer to the second part is wrong, two sliders for ρ and θ appear on the graph: as the student moves the sliders between the minimum and the maximum values previously inserted, the program draws the corresponding set in the plane, thus showing the student's mistake.

In order to evaluate the impact of our educational resource, we collected different kinds of data: video-recording of the students' interaction with the RA; questionnaires and interviews about their perception and ways of thinking; qualitative and quantitative analysis of the final written exams. These data show that students feel very positively about the digital approach for several reasons: it is intuitive and quick, it avoids the fear of judgment, and it offers the advantage of a direct interaction with a tutor (not human in this case). This is, in our opinion, the most important role of technology in this resource: each student is allowed to have an immediate and specific feedback on her/his own actions, learning process and difficulties. In addition, the available hints should become a graphical and dynamical scaffolding for the student, serving as a step-by-step guide towards the solution; obviously, this is not possible at the tertiary level of education through a traditional approach.

Some criticality emerged from the cognitive point of view, such as the students' difficulty in facing unstructured tasks, in visualizing and in choosing autonomously the optimal strategy towards the solution of a problem. Neverthless, our data show that the tool has positive effects in this respect; this results from the evolution of the behaviour of the single student in facing the RA (diminished number of mistakes and increased awareness of the difficulties) and from the comparison between the final written exams of students who faced the RA (the experimental group, EG) and the other students (the control group, CG). In the problem concerning the double integral, the EG obtained an average score of 5,25/8 points, while the CG obtained an average of 4,31/8 points. A qualitative analysis of the texts shows that only students in the EG realize that for some choices of the solution strategy there are too many difficult calculations to perform and it would be advisable to change method. This suggests that the interaction with the RA stimulates the students' autonomy in choosing the most convenient solution strategy.

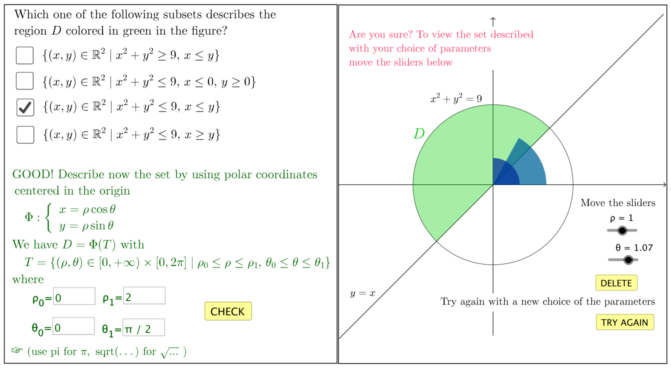

Fig. 1 A screenshot from a task of the RA

6 Dissemination of Mathematics to a Large Audience and Permanent Education for Teachers

We carried out a broad activity of mathematics communication addressed to a large audience by using several technological resources. At national and international conferences, in secondary schools, and at public meetings, we reported on contaminations between mathematics and literature (with specific focus on Leopardi's and Camus' works as well as on the classical Greek tragedy), deepened the philosophical issues concerning the nature and the development of mathematics, and described how we created a versitile approach to analyze an artistic work (such as a poem or a painting) by means of mathematical techniques in order to obtain a set of parameters that were subsequently used by a musician to compose a piece inspired by the artwork.

In particular, Chiara de Fabritiis together with Maestro Davide Amodio (Conservatorio Benedetto Marcello, Venezia) made a large use of graphic programs like Inkscape to deal with the different forms (coloured regions, brush strokes, curves, dots) that appear in "Summertime n. 9", a painting by Jackson Pollock owned by the Tate Gallery of London. After creating a reference frame on the canvas, we used a technique due to Filchakov to compute the most relevant coefficients of the transformations (under the form of power series) that map the first region of each colour (red, yellow, and blue) to the following ones (see Figure 3), while we used a frequency principle to deal with curves and dots that were "translated" in percussions [5]. Using Geogebra, a 3D plot and a spreadsheet, we applied an analogous approach to study Giacomo Leopardi's poem "L'infinito". In this case, we followed two road paths: the first follows the position of syllables in the planar scheme that represents the lyric, creating a family of graphs that suggested themes and compositional patterns for the orchestration; the second takes into account the structure of the verse with its lines, indentations and enjambements which point a 3D interpretation through a cylindrical helix [6]. These researches where presented both at workshops on the spreading of mathematics, such as “Matematica e Cultura”(Istituto Veneto di Scienze, Lettere ed Arti) and Giornata per la Ricerca Artistica e Musicale (Fondazione Teatro alla Scala), and in public lectures and performances (e.g. Istituto Italiano di Cultura in London, Guggenheim Museum in Venice, “Your Future Festival”, “Aperitivi Matematici”). The research on “L’infinito”, presented at Istituto Veneto di Scienze, Lettere ed Arti, was filmed, and is now available on YouTube (https://www.youtube.com/watch?v=cbuIJecLBO8).

Finally, on several occasions, we lectured to high schools students or professionals (engineers, architects) on the many links between mathematics, architecture and nature, with a particular focus on geometry (spirals, curves and surfaces, self-similarity, planar and spatial tesselations, regular and uniform polyhedra), making a large use of applications such as Geogebra for creating drawings and animations which improved the comprehension of the audience.

In addition, we were engaged in teachers’ education through workshops and series of lessons involving the use of softwares such as Geogebra, computational knowledge engines such as Wolfram Alpha, and learning management systems such as Moodle. The demonstration of Geogebra to first level secondary school (scuola media) teachers focused mainly on the use of sliders and the construction of polygonal figures in the plane and solid bodies in three-dimensional space, with special attention to projections and nets of polyhedrons. Secondary school (liceo) professors received a more detailed training: Geogebra was used to deal with slider and animations, graphs of real functions of one real variable, planar and spatial curves, surfaces in three dimensional space, computations with Computer Algebra System; Wolfram Alpha was employed to work with rational and algebraic expressions, polynomial equations in one variable, systems of linear equations in several variables, computation on vectors and matrices such as products, inverse, determinants. Through the test tool, Moodle gave the teachers the possibility to create an assignment for students in order to verify the comprehension of a subject explained during classhours; it was also used to get next year freshmen accustomed with the kind of quizzes they would have to pass in order to enter the university.

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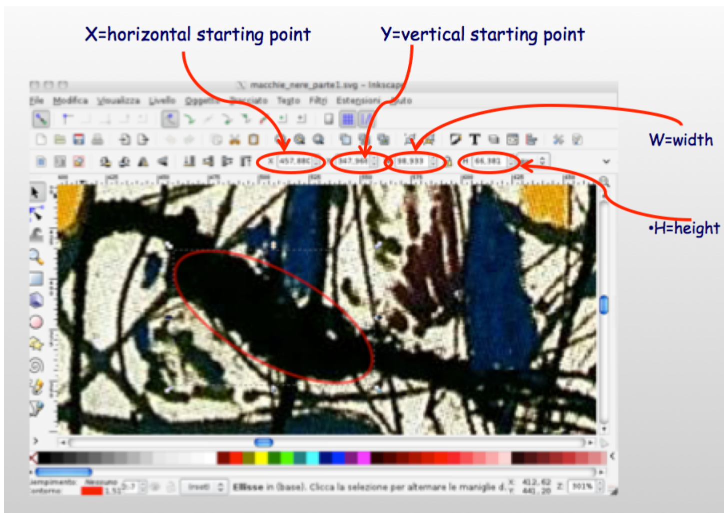

Fig. 2 Parameters of the ellipse obtained with Inkscape

Fig. 3 Deformations of yellow regions in Summertime n.9 by Jackson Pollock