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Resolution of financial crises

This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

Published Version:
Fanelli S., Gonzalez-Eiras M. (2021). Resolution of financial crises. JOURNAL OF ECONOMIC DYNAMICS & CONTROL, 133, 1-28 [10.1016/j.jedc.2021.104252].

Availability:
This version is available at: https://hdl.handle.net/11585/875599 since: 2022-03-01

Published:
DOI: http://doi.org/10.1016/j.jedc.2021.104252

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(Article begins on next page)

12 October 2024

This is the final peer-reviewed accepted manuscript of:

Fanelli, S., & Gonzalez-Eiras, M. (2021). Resolution of financial crises. *Journal of Economic Dynamics and Control*, *133*, 104252.

The final published version is available online at:

https://doi.org/10.1016/j.jedc.2021.104252

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Resolution of Financial Crises*

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Abstract

A financial crisis creates substantial wealth losses. How these losses are allocated determines the magnitude of the crisis and the path to recovery. We study how institutions and technological factors that shape default and debt restructuring decisions affect the amplification of aggregate shocks. For sufficiently large shocks, agents renegotiate. This limits the losses borne by borrowers, shutting the amplification mechanism via asset prices. The range of shocks that trigger renegotiation is decreasing in repossession costs and increasing in default costs, if the latter are public information. Private information may induce equilibrium default but, by allowing agents with high default costs to extract a larger haircut, facilitates the recovery. The model is consistent with evidence from real estate markets in the U.S. during the Great Recession; and rationalizes recent changes in U.S. Bankruptcy Code in the wake of the COVID-19 crisis.

JEL codes: E32, E44, G01.

Keywords: Financial crises; balance sheet recessions; default; renegotiation

*We thank Toni Beutler, Vivek Bhattarcharya, Enrique Kawamura, Pablo Kurlat, Dirk Niepelt, Viktoriya Semeshenko, Alp Simsek, and seminar participants at Argentine Economic Association (AAEP), Bank of Spain, Copenhagen Business School, CREI, ESEM, LACEA, MIT, Study Center Gerzensee, Universidad Adolfo Ibáñez, Universidad de San Andrés, University of Bern, and University of Copenhagen for helpful comments. We especially thank Daniel Heymann who motivated, and was initially involved in, this project. Sebastián Fanelli gratefully acknowledges funding from Spain's Ministry of Science and Innovation (grant FJC2018-038650-I).

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1 Introduction

Financial crises often are preceded by asset price booms and increased borrowing, typically against appreciating assets. Once the boom reverses and asset prices collapse, large financial losses are realized. If these losses are concentrated in the productive sector of the economy, a deep and protracted decline in economic activity follows − a balance-sheet recession (Kiyotaki and Moore, 1997; Koo, 2003).

Such a balance-sheet recession would not occur if the productive sector could write state-contigent contracts that insulates it from volatility in asset prices (Krishnamurthy, 2003; Di Tella, 2017), i.e. if they have insurance against aggregate shocks. While in reality these contracts may be unavailable, agents can still obtain partial insurance by defaulting and renegotiating debts ex post. Indeed, recent empirical studies have shown that bankrupcy law and the ease of renegotiating outstanding debt have been key determinants of the recovery in the United States during the Great Recession (Agarwal et al., 2017; Mian et al., 2015).

In this paper, we study the resolution of financial crises in an environment where agents write non-contingent contracts that are subject to default and renegotiation ex post. Our analysis emphasizes how institutions and technological factors determine the magnitude of default costs and the distribution of financial losses among agents. Among others, these factors include bargaining power, the cost of default, and the informational frictions between creditors and debtors.

To illustrate the new implications of these partial insurance mechanisms, we depart as little as possible from what is arguably a canonical model of balance sheet recessions: the Kiyotaki and Moore (1997) model − henceforth KM. The model features two kinds of agents: entrepreneurs, who are the most productive, and financiers, who are the least productive.[1] Entrepreneurs may borrow from financiers, but face a collateral constraint: They can only borrow up to the value of capital next period. In the original steady-state entrepreneurs are fully levered (i.e. the collateral constraint is binding).

We study the response of the economy against an unforeseen shock, which could be either a technology shock to entrepreneurs (as in KM) or a preference shock to financiers, which can be interpreted as a financial shock. As in KM, these shocks depress asset prices, leaving entrepreneurs underwater. Unlike KM, who assume that entrepreneurs honor their debts ex post, we allow for default and bargaining between creditors and debtors. More precisely, we assume that entrepreneurs can default, keeping their output but paying a cost to do so. Financiers can repossess the collateralized asset at a cost. To avoid these losses, entrepreneurs and financiers may bargain, with financiers offering a haircut on the

[1] In KM, entrepreneurs are "farmers" while financiers are "gatherers".

entrepreneurs' outstanding debt.

We show four main results. First, the threat of default and the possibility of renegotiation only matter if shocks are large enough. When shocks are small, the threat of default is not credible. Entrepreneurs honor existing debts, which depresses the demand for capital and leads to a collapse in asset prices. The response of the economy is, thus, the same as in the original KM model. By contrast, when shocks are large, the threat of default is credible, which triggers a renegotiation. In this case, entrepreneurs manage to extract haircuts from financiers, cutting their financial losses. This cushions the reduction in capital demand and dampens the decrease in asset prices.

Second, institutional and technological factors that shape the debt-restructuring process determine the extent of amplification of macroeconomic shocks. More precisely, we show that there is more amplification when financiers' repossession costs are low, default costs are high, or entrepreneurs have little bargaining power. Furthermore, larger shocks are required to trigger renegotiation. In addition, when agents renegotiate, bargained haircuts are smaller, leading to a larger drop in entrepreneurs' capital holdings and asset prices, slowing down the recovery.

Third, renegotiation is socially desirable. Not only does it avoid potentially deadweight losses of default, but it also accelerates the recovery from the financial crisis by increasing entrepreneurs' net worth. This suggests that policies that make renegotiation easier, such as the Home Affordable Modification Program (Agarwal et al., 2017), are particularly valuable in this environment.

Fourth, asymmetric information may exacerbate the crisis by inducing equilibrium default, but it also accelerates the recovery by shielding entrepreneurs' net worth. We make this point in an extension of our baseline model where entrepreneurs' default costs is private information, i.e. financiers only know the distribution of default costs in the population. When shocks are large they propose a haircut that is accepted only by entrepreneurs with relatively high default costs. The remaining entrepreneurs default. To the extent that default is a deadweight loss, this exacerbates the crisis. However, since financiers must offer every entrepreneur the same haircut, entrepreneurs with high default costs get a better deal than they would in an economy with perfect information. As a result, entrepreneurs' capital holdings fall less under asymmetric information, accelerating the recovery after the shock.

We also characterize how the distribution of unobservable default costs affect macroeconomic outcomes. If the set of defaulting agents is fixed, and default costs go up, then entrepreneurs extract a smaller haircut and the recovery slows down. On the other hand, financiers may also find it optimal to affect the extensive margin, i.e. who defaults. Perhaps surprisingly, we construct examples where higher *potential* default costs may translate into

lower *effective* default costs and more sizeable haircuts.

An important assumption of the model is that the ex-post resolution of debt crises does not affect the ex-ante behavior of agents. We believe this is a reasonable approximation of behavior in credit markets for rare events such as financial crises. For example, in the credit boom before the Great Recession, lenders paid little attention to borrowers' repayment capacity. Mian et al. (2015) show that in the late 1990s and early 2000s lenders did not differentiate lending based on states' foreclosure requirements. In commercial real estate markets debt was often issued with minimum covenants, and commercial real estate had low risk premia relative to other assets. These facts point to lenders assigning a very low probability to states of the world in which foreclosure requirements and covenants would be important. Furthermore, the paucity of renegotiations suggests the presence of widespread information asymmetries between borrowers and lenders (Adelino et al., 2013).

Our work contributes to the theoretical literature on the balance sheet channel, going back to the seminal work of Bernanke et al. (1999), Carlstrom and Fuerst (1997), and Kiyotaki and Moore (1997) and, more recently, the work of Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013), among others. This strand of work stresses how the concentration of aggregate risk in one sector of the economy leads to significant amplification of shocks via their effect on balance sheets. A critique of this channel is that it would disappear if agents were allowed to write contracts contingent on the aggregate state of the economy (Krishnamurthy, 2003 and Di Tella, 2017). This motivated papers to explain why insurance contracts may not be available (e.g. Cooley et al., 2004, Krishnamurthy, 2003) or why agents may optimally decide to become exposed to aggregate risk (e.g. Asriyan, 2020, Di Tella, 2017). By contrast, our paper does not seek to explain balance sheets from an ex-ante perspective. Rather, we ask how the possibility of default and bargaining ex post affect the depth and posterior recovery of a financial crisis. We derive new results characterizing the evolution of the macroeconomy ex post as a function of the size of the shock, the institutional and technological background, and the observability of default costs.

Our model is also related to the literature on the limited enforceability of debt contracts, allowing for strategic default. Cooley et al. (2004) assume lending can take the form of long term state-contingent debt contracts, borrowers can divert capital, and default is costly. They solve for the optimal dynamic contract that is self-enforceable and find that the equilibrium features amplification. Jermann and Quadrini (2012) also allow borrowers to default and derive borrowing constraints by assuming that lenders can recover the collateral with an exogenous probability (otherwise, recovery is zero). They interpret this time-varying probability as "financial shocks" and find that they can explain a large share of observed dynamics of real and financial variables. These two papers abstract from the effect of (endogenous)

asset prices on borrowing constraints, while in our model, as in KM, it is precisely this variable that drives results. Furthermore, we also allow for financial shocks as we consider a temporary increase in the discount factor of lenders (and thus in the equilibrium interest rate).[2]

The paper is organized as follows. Section 2 presents the basic framework, which introduces default costs and renegotiation into KM's model. Section 3 develops an extension with asymmetric information about default costs. Section 4 discusses how our model can be used to interpret existing empirical findings in the context of real estate markets in the United States during the Great Recession, as well as to rationalize recent changes introduced into the U.S. Bankruptcy Code as a response to the COVID-19 crisis. Section 5 concludes. Appendix A contains all proofs and detailed derivations. Appendix B describes the parametrization and calibration used to create the figures.

2 Baseline Model

We are interested in studying how the possibility of renegotiation shapes the aftermath of a financial crisis. To highlight the novel features of our analysis, we build on the work of KM, a seminal model of financial crises. To ensure clarity, we purposefully deviate from this framework as little as possible.

2.1 Setup

There are two sets of agents: entrepreneurs and financiers, each with measure 1. We use plain notation for entrepreneurs and $'$ for financiers. Both are risk neutral and maximize their utility, given respectively by

$$\sum_{t=0}^{\infty} \beta^t x_t \qquad \text{and} \qquad x_0 + (1-\epsilon) \sum_{t=1}^{\infty} \beta'^t x'_t,$$

where x_t and x'_t denote their respective consumptions, $0 < \beta < \beta' < 1$ are their respective discount factors and $\epsilon \in [0, 1 - \beta/\beta')$ is a discount factor "shock" in the first period.[3] These

[2]Other recent contributions of the effect of financial shocks are Christiano et al. (2010), Del Negro et al. (2017), and Liu et al. (2013).

[3]The discount factor shock is absent in the original KM formulation, which focused on a technology shock. We include it to capture, in reduced form, a shock to risk-aversion that induces a sharp drop in asset prices unrelated to the underlying productivity of the asset. This shock is not to be confused with the preference shocks used in the New Keynesian literature to model liquidity traps, which would have the opposite sign (e.g. Fernández-Villaverde et al., 2015). The latter is intended to capture the drop in the riskless rate experienced during the Great Recession, while ours is intended to capture the large drop in the

assumptions imply that entrepreneurs and financiers are, respectively, borrowers and lenders in equilibrium.

There is a fixed aggregate endowment of a productive asset, or capital, $\bar{K}$. Capital is the only factor of production and creates output with a one-period lag. Agents have access to different technologies. Entrepreneurs are endowed with a linear production technology,

$$y_{t+1} = (a_t + c)k_t,$$

where a_t is the "tradable" share of output, i.e. it can be used for market transactions, while c is the "nontradable" share, i.e. it can only be consumed by the entrepreneur. We will consider cases where entrepreneurs' productivity falls for one period,

$$a_0 = a(1 - \Delta) \qquad \text{and} \qquad a_t = a \quad \forall t \geq 1$$

with $a > 0$, $\Delta \in [0, \tilde{\Delta}]$ and $\tilde{\Delta} < 1$. By contrast, financiers are endowed with a standard production technology with decreasing returns:

$$y'_{t+1} = G(k'_t)$$

with $G' > 0$, $G'' < 0$, and $\beta' G'(0) > a > \beta' G'(\bar{K})$.

Agents also differ in their access to credit. Whereas financiers are unconstrained, entrepreneurs must satisfy a collateral constraint,

$$R_t b_t \leq q_{t+1} k_t, \tag{1}$$

where q_t is the price of capital, b_t is one-period debt contracted at t, and R_t is the gross interest rate. This constraint is widely used in the literature, and it is typically microfounded through the impossibility of the borrower to pre-commit to making use of the firm's assets (see, e.g. KM). In other words, it is a friction in the *interim* stage, i.e. after the financial contract is written but *before* agents commit their labor. This constraint determines how much debt the entrepreneur can take ex ante at t depending on what agents expect the future price of capital q_{t+1} to be. Therefore, it determines how fast entrepreneurs can accumulate capital. In section 2.6, we relax this constraint.

In this paper, we analyze how different institutional arrangements *ex post* affect the propagation of economic shocks. To do so, we enrich the original KM model to contemplate the possibility of default and renegotiation.[4] More precisely, we assume entrepreneurs always

price of risky assets (see Caballero and Farhi (2017) for a model where both phenomena are tightly linked).

[4]KM assume that, once the productivity shock is realized, the entrepreneur has to repay her debt in full.

have the option to renege on their debts ex post. However, if they do so, they lose D_t units of tradable output.[5] Entrepreneurs also have the possibility of renegotiating their debts with their creditors to avoid the default cost. We assume the surplus is split according to Nash bargaining and let φ denote the haircut on the outstanding value of debt.

Since agents have perfect foresight from $t = 0$ and onwards, these considerations will be of no consequence for the equilibrium at dates $t \geq 1$. However, in period 0, entrepreneurs have some legacy debt b_{-1} and capital k_{-1} and, depending on their levels and the state of the economy, renegotiation may be optimal. Henceforth, we assume that the level of legacy debt and capital are exactly their respective "steady state" levels where the economy would stabilize if $a_t = a \ \forall t$.[6] This is the outcome that would arise if agents in this economy were expecting $\Delta = \epsilon = 0$. By contrast, we analyze cases where $\Delta \neq 0$ or $\epsilon \neq 0$, i.e. we study the response to "one-time" unexpected shocks. Henceforth, we use $*$ to denote variables at the steady state. We also parametrize the default cost as a share of the steady-state level of debt to make the comparison across economies more transparent, i.e. we set $D_t = \alpha q_{t-1} K_t$.[7]

2.2 Solving the model

We solve the model backwards. We first solve for the equilibrium at dates $t \geq 1$. Then, we use these results to determine the bargained haircut. We complete this subsection with a system of two equations that characterize the equilibrium at date $t = 0$.

Continuation equilibrium $t \geq 1$ We start by characterizing financiers' demand for capital. Since they are unconstrained, they must be indifferent between lending and investing, i.e.

$$q_t = \frac{G'(\bar{K} - K_t) + q_{t+1}}{R_t} \quad \forall t, \tag{2}$$

where $K_t \equiv \int_0^1 k_t(i)di$ denotes the aggregate amount of capital in the hands of entrepreneurs. This must hold at all dates t. Furthermore, since financiers have linear utility

$$R_t^{-1} = R^{*-1} = \beta' \quad \forall t \geq 1.$$

[5] For simplicity, we assume that the nontradable output c realizes during the "interim" stage as a private benefit of investing and, thus, is unaffected by default. To ensure the equilibrium is well defined, we assume $\tilde{\Delta}$ satisfies $(1 - \tilde{\Delta})aK^* = D^* = \alpha q^* K^*$, where stars denote steady-state values. This ensures that, starting from a capital stock at the steady state or below, default is feasible.

[6] Convergence to this steady-state requires $c > (\beta^{-1} - 1)a$, which is "assumption 2" in the original KM paper (see KM for a proof). We assume this condition also holds in our environment.

[7] If the default cost were kept constant across economies, then economies with higher K^* would have lower default costs. Similarly, if it were proportional to output (instead of the value of capital), then economies with lower interest rates and hit by larger shocks would have lower default costs. The current modeling choice helps isolate the direct effect of default costs.

This completes the characterization of financiers' decisions.

Next, we solve for entrepreneurs' capital demand. Given our assumptions, entrepreneurs will borrow as much as they can and invest the proceeds in capital.[8] Since there is perfect foresight from $t = 0$ onwards, there will be no default or renegotiation and the borrowing constraint will bind at every date $t \geq 1$.[9] Letting hats denote proportional deviations from the steady state (e.g. $\hat{k} = \frac{k_t - K^*}{K^*}$) we obtain

$$1 + \hat{k}_t = \frac{a}{u(K_t)}(1 + \hat{k}_{t-1}) \quad \forall t \geq 1 \tag{3}$$

where $u(K_t) \equiv \beta' G'(\bar{K} - K_t)$, following KM's notation. Note that equation (3) already solves for equilibrium in the continuation dates $t \geq 1$, since financiers' demand is encoded in $u(K_t)$. The only remaining step is to aggregate entrepreneurs' decision, which is straightforward since (3) is linear in k_t and k_{t-1}. Iterating backwards, we may summarize the date $t \geq 1$ equilibrium via an increasing relationship $K_t = f_t(K_0)$.

Solving for the haircut At $t = 0$, the entrepreneur has two options: to renegotiate or to default. The amount of capital the entrepreneur can buy will be impacted by this decision,

$$\begin{aligned} 1 + \hat{k}_0^R &= \frac{a}{u(K_0)(1-\epsilon)} \left(1 - \Delta + \frac{R^*}{R^* - 1}(\hat{q}_0 + \varphi)\right) \\ 1 + \hat{k}_0^D &= \frac{a}{u(K_0)(1-\epsilon)} \left(1 - \Delta - \frac{R^*}{R^* - 1}\alpha\right), \end{aligned} \tag{4}$$

where $\hat{k}_0^R$ and $\hat{k}_0^D$ denote, respectively, the amount of capital that can be purchased in the case of renegotiation and default, respectively. Note that we used that $R_0^{-1} = (1-\epsilon)\beta' = (1-\epsilon)R^{*-1}$, since financiers are indifferent between consuming and lending.

Next, we compute the implied entrepreneurs' utilities of default and renegotiation given the shocks, their aggregate capital holdings K_0, and the proposed haircut φ,

$$U^i = cK^* + \beta c k_0^i + \beta^2 c k_1^i + ... + \lim_{t \to \infty} \beta^t c k_{t-1}^i$$

[8] The assumption $c > (1/\beta - 1)a$ guarantees that investing to the maximum dominates consumption around the steady state (see footnote 6). In the solution, $\{\hat{q}_t\}$ is an increasing sequence that converges to q^* (provided $\hat{K}_t < 0$, which is guaranteed by lemma 1), which increases the attractiveness of investing even further. To see this, note that (3) implies $\{\hat{K}_t\}$ is an increasing sequence, and $\hat{q}_t$ is monotonic in $\{\hat{K}_{t+s}\}$ for $t \geq 1$ and even lower at $t = 0$ if $\epsilon > 0$. Finally, lending is always dominated given $\beta \leq R_t^{-1}$ $\forall t$.

[9] Kozlowski et al. (2020) show that when economic agents do not know the true distribution of shocks, an extreme event leads to persistent changes in beliefs that feed into economic outcomes. We abstract from these considerations.

with $i = R, D$. Using our previous results, we obtain

$$U^R - U^D = \frac{acK^*}{u(K_0)(1-\epsilon)} \frac{\beta R^*}{R^* - 1} (\hat{q}_0 + \varphi + \alpha) \left(\sum_{t=0}^{\infty} \beta^t \left(\prod_{s=0}^{t} \frac{a}{u(f_s(K_0))} \right) \right). \tag{5}$$

By renegotiating, an entrepreneur saves on the default costs, $\alpha q^* K^*$ and, in exchange, accepts to keep a share of the (negative) capital gains, $(\hat{q}_0 + \varphi) q^* K^* \leq 0$.

Renegotiation gives an entrepreneur surplus $U^R - U^D$, while a financier gets surplus $(1-\varphi)q^*K^* - (1 + \hat{q}_0 - \mu)q^*K^* = -(\hat{q}_0 + \varphi - \mu)q^*K^*$, where $\mu q^* K^*$ is a repossession cost (e.g. the cost of foreclosing in the case of real estate). We assume these surpluses are split according to Nash bargaining. Letting θ denote financiers' bargaining power, the equilibrium haircut φ is given by[10]

$$\varphi = \max\{-\hat{q}_0 - \theta\alpha + (1-\theta)\mu, 0\}. \tag{6}$$

The equilibrium haircut depends on the effect that φ has on entrepreneurs' surplus. We assume that $\theta\alpha > (1-\theta)\mu$, i.e. entrepreneurs cannot extract a haircut from financiers if they do not suffer a negative shock.[11] Let $\bar{q} \equiv -\theta\alpha + (1-\theta)\mu$. When $\hat{q}_0 \geq \bar{q}$, the price of capital is sufficiently high that the threat of default is not credible even if $\varphi = 0$. Hence, entrepreneurs bear all the capital losses in this region and do not default. By contrast, when $\hat{q}_0 < \bar{q}$, entrepreneurs can bargain a positive haircut. Everything else equal, entrepreneurs extract a larger haircut when their default costs are lower (low α), their bargaining power is higher (low θ) and when financiers' repossession costs are higher (high μ).

Equilibrium at date 0 Equilibrium at date 0 is fully characterized by[12]

$$u(K_0)(1 + \hat{K}_0) = \frac{a}{1-\epsilon} \left(1 - \Delta + \frac{R^*}{R^* - 1} \max\{\hat{q}_0, \bar{q}\} \right) \tag{7}$$

$$1 + \hat{q}_0 = \frac{R^* - 1}{R^*} \frac{1-\epsilon}{a} \left(u(K_0) + \sum_{t=1}^{\infty} \frac{1}{R^{*t}} u(f_t(K_0)) \right). \tag{8}$$

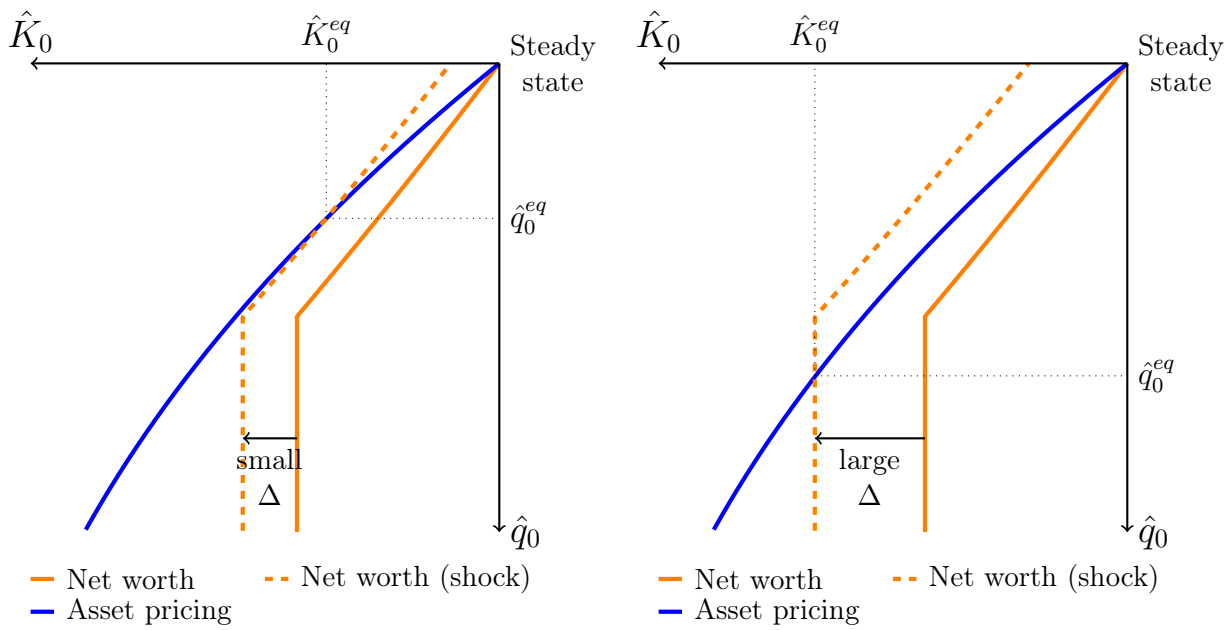
The first equation is the "net worth" relation, which links the size of the capital losses faced by the entrepreneur with the amount of capital she can retain the first period. Noting that renegotiation always dominates default from an individual's perspective, using equation (4), and that every entrepreneur is identical yields equation (7). The orange solid line in both

[10] See appendix A.1.1 for details.

[11] Of course, since financiers are rational, we would not be able to sustain levels of debt that satisfy $q_{t+1}k_t = R_t b_t$ if $(1-\theta)\mu > \theta\alpha$. Indeed, constraint (1) would have to be replaced by $R_t b_t \leq q_{t+1}k_t - (1-\theta)\mu q_t k_t + \theta\alpha q_t k_t$. In this case, even an infinitesimal shock would trigger renegotiation.

[12] Note that $K_0 = (1 + \hat{K}_0)K^*$, so we can think of equations (7) and (8) as describing the equilibrium $(\hat{q}_0, \hat{K}_0)$.

Figure 1: Technology shock.



Note. This figure illustrates the date-0 equilibrium for technology shocks Δ of different sizes. On the left, the shock is small and entrepreneurs get no haircut ($\varphi = 0$). On the right, the shock is large and entrepreneurs get a positive haircut ($\varphi > 0$). See appendix B for details.

panels of Figure 1 plots this relationship in the $(\hat{K}_0, \hat{q}_0)$ space when $\Delta = 0$ and $\epsilon = 0$. Since this is consistent with the steady state, this curve passes through $(0, 0)$. Around this point, this curve describes an increasing relationship: Since entrepreneurs are heavily levered, a lower price of capital damages their net worth more than one-to-one, decreasing their purchasing power. However, when capital becomes low enough, renegotiation is triggered putting a lower bound on how much entrepreneurs' capital holdings can fall in equilibrium:

$$u\left((1+\underline{\hat{K}}(\Delta,\epsilon))K^*\right)(1+\underline{\hat{K}}(\Delta,\epsilon)) = \frac{a}{1-\epsilon}\left(1-\Delta+\frac{R^*}{R^*-1}\bar{q}\right). \tag{9}$$

Thus, when $\hat{q}_0 \leq \bar{q}$, this curve becomes a vertical line at $\underline{\hat{K}}(\cdot)$. Note that if the repossession cost and/or the preference shock are large, entrepreneurs may find it optimal to default and take advantage of the depressed asset prices to buy even more capital than they had originally. Assumption 1 rules out this unrealistic case.

Assumption 1. *The following holds*

$$\epsilon \leq \frac{R^*}{R^*-1}\left(\theta\alpha - (1-\theta)\mu\right).$$

The second equilibrium equation (8) is a standard "asset-pricing" relation, which states that the price of capital is the present sum of future dividends. It comes from iterating forward on (2) and imposing a standard no-bubbles condition. The blue solid line in both panels of Figure 1 plots this relationship in the $(\hat{K}_0, \hat{q}_0)$ space when $\Delta = 0$ and $\epsilon = 0$. Since both u and f are increasing in $\hat{K}_0$, this curve also describes an increasing relationship between $\hat{q}_0$ and $\hat{K}_0$. The next lemma shows an equilibrium always exists.

Lemma 1. *An equilibrium exists. The equilibrium features $\hat{q}_0 \leq 0$ and $\hat{K}_0 \leq 0$.*

2.3 Shocks

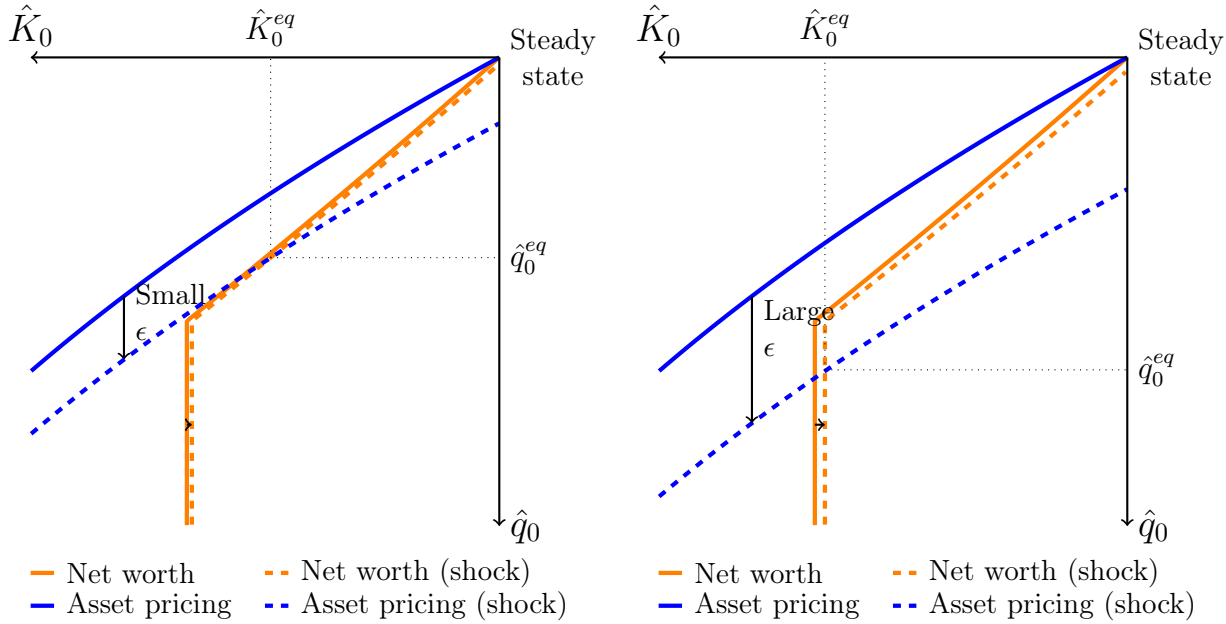
Technology shocks The left panel in Figure 1 illustrates the effect of a small negative technology shock. Given capital prices, entrepreneurs can now buy less capital, shifting the net worth curve to the left. Since the shock is small, $\hat{q}_0$ remains above $\bar{q}$. Thus, entrepreneurs bear all the losses and capital demand increases with its price. On the other hand, the asset-pricing relationship is independent of the shock. The interaction of both upward sloping curves leads to significant amplification of the original shock and a substantial drop in entrepreneur capital and asset prices - exactly as in the original KM analysis.

The right panel in Figure 1 illustrates the effect of a large negative shock. In this case, the drop in asset prices is so large that renegotiation is triggered. This puts a lower bound on the fall of entrepreneur's net worth and, thus, on their capital demand, which is now equal to $\underline{\hat{K}}(\Delta, \epsilon)$. Further shocks still have a negative effect on capital prices, but the amplification via the net worth channel is now absent. As prices fall, haircuts increase, stabilizing entrepreneurs' losses at $(\theta\alpha - (1-\theta)\mu)q^*K^*$.

Preference shocks The left panel of Figure 2 shows the date-0 equilibrium curves after a small preference shock, such that there is no renegotiation. In contrast to a technology shock, a preference shock that makes financiers more impatient moves both curves. On the one hand, it decreases the downpayment, implying that entrepreneurs can afford to buy more capital given their net worth (the net worth curve shifts to the right). On the other hand, higher discounting implies lower asset prices (the asset pricing curve shifts downwards). Which force dominates? Using equations (7) and (8), one can show that the shift in the asset-pricing curve is larger. Intuitively, this is because the change in discounting affects not only current dividends $u(K)$ but also future ones.[13] As a result, prices and entrepreneurs' capital holdings decrease with preference shocks in this region.

[13] One also needs to use $\hat{K}_0 \leq 0$ to arrive at this conclusion. For further details, see appendix A.1.3.

Figure 2: Preference shock.



Note. This figure illustrates the date-0 equilibrium for preference shocks ϵ of two different sizes. On the left, the shock is small and entrepreneurs get no haircut ($\varphi = 0$). On the right, the shock is large and entrepreneurs get a positive haircut ($\varphi > 0$). See appendix B for details.

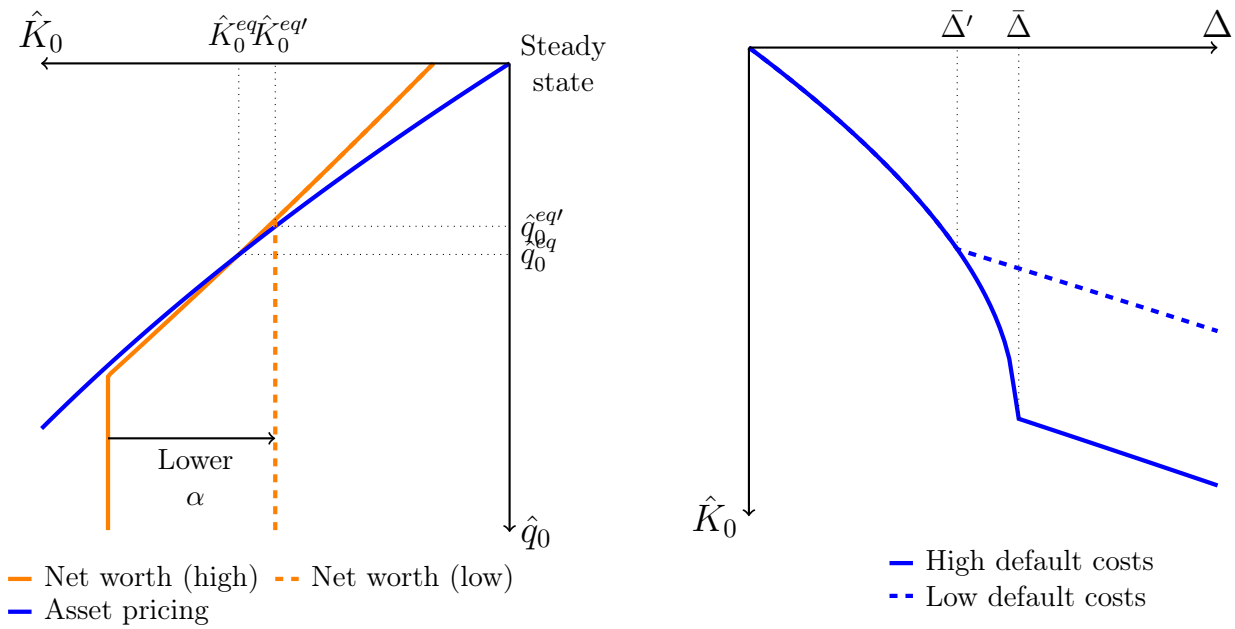
The right panel of Figure 2 shows the date-0 equilibrium curves after a large preference shock, which triggers renegotiation. Here, entrepreneurs' net worth is insulated from variations in asset prices. Thus, they only profit from a decrease in asset prices and their capital holdings increase. Since entrepreneurs' capital holdings increase, raising the marginal productivity of capital, but discounting of future dividends also increases, the effect on asset prices is ambiguous. Also, given that entrepreneurs' net worth is fixed in this region, the effect on haircuts is also ambiguous.

The following proposition summarizes the comparative statics results for technology and preference shocks.

Proposition 1. *(a) There exists a ball $\mathcal{B} \in \mathbb{R}^2$ with $(0,0) \in \mathcal{B}$ and a continuum of equilibria, $\{\hat{K}^{km}(\Delta,\epsilon), \hat{q}^{km}(\Delta,\epsilon)\}_{(\Delta,\epsilon)\in\mathcal{B}}$, such that (i) $\{\hat{K}^{km}(0,0), \hat{q}^{km}(0,0)\} = \{K^*, q^*\}$, (ii) $\hat{K}^{km}(\cdot)$ and $\hat{q}^{km}(\cdot)$ are continuous and strictly decreasing in Δ and ϵ for all $(\Delta,\epsilon) \in \mathcal{B}$.*

(b) There exists $\bar{\Delta}(\epsilon)$ such that an equilibrium with non-trivial renegotiation $\{\underline{\hat{K}}(\Delta,\epsilon), \underline{\hat{q}}(\Delta,\epsilon)\}_{(\Delta,\epsilon)}$ exists iff $\Delta > \bar{\Delta}(\epsilon)$. In this equilibrium, $\underline{\hat{K}}(\cdot)$ and $\underline{\hat{q}}(\cdot)$ are continuous in Δ and ϵ. $\underline{\hat{K}}(\cdot)$ is strictly decreasing in Δ and strictly increasing in ϵ, $\underline{\hat{q}}(\cdot)$ is strictly decreasing in Δ, and $\varphi(\Delta,\epsilon)$ is strictly increasing in Δ.

Figure 3: Changing default costs.



Note. This figure shows, for two different levels of default costs: the date-0 equilibrium (left panel) and entrepreneurs' capital holdings as a function of the technology shock (right panel). Smaller default costs expand the renegotiation region and lead to a milder crisis. See appendix B for details.

2.4 Allocating financial losses

The left panel in Figure 3 shows the date-0 equilibrium for two values of default costs: "high" (solid line) and "low" (dashed line).[14] Even though default is never realized in equilibrium, a lower default cost implies a more attractive outside option and, as a result, a larger haircut. Formally, the "vertical" branch of the net worth curve shifts to the right as default costs decrease. This has two implications. First, when the equilibrium is unique, the "amplification" region where equilibrium is characterized by the intersection of two upward-sloping curves shrinks.[15] That is, the economy features amplification for a smaller set of shocks. Second, when the shock is large enough to trigger renegotiation, the surplus extracted by entrepreneurs is larger and, as a result, the crisis is less pronounced.

The right panel in Figure 3 shows the entrepreneurs' capital holdings as a function of the

[14] Changes in bargaining costs and repossession costs are symmetric; see equations (7) and (8).

[15] Uniqueness is needed to ensure that the asset-pricing curve is flatter than the net-worth curve at the threshold $\bar{\Delta}(\epsilon)$, which is a property of the "km" equilibrium described in proposition 1 (a). With multiple equilibria, we cannot rule out cases where, at $\bar{\Delta}(\epsilon)$, the asset pricing curve is steeper and, hence, the effect of $\theta\alpha - (1-\theta)\mu$ on the size of the amplification region is the opposite.

shock for a "high" (solid line) and "low" (dashed line) level of default costs. Both economies behave similarly for small shocks. However, the low default cost economy is much more stable for large shocks: Renegotiation is triggered earlier and entrepreneurs capture a larger share of the surplus of avoiding costly default.

Proposition 2. *An increase in $\theta\alpha - (1-\theta)\mu$ (i.e. financiers' bargaining power or entrepreneurs' default cost increases or financiers' repossession cost decreases),*

(i) strictly increases $\bar{\Delta}(\epsilon)$ if the equilibrium is unique for all Δ and $\bar{\Delta} \in (0, \tilde{\Delta})$.

(ii) strictly decreases $\underline{\hat{K}}(\Delta, \epsilon)$ and $\underline{\hat{q}}(\Delta, \epsilon)$ for $\Delta > \bar{\Delta}(\epsilon)$.

How does the economy recover after the financial crisis? Approximating equation (3) around the steady state yields

$$\hat{k}_t = \frac{\eta}{\eta + 1}\hat{k}_{t-1},$$

where $\eta^{-1} \equiv d \ln u(K)/d \ln K|_{K=K^*}$. Thus, the rate of convergence back to the steady state is exactly the same as in KM. This implies that our model retains an attractive feature of the KM model: the ability to rationalize episodes of sluggish recovery after financial crises, such as Japan in the 1990s. Importantly, note that the speed of convergence is independent of the distribution of financial losses, which only affect the size of the output drop at $t = 1$.

2.5 Welfare

So far, we have characterized the equilibrium in an economy where agents can renegotiate their past commitments. We now turn to the normative question on whether allowing for renegotiation is desirable.

To answer this question, consider an economy where renegotiation is not available, i.e. agents can either default or repay the full value of their debt. To have a well-defined comparison of welfare across economies, we assume that the equilibrium is unique.

Proposition 3. *Consider an economy where renegotiation is not possible. Then, the paths for entrepreneurs' capital holdings $\{K_t\}_{t=0}^{\infty}$ and prices $\{q_t\}_{t=0}^{\infty}$ are the same as in an economy where renegotiation is possible and $\theta = 1$. When $\theta < 1$, entrepreneurs' capital holdings $\{K_t\}_{t=0}^{\infty}$ and prices $\{q_t\}_{t=0}^{\infty}$ are higher in an economy with renegotiation than in an economy where renegotiation is unavailable, strictly so when $\Delta > \bar{\Delta}(\epsilon)$.*

Proposition 3 states that the distribution of capital and asset prices in the economy with default behave as in the model where entrepreneurs have no bargaining power ($\theta = 1$). This is because, in this case, entrepreneurs' net worth is independent of whether they default or renegotiate. By contrast, when entrepreneurs have some bargaining power ($\theta < 1$), their net

worth is strictly larger if renegotiation is allowed. Using proposition 2, it follows that an economy with renegotiation features a smaller drop in entrepreneurs' capital holdings and prices.

Is renegotiation good for welfare? There is an obvious mechanical benefit to allowing for renegotiation: agents avoid paying default and repossession costs. However, these costs may not be deadweight losses, e.g. if they are payments to other sectors in the economy, such as a litigation sector. To avoid taking a stand on the nature of these costs, we define aggregate welfare net of default and repossession costs

$$\mathcal{W} = \mathcal{W}^E + \mathcal{W}^F + \mathbf{1}_d \left(\mu q^* K^* + \alpha q^* K^*\right), \tag{10}$$

where $\mathcal{W}^E$ is the welfare of a representative entrepreneur, $\mathcal{W}^F$ is the welfare of a representative financier, and $\mathbf{1}_d$ is an indicator variable that takes a value of 1 if entrepreneurs default.

As argued above, whether renegotiation is allowed is irrelevant for the distribution of capital and prices when financiers have all bargaining power ($\theta = 1$). Thus, in this case, the only possible net welfare gains would arise from avoiding repossession and default costs. By contrast, when $\theta < 1$, entrepreneurs retain more capital when they can renegotiate after large shocks. Since capital is more productive in the hands of entrepreneurs, renegotiation creates additional welfare gains. Corollary 1 summarizes these results.

Corollary 1. *Welfare net of default and repossession costs $\mathcal{W}$, given by (10), is strictly larger when renegotiation is possible if and only if $\Delta > \bar{\Delta}$ and $\theta < 1$.*

2.6 Ex-ante vs. ex-post hold-up incentives

Our model features two main moral hazard frictions. First, in the *interim* stage, after the financial contract has been written but *before* agents commit their labor, entrepreneurs can threaten to walk away with borrowed funds. Second, *ex post*, i.e. after production is realized, entrepreneurs can default at a cost $(1-\alpha)q_{t-1}k_t$ of tradable output.

In section 2, we allowed financiers to have different degrees of bargaining power ex post but none in the interim stage. Here, we relax this assumption. That is, we assume that in the interim stage financiers and entrepreneurs engage in Nash bargaining.[16] In appendix A.1.7, we show that constraint (1) is replaced by

$$R_t b_t \leq q_{t+1} k_t + \iota a_{t+1} k_t \tag{11}$$

[16]Note that this bargaining protocol, while consistent with our modeling of ex post bargaining, is different from the one in the original model of Hart and Moore (1994).

where ι is the bargaining power of financiers in the interim stage. When entrepreneurs walk away from the contract, they forego a share of the return on their investment. This outside option is irrelevant when $\iota = 0$ (entrepreneurs make a take-it-or-leave-it offer) but, when $\iota > 0$, it allows entrepreneurs to increase their leverage.

Since entrepreneurs can also default ex post,

$$R_t b_t \leq q_{t+1} k_t + \theta \alpha q_t k_t - (1-\theta)\mu q_t k_t. \tag{12}$$

Clearly, with perfect foresight, only one of the constraints may bind at any one time. Henceforth, we assume that (11) binds at the steady state, i.e.

$$\iota a \leq \theta \alpha q^* - (1-\theta)\mu q^*. \tag{13}$$

Were this inequality not to hold, agents would lever up to the point that even an infinitesimal negative shock would trigger renegotiation.[17] Thus, ι determines borrowing capacity.

The law of motion of entrepreneur's capital holdings is now given by

$$k_t = \underbrace{\frac{1}{u(K_t) - R_t^{-1}\iota a}}_{\text{higher leverage}} \underbrace{a(1-\iota)}_{\text{lower net worth}} k_{t-1}.$$

An increase in financiers' interim bargaining power, ι, has two effects. On the one hand, entrepreneurs can lever up more. That is, they can buy more capital per unit of net worth. On the other hand, since they borrowed more in the past, they have lower net worth given some level of capital. In appendix A.1.7, we show that the net-worth effect dominates the leverage effect at the steady state and, thus, entrepreneurs' capital holdings are decresing in ι. Intuitively, while ι has a one-to-one effect on net worth, the effect on leverage involves future income and, thus, is discounted by R_t.

To avoid this paradoxical result, we assume that entrepreneurs have finite lives, which weakens the net-worth effect. More precisely, we assume that each period entrepreneurs die with probability χ and are replaced by a new cohort. A representative entrepreneur in the new cohort is endowed with a unit of labor l_t, which is combined with capital to produce the final tradable good,[18]

$$y_t = \chi^{-\chi} a_t k_t l_t^{\chi}.$$

We make the following assumptions, which guarantee that the leverage effect dominates the

[17] In appendix A.1.7, we show that (13) also implies that constraint (11) binds after a negative shock since renegotiation prevents the price of capital from falling below a value such that (12) binds.

[18] Note that this production function implies that aggregate wage income satisfies $w_t L_t = \chi a_t K_t$.

net-worth effect and that investing is optimal at the steady state.

Assumption 2. *We impose the following restrictions.*

$$R^{*-1} \geq 1 - \chi$$
$$c > a(\beta^{-1} - 1)(1 - (1 - \chi)\iota)$$

Proposition 4. *Under assumption 2, the steady-state levels of entrepreneurs' capital holdings K^*, asset prices q^*, and leverage $\frac{R^* B^*}{q^* K^*}$ are increasing in ι, strictly so if $R^{*-1} > 1 - \chi$.*

Next, we study the response of the economy after a measure-zero negative productivity shock. We make two additional assumptions. The first ensures that the downpayment cannot be fully covered with claims on future output, i.e. collateral is needed, regardless of the size of the shock. The second is a condition on the curvature of the production function, which ensures that the speed of convergence is monotonic with the level of entrepreneur's capital holdings.

Assumption 3. *We impose the following restrictions.*[19]

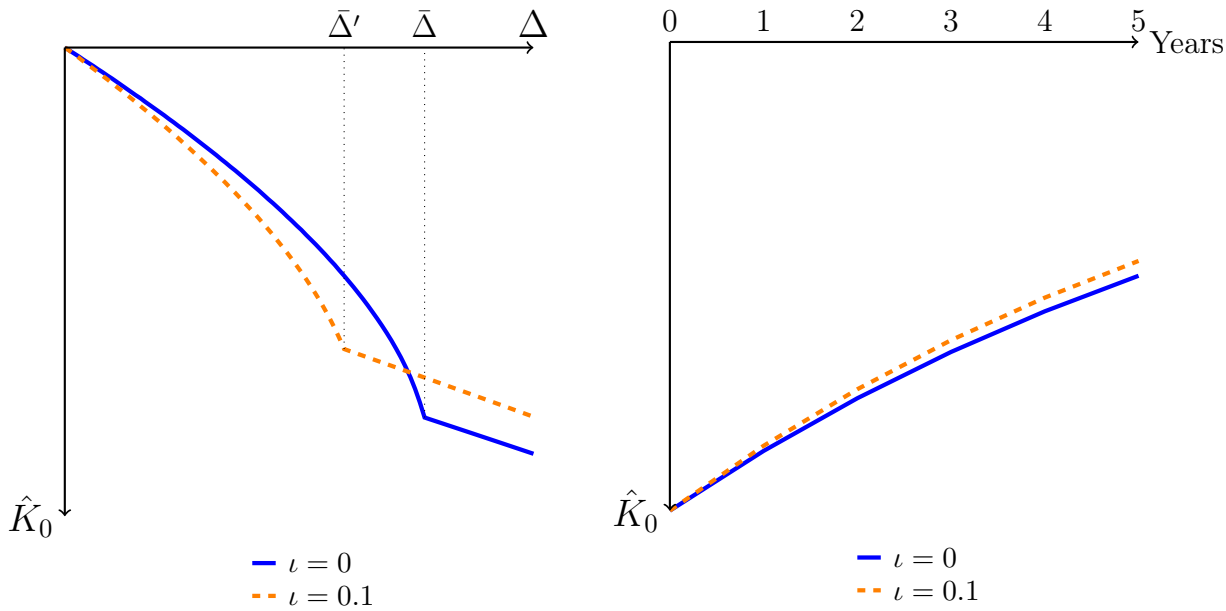
$$u(0) > R^{*-1}\iota a$$
$$u'(K)K \textit{ is increasing in } K$$

Proposition 5. *Under assumptions 2 and 3, the following holds.*
(a) There exists $\mathring{\Delta} \in \mathbb{R}$ and a continuum of equilibria, $\{\hat{K}^{km}(\Delta, \iota), \hat{q}^{km}(\Delta, \iota)\}_{\Delta < \mathring{\Delta}}$, such that (i) $\{\hat{K}^{km}(0, \iota), \hat{q}^{km}(0, \iota)\} = \{K^*(\iota), q^*(\iota)\}$, and (ii) $\hat{K}^{km}(\cdot)$ and $\hat{q}^{km}(\cdot)$ are continuous in Δ. When $\chi \to 1$, $\hat{K}^{km}(\cdot)$ is strictly increasing in ι.
(b) There exists $\bar{\Delta}(\iota)$ such that an equilibrium with non-trivial renegotiation $\{\underline{\hat{K}}(\Delta, \iota), \underline{\hat{q}}(\Delta, \iota)\}$ exists iff $\Delta > \bar{\Delta}(\iota)$. In this equilibrium, $\underline{\hat{K}}(\cdot)$ and $\underline{\hat{q}}(\cdot)$ are continuous in Δ. $\underline{\hat{K}}(\cdot)$ is strictly increasing in ι.
(c) Given $\hat{K}_0$, if $\iota' > \iota$, then $\hat{K}_t = f_t(\hat{K}_0, \iota') > \hat{K}_t = f_t(\hat{K}_0, \iota)$ for all $t \geq 1$.

Proposition 5 describes how the equilibrium changes with financiers' interim bargaining power ι. Entrepreneurs in economies with higher ι enter the financial crisis with a deeper debt-overhang problem, i.e. they have a stronger net-worth effect. On the other hand, entrepreneurs in economies with higher ι can borrow more against future output, i.e. they have a stronger leverage effect. The former becomes less important as the death rate χ increases. Indeed, in the limit $\chi \to 1$, the former is absent so the financial crisis is unambiguously mitigated by ι.

[19] A sufficient condition for $u'(K)K$ to be increasing is $G'''(k') \geq 0$.

Figure 4: Changing financiers' bargaining power in the interim stage



Note. This figure shows, for two different levels of financiers' bargaining power ι in the interim stage: the date-0 entrepreneurs' capital holdings as a function of the size of the technology shock (left panel) and the path for entrepreneurs capital holdings starting from some level $\hat{K}_0$ (right panel). See appendix B for details.

When the death rate is away from one, what effect dominates is ambiguous. Figure 4 illustrates the response of an economy after a negative productivity shock for two values of ι, $\iota = 0$ (solid-blue line) and $\iota = 0.1$ (dashed-orange line), in an economy with a small death rate ($\chi = 1 - R^{*-1} = 0.1$). Panel (a) plots entrepreneurs' capital holdings on impact. In this example, the debt-overhang problem initially dominates the leverage effect. Thus, there is more amplification in the economy with higher ι. As the crisis becomes deeper, future output becomes smaller, weakening the leverage effect and exacerbating the difference between both economies.[20] Eventually, however, the shock is large enough to trigger renegotiation. In appendix A.1.7, we show that the haircut is given by

$$\varphi = \max\left\{\frac{1}{1+\frac{R^*-1}{R^*}\frac{\iota}{1+(R^{*-1}-(1-\chi))\iota}}\left(-\hat{q}_0 - \theta\alpha + (1-\theta)\mu + \frac{R^*-1}{R^*}\frac{\iota}{1+(R^{*-1}-(1-\chi))\iota}\right), 0\right\}.$$

Since steady-state leverage is higher in the $\iota > 0$ economy, the drop in capital prices required to trigger renegotiation is smaller and, given $\hat{q}_0$, the haircut is larger. In this example, this implies that the economy with higher ι enters the renegotiation region with a smaller shock.

[20] For the same reason, if there was some persistence in the productivity process, the leverage effect would become even weaker, exacerbating the difference between both economies further.

Furthermore, in appendix A.1.7, we show that, conditional on renegotiating, entrepreneurs retain more capital in an economy with higher ι.

Finally, ι also affects the speed of the recovery. Panel (b) in Figure 4 plots the path entrepreneurs' capital holdings for the previous two values of ι starting from the same level of capital holdings $\hat{K}_0$ relative to the steady state. The ability to pledge future output unambiguously implies that the economy with $\iota = 0.1$ converges faster to the steady state, although our simulations suggest that the effect is quantitatively small.

3 A Model with Equilibrium Default

The aftermath of a financial crisis is often characterized not only by debt restructuring negotiations but also by outright default. For example, both outcomes were observed in the hotel business during the Great Recession. This sector was particularly affected by economic conditions with revenue earned per room falling by almost 17% in 2009, and stock prices of the largest publicly traded hotel chains falling by around 80% between July 2007 and March 2009.[21] A prominent example of renegotiation was the deal that Blackstone secured for Hilton's debts in April 2010. Debt was restructured from \$20 to \$16 billion and maturity extended by two years.[22] An example of default is the case of Sunstone Hotel Investors, who defaulted on \$300 million of debt in June 2009 and had 13 hotels seized by its bank, only days later announcing its intention to buy hotels at a discount.

In this section, we extend our model to rationalize why some firms default in equilibrium and characterize its implications for the allocation of capital and asset prices.

3.1 Setup

We extend our previous model to accomodate heterogeneity in the size of the default cost α_i faced by each entrepreneur i. Crucially, default costs are private information. That is, α_i is known by the entrepreneur but unknown to the financier, who only knows the cumulative distribution function $F(\alpha) \in C^2$ with support $[0, \tilde{\alpha}]$.[23]

Since financiers ignore the type of entrepreneurs they have lent to, they face a tradeoff

[21] Starwood Hotels and Resorts Worldwide, Wyndham Worldwide Corporation, and Marriott International stock prices fell 85%, 84%, and 71% respectively in this period. Source for drop in revenue per room: https://www.economist.com/finance-and-economics/2010/03/18/you-can-check-out-any-time-you-like.

[22] Hilton's deal included the repurchase of \$1.8 billion of secured debt with a 54% discount, see Phalippou and Baum (2014). Other large hotel groups that restructured their debts were MGM Mirage in April 2009, and Harrah's in March 2010.

[23] In this section, we assume default is feasible even for the agent with the highest default cost, i.e. $\tilde{\Delta} = 1 - \frac{R^*}{R^*-1}\tilde{\alpha}K^*$. We also consider a few examples with a discrete distribution function in section 3.2.2.

in the event of an unforeseen negative shock: a higher level of debt relief makes more entrepreneurs willing to accept, but the rent extracted from each entrepreneur gets smaller. Financiers will balance the two effects, recognizing that the willingness of entrepreneurs to accept a certain deal will be weaker for those with low default costs. For simplicity, we assume that financiers are identical and have all the bargaining power ex post, i.e. $\theta = 1$, while entrepreneurs have all the bargaining power ex ante, i.e. $\iota = 0$.[24]

We solve the problem by backward induction. First, an entrepreneur must decide whether to accept or decline a proposed haircut of φ, taking prices as given. From equation (5), we know that entrepreneurs will only accept an offer if $\alpha_i \geq -(\hat{q}_0 + \varphi)$. Taking this into account financiers minimize expected losses. For a given debt offer φ, a financier incurs in a cost (in percentage terms) given by $-\hat{q}_0 + \mu$ on the fraction $F(-(\hat{q}_0 + \varphi))$ of the entrepreneurs who default and deliver their collateral, whereas he loses φ (in percentage terms) on the complementary fraction $1 - F(-(\hat{q}_0 + \varphi))$ of credits that are renegotiated. Since individual financiers take prices, $\hat{q}_0$, as given, we can write their problem as,

$$\min_{\varphi \geq 0} \quad (\hat{q}_0 + \varphi)(1 - F(-(\hat{q}_0 + \varphi))) + \mu F(-(\hat{q}_0 + \varphi)).$$

The first order condition yields,[25]

$$1 - F(-(\varphi + \hat{q}_0)) + f(-(\varphi + \hat{q}_0))(\hat{q}_0 + \varphi - \mu) \geq 0, \qquad \text{with equality if } \varphi > 0. \tag{14}$$

Let $\bar{\alpha}$ denote the threshold default cost implied by the solution ignoring the non-negativity constraint. Note that $\bar{\alpha}$ is a function only of μ and $F(\cdot)$.[26] The solution to the financiers' problem can be written as

$$\varphi = \max\{-\hat{q}_0 - \bar{\alpha}, 0\}.$$

Note the symmetry with the derivation of the equilibrium haircut in the previous section, with $\bar{q}$ replaced by $-\bar{\alpha}$.

The net-worth relation now needs to take into account that there may be default in equilibrium. Entrepreneurs with $\alpha_i < \min\{\bar{\alpha}, -\hat{q}_0\}$ default while agents with $\alpha_i \geq \min\{\bar{\alpha}, -\hat{q}_0\}$

[24] Given that financiers are risk neutral, we proceed as if each one of them faces a continuum of entrepreneurs. This makes the number of entrepreneurs who default for a given debt reduction offer deterministic.

[25] We assume that (i) $(\alpha + \mu)F(\alpha)$ is strictly convex or (ii) the Mills ratio $\frac{1-F(\alpha)}{f(\alpha)}$ is weakly decreasing in α. Either of these are sufficient conditions for a unique solution to the financiers' problem. For example, a uniform distribution satisfies both requirements.

[26] More rigorously, we should write $\overline{\alpha}(\mu)$, but we omit the dependence on μ for ease of exposition.

renegotiate. Thus, the net-worth relationship for an entrepreneur of type i yields

$$u(K_0)(1+\hat{k}_0^i) = \frac{a}{1-\epsilon}\left(1-\Delta-\frac{R^*}{R^*-1}\min\left\{\alpha_i, \min\{\bar{\alpha}, -\hat{q}_0\}\right\}\right).$$

Integrating over individual capital holdings yields

$$\begin{aligned} u(K_0)(1+\hat{K}_0) =& \frac{a}{1-\epsilon}\Bigg(1-\Delta-\frac{R^*}{R^*-1}\mathbb{E}\left(\alpha|\alpha \leq \min\{\bar{\alpha}, -\hat{q}_0\}\right)\underbrace{F(\min\{\bar{\alpha}, -\hat{q}_0\})}_{\text{defaulters}} \\ &- \frac{R^*}{R^*-1}\min\{\bar{\alpha}, -\hat{q}_0\}\underbrace{(1-F(\min\{\bar{\alpha}, -\hat{q}_0\}))}_{\text{non-defaulters}}\Bigg) \end{aligned} \tag{15}$$

Agents that default face capital losses of α_i while agents who renegotiate suffer losses of $\min\{\bar{\alpha}, -\hat{q}_0\}$. Note that all agents that do not default have the same net worth, as financiers must offer everyone the same haircut. Thus, agents with high default costs that accept the offer profit from the unobservability of their default costs.

While equation (15) seems more complicated than its counterpart in the model with no heterogeneity or private information, equation (7), it has similar properties: it still describes an upward relationship between $\hat{K}_0$ and $\hat{q}_0$ for $\hat{q}_0 \geq -\bar{\alpha}$ and a vertical line when $\hat{q}_0 < -\bar{\alpha}$. The lower bound on entrepreneurs' capital holdings $\underline{\hat{K}}(\Delta, \epsilon)$ is now given by,

$$u\left((1+\underline{\hat{K}}(\cdot))K^*\right)(1+\underline{\hat{K}}(\cdot)) = \frac{a}{1-\epsilon}\left(1-\Delta-\frac{R^*}{R^*-1}\left(\mathbb{E}\left(\alpha_i|\alpha_i \leq \bar{\alpha}\right)F(\bar{\alpha})+\bar{\alpha}(1-F(\bar{\alpha}))\right)\right). \tag{16}$$

The model is closed by the same asset-pricing relationship as before.

We again make an assumption to rule out cases where the drop in asset prices triggers a default that allows entrepreneurs to increase their capital holdings on average (an analogue of assumption 1). In this case, we need not only a bound on the size of the preference shock, but also a (weak) bound on the share of defaulting entrepreneurs due to redistributional considerations (i.e. entrepreneurs with low default costs benefit from fire sales by entrepreneurs with high default costs). Lemma 2 shows an equilibrium exists in this economy.

Assumption 4. *The following holds*

$$\begin{aligned} \epsilon &\leq \frac{R^*}{R^*-1}\left(\bar{\alpha}(1-F(\bar{\alpha}))+\mathbb{E}\left(\alpha_i|\alpha_i \leq \bar{\alpha}\right)F(\bar{\alpha})\right) \\ F(\bar{\alpha}) &\leq \beta'. \end{aligned}$$

Lemma 2. *An equilibrium exists. The equilibrium features $\hat{q}_0 \leq 0$ and $\hat{K}_0 \leq 0$.*

3.2 Shocks

Next, we study the response of this economy to technology and preference shocks. Proposition 6 shows that all the main results we derived in proposition 1 carry over to this environment. This follows from showing that the net worth curve has similar properties and the asset-pricing relationship is unaltered.

More interestingly, proposition 6 characterizes the behavior of the share of defaulting entrepreneurs in this new economy. Since there are some agents with tiny default costs (i.e. $\alpha = 0$ is in the support), there is default even after small shocks. As shocks become larger, asset prices decline more and an increasingly large share of entrepreneurs default. Eventually, financiers find it optimal to offer positive haircuts $\varphi > 0$. At this point, the share of defaulting entrepreneurs stabilizes and any further drops in asset prices are offset by corresponding increases in the haircut.[27]

Proposition 6. *(a) There exists a ball $\mathcal{B} \in \mathbb{R}^2$ with $(0,0) \in \mathcal{B}$ and a continuum of equilibria, $\{\hat{K}^{km}(\Delta,\epsilon), \hat{q}^{km}(\Delta,\epsilon)\}_{(\Delta,\epsilon)\in\mathcal{B}}$, such that (i) $\{\hat{K}^{km}(0,0), \hat{q}^{km}(0,0)\} = \{K^*, q^*\}$, (ii) $\hat{K}^{km}(\cdot)$ and $\hat{q}^{km}(\cdot)$ are continuous and strictly decreasing in Δ and ϵ for all $(\Delta,\epsilon) \in \mathcal{B}$. The share of defaulting entrepreneurs strictly increases with Δ and ϵ, and is strictly positive whenever either $\Delta > 0$ or $\epsilon > 0$.*

(b) There exists $\bar{\Delta}(\epsilon)$ such that an equilibrium with non-trivial renegotiation $\{\underline{\hat{K}}(\Delta,\epsilon), \underline{\hat{q}}(\Delta,\epsilon)\}$ exists iff $\Delta > \bar{\Delta}(\epsilon)$. In this equilibrium, $\underline{\hat{K}}(\cdot)$ and $\underline{\hat{q}}(\cdot)$ are continuous in Δ and ϵ. $\underline{\hat{K}}(\cdot)$ is strictly decreasing in Δ and strictly increasing in ϵ, $\underline{\hat{q}}(\cdot)$ is strictly decreasing in Δ, and φ is strictly increasing in Δ. The share of defaulting entrepreneurs is fixed at $F(\bar{\alpha})$.

3.2.1 Speed of recovery and welfare

To understand the differential effects of asymmetric information on the equilibrium, we compare the solution to the case of perfect information. More precisely, we consider an economy where default costs are distributed according to the same distribution F, but where the entrepreneurs' type α_i is perfectly observable by the financier. In other words, financiers can tailor the haircut to each entrepreneur, offering $\varphi_i = \max\{-\hat{q}_0 - \alpha_i, 0\}$. As a result, the

[27] Since both curves are upward sloping, both types of equilibria may coexist for some Δ and ϵ. Our discussion in this paragraph describes a case with a unique equilibrium, but the results of proposition 6 are general.

individual net worth relation is given by[28]

$$u(K_0)(1+\hat{k}_0^i) = \frac{a}{1-\epsilon}\left(1-\Delta-\frac{R^*}{R^*-1}\min\{\alpha_i, -\hat{q}_0\}\right).$$

Integrating over individual capital holdings yields

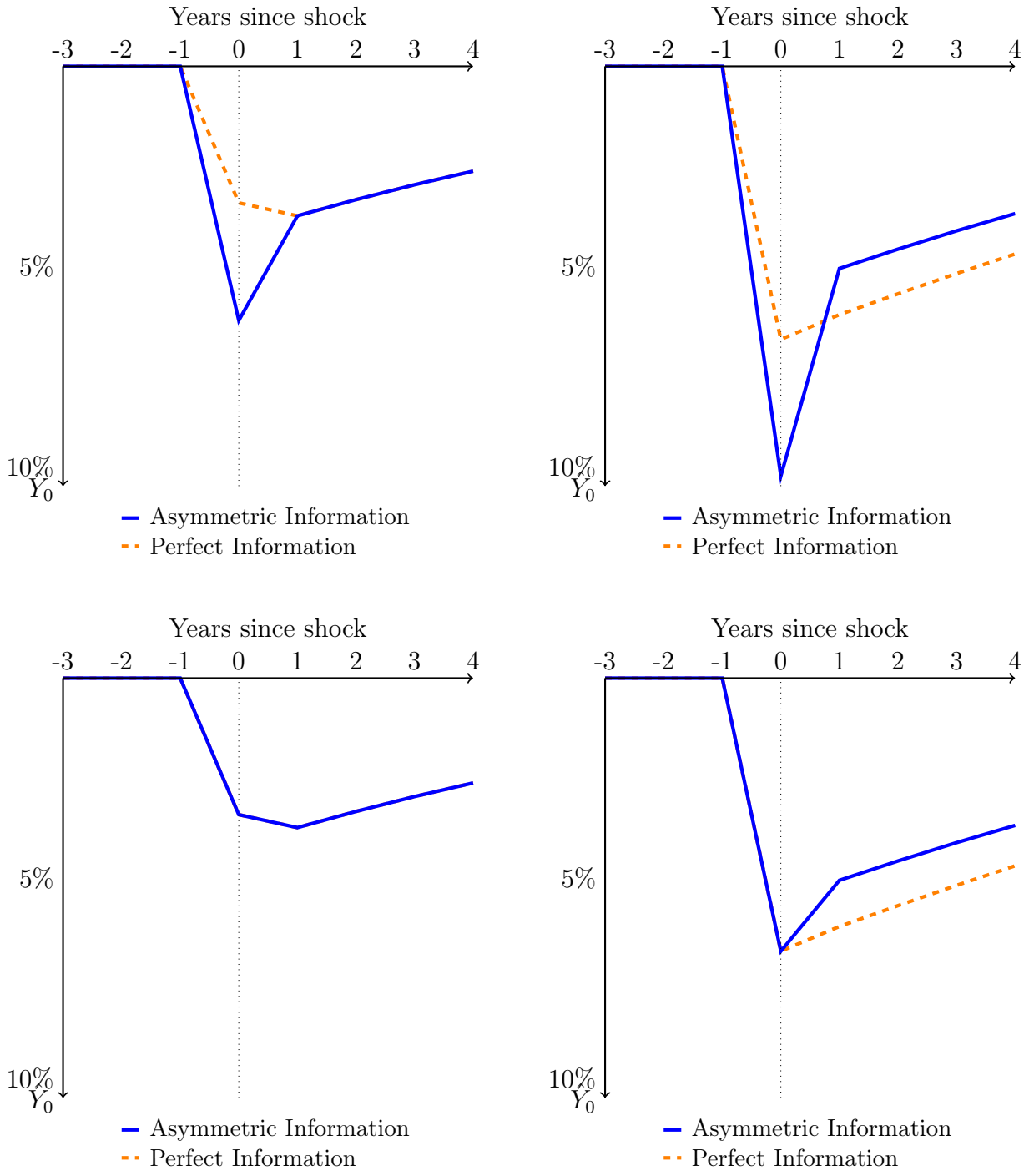
$$\begin{aligned} u(K_0)(1+\hat{K}_0) =& \frac{a}{1-\epsilon}\Bigg(1-\Delta-\frac{R^*}{R^*-1}\mathbb{E}\left(\alpha|\alpha\leq\min\{\bar{\alpha},-\hat{q}_0\}\right) \underbrace{F(\min\{\bar{\alpha},-\hat{q}_0\})}_{\text{defaulters with asymmetric information}} \\ &- \frac{R^*}{R^*-1}\underbrace{\mathbb{E}\left(\alpha|\alpha>\min\{\bar{\alpha},-\hat{q}_0\}\right)}_{\text{larger than }\bar{\alpha}} \underbrace{(1-F(\min\{\bar{\alpha},-\hat{q}_0\}))}_{\text{non-defaulters with asymmetric information}}\Bigg) \end{aligned} \tag{17}$$

Figure 5 shows the response of output to a shock in an economy with asymmetric information (blue solid line) and in one with perfect information (orange dashed line), depending on whether default is a deadweight loss (top two panels) or not (bottom two panels). In the panels on the left, the economy is hit with a small shock, i.e. the equilibrium features $\hat{q}_0 \geq -\bar{\alpha}$. In this case, agents with default costs larger than $-\hat{q}_0$ do not get any haircuts in either economy. Agents with smaller default costs get a haircut in the perfect information (PI) economy and default in the asymmetric information (AI) economy. Since financiers have all the bargaining power, entrepreneurs have the same net worth in either case, implying that they can buy the same amount of capital. Given that there is a one-period lag in production, output is the same in both economies from $t = 1$ onwards. At $t = 0$, the PI economy avoids default. To the extent that default entails deadweight losses, output is lower in the AI economy at $t = 0$. Welfare net of default and repossession costs, as defined in section 2.5, is identical in both economies.

In the panels on the right, the economy is hit with a large shock, i.e. the equilibrium features $\hat{q}_0 < -\bar{\alpha}$. Entrepreneurs with $\alpha_i < -\bar{\alpha}$ still have the same net worth in both economies. By contrast, agents with $\alpha_i > \bar{\alpha}$ have more net worth in the AI economy. This is because financiers are forced to offer everyone the same haircut, so agents with high default costs, who would get a very small haircut under PI, now obtain a more generous haircut from financiers. In other words, asymmetric information transfers wealth from creditors to debtors. This can be seen by comparing (15) and (17): under AI all these agents lose the same amount $\bar{\alpha}$ while under PI they lose their expected default cost $\mathbb{E}(\alpha|\alpha > \bar{\alpha})$, which is larger. In addition, since entrepreneurs can afford more capital in the AI economy, in equilibrium asset prices are higher, further boosting their net worth relative to the PI economy. Since

[28]Note that, with perfect information, μ is irrelevant because financiers are assumed to have all bargaining power.

Figure 5: Speed of recovery: Asymmetric vs. perfect information



Note. This figure simulates an event at $t = 0$ (a shock to both Δ and ϵ) and plots the response of output under asymmetric (AI) and perfect information (PI) for small shocks (left panels) and large shocks (right panels). In the top panels we assume that default costs are deadweight losses, while in the bottom panels they are not (i.e. they are remuneration of an unmodelled sector). See appendix B for details.

entrepreneurs' capital holdings at $t = 0$ are higher, output is higher in the AI economy from $t = 1$ onwards. At $t = 0$, the PI economy avoids default. To the extent that default entails deadweight losses, output is lower in the AI economy at $t = 0$. Welfare net of default and repossession costs, as defined in section 2.5, is larger in the AI economy since entrepreneurs retain more capital after the crisis.

Proposition 7. *In an economy with asymmetric information ("AI" superscript), a shock has the following effects, relative to an equivalent economy with perfect information ("PI" superscript):*

(i) After a small shock (i.e. when $\hat{q}_0^{AI} \geq -\bar{\alpha}$), $\varphi^{AI} = 0$ and $\hat{K}_0^{AI} = \hat{K}_0^{PI}$. There is default at $t = 0$ in the asymmetric information economy. At dates $t \geq 1$, output is equal in both economies. Welfare net of default and repossession costs, as defined in 2.5, is identical across economies.

(ii) After a large shock (i.e. when $\hat{q}_0^{AI} < -\bar{\alpha}$), $\varphi^{AI} > 0$ and $\hat{K}_0^{AI} > \hat{K}_0^{PI}$. There is default at $t = 0$ in the asymmetric information economy. At dates $t \geq 1$, output is larger in the asymmetric information economy. Welfare net of default and repossession costs is larger in the AI economy.

3.2.2 Allocating financial losses

Proposition 2 also has an analogue in this economy. Here, a higher repossession cost implies it is more costly to let agents default. Hence, to prevent agents from defaulting, financiers must offer everyone a better haircut, boosting entrepreneurs' net worth across the board.

Proposition 8. *An increase in μ,*

(i) strictly decreases $\bar{\Delta}(\epsilon)$ if the equilibrium is unique for all Δ and $\bar{\Delta}(\epsilon) \in (0, \tilde{\Delta})$.

(ii) strictly increases $\underline{\hat{K}}(\Delta, \epsilon)$ and $\underline{\hat{q}}(\Delta, \epsilon)$ for $\Delta > \bar{\Delta}(\epsilon)$,

(iii) decreases the share of defaulting entrepreneurs for $\Delta > \bar{\Delta}(\epsilon)$.

Note that, unlike proposition 2, proposition 8 is silent on the effect of default costs. The reason is that $\bar{\alpha}$ is an endogenous object. Indeed, many cases could arise: effective default costs could go up or down, and the recovery could be faster or slower (i.e. debtors could bear more or less losses). Next, we construct four examples to illustrate the different possible cases. For simplicity, we set $\mu = 0$. Table 1 summarizes the results.

Uniform F $[0, \tilde{\alpha}]$ In this case, one can show (see appendix B),

$$\bar{\alpha} = \frac{1}{2}\tilde{\alpha}, \quad , F(\bar{\alpha}) = \frac{1}{2}, \quad , \underline{\hat{K}} \propto -\tilde{\alpha}.$$

Table 1: Potential default costs, effective default costs, and amplification

Distribution	Increase in (potential) default costs:	Effective default	$\hat{K}_0$
Uniform $F\ [0, \tilde{\alpha}]$	$\uparrow \tilde{\alpha}$	$+$	$-$
Two types	$\uparrow p^H$, $\downarrow p^L$	$-$	$-$
Three types I	$\downarrow \alpha^M$, $\uparrow \alpha^L$, $p^L \Delta\alpha^L > -p^M \Delta\alpha^M$	$+$	$+$
Three types II	$\uparrow p^M$, $\downarrow p^L$	$-$	$+$

Thus, as $\tilde{\alpha}$ increases, entrepreneurs extract a smaller haircut and the ensuing recovery slows down. While the share of defaulting entrepreneurs stays constant, each entrepreneur that defaults pays a larger cost. Thus, the effective default costs are larger.

Two types Suppose there are two types, α^H and α^L with $\alpha^H > \alpha^L$ that arise with probability p and $1-p$, respectively. We cannot use the first order condition given by equation (14). Nevertheless, note that financiers will either offer a haircut such that $\varphi^H = -\hat{q}_0 - \alpha^H$, in which case only the high type will renegotiate, or they will offer $\varphi^L = -\hat{q}_0 - \alpha^L$, in which case both types will renegotiate. The profits of each strategy are

$$\begin{aligned} \varphi^H\colon &\quad \hat{q}_0 + p\alpha^H \\ \varphi^L\colon &\quad \hat{q}_0 + \alpha^L \end{aligned}$$

If $p\alpha^H > \alpha^L$, then offering a small haircut is optimal. Suppose this is the case. Note the probability of default is $1-p$, while

$$\underline{\hat{K}} = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\alpha^L(1-p) + \alpha^H p\right)\right).$$

Next, consider an increase in average potential default costs by raising p. This does not change the net worth of any entrepreneur, but tilts the composition towards higher default cost agents. Thus, entrepreneurs as a group bear a large share of the financial losses, slowing down the recovery of output at dates $t \geq 1$. However, effective default costs decrease: There are fewer low type agents, who are the only ones that default in equilibrium.

Three types I Suppose there are three types, α^H, α^M and α^L with $\alpha^H > \alpha^M > \alpha^L$ that arise with probability p^H, p^M, and p^L, respectively. We now need to compare three possible strategies: $\varphi^H = -\hat{q}_0 - \alpha^H$ (only high type accepts), $\varphi^M = -\hat{q}_0 - \alpha^M$ (high and medium type accept), and $\varphi^L = -\hat{q}_0 - \alpha^L$ (everyone accepts). The profits of each strategy

are

$$\begin{aligned}
\varphi^H&: \quad \hat{q}_0 + \alpha^H p^H \\
\varphi^M&: \quad \hat{q}_0 + \alpha^M (p^H + p^M) \\
\varphi^L&: \quad \hat{q}_0 + \alpha^L
\end{aligned}$$

Suppose $(p^M + p^H)\alpha^M > \max\{p^H\alpha^H, \alpha^L\}$. In this case, the financier offers a haircut attractive enough to attract both the high type and the medium type, φ^M. Thus,

$$\underline{\hat{K}} = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\alpha^L p^L + \alpha^M (p^M + p^H)\right)\right).$$

Next, consider an increase in average potential default costs that comes from increasing α^L and decreasing α^M such that $p^L \Delta\alpha^L = -p^M \Delta\alpha^M + \varepsilon$. As long as these changes are not very large, the financier will still offer a haircut that attracts both the high type and the medium type. This haircut will have to increase, since medium agents are now more prone to defaulting:

$$\underline{\hat{K}}' - \underline{\hat{K}} = -\frac{1}{1-\epsilon}\frac{R^*}{R^*-1}(p^L \Delta\alpha^L + (p^M + p^H)\Delta\alpha^M)$$

When $\varepsilon \to 0$,

$$\underline{\hat{K}}' - \underline{\hat{K}} = -\frac{1}{1-\epsilon}\frac{R^*}{R^*-1} p^H \Delta\alpha^M > 0.$$

The increase in potential default costs lowers the losses borne by entrepreneurs, speeding up the recovery from the crisis. However, defaulting entrepreneurs (i.e. low types) must now pay a higher cost. Therefore, effective default costs increase.

Three types II Suppose $p^M = (\alpha^M)^{-1}(\alpha^H - \alpha^M)p^H - \frac{\varepsilon}{2}$, i.e. the financier slightly prefers offering φ^H to attract high types to offering φ^M and also attract medium types. Thus,

$$\underline{\hat{K}} = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\alpha^L p^L + \alpha^M p^M + \alpha^H p^H\right)\right).$$

Next, consider an increase in average potential default costs that comes from increasing p^M to $p^M + \varepsilon$ and decreasing p^L to $p^L - \varepsilon$. Clearly, this increases average default costs. Furthermore, now there are enough intermediate types that it is profitable for financiers to offer a more attractive haircut to induce them to renegotiate. The lower bound on entrepreneurs' capital

holdings after renegotiation is given by

$$\underline{\hat{K}}' = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\alpha^L(p^L - \varepsilon) + \alpha^M(p^M + \varepsilon + p^H)\right)\right).$$

Thus, when $\varepsilon \to 0$,

$$\underline{\hat{K}}' - \underline{\hat{K}} = \frac{1}{1-\epsilon}\frac{R^*}{R^*-1}(\alpha^H - \alpha^M)p^H > 0.$$

In other words, even though the average potential default cost went up, the financier, in order to attract intermediate types, offers a much larger haircut. Thus, entrepreneurs have a larger net worth as a group and the recovery of output at dates $t \geq 1$ is faster. Furthermore, there is also less effective default as fewer agents default (i.e. entrepreneurs of type M stop defaulting).

4 Discussion

Our model is well suited to describe events where a large number of agents find themselves in a dire situation they are not insured against. For example, when a big negative shock hits and agents have sizeable uncontingent liabilities.[29] Prominent recent examples include: the real estate market in the U.S. after the Great Recession of 2008, where a large number of borrowers had bought property using residential mortgage loans and secured commercial property loans, which are typically nonrecourse; and business failures due to the forced shutdown of non-essential businesses during the first months of the COVID-19 pandemic.

An ideal experiment for our paper would compare two economies that are identical ex ante, but different ex post, i.e. economies that differ only in how they allocate unforeseen capital losses. To see this, note that in our model the parameters of interest that determine the ex-post resolution of financial crises, α, μ, and θ, do not affect the steady state of the economy (q^*, K^*, B^*). That is, since the threat of default and renegotiation are triggered in crisis, they do not affect the "normal" debt capacity of a borrower.

In this sense, the work of Mian et al. (2015) suggests such an experiment. They exploit variation in foreclosure laws across U.S. states: While some states have a judicial requirement to foreclose homes, others do not. Getting a judicial requirement consumes time and resources, making it more costly for the lender to foreclose on the property. In this sense, it is akin to an increase in μ. In addition, the authors show that there are no significant

[29] A separate yet interesting question is why agents do not insure in the first place. One possibility, particularly relevant for our application, is that agents underestimate the true probability of a crisis. This is consistent with the evidence in Mian et al. (2015), as we discuss below.

differences between judicial and nonjudicial states in terms of house prices, leverage, loan-to-value ratios, and household characteristics between 2002 and 2005, i.e. in the "pre-crisis" period. This observation is consistent with μ not affecting the steady state of the model.

In our model, a higher μ leads to fewer defaults (foreclosures), larger haircuts, and smaller amplification after a sizeable shock (proposition 8). Mian et al. (2015) show there is a strong correlation between foreclosure laws and foreclosure propensities during the Great Recession, i.e. there are fewer foreclosures in states with a judicial requirement. Then, using judicial requirements as an instrument for foreclosures, they find a strong negative effect of foreclosures on house prices.[30] Our model is consistent with these results and suggests a new transmission mechanism: There are fewer distressed home sales not only because the judicial requirement prevented inefficient foreclosures, but also because it allowed other agents, who would have renegotiated anyway, to extract a larger haircut.

Another relevant experiment, also in the context of the Great Recession, is provided by Agarwal et al. (2017). They exploit the exposure of different zip codes to the Home Affordable Modification Program (HAMP) to study the effect of renegotiations on economic outcomes. HAMP provided financial incentives for intermediaries to renegotiate distressed financial loans, which we interpret as a higher μ since it increases the opportunity cost of letting agents default. They use investor-owned properties, which were initially not eligible, as a control for the effect of HAMP on renegotiations. They find that the program led to a net increase in the annual rate of temporary and permanent contract modifications, and reduced the foreclosure rate. They also show that regions with higher shares of mortgage renegotiations had lower house price declines.[31] Our model is consistent with these results.

Finally, our model can also shed light on some recent changes introduced into U.S. Bankruptcy Code. The Small Business Reorganization (SBR) Act of 2019 created Subchapter V of Chapter 11 of this Code to facilitate the rescue of small businesses. It seeks to achieve this aim by giving more bargaining power to borrowers in distress, by letting the debtor to remain in possession instead of having a creditors' committee, and by allowing more room for confirmation of a restructuring plan based on statutory entitlements rather than a vote.[32] When enacted, the SBR Act defined small businesses as those with fewer than USD 2.7 million in debt. In the wake of the COVID-19 pandemic, the Coronavirus

[30] They also find a negative effect on other measures of economic activity, such as residential investment and auto sales. Ghent and Kudlyak (2011) show that in states where recourse is available, its threat affects borrower behavior. This highlights another dimension along which to explore the heterogeneous response to the Great Recession.

[31] They also find lower consumer debt delinquency rates and a modest increase in auto sales.

[32] Usual requirements for secured creditor cramdown applies: they are still entitled to the value of collateral. Also, the SBR Act makes it possible to modify a residential mortgage where the proceeds of it were used to fund a business. See Janger (2020).

Aid, Relief, and Economic Security (CARES) Act of March 27, 2020, increased for one year the elegibility threshold to USD 7.5 million in debt. Given the expected effects of lockdown restrictions on non-essential businesses' financial health, our model provides a theoretical basis for the relaxation of elegibility criteria to qualify for Subchapter V as a way to increase distressed borrowers' bargaining power, and thus cushion output losses.[33]

5 Conclusions

Modelling the resolution of financial crises requires specifying how counterparties, and the legal system itself, deal with widespread broken promises. To this end, we provided a framework to study how institutions and technological factors determine the way the economy allocates financial losses and examined their macroeconomic implications. Our model emphasized the size and observability of debtors' default costs and the size of creditors' repossession costs. These costs were meant to capture, in reduced form, the frictions surrounding bankruptcy procedures that prevent creditors from collecting debts and discourage borrowers from starting new businesses.

We found that renegotiation and default put a lower bound on the financial losses borne by the most productive sector, thereby limiting the depth of the financial crisis. For this reason, renegotiation is welfare enhancing. Renegotiation is only triggered when shocks exceed a specific size, which is smaller if repossession costs are substantial. Entrepreneurs' capital holdings after renegotiation increase with repossession costs and borrowing capacity, which accelerates the recovery. The converse is true of entrepreneurs' default cost, but only if these costs are perfectly observable. Indeed, we constructed examples where these costs are private information, and an increase did not lead to less favourable haircuts or more equilibrium default.

We also showed that the unobservability of default costs imprints interesting dynamics on output after a financial crisis. On the one hand, when default costs are private information, some agents default in equilibrium, exacerbating the crisis to the extent that default entails deadweight losses. On the other hand, asymmetric information prevents financiers from effectively extracting surplus from entrepreneurs. In particular, entrepreneurs with high default costs obtain a larger haircut than in an economy where these costs are public information. Since entrepreneurs retain more capital after the crisis, welfare (net of default and repossession costs) is larger.

The model is useful to interpret developments in real estate markets in the United States

[33] Gourinchas et al. (2020) estimate the increase in the failure rates of SMEs to be of about 9 percentage points absent government support.

during the Great Recession. In particular, it suggests exceptional interventions in debt markets, such as HAMP, might have significant macroeconomic effects. These interventions may help not only by reducing inefficient liquidation but also by allowing entrepreneurs who would have renegotiated anyway extract a more substantial haircut. The model also helps rationalize the enlargement of elegibility criteria for Subchapter V of Chapter 11 of the U.S. Bankruptcy Code in the wake of the COVID-19 crisis, as this allows small non-essential businesses to emerge from the current recession with stronger balance sheets. We leave for future work an estimation of the effect of enhancing entrepreneurs' bargaining power by contrasting performance of firms that barely qualified to Subchapter V and those that barely did not.

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A Proofs and derivations

A.1 Section 2

A.1.1 Nash bargaining solution

Given our assumption of Nash bargaining, φ solves

$$\max_{\varphi \geq 0} (-(\hat{q}_0 + \varphi - \mu) q^* K^*)^{\theta} (U^R - U^D)^{1-\theta}$$

where $\theta \in [0, 1]$ is the financiers' bargaining power. Since $\hat{K}_0$ is taken as given, this program has the same solution as

$$\max_{\varphi \geq 0} \theta \ln(-\hat{q}_0 - \varphi + \mu) + (1 - \theta) \ln(\hat{q}_0 + \varphi + \alpha)$$

Note that the objective is concave. Thus, we can characterize the solution using the first order condition, which simplifies to

$$-\hat{q}_0 - \varphi - \theta\alpha + (1 - \theta)\mu \leq 0 \quad \text{with equality if } \varphi > 0.$$

Rearranging yields equation (6).

A.1.2 Lemma 1

First, guess $\hat{K}_0 = \underline{\hat{K}}(\Delta, \epsilon)$, which is defined by equation (9) (note $\underline{\hat{K}}(\cdot) \leq K^*$ since $\Delta \geq 0$ and $\epsilon \geq 0$). Plugging in $\underline{\hat{K}}(\cdot)$ into equation (8) implies some $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot))$. If $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) \leq \bar{q}$, then $\left(\underline{\hat{K}}(\cdot), \hat{q}_0^{ap}(\underline{\hat{K}}(\cdot))\right)$ is an equilibrium.

If $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) > \bar{q}$, we can ignore the max in the net worth relation. That is, we can define two functions $\hat{q}^{nw}(\hat{K}_0)$ and $\hat{q}^{ap}(\hat{K}_0)$ that describe asset prices that satisfy equations (7) and (8), respectively,

$$\hat{q}_0^{nw}(\hat{K}_0) = \frac{R^* - 1}{R^*} \left(\frac{u\left((1 + \hat{K}_0)K^*\right)}{a} (1 + \hat{K}_0)(1 - \epsilon) - (1 - \Delta) \right) \tag{18}$$

$$\hat{q}_0^{ap}(\hat{K}_0) = \frac{R^* - 1}{R^*} (1 - \epsilon) \left(\frac{u\left((1 + \hat{K}_0)K^*\right)}{a} + \sum_{t=1}^{\infty} \frac{1}{R^{*t}} \frac{u\left(f_t\left((1 + \hat{K}_0)K^*\right)\right)}{a} \right) - 1. \tag{19}$$

Note $\hat{q}_0^{nw}(\underline{\hat{K}}(\cdot)) = \bar{q}$ by definition of $\underline{\hat{K}}(\cdot)$, implying $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) > \hat{q}_0^{nw}(\underline{\hat{K}}(\cdot))$. At $\hat{K} = 0$,

$$\hat{q}_0^{nw}(0) = (\Delta - \epsilon)\frac{R^* - 1}{R^*}$$
$$\hat{q}_0^{ap}(0) = -\epsilon.$$

Since $\hat{q}_0^{ap}(0) \leq \hat{q}_0^{nw}(0)$, and both $\hat{q}_0^{nw}(\cdot)$ and $\hat{q}_0^{ap}(\cdot)$ are continuous functions, the intermediate value theorem implies there exists $\hat{K}^{eq} \in (\underline{\hat{K}}(\cdot), 0]$ such that $\hat{q}_0^{nw}(\hat{K}^{eq}) = \hat{q}_0^{ap}(\hat{K}^{eq})$. Thus, $\left(\hat{K}^{eq}, \hat{q}^{nw}(\hat{K}^{eq})\right)$ is an equilibrium.

A.1.3 Proposition 1

Part (a) Consider the system of equations (18) and (19). These equations describe equilibria with $\varphi = 0$, i.e. as long as $\hat{q}_0^{nw}(\hat{K}_0) > \bar{q}$. When $\Delta = 0$ and $\epsilon = 0$, $\hat{q}_0 = 0$ and $\hat{K}_0 = 0$ solve this system. Note that

$$\frac{d\hat{q}_0^{nw}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta,\epsilon)=\mathbf{0}} = \frac{R^* - 1}{R^*}(\frac{1}{\eta} + 1) \tag{20}$$
$$\frac{d\hat{q}_0^{ap}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta,\epsilon)=\mathbf{0}} = \frac{R^* - 1}{R^*}(\frac{1}{\eta} + \frac{1}{(R^* - 1)\eta + R^*}) \tag{21}$$

where $1/\eta \equiv \frac{d\log u(K)}{d\log K}|_{K=K^*}$ as in the original KM paper. Note $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta,\epsilon)=\mathbf{0}} > \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta,\epsilon)=\mathbf{0}}$. Thus, we can apply the implicit function theorem at the steady state to establish the existence of a unique continuously differentiable solution $\{\hat{K}_0^{km}(\Delta,\epsilon), \hat{q}_0^{km}(\Delta,\epsilon)\}$ in an open ball $\mathcal{B}$ around $(\Delta,\epsilon) = (0,0)$. The result then follows from the fact that $\hat{q}_0^{km}$ is continuous and $\hat{q}_0^{km}(0,0) > \bar{q}$. Note that this equilibrium exists until $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} = \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}$ (the curves become tangent), or $\hat{q}_0^{km} = \bar{q}$, whichever occurs first.

It remains to show that entrepreneurs' capital holdings and asset prices are decreasing

in Δ and ϵ in these equilibria. We rely again on the implicit function theorem to compute:

$$\frac{d\hat{q}_0^{km}}{d\Delta} = -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}} \frac{d\hat{q}_0^{ap}}{d\hat{K}_0} < 0 \tag{22}$$

$$\frac{d\hat{K}_0^{km}}{d\Delta} = -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}} < 0 \tag{23}$$

$$\frac{d\hat{q}_0^{km}}{d\epsilon} = -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}} \left(\sum_{s=0} \frac{1}{R^{*s}} \underbrace{\frac{u(K_s)}{a}}_{\geq u(K_0)/a} - \underbrace{(1+\hat{K}_0)}_{\leq 1} \frac{u(K_0)}{a} \right) \frac{d\hat{q}_0^{ap}}{d\hat{K}_0} \tag{24}$$

$$- \frac{R^*-1}{R^*} \left(\sum_{t=0}^{\infty} \frac{1}{R^{*t}} \frac{u(K_t)}{a} \right) < 0$$

$$\frac{d\hat{K}_0^{km}}{d\epsilon} = -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}} \left(\sum_{s=0} \frac{1}{R^{*s}} \underbrace{\frac{u(K_s)}{a}}_{\geq u(K_0)/a} - \underbrace{(1+\hat{K}_0)}_{\leq 1} \frac{u(K_0)}{a} \right) < 0, \tag{25}$$

where we used the fact that we are focusing on equilibria with $K_0 \leq K^*$, $\{K_t\}$ is an increasing sequence, $u(K_t)$ is increasing in K_t, $d\hat{q}_0^{ap}/dK_t > 0$, and, by continuity, $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}} > 0$.

Part (b) In an equilibrium with renegotiation, $\hat{K}_0 = \underline{\hat{K}}(\Delta, \epsilon)$. Clearly, $\underline{\hat{K}}(\cdot)$ is strictly decreasing in Δ and strictly increasing in ϵ. An equilibrium with renegotiation exists if

$$\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) = \frac{R^*-1}{R^*}(1-\epsilon) \left(\frac{u\left((1+\underline{\hat{K}}(\cdot))K^*\right)}{a} + \sum_{t=1}^{\infty} \frac{1}{R^t} \frac{u\left(f_t\left((1+\underline{\hat{K}}(\cdot))K^*\right)\right)}{a} \right) - 1 \leq \bar{q}.$$

Since $\underline{\hat{K}}(\cdot)$ is strictly decreasing in Δ, while $\hat{q}_0^{ap}(\hat{K})$ is strictly increasing in $\hat{K}$, it follows that if this condition is satisfied for some Δ, it is also satisfied for all $\Delta' < \Delta$. Three cases may arise. First, it could be that evaluated at $\Delta = 0$, $\hat{q}^{ap}(\underline{\hat{K}}(\cdot)) < \bar{q}$. In this case, there will be a renegotiation equilibrium $\forall \Delta$ so $\bar{\Delta} = 0$. Second, it could be that even evaluating the previous expression at $\Delta = \tilde{\Delta}$, $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) > \bar{q}$. In this case there will be no renegotiation equilibrium so trivially $\bar{\Delta} = \tilde{\Delta}$. If neither of these cases arise, then since $\underline{\hat{K}}(\cdot)$ is continuous and monotone, we can define $\bar{\Delta}(\epsilon)$ as the solution to

$$\bar{q} = \frac{R^*-1}{R^*}(1-\epsilon) \left(\frac{u\left(\left(1+\underline{\hat{K}}(\bar{\Delta}(\epsilon),\epsilon)\right)K^*\right)}{a} + \sum_{t=1}^{\infty} \frac{1}{R^t} \frac{u\left(f_t\left(\left(1+\underline{\hat{K}}(\bar{\Delta}(\epsilon),\epsilon)\right)K^*\right)\right)}{a} \right) - 1 \tag{26}$$

Finally, since $\hat{q}_0^{ap}(\cdot)$ does not directly depend on Δ, $\underline{\hat{q}}_0(\Delta,\epsilon) = \hat{q}_0^{ap}(\underline{\hat{K}}(\Delta,\epsilon))$ is strictly decreasing in Δ. Finally, equation (6) immediately implies φ is strictly increasing in Δ.

A.1.4 Proposition 2

When $\bar{\Delta}(\epsilon) \in (0, \tilde{\Delta})$, i.e. when it is interior, it must solve equation (26). Consider a decrease in $\bar{q}$, which is equivalent to an increase in $\alpha\theta - (1-\theta)\mu$. Equation (26) implies that $\underline{\hat{K}}(\Delta,\epsilon)$ decreases. Since $\hat{q} = \bar{q}$, (9) is the same as net-worth relation (7). Thus,

$$d\bar{q} = \frac{d\hat{q}_0^{nw}}{d\hat{K}_0} d\underline{\hat{K}}(\cdot) + \frac{R^*}{R^*-1} d\bar{\Delta}(\epsilon)$$
$$d\bar{q} = \frac{d\hat{q}_0^{ap}}{d\hat{K}_0} d\underline{\hat{K}}(\cdot).$$

Solving,

$$\frac{d\bar{\Delta}(\epsilon)}{d\bar{q}} = -\frac{R^*-1}{R^*}\left(\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}\right)\frac{d\underline{\hat{K}}(\cdot)}{d\bar{q}}.$$

If the equilibrium is unique for all Δ, then at the threshold it must belong to the equilibrium set described in proposition (1) part (a), $\underline{\hat{K}}(\cdot) = \hat{K}^{km}(\bar{\Delta}(\epsilon),\epsilon)$. Thus, generically, $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} > \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}$, which implies $\frac{d\bar{\Delta}(\epsilon)}{d\bar{q}} < 0$.

Next, focus on the interior of the renegotiation region $\Delta > \bar{\Delta}(\epsilon)$. Equation (9) implies that $\underline{\hat{K}}(\Delta,\epsilon)$ strictly decreases when $\bar{q}$ decreases. Then, equation (8) implies $\underline{\hat{q}}(\Delta,\epsilon)$ decreases as well.

A.1.5 Proposition 3

When renegotiation is not available, default is optimal if

$$U^R(\varphi = 0) - U^D = \frac{acK^*}{u(K_0)(1-\epsilon)}\frac{\beta R^*}{R^*-1}(\hat{q}_0 + \alpha)\left(\sum_{t=0}^{\infty}\beta^t\left(\prod_{s=0}^{t}\frac{a}{u(f_s(K_0))}\right)\right) < 0.$$

Thus, entrepreneurs default if

$$\hat{q}_0 < -\alpha.$$

This is the same default threshold as an economy where renegotiation is available and $\theta = 1$. Since entrepreneurs' net-worth is the same with or without renegotiation in this case, the result follows. The case $\theta < 1$ is an immediate consequence of proposition 2.

A.1.6 Corollary 1

Total output net of default costs at $t = 0$ is predetermined. In periods $t \geq 1$,

$$Y_t = (a + c)K_t + G(\bar{K} - K_t)$$

Thus,

$$\begin{aligned}\frac{dY_t}{dK_t} &= a + c - G'(\bar{K} - K_t) \\ &\geq a + c - G'(\bar{K} - K^*) \\ &\geq (1 - R)\, a + c \\ &> 0,\end{aligned}$$

where the first inequality follows from $K_t \leq K^*$ and last inequality follows from the assumption that guarantees that investing is optimal at the steady state (assumption 2 in KM).

Since agents have linear utilities, it follows that welfare is proportional to output. By proposition 3, the path for K_t is higher when agents renegotiate and $\theta < 1$. Thus, the result follows.

A.1.7 Section 2.6

We first present and solve the bargaining problem in the interim stage. Then, we derive the steady state of the extended model and prove proposition 4. Third, we derive a system of equations that describes the equilibrium after a negative productivity shock. Finally, we prove proposition 5.

Bargaining in the interim stage In this section, we characterize the solution to the bargaining problem between financiers and entrepreneurs *before* tradable output is realized. If entrepreneurs walk away from the contract, they do not have to pay $R_t B_t$ tomorrow, but they also lose their capital stock and the tradable output this capital would produce. Recall that, for simplicity, we assume that they would not lose their nontradable output (see footnote 5).[34] Given a haircut of ϕ, by staying in the contract entrepreneurs get $(a_{t+1} + q_{t+1})k_t - (1 - \phi)R_t B_t$. On the other hand, if financiers successfully prevent entrepreneurs from walking away, they obtain $(1 - \phi)R_t B_t - q_{t+1}k_t$. (For simplicity, we assume there are

[34] This assumption is important for analytical tractability. Without it, we would need to keep track of the entrepreneurs' marginal value of wealth, which would make the problem more complicated.

no costs of repossessing the capital stock at this stage.)

The optimal ϕ solves the following Nash-bargaining problem,

$$\max_{\phi \geq 0} \iota \ln\left((1-\phi)R_t B_t - q_{t+1}k_t\right) + (1-\iota)\ln\left((q_{t+1}+a_{t+1})k_t - (1-\phi)R_t B_t\right)$$

For the original contract to be renegotiation-proof, the first derivative of the objective function evaluated at $\phi = 0$ should be negative,

$$-\frac{\iota R_t B_t}{R_t B_t - q_{t+1}k_t} + \frac{(1-\iota)R_t B_t}{(q_{t+1}+a_{t+1})k_t - R_t B_t} \leq 0$$

Solving,

$$R_t B_t \leq q_{t+1}k_t + \iota a_{t+1}k_t.$$

Steady state We start by characterizing the steady state. Since $\beta < \beta'$, entrepreneurs borrow as much as they can,

$$R^* B^* = q^* K^* + \iota a K^*. \tag{27}$$

The net worth of entrepreneurs that survive until the following period is given by

$$N^* = aK^* + (1-\chi)\left(q^* K^* - R^* B^*\right). \tag{28}$$

Replacing,

$$N^* = aK^* - (1-\chi)\iota a K^*$$

We conjecture, and later verify, that surviving entrepreneurs invest all their net worth into capital such that

$$q^* K^* = N^* + B^*.$$

Next, use (27) to substitute B^* and (28) to substitute N^*. After some algebra, we obtain

$$q^* = \frac{R^*}{R^*-1}\left(1-(1-\chi)\iota + R^{*-1}\iota\right)a. \tag{29}$$

Under assumption 2, $R^{*-1} \geq 1-\chi$, higher financiers' bargaining power in the interim stage ι leads to higher steady state asset price q^*, strictly so if $R^{*-1} > 1-\chi$. Note that, if $\chi = 0$, higher financiers' bargaining power in the interim stage would imply lower steady-state entrepreneur capital holdings and asset prices. In other words, the net-worth effect would dominate.

Entrepreneur's capital holdings can be found from the asset-pricing relation,

$$q^* = \frac{1}{R^* - 1} G'(\bar{K} - K^*).$$

Thus, K^* also increases with ι. Finally, note that aggregate leverage is given by

$$\frac{R^* B^*}{q^* K^*} = 1 + \left(\frac{R^* - 1}{R^*}\right)\left(\frac{\iota}{1 - (1-\chi)\iota + R^{*-1}\iota}\right).$$

Therefore, higher financiers' bargaining power in the interim stage ι leads to higher steady state leverage.

It remains to verify that maximum investment by entrepreneurs is optimal. Consider an entrepreneur that reinvests all the returns and consumes at her death. The returns of this strategy are given by

$$\pi = \beta \sum_{s=0}^{\infty} \underbrace{\beta^s (1-\chi)^s c k_s}_{\text{survive } s \text{ more periods}} + \beta \sum_{s=0}^{\infty} \underbrace{\beta^s (1-\chi)^s \chi a k_s}_{\text{die at } s}$$

Noting that the law of motion for individual capital is

$$k_s = \left(\frac{(1-\iota)\, a}{q^* - \frac{1}{R^*} q^* - \frac{1}{R^*}\iota a}\right)^s k_0$$

and $k_0 = 1/q^* - \frac{1}{R^*} q^* - \frac{1}{R^*}\iota a$,

$$\pi = \frac{\beta}{q^* - \frac{1}{R^*} q^* - \frac{1}{R^*}\iota a} \sum_{s=0}^{\infty} \left(\frac{\beta\,(1-\iota)\,(1-\chi)\, a}{q^* - \frac{1}{R^*} q^* - \frac{1}{R^*}\iota a}\right)^s (c + \chi a)\,.$$

This dominates consumption if $\pi > 1$. After some algebra, we can write this condition as,

$$\beta\,(c + \chi a) > q^* - \frac{1}{R^*} q^* - \frac{1}{R^*}\iota a - \beta\,(1-\iota)\,(1-\chi)\, a.$$

Using (29) and rearranging we obtain

$$c > a(\beta^{-1} - 1)(1 - (1-\chi)\iota).$$

When $\iota \to 0$, this becomes $c > a(\beta^{-1} - 1)$ (assumption 2 in KM).

We also assume that the minimum return of capital in the hands of financiers, $\beta' G'(\bar{K})$, is above $R^{*-1}\iota a$, i.e.

$$R^* u(0) = G'(\bar{K}) > \iota a. \tag{30}$$

This ensures that for any distribution of capital in the economy, collateral is needed, i.e. the downpayment can never be fully paid by the promise of future fruit a.

Equilibrium after a negative productivity shock Next, we analyze the case of a temporary productivity shock that lowers a_t from a to $a(1-\Delta)$ at $t=0$.

Continuation equilibrium After $t \geq 1$, the economy is deterministic so there is no default. For an individual entrepreneur,

$$n_t = a(1-\iota)k_{t-1}.$$

We conjecture, and later verify, that (i) entrepreneurs invest all their tradable income and, (ii) constraint (11) is the binding constraint. Under these conjectures, the budget constraint implies

$$1+\hat{k}_t = \frac{a(1-\iota)}{u\left((1+\hat{K}_t)K^*\right) - R^{*-1}\iota a}(1+\hat{k}_{t-1}).$$

Aggregating across entrepreneurs and taking into account births and deaths, we obtain

$$1+\hat{K}_t = \frac{a}{u\left((1+\hat{K}_t)K^*\right) - R^{*-1}\iota a}\left(1-(1-\chi)\iota\right)(1+\hat{K}_{t-1}). \tag{31}$$

Equation (30) implies $u\left((1+\hat{K}_t)K^*\right) > R^{*-1}\iota a$. This, together with the fact that $u' > 0$, imply that $d\hat{K}_t/d\hat{K}_{t-1} > 0$. In other words, (31) describes an increasing relation $\hat{K}_t = g_t(\hat{K}_{t-1})$. Iterating forward, we obtain an increasing relation $\hat{K}_t = f_t(\hat{K}_0)$. Note that, if $\hat{K}_0 < 0$, then $\hat{K}_t < 0$. Thus, since $u\left((1+\hat{K}_t)K^*\right) < u\left(K^*\right)$, it follows that $\hat{K}_t > \hat{K}_{t-1}$ if $\hat{K}_0 < 0$. Thus, asset prices are increasing and investing is indeed optimal, verifying conjecture (i) if $\hat{K}_0 < 0$.

Solving for the haircut At $t=0$, the entrepreneur has two options: to renegotiate or to default. The amount of capital the entrepreneur can buy will be impacted by this decision,

$$1+\hat{k}_0^R = \frac{a}{u\left((1+\hat{K}_0)K^*\right) - R^{*-1}\iota a}\left((1-\iota-\Delta) + \frac{q^*}{a}(\hat{q}_0+\varphi) + \varphi\iota\right)$$

$$1+\hat{k}_0^D = \frac{a}{u\left((1+\hat{K}_0)K^*\right) - R^{*-1}\iota a}\left((1-\Delta) - \frac{q^*(\iota)}{a}\alpha\right)$$

where $\hat{k}_0^R$ and $\hat{k}_0^D$ denote, respectively, the amount of capital that can be purchased in the case of renegotiation and default, respectively.

Next, we compute the implied entrepreneurs' utilities of default and renegotiation given the shocks, their aggregate capital holdings K_0, and the proposed haircut φ,

$$U^i = \beta c k_0^i + \beta^2 c k_1^i + ... + \lim_{t\to\infty} \beta^t c k_{t-1}^i$$

with $i = R, D$. Using our previous results, we obtain

$$U^R - U^D = \frac{acK^*}{u\left((1+\hat{K}_0)K^*\right) - R^{*-1}\iota a}\beta\left(-(1-\varphi)\iota + \frac{q^*}{a}(\hat{q}_0 + \varphi + \alpha)\right) \tag{32}$$
$$\times \left(\sum_{t=0}^{\infty}(1-\chi)^t\beta^t\left(\prod_{s=0}^{t}\frac{a}{u((1+f_s(\hat{K}_0))K^*) - R^{*-1}\iota a}\right)\right).$$

Renegotiation gives an entrepreneur surplus $U^R - U^D$ while a financier gets surplus $-(\hat{q}_0 + \varphi - \mu - (1-\varphi)\iota\frac{a}{q^*})q^*K^*$. Given our assumption of Nash bargaining, φ solves

$$\max_{\varphi\geq 0}\left(-(\hat{q}_0 + \varphi - \mu - (1-\varphi)\iota\frac{a}{q^*})\right)^{\theta}\left(-(1-\varphi)\iota + \frac{q^*}{a}(\hat{q}_0 + \varphi + \alpha)\right)^{1-\theta}$$

where $\theta \in [0,1]$ is the financiers' bargaining power. Since $\hat{K}_0$ is taken as given, this program has the same solution as

$$\max_{\varphi\geq 0}\theta\ln\left(-\hat{q}_0 - \varphi + \mu + (1-\varphi)\iota\frac{a}{q^*})\right) + (1-\theta)\ln\left(-(1-\varphi)\iota\frac{a}{q^*} + (\hat{q}_0 + \varphi + \alpha)\right)$$

Note that the objective is concave. Thus, we can characterize the solution using the first order condition. After some algebra, we obtain

$$\varphi = \max\left\{\frac{1}{1+\iota\frac{a}{q^*}}\left(-\hat{q}_0 - \theta\alpha + (1-\theta)\mu + \iota\frac{a}{q^*}\right), 0\right\}.$$

Replacing q^* we obtain the expression in the main text. Note that renegotiation is triggered when

$$\hat{q}_0 \leq -\theta\alpha + (1-\theta)\mu + \iota\frac{a}{q^*} = \bar{q}. \tag{33}$$

Now we are ready to verify conjecture (ii), conditional on $\hat{K}_0 \leq 0$. Suppose that the ex

post constraint binds, i.e.

$$R^* b_t = q_{t+1} k_t + \theta \alpha q_t k_t - (1-\theta)\mu q_t k_t.$$

Since the path for asset prices is increasing when $\hat{K}_0 \leq 0$, renegotiation puts a floor on the fall in asset prices, i.e. $q_t \geq (1+\overline{q})q^*$, and $\theta\alpha > (1-\theta)\mu$,

$$R^* b_t \geq q_{t+1} k_t + \theta \alpha (1+\overline{q}) q^* k_t - (1-\theta)\mu(1+\overline{q})q^* k_t.$$

Substituting in (33), and adding and substracting $\iota a k_t$,

$$R^* b_t \geq q_{t+1} k_t + \iota a k_t - (1 - \theta\alpha + (1-\theta)\mu) \underbrace{\left(-\theta\alpha q^* + (1-\theta)\mu q^* + \iota a\right)}_{<0} q^* k_t,$$

where the term in braces is negative given (13). Thus,

$$R^* b_t > q_{t+1} k_t + \iota a k_t,$$

which violates constraint (11). Thus, (11) binds.

Equilibrium at date 0 Using the solution for φ, we can fully characterize the equilibrium at date 0 by

$$\left(u\left((1+\hat{K}_0)K^*\right) - R^{*-1}\iota a\right)(1+\hat{K}_0) = a\left(1 - \Delta - (1-\chi)\iota + (1-\chi)\frac{q^*}{a}\max\{\hat{q}_0, \overline{q}\}\right) \tag{34}$$

$$1 + \hat{q}_0 = \frac{1}{q^*}\left(u\left((1+\hat{K}_0)K^*\right) + \sum_{t=1}^{\infty}\frac{1}{R^{*t}} u\left(\left(1 + f_t(\hat{K}_0)\right)K^*\right)\right). \tag{35}$$

Note that the system is very similar the one we obtained in section 2, given by equations (7) and (8).

Let $\underline{\hat{K}}(\Delta, \iota)$ denote the lower bound on entrepreneur's capital holdings given by renegotiation:

$$\left(u\left(K^*\left(1 + \underline{\hat{K}}(\cdot)\right)\right) - R^{*-1}\iota a\right)(1 + \underline{\hat{K}}(\cdot)) = a\left(1 - \Delta - (1-\chi)\iota + (1-\chi)\frac{q^*}{a}\overline{q}\right). \tag{36}$$

Note that $\underline{\hat{K}}(\cdot) < 0$ since $\Delta > 0$ and $\bar{q} < 0$. Plugging in $\underline{\hat{K}}(\cdot)$ into (35) implies some $\underline{\hat{q}}(\cdot) = \hat{q}_0^{ap}(\underline{\hat{K}}(\cdot))$. If $\underline{\hat{q}}(\cdot) \leq \bar{q}$, then $\left(\underline{\hat{K}}(\cdot), \underline{\hat{q}}(\cdot)\right)$ is an equilibrium.

If $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) > \bar{q}$, we can ignore the max in the net-worth relation. That is, we can define two functions

$$\hat{q}_0^{nw}(\hat{K}_0) = \frac{a}{q^*(1-\chi)} \left\{ \left(\frac{u\left((1+\hat{K}_0)K^*\right) - R^{*-1}\iota a}{a} \right) (1+\hat{K}_0) - (1-\Delta-(1-\chi)\iota) \right\} \tag{37}$$

$$\hat{q}_0^{ap}(\hat{K}_0) = \frac{1}{q^*} \left(u\left((1+\hat{K}_0)K^*\right) + \sum_{t=1}^{\infty} \frac{1}{R^{*t}} u\left(\left(1+f_t(\hat{K}_0)\right)K^*\right) \right) - 1. \tag{38}$$

Note that $\hat{q}_0^{nw}(\underline{\hat{K}}(\cdot)) = \bar{q}$ by definition of $\underline{\hat{K}}(\cdot)$, implying $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) > \hat{q}_0^{nw}(\underline{\hat{K}}(\cdot))$. At $\hat{K} = 0$,

$$\begin{aligned} \hat{q}_0^{nw} &= \frac{a}{q^*(1-\chi)}\Delta \\ \hat{q}_0^{ap} &= 0. \end{aligned}$$

Since $\hat{q}_0^{ap}(0) \leq \hat{q}_0^{nw}(0)$, and both $\hat{q}_0^{nw}$ and $\hat{q}_0^{ap}$ are continuous functions, the intermediate value theorem implies that there exists $\hat{K}_0^{eq} \in (\underline{\hat{K}}, 0]$ such that $\hat{q}_0^{ap}(\hat{K}_0^{eq}) = \hat{q}_0^{nw}(\hat{K}_0^{eq})$. Thus, $(\hat{K}_0^{eq}, \hat{q}_0^{eq})$ is an equilibrium. Since $\hat{K}_0 \leq 0$, this completes our verification of conjectures (i) and (ii).

Proof of proposition 5

Part (a) Consider the system of equations (37) and (38). These equations describe equilibria with $\varphi = 0$, i.e. as long as $\hat{q}_0^{nw}(\cdot) > \bar{q}$. When $\Delta = 0$, $\hat{q}_0(\cdot) = 0$ and $\hat{K}_0(\cdot) = 0$ solve this system, proving (i). Clearly, we get, generically, that $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta)=\mathbf{0}} \neq \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta)=\mathbf{0}}$. Thus, we can apply the implicit function theorem at the steady state to establish the existence of a unique continuously differentiable solution $\{\hat{K}_0^{km}(\Delta;\iota), \hat{q}_0^{km}(\Delta;\iota)\}$ for $\Delta < \mathring{\Delta}$ for some $\mathring{\Delta} > 0$. Result (ii) then follows from the fact that $\hat{q}_0^{km}(\cdot)$ is continuous and $\hat{q}_0^{km}(0,0) > \bar{q}$.

Next, consider a shock Δ in the interior of the $\varphi = 0$ region, i.e. such that $\hat{q}_0 > \bar{q}$. Differentiating (37) around $(\hat{K}_0^{km}, \Delta)$,

$$\begin{aligned} \left(u'(K_0^{km})K_0^{km} + u(K_0^{km}) - R^{*-1}\iota a\right) d\hat{K}_0^{km} + a\frac{K_0^{km}}{K^*}\frac{u'(K_0^{km})K_0^{km}}{u'(K^*)K^*}\left(R^{*-1} - (1-\chi)\right) d\iota - R^{*-1}a\frac{K_0^{km}}{K^*}d\iota = \\ -a(1-\chi)d\iota + (1-\chi)(\frac{q_0^{km} - q^*}{q^*})\frac{R^*}{R^*-1}\left(R^{*-1} - (1-\chi)\right) a d\iota + (1-\chi)q^* d\hat{q}_0^{km}, \end{aligned}$$

where we used

$$\frac{dq^*(\iota)}{d\iota} = \frac{R^*}{R^*-1}\left(R^{*-1} - (1-\chi)\right)a \tag{39}$$

$$\frac{dK^*(\iota)}{d\iota} = \left(\frac{R^{*-1} - (1-\chi)}{u'(K^*)}\right)a. \tag{40}$$

Solving,

$$\frac{d\hat{K}_0}{d\iota} = \frac{a}{u'(K_0)K_0 + u(K_0) - R^{*-1}\iota a}\{-\frac{K_0}{K^*}\frac{u'(K_0)K_0}{u'(K^*)K^*}\left(R^{*-1} - (1-\chi)\right) + R^{*-1}\frac{K_0}{K^*} - (1-\chi)$$
$$+ (1-\chi)(\frac{q_0 - q^*}{q^*})\frac{R^*}{R^*-1}\left(R^{*-1} - (1-\chi)\right) + (1-\chi)\frac{q^*}{a}\frac{d\hat{q}_0}{d\iota}\}.$$

Taking $\chi \to 1$,[35]

$$\frac{d\hat{K}_0}{d\iota} = \frac{a}{u'(K_0)K_0 + u(K_0) - R^{*-1}\iota a}\frac{K_0}{K^*}\left(1 - \underbrace{\frac{u'(K_0)K_0}{u'(K^*)K^*}}_{\leq 1}\right)R^{*-1} \geq 0.$$

where we used the assumption that $u'(K)K$ is increasing.

Part (b) In an equilibrium with renegotiation, $\hat{K}_0 = \underline{\hat{K}}(\cdot)$, which is defined by equation (36). Thus, an equilibrium with renegotiation exists if

$$\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) = \frac{1}{q^*(\iota)}\left(u\left((1+\underline{\hat{K}}(\cdot))K^*(\iota)\right) + \sum_{t=1}^{\infty}\frac{1}{R^{*t}}u\left(\left(1 + f_t(\underline{\hat{K}}(\cdot))\right)K^*(\iota)\right)\right) - 1 \leq \bar{q}.$$

Since $\underline{\hat{K}}(\Delta, \iota)$ is strictly decreasing in Δ, while $\hat{q}_0^{ap}(\hat{K})$ is strictly increasing in $\hat{K}$, it follows that if this condition is satisfied for some Δ, it is also satisfied for all $\Delta' < \Delta$. Three cases may arise. First, it could be that evaluated at $\Delta = 0$, $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) < \bar{q}$. In this case, there will be a renegotiation equilibrium $\forall\Delta$ so $\bar{\Delta} = 0$. Second, it could be that even evaluating the previous expression at $\Delta = \tilde{\Delta}$, $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) > \bar{q}$. In this case there will be no renegotiation equilibrium so trivially $\bar{\Delta} = \tilde{\Delta}$. If neither of these cases arise, then since $\underline{\hat{K}}(\cdot)$ is continuous

[35] Formally, one needs to show that $\frac{d\hat{q}_0}{d\iota}$ is bounded. This follows from differentiating equations (35) and (31).

and monotone, we can define $\bar{\Delta}(\iota)$ as the solution to

$$\bar{q} = \frac{1}{q^*(\iota)}\left(u\left((1+\underline{\hat{K}}(\bar{\Delta}(\iota),\iota))K^*(\iota)\right) + \sum_{t=1}^{\infty}\frac{1}{R^{*t}}u\left(\left(1+f_t(\underline{\hat{K}}(\bar{\Delta}(\iota),\iota))\right)K^*(\iota)\right)\right) - 1. \tag{41}$$

To prove the second part of the statement, replace $\bar{q}$ into (36) to obtain

$$\left(u\left(K^*\left(1+\underline{\hat{K}}(\cdot)\right)\right) - R^{*-1}\iota a\right)(1+\underline{\hat{K}}(\cdot)) = a\left(1-\Delta+(1-\chi)\frac{q^*(\iota)}{a}(-\theta\alpha+(1-\theta)\mu)\right).$$

Totally differentiating this equation yields

$$\left(u'\left(\underline{K}(\cdot)\right)\underline{K}(\cdot) + \left(u\left(\underline{K}(\cdot)\right) - R^{*-1}\iota a\right)\right)d\underline{\hat{K}}(\cdot) + u'\left(\underline{K}(\cdot)\right)\frac{\underline{K}(\cdot)^2}{K^{*2}}\frac{dK^*(\iota)}{d\iota}d\iota - \frac{\underline{K}(\cdot)}{K^*}R^{*-1}ad\iota = (1-\chi)\frac{dq^*(\iota)}{d\iota}(-\theta\alpha+(1-\theta)\mu)d\iota$$

Using (39)and (40) and rearranging,

$$\frac{d\underline{\hat{K}}(\cdot)}{d\iota} = \frac{a}{u'\left(\underline{K}(\cdot)\right)\underline{K}(\cdot) + u\left(\underline{K}(\cdot)\right) - R^{*-1}\iota a} \times \left((1-\chi)\frac{R^*}{R^*-1}\left(R^{*-1}-(1-\chi)\right)(-\theta\alpha+(1-\theta)\mu) - \frac{u'\left(\underline{K}(\cdot)\right)\underline{K}(\cdot)}{u'(K^*)K^*}\frac{\underline{K}(\cdot)}{K^*}\left(R^{*-1}-(1-\chi)\right) + \frac{\underline{K}(\cdot)}{K^*}R^{*-1}\right).$$

Since $u'(K)K$ is increasing and $-\theta\alpha+(1-\theta)\mu \leq -\alpha$,

$$\frac{d\underline{\hat{K}}(\cdot)}{d\iota} \geq \frac{a(1-\chi)}{u'\left(\underline{K}(\cdot)\right)\underline{K}(\cdot) + u\left(\underline{K}(\cdot)\right) - R^{*-1}\iota a}\left(-\frac{R^*}{R^*-1}\left(R^{*-1}-(1-\chi)\right)\alpha + \frac{\underline{K}(\cdot)}{K^*}\right).$$

Next, we use that there is a lower bound on K_0, which we denote $\langle K\rangle$. That is, we evaluate the net-worth relation at the maximum possible shock such that default remains feasible:

$$\begin{aligned}\frac{\langle K\rangle}{K^*} &= \frac{R^*}{R^*-1}\frac{1-(1-\chi)\iota + R^{*-1}\iota}{u\left(\langle K\rangle\right) - R^{*-1}\iota a}a\left(\alpha+(1-\chi)(-\theta\alpha+(1-\theta)\mu)\right)\\ &\geq \frac{R^*}{R^*-1}\alpha\chi.\end{aligned}$$

Replacing,

$$\frac{d\underline{\hat{K}}(\cdot)}{d\iota} \geq \frac{a(1-\chi)}{u'\left(\underline{K}(\cdot)\right)\underline{K}(\cdot) + u\left(\underline{K}(\cdot)\right) - R^{*-1}\iota a}\frac{R^*}{R^*-1}\alpha\left(-R^{*-1}+1-\chi+\chi\right) > 0.$$

Part (c) Next, we totally differentiate (31),

$$u'(K_t)(1+\hat{K}_t)^2\frac{dK^*}{d\iota}d\iota + (u'(K_t)K_t + u(K_t) - R^{*-1}\iota a)d\hat{K}_t - (1+\hat{K}_t)R^{*-1}ad\iota = \\ a(1-(1-\chi)\iota)d\hat{K}_{t-1} - a(1-\chi)(1+\hat{K}_{t-1})d\iota$$

Solving,

$$d\hat{K}_t = \underbrace{\frac{a(1-(1-\chi)\iota)}{(u'(K_t)K_t + u(K_t) - R^{*-1}\iota a)}}_{+} d\hat{K}_{t-1} + a\underbrace{\frac{\frac{K_t}{K^*}R^{*-1} - \frac{(1-\chi)K_{t-1}}{K^*} - \frac{u'(K_t)K_t^2}{u'(K^*)K^{*2}}\left(R^{*-1}-(1-\chi)\right)}{u'(K_t)K_t + u(K_t) - R^{*-1}\iota a}}_{+} d\iota.$$

Since $K_t > K_{t-1}$, $R^{*-1} > 1-\chi$, and $u'(x)x$ is increasing in x, these terms are unambiguously positive. Replacing backwards, we obtain $\frac{d\hat{K}_t}{d\iota}|_{\hat{K}_0} > 0$.

A.2 Section 3

A.2.1 Lemma 2

First, guess $\hat{K}_0 = \underline{\hat{K}}(\Delta, \epsilon)$. Note $\underline{\hat{K}}(\cdot) \leq 0$ since $\Delta \in [0, \tilde{\Delta}]$ and $\epsilon \in [0, \frac{R^*}{R^*-1}((\bar{\alpha}(1-F(\bar{\alpha})) + \mathbb{E}(\alpha_i|\alpha_i \leq \bar{\alpha})F(\bar{\alpha}))]$. Plugging $\underline{\hat{K}}(\cdot)$ into equation (8) implies some $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot))$. If $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) \leq -\bar{\alpha}$, then $\left(\underline{\hat{K}}(\cdot), \hat{q}_0^{ap}\left(\underline{\hat{K}}(\cdot)\right)\right)$ is an equilibrium.

If $\hat{q}_0^{ap}(\underline{\hat{K}}(\cdot)) > -\bar{\alpha}$, then $\underline{\hat{K}}(\cdot)$ is not an equilibrium. We proceed as in the proof of lemma 1 and define $\hat{q}_0^{nw}(\hat{K})$ as the solution to (15) in the region $\hat{q}_0^{nw}(\hat{K}) \geq -\bar{\alpha}$, i.e. replacing $\min\{\hat{q}_0, \bar{\alpha}\}$ by $\hat{q}_0^{nw}(\hat{K})$. Clearly, by definition, $\hat{q}_0^{nw}(\underline{\hat{K}}(\cdot)) = -\bar{\alpha}$. Thus, $\hat{q}_0^{nw}(\underline{\hat{K}}(\cdot)) < \hat{q}_0^{ap}(\underline{\hat{K}}(\cdot))$. Furthermore, since $\hat{q}_0^{nw}(\hat{K})$ increases with $\hat{K}$, we have $-\hat{q}_0^{nw}(\hat{K}) \geq \bar{\alpha}\ \forall \hat{K} \in [\underline{\hat{K}}(\cdot), 0]$, so $\hat{q}_0^{nw}(\hat{K})$ describes the net-worth relation on this interval. In particular, at $\hat{K} = 0$, we have

$$\Delta - \epsilon = -\frac{R^*}{R^*-1}\left(\mathbb{E}\left(\alpha|\alpha \leq -\hat{q}_0^{nw}(0)\right)F(-\hat{q}_0^{nw}(0)) + (-\hat{q}_0^{nw}(0))\left(1-F(-\hat{q}_0^{nw}(0))\right)\right)$$
$$\hat{q}_0^{ap}(0) = -\epsilon$$

Since $\Delta \geq 0$, this implies

$$\hat{q}_0^{ap}(0) \leq \frac{R^*}{R^*-1}\hat{q}_0^{nw}(0)\left(1-F(-\hat{q}_0^{nw}(0))\right).$$

Using that $-\hat{q}_0^{nw}(0) \geq \bar{\alpha}$ and $R^* = 1/\beta'$,

$$\hat{q}_0^{ap}(0) \leq \hat{q}_0^{nw}(0)\frac{1-F(\bar{\alpha})}{1-\beta'},$$

which, using assumption 4 implies $\hat{q}_0^{ap}(0) \leq \hat{q}_0^{nw}(0)$. Given that both $\hat{q}_0^{nw}(\hat{K})$ and $\hat{q}_0^{ap}(\hat{K})$ are continuous functions on $[\underline{\hat{K}}(\cdot), 0]$, the intermediate value theorem implies there exists $\hat{K}^{eq} \in (\underline{\hat{K}}(\cdot), 0]$ such that $\hat{q}_0^{nw}(\hat{K}^{eq}) = \hat{q}_0^{ap}(\hat{K}^{eq})$. Thus, $(\hat{K}^{eq}, \hat{q}^{nw}(\hat{K}^{eq}))$ is an equilibrium.

A.2.2 Proposition 6

Part (a) We start by defining the implicit relationship $\hat{q}_0^{nw}(\hat{K}_0)$, which is the net worth relationship when agents do not renegotiate ($\varphi = 0$):

$$u\left((1+\hat{K}_0)K^*\right)(1+\hat{K}_0)(\frac{1-\epsilon}{a}) = 1 - \Delta - \frac{R^*}{R^*-1}\left(\int_0^{-\hat{q}_0^{nw}} \alpha dF(\alpha) + (1-F(-\hat{q}_0^{nw}))(-\hat{q}_0^{nw})\right) \tag{42}$$

and consider the system formed by this equation and the asset-pricing relationship (8).

When $\Delta = 0$, and $\epsilon = 0$, $\hat{q}_0 = 0$ and $\hat{K}_0 = 0$ solve this system. Furthermore, $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta,\epsilon)=\mathbf{0}}$ and $\frac{d\hat{q}_0^{ap}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta,\epsilon)=\mathbf{0}}$ are still given by (20) and (21), so $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta,\epsilon)=\mathbf{0}} > \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}|_{(\hat{K}_0,\Delta,\epsilon)=\mathbf{0}}$ (use $F(0) = 0$). Thus, we can apply the implicit function theorem at the steady state to establish the existence of a unique continuously differentiable solution $\{\hat{K}_0^{km}(\Delta,\epsilon), \hat{q}_0^{km}(\Delta,\epsilon)\}$ in an open ball $\mathcal{B}$ around $(\Delta,\epsilon) = (0,0)$. The result then follows from the fact that $\hat{q}_0^{km}(\cdot)$ is continuous and $\hat{q}_0^{km}(0,0) > \bar{q}$. Note that this equilibrium exists until $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} = \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}$ (the curves become tangent), or $\hat{q}_0^{km}(\cdot) = \bar{q}$, whichever occurs first.

It remains to show that entrepreneurs' capital holdings and asset prices are decreasing in Δ and ϵ in these equilibria. We rely again on the implicit function theorem to compute,

$$\begin{aligned}
\frac{d\hat{q}_0^{km}}{d\Delta} &= -\frac{1}{(1-F(-\hat{q}_0^{nw}))}\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}}\frac{d\hat{q}_0^{ap}}{d\hat{K}_0} < 0 \\
\frac{d\hat{K}_0^{km}}{d\Delta} &= -\frac{1}{(1-F(-\hat{q}_0^{nw}))}\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}}d\Delta < 0 \\
\frac{d\hat{q}_0^{km}}{d\epsilon} &= -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}}\left(\sum_{t=0}\frac{1}{R^{*t}}\frac{u(K_t)}{a} - \left(\frac{1+\hat{K}_0}{1-F(-\hat{q}_0^{nw})}\right)\frac{u(K_0)}{a}\right)\frac{d\hat{q}_0^{ap}}{d\hat{K}_0} \\
&\quad - \frac{R^*-1}{R^*}\left(\sum_{t=0}^{\infty}\frac{1}{R^{*t}}\frac{u(K_t)}{a}\right)d\epsilon < 0 \\
\frac{d\hat{K}_0^{km}}{d\epsilon} &= -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}}\left(\sum_{t=0}\frac{1}{R^{*t}}u(K_t) - \left(\frac{1+\hat{K}_0}{1-F(-\hat{q}_0^{nw})}\right)u(K_0)\right) < 0.
\end{aligned}$$

To sign the last two derivatives, we used that

$$\sum_{t=0} \frac{1}{R^{*t}} \frac{u(K_t)}{a} - \left(\frac{1+\hat{K}_0}{1-F(-\hat{q}_0^{nw})} \right) \frac{u(K_0)}{a} \geq \left(\frac{R^*}{R^*-1} - \frac{1}{1-F(-\hat{q}_0^{nw})} \right) \frac{u(K_0)}{a} \geq 0,$$

where the first inequality follows from $\hat{K}_0 < 0$ and the fact that $\{K_t\}$ is an increasing sequence, and the last inequality follows from assumption 4.

Finally, note that since $\hat{q}_0^{km}(\cdot)$ strictly decreases with Δ and ϵ, the share of defaulting entrepreneurs must strictly increase according to $F(-\hat{q}_0^{km}(\cdot))$. Furthermore, since $\hat{q}_0^{km}(\cdot) < 0$ whenever either $\Delta > 0$ or $\epsilon > 0$, and $f(0) > 0$, the share of defaulting entrepreneurs is always nonzero.

Part (b) The proof of the first part of the result is exactly analogous to the one of proposition (1) (replace $\bar{q}$ in the proof by $\mathbb{E}\left(\alpha|\alpha \leq \bar{\alpha}\right) F(\bar{\alpha}) + \bar{\alpha}\left(1 - F(\bar{\alpha})\right)$ - the rest is identical).

It only remains to show that the share of defaulting entrepreneurs is constant. This is an immediate implication of the financiers' first order condition - given by equation (14) - holding with equality.

A.2.3 Proposition 7

(i) Since $\hat{q}_0^{AI} \geq -\bar{\alpha}$, agents either default or pay the full value of debt. Guess $\hat{q}_0^{PI} = \hat{q}_0^{AI}$. Then, those agents that repay fully under asymmetric information also repay fully under perfect information (since financiers have all the bargaining power). By contrast, agents that default in the AI economy now get a haircut. However, since financiers' have all bargaining power, entrepreneurs in the PI economy are not better off than their counterparts in the AI economy, i.e. they can afford the same amount of capital. Since entrepreneurs get the same amount of capital in both economies, $\hat{q}_0^{PI}$ effectively satisfies the asset-pricing relationship so $(\hat{K}_0^{PI}, \hat{q}_0^{PI})$ constitutes an equilibrium of the PI economy.

Since the path of entrepreneurs' capital holdings is the same in both economies, and output is produced with a one-period lag, output is equal in both economies $\forall t \geq 1$. Furthermore, $F(-\hat{q}_0^{AI})$ agents default in the AI economy. As discussed before in the proof of corollary 1, welfare net of default and repossession costs only depends on the distribution of capital in the economy. Thus, it is identical across economies.

(ii) Guess $\hat{q}_0^{PI} = \hat{q}_0^{AI} < -\bar{\alpha}$. Given this asset price, agents with $\alpha_i > \bar{\alpha}$ would renegotiate their debts in the PI economy. However, since financiers' know entrepreneurs' default costs, they offer a smaller haircut (note the haircut offered under AI makes the agent with $\alpha_i = \bar{\alpha}$ indifferent). On the other hand, agents with $\alpha_i < \bar{\alpha}$ default under AI and renegotiate under

PI. As in (i), this set of agents would end up with the same net worth. It follows that, if $\hat{q}_0^{PI} = \hat{q}_0^{AI}$, agents in the PI economy would accumulate less capital at $t = 0$ relative to the AI economy. This implies that $\hat{q}_0^{PI} = \hat{q}_0^{AI}$ is a contradiction. Indeed, at $\hat{q}_0^{AI}$ the net worth curve is to the left (equivalently, above) of the asset-pricing curve, implying there is an equilibrium of the PI economy with even lower asset prices $\hat{q}_0^{PI} < \hat{q}_0^{AI}$, which in turn implies even lower capital accumulation by entrepreneurs,$\hat{K}_0^{PI} < \hat{K}_0^{AI}$. Since future capital stocks are monotone in current capital stocks and economies do not differ in their continuation equilibria, it follows that the AI economy has larger output for all $t \geq 1$. At $t = 0$, $F(-\hat{q}_0^{AI})$ agents default in the AI economy.[36] For the same reason as before, welfare is larger in the AI economy.

A.2.4 Proposition 8

In an equilibrium with renegotiation, entrepreneurs' capital holdings are equal to $\underline{\hat{K}}(\Delta, \epsilon)$, which solves

$$u\left((1+\underline{\hat{K}}(\cdot))K^*\right)(1+\underline{\hat{K}}(\cdot)) = \frac{a}{1-\epsilon}\left(1-\Delta-\frac{R^*}{R^*-1}\mathbb{E}\left(\alpha|\alpha \leq \bar{\alpha}\right)F(\bar{\alpha})-\frac{R^*}{R^*-1}\bar{\alpha}\left(1-F(\bar{\alpha})\right)\right). \tag{43}$$

When $\bar{\Delta}(\epsilon) \in (0, \tilde{\Delta})$, it must be that $\bar{\Delta}(\epsilon)$ solves

$$-\bar{\alpha} = \frac{1}{q^*}\left(u\left((1+\underline{\hat{K}}(\bar{\Delta}(\epsilon),\epsilon))K^*\right) + \sum_{t=1}^{\infty}\frac{1}{R^{*t}}u\left(\left(1+f_t\left(\underline{\hat{K}}(\bar{\Delta}(\epsilon),\epsilon)\right)\right)K^*\right)\right) - 1. \tag{44}$$

Consider an increase in μ. Applying the implicit function theorem to equation (14), we obtain $\frac{d\bar{\alpha}}{d\mu} < 0$. Since $\bar{\alpha}$ decreases, equation (44) then implies $\underline{\hat{K}}(\cdot)$ increases. Since $\hat{q}_0 = -\bar{\alpha}$ at $\bar{\Delta}(\epsilon)$, equation (43) has the same form as the net-worth relation (42). Thus,

$$d\bar{\alpha} = -\frac{d\hat{q}_0^{nw}}{d\hat{K}_0}d\underline{\hat{K}} + \frac{1}{(1-F(\bar{\alpha}))}\frac{R^*}{R^*-1}d\bar{\Delta}(\epsilon)$$
$$d\bar{\alpha} = -\frac{d\hat{q}_0^{ap}}{d\hat{K}_0}d\underline{\hat{K}}.$$

Solving,

$$d\bar{\Delta}(\epsilon) = \frac{R^*-1}{R^*}(1-F(\bar{\alpha}))\left(\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}\right)d\underline{\hat{K}} < 0$$

[36] There may be more PI (and AI) equilibria with larger entrepreneurs' capital holdings and asset prices. Our analysis compares the equilibrium with lowest entrepreneurs' capital holdings in each model to ensure it is an "apples-to-apples" comparison even if there are multiple equilibria.

If the equilibrium is unique for all Δ, then at the threshold it must belong to the equilibrium set described in proposition 6 part (a), $\underline{\hat{K}}(\cdot) = \hat{K}^{km}(\bar{\Delta}(\epsilon), \epsilon)$. Thus, generically, $\frac{d\hat{q}_0^{nw}}{d\hat{K}_0} > \frac{d\hat{q}_0^{ap}}{d\hat{K}_0}$, which implies $d\bar{\Delta}(\epsilon) > 0$.

Next, focus on the interior of the renegotiation region $\Delta > \bar{\Delta}(\epsilon)$. Since $\bar{\alpha}$ decreases, equation (43) implies that $\underline{\hat{K}}(\cdot)$ strictly increases. Then, equation (8) implies $\underline{\hat{q}}(\cdot)$ increases as well. Finally, since the share of defaulting entrepreneurs is $F(\bar{\alpha})$, it strictly decreases with μ.

B Parametrization and calibration for figures

B.1 Parametrization

We parametrize the financiers' production function as

$$G = -\frac{1}{2} g_2 \tilde{k}'^2 + g_1 \tilde{k}'$$

and set $\bar{K} = 1$. The steady state is given by

$$\begin{aligned} R^* &= (\beta')^{-1} \\ K^* &= g_2^{-1} R^* a - g_2^{-1}(g_1 - g_2) \\ q^* &= \frac{R^*}{R^* - 1} a \\ B^* &= R^{*-1} q^* K^*. \end{aligned}$$

The user cost function is, then,

$$u = a(1 + \beta' g_2 a^{-1} K^* \hat{K}_t).$$

This implies that the future path of entrepreneurs' capital holdings solves (using equation 3),

$$\hat{K}_t = \frac{-(1 + \frac{a}{\beta' g_2 K^*}) + \sqrt{(1 + \frac{a}{\beta' g_2 K^*})^2 + 4 \frac{a}{\beta' g_2 K^*} \hat{K}_{t-1}}}{2}.$$

For section 3, we assume F is uniform, i.e.

$$F(\alpha) = \frac{\alpha}{\tilde{\alpha}}.$$

This implies

$$\bar{\alpha} = \frac{1}{2}(\tilde{\alpha} - \mu).$$

To compute the path of output, note

$$\begin{aligned}
Y^* &= (a+c)K^* + G(\bar{K} - K^*) \\
Y_0 &= (a(1-\Delta)+c)K^* + G(\bar{K} - K^*) - \frac{1}{2}\frac{1}{\tilde{\alpha}}\hat{q}_0^2 q^* K^* \mathbf{1}_{\mathrm{AI}} \mathbf{1}_{\mathrm{dw}} \\
Y_t &= (a+c)K_{t-1} + G(\bar{K} - K_{t-1}) \quad \text{for } t \geq 1
\end{aligned}$$

where $\mathbf{1}_{\mathrm{AI}}$ is an indicator function that is equal to one if the economy under consideration features asymmetric information and $\mathbf{1}_{\mathrm{dw}}$ is an indicator function that is equal to one if default costs are deadweight losses.

Finally, we define $\hat{Y}_t \equiv \frac{Y_t - Y^*}{Y_t}$.

B.2 Calibration

B.2.1 Figures from section 2

At $t = 0$, the system of equations (7) and (8) becomes

$$\begin{aligned}
(1 + \beta' g_2 a^{-1} K^* \hat{K}_t)(1 + \hat{K}_t) &= \frac{1}{1-\epsilon}(1 - \Delta + \frac{R^*}{R^* - 1}\max\{\hat{q}_0, \bar{q}\}) \\
1 + \hat{q}_0 &= (1-\beta')(1-\epsilon)\left(\sum_{t=0}^{\infty} \beta'^t (1 + \beta' g_2 a^{-1} K^* \hat{K}_t)\right).
\end{aligned}$$

The lower bound on $\hat{K}$ is given by

$$\underline{\hat{K}}(\Delta, \epsilon) = \frac{-(1 + \frac{a}{\beta' g_2 K^*}) + \sqrt{(1 + \frac{a}{\beta' g_2 K^*})^2 - 4\frac{a}{\beta' g_2 K^*}\left(\frac{1}{1-\epsilon}(1 - \Delta + \frac{R^*}{R^*-1}\bar{q}) - 1\right)}}{2}.$$

For all the plots in this section we set

$$g_2 = 0.3; g_1 = 1; a = 0.75; c = 0.3; \beta' = 0.9; \beta = 0.8; \theta = 0.5; \mu = 0.$$

These parameters satisfy the required assumptions.

In Figure 1, we set $\Delta = 0.1$ for the small shock and $\Delta = 0.2$ for the large shock. Default costs are set to $\alpha = 0.08$ and preference shocks are zero ($\epsilon = 0$). In Figure 2, we set $\epsilon = 0.01$ for the small shock and $\epsilon = 0.02$ for the large shock. Our value for ϵ for the large shock

is motivated by the evidence by Gilchrist and Zakrajšek, 2012, who find that during the Great Recession the excess bond premium for financial and non-financial firms increased by approximately 2.5 percentage points. Default costs are set to $\alpha = 0.08$ and technology shocks are zero ($\Delta = 0$). In Figure 3, low default costs correspond to $\alpha = 0.04$, while high default costs are $\alpha = 0.08$. In the left panel, shocks are set to $\Delta = 0.1$ and $\epsilon = 0$. In the right panel, the technology shock Δ varies from 0 to 0.2, while $\epsilon = 0$.

B.2.2 Figures from section 2.6

We keep the parameters from the baseline model at the same values as in the case of high default costs. We set the death rate χ at its minimum possible value, i.e. $\chi = 1 - R^{*-1}$. We set $\iota = 0$ as the "baseline" case and $\iota = 0.1$ as the high borrowing capacity case. In the left panel, the technology shock Δ varies from 0 to 0.2. In the right panel, we start both economies at $\hat{K}_0 = -0.1$ and simulate the path for capital in the subsequent 5 periods.

B.2.3 Figures from section 3

With asymmetric information, equation (15) becomes

$$(1 + \beta' g_2 a^{-1} K^* \hat{K}_t^{AI})(1 + \hat{K}_t^{AI}) = \frac{R^*}{R^* - 1} \min\{\bar{\alpha}, -\hat{q}_0^{AI}\} \left(1 - \frac{1}{2} \frac{\min\{\bar{\alpha}, -\hat{q}_0^{AI}\}}{\tilde{\alpha}}\right).$$

The lower bound on $\hat{K}$ is given by

$$\underline{\hat{K}}^{AI}(\Delta, \epsilon) = \frac{-(1 + \frac{a}{\beta' g_2 K^*}) + \sqrt{(1 + \frac{a}{\beta' g_2 K^*})^2 - 4 \frac{a}{\beta' g_2 K^*} \left(\frac{1}{1-\epsilon} \left(1 - \Delta - \frac{R^*}{R^*-1} \bar{\alpha} \left(1 - \frac{1}{2} \frac{\bar{\alpha}}{\tilde{\alpha}}\right)\right) - 1\right)}}{2}.$$

With perfect information, equation (17) becomes

$$(1 + \beta' g_2 a^{-1} K^* \hat{K}_t^{PI})(1 + \hat{K}_t^{PI}) = \frac{1}{1 - \epsilon} \left(1 - \Delta + \frac{R^*}{R^* - 1} \max\{\hat{q}_0, -\tilde{\alpha}\} + \frac{R^*}{R^* - 1} \frac{1}{\tilde{\alpha}} \frac{1}{2} \{\hat{q}_0, -\tilde{\alpha}\}^2\right).$$

The lower bound on $\hat{K}$ is given by

$$\underline{\hat{K}}^{PI}(\Delta, \epsilon) = \frac{-(1 + \frac{a}{\beta' g_2 K^*}) + \sqrt{(1 + \frac{a}{\beta' g_2 K^*})^2 - 4 \frac{a}{\beta' g_2 K^*} \left(\frac{1}{1-\epsilon} \left(1 - \Delta - \frac{1}{2} \tilde{\alpha} \frac{R^*}{R^*-1}\right) - 1\right)}}{2}.$$

We keep the same parameters for preferences and technology as in the previous section. In addition, we set $\tilde{\alpha} = 0.08$ implying $\bar{\alpha} = 0.04$. In Figure 5, we set $\Delta = 0.1$ and $\epsilon = 0.01$

for the small shock and $\Delta = 0.2$ and $\epsilon = 0.02$ for the large shock. Δ is chosen to create a drop in output in the crisis period of around 5% and 10%, respectively.