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A Nonparametric Approach for Testing Long Memory in Stock Returns' Higher Moments

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Abstract: In this paper, by considering a model-based approach for conditional moment estimation, a nonparametric test was performed to study the long-memory property of higher moments. We considered the daily returns of the stocks included in the S&P500 index in the last ten years (for the period running from the 1st of January 2011 to the 1st of January 2021). We found that mean and skewness were characterized by short memory, while variance and shape had long memory. These results have deep implications in terms of asset allocation, option pricing and market efficiency evaluation.

Keywords: generalized autoregressive score; skewness and shape; nonparametric test; self-similarity; long-range dependence; financial market

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1. Introduction

The concept of long memory, also known as long-range dependence or self-similarity of a time series, is related to the persistency of the auto-correlation function. Briefly, it describes whether and how much past events influence the future evolution of the time series.

One of the first studies demonstrating the existence of such phenomenon in real time series data was published by [1]. Later on, Ref. [2] developed the Fractional Brownian motion, a type of stochastic process that incorporates the concept of self-similarity.

In econometrics, Ref. [3] was the first to show that economic time series also share long memory. In financial economics, instead, Ref. [4] presented one of the first studies questioning about the presence of long memory in the stock market. The presence of long memory in stock returns has important implications in finance. For instance, portfolio decisions may become extremely sensitive to the investment horizons [5], the pricing of options under standard assumption is not anymore reliable [6] and the same applies to traditional tests of market efficiency that are no longer valid for time series affected by self-similarity.

Since the outstanding work by [4], hundreds of papers have been focusing on testing the long-memory property of stock returns, developing ad hoc econometric methods for more accurate estimation and forecasting (e.g., [7,8]). Starting from [9], researchers began to investigate whether or not the conditional variance of stock returns is affected by long memory.

Overall, the empirical evidence about long memory in stock markets is mixed. This is especially true concerning returns (i.e., first moment) rather than volatility (i.e., second moment). Indeed, Ref. [4] found positive evidence for most of the stocks included in the NYSE. Similarly, Refs. [10,11] found evidence favorable to short-term dependence. However, in a further study that was published shortly later, Ref. [5] provided empirical evidence against the presence of long memory, gained by means of improved test statistics. Later on, Refs. [12–16] found similar evidence. Nevertheless, favorable evidence was provided by [7], as well as by [17,18], concerning the stock market; by [19], for commodities;

and by [20], for cryptocurrencies. These examples suggest that there is mixed evidence about the presence of long memory in the first moment, with a strand of literature that claims stock markets to be characterized by long memory and another one that does not. Conversely, on the side of volatility, the academic literature is rich with articles proving the existence of self-similarity; see [8,21–25] for some examples.

Scholars questioned why it is common to find long memory in economic time series. The authors of Ref. [26] investigated whether the observed evidence of long memory is due to nonstationarity in the long period. Nowadays, structural breaks seem to be the most important cause of long memory. In this respect, considering volatility, Ref. [27] discussed the possibility that occasional structural breaks generate long memory in the time series. Similarly, Ref. [28] found that the time series in the sample showing long memory experienced several breaks; moreover, they showed the absence of long memory in the volatility processes after accounting for breaks.

Despite the relevance of higher-order moments in finance is, nowadays, quite well understood (see, e.g., [29–32]), no scholars, to the best of our knowledge, have studied the long-memory property of conditional skewness and shape yet.

Testing long memory in conditional higher moments is relevant in portfolio selection [31–33] and efficient market testing [34]. For example, it is well known that rational investors prefer assets showing high skewness and low shape [35]. If their values change over time, as it usually is [36–39], investors need to closely monitor their over-time fluctuations to be better off.

Through this paper, we aim at testing the existence of long memory in conditional higher moments, by using all the time series of stocks currently quoted in and delisted from the S&P500 Index. In particular, we consider a nonparametric frequency domain test for long-range dependence [40], focusing on the most relevant higher-order moments in finance, i.e., skewness and shape.

The first step in our analysis consisted in the estimation of the time-varying values of skewness and shape. To this aim, we considered the generalized autoregressive score (GAS) model by [41]. Despite the existence of several statistical methods that can be used for estimating conditional higher moments (e.g., see [38]), the GAS became very popular due to its flexibility (currently, there are more than 250 published papers that use the GAS for modeling conditional moments and/or time-varying distribution parameters (see http://www.gasmodel.com/gaspapers.htm, accessed on 12 February 2022)). Indeed, by using the density score for updating the time variation in the distribution parameters, there is a large set of probability distributions that can be chosen a priori for modeling. Moreover, many well-known processes, such as the GARCH, are special cases of the GAS [41].

Once the variation in higher moments over time is exploited, a statistical procedure for long-memory testing also needs to be defined. In this respect, Ref. [40] recently developed an elegant and simple nonparametric test for long-range dependence by considering the frequency domain representation of the time series. Such test is more robust than alternative tests in terms of size, when the time series are not Gaussian [40]. This is the case of stock returns and market indices, as well as new financial assets such as cryptocurrencies (see, e.g., [42–48]).

The remainder of this paper is structured as follows: Section 2 describes the methodological framework related to both time-varying moment estimation and long-memory testing procedure. Section 3 contains details about the data used in our analysis, consisting of several time series taken from the S&P500 index. Section 4 provides the main results from the long-memory test. Finally, Section 5 offers some concluding remarks.

2. Methodology

2.1. Conditional Moments Estimation

In order to estimate the time variation in stock returns' higher moments, we made use of the generalized autoregressive score (GAS) model [41]. The GAS model considers the score of the data density function as the main force behind the variation in the distribution parameters' values over time.

Let y_t be a time series generated by the following observation density function $p(\cdot)$:

$$y_t \sim p(y_t | f_t, \mathcal{F}_t; \theta_n), \tag{1}$$

where θ_n is a vector of static parameters, $\mathcal{F}_t$ is the information set at time t and f_t is a vector of length $J(j = 1, \ldots, J)$ of time-varying parameters depending on the probability distribution specification. The model's information set at a given point in time t, denoted as $\mathcal{F}_t$, is obtained by the previous realizations of the time series y_t and the time-varying parameters f_t.

In this context, the role of the time-varying parameters' vector f_t is crucial, as it contains the measures used to proxy the time variation in the distribution's higher moments.

Let us suppose, for simplicity, that the time series are generated by a Gaussian density $\mathbf{Y} \sim \mathcal{N}(\mu_t, \sigma_t^2)$. Therefore, each n-th time series has the following predictive density:

$$p(y_t | f_t, \mathcal{F}_t; \theta) = \frac{1}{\sigma_t \sqrt{2\pi}} e^{-(y_t - \mu_t)^2 / 2\sigma_t^2} \tag{2}$$

where the $J = 2$ time-varying parameters are $f_t = (\mu_t, \sigma_t^2)$. The $GAS(1,1)$, for any specification of the density in (1) can be written as

$$f_{j,t} = \omega_j + \mathbf{A}_{j,1} s_{j,t-1} + \mathbf{B}_{j,1} f_{j,t-1} \tag{3}$$

where ω_j is a real vector and $\mathbf{A}_{j,1}$ and $\mathbf{B}_{j,1}$ are diagonal matrices. All the scalar parameters ω_j, $\mathbf{A}_{j,1}$ and $\mathbf{B}_{j,1}$ are collected in the vector θ. An appealing feature of the GAS model is that the vector of parameters θ_n is estimated by maximum likelihood (for the details, see [41]). Moreover, $s_{j,t}$ is the *scaled* score of the conditional density (1) at time t with respect to a j-th parameter of the time series.

Clearly, the choice of the underlying probability distribution in (1) is very important, since it changes the kind of score considered in (3), thus the considered GAS model. The scaled score $s_{j,t}$ is given by

$$s_{j,t} = S_{j,t} \cdot \nabla_{j,t} \tag{4}$$

where $\nabla_{j,t}$ is the conditional j-th score at time t for the n-th time series, computed as

$$\nabla_{j,t} = \frac{\partial \log p(y_t | f_t, \mathcal{F}_t; \theta_n)}{\partial f_t} \tag{5}$$

and $S_{j,t}$ is a *scaling matrix* of appropriate dimension that is usually given by the inverse of the Fisher information matrix, as follows:

$$S_{j,t} = \left(E\left[\nabla_{j,t} \nabla'_{j,t} \right] \right)^{-1} \tag{6}$$

The aforementioned approach is the standard GAS proposed by [41]. However, the identity matrix can also be considered (e.g., see [49]). Moreover, different GAS models can be used, for instance, by assuming a different scaling matrix $S_{j,t}$.

Considering the Gaussian distribution, the conditional score vector (5) is given by

$$\nabla_{n,1,t} = \frac{(y_t - \mu_t)}{\sigma_t^2}$$
$$\nabla_{n,2,t} = \frac{(y_t - \mu_t)^2}{2\sigma_t^4} - \frac{T}{2\sigma_t^2}$$

with $\nabla_{j,t} = (\nabla_{n,1,t}, \nabla_{n,2,t})$, where $\nabla_{n,1,t}$ is the conditional score for the first moment (i.e., the conditional mean) and $\nabla_{n,2,t}$ is the one for the second moment (i.e., the conditional variance). Therefore, the model's variables and parameters are given by

$$f_t = \begin{pmatrix} \mu_t \\ \sigma_t^2 \end{pmatrix}, \quad \omega = \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}$$

Once the scores are computed, the parameters in (3) are estimated by maximum likelihood. Then, the conditional moments can be obtained by the in-sample predictions $\hat{f}_{j,t}$ as in Cerqueti et al. [39], namely,

$$\hat{f}_{j,t} = \hat{\omega}_j + \hat{\mathbf{A}}_{j,1} s_{j,t-1} + \hat{\mathbf{B}}_{j,1} f_{j,t-1} \tag{7}$$

Nonetheless, assuming a Gaussian distribution in this framework only allows us to study the long memory of the first two distribution moments, while our aim is to study the long-memory property of the first four moments. Therefore, we consider the Skew-t distribution by [50], characterized by the following density:

$$\begin{aligned} p(y_t|f_t, \mathcal{F}_t; \theta) = \frac{2}{\gamma_t + \frac{1}{\gamma_t}} \frac{\Gamma\left(\frac{\nu_t+1}{2}\right)}{\Gamma\left(\frac{\nu_t}{2}\right)(\pi\nu_t)^{1/2}} \sigma_t^{-1} \\ \times \left[1 + \frac{(y_t-\mu_t)^2}{\nu_t \sigma_t^2} \left\{ \frac{1}{\gamma_t^2} I_{[0,\infty)}(y_t - \mu_t) + \gamma_t^2 I_{(-\infty,0)}(y_t - \mu_t) \right\} \right]^{-(\nu_t+1)/2} \end{aligned} \tag{8}$$

where μ_t is the location, σ_t the scale, ν_t the degrees of freedom and γ_t the skewness. The thickness of the distribution's tails is determined by the parameter ν_t, while γ_t determines the amount of mass on both sides of location μ_t.

Hence, we use the GAS model to obtain estimates of the time series of the parameters in (8). Then, based on such estimates, we test for long memory.

2.2. Nonparametric Test for Long-Range Dependence

As previously mentioned, a frequency-domain nonparametric approach [40] is considered for testing short versus long memory.

Let us consider a stationary process Y_t with a spectral density of some semi-parametric form as follows:

$$f(\lambda) = |\lambda|^{-2d} g(\lambda) \tag{9}$$

where $0 \le d < 1/2$ and g is an even, positive, continuous function on $\lambda \in [-\pi, \pi]$. We can affirm that the process Y_t does not show long memory, if $d = 0$, such that the spectral density is continuous in $[-\pi, \pi]$. Conversely, we say that the process has long memory if $0 < d < 1/2$, such that the spectral density is unbounded at zero.The authors of Ref. [40] start from the well-known fact that, if Y_t are weakly dependent stationary processes, the normalized periodogram is asymptotically exponentially distributed.

$$\frac{I_n(\lambda_j)}{f(\lambda_j)} \xrightarrow{d} E, \quad \text{as} \quad n \to \infty \tag{10}$$

with $\mathrm{I}(\lambda)$ being the periodogram as follows:

$$\mathrm{I}(\lambda) = \frac{1}{T}\left|\sum_{t=1}^{T} Y_t e^{-i\lambda t}\right|^2 \tag{11}$$

that is, a consistent estimator for the spectral density of the time series under the absence of long memory. By estimating $f(\lambda_j)$ as the sample average of the periodograms, the test statistics is given by [40]

$$Q_{n,m}(s) = \sum_{j=1}^{s} \frac{I(\lambda_j)}{\frac{1}{m}\sum_{i=1}^{m} I_i(\lambda_j)} \tag{12}$$

The authors of Ref. [40] showed that the statistics converges in distribution to Gamma with parameters $(s, 1)$, i.e., $Q_{n,m}(s) \to \Gamma(s, 1)$.

Since the procedure involves sample splitting, m represents the number of sub-samples, each of length ℓ, and s is the number of frequencies included in the test. Clearly, m and ℓ increase along with sample size T, such that $m = T/\ell$. According to the authors' suggestion, in what follows, we consider $m = \sqrt{T}$.

Moreover, we note that the choice of s, which is crucial for testing, depends on the sample size T. Indeed, for short time series, it is possible to build sub-samples characterized by very short length ℓ, so few frequencies are appropriate for describing whether or not the time series is characterized by long memory. In the case of particularly long time series, Ref. [40] recommends $s \leq 5$ for $T = 5000$ and $s \leq 10$ for $T = 10{,}000$.

3. Data and Descriptive Statistics

In order to study the long-memory property of the returns' higher moments, we collected data about a large set of stocks. Particularly, we considered the daily time series of the 500 constitutes belonging to the S&P500 index in the period going from 1 January 2011 to 1 January 2021.

In order to facilitate the presentation of the results, we grouped the stock returns into 11 industrial sectors. The main descriptive statistics by sector are presented in Table 1.

Table 1. Descriptive statistics for group-based stock returns.

Sector	Length	Mean	Variance	Min	Max
Industrial	74	0.000650	0.004728	−0.360829	0.344278
Health Care	62	0.000652	0.004545	−0.395290	0.481849
Information Technology	75	0.000737	0.006036	−0.467855	0.420617
Communication Services	26	0.000434	0.005322	−0.455523	0.352230
Consumer Discretionary	63	0.000549	0.006017	−0.594142	0.365733
Utilities	28	0.000444	0.002341	−0.234522	0.257634
Financial	64	0.000442	0.003358	−0.312458	0.275358
Materials	28	0.000409	0.005236	−0.321690	0.259935
Real Estate	29	0.000379	0.002496	−0.336772	0.299941
Consumer Staples	31	0.000466	0.003077	−0.321026	0.316513
Energy	23	−0.000028	0.004668	−0.773593	0.295550

From Table 1, we can observe that the groups have different characteristics. For example, the cluster related to the Energy sector is the one with the lowest (negative) average return and a relatively high variance. The Utilities and Financial sectors are characterized by a very similar mean and a markedly different volatility. Moreover, the Information Technology (IT) sector is the one with the highest number of stocks (75), as well as the highest average return over the considered period.

Clearly, within each group there is also a certain degree of heterogeneity. Indeed, Table 1 shows the average descriptive statistics by group, that is equivalent to considering the statistics related to an industry portfolio based on equal weighting. However, industry portfolios are only presented for the sake of exposition simplicity.

To show the degree of heterogeneity in the stock returns' distribution, we show the empirical density of $N = 2$ randomly selected stocks for each group (see Figures 1–4).

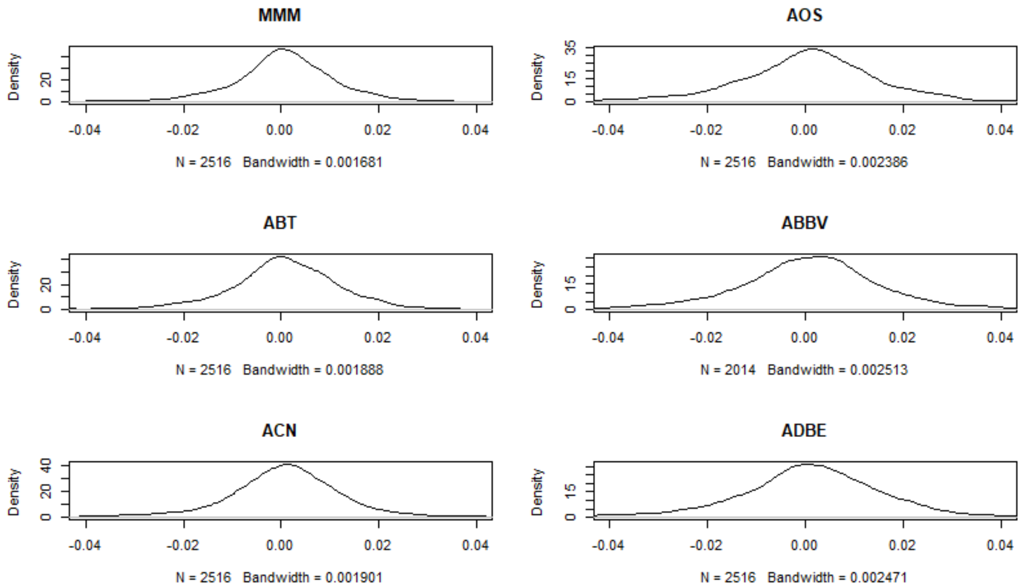

Figure 1. Randomly selected stocks for Industrial, Health Care and IT sectors.

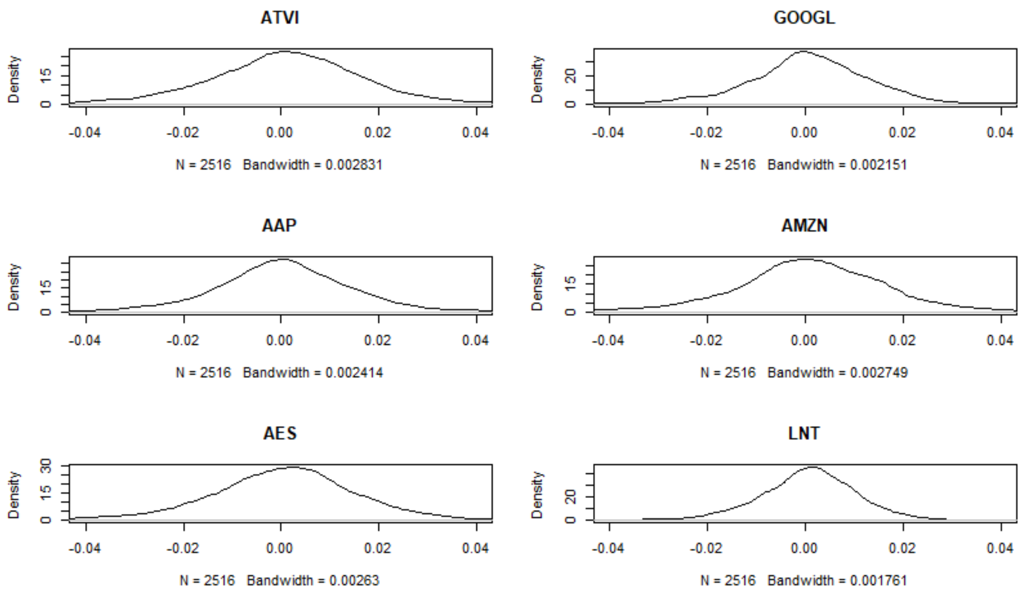

Figure 2. Randomly selected stocks for Communication Services, Consumer Discretionary and Utilities sectors.

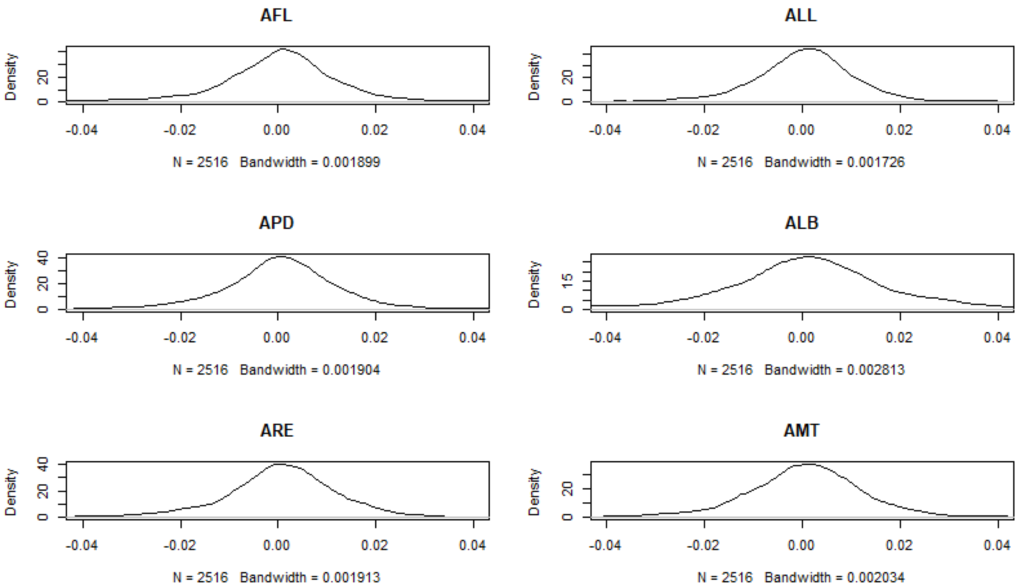

Figure 3. Randomly selected stocks for Financial, Materials and Real Estate sectors.

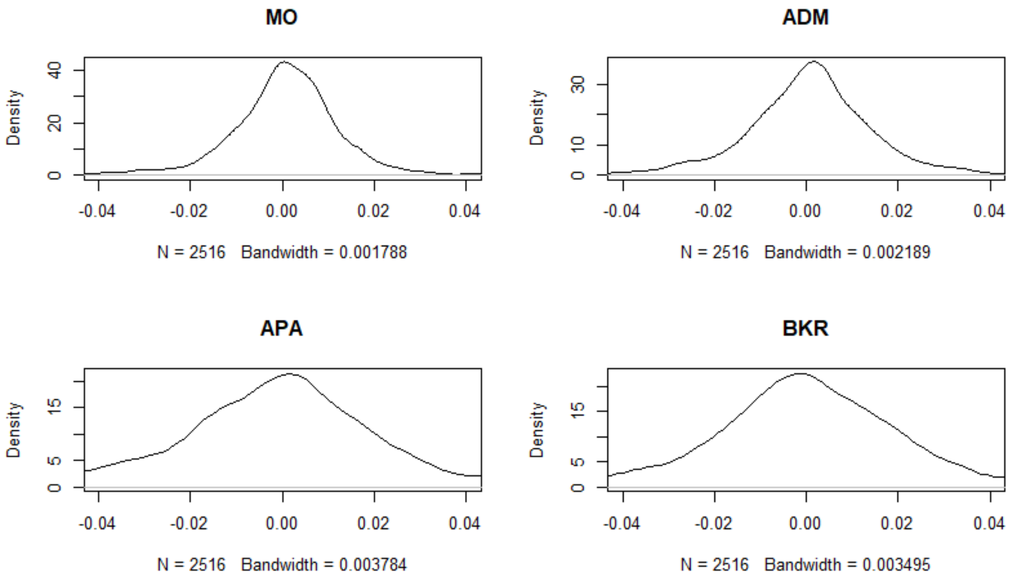

Figure 4. Randomly selected stocks for Consumer Staples and Energy sectors.

It is evident that the empirical distributions are not normally distributed, since they are asymmetric and heavy-tailed. This justifies the opportunity of modeling the stock returns according to a probability distribution that accommodates such features [48,51]. In doing so, it is relevant to investigate whether these higher moments, that are time-varying [37], are characterized by short- or long-memory processes.

Hence, as explained in the previous section, we estimated the conditional moments by means of the GAS model. To provide some examples, Figure 5 shows the estimated time-varying moments for the IBM stock (IT sector), while Figure 6 illustrates those for JPM (Financial sector).

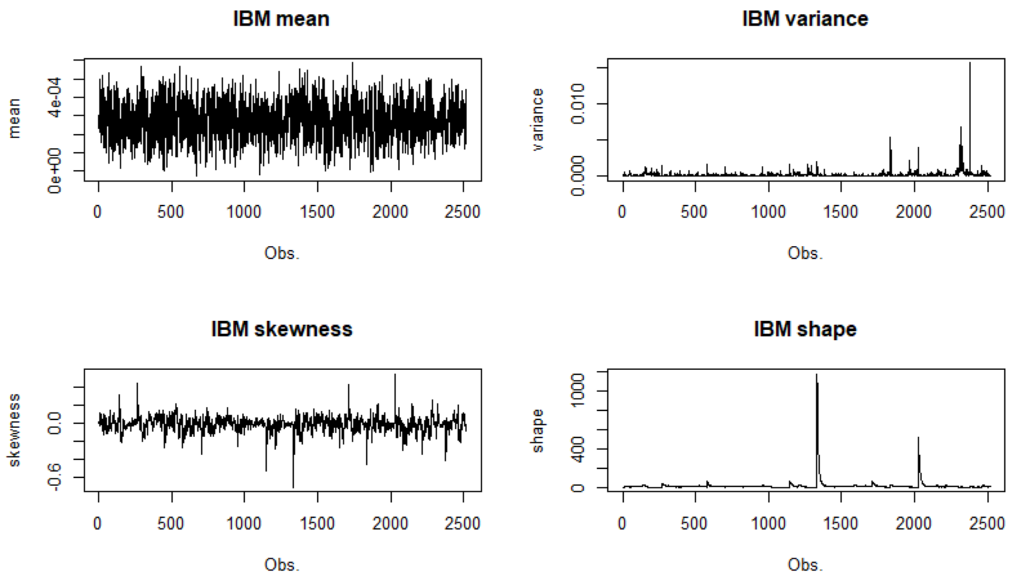

Figure 5. Estimated moments for IBM stock.

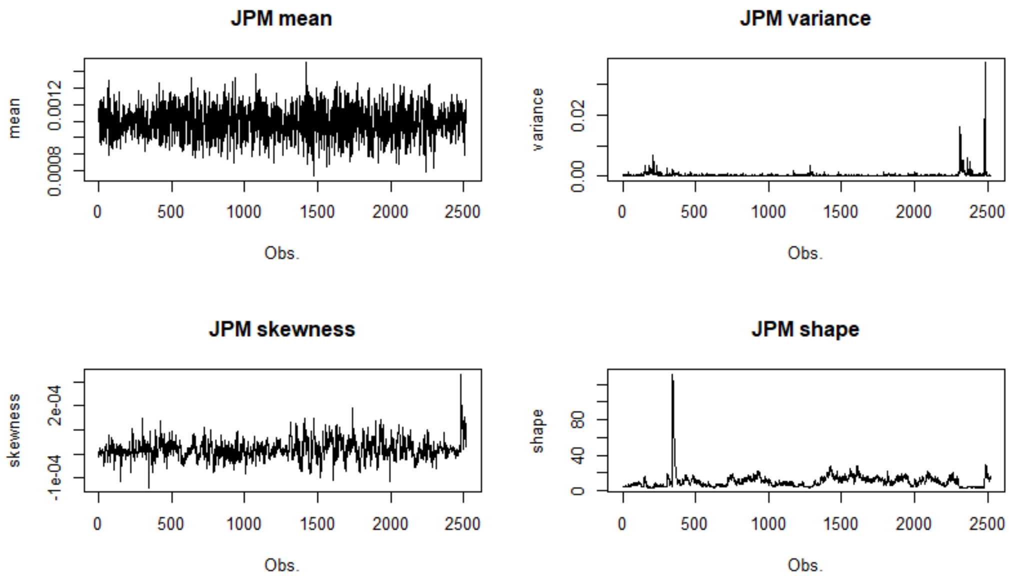

Figure 6. Estimated moments for JPM stock.

It appears clear that, for both stocks, the time-varying skewness evolved over time with a pattern similar to the one of the conditional mean, while shape showed a time variation similar to that of conditional variance. These features hold true for all the stocks in the considered sample; however, we cannot show them all, given the very large sample size.

4. Main Results

As previously stated, in order to facilitate the analysis, in what follows, we describe the results of the long-memory test by sector-based groups of stocks. The tables are reported in the Appendix A.

Table A1 shows the nonparametric long-memory results for mean, variance, skewness and shape time series in the case of the Industrial sector.

Table A1 highlights that most of the stocks belonging to the Industrial sector had short memory in the conditional mean, since only 15% of stocks showed long memory. Therefore, we can argue that the Industrial sector stocks confirmed previous literature findings in terms of absence of long memory. Conversely, in terms of conditional variance, the results are different. Indeed, the majority of stocks (61%) showed long memory in the variance, confirming the findings of a substantial body of research.

The results concerning skewness and shape are the main novelty introduced in the present paper. For the whole set of stocks, skewness showed a time pattern similar to the one of the conditional mean; therefore, we did not expect to find long memory in conditional skewness. By looking at Table A1, we can observe that 51% of the stocks showed short memory in the skewness. Here, the evidence is mixed and it is not straightforward to obtain a conclusion within this group of stocks. On the side of conditional shape, instead, the evidence is clear, with 97% of the stocks showing long memory. This happens because the behavior of the shape time series is similar to the one of conditional variance, making it reasonable to assume a sort of "conditional shape clustering" in the returns' time series.

Table A2 shows the results for the Health Care sector stocks. In the analysis of the results, we are interested in confirming those of Table A1. For this group of stocks, as expected, we observed similar results in terms of long memory in higher moments.

More in detail, in terms of the conditional mean, we found that 95% of stocks were characterized by short memory. Therefore, the evidence suggesting the absence of long memory for the conditional mean is much stronger for these stocks than for those in Table A1. In addition, in terms of conditional variance, the results do not change; the majority (66%) of the stocks showed long memory in the variance, again confirming the results of earlier research.

Moving on to skewness, instead, the results are different. Indeed, while Table A1 shows mixed evidence, Table A2 suggests that most of the stocks in the Health Care sector (60%) had long memory in conditional skewness. Nevertheless, in terms of conditional shape, only 10% of the stocks showed short memory, hence confirming the idea that the presence of long memory in shape can be explained by the fact that conditional shape and variance behave in a similar manner.

These results show that, despite an overall picture, there is a certain degree of heterogeneity within the groups.

Table A3 shows the results for the Information Technology stocks. For this group, the conclusions highlighted in Tables A1 and A2 also hold. Indeed, most of the stocks in Table A3 had short memory in the conditional mean (about 83%) and long memory in the conditional variance (65%).

In terms of conditional skewness, there is mixed evidence here as well, with slightly more than half (56%) of the stocks showing long memory. On the contrary, we confirm that most of the stocks (91%) were characterized by long memory in conditional shape.

Table A4 reports the results for the Communication Services stocks. In this case, some results are different than the previous ones.

As regards the conditional mean, most of the stocks (about 85%) showed short memory, while 77% of them were characterized by long memory in variance.

In terms of conditional skewness, we found that 62% of the stocks had long memory. Hence, Communication Services stocks differed from those belonging to the Industrial, Health Care and IT sectors. In terms of conditional shape, however, the results are similar, with 96% of the stocks showing long memory.

We present the results for the Consumer Discretionary sector in Table A5. For this cluster, a larger amount of stocks showed long memory in the conditional mean (22%); nonetheless, as in the aforementioned groups, most stocks appeared to be characterized by short memory. Considering the conditional variance, we found that about 79% of the stocks were characterized by long memory.

The long-memory nonparametric tests for conditional skewness, instead, provided mixed evidence, since 56% of the stocks in this group showed long memory. As regards shape, however, we still found that a huge amount of stocks (95%) had long memory.

Table A6 shows the results concerning the stocks belonging to the Utilities sector. In this case, we have strong evidence suggesting that the considered moments had long memory. Indeed, while only 4% of the stocks showed long-range dependence for the conditional mean, for the majority of stocks, we were led to reject the null hypothesis of short-memory processes for the other conditional moments.

Indeed, according to Table A6, we found that 96% of the stocks showed long memory in the variance, 68% in the skewness and 93% in the shape.

The case of the Financial sector stocks is shown in Table A7. In this group, only 9% of the stocks had long memory in the conditional mean process, while about 89% of them showed long memory in conditional variance. Conditional skewness showed, instead, mixed results, with half of the stocks having been characterized by short memory. The outcomes related to conditional shape show that over 92% of the stocks had long memory.

In light of this evidence, while stock returns were mainly characterized by short (for the mean) and long (for variance and shape) processes, the case of skewness deserved deeper investigation.

The results from Table A8, related to the Materials sector, suggest that skewness did not show long memory for most of the stocks (68%) and the same applies to the mean (86%). On the other hand, conditional variance and shape showed long memory with a proportion of 71% and 100%, respectively.

The conditional skewness results shown in Table A8 are very different from those presented in the previous tables; they suggest that, on average, stocks from the Materials sector showed short memory in the skewness process. This fact highlights the heterogeneity across business sectors.

The following table (Table A9) presents the results for the Real Estate sector.

Overall, Real Estate stocks showed short memory in the mean process (only 7% of them led us to reject the null hypothesis of the presence of long memory). However, as already seen as regards the Utilities sector, the majority of stocks were characterized by long-memory processes for the other higher moments. Indeed, 79% of the stocks placed in the Real Estate group showed long memory in the variance, 72% in the skewness and 86% in the shape.

Hence, for this group, we found that, despite the skewness behaved similarly to the mean process, its time series was, on average, characterized by long memory.

The Consumer Staples stocks are reported in Table A10 below.

Consumer Staples stocks showed, again, short memory in the mean process (87%). However, higher moments were fully characterized by long-memory processes; most stocks showed long memory in the variance (68%), skewness (61%) and shape (97%).

The last group is based on the Energy sector stocks, the results of which are reported in Table A11.

The Energy sector presented some peculiarities. First of all, it is the business sector with the largest amount of stocks with long memory in the mean (around 35%). Still, the majority presented short memory in the conditional mean processes.

As regards the other higher moments, the time series proved to be characterized by long-memory properties, with 87% of the stocks showing long memory in the variance, 74% in the skewness and 96% in the shape.

A summary of the results, in terms of both averages and medians, considering all the business sectors taken into account, is provided in Table 2.

Medians were considered in order to account for possible outlier sectors, such as Materials (low proportion) and Real Estate (high proportion) concerning skewness, or the Energy sector concerning the mean. Averages and medians are, however, not particularly distant from one another.

Table 2 confirms the hypothesis that S&P500 stocks were characterized by a short-memory conditional mean process overall, while most of the stocks showed long memory in the variance. These two results are in line with the previous literature on the topic.

However, the most interesting results are those provided in the last two columns of Table 2. First, nearly all of the stocks showed long memory in the conditional shape. This can be explained by the fact that the shape patterns were similar to those characterizing the variance. Hence, if structural breaks can be seen as the cause of long memory in the variance, this could also hold true for the conditional shape.

Table 2. Long-memory nonparametric tests: summary of the results for all the business sectors.

	Mean	Variance	Skewness	Shape
Industrials	14.86%	60.81%	51.35%	97.30%
Health Care	4.84%	66.13%	59.68%	90.32%
Information Technology	17.33%	65.33%	56.00%	90.67%
Communication Services	15.38%	76.92%	61.54%	96.15%
Consumer Discretionary	22.22%	79.37%	55.56%	95.24%
Utilities	3.57%	96.43%	67.86%	92.86%
Financials	9.38%	89.06%	50.00%	92.19%
Materials	14.29%	71.43%	32.14%	100.00%
Real Estate	6.90%	79.31%	72.41%	86.21%
Consumer Staples	12.90%	67.74%	61.29%	96.77%
Energy	34.78%	86.96%	73.91%	95.65%
Average	14.22%	76.31%	58.34%	93.94%
Median	14.28%	76.92%	59.67%	95.23%

Second, more than half of the stocks showed long memory in the skewness. This deserves deeper exploration, as this result is weaker than those concerning variance and shape, whose proportions were indeed much higher.

5. Conclusions

In this paper, we propose the use of the nonparametric test by [40] to study the long-memory property of stock returns' conditional moments. To this aim, we analyzed the daily returns of the S&P500 constitutes during the last 10 years (from 1 January 2011 to 1 January 2021).

We estimated the conditional moments by means of the GAS model by [41], a model-based approach in which the time variation in the distribution parameters is driven by the score of the specified distribution density function.

In order to account for skewness and tails' behavior, we considered the generalized Skew-t distribution by [50]. After estimating the moments' time series, we tested for long memory by using the nonparametric frequency-domain approach proposed by [40].

The results indicate that most of the S&P500 stocks were characterized by a short-memory conditional mean process and long memory in the variance, confirming the findings of earlier research on the topic.

In conclusion, our findings confirm that the nonparametric frequency-domain approach should be used in combination with classical time series methods for testing long memory.

Nevertheless, this paper adds noteworthy evidence with respect to the shape properties, finding that it was characterized by long-memory processes for most of the stock returns. As regards skewness, instead, the evidence is mixed. Therefore, we believe that further studies should be devoted to the analysis of conditional skewness.

Our findings are especially relevant in the context of asset allocation problems. Indeed, as noted by [32], predicting time-varying higher moments is crucial, since it allows for the so-called distribution timing, which is defined as the ability of using forecasts for moments up to the fourth one and is proved to generate significant incremental economic value to investors [32]. In this respect, if higher moments show long memory, it is possible to use appropriate statistical models for their prediction. Moreover, our findings represent a first step for the mathematical modeling of the stochastic behavior of conditional higher moments.

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Appendix A

Table A1. Long memory nonparametric test: results for stocks belonging to the Industrial sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	p-Value	Q-Stat	p-Value	Q-Stat	p-Value	Q-Stat	p-Value
2	AAL	19.39	0.00	2.60	0.52	106.72	0.00	834.80	0.00
25	ALK	0.38	0.99	4.49	0.17	1.91	0.70	43.09	0.00
27	ALLE	0.05	1.00	17.02	0.00	1.79	0.73	280.22	0.00
32	AME	0.19	1.00	1.95	0.69	0.03	1.00	846.44	0.00
41	AOS	0.47	0.99	30.05	0.00	8.41	0.01	514.98	0.00
55	BA	2.63	0.51	341.48	0.00	2.17	0.63	122.04	0.00
76	CARR	1197.31	0.00	0.26	1.00	79.86	0.00	43.25	0.00
77	CAT	356.69	0.00	104.17	0.00	1.87	0.71	14,056.86	0.00
90	CHRW	0.39	0.99	8.09	0.01	0.42	0.99	4,676,493.63	0.00
100	CMI	134.92	0.00	6.03	0.06	4.50	0.17	79.86	0.00
110	CPRT	0.70	0.97	39.95	0.00	164.06	0.00	897.75	0.00
113	CSX	2.00	0.68	2.97	0.43	101.93	0.00	988.16	0.00
114	CTAS	0.22	1.00	8.75	0.01	1.72	0.75	9.61	0.00
123	DAL	2.23	0.62	16.82	0.00	23.26	0.00	2.23	0.61
125	DE	0.38	0.99	16.61	0.00	569.78	0.00	3,944,620.35	0.00
137	DOV	2.88	0.45	7.12	0.03	2.65	0.51	37,981,286.37	0.00
152	EFX	0.13	1.00	9.44	0.00	3.03	0.42	5418.77	0.00
156	EMR	0.88	0.94	25.13	0.00	1.93	0.70	53.61	0.00
163	ETN	0.87	0.94	14.28	0.00	2.16	0.63	25,829.63	0.00
169	EXPD	0.44	0.99	9.16	0.01	164.38	0.00	205.36	0.00
174	FAST	0.12	1.00	2269.57	0.00	25.30	0.00	3,294,249,843,584,581.00	0.00
176	FBHS	7.46	0.02	6.12	0.06	6.33	0.05	4847.75	0.00
178	FDX	1.60	0.78	10.16	0.00	0.34	0.99	153,756,604,383,877,056.00	0.00
192	FTV	0.16	1.00	293.62	0.00	1.84	0.72	26.71	0.00
193	GD	3.86	0.26	0.64	0.97	0.14	1.00	37.02	0.00
194	GE	0.88	0.94	50.91	0.00	209.29	0.00	28.00	0.00
200	GNRC	4.34	0.19	6.39	0.05	3.45	0.33	290,470,898.76	0.00
208	GWW	0.04	1.00	2.94	0.44	0.09	1.00	253,267.41	0.00
218	HII	0.72	0.96	8.61	0.01	9.49	0.00	8134.98	0.00
221	HON	0.05	1.00	2.43	0.56	1.84	0.72	3589.42	0.00
229	HWM	821.51	0.00	7.03	0.03	71.74	0.00	38,025.97	0.00
233	IEX	1.88	0.71	3.95	0.25	42.20	0.00	2202.18	0.00
237	INFO	0.45	0.99	140.01	0.00	14.80	0.00	95,396.69	0.00
244	IR	692.97	0.00	10.59	0.00	7.73	0.02	8827.31	0.00
248	ITW	0.86	0.94	49.75	0.00	479.58	0.00	90.31	0.00
250	J	1.80	0.73	2.47	0.55	6.91	0.03	73,930.54	0.00
251	JBHT	0.08	1.00	9.62	0.00	2.10	0.65	4,100,854.84	0.00
252	JCI	0.54	0.98	1.32	0.85	53.17	0.00	49,691.69	0.00
268	KSU	1.50	0.81	40.85	0.00	10.10	0.00	37,086,905.93	0.00
271	LDOS	9.30	0.00	428.28	0.00	9.56	0.00	4078.81	0.00
275	LHX	1.24	0.87	5.54	0.09	0.21	1.00	106.47	0.00
279	LMT	3.15	0.39	3.34	0.35	67.94	0.00	18,479.57	0.00
285	LUV	533.76	0.00	4.42	0.18	0.05	1.00	607,402.71	0.00
293	MAS	1.42	0.83	1.87	0.71	0.20	1.00	9.37	0.00
307	MMM	1.27	0.86	17.90	0.00	993.30	0.00	488.30	0.00
331	NLSN	5.41	0.09	5.99	0.06	49.15	0.00	14,949.03	0.00
332	NOC	1.81	0.73	0.56	0.98	19.71	0.00	30,549.28	0.00
336	NSC	1.53	0.80	24.79	0.00	1.82	0.73	19,050,673.08	0.00
347	ODFL	1.40	0.83	6.86	0.03	8.69	0.01	51.17	0.00
352	OTIS	4.88	0.14	0.11	1.00	2.61	0.52	16.14	0.00
357	PCAR	0.37	0.99	1.85	0.72	222.78	0.00	67,386,429.24	0.00
366	PH	7.91	0.01	27.54	0.00	0.51	0.98	521.89	0.00
373	PNR	1.07	0.91	165.99	0.00	516.89	0.00	195.39	0.00
384	PWR	1.14	0.89	0.88	0.94	0.71	0.96	22.61	0.00

Table A1. *Cont.*

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	p-Value	Q-Stat	p-Value	Q-Stat	p-Value	Q-Stat	p-Value
394	RHI	52.18	0.00	2.97	0.43	0.21	1.00	83,337.44	0.00
398	ROK	0.91	0.94	2.02	0.67	0.44	0.99	22.88	0.00
399	ROL	0.47	0.99	41.18	0.00	35.74	0.00	37,674,798,980.54	0.00
400	ROP	0.25	1.00	16.68	0.00	1265.39	0.00	4782.85	0.00
402	RSG	0.95	0.93	1.63	0.78	2.28	0.60	293,363,986.16	0.00
403	RTX	2.62	0.51	56.57	0.00	137.65	0.00	28,007.85	0.00
412	SNA	0.32	1.00	7.24	0.02	772.21	0.00	1,169,932.21	0.00
422	SWK	0.77	0.96	112.95	0.00	0.23	1.00	93.82	0.00
429	TDG	0.22	1.00	373.46	0.00	4.84	0.14	398.92	0.00
430	TDY	0.37	0.99	68.68	0.00	0.89	0.94	609.31	0.00
446	TT	2.20	0.62	2.19	0.63	18.73	0.00	51.96	0.00
450	TXT	1.24	0.87	14.92	0.00	0.13	1.00	403.95	0.00
454	UAL	0.39	0.99	29.73	0.00	518.49	0.00	60.79	0.00
460	UNP	1.09	0.90	15.96	0.00	2.87	0.45	12,243.78	0.00
461	UPS	0.56	0.98	4.22	0.21	8.20	0.01	12,253.66	0.00
462	URI	2.84	0.46	23.39	0.00	74.23	0.00	26,727.20	0.00
470	VRSK	1.49	0.81	22.06	0.00	16.19	0.00	1,922,017.73	0.00
476	WAB	2.49	0.55	3.57	0.31	0.25	1.00	1.62	0.78
485	WM	0.35	0.99	3.62	0.30	2.46	0.55	32,050,262,904.22	0.00
498	XYL	3.62	0.30	6.61	0.04	8.27	0.01	22,587.86	0.00

Table A2. Long-memory nonparametric test: results for stocks belonging to the Health Care sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	p-Value	Q-Stat	p-Value	Q-Stat	p-Value	Q-Stat	p-Value
1	A	0.97	0.93	6.12	0.06	105.67	0.00	841,079,388.58	0.00
5	ABBV	24.86	0.00	43.11	0.00	6.53	0.04	3682.11	0.00
6	ABC	0.52	0.98	10.24	0.00	108.28	0.00	8341.98	0.00
7	ABMD	0.53	0.98	15.49	0.00	109.00	0.00	25.71	0.00
8	ABT	1.67	0.76	60.63	0.00	173.47	0.00	190,949,740.36	0.00
24	ALGN	181.94	0.00	251.73	0.00	0.78	0.95	4,750,3316,609,669.00	0.00
28	ALXN	0.11	1.00	1004.47	0.00	753.84	0.00	15,279.64	0.00
33	AMGN	1.56	0.79	8.80	0.01	75.27	0.00	420.89	0.00
39	ANTM	2.83	0.46	34.58	0.00	0.87	0.94	2151.78	0.00
57	BAX	2.61	0.51	11.83	0.00	23.20	0.00	10,111.26	0.00
59	BDX	0.77	0.96	2.58	0.52	4.11	0.22	9.40	0.00
61	BIIB	0.00	1.00	4.87	0.14	20.11	0.00	1319.21	0.00
62	BIO	3.32	0.36	13.66	0.00	123.32	0.00	16,644,212,815,952,544.00	0.00
68	BMY	0.24	1.00	13.71	0.00	0.70	0.97	642,853,886,634,605.00	0.00
70	BSX	0.28	1.00	0.00	1.00	25.14	0.00	0.24	1.00
75	CAH	0.34	0.99	7.70	0.02	0.38	0.99	135,912,232,695.16	0.00
86	CERN	0.01	1.00	14.02	0.00	0.88	0.94	140,223,765,921.39	0.00
92	CI	0.55	0.98	227.47	0.00	2.90	0.45	1469.98	0.00
102	CNC	0.51	0.98	39.77	0.00	3.23	0.37	142,257.20	0.00
106	COO	0.29	1.00	6.48	0.04	1.69	0.76	47,335,590.74	0.00
115	CTLT	2.91	0.44	27.19	0.00	8.53	0.01	152,172,724.91	0.00
119	CVS	2.63	0.51	5.63	0.08	0.56	0.98	1221.33	0.00
128	DGX	1.81	0.73	4.28	0.20	0.90	0.94	96,007,333.51	0.00
130	DHR	0.66	0.97	12.23	0.00	2.39	0.57	26,723.56	0.00
144	DVA	0.00	1.00	120.36	0.00	10.71	0.00	186,548.11	0.00
147	DXCM	0.20	1.00	3.42	0.34	2.35	0.58	5442.55	0.00
167	EW	1.43	0.83	22.52	0.00	1.00	0.92	20,838.72	0.00
195	GILD	0.08	1.00	23.89	0.00	30.95	0.00	1,055,355.57	0.00
213	HCA	0.13	1.00	16.12	0.00	185.60	0.00	1.88	0.71
220	HOLX	1.93	0.70	2.18	0.63	1.51	0.81	148.08	0.00
225	HSIC	0.75	0.96	5.69	0.08	35.08	0.00	114.84	0.00
228	HUM	2.92	0.44	17.91	0.00	283.55	0.00	22.55	0.00
232	IDXX	1.40	0.83	59.24	0.00	11.88	0.00	997,701.95	0.00
235	ILMN	0.17	1.00	48.87	0.00	141.30	0.00	82,783,148,014,973,681,664.00	0.00
236	INCY	0.21	1.00	668.29	0.00	1.34	0.85	556,364,652.25	0.00
243	IQV	2.84	0.46	49.71	0.00	67.13	0.00	2215.75	0.00
246	ISRG	1.04	0.91	0.18	1.00	544.34	0.00	0.05	1.00
254	JNJ	1.59	0.78	2.23	0.61	103.32	0.00	1016.69	0.00
274	LH	0.70	0.97	345.63	0.00	2.09	0.65	130.21	0.00
278	LLY	0.22	1.00	63.67	0.00	194.52	0.00	747,225.92	0.00
296	MCK	0.61	0.98	45.09	0.00	2.69	0.50	6831.38	0.00
299	MDT	1.79	0.73	0.40	0.99	121.33	0.00	1,640,410,921.56	0.00
313	MRK	0.95	0.93	8.50	0.01	168.78	0.00	0.13	1.00
320	MTD	3.11	0.40	5.95	0.06	40.66	0.00	9,601,115.42	0.00
362	PFE	5.26	0.10	4.35	0.19	110.47	0.00	151.17	0.00
369	PKI	0.02	1.00	6.81	0.03	14.71	0.00	12,416.89	0.00
378	PRGO	1.15	0.89	3.42	0.34	102.30	0.00	11,780.07	0.00
392	REGN	1.06	0.91	59.60	0.00	44.37	0.00	0.01	1.00
397	RMD	0.66	0.97	3.93	0.25	1.90	0.70	16,150,481,436.89	0.00
418	STE	0.04	1.00	35.83	0.00	1.70	0.76	1452.73	0.00

Table A2. *Cont.*

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
425	SYK	0.04	1.00	34.84	0.00	150.53	0.00	8744.73	0.00
434	TFX	0.94	0.93	15.40	0.00	2.55	0.53	9592.07	0.00
437	TMO	0.41	0.99	72.37	0.00	0.20	1.00	844.85	0.00
456	UHS	0.03	1.00	1.24	0.87	0.32	1.00	0.28	1.00
458	UNH	0.73	0.96	4.74	0.15	320.08	0.00	437.51	0.00
472	VRTX	0.19	1.00	3.58	0.31	1.25	0.87	1,838,838,485,805,622,347,462,806.00	0.00
474	VTRS	1.26	0.87	39.47	0.00	12.60	0.00	31,779,637,199.41	0.00
477	WAT	0.47	0.99	23.14	0.00	7.15	0.03	843,850.13	0.00
490	WST	1.15	0.89	3.79	0.27	2.44	0.56	552.13	0.00
497	XRAY	0.46	0.99	9.83	0.00	118.46	0.00	17,242,480.93	0.00
500	ZBH	6.88	0.03	5.70	0.08	40.00	0.00	55,839.61	0.00
503	ZTS	5.54	0.09	39.06	0.00	21.86	0.00	29,206.43	0.00

Table A3. Long-memory nonparametric test: results for stocks belonging to the Information Technology sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
4	AAPL	0.27	1.00	14.42	0.00	2.65	0.51	806,704.86	0.00
9	ACN	0.05	1.00	4.13	0.22	28.10	0.00	81,229.80	0.00
10	ADBE	1.18	0.88	6.09	0.06	44.04	0.00	2215.62	0.00
11	ADI	0.59	0.98	28.09	0.00	5.54	0.09	3465.60	0.00
13	ADP	0.07	1.00	37.67	0.00	3.10	0.40	621.40	0.00
14	ADSK	1.20	0.88	2.47	0.55	0.42	0.99	555.46	0.00
22	AKAM	0.93	0.93	5.65	0.08	17.18	0.00	0.00	1.00
29	AMAT	14.95	0.00	7.84	0.02	158.39	0.00	162.53	0.00
31	AMD	235.71	0.00	98.95	0.00	7.44	0.02	857,785,903,381,856.00	0.00
37	ANET	0.46	0.99	21.05	0.00	29.95	0.00	2.49	0.55
38	ANSS	0.04	1.00	10.50	0.00	0.16	1.00	976.27	0.00
44	APH	2.83	0.46	3.93	0.25	5877.73	0.00	200,425,249,067.64	0.00
50	AVGO	1.94	0.69	3.22	0.38	5.50	0.09	3029.97	0.00
69	BR	1.45	0.82	0.54	0.98	5.31	0.10	3.73	0.28
83	CDNS	32.93	0.00	0.82	0.95	6.41	0.05	574.11	0.00
84	CDW	4.09	0.22	13.50	0.00	34.54	0.00	168.15	0.00
111	CRM	1.18	0.88	1.88	0.71	3.57	0.31	274,433.66	0.00
112	CSCO	1.17	0.89	69.56	0.00	2.37	0.58	27.53	0.00
116	CTSH	1.52	0.80	15.92	0.00	326.22	0.00	94,814.79	0.00
118	CTXS	0.45	0.99	7.38	0.02	0.03	1.00	217,336,217.27	0.00
146	DXC	2.62	0.51	1113.92	0.00	0.10	1.00	5938.11	0.00
157	ENPH	218.99	0.00	326.29	0.00	337.15	0.00	1785.69	0.00
180	FFIV	7.17	0.03	0.75	0.96	35.92	0.00	0.01	1.00
181	FIS	0.33	1.00	120.28	0.00	2.28	0.60	63.67	0.00
182	FISV	0.02	1.00	69.16	0.00	1.49	0.81	134.65	0.00
184	FLIR	2.06	0.66	5.32	0.10	2.92	0.44	28,464.51	0.00
185	FLT	0.37	0.99	17.82	0.00	3.55	0.31	187.63	0.00
191	FTNT	4.32	0.19	0.71	0.96	12.76	0.00	0.01	1.00
198	GLW	1.26	0.87	21.97	0.00	5.92	0.07	26,897.60	0.00
204	GPN	0.12	1.00	35.81	0.00	17.03	0.00	48.40	0.00
222	HPE	1.08	0.91	10.28	0.00	0.60	0.98	21.89	0.00
223	HPQ	3.71	0.28	18.75	0.00	296.60	0.00	2770.01	0.00
230	IBM	0.74	0.96	9.21	0.01	2.33	0.59	9634.67	0.00
238	INTC	0.30	1.00	38.78	0.00	0.14	1.00	35,716.06	0.00
239	INTU	0.71	0.96	30.64	0.00	1.37	0.84	1,231,879.28	0.00
242	IPGP	685.87	0.00	4.55	0.17	1.76	0.74	0.00	1.00
247	IT	0.02	1.00	4.47	0.18	82.55	0.00	33.71	0.00
253	JKHY	0.05	1.00	4.41	0.18	249.09	0.00	15,049.62	0.00
255	JNPR	0.70	0.97	6.97	0.03	408.45	0.00	4.78	0.14
259	KEYS	2.75	0.48	37.70	0.00	7.50	0.02	4594.59	0.00
262	KLAC	7.80	0.02	1.71	0.75	8.90	0.01	12,236,802,372.94	0.00
283	LRCX	3.50	0.32	0.91	0.94	117.59	0.00	4283.25	0.00
290	MA	0.27	1.00	4.14	0.22	1.43	0.83	365,171.12	0.00
295	MCHP	3.27	0.37	27.67	0.00	64.60	0.00	371,984.08	0.00
312	MPWR	0.35	0.99	5.15	0.11	122.82	0.00	84.93	0.00
317	MSFT	0.41	0.99	54.36	0.00	225.41	0.00	7,082,818.52	0.00
318	MSI	0.37	0.99	95.94	0.00	28.33	0.00	201,336,442,869.61	0.00
321	MU	950.20	0.00	3474.96	0.00	19.31	0.00	24,891.98	0.00
322	MXIM	1.76	0.74	6.18	0.05	771.23	0.00	458.84	0.00
330	NLOK	7.41	0.02	54.26	0.00	13.14	0.00	163,613.06	0.00
334	NOW	0.78	0.96	0.09	1.00	748.15	0.00	793.38	0.00
337	NTAP	0.88	0.94	5.50	0.09	0.32	1.00	5,398,359,121,234.75	0.00
340	NVDA	2.15	0.64	1.32	0.85	4.58	0.16	113.50	0.00
345	NXPI	4.35	0.19	10.40	0.00	16.08	0.00	649,872.58	0.00
350	ORCL	0.60	0.98	38.60	0.00	47.89	0.00	839,023.77	0.00
354	PAYC	0.29	1.00	8.83	0.01	46.70	0.00	8.86	0.01
355	PAYX	0.60	0.98	44.50	0.00	4.11	0.22	796.33	0.00

Table A3. *Cont.*

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
382	PTC	25.18	0.00	54.22	0.00	20.36	0.00	9823.37	0.00
386	PYPL	490.43	0.00	100.24	0.00	5.81	0.07	23,985.04	0.00
387	QCOM	0.15	1.00	15.87	0.00	7.12	0.03	12,228.67	0.00
388	QRVO	0.36	0.99	26.33	0.00	76.56	0.00	105,343.95	0.00
413	SNPS	0.10	1.00	24.63	0.00	8.51	0.01	22,961.41	0.00
420	STX	188.36	0.00	271.63	0.00	3.26	0.37	216,750,037,595.95	0.00
423	SWKS	0.75	0.96	13.35	0.00	3.04	0.41	78,373,584,126,967,520.00	0.00
431	TEL	1.84	0.72	4.38	0.19	4.26	0.20	601.72	0.00
432	TER	4.85	0.14	5.51	0.09	362.28	0.00	57,117.61	0.00
440	TRMB	1.82	0.73	1.06	0.91	2.22	0.62	8488.83	0.00
449	TXN	4.24	0.20	229.26	0.00	18.11	0.00	85,883.74	0.00
451	TYL	0.21	1.00	21.45	0.00	29.17	0.00	82,591.72	0.00
464	V	0.14	1.00	23.16	0.00	17.49	0.00	782,385.57	0.00
471	VRSN	0.55	0.98	15.45	0.00	124.20	0.00	778,422.22	0.00
479	WDC	523.80	0.00	9.77	0.00	41.42	0.00	87,334,393.43	0.00
491	WU	0.72	0.96	6.64	0.04	1.21	0.88	16,968,419.06	0.00
495	XLNX	2.61	0.52	15.46	0.00	5.50	0.09	10,654,035.18	0.00
501	ZBRA	0.46	0.99	40.09	0.00	1.44	0.82	8746.86	0.00

Table A4. Long-memory nonparametric test: results for stocks belonging to the Communication Services sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
48	ATVI	0.17	1.00	12.54	0.00	5.67	0.08	448,344.09	0.00
91	CHTR	1.58	0.79	74.07	0.00	93.26	0.00	2,807,626,133.76	0.00
97	CMCSA	0.08	1.00	12.87	0.00	3.87	0.26	9419.82	0.00
131	DIS	0.00	1.00	6.65	0.04	4.14	0.22	9.64	0.00
132	DISCA	4.21	0.21	23.46	0.00	35.27	0.00	1432.35	0.00
133	DISCK	4.67	0.16	12.85	0.00	59.46	0.00	14,352.55	0.00
134	DISH	2.77	0.48	9.38	0.00	5.48	0.09	13.04	0.00
148	EA	6.35	0.05	17.39	0.00	0.21	1.00	91.69	0.00
175	FB	0.35	0.99	0.23	1.00	10.47	0.00	879,304,248,858,196,049,920.00	0.00
187	FOX	12.24	0.00	251.00	0.00	16.34	0.00	48.71	0.00
188	FOXA	9.38	0.00	234.26	0.00	3.90	0.25	63.24	0.00
201	GOOG	0.57	0.98	47.59	0.00	7.98	0.01	196.52	0.00
202	GOOGL	0.96	0.93	61.39	0.00	8.02	0.01	204.77	0.00
241	IPG	0.31	1.00	0.20	1.00	0.18	1.00	833.91	0.00
284	LUMN	3.38	0.34	4.99	0.13	0.62	0.97	21,663.30	0.00
289	LYV	1.77	0.74	87.10	0.00	22.53	0.00	1,558,205.32	0.00
327	NFLX	0.52	0.98	44.82	0.00	33.86	0.00	5081.28	0.00
343	NWS	2.27	0.60	112.28	0.00	653.99	0.00	8,605,918.51	0.00
344	NWSA	0.42	0.99	72.51	0.00	1728.27	0.00	12,074,042,902.95	0.00
349	OMC	0.49	0.99	12.49	0.00	0.48	0.99	111.46	0.00
427	T	1.46	0.82	15.70	0.00	225.71	0.00	3,282,572.27	0.00
438	TMUS	6.05	0.06	7.73	0.02	14.48	0.00	1,393,212.93	0.00
447	TTWO	2.46	0.55	25.03	0.00	426.53	0.00	387.81	0.00
448	TWTR	64.62	0.00	2.65	0.50	2.96	0.43	0.02	1.00
466	VIAC	2.09	0.65	3.35	0.35	15.11	0.00	311,799,076,512,593,543,168.00	0.00
475	VZ	0.65	0.97	0.91	0.93	11.90	0.00	662,223.66	0.00

Table A5. Long-memory nonparametric test: results for stocks belonging to the Consumer Discretionary sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
3	AAP	1.69	0.76	16.81	0.00	8.40	0.01	922.06	0.00
36	AMZN	0.63	0.97	8.17	0.01	28.05	0.00	351.68	0.00
45	APTV	0.64	0.97	30.94	0.00	27.57	0.00	102,225.66	0.00
54	AZO	1.54	0.80	19.22	0.00	6.77	0.04	940,373.40	0.00
58	BBY	1432.42	0.00	69.71	0.00	88.99	0.00	63,814,528,225.04	0.00
64	BKNG	0.39	0.99	1.42	0.83	4.76	0.15	2,378,793,118,270.88	0.00
71	BWA	2.88	0.45	46.02	0.00	698.52	0.00	98,806.82	0.00
82	CCL	6.79	0.03	145.67	0.00	6.73	0.04	646.21	0.00
99	CMG	1.10	0.90	14.41	0.00	175.62	0.00	28,421,583.74	0.00
121	CZR	152.55	0.00	2233.19	0.00	0.13	1.00	10,800.24	0.00
127	DG	9.42	0.00	495.31	0.00	18.32	0.00	103.84	0.00
129	DHI	39.76	0.00	17.04	0.00	2.92	0.44	719.23	0.00
136	DLTR	6.94	0.03	65.00	0.00	288.24	0.00	302,860,661.73	0.00

Table A5. *Cont.*

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
139	DPZ	6.21	0.05	16.22	0.00	8.93	0.01	34,807.52	0.00
141	DRI	0.38	0.99	69.63	0.00	0.50	0.99	75.82	0.00
149	EBAY	0.29	1.00	1.91	0.70	0.01	1.00	63,267,093.69	0.00
165	ETSY	0.52	0.98	0.69	0.97	0.82	0.95	0.06	1.00
170	EXPE	0.55	0.98	0.79	0.95	0.24	1.00	155,780,687,837,237,575,680.00	0.00
172	F	1.20	0.88	8.49	0.01	18.99	0.00	9.25	0.01
199	GM	58.12	0.00	6.02	0.06	154.47	0.00	18,879.23	0.00
203	GPC	0.48	0.99	49.54	0.00	0.09	1.00	169.79	0.00
205	GPS	62.56	0.00	636.64	0.00	7.73	0.02	36.52	0.00
206	GRMN	0.77	0.96	31.05	0.00	165.38	0.00	2,257,159.82	0.00
210	HAS	0.13	1.00	11.84	0.00	4.62	0.16	123.55	0.00
212	HBI	0.90	0.94	7.45	0.02	191.96	0.00	275,707.95	0.00
214	HD	0.17	1.00	11.47	0.00	3.63	0.30	1632.73	0.00
219	HLT	0.34	0.99	269.83	0.00	1.92	0.70	54.36	0.00
265	KMX	13.41	0.00	5.07	0.12	1.57	0.79	165.34	0.00
270	LB	1.17	0.89	82.90	0.00	0.69	0.97	564.99	0.00
272	LEG	0.10	1.00	21.86	0.00	15.09	0.00	8.88	0.01
273	LEN	0.09	1.00	36.81	0.00	11.64	0.00	6503.77	0.00
277	LKQ	0.01	1.00	18.91	0.00	1.02	0.92	803.76	0.00
282	LOW	1.05	0.91	9.59	0.00	4.43	0.18	24,755.76	0.00
286	LVS	4.24	0.21	2.16	0.63	192.08	0.00	384.87	0.00
292	MAR	869.23	0.00	9.76	0.00	0.55	0.98	286.77	0.00
294	MCD	0.47	0.99	67.09	0.00	16.64	0.00	829,057.86	0.00
301	MGM	1333.20	0.00	44.74	0.00	48.77	0.00	715.34	0.00
302	MHK	1.58	0.79	152.14	0.00	0.10	1.00	4,535,476,500,387,485.00	0.00
323	NCLH	0.09	1.00	9198.76	0.00	198.22	0.00	31.27	0.00
329	NKE	0.68	0.97	1.52	0.80	12.34	0.00	1.97	0.69
341	NVR	0.59	0.98	60.81	0.00	0.92	0.93	93,020,233,148,373,296.00	0.00
342	NWL	2.84	0.46	13.58	0.00	1.21	0.88	2,484,344,665,315.43	0.00
351	ORLY	1.72	0.75	2.23	0.62	92.87	0.00	21,714,590.91	0.00
360	PENN	2.89	0.45	1466.13	0.00	75.61	0.00	438.73	0.00
367	PHM	0.77	0.96	16.30	0.00	42.25	0.00	147.21	0.00
375	POOL	2.35	0.58	1.02	0.92	3.30	0.36	579.53	0.00
383	PVH	0.12	1.00	62.90	0.00	31.36	0.00	2239.87	0.00
389	RCL	1.61	0.78	72.95	0.00	33.39	0.00	303.48	0.00
396	RL	361.98	0.00	24.53	0.00	409.19	0.00	11,324.32	0.00
401	ROST	1.76	0.74	24.10	0.00	15.37	0.00	275.04	0.00
405	SBUX	4.96	0.13	15.01	0.00	3.03	0.42	4.00	0.24
435	TGT	2.15	0.64	3.66	0.29	12.99	0.00	502,172,473,158.29	0.00
436	TJX	1.58	0.79	12.04	0.00	0.58	0.98	739.52	0.00
439	TPR	0.59	0.98	80.78	0.00	7.53	0.02	3,323,250.43	0.00
443	TSCO	0.61	0.98	411.84	0.00	46.59	0.00	555,762,781,736.86	0.00
444	TSLA	330.77	0.00	5.23	0.11	1.98	0.68	293.58	0.00
452	UA	1.06	0.91	57.25	0.00	1.40	0.83	38.91	0.00
453	UAA	0.16	1.00	1.43	0.83	2.26	0.61	2517.59	0.00
457	ULTA	3.13	0.39	10.29	0.00	22.49	0.00	574,909,833.31	0.00
465	VFC	1.87	0.71	7.53	0.02	3.07	0.41	133,073.09	0.00
483	WHR	1.07	0.91	25.71	0.00	0.57	0.98	8124.45	0.00
493	WYNN	835.20	0.00	139.23	0.00	103.23	0.00	4,842,660.01	0.00
499	YUM	1.14	0.89	66.36	0.00	1.34	0.85	1,397,654.23	0.00

Table A6. Long-memory nonparametric test: results for stocks belonging to the Utilities sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
15	AEE	0.11	1.00	42.83	0.00	4.33	0.19	4.76	0.15
16	AEP	0.27	1.00	59.10	0.00	172.76	0.00	2732.80	0.00
17	AES	5.60	0.08	6.48	0.04	72.73	0.00	2813.28	0.00
47	ATO	0.10	1.00	52.67	0.00	255.88	0.00	269.79	0.00
52	AWK	0.76	0.96	41.18	0.00	0.19	1.00	8600.55	0.00
101	CMS	1.29	0.86	40.46	0.00	2.43	0.56	6517.93	0.00
103	CNP	0.63	0.97	8.04	0.01	0.17	1.00	0.26	1.00
122	D	0.09	1.00	233.52	0.00	27.54	0.00	103,706.42	0.00
142	DTE	0.05	1.00	1197.20	0.00	1.96	0.69	18.78	0.00
143	DUK	0.79	0.95	28.24	0.00	693.59	0.00	111.11	0.00
151	ED	0.34	0.99	123.61	0.00	181.39	0.00	829.47	0.00
153	EIX	0.84	0.95	9.56	0.00	11.82	0.00	205,368.73	0.00
161	ES	0.28	1.00	751.24	0.00	112.47	0.00	37.22	0.00
164	ETR	0.01	1.00	38.38	0.00	209.86	0.00	136.07	0.00
166	EVRG	1.70	0.76	216.15	0.00	8.98	0.01	632.36	0.00
168	EXC	0.55	0.98	37.45	0.00	190.16	0.00	1693.43	0.00
179	FE	1.63	0.78	70.87	0.00	863.97	0.00	1,500,106.76	0.00
281	LNT	0.01	1.00	27.78	0.00	71.67	0.00	10,822.32	0.00
325	NEE	0.02	1.00	104.90	0.00	0.94	0.93	2696.38	0.00
328	NI	0.12	1.00	17.96	0.00	0.86	0.94	431.30	0.00

Table A6. *Cont.*

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
335	NRG	189.05	0.00	304.58	0.00	444.61	0.00	1,554,966.54	0.00
359	PEG	0.76	0.96	5.62	0.08	1026.24	0.00	274.89	0.00
374	PNW	0.26	1.00	61.75	0.00	18.88	0.00	107.96	0.00
377	PPL	0.26	1.00	12.51	0.00	0.59	0.98	59.27	0.00
414	SO	0.61	0.98	201.24	0.00	1427.79	0.00	132.94	0.00
417	SRE	0.64	0.97	102.47	0.00	3.45	0.33	951.22	0.00
480	WEC	0.45	0.99	23.66	0.00	458.24	0.00	158.55	0.00
494	XEL	0.04	1.00	44.12	0.00	261.50	0.00	109.60	0.00

Table A7. Long-memory nonparametric test: results for stocks belonging to the Financial sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
18	AFL	0.10	1.00	7.46	0.02	0.28	1.00	413.14	0.00
19	AIG	22.02	0.00	18.39	0.00	0.99	0.92	44.49	0.00
20	AIZ	5.69	0.08	28.76	0.00	16.24	0.00	30,825,589.73	0.00
21	AJG	2.07	0.66	55.11	0.00	9.96	0.00	5370.54	0.00
26	ALL	1.38	0.84	1.43	0.83	195.99	0.00	0.03	1.00
34	AMP	10.21	0.00	46.94	0.00	2.29	0.60	432.26	0.00
40	AON	0.45	0.99	1.52	0.80	0.65	0.97	401.82	0.00
53	AXP	0.03	1.00	95.26	0.00	431.93	0.00	1,201,746,531,403,824,896.00	0.00
56	BAC	0.64	0.97	82.53	0.00	0.38	0.99	373.49	0.00
60	BEN	2.00	0.68	66.81	0.00	88.36	0.00	33.25	0.00
63	BK	0.27	1.00	10.47	0.00	0.08	1.00	1451.15	0.00
66	BLK	2.57	0.53	8.83	0.01	4.56	0.17	15.95	0.00
73	C	0.31	1.00	97.49	0.00	0.23	1.00	65.83	0.00
78	CB	0.12	1.00	27.94	0.00	0.78	0.95	127.58	0.00
79	CBOE	1.57	0.79	2.06	0.66	4.76	0.15	15.23	0.00
88	CFG	0.26	1.00	3682.07	0.00	3.01	0.42	111.33	0.00
93	CINF	4.32	0.19	177.99	0.00	72.55	0.00	138,326.80	0.00
96	CMA	0.68	0.97	255.01	0.00	10.00	0.00	88.86	0.00
98	CME	0.25	1.00	6.61	0.04	2177.29	0.00	5808.24	0.00
104	COF	1.17	0.89	351.82	0.00	18.21	0.00	387,608.07	0.00
126	DFS	0.57	0.98	106.23	0.00	14.74	0.00	2131.94	0.00
183	FITB	0.72	0.96	81.80	0.00	31.18	0.00	201.83	0.00
189	FRC	0.78	0.96	14.64	0.00	6.83	0.03	3,955,982.64	0.00
197	GL	1.24	0.87	434.80	0.00	349.22	0.00	1213.73	0.00
207	GS	3.71	0.28	75.17	0.00	2.89	0.45	3915.39	0.00
211	HBAN	0.11	1.00	23.49	0.00	8.66	0.01	1971.27	0.00
217	HIG	0.28	1.00	7.32	0.02	1.59	0.79	4260.02	0.00
231	ICE	1.17	0.89	0.05	1.00	20.08	0.00	10,837.07	0.00
249	IVZ	4.45	0.18	98.27	0.00	13.24	0.00	361,567,880.95	0.00
256	JPM	0.08	1.00	7.99	0.01	0.64	0.97	53.59	0.00
258	KEY	0.88	0.94	193.85	0.00	1.21	0.88	2359.86	0.00
269	L	2.97	0.43	141.24	0.00	4.00	0.24	80.91	0.00
280	LNC	1.55	0.80	136.50	0.00	0.05	1.00	145.25	0.00
297	MCO	2.17	0.63	24.90	0.00	5.92	0.07	627,170.28	0.00
300	MET	0.12	1.00	13.34	0.00	506.48	0.00	11,324.09	0.00
304	MKTX	288.88	0.00	3.53	0.32	32.09	0.00	26,536.35	0.00
306	MMC	0.56	0.98	0.17	1.00	0.46	0.99	6.02	0.06
315	MS	30.11	0.00	72.03	0.00	0.74	0.96	110.90	0.00
316	MSCI	0.26	1.00	17.33	0.00	525.90	0.00	31,422,615,411.73	0.00
319	MTB	0.08	1.00	124.24	0.00	153.78	0.00	8913.73	0.00
324	NDAQ	0.42	0.99	0.35	0.99	1.04	0.91	0.80	0.95
338	NTRS	1.17	0.89	31.54	0.00	0.32	1.00	30.90	0.00
356	PBCT	1.55	0.80	238.25	0.00	2558.81	0.00	6255.71	0.00
363	PFG	3.47	0.33	25.39	0.00	126.03	0.00	12,093.15	0.00
365	PGR	0.76	0.96	19.71	0.00	323.52	0.00	64,294.76	0.00
372	PNC	0.14	1.00	137.22	0.00	1.95	0.69	1199.57	0.00
379	PRU	0.30	1.00	99.70	0.00	16.01	0.00	20,748,890,009.28	0.00
390	RE	1.74	0.75	12.29	0.00	572.82	0.00	0.01	1.00
393	RF	0.60	0.98	25.74	0.00	6.47	0.04	680.50	0.00
395	RJF	0.42	0.99	24.43	0.00	0.03	1.00	53.72	0.00
406	SCHW	2.14	0.64	34.52	0.00	0.02	1.00	532.19	0.00
409	SIVB	0.56	0.98	82.25	0.00	1.27	0.86	19,650.78	0.00
416	SPGI	6.13	0.06	8.24	0.01	5.46	0.09	905.36	0.00
419	STT	2.33	0.59	50.81	0.00	7.83	0.02	658.05	0.00
424	SYF	10.09	0.00	2632.83	0.00	10.76	0.00	1337.48	0.00
433	TFC	0.20	1.00	163.44	0.00	2.49	0.55	0.40	0.99
441	TROW	1013.25	0.00	14.75	0.00	2.31	0.59	407.78	0.00
442	TRV	0.93	0.93	107.18	0.00	0.38	0.99	4663.98	0.00
459	UNM	1.76	0.74	242.95	0.00	6.06	0.06	10,564.73	0.00
463	USB	0.05	1.00	87.10	0.00	202.19	0.00	477.28	0.00
482	WFC	0.01	1.00	25.70	0.00	5.14	0.11	574.50	0.00
484	WLTW	1.75	0.74	7.41	0.02	12.33	0.00	210.72	0.00
488	WRB	0.06	1.00	58.69	0.00	70.09	0.00	354.54	0.00
502	ZION	1.71	0.75	9.46	0.00	17.12	0.00	1213.46	0.00

Table A8. Long-memory nonparametric test: results for stocks belonging to the Materials sector.

Stock		Mean		Variance		Skewness		Shape	
ID	**Ticker**	**Q-Stat**	***p*-Value**	**Q-Stat**	***p*-Value**	**Q-Stat**	***p*-Value**	**Q-Stat**	***p*-Value**
23	ALB	1.04	0.91	23.58	0.00	42.90	0.00	50.53	0.00
30	AMCR	19.24	0.00	158749.17	0.00	0.31	1.00	15.43	0.00
43	APD	0.83	0.95	4.47	0.18	1.04	0.91	387.64	0.00
51	AVY	0.21	1.00	10.23	0.00	0.06	1.00	2,282,202,533.61	0.00
67	BLL	0.41	0.99	48.50	0.00	203.70	0.00	134.49	0.00
85	CE	0.29	1.00	6.00	0.06	137.50	0.00	155,026.65	0.00
87	CF	2.21	0.62	19.38	0.00	3.54	0.31	4724.43	0.00
117	CTVA	0.82	0.95	6.71	0.04	1135.99	0.00	1571.97	0.00
124	DD	0.72	0.96	21.97	0.00	12.71	0.00	171,927.86	0.00
138	DOW	2.86	0.46	105.60	0.00	5.36	0.10	172.94	0.00
150	ECL	0.03	1.00	3.01	0.42	0.17	1.00	207.26	0.00
155	EMN	1.41	0.83	74.64	0.00	26.98	0.00	32,498.79	0.00
177	FCX	468.69	0.00	156.78	0.00	18.30	0.00	7587.42	0.00
186	FMC	1.07	0.91	27.93	0.00	1.97	0.69	70,699.93	0.00
234	IFF	0.98	0.92	15.37	0.00	0.47	0.99	30,824.92	0.00
240	IP	3.55	0.31	9.18	0.01	1.22	0.87	4870.79	0.00
276	LIN	1.70	0.76	23.52	0.00	2.88	0.45	21,444.77	0.00
288	LYB	840.24	0.00	5.41	0.09	4.96	0.13	3011.29	0.00
305	MLM	1.51	0.81	2.81	0.47	0.10	1.00	132,421,508.70	0.00
310	MOS	0.55	0.98	105.00	0.00	0.19	1.00	2623.76	0.00
326	NEM	72.63	0.00	5.28	0.10	0.15	1.00	33.99	0.00
339	NUE	0.48	0.99	840.43	0.00	3.52	0.32	401,742,062.49	0.00
368	PKG	2.40	0.57	3.50	0.32	77.31	0.00	19,504.28	0.00
376	PPG	0.10	1.00	23.30	0.00	0.14	1.00	1555.03	0.00
407	SEE	0.16	1.00	58.50	0.00	0.50	0.99	12,707.18	0.00
408	SHW	0.00	1.00	197.59	0.00	3.47	0.33	689,594.11	0.00
468	VMC	6.11	0.06	2.14	0.64	4.94	0.13	191.93	0.00
489	WRK	1.74	0.75	164.38	0.00	9.28	0.00	13.10	0.00

Table A9. Long-memory nonparametric test: results for stocks belonging to the Real Estate sector.

Stock		Mean		Variance		Skewness		Shape	
ID	**Ticker**	**Q-Stat**	***p*-Value**	**Q-Stat**	***p*-Value**	**Q-Stat**	***p*-Value**	**Q-Stat**	***p*-Value**
35	AMT	0.58	0.98	2.90	0.45	1.86	0.71	0.07	1.00
46	ARE	0.07	1.00	1.92	0.70	86.33	0.00	362.03	0.00
49	AVB	1.60	0.78	13.19	0.00	1392.96	0.00	1852.41	0.00
72	BXP	0.05	1.00	195.25	0.00	0.01	1.00	687.96	0.00
80	CBRE	1.04	0.91	11.00	0.00	56.40	0.00	1,075,058,567.14	0.00
81	CCI	1.89	0.71	11.40	0.00	18.79	0.00	4765.86	0.00
135	DLR	0.40	0.99	184.57	0.00	111.93	0.00	1,996,169.04	0.00
140	DRE	1.92	0.70	1.37	0.84	5.07	0.12	114.05	0.00
159	EQIX	0.50	0.99	15.59	0.00	0.33	1.00	513.91	0.00
160	EQR	0.45	0.99	13.34	0.00	2135.07	0.00	2673.21	0.00
162	ESS	0.29	1.00	10.45	0.00	11.08	0.00	1.28	0.86
171	EXR	0.28	1.00	2.73	0.49	1.01	0.92	249.22	0.00
190	FRT	0.25	1.00	90.68	0.00	0.79	0.95	185.83	0.00
226	HST	1.29	0.86	9.56	0.00	185.19	0.00	1618.23	0.00
245	IRM	2.71	0.49	3.78	0.27	402.35	0.00	153,389.74	0.00
261	KIM	101.66	0.00	72.41	0.00	7.60	0.02	366.48	0.00
291	MAA	0.06	1.00	24.87	0.00	121.52	0.00	331.69	0.00
346	O	0.78	0.95	725.34	0.00	14.55	0.00	37.77	0.00
358	PEAK	0.90	0.94	137.87	0.00	183.60	0.00	100.41	0.00
370	PLD	1.84	0.72	2.82	0.46	46.41	0.00	707.18	0.00
380	PSA	0.26	1.00	44.84	0.00	13.88	0.00	2020.36	0.00
391	REG	0.16	1.00	84.56	0.00	1.42	0.83	4.42	0.18
404	SBAC	0.14	1.00	20.17	0.00	41.75	0.00	0.03	1.00
415	SPG	0.41	0.99	464.66	0.00	25.80	0.00	53.20	0.00
455	UDR	0.39	0.99	52.33	0.00	62.97	0.00	2895.68	0.00
469	VNO	0.25	1.00	74.77	0.00	0.58	0.98	431.73	0.00
473	VTR	2.42	0.56	117.14	0.00	67.70	0.00	1,462,488.27	0.00
481	WELL	0.20	1.00	258.05	0.00	121.19	0.00	7311.74	0.00
492	WY	9.29	0.00	134.50	0.00	15.74	0.00	38.15	0.00

Table A10. Long-memory nonparametric test: results for stocks belonging to the Consumer Staples sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
12	ADM	1.66	0.77	11.45	0.00	16.89	0.00	15.16	0.00
74	CAG	0.17	1.00	35.57	0.00	196.12	0.00	552,087.57	0.00
89	CHD	3.46	0.33	5.78	0.07	13.87	0.00	30.36	0.00
94	CL	1.13	0.89	115.07	0.00	2.29	0.60	41.63	0.00
95	CLX	1.47	0.82	1.21	0.88	48.10	0.00	1.11	0.90
108	COST	1.34	0.85	0.93	0.93	155.15	0.00	923.19	0.00
109	CPB	2.46	0.55	18.84	0.00	13.10	0.00	14,191,017.32	0.00
154	EL	0.21	1.00	9.28	0.00	0.18	1.00	12.26	0.00
196	GIS	1.98	0.68	2.16	0.63	0.75	0.96	869,261,422,467.77	0.00
224	HRL	0.62	0.98	2.97	0.43	41.36	0.00	266,212.51	0.00
227	HSY	3.20	0.38	1.58	0.79	4.39	0.19	13,408.11	0.00
257	K	1.33	0.85	19.70	0.00	4.06	0.23	20,494,513.99	0.00
260	KHC	0.13	1.00	38.44	0.00	2.23	0.61	7734.06	0.00
263	KMB	1.86	0.71	22.93	0.00	11.99	0.00	87.97	0.00
266	KO	0.13	1.00	41.31	0.00	102.34	0.00	8656.05	0.00
267	KR	1.81	0.73	0.09	1.00	185.45	0.00	206,368.10	0.00
287	LW	0.27	1.00	844.20	0.00	316.08	0.00	1898.22	0.00
298	MDLZ	0.99	0.92	270.91	0.00	2.80	0.47	232.49	0.00
303	MKC	3.39	0.34	15.03	0.00	10.65	0.00	78,669,023,663,671,856.00	0.00
308	MNST	212.71	0.00	7.37	0.02	1.27	0.86	415.85	0.00
309	MO	6.86	0.03	16.16	0.00	99.86	0.00	63.68	0.00
361	PEP	0.10	1.00	7.88	0.02	5.49	0.09	1127.60	0.00
364	PG	4.83	0.14	55.27	0.00	163.60	0.00	65.11	0.00
371	PM	0.30	1.00	15.44	0.00	1.69	0.76	154,030,094,291.82	0.00
410	SJM	57.29	0.00	0.78	0.95	509.02	0.00	7280.62	0.00
421	STZ	1.46	0.82	57.67	0.00	36.10	0.00	10,475,104.79	0.00
426	SYY	0.01	1.00	291.70	0.00	0.27	1.00	18.41	0.00
428	TAP	0.97	0.92	33.64	0.00	54.26	0.00	264,885,029,611.89	0.00
445	TSN	333.93	0.00	8.46	0.01	1100.99	0.00	24,018.10	0.00
478	WBA	0.11	1.00	0.05	1.00	22.78	0.00	171.48	0.00
487	WMT	0.23	1.00	1.78	0.74	5.42	0.09	4337.34	0.00

Table A11. Long-memory nonparametric test: results for stocks belonging to the Energy sector.

Stock		Mean		Variance		Skewness		Shape	
ID	Ticker	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value	Q-Stat	*p*-Value
42	APA	18.44	0.00	108.43	0.00	25.19	0.00	1041.36	0.00
65	BKR	0.24	1.00	4.57	0.17	0.21	1.00	2113.11	0.00
105	COG	0.78	0.96	5.24	0.11	540.21	0.00	0.17	1.00
107	COP	127.06	0.00	69.03	0.00	249.34	0.00	485.46	0.00
120	CVX	0.08	1.00	11.10	0.00	11.40	0.00	408,723.10	0.00
145	DVN	73.89	0.00	179.59	0.00	4.45	0.18	341.66	0.00
158	EOG	0.14	1.00	13.09	0.00	1763.58	0.00	737.63	0.00
173	FANG	0.10	1.00	876.44	0.00	30.68	0.00	103.99	0.00
209	HAL	5.12	0.11	38.16	0.00	4.06	0.23	76.40	0.00
215	HES	68.38	0.00	44.80	0.00	17.57	0.00	154.14	0.00
216	HFC	434.33	0.00	6.87	0.03	28.83	0.00	482.16	0.00
264	KMI	0.69	0.97	343.00	0.00	27.28	0.00	582.49	0.00
311	MPC	1.29	0.86	4.13	0.22	522.03	0.00	8650.36	0.00
314	MRO	92.63	0.00	265.39	0.00	30.26	0.00	40.33	0.00
333	NOV	121.13	0.00	72.49	0.00	8.35	0.01	99,678.31	0.00
348	OKE	3.96	0.24	612.81	0.00	997.01	0.00	606.72	0.00
353	OXY	4.15	0.22	134.08	0.00	566.34	0.00	184,685.22	0.00
381	PSX	16.35	0.00	9.51	0.00	5.50	0.09	11,185,126.83	0.00
385	PXD	1.32	0.85	6.69	0.04	7.56	0.02	590.97	0.00
411	SLB	3.93	0.25	22.63	0.00	37.13	0.00	3167.17	0.00
467	VLO	3.17	0.39	17.30	0.00	79.05	0.00	925,056,523.49	0.00
486	WMB	0.23	1.00	242.48	0.00	0.41	0.99	873.35	0.00
496	XOM	0.01	1.00	7.76	0.02	2.09	0.65	77.50	0.00

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