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A spatial econometric multivariate model of Okun's law

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ABSTRACT

A system of two dynamic spatial panel data model equations is developed in which output growth and the change in the unemployment rate are interdependent. The parameters of the model are estimated by recently developed maximum likelihood techniques for multivariate spatial econometric models, using data of twelve provinces in the Netherlands over the period 1974–2018, covering four major economic downturns of the Dutch economy. By using time-cumulative marginal effects derived from the impulse response function of this model, it is found that Okun's law is dominated by the relationship that runs from output growth to unemployment. The amount of growth that is needed to reduce unemployment by one percentage point is shown to depend on the extent to which spillover effects to neighboring regions and output multiplier effects are accounted for.

1. Introduction

Okun's law, named after the economist Arthur Okun (1962), is one of the basic rules of thumb of macroeconomics. It measures the trade-off between unemployment and output. One reading of Okun's law tells how much growth is needed to reduce unemployment by one percentage point. Another reading measures the cost of unemployment in terms of forgone output. Both readings and the functional form of Okun's law form the core of this paper.

Since Okun's original publication, the existence of a trade-off between unemployment and output growth has been studied extensively. Recent reviews are provided by Perman et al. (2015) and Ball et al. (2017). Studies that appeared in the last three decades can be classified into three broad groups:

A. Studies that use data from one country. Prachowny (1993), Weber (1995), Weber and West (1996), Moosa (1999), Cuaresma (2003), Silvapulle et al. (2004), and Huang and Lin (2006, 2008), Valadkhani and Smyth (2015) use U.S. data, Attfield and Silverstone (1997, 1998) U.K. data, and Sögner (2001) Austrian data;
B. Studies that use data from more than one country or that use regional instead of country data. Examples of the first are Moosa (1997), Lee (2000), Virén (2001), Freeman (2001), Sögner and Stiassny (2002), Izyumov and Vahaly (2002), Perman and Tavera (2005, 2007), Huang and Yeh (2013), Tang and Bethencourt (2017), Ibragimov and Ibragimov (2017), Nebot et al. (2019), and Huang et al. (2020). Examples of the second are Freeman (2000), Apergis and Rezitis (2003), Christopoulos (2004), Adanu (2005), Kosfeld and Dreger (2006), Villaverde and Maza (2007), Kangasharju et al. (2012), Binet and Facchini (2013) and Durech et al. (2014). The main reason for examining data for multiple units is to test for spatial differences in the responsiveness of output to changes in unemployment, or vice versa. Differences among countries or regions point to institutional differences that determine the rigidity or flexibility of national or regional labor markets (Moosa, 1997).
C. Studies that use data from multiple units, often regions, which control for spatial dependence and quantify spatial spillover effects, among which Niebuhr (2003), Basistha and Kuscevic (2017), and Palombi et al. (2017).

One the most widely used empirical specifications takes the form

$$(u_{it} - u^*_{it}) = \beta(y_{it} - y^*_{it}) + \varepsilon_{it}, i = 1, \dots, N; t = 1, \dots, T \tag{1}$$

where the index i refers to a country or region and the index t to a time period, u is the actual rate of unemployment, u^* the natural or the

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equilibrium rate of unemployment, y is the logarithm of actual output, and y^* of potential or equilibrium output (both measured as real gross domestic product). The difference between the actual and the potential value of a variable is called the cyclical component, or shorter the gap. Okun's coefficient of output on unemployment is represented by β, the increase/decrease in the unemployment gap (in percentage points) for every percentage point decrease/increase in the output gap. Generally, there are three reasons why unemployment may decrease (or increase) more rapidly than output growth increases (or decreases) and thus why Okun's coefficient will be negative and smaller than 1. As unemployment increases, (i) unemployed persons may drop out of the labor force, after which they are no longer counted in unemployment statistics, (ii) employed persons may work shorter hours, and (iii) labor productivity may decrease, perhaps because employers retain more workers than they need.

Originally, Okun (1962) regressed the unemployment gap on the output gap and then used the reciprocal of the slope of this regression to predict the impact of unemployment on output. However, Barreto and Howland (1993) demonstrate that this leads to an overestimation of Okun's coefficient of unemployment on output. This is because the reciprocal of the slope of this regression not only measures the impact of unemployment on output, but also the expected value of the error term given the unemployment rate, which are not independent of each other, $E(\varepsilon|u\text{-}u^*)\neq 0$.[1] Barreto and Howland (1993) find that, had Okun adopted the model $(y_{it}-y_{it}^*)=\beta'(u_{it}-u_{it}^*)$, the slope (β') would fall to 2, instead of the 3.2 reported in his study (Okun, 1962).[2] In line with this, the literature can be divided into studies that take the unemployment gap (e.g. Lee, 2000; Dixon and Van Ours, 2017; Ball et al., 2017) and studies that take the output gap (e.g. Gordon, 2010; Basu and Foley, 2013; Guisinger et al., 2018) as the dependent variable. In this paper, we avoid this choice by doing both, i.e., we develop a two-equations system of Okun's law that controls for potential endogeneity of unemployment with respect to output and vice versa. This extension is motivated by studies of Ibragimov and Ibragimov (2017) and Huang et al. (2020), which provide empirical evidence that the estimation of Okun's coefficient of output on unemployment, or of unemployment on output, using ordinary least squares (OLS) produces inconsistent estimates since the assumption of strict exogeneity of the right-hand side variable when measured at the same moment of time is violated in most existing studies.

The studies earlier classified in groups A and B commonly estimate Okun's law for each country or region in the sample separately from the others, producing a different β_i for each unit i, thereby, assuming that these countries or regions are independent of each other. This is a second reason why Okun's coefficient estimated by OLS might be inconsistent. According to the studies earlier classified in group C, this is because countries or regions affect each other through spatial interaction effects. Output interaction occurs when output in one economy affects output in other economies. If output increases in one economy, output in other economies might also increase due input-output relationships between firms located in these different economies. Trade between firms due to these input-output relationships may also encourage output growth in neighboring economies through diffusion of knowledge, since it opens up the possibility of cross-border learning-by-doing, as well as investment in research and development (Helpman, 2004). The hypothesis that the relative location of an economy, the effect of being located closer or further away from other economies, is a determinant of economic growth due to diffusion of knowledge has been underpinned by economic-theoretical models (Ertur and Koch, 2007) and a vast empirical literature (for an overview see LeGallo and Fingleton, 2014). Similarly, regional science literature has paid considerable attention to explaining why unemployment interactions among neighboring economies can occur (e.g. Burridge and Gordon, 1981; Molho, 1995; Overman and Puga, 2002; Patacchini and Zenou, 2007; Halleck-Vega and Elhorst, 2016).

Interaction effects may also occur between output and unemployment, for example, because people do not necessarily live and work in the same economy. This holds especially when estimating Okun's law at a lower level of scale such as regions. Households may change their labor supply decisions depending on the market conditions in the home region compared to other regions. Any person, employed or unemployed, may supply his labor outside his home region when the wage rate in a nearby region is higher and this higher wage rate compensates for the greater time and commuting costs. Migration is another adjustment mechanism. Pissarides and Wadsworth (1989) demonstrate that unemployment encourages migration of unemployed workers relatively more than that of employed workers, especially of those living in high-unemployment regions to low-unemployment regions with higher prosperity levels.

There are also reasons why time lags appear in econometric models of Okun's law. First, a household may not change its labor supply immediately in response to a change in unemployment in the own region or elsewhere. Similarly, a firm may react with some delay to changes in costs and the demand for its product. Second, lags can arise because of imperfect information. Economic agents require time to gather relevant information, and this delays the making of decisions. There are also occasions when institutional factors can result in lags. Households may be contractually obliged to supply a certain level of labor hours, even though other conditions would indicate a reduction or increase in labor supply.

In sum, the analysis of Okun's law requires a multivariate model with dynamic effects in both space and time. In the spatial econometric literature, only a few examples of multivariate models have been established, highlighting the importance of simultaneous estimation with spatial, temporal and spatiotemporal lags from an econometric-theoretical perspective: Kelejian and Prucha (2004), Cohen-Cole et al. (2013), Liu (2014), Baltagi and Deng (2015), and Yang and Lee (2017, 2019). The last two studies formalize a system of dynamic spatial panel data equations that is able to deal with temporal, spatial and spatiotemporal lags, with fixed effects in space and time, and to a limited extent with different spatial weight matrices. Despite the relative scarcity of this type of econometric-theoretical studies, empirical motivations for multivariate dynamic spatial panel data models have been well depicted in the literature, especially in regional science, such as Allers and Elhorst (2011), Baltagi and Bresson (2011), de Graaff et al. (2012), and Elhorst et al. (2021). The last study integrates multivariate dynamic spatial panel data models and global vector autoregressive (GVAR) models, popular in the macro-econometric literature, and shows that spillover effects measured by the so-called indirect effects in the spatial econometric literature and impulse responses in the GVAR literature are analytically equivalent. The studies of Yang and Lee (2017, 2019) and Elhorst et al. (2021) are used in this study as point of departure to develop a multivariate model of Okun's law with dynamic effects in both space and time and to determine the impact of shocks to unemployment and output growth along three channels: (i) the spatial channel covering the extent to which a shock to one variable in a particular economy affects other economies; (ii) the time channel focusing on the speed of shock transmissions; and (iii) the mutual impact of the variables on each other, which will be used to answer the questions how much growth is needed to reduce unemployment by one percentage point and how large the costs of unemployment are in terms of forgone output growth.

The empirical analysis in this paper is based on a spatial panel of unemployment and growth rates of the twelve provinces in the Netherlands over the period 1974–2018. The spatial weight matrices are

[1] For this reason, the coefficient of regressing y on x, where y and x are two variables, is generally not identical to the reciprocal of the coefficient of regressing x on y.

[2] The average estimate of Okun's coefficient taken from studies based on U.S. data in which the unemployment gap is taken as the dependent variable is 2.85 (Moosa, 1997, 1999; Sögner, 2001; Cuaresma, 2003; Silvapulle et al., 2004; Huang and Lin, 2008). For studies that take the output gap as the dependent variable this average estimate amounts to 1.74 (Prachowny, 1993; Attfield and Silverstone, 1997, 1998; Freeman, 2001; Lee, 2000).

based on the binary contiguity principle of sharing a common border and on the travel times between provinces, though not on the travel times between their capitals or centroids, but on the travel times between the municipalities located in these provinces and weighted by their population sizes. The advantage of this setup is that it also offers the opportunity to calculate internal travel times within provinces, an opportunity that will be utilized to model spatial interaction effects across variables.

The paper is organized as follows. In Section 2, we provide an overview of that part of the literature on Okun's law that already paid attention to multivariate systems and dynamics in space and time. This overview is used to explain our contribution to the literature in more detail. In Section 3, we set out the model specification, the estimation approach, and the determination of own-variable and cross-variable spillover effects. Data, its implementation, and results are presented in Section 4. Finally, conclusions, discussions, limitations and options for further research are the subject of Section 5.

2. Okun's law and the triangle between simultaneity, space and time

Since the pioneering work of Okun, several extensions of his law have been proposed and reviewed. However, up to now not one single study has considered a multivariate model with dynamic effects in both space and time, even not when analyzing Okun's law for multiple countries or regions.

Durech et al. (2014) estimate the relationship between the output gap (dependent variable) and the unemployment gap for Czech Republic and Slovakia at the regional level over the period 1995–2011, treating the 22 regions in their analysis as independent entities. It is a recent but classical example of all studies that we classified under group B in the introduction to this paper. Huang et al. (2020) estimate the annual change in unemployment by the annual change in the level of GDP (growth rate), using a model with a homogenous slope for Okun's coefficient but with country and time period fixed effects, for a space-time data set of up to 66 countries over the period 1960–2016. To deal with endogeneity, they apply two-stage least square estimation with weighted annual variation in the international oil price as exogenous instrument for economic growth. Using a similar model based on quarterly data, Ibragimov and Ibragimov (2017) adopt an IV estimator to instrument the growth rate for six countries of the former USSR. Importantly, all three studies ignore potential spatial interactions in their model and, except for Huang et al. (2020), focus on cross-country or region comparative analyses.

Kangasharju et al. (2012) test for and find evidence in favor of cointegration of Okun's law, using a wide set of Finnish travel-to-work areas (74 regions) over the period 1976–2006. To control for cross-sectional dependence in both unemployment and output, the change at the regional level is accounted for, though only if it exceeds the corresponding change at the national level in magnitude. Huang and Yeh (2013) adopt an autoregressive distributed lag (ARDL) model of order (2, 1) and rewrite it as an error correction model to allow for temporal dynamics in the dependent variable (change in unemployment rate) and the explanatory variable (GDP growth rate). The model is estimated for 53 countries (21 OECD and 32 non-OECD) and for 50 U.S. states. The authors use the pooled mean group (PMG) estimator developed by Pesaran et al. (1999) to end up with one long-run estimate of the impact of output growth on unemployment for the whole sample. In sum, both studies recognize simultaneity and control for either cross-sectional dependence or time-series dependence, but not both.

Basistha and Kuscevic (2017) estimate a homogenous version of Okun's law for 48 U.S. states over the period 1987–2014 with state fixed effects but without time fixed effects. They enhance previous studies by regressing the change in unemployment not only on the growth rate in the own state, but also in neighboring states and even in non-neighboring states. This study is a recent example of all studies that we classified under group C in the introduction to this paper.

Probably the most complete work up to now is of Palombi et al. (2017). They examine Okun's law for 128 British NUTS3 areas over the period 1985–2011. The authors adopt a dynamic spatial Durbin model with homogenous coefficients and control for spatial-specific effects, and test whether these effects should be fixed or may be modeled as being random. In addition, they use instrumental variables to control for endogeneity of the right-hand side variables, which in their case is GDP growth in the own and in neighboring areas, since the change in unemployment is taken as the dependent variable. Yet they limit themselves to a univariate model and do not control for time-period fixed (or random) effects. The latter is remarkable since the results of the cross-sectional dependence test of Pesaran (2015) applied to both the change in unemployment and output growth (Palombi et al., 2017, Table 1) turn out to take values outside the critical interval (−1.96,+1.96) of this test. This points to a degree of cross-sectional dependence beyond that generally considered in spatial econometric type of studies. Time dummies are the minimum to control for this.

In this paper a multivariate model of Okun's law with dynamic effects in both space and time will be developed. Just as in the studies of Kangasharju et al. (2012), Huang and Yeh (2013), Basistha and Kuscevic (2017), Palombi et al. (2017), and Huang et al. (2020), the coefficients of this model are assumed to be homogenous rather than heterogeneous. The explanation for this is twofold. The model that will be set out in the next section contains 13 parameters, of which 10 would be unit-specific when allowing for heterogeneous slopes. Our data set consists of $N = 12$ provinces in the Netherlands. This implies that the number of parameters to be estimated in a heterogeneous model would increase to $12 \times 10 + 3 = 123$ parameters. This number would increase even further if the elements of the variance-covariance to be introduced below are also assumed to be heterogeneous (see Aquaro et al., 2021). Since we will estimate the parameters by maximum likelihood (ML) and the log-likelihood of a multivariate spatial econometric model cannot be concentrated with respect to a subset of the parameters, except for the parameters of the variance-covariance matrix (in our case 3 of the 123 parameters), maximizing the log-likelihood function including the Jacobian term accounting for so many parameters is still a bridge too far at the moment, both from a programming and a numerical viewpoint. This might change when the estimation of multivariate spatial econometric models becomes more common and faster algorithms become available.

3. Methodology

3.1. The econometric model

The system of dynamic spatial panel data models introduced by Yang and Lee (2019) reads as

$$
\begin{aligned}
z_t\Gamma = {} & [W_1 z_{1t}, \ldots, W_m z_{mt}]\Psi + [W_1 z_{1t-1}, \ldots, W_m z_{mt-1}]\Phi + \\
& + z_{t-1}P + x_t\Pi + C_n + \alpha_t \otimes \iota_N + \varepsilon_t, \quad \varepsilon_t \sim (0, \Sigma)
\end{aligned} \tag{2}
$$

where $z_t = (z_{1t}, z_{2t}, \ldots, z_{mt})$ is an $(N \times m)$ vector of dependent variables. In this study, $m = 2$ and $z_t = (y_t - y_t^*, u_t - u_t^*)$. Γ is an $(m \times m)$ matrix of mutual impacts among the m dependent variables. Its diagonal elements are normalized to one since an independent variable can only affect other variables and not itself, which in case of $m = 2$ implies that $\mathrm{vec}(\Gamma) = (1, \gamma_{2,1}, \gamma_{1,2}, 1)'$. Ψ is an $(m \times m)$ matrix, with $\mathrm{vec}(\Psi) = (\psi_{1,1}, \psi_{2,1}, \psi_{1,2}, \psi_{2,2})'$ for $m = 2$. This matrix is used as part of the term $[W_1 z_{1t}, \ldots, W_m z_{mt}]\Psi$ to denote the impact of the spatially lagged dependent variables measured at time t of both own-variable spatial lags along the diagonal of Ψ, and of cross-variable spatial lags along the off-diagonal of Ψ. Yang and Lee (2019) assume that each dependent variable has its own row-normalized $(N \times N)$ spatial weight matrix with zero diagonal describing the spatial arrangement of the units in the sample. Similarly, the term $[W_1 z_{1t-1}, \ldots, W_m z_{mt-1}]\Phi$ denotes the impact of the spatially lagged dependent variables measured at time t-1, where Φ is an $(m\times\ m)$ matrix and $\mathrm{vec}(\Phi) = (\varphi_{1,1}, \varphi_{2,1}, \varphi_{1,2}, \varphi_{2,2})'$ for $m = 2$. Temporal dynamics are controlled for by the term $z_{t-1}\mathrm{P}$, where P is an $(m\times\ m)$ matrix of autoregressive coefficients with $\mathrm{vec(P)} = (p_{1,1}, p_{1,2}, p_{2,1}, p_{2,2})'$ for $m = 2$. The term $x_t\Pi$ represents the impact of K exogenous regressors x_t through the $(K\times m)$ matrix Π with $\mathrm{vec}(\Pi) = (\pi_{1,1}, \ldots, \pi_{K,1}, \pi_{1,2}, \ldots, \pi_{K,2})'$ for $m =$ 2. The $(N\times\ m)$ vector ε_t reflects the independently and identically distributed idiosyncratic disturbances of the model with covariance matrix Σ, and $\mathrm{vec}(\Sigma) = (\sigma_{1,1}, \sigma_{2,1}, \sigma_{1,2}, \sigma_{2,2})'$ and $\sigma_{1,2} = \sigma_{2,1}$ for $m = 2$. The off-diagonal entries of this covariance matrix allow the error terms of the two equations to be correlated for each unit at the same moment in time. The $(N\times m)$ vector C_n represents individual fixed effects and controls for all unit-specific, time-invariant variables in each of the m equations whose omission could bias the parameter estimates in a typical time-series application. Similarly, the $(N\times m)$ vector $\alpha_t\otimes\ \iota_N$, where α_t is an $(1\ \times m)$ vector and ι_N is an $(N\times 1)$ vector of ones, represents time fixed effects and controls for all time-specific, unit-invariant variables in each of the m equations whose omission could bias the parameter estimates in a typical cross-sectional application.

Yang and Lee (2019) derive the conditions that need to be satisfied for the full information maximum likelihood (FIML) estimator set out in their paper to be identified, consistent, and asymptotically normal. First, T needs to be sufficiently large to avoid that the initial values of the dependent variables in all units also need to be explained, such as in Elhorst (2010) and Parent and LeSage (2011) for single equation models. If $N/T\to C$, where C is a finite positive constant, or $N/\ T\to\infty$, the proposed FIML estimator needs to be bias-corrected. Details about the mathematical expression of this bias-correction can be found in Yang and Lee (2019, theorem 2). By contrast, if $N/T\to 0$, the bias disappears.

Second, the weight matrices of the spatial lags should have zero diagonal elements, be row-normalized, and row and columns sums should be uniformly bounded in absolute value. Row-normalization is necessary to concentrate out the time period fixed effects. There are two techniques to concentrate out the time fixed effects in combination with the individual fixed effects. One is the standard demeaning procedure set out in Baltagi (2013). For $z_{m,t}$ this procedure reads as $\overline{\overline{z}}_{m,t} = z_{m,t} - \dot{z}_{m,t} - \ddot{z}_{m,i} + \overline{z}_m$, where $\dot{z}_{m,t} = \frac{1}{N}\sum_{i=1}^{N} z_{m,it}$ for each $t = 1, \ldots, T$, $\ddot{z}_{m,i} = \frac{1}{T}\sum_{t=1}^{T} z_{m,it}$ for each $i = 1,\ \ldots, N$, and $\overline{z}_m = \frac{1}{NT}\sum_i\sum_t z_{m,it}$. Similar transformations need to be carried out on $z_{m,t}$ lagged in space, time and space-time. The second technique is the orthogonal Helmert transformation, part of the FIML estimator of Yang and Lee (2019). Let $J_N = I_N - \frac{1}{N}\iota_N\iota_N'$ denote the cross-sectional mean transformation matrix normally used to wipe out the time fixed effects, and $F_{N,N-1}$ the matrix of eigenvectors corresponding to the $N-1$ eigenvalues of J_N that equal one. Then multiplying all vectors $z_{m,t}$ and their counterparts lagged in space, time and space-time on the left and right-hand side of Equation (2), yielding $\tilde{z}_{m,t} = F_{N,N-1}'z_{m,t}$, eliminates the time fixed effects of the model. The individual fixed effects can subsequently be concentrated out by applying the standard transformation $J_T = I_T - \frac{1}{T}\iota_T\iota_T'$, but then to the transformed variables $\tilde{z}_{m,t}$ to get $\tilde{\tilde{z}}_{m,t} = J_T\tilde{z}_{m,t} = J_T F_{N,N-1}' z_{m,t}$. Importantly, Lee and Yu (2010) demonstrate that the disturbances resulting from the standard demeaning procedure are linearly dependent over the time dimension. The orthogonal Helmert transformation avoids this linear dependence, but has the effect that the $N\times 1$ vector $z_{m,t}$ changes into a vector $\tilde{z}_{m,t}$ of length $(N-1)\times 1$. Instead of N observations on each variable at a particular moment in time, $N-1$ observations remain available for estimation.

The concentrated log-likelihood function of the model with both the individual and time period fixed effects concentrated out by the orthogonal Helmert transformation reads as

$$\ln L = -\frac{m(N-1)T}{2}\ln(2\pi) + T\ln|S| - T\ln|\Gamma-\Psi| - \frac{(N-1)T}{2}\ln|\Sigma| - \frac{1}{2}\sum_{t=1}^{T} e_t'\Sigma^{-1}e_t. \tag{3}$$

where $S = \Gamma'\otimes I_N - (\Psi'\otimes I_N)\boldsymbol{W}$, $\boldsymbol{W} =$ diag$(W_1,\ W_2,\ \ldots,\ W_m)$, the $((N-1)m\times 1)$ vector of residuals at time t are

$$e_t = S^*\mathrm{vec}(\tilde{\tilde{z}}_{m,t}) - [(\Phi'\otimes I_{N-1})\boldsymbol{W}^* + P'\otimes I_{N-1}]\ \mathrm{vec}(\tilde{\tilde{z}}_{m,t-1}) - (I_m\otimes\tilde{\tilde{x}}_t)\mathrm{vec}(\Pi), \tag{4}$$

$S^* = \Gamma'\otimes I_{N-1} - (\Psi'\otimes I_{N-1})\boldsymbol{W}^*$, $\boldsymbol{W}^* =$ diag$(W_1^*, W_2^*, \ldots, W_m^*)$, and $W_m^* = F_{N,N-1}'W_m F_{N,N-1}$. The derivation of this concentrated log-likelihood function is based on two properties. First, $F_{N,N-1}'W_m z_{m,t} = W_m^* F_{N,N-1}' z_{m,t}$. Second, $\ln|S| - \ln|\Gamma-\Psi| = \ln|\Gamma'\otimes \mathrm{I}_{N-1} - (\Psi'\otimes I_{N-1})\mathbf{W}^*| = \ln|S^*|$, as a result of which $\ln|S| - \ln|\Gamma-\Psi|$ can be used to replace $\ln|S^*|$. Both properties only hold when W_m is row-normalized and its diagonal elements are zero (see Yang and Lee, 2019 for details). Finally, it is to be noted that the concentrated log-likelihood function is expressed in terms of the transformed observations $\tilde{\tilde{z}}_{m,t}$, while Yang and Lee (2019, Equation (3)) did not substitute out J_N by the matrix $F_{N,N-1}'$.

3.2. Implementation of Okun's law

To estimate a multivariate model of Okun's law with dynamic effects in both space and time, we rewrite and adjust the setup proposed by Yang and Lee (2019) as follows:

$$\begin{vmatrix} I_N & -\psi_{1,2}R \\ -\psi_{2,1}R & I_N \end{vmatrix}\begin{vmatrix}\Delta y_t \\ \Delta u_t\end{vmatrix} = \begin{vmatrix}\psi_{1,1}W_a & \psi_{1,2}W_b \\ \psi_{2,1}W_b & \psi_{2,2}W_a\end{vmatrix}\begin{vmatrix}\Delta y_t \\ \Delta u_t\end{vmatrix} + \begin{vmatrix}\varphi_{1,1}W_a & \varphi_{1,2}W_b \\ \varphi_{2,1}W_b & \varphi_{2,2}W_a\end{vmatrix}\begin{vmatrix}\Delta y_{t-1} \\ \Delta u_{t-1}\end{vmatrix} + \begin{vmatrix}p_{1,1}I_N & \varphi_{1,2}R \\ \varphi_{2,1}R & p_{2,2}I_N\end{vmatrix}\begin{vmatrix}\Delta y_{t-1} \\ \Delta u_{t-1}\end{vmatrix} + \begin{vmatrix}C_{yn} \\ C_{un}\end{vmatrix} + \begin{vmatrix}\gamma_{yt} \\ \gamma_{ut}\end{vmatrix} + \begin{vmatrix}\varepsilon_{yt} \\ \varepsilon_{ut}\end{vmatrix}, \text{ and } \begin{vmatrix}\varepsilon_{yt} \\ \varepsilon_{ut}\end{vmatrix} \sim iidN\left(\begin{vmatrix}0 \\ 0\end{vmatrix}, \begin{vmatrix}\sigma_y^2 I_N & \sigma_{yu}I_N \\ \sigma_{uy}I_N & \sigma_y^2 I_N\end{vmatrix}\right). \tag{5}$$

Below all changes with respect to Yang and Lee's original specification are discussed one by one. First, the model does not contain any exogenous regressors x_t. Using a production function approach, Prachowny (1993) has argued that Okun's law in (1) should be extended to include variables measuring the difference between the actual and the potential utilization rate of capital (c-c*), the difference between the actual and the potential supply of workers (l-l*), and the difference between the actual and the potential number of working hours (h-h*). However, this study has not only been criticized (Attfield and Silverstone, 1997), it has also failed to gain a firm foothold in later studies (see also Perman et al., 2015; Ball et al., 2017). It has been criticized because the levels and especially the potential values of c, l and h are extremely difficult to measure, and because the labor supply gap and the capacity utilization gap are highly correlated. The overall conclusion from most studies is that the mutual relationships between output growth and unemployment are correctly specified when it is assumed that all other variables are either on their equilibrium paths or change pari passu with unemployed labor (see Freeman, 2001; Christopoulos, 2004).

Despite this, Okun's law remains difficult to put into practice because y* and u*, representing potential output and unemployment, can only be estimated, not observed. Modern empirical work offers a number of alternatives to the separation of trends and cycles in economic time series and thus for the derivation of these two variables. Examples are linear or quadratic trends, first differencing, or more complex methods such as the Beveridge-Nelson method, the Harvey structural time series approach, the Baxter-King bandpass filter and the Hodrick-Prescott (HP) filter. The HP filter has become standard to remove cyclical trends in many studies.

However, it has also been criticized (Freeman, 2001; Sögner, 2001; Silvapulle et al., 2004). Recently, Hamilton (2018, abstract) statistically formalized its inadequacy from an econometric-theoretical viewpoint. He showed that the HP filter introduces spurious dynamics "that have no basis in the underlying data-generating process". In addition, Halleck-Vega and Elhorst (2016) demonstrated that two-stage modeling approaches that first filter out (part of the) time-series variation in the data and then focus on the explanation of spatial dynamics among the filtered variables are likely to produce biased results since serial, cyclical and spatial dynamics are interdependent. For these reasons, we do not filter GDP and unemployment, but take first-differences, thereby following many previous studies, among which the overview studies of Perman et al. (2015) and Ball et al. (2017), as well as the recent empirical study of Palombi et al. (2017) based on regional data. In mathematical terms, if $\Delta\cdot_{it}$ denotes the first-difference operator, $\Delta\cdot_{it} = (\cdot_{it} - \cdot_{it-1})$, then the output gap $y_t - y_t^*$ and the unemployment gap $u_t - u_t^*$ are approached by Δy_t and Δu_t. In addition, we control for business cycle effects using time period effects. By using this approach, the impact of serial, cyclical and spatial dynamics are analyzed simultaneously.

The second change with respect to Yang and Lee's original specification concerns the spatial weight matrices. Yang and Lee (2019) allow each dependent variable to have its own spatial weight matrix, compared to Yang and Lee (2017) where all m dependent variables have only one common spatial weight matrix. By contrast, we allow the own-variable spatial lag of a variable and the cross-variable spatial lag of that variable in another equation to be different. More specifically, given that GDP and unemployment are both affected by GDP in neighboring units, the own-variable spatial lag of GDP on GDP is assumed to run through the spatial weight W_a and the cross-variable spatial lag of GDP on unemployment through the spatial weight matrix W_b. A similar setup is used with respect to unemployment.

In this study, W_a takes the form of a doubly-stochastic first-order binary contiguity matrix, and W_b the form of a doubly-stochastic inverse distance matrix conceptualized by the market potential. A matrix is doubly-stochastic if both the elements in each row and in each column sum to one. This property is needed to be able to compute spill-in and spill-out effects, an issue to which we come back in section 3.4.[3] This setup where the spatial weight matrix is different for own and cross-variable spatial lags also provides a solution to the identification problem that may occur in Yang and Lee's original specification when exogenous explanatory variables are left aside. The parameters in the model of Okun's law are identified since the cross-variable spatial lags $W_b\Delta u_t$ in the first equation and $W_b\Delta y_t$ in the second equation bring in neighboring characteristics that are unique in both equations. Whereas Δy_t depends on $W_a\Delta y_t$, i.e., output growth rates in neighboring units sharing a common border with the focal unit, Δu_t depends on $W_b\Delta y_t$, which also captures output growth rates in more distant units. Similarly, whereas Δu_t only depends on $W_a\Delta u_t$ representing changes in unemployment rates in neighboring units sharing a common border with the focal unit, Δy_t also depends on $W_b\Delta u_t$ capturing changes in unemployment rates observed in more distant units.

In line with Lee (2004) and Yang and Lee (2019), a pre-condition for obtaining consistent parameter estimates is that the diagonal elements of the spatial weight matrix (W_a) of the own-variable spatial lags are zero and that this matrix is row-normalized. This leaves open whether this matrix is sparse or dense. When testing different specifications of the spatial weight matrix against each other, however, it generally appears that this matrix should be sparse. For example, Halleck-Vega and Elhorst (2014, 2016) find that the first-order binary contiguity matrix gives the best performance when explaining regional unemployment rates by regional unemployment rates observed in surrounding regions. The reason is that own-variable spatial lags produce global spillover effects, i.e., even if regions are not connected to each other they might still affect each other. This can be seen by rewriting the spatial multiplier matrix as $(I-\psi W)^{-1} = I + \psi W + (\psi W)^2 + (\psi W)^3 + \cdots$; if region A affects B, and B affects C, then A also affects C via the second-order term $(\psi W)^2$ even if A and C are unconnected. The same applies for all the other higher-order terms. Eventually, all regions influence each other, though nearby regions more strongly than distant regions, in line with Tobler's first law of geography. In this study, we will compare the performance of two sparse spatial weight matrices with each other: a first and a second-order binary contiguity matrix.

By contrast, cross-variable spatial lags do not have the property of producing global spillovers. If two regions are unconnected according to the spatial weight matrix, GDP in one region will not affect unemployment in the other region, and vice versa. It is unlikely that a sparse matrix will perform well in this case. If output grows in one region, unemployed people even living in distant regions may benefit from this. Although the willingness to commute to a job in another region decreases with distance, there will always be people who seize this opportunity. Conversely, people who due to unemployment have less money to spend on goods and services will not only cause a loss of production (GDP) in their own region, but also in neighboring and even distant regions. For example, they might diminish their consumption of commodities produced elsewhere or not go on holiday anymore to distant regions because of less income. Consequently, the spatial weight matrix of cross-variable spatial lags is more likely to be dense. In line with this, we depart from an inverse distance matrix for cross-variable spatial lags, but in contrast to previous studies, this matrix is not simply based on the inverse of the Euclidian distance between the capitals or centroids of all regions. Researchers highlighted several problems of this approximation, especially the ignorance of population density (Head and Mayer, 2006), and several geographical characteristics of particular areas (Dijkstra et al., 2011). For example, it is quite unusual that the main city of coastal regions is located in the middle of the region. For this reason, we collected data on the population size of all municipalities within each region and all regions across the country and in different time periods. Suppose that region A consists of I municipalities ($i = 1,\ldots,I$) and region B of J municipalities ($j = 1,\ldots,J$), that the distance between two municipalities is d_{ij}, and that the population size of a municipality at a particular point in time is P_{it} or P_{jt}. Then the distance between two regions is determined as the average distance between each pair of municipalities weighted by the population sizes of these municipalities: $d_{AB,t} = \sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}P_{it}P_{jt} / \sum_{i=1}^{I}\sum_{j=1}^{J} P_{it}P_{jt}$. This approach produces a more accurate measure of the distance between two regions, as it accounts for the distribution of the population over the regions. To demonstrate this, we will also investigate the performance of the model using a standard inverse distance matrix.

Importantly, this setup also offers the opportunity to determine the internal distance within each region, i.e., by considering all pairs of municipalities within a region and weighting the distances between these municipalities within this region by their population sizes. In principle, the diagonal elements of matrix W_b could be re-specified as the inverted internal distance, such that the two cross-variable spatial lags, $W_b\Delta y_t$ and $W_b\Delta u_t$, might also viewed as market potentials, the first with respect to GDP and the second with respect unemployment. In general, the market potential stands for the accessibility of a specific region, i.e., the sum of economic (in)activity of other regions dependent on the distance to these regions, plus the potential of the region itself based on its internal distance. However, since the FIML estimator developed by Yang and Lee (2019) requires that the diagonal elements of the spatial weight matrices are zero, we developed another solution. Judging by the coefficient and covariance matrices, the system in Equation (2) contains $m + 3m^2 + m(m+1)/2$ parameters to be estimated when exogenous regressors x_t are left aside, which amounts to 17 for $m = 2$, i.e., $\vartheta = (\gamma_{2,1}, \gamma_{1,2}, \psi_{1,1}, \psi_{2,1}, \psi_{1,2}, \psi_{2,2}, \varphi_{1,1}, \varphi_{2,1}, \varphi_{1,2}, \varphi_{2,2}, p_{1,1}, p_{2,1}, p_{1,2}, p_{2,2}, \sigma_y^2, \sigma_{uy}, \sigma_u^2)$. Let R denote the diagonal matrix or order N whose elements measure the inverted internal distance

[3] We thank one of the reviewers for pointing this out to us.

Table 1
Spatial weight matrix W_a (lower diagonal), W_b (upper diagonal) and R (diagonal). W_a and W_b are symmetric with row and column sums of one.

Elements												
0.37		0.17	0.18	0.11	0.08	0.07	0.06	0.07	0.06	0.07	0.06	0.07
	0.38		0.17	0.12	0.09	0.07	0.06	0.08	0.06	0.07	0.05	0.07
0.46		0.27		0.15	0.08	0.07	0.05	0.06	0.05	0.06	0.05	0.07
0.54	0.15		0.25		0.09	0.12	0.07	0.07	0.06	0.07	0.07	0.09
0	0.26	0.31		0.25		0.10	0.12	0.13	0.08	0.07	0.07	0.07
0	0.13	0	0.27		0.19		0.11	0.09	0.07	0.08	0.10	0.11
0	0	0	0.16	0.08		0.27		0.14	0.11	0.09	0.10	0.09
0	0	0	0	0.22	0.13		0.28		0.12	0.09	0.08	0.08
0	0	0	0	0.30	0	0.50		0.35		0.16	0.12	0.09
0	0	0	0	0	0.05	0.15	0.20		0.67		0.13	0.11
0	0	0	0	0	0	0	0	0.56		0.21		0.15
0	0	0	0	0	0.04	0	0	0.05	0.44		0.58	
0	0	0	0	0	0.53	0	0	0	0	0.47		

within each region. These elements are determined in the same way as the elements of W_b. After having determined the elements of W_{raw} and R_{raw} in raw form, the Sinkhorn-Knopp algorithm developed by Knight (2008) is used to get the corresponding doubly-stochastic matrix W_b (this approach is also used to get W_a). To this end, two diagonal matrices S_1 and S_2 are determined such that $W_b = S_1 W_{raw} S_2$. Starting from these two matrices, we next construct $R = S_1 R_{raw} S_2$ to get an R matrix whose elements are scaled in the same way as those of W_b. To clarify the spatial weight matrices, their numerical values are reported in Table 1. Since the matrices W_a and W_b are symmetric, we only report the lower-diagonal elements of the first and the upper-diagonal elements of the second matrix. The reported numbers show that the matrix W_a is sparse. When excluding the zero diagonal, 87 of the remaining 132 elements turn out to be zero. Conversely, W_b is dense. None of its off-diagonal elements are zero. The row and columns of both matrices sum up to one. The matrix R only has non-zero diagonal elements, reflecting the inverse internal distance within each of the twelve regions. These elements are larger than the off-diagonal elements of W_b since interregional distances are generally greater than intraregional distances.

When employing the standard demeaning procedure set out in Baltagi (2013), denoted by D below, the log-likelihood function of Eq. (5) with both the individual and time-period fixed effects concentrated out takes the form

$$\ln L = -\frac{mNT}{2}\ln(2\pi) + T\ln|S_D| - \frac{NT}{2}\ln|\Sigma| - \frac{1}{2}\sum_{t=1}^{T} e_t^{'}\Sigma^{-1}e_t, \tag{6}$$

where the $(Nm \times 1)$ vector of residuals at time t is $e_t = S_D \mathrm{vec}(\Delta\overline{\overline{y}}_t, \Delta\overline{\overline{u}}_t) - S_{D\Phi P}\mathrm{vec}(\Delta\overline{\overline{y}}_{t-1}, \Delta\overline{\overline{u}}_{t-1})$,

$$S_D = \begin{vmatrix} I_N & -\psi_{1,2}R \\ -\psi_{2,1}R & I_N \end{vmatrix} - \begin{vmatrix} \psi_{1,1}W_a & \psi_{1,2}W_b \\ \psi_{2,1}W_b & \psi_{2,2}W_a \end{vmatrix} \text{ and}$$

$$S_{D\Phi P} = \begin{vmatrix} \varphi_{1,1}W_a & \varphi_{1,2}W_b \\ \varphi_{2,1}W_b & \varphi_{2,2}W_a \end{vmatrix} + \begin{vmatrix} p_{1,1}I_N & \varphi_{1,2}R \\ \varphi_{2,1}R & p_{2,2}I_N \end{vmatrix}.$$

This implies that the parameters $(\gamma_{2,1}, \gamma_{1,2})$ and $(p_{2,1}, p_{1,2})$ do not have to be estimated anymore, and thus that this number reduces to 13. However, the objection to this approach is that the disturbances will be linearly dependent over the time dimension and might bias the parameter estimates. Lee and Yu (2010) analyzed these biases for a single equation approach. They find that especially the bias in the spatial autoregressive parameter of the spatially lagged dependent variable may grow large if N is small. To be able to employ the orthogonal Helmert transformation instead and to avoid this linear dependence over the time dimension that occurs when reducing the numbers of parameters to be estimated, we need to impose the restriction

$$\begin{vmatrix} I_N & -\psi_{1,2}R \\ -\psi_{2,1}R & I_N \end{vmatrix} = \Gamma \otimes I_N. \tag{7}$$

This restriction is necessary since the property that $F_{N,N-1}^{'}Rz_{m,t} = R_m^{*}F_{N,N-1}^{'}z_{m,t}$ does not hold for a diagonal matrix.[4] This reduction of R to the matrix Γ of order $(m \times m)$ is possible by scaling the ψ parameters by the average inverted internal distance $(\overline{R})$ over all regions in the sample. The φ parameters that are part of the matrix P are scaled in the same way. The objection to imposing this restriction is that it might also bias the parameters, but at least it gives the opportunity to apply the orthogonal Helmert transformation, denoted by H below, advocated by Yang and Lee (2019). The log-likelihood function of Equation (5) with both the individual and time-period fixed effects concentrated out then takes the form

$$\ln L = -\frac{m(N-1)T}{2}\ln(2\pi) + T\ln|S_H| - T\ln|\Gamma - \Psi| - \frac{(N-1)T}{2}\ln|\Sigma| - \frac{1}{2}\sum_{t=1}^{T} e_t^{'}\Sigma^{-1}e_t. \tag{8}$$

where the $((N-1)m \times 1)$ vector of residuals at time t is $e_t = S_H^{*}\mathrm{vec}(\Delta\tilde{\tilde{y}}_t, \Delta\tilde{\tilde{u}}_t) - S_{H\Phi P}^{*}\mathrm{vec}(\Delta\tilde{\tilde{y}}_{t-1}, \Delta\tilde{\tilde{u}}_{t-1})$,

$$S_H = \begin{vmatrix} 1 & -\psi_{1,2}\overline{R} \\ -\psi_{2,1}\overline{R} & 1 \end{vmatrix} \otimes I_N - \begin{vmatrix} \psi_{1,1}W_a & \psi_{1,2}W_b \\ \psi_{2,1}W_b & \psi_{2,2}W_a \end{vmatrix},$$

$$S_H^{*} = \begin{vmatrix} 1 & -\psi_{1,2}\overline{R} \\ -\psi_{2,1}\overline{R} & 1 \end{vmatrix} \otimes I_{N-1} - \begin{vmatrix} \psi_{1,1}W_a^{*} & \psi_{1,2}W_b^{*} \\ \psi_{2,1}W_b^{*} & \psi_{2,2}W_a^{*} \end{vmatrix}, \text{ and}$$

$$S_{H\Phi P}^{*} = \begin{vmatrix} \varphi_{1,1}W_a^{*} & \varphi_{1,2}W_b^{*} \\ \varphi_{2,1}W_b^{*} & \varphi_{2,2}W_a^{*} \end{vmatrix} + \begin{vmatrix} p_{1,1} & \varphi_{1,2}\overline{R} \\ \varphi_{2,1}\overline{R} & p_{2,2} \end{vmatrix} \otimes I_{N-1}.$$

3.3. Monte Carlo simulation

To estimate the parameters of Equation (5) by the FIML estimator of Yang and Lee (2019) based on the formulas set out in their paper (including the appendix of their paper), we developed a computer program, written in Matlab. To evaluate this estimator and computer program, we also carried out a simple Monte Carlo (MC) simulation experiment, with $m = 2$, $N = 12$, and $T = 20, 40$. This experiment is also meant to test whether the small sample properties of Yang and Lee's FIML estimator remain intact, despite the changes made in the setup of their model, discussed in section 3.2. The bias corrections set out in Yang and Lee (2019) are left aside since we only consider values of T larger than N. Finally, it offers the opportunity to test whether the simpler standard demeaning procedure to concentrate out the time fixed effects is an acceptable alternative to the orthogonal Helmert transformation. The error terms are randomly generated from a normal distribution with zero mean and variance-covariance matrix Σ, where $vech(\Sigma) = (1.2, 0.4, 1.3)$. The individual and time period fixed effect C_n and $\alpha_t \otimes \iota_N$ are generated by uniformly distributed random variables on the interval [0,1].

[4] We thank the other reviewer for pointing this out to us.

Table 2
Results Monte Carlo simulations based on N = 12, T = 20,40, 500 replications, and two demeaning procedures.

Parameter	True	Orthogonal Helmert transformation				Standard demeaning			
		T = 20		T = 40		T = 20		T = 40	
		Bias	MCse	Bias	MCse	Bias	MCse	Bias	MCse
$\psi_{1,1}$	0.100	−0.036	0.015	−0.022	0.006	0.009	0.008	0.010	0.004
$\psi_{2,1}$	−0.150	−0.054	1.262	−0.025	0.593	0.013	0.201	0.000	0.084
$\psi_{1,2}$	−0.050	0.085	0.824	0.021	0.375	0.017	0.133	0.007	0.071
$\psi_{2,2}$	0.200	−0.042	0.016	−0.026	0.007	0.014	0.007	0.009	0.004
$\varphi_{1,1}$	0.200	0.011	0.017	−0.004	0.007	0.016	0.013	0.003	0.006
$\varphi_{2,1}$	−0.200	0.053	0.122	0.025	0.069	0.019	0.058	0.009	0.031
$\varphi_{1,2}$	−0.100	−0.013	0.099	−0.006	0.050	0.002	0.049	−0.007	0.025
$\varphi_{2,2}$	0.200	−0.015	0.014	−0.013	0.006	−0.004	0.012	0.007	0.007
$p_{1,1}$	0.150	0.064	0.007	0.030	0.002	0.061	0.005	0.028	0.002
$p_{2,2}$	−0.300	0.033	0.005	0.017	0.003	0.036	0.004	0.019	0.002
σ_1^2	1.200	−0.009	0.058	0.000	0.025	0.166	0.016	0.127	0.008
$\sigma_{1,2}$	0.400	0.020	0.040	0.018	0.018	0.051	0.017	0.039	0.013
σ_2^2	1.300	0.007	0.076	−0.004	0.035	0.179	0.017	0.148	0.008

The results of the MC simulation are recorded in Table 2. For each parameter of ϑ, we report its true value, bias and standard error. The latter is obtained as the square root of $E(\widehat{\vartheta}^2) - [E(\widehat{\vartheta})]^2$, where $E(\widehat{\vartheta}^2)$ is the empirical mean over all replications of the squared estimated parameter vector and $[E(\widehat{\vartheta})]^2$ is the empirical mean over all replications squared.

The magnitude of the biases and MC standard errors when applying the orthogonal Helmert transformation (left panel) decrease when increasing T, in line with Yang and Lee (2019) econometric-theoretical finding that no bias correction is necessary when $N/T \to 0$. The same applies to the standard demeaning procedure (right panel). The orthogonal Helmert transformation produces smaller biases in the σ parameters, comparable biases in the p parameters, but larger biases in the ψ and φ parameters than the standard demeaning approach. The standard errors (MCse) of the standard demeaning approach are also smaller. There are two explanations for this result. First and most importantly, T is large relative to N. Second, the standard demeaning approach benefits from the fact that in our particular empirical setting the full matrix R of inverted internal distances within regions can be utilized, whereas the orthogonal Helmert transformation needs to impose the restriction specified in Equation (7).

3.4. Own-variable and cross-variable spillovers

After having estimated the parameters of the spatial econometric multivariate model of Okun's law, either by the standard demeaning approach or by the orthogonal Helmert transformation, it is tempting to consider to parameter estimates of $\psi_{1,2}$, $\varphi_{1,2}$ and $p_{1,1}$ to analyze the impact of a change in unemployment on GDP growth, and conversely, to consider $\psi_{2,1}$, $\varphi_{2,1}$ and $p_{2,2}$ to analyze the impact of a change in GDP growth on unemployment. However, the problem is that these parameters do not represent the marginal effects of these changes in a multivariate dynamic spatial panel data model. Instead, one should derive the marginal effects from the reduced form of the model. The determination of marginal effects within a system of dynamic spatial panel data models is not considered in Yang and Lee (2019), but in LeSage and Chih (2016) for a single equation model and in Elhorst et al. (2021) for a simultaneous equation model. LeSage and Chih (2016, p.4) further recommend using a doubly-stochastic weight matrices when comparing the marginal effects of different units of observation rather than when averaging these effects over all units in the sample. This is the reason why we adopted doubly-stochastic weight matrices when estimating the parameters of the model.

The short-term (time horizon $h = 0$) marginal effects of the system in Equation (5) take the form

$$\begin{bmatrix} \dfrac{\partial \Delta y_t}{\partial \varepsilon_{\Delta y,t,s}} & \dfrac{\partial \Delta y_t}{\partial \varepsilon_{\Delta u,t,s}} \\ \dfrac{\partial \Delta u_t}{\partial \varepsilon_{\Delta y,t,s}} & \dfrac{\partial \Delta u_t}{\partial \varepsilon_{\Delta u,t,s}} \end{bmatrix} = G_0^{-1} \begin{bmatrix} \varepsilon_{\Delta y,t,s} \\ \varepsilon_{\Delta u,t,s} \end{bmatrix}, \quad \text{where } G_0 = \begin{bmatrix} I_N - \psi_{1,1} W_a & -\psi_{1,2} R - \psi_{1,2} W_b \\ -\psi_{2,1} R - \psi_{2,1} W_b & I_N - \psi_{2,2} W_a \end{bmatrix} \tag{9}$$

where $\varepsilon_{\Delta y,t,s}$ and $\varepsilon_{\Delta u,t,s}$ are both $N \times 1$ vectors reflecting the unit where a change takes place, i.e., the shock s that occurs in unit i at time t. One can shock GDP growth first in unit 1, then in unit 2, and so forth to unit N, and then shock the unemployment rate first in unit 1, then in unit 2, and so forth to unit N. This results in $2N$ different outcomes for each shock (read: each change) with respect to GDP growth in the own and each of the other units, as well as unemployment in the own and each of the other units. Generally, the size of a shock is set to one standard deviation of the error term (based on the estimates of σ_y^2 and σ_u^2), but in this case it is more obvious to work with GDP growth shocks of 1 percentage point and unemployment rate shocks of 1 percentage point.

The point-in-time ($h > 0$) marginal effects of the system are

$$\begin{bmatrix} \dfrac{\partial \Delta y_{t+h}}{\partial \varepsilon_{\Delta y,t,s}} & \dfrac{\partial \Delta y_{t+h}}{\partial \varepsilon_{\Delta u,t,s}} \\ \dfrac{\partial \Delta u_{t+h}}{\partial \varepsilon_{\Delta y,t,s}} & \dfrac{\partial \Delta u_{t+h}}{\partial \varepsilon_{\Delta u,t,s}} \end{bmatrix} = G_0^{-1} G_1 \begin{bmatrix} \dfrac{\partial \Delta y_{t+h-1}}{\partial \varepsilon_{\Delta y,t,s}} & \dfrac{\partial \Delta y_{t+h-1}}{\partial \varepsilon_{\Delta u,t,s}} \\ \dfrac{\partial \Delta u_{t+h-1}}{\partial \varepsilon_{\Delta y,t,s}} & \dfrac{\partial \Delta u_{t+h-1}}{\partial \varepsilon_{\Delta u,t,s}} \end{bmatrix}, \quad \text{where } G_1 = \begin{bmatrix} p_{1,1} I_N + \varphi_{1,1} W_a & \varphi_{1,2} W_b + \varphi_{1,2} R \\ \varphi_{2,1} W_b + \varphi_{2,1} R & p_{2,2} I_N + \varphi_{2,2} W_a \end{bmatrix} \tag{10}$$

Finally, the time-cumulative marginal effects (ME) are the sum of the point-in-time marginal effects over the time horizon $\tilde{h}$ at which the point-in-time marginal effects have (almost) converged to zero after a shock.

$$\sum_{h=0}^{\tilde{h}} \begin{bmatrix} \dfrac{\partial \Delta y_{t+h}}{\partial \varepsilon_{\Delta y,t,s}} & \dfrac{\partial \Delta y_{t+h}}{\partial \varepsilon_{\Delta u,t,s}} \\ \dfrac{\partial \Delta u_{t+h}}{\partial \varepsilon_{\Delta y,t,s}} & \dfrac{\partial \Delta u_{t+h}}{\partial \varepsilon_{\Delta u,t,s}} \end{bmatrix} = \begin{bmatrix} ME_{\Delta y \Delta y} & ME_{\Delta y \Delta u} \\ ME_{\Delta u \Delta y} & ME_{\Delta u \Delta u} \end{bmatrix} \tag{11}$$

This $2N \times 2N$ matrix of marginal effects can be partitioned into four $N \times N$ matrices. The submatrix $ME_{\Delta y \Delta y}$ measures the change in GDP growth rates in all N units due to a GDP growth rate shock in one of these units. Similarly, $ME_{\Delta u \Delta u}$ measures the change in unemployment in all N units due to an unemployment shock in one of these units. Following LeSage and Pace (2009), each diagonal element of these two submatrices represents a direct effect, i.e., the extent to which a unit is affected by the shock in that unit itself, and every off-diagonal element an indirect effect, i.e., to extent to which other units are affected by the shock in that unit. Elhorst (2014) labels these indirect effects as spatial spillover effects. LeSage and Chih (2016) argue that these spillover effects can also be split into spill-in and spill-out effects. When reading the off-diagonal elements by row, each spillover effect represents the vulnerability of a unit to a GDP growth or unemployment shock in another unit, i.e., the extent to which it spills in. When reading the off-diagonal elements by column, each spillover effect represents the impact a GDP growth or unemployment shock in one unit has on another unit, i.e., the extent to which it spills out. This distinction might be relevant since it is possible that some units have a large impact on other units, but are hardly sensitive to what happens in other units, and vice versa.

The submatrix $ME_{\Delta y \Delta u}$ measures the change in GDP growth rates in all N units due to an unemployment shock in one of these units, and the submatrix $ME_{\Delta u \Delta y}$ measures the change in unemployment in all N units due to a GDP growth rate shock in one of these units. In line with the two readings of Okun's law, the first matrix can be used to answer the question how much growth is needed to reduce unemployment by one percentage point, and the second matrix to answer the question what the cost of unemployment is in terms of forgone output. In contrast to the two submatrices $ME_{\Delta y \Delta y}$ and $ME_{\Delta u \Delta u}$, the diagonal elements of $ME_{\Delta y \Delta u}$ and $ME_{\Delta u \Delta y}$ do not represent direct effects but cross-variable spillover effects, i.e., the marginal effect of changing one variable on the other variable in the same unit. In line with this, we will make a distinction in the next section between cross-variable spillover effects that occur in the own and in other units. In addition, we will make a distinction between neighboring and non-neighboring units when presenting the own-variable and cross-variable spillover effects. This distinction is based on the first-order binary contiguity matrix used in the estimations, the units with which each unit shares a common border and the remaining units with which it does not share a common border.

Elhorst et al. (2021) show that the marginal effects set out above are equivalent to impulse responses used in the global vector autoregressive (GVAR) literature, and direct and indirect (and related to that spill-in and spill-out) effects in the spatial econometric literature. They also show that direct and indirect effects in a multivariate model, and especially cross-variable spillovers, have not yet been considered in the spatial econometric literature, but that a generalization of these effects from one to multiple dependent variables is straightforward since both formalizations are mathematically equivalent. This study is among the first to determine these cross-variable spillovers in an empirical setting. The results will be used to answer the questions related to two basic readings of Okun's law.

4. Local Okun's law for the Netherlands

Annual data on unemployment and GDP for the 12 provinces in the Netherlands are collected from the Dutch Central Bureau of Statistics (CBS) over the period 1974–2018 (data of the year 1973 is also used but only to determine lagged values of the variables in the model). To determine the generalized inverse distance matrix W_b, we used data of all municipalities in the Netherlands and travel distances among them in 1970, 1980, 1990, 2001 and 2012, i.e., spatial lags constructed in the period up to and including 1979 are based on figures of 1970, up to and including 1989 on figures of 1980, and so on.

The average growth rate calculated over all provinces and all years amounts to 1.95% with a standard deviation of 2.79%. The average change in the unemployment rate is 0.03% with a standard deviation of 1.03%. The ratio between these two standard deviations shows that fluctuations in output growth rates are almost twice as large as fluctuations in unemployment changes over this period, both measured at the regional level. Fig. 1 shows the development of the correlation coefficient between the output growth rate and the change in the unemployment rate of the twelve provinces in each year over time. As expected, this correlation coefficient is mainly negative. The reason to also investigate the time-cumulative marginal effects is because the response to major downturns of the Dutch economy may also have medium-term rather than just immediate effects, among which the first and the second oil crises in 1973 and 1979, the early 1990s and 2000-01 recessions, and the financial crisis in 2008 and eurozone crisis that started at the end of 2009. Fig. 2 graphs the development of the national unemployment rate over the period 1974–2018.[5] This graph shows that the national unemployment rate reached its peak of 10.5% in 1983 after the two oil crises that occurred in the 1970s. Especially since then the question of how much growth is needed to reduce unemployment by one percentage point became topical. By analyzing the period 1974–2018, we can answer this question based on four major downturns that affected the Dutch Economy, one in every decade. In line with the overview study of Perman et al. (2015), we assume Okun's law to be stable over time, which according to the overview study of Ball et al. (2017) appears to be a good approximation of reality (p.1424). Also note that we would end up with too little observations to estimate the parameters of Okun's law accurately for separate time periods (see Table 2).

Table 3 reports the estimation results of Okun's law based on these data, the first-order doubly-stochastic binary contiguity matrix and the doubly-stochastic time-varying inverse distance matrix based on travel times, and in view of the results of our Monte Carlo simulation experiment when applying the standard demeaning approach. Table 4 contains the different marginal effects based on these estimation results, using the formulas set out in the previous section. The system is dynamically stable since the largest eigenvalue of the matrix $G_0^{-1}G_1$ used in Equation (10) to calculate how each shock evolves over time is smaller than one: 0.208 (see Table 3). The half-life of a shock turns out to be 1.372 years (Table 3), which implies that each shock has largely died out over a period of 4 years. For this reason, a period of 4 years is used in Table 4 to compute the time-cumulative marginal effects. Even though the coefficients of the system are homogeneous, the direct effects, own-variable and cross-variable spillover effects are unit-specific and thus heterogeneous. For this reason, Table 3 not only reports the mean of each marginal effect, but in parentheses also the smallest and the largest marginal effects that have been found among the twelve provinces. When replacing one of the spatial weight matrices by another one, such as a second-order binary contiguity matrix, a standard inverse distance matrix between the capitals of each province or a time-invariant travel time matrix, or by adopting row-normalized rather than doubly-stochastic matrices, the McElroy (1977) R^2 of the system falls, indicating that the two adopted spatial weight matrices give the best performance.

The direct effect of a local output shock (ΔY) of 1% in the own region appears to be 1.614%. This figure is greater than one due to multiplier effects. If output of some firms or industries in the own region increases, other firms and industries in this region also benefit due to input-output relationships with these firms or industries. By contrast, other regions see their output growth decline by approximately 0.06%, which points to the existence of competitive effects between regions. Note that we hardly find any difference between neighboring and non-neighboring regions and between spill-in and spill-out effects. Since the Netherlands has 12 provinces, a local output shock of 1% in one of them thus causes a loss of output of approximately 11*0.06 = 0.66% elsewhere. This finding makes clear that output growth is not an isolated phenomenon; for two-fifths (0.66/1.614) it is at the expense of production elsewhere in the country.

The direct effect of a local unemployment shock of 1% appears to be 0.345%. This figure is smaller than one because part of the people who become unemployed are able to find another job within one year. The unemployment spillover effects (spill-in and spill-out and between neighboring and non-neighboring regions) appear to be negative and small (<-0.020).

[5] Halleck-Vega and Elhorst (2016) show that regional unemployment rates in the Netherlands follow a similar pattern.

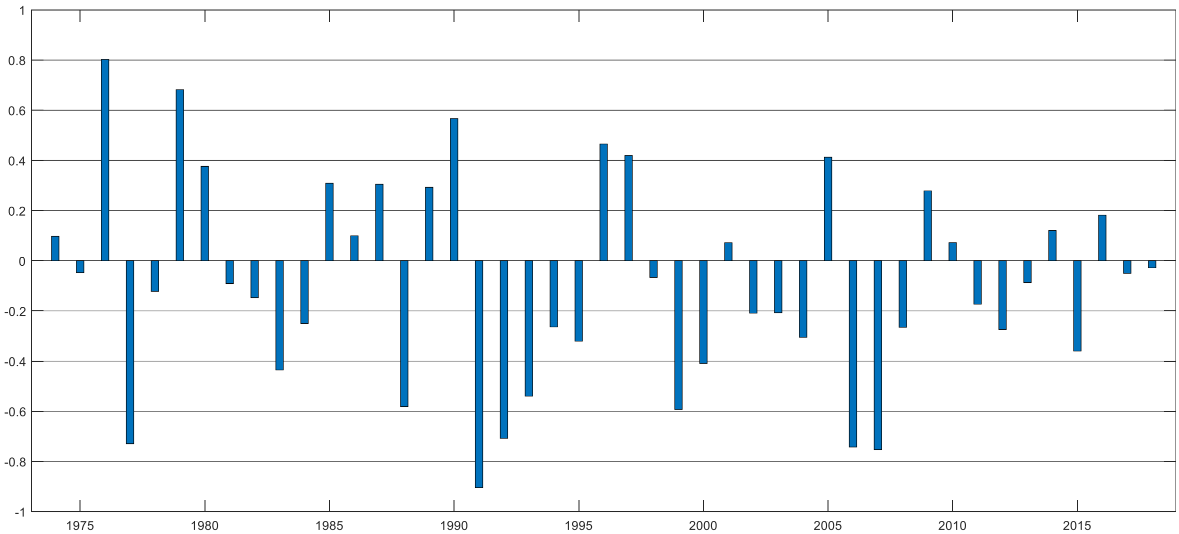

Fig. 1. The correlation coefficient between the output growth rate and the change in the unemployment rate of the twelve provinces over time (1974–2018).

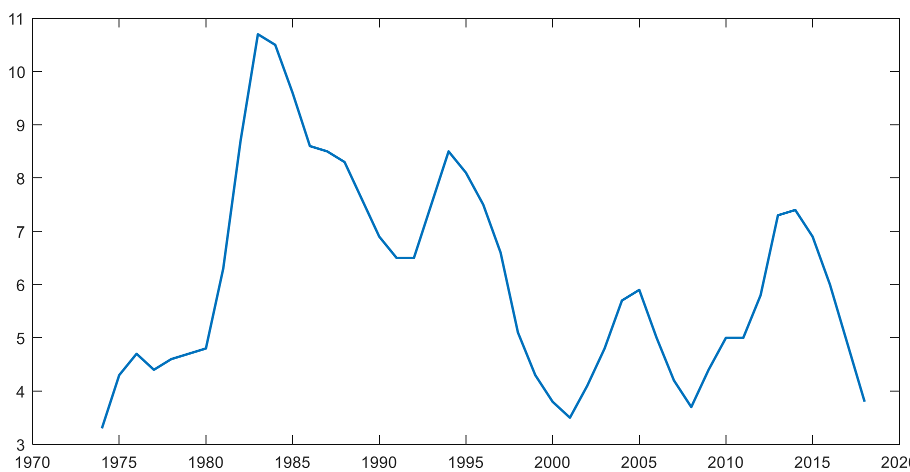

Fig. 2. The development of the national unemployment rate over time.

The average cross-variable spill-in and spill-out effects of a local output shock (ΔY) of 1% in a particular region on the unemployment rate (ΔU) in the own, neighboring and non-neighboring regions differs widely. It turns out be −0.400% on average in the own region, −0.047 (both spill-in and spill-out) in neighboring regions, and 0.010 (both spill-in and spill-out) in non-neighboring regions. The intervals with the smallest and largest effects that have been found also point to heterogeneity. More specifically, the spillover effect in the own region ranges from −0.286 to −0.501%, in neighboring regions from −0.025 to −0.081, and in non-neighboring regions from 0.003 to 0.018%. This implies that the question of how much growth is needed to reduce unemployment by one percentage point is difficult to answer in general by one single number. If a local output shock of 1% reduces the local unemployment rate by −0.400%, a naive answer would be that approximately 2.5% (1/0.400) growth is needed. However, this number ignores the fact that the local output shock also causes unemployment to change in neighboring and non-neighboring regions. An important issue here is whether local politicians care about cross-border spillover effects, especially if it concerns more distant regions. If they do care about their direct neighbors, that is, if they account for unemployment spillovers in neighboring regions, approximately 1.74% (1/(0.400 + 3.7*0.047)) growth is needed, where the number of 3.7 reflects the average number of neighbors of each single region. This reduction for growth needed demonstrates that policy coordination among adjacent regions is important. Another issue is that this number excludes the multiplier effect of 0.614% on output growth in the region itself, and of −0.06% with respect to neighboring regions. When these multiplier effects are also included, the ratio between output growth and the fall in unemployment changes into 2.43% ((1.614–3.7*0.06)/(0.400 + 3.7*0.047)), which is close to the naive number of 2.5%. When performing this calculation for each single region, taking into account the exact number of neighbors of each region, these percentages range from 2.06% for the province of Limburg to 3.12% for the province in Groningen. It should be noted that these two extremes concern two regions both located at the periphery of the country, the first in the southeast and the second in the northeast. It indicates that the output growth needed to reduce unemployment by 1% can vary by approximately 1 percentage point, even if two regions initially appear to be identical.

The average cross-variable spillover effects of a local unemployment shock (ΔU) of 1% in a particular region on the output growth rate in the own, neighboring and non-neighboring regions turn out to be relatively small. The cost of local unemployment in terms of forgone output appears to be approximately 0.11% in the own region and in neighboring regions, and negligible in non-neighboring regions. The conclusion must be that Okun's law in the Netherlands is dominated by the relationship that runs from output growth to unemployment, i.e., the relationship specified in Equation (1). This finding is also reflected by the R^2 of the unemployment rate equation, which is much higher than that of the output equation:

Table 3
Estimation Results of Okun's law for the Netherlands (N = 12, T = 45).

Parameter	Estimate	t-value
$\psi_{1,1}$	0.028	0.06
$\psi_{2,1}$	0.299	4.11
$\psi_{1,2}$	−2.389	−6.24
$\psi_{2,2}$	0.140	0.63
$\varphi_{1,1}$	0.107	0.40
$\varphi_{2,1}$	−0.099	−0.12
$\varphi_{1,2}$	1.061	0.73
$\varphi_{2,2}$	0.036	0.36
$p_{1,1}$	0.067	1.62
$p_{2,2}$	−0.313	−0.47
σ_1^2	1.863	31.41
$\sigma_{1,2}$	−0.283	−1.53
σ_2^2	0.561	4.78
Largest eigenvalue		0.208
Half-life (years)		1.372
R^2 output eq.		0.359
R^2 unemployment eq.		0.862
R^2 system		0.810
R^2 system W_a replaced by 2nd–order binary contiguity matrix		0.755
R^2 system W_b replaced by inverse distance matrix		0.639
R^2 system W_b replaced by time-invariant matrix		0.807
R^2 system W_a and W_b both row-normalized		0.770

Table 4
Mean, smallest and largest regional-specific cumulative (4-year horizon) marginal effects of local output (ΔY) and local unemployment (ΔU) shock of 1% at the regional level.

Marginal effect	Local output shock of 1%	Local unemployment shock of 1%
Direct effect own region	1.614 (1.349/1.810)	0.345 (0.297/0.384)
Spill-in ΔY		
Own region	- (direct effect)	0.114 (0.081/0.145)**
Neighboring regions	−0.062 (−0.021/-0.075)	0.110 (0.006/0.020)
Non-neighboring regions	−0.064 (−0.056/-0.071)	−0.002 (−0.001/-0.004)
Spill-in ΔU		
Own region	−0.400 (−0.286/-0.501)*	- (direct effect)
Neighboring regions	−0.047 (−0.025/-0.081)	−0.006 (−0.000/-0.015)
Non-neighboring regions	0.010 (0.003/0.018)	−0.014 (−0.012/-0.015)
Spill-out ΔY		
Own region	- (direct effect)	0.114 (−0.081/0.145)**
Neighboring regions	−0.057 (−0.002/-0.095)	0.012 (0.006/0.018)
Non-neighboring regions	−0.066 (−0.043/-0.091)	−0.002 (0.000/-0.004)
Spill-out ΔU		
Own region	−0.400 (−0.286/-0.501)*	- (direct effect)
Neighboring regions	−0.047 (−0.034/-0.073)	−0.009 (0.004/-0.017)
Non-neighboring regions	0.010 (0.003/0.015)	−0.014 (−0.009/-0.019)

Notes: Smallest and largest effects across the twelve provinces in parentheses, * Spill-in ΔU own region = Spill-out ΔU own region, ** Spill-in ΔY own region = Spill-out ΔY own region.

0.862 versus 0.359 (Table 3). This finding further indicates that the work of Palombi et at. (2017), who limited themselves to a univariate model in which the change in the rate of unemployment each year is explained by the GDP growth rate for 128 British NUTS3 areas over the period 1985–2011, is consistent with our study in terms of methodology (estimated equation) but not in terms of outcome. In Section 4.2 of their paper, they find Okun's law coefficient to be −0.2798, which implies that 3.57% output growth (1/0.2798) is needed to reduce the unemployment rate by 1 percentage point. For the Netherlands over the period 1974–2018, we find that, on average, 2.4% output growth is needed when spillover effects to neighboring regions and output multiplier effects are accounted for, and this percentage ranges between 2.1 and 3.1%.

One potential limitation is that spillover effects of Dutch border regions with neighboring regions in Germany or Belgium are not included, known as the boundary value problem, potentially leading to an omitted variable bias.[6] This problem received considerable attention in the spatial statistics literature, though mainly in the 1980s (e.g. Griffith, 1983a, 1983b, 1985; Haining, 1990), and relatively little attention in the spatial econometric literature. One notable exception is Kelejian and Prucha (2010), who propose to estimate the parameters of the model on the interior units of observation in the study area, and to use observations located at the border of the study area to determine spatially lagged values of the variables in the model. Another relevant study is of Halleck Vega and Elhorst (2014), who estimated three dynamic spatial panel data model equations separately from each other, one explaining the unemployment rate, one the labor force participation rate and one the employment growth rate, and compared the performance of ten different specifications of the spatial weight matrices, using data of 112 regions across eight EU countries over the period 1986–2010. Among these spatial weight matrices both binary contiguity matrices covering neighboring regions across national borders and binary contiguity matrices limited to linkages within countries only are considered. Their argument in favor of the first matrix is that increased integration among EU member states might make national boundaries less relevant, and in favor of the second that it is still realistic to assume that there are barriers (social, political, cultural, etc.) between neighboring countries. Their main finding is that the first-order binary contiguity matrix limited to within country neighbors gives the best performance for the unemployment rate and the participation rate equations, and that higher order matrices covering neighboring regions across national borders give the best performance for the employment growth equation. This finding in relation to our observation that the unemployment rate equation is dominating Okun's law in the Netherlands indicates that any bias due to the boundary value problem is likely to be small. Nevertheless, it is worthwhile to investigate this issue further in future research.

5. Conclusion

A system of two dynamic spatial panel data model equations is developed. Output growth in the first equation is taken to depend on output growth in the previous time period, output growth in neighboring units in the same and the previous time period, and the change in the unemployment rate in the same and in neighboring units in the same and the previous time period. Similarly, the change in the unemployment rate in the second equation is taken to depend on the change in the unemployment rate in the previous time period, the change in the unemployment rate in neighboring units in the same and the previous time period, and output growth in the same and in neighboring units in the same and the previous time period. Whereas most previous studies estimate only one of these two equations, we adopt this two-equations system of Okun's law to control for potential endogeneity of unemployment with respect to output and vice versa. The parameters of this model are estimated by maximum likelihood techniques for multivariate spatial econometric models, using data of twelve provinces in the Netherlands over the period 1983–2018, capturing four major economic downturns of the Dutch economy. Data and Matlab routines developed for this purpose will be made available. The coefficients of this model are assumed to be homogeneous rather than heterogeneous across these provinces, because maximizing the log-likelihood function in these cases is still a bridge too far at the moment, both from a programming and a numerical viewpoint. On the other hand, even though the coefficients of the system are homogeneous, the direct effects, own-variable and cross-variable spillover effects derived from this system of equations are unit-specific and thus heterogeneous.

The main results of this study are the following. First, Okun's law is

[6] This problem is less relevant for the UK since it is surrounded by sea.

dominated by the relationship that runs from output growth to unemployment; the cost of local unemployment in terms of forgone output appears to be relatively small. Second, output growth in a particular region is not an isolated phenomenon; for two-fifths it is at the expense of production elsewhere in the country. Third, policy coordination among adjacent regions is important due to spillover effects; a local output shock of 1% reduces the unemployment rate in the own region by 0.40% and in each adjacent region by 0.05%. In addition, it causes a multiplier effect of 0.6% on output growth in the region itself, and of −0.06% in adjacent regions. Fourth, the answer to the question of how much growth is needed to reduce unemployment by 1% depends on the extent to which neighboring regions and these output multiplier effects are included or not. We find that, on average, 2.4% output growth is needed when spillover effects to neighboring regions and output multiplier effects are accounted for, and this percentage ranges between 2.1 and 3.1%.

Important topics for further research are the (non-)stability of Okun's law over time due to structural breaks, the existence of a potential omitted variable bias due to the boundary value problem, and the estimation of a similar multivariate system with unit-specific coefficients.

Credit author statement

J. Paul Elhorst: Data curation, Conceptualization, Formal analysis, Investigation, Writing – original draft, review and editing. Silvia Emili: Formal analysis, Investigation, Software, Visualization, Simulation, Writing – original draft.

Declaration of competing interest

We declare that there is not any conflict of interest.

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