Estimating redenomination risk under Gumbel–Hougaard survival copulas

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Abstract

We study the dependence between redenomination and default risk for Italy, France, Germany and the Netherlands exploiting the 2014 revision of ISDA CDS contract standard. For these countries redenomination has become a credit event under the new standard, while it was not so under the old standard. Both contracts are currently traded. Using MLE on transformed data we simultaneously estimate the dependence of the two risks and an unbiased measure of redenomination risk. This allows cleaning the estimate of redenomination risk from the bias due to dependence. The unbiased estimate unveils a feature common to all four countries. Redenomination risk has risen and remained above default risk since February 2017, when the U.K. Parliament officially approved the formal request to start the Brexit procedure. We find that after February 2017 the CDS market has mostly priced redenomination as the most likely outcome of the next sovereign crisis in the Euro area. Finally, we use a Marshall-Olkin approach to model the dependence of redenomation risk of the countries and to estimate a measure of the end of the Euro. The measure obtained is very close to the value of the German CDS, in support of the widespread market practice of using that contract as tail-hedge against end of the Euro.

Keywords: Redenomination Risk, Default Risk, Credit Defualt Swaps, Sovereign Bonds, Euro crisis

JEL Codes: C002,C58,E44,F45,G01,G13

1   Introduction

A sovereign debt crisis in Europe could materialize in terms of outright default on public debt obligations and/or redenomination of the currency. It is natural to expect dependence between these two events since they typically depend on some common frailty factors in the sustainability of public debt. The purpose of this paper is to estimate this dependence, and to show that ignoring it induces a bias in the measure of redenomination risk.

On technical grounds, if default and redenomination risk are dependent, the instantaneous conditional probability that either of the two could happen is not equal to the sum of the marginal intensities (that is the marginal instantaneous conditional probabilities). Instead, it could be broken in the sum of the conditional probabilities of either of the two events occcuring before the other. These intensities (called i-intensities in Bielecki et al. 2007) collapse to the marginal intensities only if default and redenomination risk are independent.

Since 2014, an institutional innovation in the CDS market has made possible the estimation of dependence between default and redenominaton risk and the unbiased estimation of the latter. In 2014 ISDA introduced a vast revision of the CDS standards. Among other changes, the ISDA 2014 standard stated that for all the European countries, sovereign CDS can be triggered the first time that either default or redenomination occurs [1]. This marked a change with respect to the previous standard, dated 2003, for those European countries that are part to the G7 group (Italy, France and Germany) and for OECD countries with AAA-rated debt (the Netherlands)[2]. For these 4 European countries, under the 2003 standard rede-

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[1]The 2014 ISDA credit derivatives definitions at page 42 state a redenomination credit event as "...*any change in the currency of any payment of interest, principal or premium, to any currency other than the lawful **currency of Canada, Japan, Switzerland, the United Kingdom and the United States of America and the euro** and any successor currency to any of the aforementioned currencies (which in the case of the euro, shall mean the currency which succeeds to and replaces the euro as a whole).*", ISDA (2014), page 42.

[2]The 2003 ISDA credit derivatives definitions, at page 32, state that a credit event is not triggered if redenomination is to a *permitted currency*: *"Permitted Currency" means (1) the legal tender of any Group of 7 country*

nomination was not a credit event and has become a credit event under the 2014 one.

Both the contracts under the 2003 and 2014 are currently traded in the market, with the 2003 standard trading at a lower premium than the 2014 one. Unfortunately, to the best of our knowledge a formal analysis of the relative level of trading acticity is not available. However, a casual observation of the number of contracts that we performed for July 20, 2021 and July 21, 2021 showed that the trading activity on contracts underwritten under the 2003 contract remains significant, even though lower than that on the 2014 standard contract. In the two days we reckoned 142 new contracts under the 2014 standard and 96 under the 2003 standard. The 2003 contract is still traded for two reasons: first, the 2003 contract is cheaper than the other, even though it provides less protection; second, when redenomination risk is material, it can be used to build a synthetic hedge. The evidence for Italy seems to be consistent with the latter argument. In fact, for Italy, where one may expect higher redenomination risk, we counted 52 contracts under the 2014 standard and 47 under the old one. So, the trading activity in the two contracts is comparable (one is more than 90% of the other). Moreover, the trading volume on Republic of Italy represented more than 40% of the trading activity of the four countries, confirming a robust demand for hedging associated to higher risk.

Following this innovation, a measure of redenomination risk called ISDA basis was proposed in blog posts by Minenna (2017) and Gros (2018). The ISDA basis is the difference between CDS premia under the 2014 ISDA standard and the corresponding ones in the 2003 definition. The robustness of this measure with respect to liquidity considerations was proved by Kremens (2019). Bonaccolto, Borri and Consiglio (2020) also exploit the ISDA basis, exploring the information content of

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*(or any country that becomes a member of the **Group of 7** if such group of 7 expands its membership) or (2) the legal tender of any country which, as of date of such change, is a member of the **OECD** and has a local currency long-term debt rating at **AAA**..."*

CDS denoted in different currencies. In this paper we build on this idea to show that using the CDS quotes under the two definitions makes it possible to estimate the dependence between default and redenomination risk and to compute an unbiased meaure of redenomination risk. In other words, while in general the ISDA basis considered as the outright difference between the CDS spreads in the alternative ISDA standards would be biased, we illustrate here that it is possible, and relatively easy, to correct this bias, by Maximum Likelihood Estimation (MLE) on transformed data (Duan, 1996, 2000).

Technically, the cases of Germany, France, Italy and the Netherlands provide an interesting inverse derivative estimation problem: we observe the price of *First-to-Default* (FTD) derivative contract on two events, default or redenomination, and the price of just one of the two, namely default. The former is the CDS quote under the 2014 ISDA standard, the latter it the CDS quote under the 2003 one. The estimation procedure allows to recover the premium of a CDS contract representing protection against redenomination only.

Of course, the bias generated by dependence between default and redenomination could also affect other measures of redenomination. In particular, De Santis (2019) proposes a Quanto-CDS measure, defined as the difference of Quanto-CDS (the spread between CDS quotes in dollars and euros) for several countries and those of Germany. Also in this case the purpose is to clean the CDS quotes from credit risk. This may be subject to debate since Quanto CDS is not a direct measure of redenomination, and actually measures the conditional depreciation of the currency given a credit event, including those occurring to corporate borrowers (see Ehlers and Schoenbucher2004, El-Mohammadi, 2009 and Brigo, Pede and Petrelli, 2018). However, for what matters here, even if one accepted that a Quanto-CDS measure could clear credit risk, the outright subtraction of CDS in euros from CDS in dollars

would produce a biased measure of redenomination risk if default and redenomination risk are dependent. The same applies to similar measures based on prices of other products denominated in different currencies (Eichler and Rövekamp, 2017).

Of course, the measure extracted from the ISDA standard innovation is available only for four countries (even though a core part of the Euro area market), while while the other measures proposed in the literature are available for all the countries of the Euro, at the cost of being less direct and more noisy. We will show that the measure obtained for the four countries affected by the ISDA change can play a role as a reference model to check the robustness of the other measures. Finally, recognizing dependence among risks may also be relevant for the related literature that considers redenomination among other risk factors explaining the dynamics of sovereign bond spreads (Di Cesare et al. 2012, Dewachter et al., 2015, Afonso et al. 2018, Krishanmurthy, Nagel and Vissing-Jorgensen, 2017, Bayer, Kim and Krivolutsky, 2018).

Having addressed the question of the unbiased estimation of redenomination risk for every country, the next obvious goal would be to study the cross-border link of redenomination events and the relevance of a catastrophic event of break-up of the Euro. Among practitioners it is well known that tail-hedgers use the CDS on Republic of Germany as a proxy for this extreme risk. Here we propose to model the break-up of the Euro as a common shock Marshall-Olkin model. Redenomination of each country is decomposed in a break-up of the Euro component and a idiosyncratic component. We show that this model produces an estimate of the risk of Euro break-up that is quite close to the German CDS, confirming the market practice.

The structure of the paper is as follows. In section 2 we model dependence between default and redenomination risk with constant intensities. In section 3 we show how to estimate the redenomination measure allowing for dependence with the

risk of default. In section 4 we report empirical evidence on redenomination risk for Italy, France, Germany and the Netherlands, both at the country level and in a cross-country analysis including a scenario of simultaneous redenomination events leading to the end of the Euro. Section 5 concludes.

2 CDS and the Financial Mathematics of Redenomination

2.1 The CDS Market

Here we describe the basic financial mathematics of the CDS markets that will be used in the following sections. We will use both CDS on a single event, i.e. default, and a first-to-default (FTD) CDS paying protection the first time that either of the two credit events (default or redenomination) occurs. The analysis is general and preliminary to any implementatiion (ISDA basis, Quanto CDS and others). It merely addresses the correct way to decompose a CDS that pays the first time that either default or redenomination occurs in CDS contracts on each event.

In continuous time, the CDS premium on a single credit event i is

$$CDS_t = -\frac{L_i \int_0^t B(u)dG_i(u)}{\int_0^t B(u)G_i(u)du}$$

where $B(t)$ is the risk free discount factor, $G_i(t)$ denotes the survival function of event i and L_i is the loss triggered by the event. The survival function $G_i(t)$ is defined from the instantaneous conditional default probability called intensity $\lambda_i(t)$

$$\lim_{h\to 0}\Pr(t \le \tau < t+h|\tau > t) = \lim_{h\to 0}\frac{G_i(t) - G_i(t+h)}{G_i(t)} = -\frac{\partial G_i(t)}{\partial t G_i(t)} \equiv \lambda_i(t).$$

Applied to our problem, we set $i = D, R$ where D is default and R is redenomination:

$$-\frac{\partial G_D(t)}{\partial t}\frac{1}{G_D(t)} = \lambda_D(t) \qquad -\frac{\partial G_R(t)}{\partial t}\frac{1}{G_R(t)} = \lambda_R(t) \tag{1}$$

Before proceeding with the analysis, it is important to discuss the identification of the intensity of the event λ_i and loss severity L_i, Typically, in CDS applications it is customary to use a value of 60% by market convention. This cannot be done here, where a key issue is to compare both intensity and severity of two events. Since it is impossible to identify intensity and severity of both events, the most optimal feasible solution is to adopt the approach in Duffie and Singleton (1999). In their model the severity is included in the definition of intensity:

$$\mu_D(t) = \lambda_D(t)L_D \qquad \mu_R(t) = \lambda_R(t)L_R \qquad (2)$$

and the new intensities $\mu_i(t)$ represent a comprehensive measure of default and redenomination risk. From now on, in this paper we will use this new definition of intensity, with the definition of CDS updated to

$$CDS_t = -\frac{\int_0^t B(u)dF_i(u)}{\int_0^t B(u)F_i(u)du} \qquad (3)$$

where $F_i(t)$ is the survival probability corresponding to intensity $\mu_i(t)$.

In general the intensity is modelled as a stochastic process in the so called *double stochastic* models. Here we restrict the analysis to constant intensities $\mu_D(t) = \mu_D$ and $\mu_R(t) = \mu_R$. It may be proved that in this case the term structure of CDS premia is flat and the CDS premium is equal to intensity (see Brigo and Mercurio,2006 page 735-736). In applications, the constant intensity approach, known as the *simple rule*, is used as an approximation, particularly in cases, like our analysis, in which liquidity considerations advise to stick to the maturity where the trading activity is mostly concentrated, that is 5 years in the CDS market.

In what follows, we will need a multivariate extension of the concept of CDS, namely a bivariate *First-to-Default* (FTD) contract written on default D or rede-

nomination R. The extension is immediate if we define $F_{DR}(t)$ the joint survival probability beyond time t, that is the probability that neither default or redenomination would occur before time t. Care should be taken when extending this concept to our specific problem. In fact, while in standard FTD swaps it is usual to assume the same loss given default so that it is immaterial which credit event occurs before the other, here it would make a difference if redenomination or default occurs first. For this purpose, we follow Bielecki et al. (2007) and define the concept of i-FTD-intensity. This is the instantaneous conditional probability of credit event i taking place, given that no other credit event has taken place yet. We term this the intensity of a credit event occurring first.

In our case we have

$$\lim_{h \to 0} \frac{G_{DR}(t_D, t_R) - G_{DR}(t_D + h, t_R)}{G_{DR}(t_D, t_R)} = -\frac{\partial G_{DR}}{\partial t_D} \frac{1}{G_{DR}} = \tilde{\lambda}_D(t_D)$$

$$\lim_{h \to 0} \frac{G_{DR}(t_D, t_R) - G_{DR}(t_D, t_R + h)}{G_{DR}(t_D, t_R)} = -\frac{\partial G_{DR}}{\partial t_R} \frac{1}{G_{DR}} = \tilde{\lambda}_R(t_R)$$

where $\tilde{\lambda}_i, i = D, R$ is the i-FTD-intensity of default and redenomination.

As for a comparison between these intensities and the marginal ones, in Appendix, Lemma 5.1 we proved that

$$\tilde{\lambda}_i \leq \lambda_i, \tag{4}$$

$i = D, R$ with equality holding only if redenomination and default are independent.

Now, if we simply take the total derivative of $G_{DR}(t_D, t_R)$ it is evident that the FTD-intensity is the sum of the instantaneous conditional probabilities that each one occurs before the other. Indeed, this requires to add the assumptions that the joint survival function is absolutely continuous, so that the probability that redenomination and default occur at the same time is zero. We can then specify the

FTD intensity and the way it can be decomposed into the individual intensities.

Proposition 2.1. *If the joint survival function $G_{DR}(\mathbf{t}) \equiv G_{DR}(t,t)$ is absolutely continuous the FTD intensity is defined as*

$$\lim_{h \to 0} \Pr(t \leq \tau_D, \tau_R < t + h | \tau_D > t, \tau_R > t) = \lim_{h \to 0} \frac{G_{DR}(\mathbf{t}) - G_{DR}(\mathbf{t} + h)}{G_{DR}(\mathbf{t})} = \Lambda(t),$$

with $G_{DR}(\mathbf{t} + h) \equiv G_{DR}(t + h, t + h)$. The FTD default intensity can be decomposed in the sum of the i-FTD intensities:

$$\Lambda(t) = \tilde{\lambda}_D(t) + \tilde{\lambda}_R(t) \tag{5}$$

Remark 2.1. *It is easy to show that this is not true if the joint survival function is not absolutely continuous, that is singular. In this case, we have $\lim_{h \to 0} \Pr((t \leq \tau_D = \tau_R < t + h | \tau_D > t, \tau_R > t) = \tilde{\lambda}_0(t) > 0$, and*

$$\Lambda(t) = \tilde{\lambda}_D(t) + \tilde{\lambda}_R(t) + \tilde{\lambda}_0(t) > \tilde{\lambda}_D(t) + \tilde{\lambda}_R(t)$$

where $\tilde{\lambda}_i(t), i = D, R$ has to be redefined as the probability of event i alone *to take place before the other. Alternatively, we could say that it represents the instantaneous conditional probability of event i occurring* strictly *before the other.*

We can now introduce the loss severity of default L_D and redenomination L_R. As for marginal intensities, we redefine the i-FTD intensities embedding the percentage of loss:

$$\tilde{\mu}_D(t) = \tilde{\lambda}_D(t) L_D \qquad \tilde{\mu}_R(t) = \tilde{\lambda}_R(t) L_R \tag{6}$$

The corresponding FTD intensity is redefined accordingly as:

$$\mu_{DR}(t) = \tilde{\mu}_D(t) + \tilde{\mu}_R(t) \tag{7}$$

Now, the FTD swap premium can finally be written just like a univariate CDS as

$$FTD_t = -\frac{\int_0^t B(u)dF_{DR}(u)}{\int_0^t B(u)F_{DR}(u)du}$$

Note that the definition $F_{DR}(t)$ recalls the marginal survival functions $F_i(t)$ meaning that the severity of loss is included in the intensity.

As for our task of recovering a measure of redenomination risk from information implied in market prices we may summarize the results of this section as follows:

Proposition 2.2. *Assume that for some maturity t we observe the joint intensity μ_{DR} and the corresponding default risk intensity μ_D. Then,*

$$\mu_{DR} - \mu_D = \lambda_R L_R \Leftrightarrow F_{DR} = F_D F_R$$

If redenomination and default are dependent $(F_{DR} > F_D F_R)$, the bias is

$$\mu_{DR} - \mu_D = \tilde{\mu}_R - (\mu_D - \tilde{\mu}_D)$$

Note that if there is positive dependence between default and redenomination risk the plain difference between the FTD intensity and the marginal intensity of default underestimates the redenomination intensity: this simply follows from $\mu_R > \tilde{\mu}_R$ and $\mu_D > \tilde{\mu}_D$.

The next goal is then to specify a dependence model for redenomination and default.

2.2 Dependence between Default and Redenomination

We demonstrated that if redenomination risk and default risk are dependent the intensity of a FTD on them (that is the conditional probability of the first time that either of the two occurs) cannot be decomposed as the sum of the marginal

intensities. We now investigate a specific dependence model allowing to split the FTD intensity in a non linear function of the marginal intensities, with the non linearity linked to the degree of dependence.

We work in the most general setting for the representation of dependence, that is copula function theory (see Nelsen, 2006 for a mathematical introduction and Cherubini et al. 2004 for financial applications). This is a way to break a joint distribution, or survival function, into a function of the marginal distributions, or survival functions, and their dependence structure. Here we apply the copula tool to survival functions, the so called *survival copulas*. We write

$$F_{DR}(t_D, t_R) = C\left(F_D(t_D), F_R(t_R)\right) \tag{8}$$

where $C(u, v)$ is a copula function.

While copula functions represent in full generality the link between marginal and FTD survival functions, some restrictions on the choice is needed to express the dependence link in terms of intensities. In our case, we want to express $F_{DR}(t, t) = \exp(-\mu_{DR}t)$ as a function of $F_i(t) = \exp(-\mu_i t), i = D, R$. Denote: $u_i = \exp(-\mu_i)$. If we want to express the link in terms of intensities we need functions that satisfy:

$$F_{DR}(t, t) = \exp(-\mu_{DR})^t = C\left(u_D^t, u_R^t\right) \tag{9}$$

This gives a function for μ_{DR}

$$\mu_{DR} = -\log\left[C\left(u_D^t, u_R^t\right)^{1/t}\right] \tag{10}$$

Now, if we assume that marginal intensities are flat, that is constant across maturi-

ties, and we require the same for μ_{DR}, this clearly requires

$$C(u_D, u_R) = C\left(u_D^t, u_R^t\right)^{1/t}$$

Copula functions satisfying this property are called *Extreme Value Copulas* (EVC). In this analysis, we use the most famous EVC model, and the only one that is also part of the family of Archimedean copulas (whose advantage for estimation will be discussed below). This is the Gumbel-Hougaard copula and is defined as

$$C(F_D, F_R) = \exp\left(-\left[(-\log F_D)^\theta + (-\log F_R)^\theta\right]^{1/\theta}\right) \tag{11}$$

where $\theta \in [1, \infty)$ is a parameter representing dependence. It is immediate to check that $\theta = 1$ corresponds to independence, that is $F_D F_R$. It may also be seen that as θ grows to infinity the copula function converges to perfect dependence, that is $\min(F_D, F_R)$. The copula function cannot represent negative dependence. We may immediately check that the Gumbel copula can be fully specified in terms of intensities:

$$C(F_D(t), F_R(t)) = \exp(-\mu_{DR}t) = \exp\left(-\left[(\mu_D t)^\theta + (\mu_R t)^\theta\right]^{1/\theta}\right)$$
$$= \exp\left(-\left[(\mu_D)^\theta + (\mu_R)^\theta\right]^{1/\theta} t\right) \tag{12}$$

The link between FTD and marginal intensities is then

$$\mu_{DR} = \left[\mu_D^\theta + \mu_R^\theta\right]^{1/\theta} \tag{13}$$

Note that the relationship is linear only if $\theta = 1$, that is redenomination default risk are independent. At the other extreme we have that when the two risks become comonotonic, that is $\theta \to \infty$, we obtain $\mu_{DR} = \max(\mu_D, \mu_R)$.

What remains to be done is to recover a function for $\tilde{\mu}_i, i = D, R$, that we called the i-FTD intensity and represents the instantaneous conditional probability of event i occurring first. We have that

$$\tilde{\mu}_i = \mu_i^\theta \mu_{DR}^{1-\theta} \tag{14}$$

The proof is reported in Appendix. Here we only note that

$$\mu_{DR} = \tilde{\mu}_d + \tilde{\mu}_R \tag{15}$$

as can be easily verified using equation (13). Again, we have $\tilde{\mu}_i = \mu_i$ only if $\theta = 1$. We can then summarize our results on dependence of intensities in a Gumbel-Hougaard model as follows.

Proposition 2.3. *In the Gumbel-Hougaard dependence model with marginal intensities $\mu_i, i = D, R$ and dependence parameter $\theta \in [1, \infty)$ we have*

1. $\mu_{DR} = \left[\mu_D^\theta + \mu_R^\theta \right]^{1/\theta}$

2. $\tilde{\mu}_R = \mu_R^\theta \mu_{DR}^{1-\theta}$ *and* $\tilde{\mu}_D = \mu_D^\theta \mu_{DR}^{1-\theta}$

3. $\mu_{DR} = \tilde{\mu}_D + \tilde{\mu}_R$

3 Maximum Likelihood Estimation

So far, we have explored the relationship between marginal intensities of default and redenomination and their FTD intensity. The analysis applies to all the approaches that are used to recover or estimate redenomination risk and could be also used in other applications. Here we exploit the ideal opportunity for an econometric application that is offered by the revision of the CDS standard undertaken by the ISDA in 2014 discussed in the Introduction. This new standard provides new data that

can be used to simultaneously estimate the redenomination risk and its dependence with default. More precisely, if we denote μ_{CR03} and μ_{CR14} the CDS quotes under the 2003 and 2014 standards, we have:

1. $\mu_{CR03} = \mu_D$

2. $\mu_{CR14} = \mu_{DR} = \left[\mu_D^\theta + \mu_R^\theta\right]^{1/\theta}$

If redenomination and default risk are linked by a Gumbel-Hougaard copula with dependence parameter $\theta \in [1, \infty)$ the redenomination intensity is

$$\mu_R = \left[\mu_{DR}^\theta - \mu_D^\theta\right]^{1/\theta} \tag{16}$$

So, μ_R is the premium that an investor would pay for a CDS that pays protection against redenomination only.

Note that we face an inverse problem. While typically we observe the marginals and we estimate the dependence structure and the FTD, here we observe the FTD intensity and the marginal intensity of default. We know that in cases like this Maximum Likelihood Estimation (MLE) solution is available, and is called *MLE on transformed data* (Duan,1994, 2000).

To show how the approach work, let us start asking what the ML estimation problem would be if we were allowed to observe all the underlyings of the FTD contract. In our setting, this would be a standard copula estimation problem. The copula to be estimated would be

$$F_{DR} = C(F_D, F_R) \tag{17}$$

Assuming to observe time series of the marginal probabilities F_D and F_R for dates

$\{t_1, \ldots, t_n\}$, the MLE problem would be written as

$$
\max_{\Theta} \mathcal{L} = \log L \left(F_{R,t_1}, \ldots, F_{R,t_n}, F_{D,t_1}, \ldots, F_{D,t_n}; \Theta \right) =
$$
$$
= \sum_{i=1}^{n} \log \mathbf{c}(F_{D,(t_i)}, F_{R,t_i}) + \sum_{i=1}^{n} \log f_{D,t_i} + \sum_{i=1}^{n} \log f_{R,t_i} \tag{18}
$$

where Θ is the parameters set, $f_{j,t_i}(.)$, $j = D, R$ denote the marginal densities of default (D) and redenomination (R) at time t_i and $\mathbf{c}(u, v)$ is the density of the copula function defined as:

$$
\mathbf{c}(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v} \tag{19}
$$

It is of course assumed that the copula function is absolutely continuous, so that its density is defined almost everywhere in the unit square.

We are now going to transform our estimation problem into an inverse one. We then assume, as it happens in the case at hand, to observe a time series of F_{DR,t_i}, $i = 1, 2, \ldots, n$, of joint survival functions of the default and redenominations events and the corresponding F_{D,t_i} survival functions of the default event. The task is to estimate the parameter set and the series $\hat{F}_{R,t_i}$ of the redenomination survival functions. The MLE problem is now written as

$$
\max_{\Theta} \mathcal{L} = \log L \left(F_{DR,t_1}(t_1), \ldots, F_{DR,t_n}, F_{D,t_1}, \ldots, F_{D,t_n}; \Theta \right) =
$$
$$
= \sum_{i=1}^{n} \log \mathbf{c}(F_{D,t_i}, \hat{F}_{R,t_i}(\Theta)) + \sum_{i=1}^{n} \log f_{D,t_i} + \sum_{i=1}^{n} \log \hat{f}_{R,t_i}(\Theta)
$$
$$
- \sum_{i=1}^{n} \log \left| \frac{\partial \tilde{C}(F_{D,t_i}, \hat{F}_{R,t_i}(\Theta))}{\partial \hat{F}_{R,t_i}(\Theta)} \right| \tag{20}
$$

where the differences with respect to equation (18) are marked in red. For the proof of the result the reader is referred to the original work by Duan (1994). The application to the copula function estimation theorem is straightforward. The ML equation is augmented by one new element, which involves the partial derivative

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of the survival copula with respect to the redenomination survival function, that is the conditional probability of default given redenomination. Moreover, at every run of the optimisation problem the marginal survival function and the density of the redenomination time must be estimated for every observation at time t_i, given the parameter estimate and the corresponding observed survival functions F_{DR,t_i} and F_{D,t_i}. For this reason we use the *hat* notation in redenomination survival functions and densities.

As a final remark, consider that it is no longer possible to resort to shortcuts that are typical of the MLE approach to copula functions, such as the estimation in two steps of the marginal densities and the copula density (*Inference Functions for Margins*, IFM) or the estimation of the copula density only, avoiding the specification of the marginal densities and using empirical ranks (*canonical MLE*). In the copula MLE on transformed data the density of the unobserved marginal must be kept in the ML specification, because it depends on the parameters.

We now exploit the Gumbel-Hougaard specification that allows us to write the log-likelihood in terms of intensities only. The derivation is reported in Appendix.

Proposition 3.1. *If the default and redenomination times are exponentially distributed and their dependence is given by a Gumbel-Hougaard copula, the ML estimator on transformed data is obtained by maximizing the log-likelihood:*

$$\mathcal{L} = \log L\left(\mu_{CR14}(t_1), \ldots, \mu_{CR14}(t_n), \mu_{CR03}(t_1), \ldots, \mu_{CR03}(t_n); \theta\right) =$$
$$= (1 - \theta) \sum_{i=1}^{n} \log \mu_{\mu_{CR14}}(t_i) + \sum_{i=1}^{n} \log \left(1 + \frac{\theta - 1}{\mu_{CR14}(t_i)}\right) + \tag{21}$$
$$+ \theta \sum_{i=1}^{n} \log \mu_{CR03}(t_i) + \sum_{i=1}^{n} \log \hat{\mu}_{R,t_i}(\theta) - \sum_{i=1}^{n} \hat{\mu}_{R,t_i}(\theta)$$

and

$$\hat{\mu}_{R,t_i}(\theta) = \left(\mu_{CR14}(t_i)^\theta - \mu_{CR03}(t_i)^\theta\right)^{1/\theta}$$

4 Empirical Evidence

We are now ready to put our model at work. We downloaded from Datastream the time series of 5 year CDS contracts for the two different standards, the 2014 and the older one for the three G-7 countries of the Euro area, that is Italy, France and Germany, and for the Netherlands. The sample spans from January 2015 to May 2020. The estimates were carried out using contracts both in dollars and euros, with almost identical results. If fact, remember that the redenomination to a "permitted currency" or not refers to the debt of the name underlying the contract, and has nothing to do with the currency of the contract itself.

4.1 A Single Country Analysis

The MLE analysis in section 3, with Gumbel-Hougaard dependence, was applied to the two series of contracts under the 2014 and the older ISDA standard for each country. Table 1 reports the estimates of the dependence parameter θ and their standard errors, along with the corresponding dependence measure, represented by the Kendall's τ statistics, as it is usual in copula functions applications [3]. We also report the estimates for a subsample starting February 2017 for a reason that will be clear very soon. We see that dependence is quite high for all countries. The Kendall's τ figures are above 70% for Italy and France, and higher than 80% for Germany and Netherlands.

The main results are portrayed in figures (1), (2), (3) and (4) where we report the estimate and the 5% confidence intervals of the marginal redenomination intensities. This corresponds to the fair value premium that ought to be paid in a CDS for

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[3]It may be proved (see Nelsen, 2006, Theorem 5.1.3 on page 161) that the Kendall's τ statistics is linked to the copula function by the following relationship

$$\tau = 4 \int_0^1 \int_0^1 C(u,v)dC(u,v) - 1$$

protection against a redenomination event only. A comparison with the ISDA basis, that is the plain difference between the 2014 CDS standard and the 2003 one, shows that the latter largely underestimates the price of insurance against redenomination risk that allows for dependence with default. Nevertheless, both the ISDA basis and the measure corrected for the dependence are able to spot the rise in redenomination risk when this becomes extreme. We in fact recognise both the rise in redenomination risk during the Presidential French elections in March 2017, and the sudden decrease after the first round, as well as the sharp increase at inception of the so-called *yellow-green* Italian government (from the colours of the 5Star Movement and the Northern League) in May 2018. We also find an increase in redenomination risk at the beginning of the COVID crisis, in March 2020.

Accounting for dependence makes a difference if one is interested in the fair price of the redenomination CDS and not only its dynamics, and also if he wants to compare the relative weight of redenomination and default risk by means of a decomposition of the 2014 standard contract. Remember that in the presence of dependence, the correct way to compute this decomposition is to use the intensities of redenomination or default occurring first, that are lower than the marginal intensities. Figures (5), (6), (7) and (8) report this decomposition. The behaviour is remarkably similar in all four cases. There is an evident structural breakpoint in February 2017. Before that period default risk was more relevant than redenomination risk in the pricing of the CDS. In other words, default was expected to happen before redenomination, even in presence of material risk of redenomination. After February 2017 redenomination risk jumps up becoming by far the most relevant risk. We may even say that in the later part of the sample the CDS in the 2014 standard is mostly pricing redenomination risk, while default risk is almost irrelevant. It is to be reminded that this does not mean that default risk vanishes, but simply that

it is not expected to occur before redenomination. The fact that default is priced in the CDS under the old standard implies that default risk is present, and the high dependence with redenomination risk signals that default risk would substantially increase following redenomination. In other words, conditional default risk would be substantially higher than marginal default risk.

A possible explanation for the same position of the breakpoint in the four time series can be found in the event that on February 1st Theresa May received approval from the UK Parliament to trigger the Article 50 of the EU Treaty and to begin the Brexit procedure. Even though of course this was not a redenomination event it was the first time that a country effectively started a procedure to leave the EU, and every country wishing to leave the Eurozone would have to undergo the procedure of leaving the EU.

Af for the statistical nature of this breakpoint, we investigated whether this is due to: i) a change in dependence, namely an increase in the degreee of association between redenomination and default risk; ii) a change in the marginal risks of default and redenomination.

In order to check for a structural break in dependence, we estimated the model on a sub-sample starting on February 2017. The results excluded in all cases an increase in dependence. On the contrary, dependence appears to be marginally lower than in the full sample in all cases. An explanation of the breakpoint can instead be found in pictures (9), (10), (11) and (12). This evidence testifies that there is a marked decrease of the intensity of the default event occurring first. For Italy this decreases to about 50 basis points at the end of the sample from about 80 in the beginning. For France the decrease is from 35 basis points to about 4. For Germany and the Netherlands it has decreased from 11 and 13 to 4 and 2.5 respectively.

It is worth noting that while the decrease in the risk of default occurring first is

Table. 4.1 MLE estimates

		01/01/15-29/05/20	01/02/17-29/05/20
Italy	θ	4.943876	3.749147
	S.D.	0.09072041	0.08186898
	Kendall's τ	0.797729555	0.733272662
France	θ	4.513953	3.355271
	S.D.	0.0797664	0.06976977
	Kendall's τ	0.778464685	0.701961481
Germany	θ	6.958329	5.237983
	S.D.	0.138716	0.1260464
	Kendall's τ	0.856287336	0.809086818
Netherlands	θ	8.015144	6.141835
	S.D.	0.1655264	0.1541326
	Kendall's τ	0,875236178	0.83718221

computed using the model, the decrease of marginal default risk that triggers this behavior is in the data, since after 2017 the CDS under the 2003 standard decreased substantially for all countries. This does not depend on the model. Moreover, comparing the marginal default risk and the risk of default occurring first, we see that this gap remains constant in France and increases in Italy from the period of the populist government. Overall, as suggested above, this evidence shows that while redenomination is definitely expected to occur first, this would not rule out an event of default. Rather, positive dependence between redenomination and default implies that the event of redenomination would actually increase the risk of default.

4.2 Redenomination Risk: A Longer View

An obvious limitation of the analysis based on the CDS revision is that it cannot be applied to data before 2015. And indeed it would have been very interesting if one could extend the analysis backwards to the time of sovereign crisis from 2010 to 2012.

It is well known that in the market it is usual to use bond spreads as a proxy for credit risk if a CDS market is not available. More precisely, the market practice is

to look at the *Asset Swap* spread (ASW). This is defined as

$$\text{ASW} = \text{coupon} - \text{swap rate} + \frac{1 - \text{Price of the Bond}}{\sum_i^N B(t_i)}$$

In the market there is a lively arbitrage activity on the difference between CDS and ASW spreads, that is called *CDS-ASW basis* (Chowdry, 2006, De Wit, 2006). Unfortunately for our application, it is not always the case that this difference is equal to zero. Rather, it is more the exception than the rule. This in general is due to the fact that the ASW spread is defined with respect to a specific bond, while the CDS spread is referred to the whole debt oustanding amount of an issuer, including bonds with different degree of liquidity.

In our application, we use the ASW spread as a substitute for the CDS under the 2014 standard. In fact, the ASW spread would incorporate redenomination risk while the CDS under the old standard would not. If we do not consider the liquidity issues that we quoted above, this would predict a negative basis, and

$$\text{ASW} \geq \mu_{CR03} \tag{22}$$

and we could perform the same analysis as before using ASW as a proxy for μ_{CR14}. Unfortunately, Fontana and Scheicher (2016) document that the basis is negative mostly in periods of crisis, and more so for countries with lower credit standing.

Coming to our analysis, we approximated the asset swap spread (ASW) by taking the difference between the Thompson Reuters 5 year government bond yield and the swap rate for the same maturity. Moreover, in order to reduce the impact on the measure from issues linked to liquidity and to other technical aspects, we computed the measure relative to a benchmark country, namely Germany. Moreover, we restricted ourselves to the cases of Italy and France, for which the dimension of

the spreads is material enough to justify the approximation. We refer to this measure as the *Relative ASW spread*. We found a negative basis 60% of the times for Italy, both in the period 2008-2015 and in the following period (that we used to check the level of approximation). For France, we remained with much fewer cases in which condition (22) is verified: only about 25% of the cases in the period before 2015 and 40% of the cases afterwards.

We then estimated the model with MLE on transformed data for Italy and France using only the cases in which condition (22) was satisfied. The dependence estimates obtained are comparable with the those obtained with CDS data. They are somewhat lower in the period 2015-2020 but they give a Kendall's tau of 78% for Italy and 72% for France in the whole period.

Using the estimates we performed the same decomposition as before. Figures (13) and (14) report the decomposition for Italy and France from January 2008 to end of May 2020. Straight lines refer to periods in which the measures could not be computed.

The picture for Italy represents quite clearly the upsurge in redenomination risk during the crisis, even though the decomposition is quite volatile. Overall, however, redenomination risk increases massively during the Italian crisis, starting in the Summer of 2011. Most importantly, it reaches its peak corresponding to the end of Berlusconi's government in Italy and to the "whatever it takes" speech, on July 26th 2012. After that, it steadily declined, all through 2013 and 2014. Since 2015, it remained lower than default risk until February 2017, confirming that the ASW measure gives results that are qualitatively similar to those generated by CDS data.

We also propose a quantitative comparison of the two measures in pictures (15) and (16). The main regularity that we find is that the ASW based measure produces results that are quite close to those obtained with the CDS measure mostly when

Table. 4.2 MLE estimates: Relative ASW spread and CDS 2003 Standard

		2008-2020	**2008-2014**	**2015-2020**
Italy	θ	4.496559	6.6241257	3.269424
	S.D.	0.0659081	0.1440191	0.06598034
	Kendall's τ	0.7776077	0.8490367	0.69413573
France	θ	3.542788	5.7808881	2.934597
	S.D.	0.05731532	0.1788058	0.05616793
	Kendall's τ	0.71773643	0.8270162	0.65923771

and where redenomination risk is higher. When risk is low either the ASW measure cannot be computed or is dominated by other technical factors. This explains why the ASW measure works much better in the Italian case, when it explains 90% of the variance of redenomination risk. The degree of association is much lower for France, when it falls to 66%, Moreover, in both cases the measure is obtained in the period starting from 2017 because before, when redenomination risk was lower, it was almost always impossible to compute the ASW based measure.

4.3   Use of the Measure for Other Countries

An obvious flaw of the CDS based measure studied in this paper is the limitation to four countries of the Euro area. It may be argued that these countries account for a relevant share of the area. In quantitative terms, this can be evaluated in terms of the so called *capital key*, that is the share of ownership of the European Central Bank (ECB) attributed to each country. This was established based on a quantitative assessment of the economic dimension of the country. In terms of capital key, the four countries in question account for 56.64% of the ECB capital (21.44% Germany, 16.61% France, 13.82% Italy and 4.77% the Netherlands). Other measures may corroborate these data. Cherubini and Violi (2015) compute the relative dimension of the first ten Euro area sovereign debt markets and find that our four countries represented 75.75% of the overall amount of bonds outstanding

at the beginning of the Quantitative Easing (QE) in 2015.

Nevertheless, it may be useful to learn what the CDS based model can suggest for countries that are neither part of the G7 group nor OECD countries with AAA rating. In particular, one would like at least to cover Spain, that under the measures above represents around 10% of the Euro area (9.70% in terms of capital key and 11.05% in terms of market dimension measure). The other countries account for about 2% each.

For the countries that are not part of our sample the CDS measure is not available because both under the 2003 and the 2014 standard the CDS is triggered the first time that either default or redenomination occurs. For these countries we then need a model based on observable variables, from which a proxy for redenomination risk could be computed. Such model could then be used to extract information about redenomination from the two markets available, that is the ASW and the CDS market. The cases of Italy and France, for which the estimate of redenomination risk is available, can be used to gauge the reliability of such model.

In this analysis we propose the simplest model consisting of two variables. The first is the Quanto-CDS measure discussed above, that represents a proxy for idiosyncratic redenomination risk. The second is the German CDS that in tail hedging practices is typically used as a hedge against a scenario of end of the Euro. Our model for redenomination risk is then

$$\mu_{R,it} = a_{0i} + a_{1i}QuantoCDS_{it} + a_{2i}GerCDS_{it} + \epsilon_{it} \tag{23}$$

where a_{0i}, a_{1i} and a_{2i} are parameters and ϵ_{it} are disturbances. For Italy and France this model can be estimated. The results reported in table 4.3a show that the variables are highly significant and the model provides a good fit, with adjusted R^2 figures around 90% for Italy and 80% for France. Also in this case the goodness of

fit of the model improves in the case in which redenomination risk is higher. As for the relevance of the two measures, the Quanto CDS measure alone accounts for 78% for Italy and 64% for France.

The next step of the analysis is to try the model on credit risk data that are available for all the countries in the Euro area, that is ASW spreads and the CDS contracts that are triggered by both redenomination and default. For France, Italy and Spain we then estimate

$$ASW_{it} = b_{0i} + b_{1i}QuantoCDS_{it} + b_{2i}GerCDS_{it} + \omega_{ASW,it} \tag{24}$$

$$\mu_{CR14,it} = c_{0i} + c_{1i}QuantoCDS_{it} + c_{2i}GerCDS_{it} + \omega_{CDS,it} \tag{25}$$

where b_{ki} and $c_{ki}, k = 0, 1, 2$ are parameters and $\omega_{ASW,it}$ and $\omega_{CDS,it}$ are disturbances.

The estimates of these regressions are also reported in table 4.3a. The percentage of variance explained is quite high for Italy for both the ASW and CDS spreads. This is consistent with redenomination risk being particularly relevant in the Italian case. In the French case, the evidence is mixed, and the model explains a relatively low percentage of variance of the ASW spread and a very high percentage of the CDS spread. The variance of ASW and CDS spreads explained by the model is somewhat lower for Spain, even though the model is confirmed to be largely statistically significant.

Now, for Italy and France we can study the link between the predictive power of the model with respect to the CDS based redenomination risk measure and that of the same model in explaining the dynamics of ASW and CDS spreads. In order to do that we regress the values of redenomination intensity predicted by the model on the predicted values of the ASW and CDS spreads. In both cases, the results are quite promising if we run the regression on each of the predicted values, but they become outstanding if we use the average of the predicted values of the ASW and

Table. 4.3a Redenomination Models: Regressions

Country	Dep. Variable	Constant	Quanto CDS	ASW	Adj. R^2
Italy	μ_R	-9.5118	3.7325	4.37183	0.8921
	S.D.	(2.14606)	(0.03457)	(0.11832)	
	ASW	-34,05224	4.49819	3.79118	0.8581
	S.D.	(3.04847)	(0.04911)	(0.16168)	
	μ_{CR14}	-7,80148	2.43576	5.17424	0.7823
	S.D.	(2,20934)	(0.03559)	(0.11717)	
France	μ_R	4.02347	2.28548	0.94551	0.8061
	S.D.	(0.38020)	(0.04537)	(0.02693)	
	ASW	6.63332	2.14346	0,33747	0.4059
	S.D.	(0.70065)	(0.08361)	(0.04963)	
	μ_{CR14}	0.07018	2.07587	1.50638	0.8625
	S.D.	(0,3672)	(0.04382)	(0.02601)	
Spain	ASW	18.7289	1.989512	1.064502	0.4837
	S.D.	(1.346898)	(0.143587)	(0.122998)	
	μ_{CR14}	4.706327	1.285471	2.976478	0.6761
	S.D.	(1.340074)	(0.142859)	(0.122375)	

CDS spreads. So, for France and Italy we estimate

$$\hat{\mu}_{R,it} = p_{0i} + p_{1i}\frac{A\hat{S}W_{it} + \hat{\mu}_{CR14,it}}{2} + z_{it} \qquad (26)$$

where $\hat{y}$ denotes the predicted value of variable y and $p_{0,i}$ are parameters. Table 4.3b reports the results of these regressions and we find that the average of the predicted values produced by the models explain in both cases close to 100% of the variance of the predicted redenomination risk measure. This new measure represented by the average of the predictions estimated on ASW and CDS spreads could provide a good proxy of the redenomination risk estimated for Italy and France. Namely, this measure would explain 90% of the redenomination risk for Italy and 80% for France.

Based on this analysis, one could extend the model to cases like Spain, for which we do not observe the redenomination measure based on CDS and we cannot gauge the percentage of variance explained. Nevertheless, we report the result in picture 17, in which we compare the redenomination risk predicted for Spain with the corre-

Table. 4.3b Redenomination Models: Regressions

Country	Dep. Variable	Constant	$\frac{\hat{ASW}_{it}+\hat{\mu}_{CR14,it}}{2}$	R^2
Italy	$\hat{\mu}_R$	6.467206	1.07593	0.9984
	S.D.	(0.151509)	(0.001145)	
France	$\hat{\mu}_R$	0.178354	1.060795	0.9997
	S.D.	(0.013117)	(0.00503)	

sponding predictions for Italy and France, and with the CDS based redenomination measures computed for those countries. It is not surprising that the redenomination risk predicted for Spain runs between those of Italy and France, being closer to that of Italy at the beginning of the sample and moving downward towards that of France at the end of the sample. Nothing could be said about the variance of this estimate, even though we can expect that since the percentage of variance of the spreads explained is lower than for Italy and France, most likely the measure would explain less than in those cases. However, this is more than what we could do if we could not observe a reliable measure of redenomination risk for Italy and France.

4.4   A Cross-Country Analysis: A Scenario of End of the Euro

It is well known that tail-hedgers use the German CDS as protection from an extreme scenario of end of the Euro. Here we use the results of the single country analysis to estimate a common shock model of redenomination risk, in which the common shock is meant to be the end of the Euro. To this aim we apply the most common model of this kind, that is the Marshall-Olkin model.

The Marshall-Olkin model in its simplest version can be easily explained in our constant intensity model. In fact, define μ_{EoE} the constant intensity of the event of "end of the Euro", that is the simultaneous redenomination of the Euro back to national currencies. Moreover if the event of redenomination of a single country alone is assumed to be independent of the end of the Euro, the observed redenomination

intensities of the countries in our analysis are assumed to be given by

$$\mu_i = \mu_{EoE} + \breve{\mu}_i \tag{27}$$

where $\breve{\mu}_i$ denotes the idiosyncratic redenomination intensity for Italy, France, Germany and the Netherlands, that is the conditional probability that country i would redenominate on its own.

Of course, a more general model would allow for a Gumbel-Hougaard copula linking the idiosyncratic and the common redenomination event, in which case one would obtain the more general representation

$$\mu_i = (\mu_{EoE}^\gamma + \breve{\mu}_i^\gamma)^{1/\gamma} \tag{28}$$

with $\gamma \in [1, \infty)$ the cross-section dependence parameter. This model is called Gumbel-Marshall-Olkin (GMO) model and was proposed in Cherubini and Mulinacci (2017).

In the preliminary analysis of this application, the model was tested, and we found $\gamma = 1$. Details on the estimation procedure are reported in Appendix B. So, the dependence among redenomination times is given by the fact that all countries are exposed to the same common shock, given by the end of the Euro. A crucial parameter for the determination of the dependence structure in the Marshall-Olkin model is given, for every country i, by the ratio:

$$\alpha_i = \frac{\mu_{EoE}}{\mu_{EoE} + \breve{\mu}_i} \tag{29}$$

If the ratio is close to 1 the common shock dominates over the idiosyncratic one. The

Table 4.4. Dependence Table (Kendall's τ)

	Italy	France	Germany	Netherlands
Italy	-			
France	0.24301622	-		
Germany	0.08324528	0.5534749	-	
Netherlands	-0.03841769	0.3569804	0.68144244	-

bivariate Kendall's τ statistics can be proved to be functions of the α parameters:

$$\tau_{ij} = \frac{\alpha_i \alpha_j}{\alpha_i + \alpha_j - \alpha_i \alpha_j} \tag{30}$$

The natural way to estimate these models is to fit the estimated Kendall's τ measures to the theoretical ones. This is also the only estimation technique available for singular distributions, since the density of the copula function is not defined in the singular region.

Kendall's τ measures are reported in table 4.4. On one hand we find that redenomination risk of Italy is almost independent from that of Germany and Netherlands and mildly dependent on that of France. On the other hand, dependence of the redenomination risk measures of Germany, Netherlands and France are quite strong. Overall, this evidence confirms the view according to which the Euro could survive to Italy leaving (Kremens, 2019).

We then estimated the paramters α that minimize the Euclidean distance of theoretical and observed Kendall tau's. For Italy we have a value quite close to zero (0.096), while the parameter for Germany is equal to one. The α parameters for France and Netherlands are 0.525 and 0.657 respectively.

Once the estimation is carried out, an estimate of the intensity of the common shock at each point in time can be obtained as a proportional fraction of the average

observed intensities. More specifically, in our case

$$\mu_{EoE} = \bar{\alpha} \sum_{i=1}^{n} \frac{\mu_i}{n} \tag{31}$$

where μ_i is the intensity of redenomination estimated for each country i (Italy, France, Germany and the Netherlands), and $\bar{\alpha}$ denotes the harmonic mean of the α_i's. This result was first introduced in Baglioni and Cherubini (2013a, 2013,b). The parameters estimated gave a harmonic mean $\bar{\alpha}$ around 0.27).

In Figure 18 we superpose the intensities of redenomination of Italy, France and Germany on the intensity of a simultaneous redenomination of the four currencies, meant to represent a scenario of end of the Euro. We did not report the redenomination risk of the Netherlands because it is quite close to the German one and would have confused the graph. The figure confirms the idiosyncratic nature of redenomination of Italy. Overall, it turns out that the index of the end of the Euro is very close to the CDS of Germany. This confirms the practice widespread among tail-hedgers, that typically use the German CDS as insurance against an extreme scenario of a break-up of the Euro.

5   Conclusions

We have shown that exploiting the innovation in the CDS contract standard introduced by ISDA in 2014 it is possible to simultaneously estimate the redenomination risk and its dependence with default risk. Technically, under the new standard issued in 2014 the sovereign CDS of all countries of the Euro area are first-to-default (FTD) swaps that could be triggered the first time that either default or redenomination occurs. Under the old standard, dated back to 2003, it was not so for Germany, France and Italy (members of the G7 group) and for the Netherlands (OECD country with

AAA rating): for these 4 countries, the CDS in the 2003 definition is triggered only if there is a default, and is not triggered if there is redenomination.

The problem of recovering the redenomination marginal probability knowing the FTD and marginal default probabilities is solved using the MLE on transformed data methodology, allowing also to estimate the dependence between the two risks. The model can be specified in terms of intensities, that is instantaneous conditional probabilities, if we assume the Gumbel-Hougaard copula. If redenomination and default risk are found to be dependent, then the intensity of the FTD contract cannot be decomposed in the sum of marginal redenomination and default risk. In the Gumbel-Hougaard copula the correct decomposition is a distorted sum. This implies that any measure based on the plain difference between the joint redenomination and default probability and the default marginal one are biased. Exploiting the comovement of the 2014 and 2003 it is possible to clean the redenomination measure from this bias.

The empirical analysis shows evidence of substantial dependence between redenomination and default risk for all four countries. The estimates of dependence are robust to a cross-check substituting asset swap spreads to CDS in the 2014 definition. As for the dynamics of redenomination risk, cleaning the bias unveils a remarkably similar behavior across the 4 countries. In all cases, since the beginning of February 2017 redenomination risk has become more relevant in the decomposition. This is not only a result of the model, since it is based on the actual decoupling of the paths of the two CDS contracts, with the CDS in the 2003 standard decreasing after February 2017 in all countries.

Finally, in a cross-country analysis of redenomination risk performed with a Marshall-Olkin model we provide an estimate of the common redenomination event that can be naturally interpreted as the risk of end of the Euro. The measure ob-

tained is very close to the German CDS, supporting the widespread practice among professional tail-hedgers of using exactly that contract as protection against break up of the Euro.

Appendix A

Lemma 5.1. *Denote $\tilde{\lambda}_i$ the* i*-FTD-intensity and λ_i the marginal intensity of credt event* i*. Then,*

$$\tilde{\lambda}_i \leq \lambda_i \tag{32}$$

with equality holding if the credit events are independent.

Proof. The fact that i-FTD intensity cannot be higher tham the corresponding marginal intensity is obvious. As for the case of equality we have that

$$\tilde{\lambda}_i = -\frac{\partial G_{DR}}{\partial t_i}\frac{1}{G_{DR}} = -\frac{\partial G_{DR}}{\partial G_i}\frac{\partial G_i}{\partial t_i}\frac{1}{G_{DR}} = \frac{\partial G_{DR}}{\partial G_i}\frac{1}{G_{DR}}\lambda_i G_i$$

with $i = D, R$. If default and redenomination are independent, we have $G_{DR} = G_D G_R$. Without loss of generality set $i = R$:

$$\tilde{\lambda}_R = \frac{\partial(G_D G_R)}{\partial G_R}\frac{1}{G_D G_R}\lambda_R G_R = G_D \frac{1}{G_D G_R}\lambda_R G_R = \lambda_R$$

and the same holds for $i = D$. $\qquad\square$

Lemma 5.2. *Take a Gumbel-Hougaard function $C(u,v)$ with $\theta \in [1,\infty$ linking two survival functions F'_D, F'_R with intensities μ_D, μ_R. Then, the i-FTD to default intensity $\tilde{\mu}_i, i = D, R$ is*

$$\tilde{\mu}_i = \mu_i^{\theta}\mu_{DR}^{1-\theta}$$

Proof. We use the link between the i-FTD intensity $\tilde{\mu}_i$ and marginal intensity μ_i:

$$\tilde{\mu}_i = \frac{\partial F_{DR}}{\partial F_i} \frac{1}{F_{DR}} \mu_i F_i = \frac{\partial C(F_D, F_R)}{\partial F_i} \frac{F_i}{C(F_D, F_R)} \mu_i \tag{33}$$

In the Gumbel-Hougaard case, the partial derivative of the copula, that is the conditional probability, can be expressed in term of intensities as

$$\begin{aligned}
\frac{\partial C(F_D, F_R)}{\partial F_i} &= F_{DR} \left[(-\log(F_{DR}))^\theta \right]^{1/\theta - 1} \frac{(-\log(F_i))^{\theta - 1}}{F_i} \\
&= C(F_D, F_R) \mu_{DR}^{1-\theta} \frac{\mu_i^{\theta-1}}{F_i}
\end{aligned} \tag{34}$$

Substituting (34) in (33) yields the result. $\square$

Lemma 5.3. *In the Gumbel-Hougaard copula model with exponential margins the MLE on transformed data is given by:*

$$\begin{aligned}
\mathcal{L} &= \log L \left(\mu_{CR14}(t_1), \ldots, \mu_{CR14}(t_n), \mu_{CR03}(t_1), \ldots, \mu_{CR03}(t_n); \theta \right) = \\
&= (1-\theta) \sum_{i=1}^{n} \log \mu_{\mu_{CR14}}(t_i) + \sum_{i=1}^{n} \log \left(1 + \frac{\theta - 1}{\mu_{CR14}(t_i)} \right) + \\
&\quad + \theta \sum_{i=1}^{n} \log \mu_{CR03}(t_i) + \sum_{i=1}^{n} \log \hat{\mu}_{R,t_i}(\theta) - \sum_{i=1}^{n} \hat{\mu}_{R,t_i}(\theta)
\end{aligned} \tag{35}$$

Proof. The density of the Gumbel copula is

$$\begin{aligned}
\frac{\partial C(F_D, F_R)}{\partial F_R \partial F_D} &= \frac{F_{DR}}{F_D F_R} \left(\mu_{DR}^\theta \right)^{-2+2/\theta} (\mu_D \mu_R)^{\theta - 1} \left[1 + (\theta - 1) \left(\mu_{DR}^\theta \right)^{-1/\theta} \right] \\
&= \frac{F_{DR}}{F_D F_R} (\mu_{DR})^{2-2\theta} (\mu_D \mu_R)^{\theta-1} \left[1 + \frac{\theta - 1}{\mu_{DR}} \right]
\end{aligned} \tag{36}$$

The partial derivative with respect to F_R, that is the conditional distribution of the

default time with respect to redenomination is

$$\frac{\partial C(F_D, F_R)}{\partial F_R} = F_{DR} \left[(-\log(F_{DR}))^\theta \right]^{1/\theta - 1} \frac{(-\log(F_R))^{\theta - 1}}{F_R}$$
$$= FDR \left(\mu_{DR} \right)^{1-\theta} \frac{\mu_R^{\theta - 1}}{F_R}$$

(37)

Since the margins are exponential the density functions are $f_R = \mu_R F_R$ and $f_D = \mu_D F_D$ respectively. Taking logharitms and simplifying gives the formula in the paper.

□

Appendix B

In this Appendix we document the estimation technique used in the multivartiate analysis. Assume d observable intensities mu_i (the redenomination intensities of the four countries in our model), and $d + 1$ intensities corresponding to hidden shocks (idiosyncratic redenomination events $\check{\mu}_i$ and simultaneous redenomination μ_{EoE}). Each observable intensity is linked to the hidden intensities by the relationship

$$\mu_i = (\mu_{EoE}^\gamma + \check{\mu}_i^\gamma)^{1/\gamma}$$

The theoretical Kendall's τ between the observed redenomination times of countries i and j is given by

$$\tau_{ij} = \frac{\gamma - 1}{\gamma} + \frac{1}{\gamma} \frac{\alpha_i \alpha_j}{\alpha_i + \alpha_j - \alpha_i \alpha_j}$$

with

$$\alpha_i = \frac{\mu_{EoE}}{\mu_{EoE} + \check{\mu}_i}$$

Estimation is carried out by matching theoretical and empirical Kendall's τ. Formally we estimate the set of parameters $\boldsymbol{\Theta} = \{\alpha_1, \alpha_2, \ldots, \alpha_d, \theta\}$ by solving

$$\hat{\boldsymbol{\Theta}} = \underset{\{\alpha_1, \alpha_2, \ldots, \alpha_d, \gamma\}}{\operatorname{argmin}} \sum_{i=1}^{d-1} \sum_{j=i+1}^{d} \left(\hat{\tau}_{i,j} - \tau_{i,j}(\alpha_i, \alpha_j, \gamma)\right)^2$$

where $\tau_{i,j}(\alpha_i, \alpha_j, \gamma)$ is the theoretical Kendall's τ and $\hat{\tau}_{i,j}$ is the corresponding empirical Kendall's τ statistics. The parameters of the estimation obtained are $\alpha_{IT} = 0.09640701, \alpha_{FR} = 0.52509536, \alpha_{GER} = 1, \alpha_{NL} = 0.65718309, \gamma = 1$.

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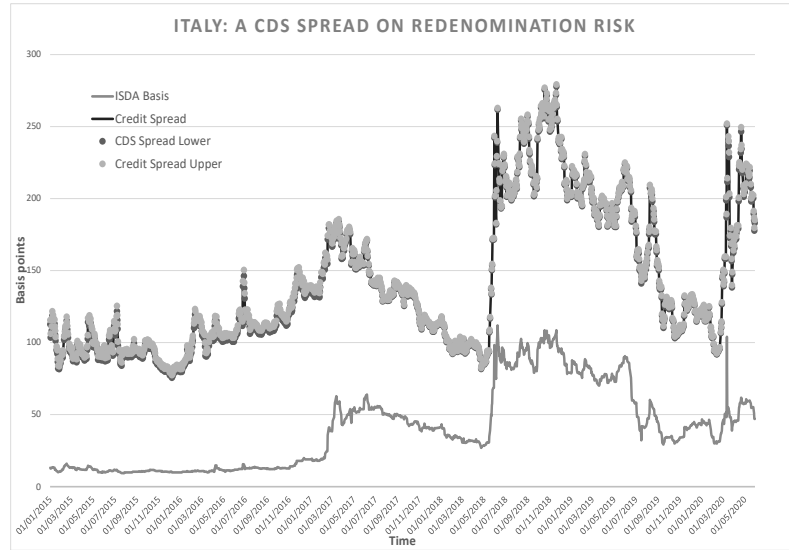
Figure 1: Italy. Value of a CDS spread on currency redenomination (with confidence interval) and the ISDA basis
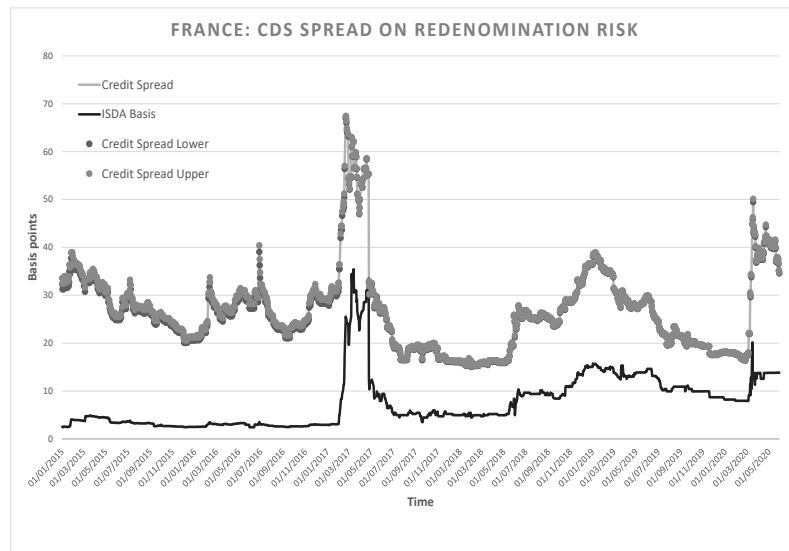
Figure 2: France. Value of a CDS spread on currency redenomination (with confidence interval) and the ISDA basis
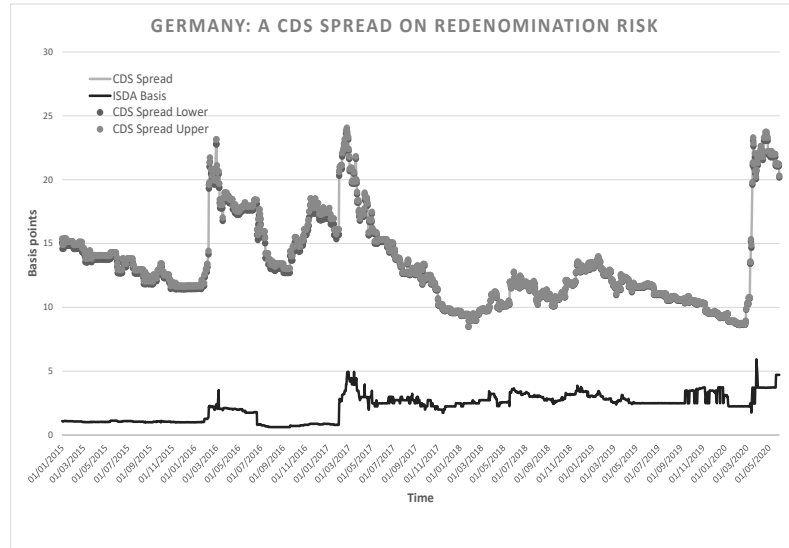
Figure 3: Germany. Value of a CDS spread on currency redenomination (with confidence interval) and the ISDA basis
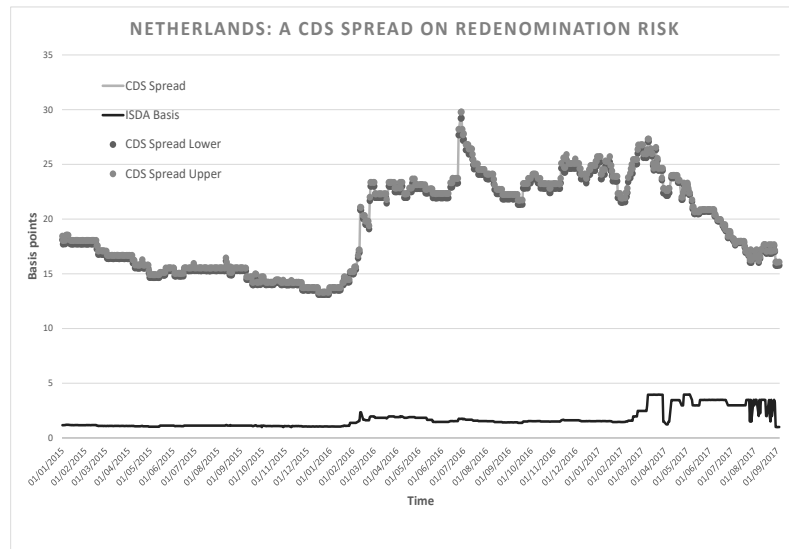
Figure 4: Netherlands. Value of a CDS spread on currency redenomination (with confidence interval) and the ISDA basis
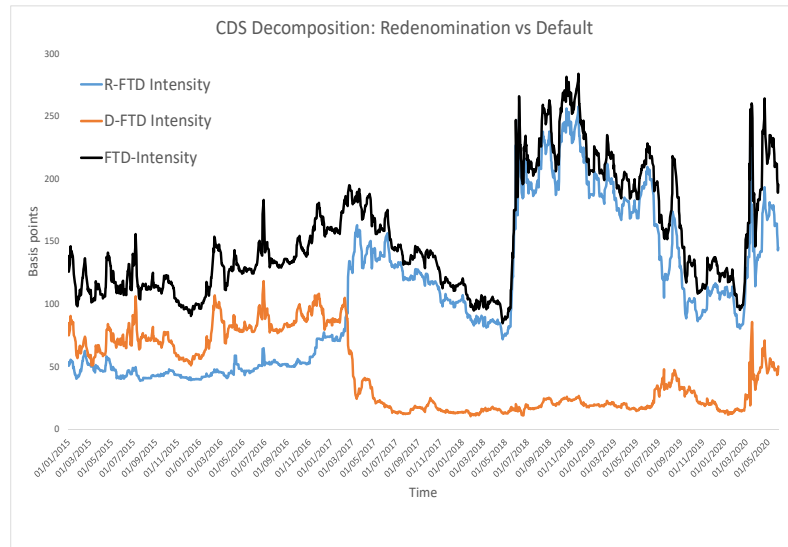
Figure 5: CDS 2014 decomposition: Italy. FTD-intensity is the CDS 2014-style; D-FTD intensity (R-FTD intensity) is the instantaneous conditional probability of a default (redenomination) event occurring first.
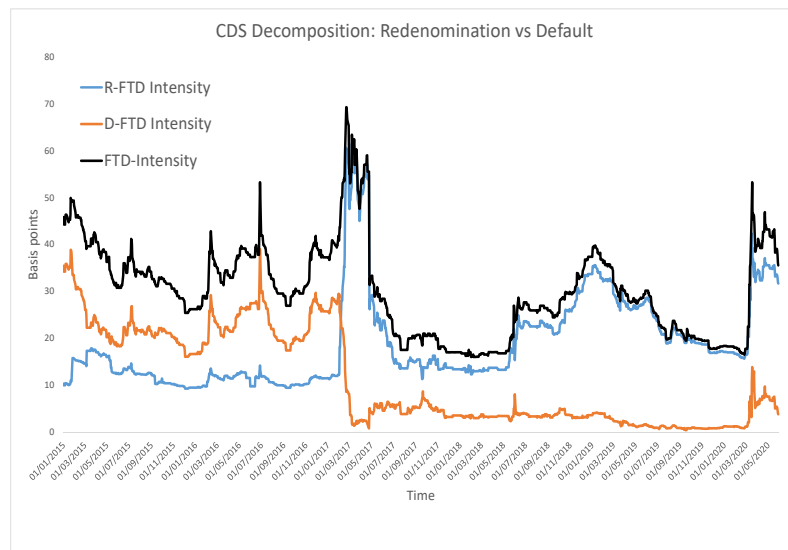
Figure 6: CDS 2014 decomposition: France. FTD-intensity is the CDS 2014-style; D-FTD intensity (R-FTD intensity) is the instantaneous conditional probability of a default (redenomination) event occurring first.
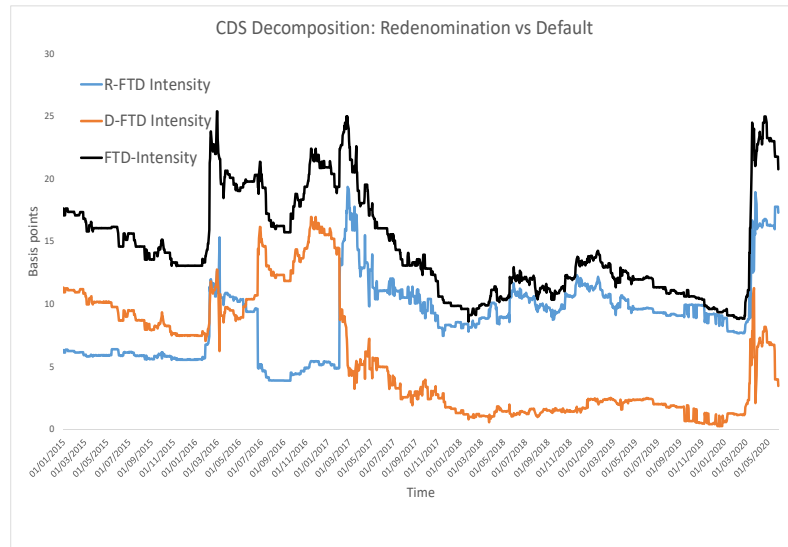
Figure 7: CDS 2014 decomposition: Germany. FTD-intensity is the CDS 2014-style; D-FTD intensity (R-FTD intensity) is the instantaneous conditional probability of a default (redenomination) event occurring first.
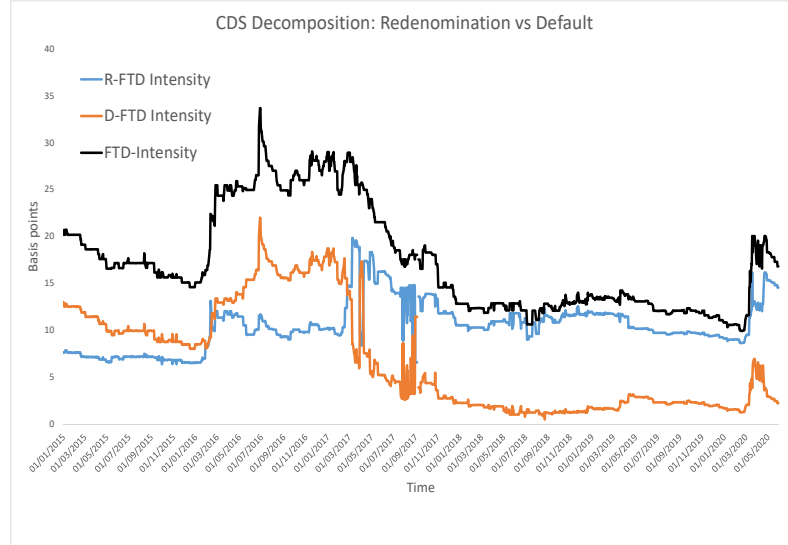
Figure 8: CDS 2014 decomposition: Netherlands. FTD-intensity is the CDS 2014-style; D-FTD intensity (R-FTD intensity) is the instantaneous conditional probability of a default (redenomination) event occurring first.
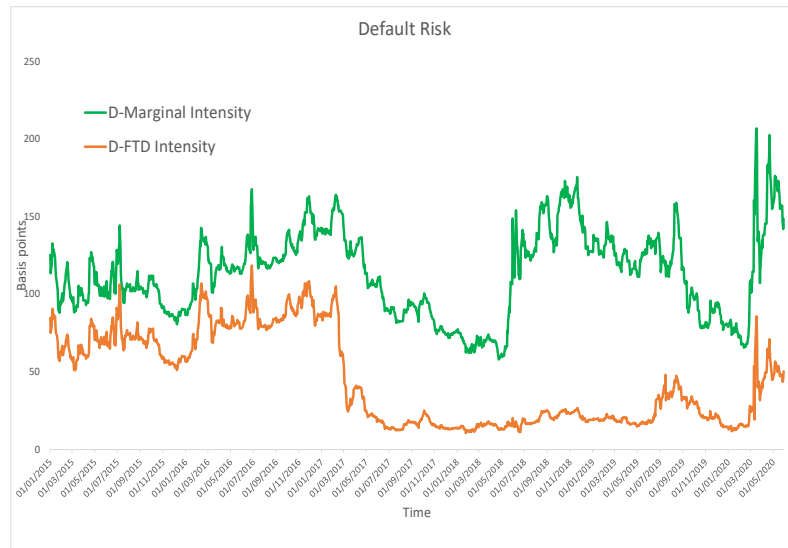
Figure 9: Default risk: Italy. Marginal default risk is the CDS 2003-style; D-FTD intensity is the instantaneous conditional probability of a default event occurring before redenomination.
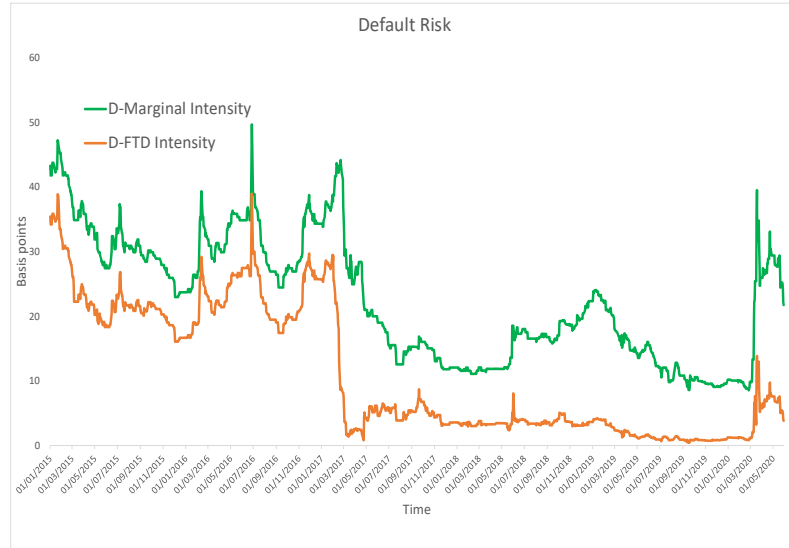
Figure 10: Default risk: France. Marginal default risk is the CDS 2003-style; D-FTD intensity is the instantaneous conditional probability of a default event occurring before redenomination.
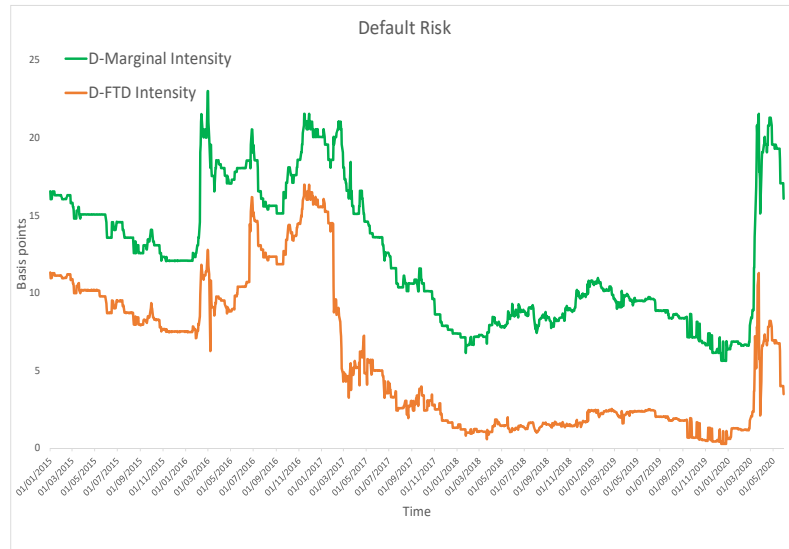
Figure 11: Default risk: Germany. Marginal default risk is the CDS 2003-style; D-FTD intensity is the instantaneous conditional probability of a default event occurring before redenomination.
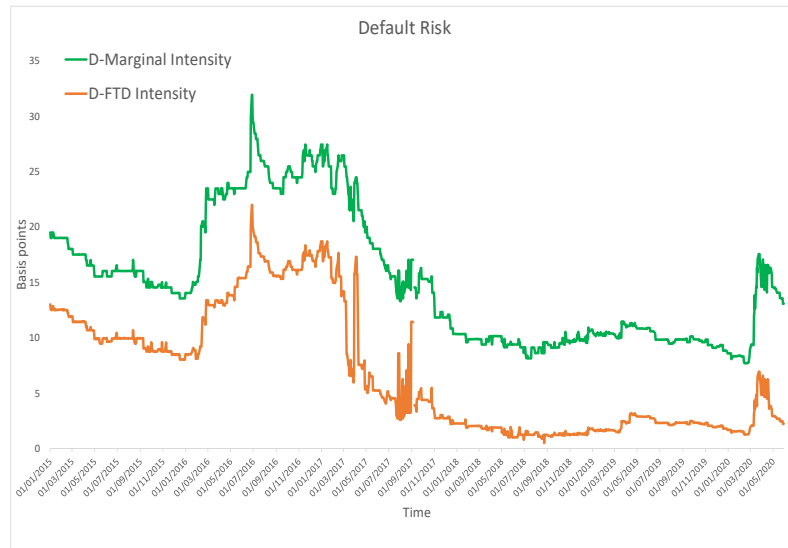
Figure 12: Default risk: Netherlands. Marginal default risk is the CDS 2003-style; D-FTD intensity is the instantaneous conditional probability of a default event occurring before redenomination.
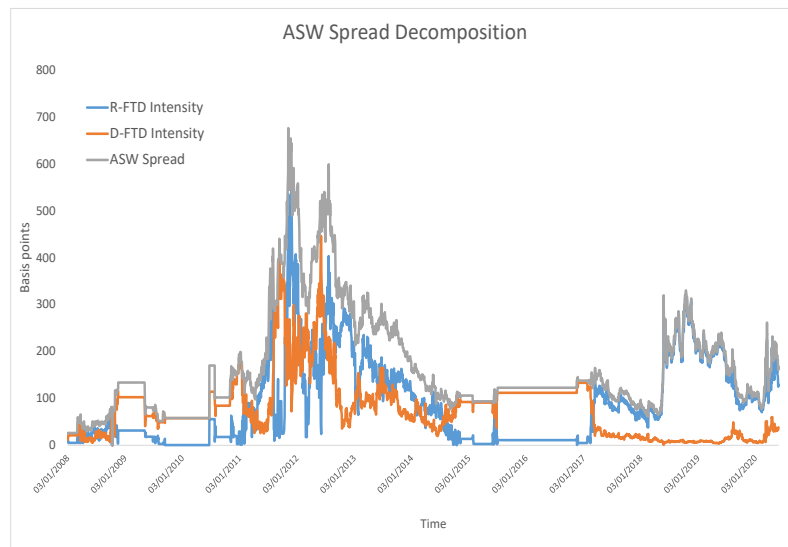
Figure 13: CDS 2014 decomposition: Italy. FTD-intensity is the R-ASW spread; D-FTD intensity (R-FTD intensity) is the instantaneous conditional probability of a default (redenomination) event occurring first. Period 2008-2020.
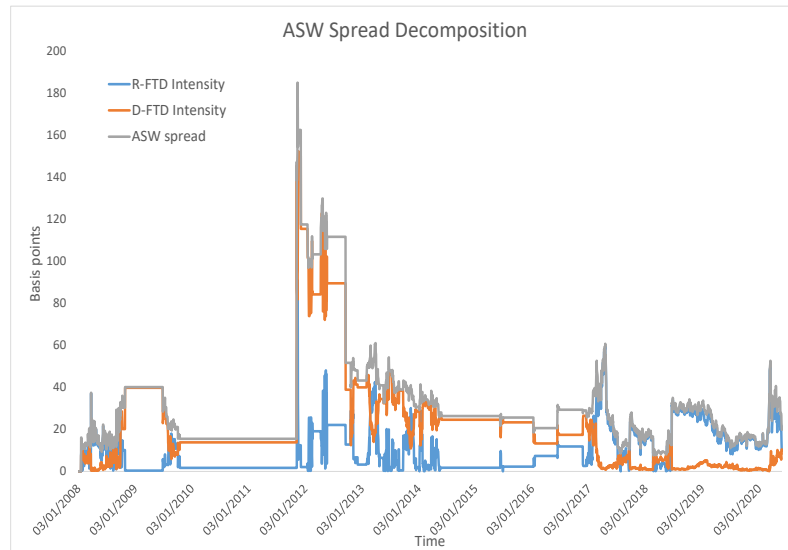
Figure 14: CDS 2014 decomposition: France. FTD-intensity is the R-ASW spread; D-FTD intensity (R-FTD intensity) is the instantaneous conditional probability of a default (redenomination) event occurring first. Period 2008-2020.
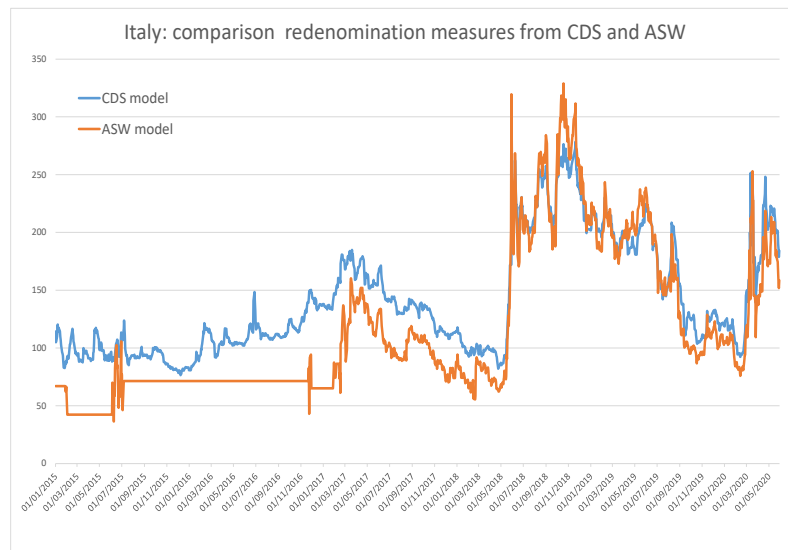

Figure 15: CDS 2014 decomposition: Italy. Comparison of redenomination risk estimated from CDS and ASW in the period 2015-2020.
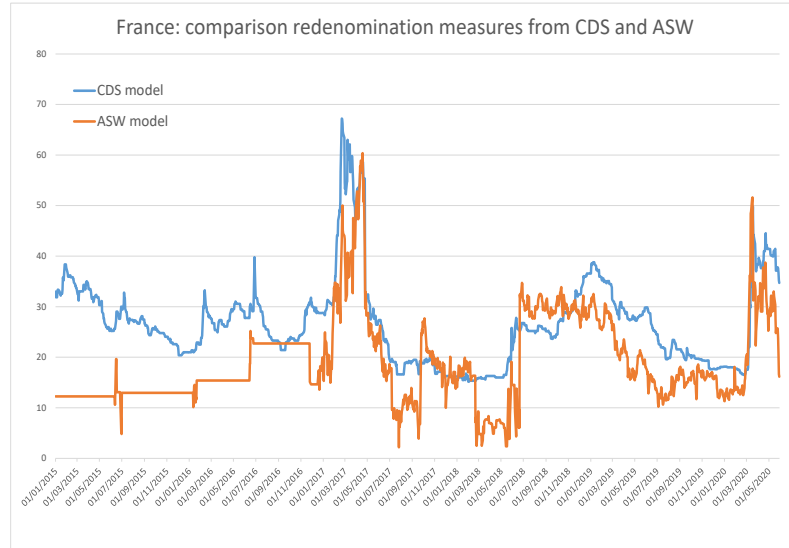
Figure 16: CDS 2014 decomposition: France. Comparison of redenomination risk estimated from CDS and ASW in the period 2015-2020.
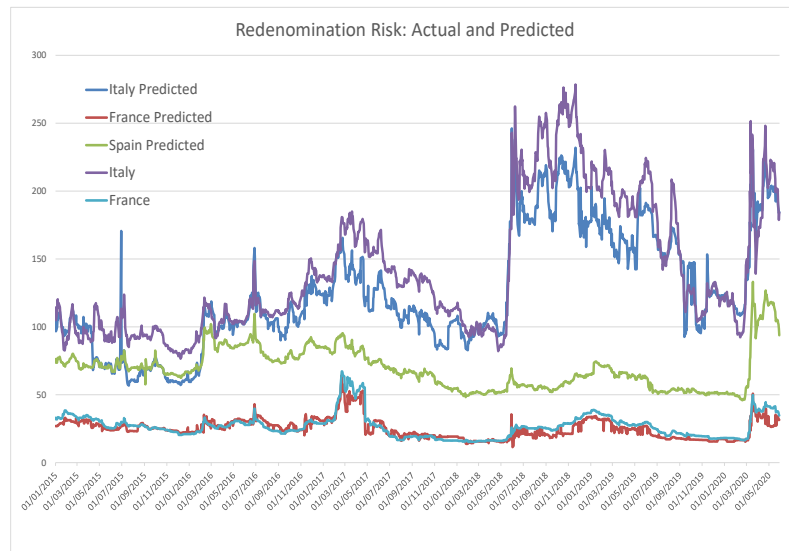

Figure 17: Redenomination risk estimate for Spain, compared with Italy and France (for which we reported both redenomination risk predicted from the model and estimated from the CDS market.
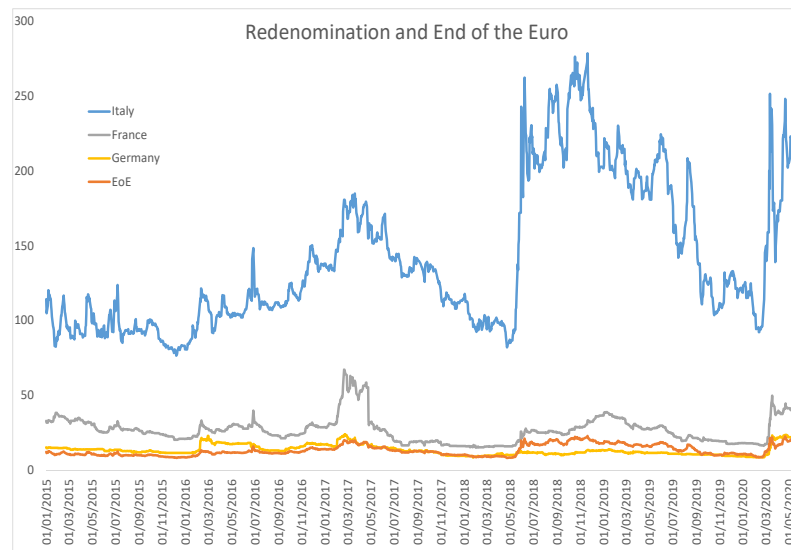
Figure 18: Redenomination risk of Italy, France and Germany and the end of the Euro