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Errata to “Global L^p estimates for degenerate
Ornstein-Uhlenbeck operators with variable
coefficients”, Math. Nachr. 286, No. 11–12, 1087
– 1101 (2013)

Marco Bramanti, Giovanni Cupini,
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May 24, 2021

In this note we want to point out and correct a mistake in our paper [1], where we consider a class of Ornstein-Uhlenbeck operators on \mathbb{R}^N

$$\mathcal{A} = \sum_{i,j=1}^{p_0} a_{ij}(x) \partial_{x_i x_j}^2 + \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j},$$

together with the corresponding Kolmogorov-Fokker-Planck operators on \mathbb{R}^{N+1}

$$L = \sum_{i,j=1}^{p_0} a_{ij}(z) \partial_{x_i x_j}^2 + \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j} - \partial_t$$

(here $z = (x, t)$). We will not recall here the structural assumptions on the matrix $B = (b_{ij})_{i,j=1}^N$. The matrix $A_0 = (a_{ij}(x))_{i,j=1}^{p_0}$ is a $p_0 \times p_0$ ($p_0 \leq N$) symmetric, bounded and uniformly positive definite matrix:

$$\frac{1}{\Lambda} |\xi|^2 \leq \sum_{i,j=1}^{p_0} a_{ij}(x) \xi_i \xi_j \leq \Lambda |\xi|^2 \quad (1)$$

for all $\xi \in \mathbb{R}^{p_0}$, $x \in \mathbb{R}^N$ (or $z \in \mathbb{R}^{N+1}$) and for some constant $\Lambda \geq 1$. The entries a_{ij} are assumed to satisfy a continuity condition which will be clarified in a moment. The main result in [1] is the following:

Theorem 0.1 (see [1, Thm. 1.1]) *For every $p \in (1, \infty)$ there exists a constant $c > 0$, depending on p, N, p_0 , the matrix B , the number Λ in (1) and the continuity modulus ω ,*

$$\omega(r) = \max_{i,j=1,\dots,p_0} \sup_{\substack{x,y \in \mathbb{R}^N \\ |x-y| \leq r}} |a_{ij}(x) - a_{ij}(y)|, \quad (2)$$

such that for every $u \in C_0^\infty(\mathbb{R}^N)$ one has:

$$\begin{aligned} \sum_{i,j=1}^{p_0} \left\| \partial_{x_i x_j}^2 u \right\|_{L^p(\mathbb{R}^N)} &\leq c \left\{ \|\mathcal{A}u\|_{L^p(\mathbb{R}^N)} + \|u\|_{L^p(\mathbb{R}^N)} \right\}, \\ \left\| \sum_{i,j=1}^N b_{ij} x_i \partial_{x_j} u \right\|_{L^p(\mathbb{R}^N)} &\leq c \left\{ \|\mathcal{A}u\|_{L^p(\mathbb{R}^N)} + \|u\|_{L^p(\mathbb{R}^N)} \right\}. \end{aligned}$$

This result is derived from an analogous L^p estimate holding for L on a strip $S_T = \mathbb{R}^N \times [-T, T]$:

Theorem 0.2 (see [1, Thm. 3.1]) *Let L be as above, with uniformly continuous coefficients a_{ij} . For every $p \in (1, \infty)$ there exist constants $c, T > 0$ depending on p, N, p_0 , the matrix B , the number Λ in (1), c also depending on the modulus of continuity ω*

$$\omega(r) = \max_{i,j=1,\dots,p_0} \sup_{\substack{z_1, z_2 \in \mathbb{R}^{N+1} \\ |z_1 - z_2| \leq r}} |a_{ij}(z_1) - a_{ij}(z_2)|, \quad (3)$$

such that

$$\sum_{i,j=1}^{p_0} \left\| \partial_{x_i x_j}^2 u \right\|_{L^p(S_T)} \leq c \left\{ \|Lu\|_{L^p(S_T)} + \|u\|_{L^p(S_T)} \right\}$$

for every $u \in C_0^\infty(S_T)$.

Now, in the statement of the above theorem we assumed the coefficients a_{ij} to be uniformly continuous in S_T in Euclidean sense. However, the assumption that we actually use in the proof of Proposition 3.2 (which implies the above theorem), is the global uniform continuity of the coefficients with respect to the local quasidistance d in \mathbb{R}^{N+1} which is introduced in the paper. Although the topology induced by d coincides with the Euclidean topology, so that continuity with respect to the two structures is the same thing, global uniform continuity is a different issue. In particular, global uniform continuity in Euclidean sense does not imply global uniform continuity with respect to d . Accordingly, also the assumption in the statement of [1, Thm. 1.1] must be corrected. This means that the coefficients $a_{ij}(x)$, which now are defined in \mathbb{R}^N , must be uniformly continuous with respect to d if they are regarded as defined in a strip S_T . Let us make precise the above corrections.

The definition (3) of the continuity modulus $\omega(r)$ (when the coefficients are defined in the strip S_T) must be changed to:

$$\omega_{S_T}(r) = \max_{i,j=1,\dots,p_0} \left\{ \sup |a_{ij}(z) - a_{ij}(\zeta)| : d(z, \zeta) < r, z, \zeta \in S_T \right\}.$$

The definition (2) of the continuity modulus $\omega(r)$ (when the coefficients are defined in \mathbb{R}^N) must be changed to:

$$\omega_{\mathbb{R}^N}(r) = \max_{i,j=1,\dots,p_0} \left\{ \sup |a_{ij}(x) - a_{ij}(y)| : \inf_{|s|<T, |t|<T} d((x, t), (y, s)) < r, x, y \in \mathbb{R}^N \right\} \quad (4)$$

(where $T > 0$ must be small enough). The global estimates proved in [1, Thm. 1.1, Thm. 3.1] hold, with the same proof, with the constant depending on these moduli.

We end with a remark and an example that should better enlighten the relation between uniform continuity in the two senses.

Remark 0.3 *In the case of time-independent coefficients, let us compare global uniform continuity in Euclidean sense with global uniform continuity w.r.t. $\omega_{\mathbb{R}^N}(r)$ in (4). Since*

$$\inf_{|s|<T, |t|<T} d((x, t), (y, s)) \leq d((x, 0), (y, 0)) = \sum_{i=1}^N |(x - y)_i|^{1/q_i}.$$

(where q_i are the positive integers defined in [1, p.1091]), for $r < 1$ we have

$$|x - y| < r \Rightarrow \inf_{|s|<T, |t|<T} d((x, t), (y, s)) < Nr^{1/q_N}$$

so that

$$\max_{i,j=1,\dots,p_0} \{\sup |a_{ij}(x) - a_{ij}(y)| : |x - y| < r\} \leq \omega_{\mathbb{R}^N}(Nr^{1/q_N}).$$

This shows that global uniform continuity w.r.t. $\omega_{\mathbb{R}^N}(t)$ implies global uniform continuity in Euclidean sense.

Example 0.4 *Let us consider the simplest example of degenerate KFP operator,*

$$\mathcal{L}u = u_{x_1x_1} + x_1u_{x_2} - u_t.$$

We have (keeping the notation in [1, p.1089])

$$E(s) = \begin{bmatrix} 1 & 0 \\ -s & 1 \end{bmatrix}$$

and, letting $y = (y_1, y_2)$,

$$E(s)y = (y_1, y_2 - sy_1).$$

Choosing, for $\varepsilon > 0$,

$$x = \left(\frac{1}{\varepsilon}, 1\right); y = \left(\frac{1}{\varepsilon}, 2\right); s = 0; t = \varepsilon$$

we have $(y, s) - (x, t) = (0, 1, -\varepsilon)$, so that $|(y, s) - (x, t)| \rightarrow 1$ as $\varepsilon \rightarrow 0$. On the other hand,

$$\begin{aligned} (y, s)^{-1} \circ (x, t) &= (x - E(t - s)y, t - s) = (x - E(\varepsilon)y, \varepsilon) \\ &= \left(\left(\frac{1}{\varepsilon}, 1\right) - \left(\frac{1}{\varepsilon}, 2 - \varepsilon\frac{1}{\varepsilon}\right), \varepsilon \right) = (0, 0, \varepsilon) \\ \left\| (y, s)^{-1} \circ (x, t) \right\| &= \|(0, 0, \varepsilon)\| = \sqrt{\varepsilon} \rightarrow 0. \end{aligned}$$

This shows that, if x, y are free to move in the whole \mathbb{R}^N ,

$$\left\| (y, s)^{-1} \circ (x, t) \right\| \rightarrow 0 \text{ does not imply } |(y, s) - (x, t)| \rightarrow 0.$$

For instance, let

$$a(x_1, x_2, t) = \sin\left(\frac{\pi}{2}x_2\right),$$

then a is uniformly continuous in \mathbb{R}^{N+1} , in Euclidean sense. Nevertheless, for (x_1, x_2, t) and (y_1, y_2, s) as above we have

$$d((x_1, x_2, t), (y_1, y_2, s)) = \sqrt{\varepsilon} \rightarrow 0$$

but

$$|a(x_1, x_2, t) - a(y_1, y_2, s)| = \left| \sin\left(\frac{\pi}{2}x_2\right) - \sin\left(\frac{\pi}{2}y_2\right) \right| = \left| \sin\left(\frac{\pi}{2}\right) - \sin\pi \right| = 1,$$

so that the function a is not uniformly continuous in any strip S_T , w.r.t. d .

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References

- [1] M. Bramanti, G. Cupini, E. Lanconelli, E. Priola: Global Lp estimates for degenerate Ornstein-Uhlenbeck operators with variable coefficients. *Mathematische Nachrichten*, Vol. 286, Issue 11-12 (2013), 1087–1101.

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