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Secret contracting and Nash-in-Nash bargaining*

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Abstract

In take-it-or-leave-it vertical contracting, an equilibrium with passive beliefs may fail to exist. We argue that this problem can be alleviated by tackling the contracting stage of the game through a cooperative approach. The outcome of the take-it-or-leave-it game then coincides with the *limit* of the cooperative solution when the bargaining power of the downstream firms tends to zero. We argue that the cooperative approach, which requires a different interpretation of the out-of-equilibrium beliefs, is not affected by the well-known existence problems of the non-cooperative one.

Keywords: Multilateral vertical contracting; Passive beliefs; Nash Bargaining Solution.

JEL classification: D43, L13, L14.

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1 Introduction

The games where an upstream monopolist makes offers over two-part tariffs to downstream competitors endowed with passive beliefs are potentially affected by issues of non-existence of a Perfect Bayesian Equilibrium, as first pointed out by Rey and Vergé (2004) for the case of take-it-or-leave-it (TIOLI) offers. O'Brien and Shaffer (1992) show that the optimal wholesale prices under TIOLI offers coincide with those obtained through simultaneous negotiations modeled through the Nash bargaining solution. We argue that in this latter case, equilibrium nonexistence problems disappear. Consequently, if TIOLI offers are interpreted as the limiting case of Nash negotiations with one party endowed with all the bargaining power, the equilibrium non-existence of this setup is defused.

In contexts of vertical relationships with secret TIOLI negotiations and non-linear contracts, a crucial role in determining the outcomes of the negotiations themselves is played by the beliefs of the downstream firms about the contracts offered by the upstream monopolist to their rivals. The literature has proposed different types of such beliefs, among which the most common are the so-called *passive beliefs*, whereby, when receiving an out-of-equilibrium offer, a downstream firm does not revise its beliefs about the (equilibrium) offers received by its rivals. The diffusion of these beliefs is due to the fact that, unlike many of the alternative ones, they lead to tractable results. In an influential paper, however, Rey and Vergé (2004) warn of the problems that might arise when using passive beliefs in the presence of opportunism (Hart and Tirole, 1990). In particular these authors (p. 740) show that "(i) an equilibrium with passive beliefs may not exist, due to the fact that multilateral deviations may become attractive, and (ii) passive beliefs differ from and are arguably less plausible than wary beliefs". Point (ii) is related to the fact that passive beliefs do not take into account the fact that under price competition the upstream monopolist has an incentive to renegotiate the contractual terms with one downstream firm in order to influence the pricing behavior of the *other* downstream producers. Each downstream firm should rationally acknowledge this and, therefore, disregard the belief that the offers made to the rivals do not change when it receives an out-of-equilibrium offer. Wary beliefs (McAfee and Schwartz, 1994), by contrast, are such that the downstream firm believes that the rivals have received offers that are optimal given the out-of-equilibrium one it has received.[1] This notwithstanding, passive beliefs are largely used in the literature (see e.g. O'Brien and Shaffer, 1992, de Fontenay and Gans, 2005) because they are intuitive and easy to implement.

Point (i) is more technical, and this note deals with it. Passive beliefs are strictly related to the concept of *contract equilibrium* (Crémer and Riordan, 1987) which, in turn, builds on the idea that the upstream monopolist and one downstream firm sign an optimal contract *given* the contracts of the other downstream firms. Contract equilibria need to be immune from unilateral deviations only, i.e. re-negotiations of the contractual offer of the upstream firm to a single downstream firm. Yet, because the upstream firm is entitled to simultaneously make the contractual offers to all the downstream firms, it can also propose simultaneous deviations to more than one of them. Such deviations are by definition not contemplated under the contract equilibrium concept, but the Perfect Bayesian Equilibrium (PBE) concept requires that they are not profitable for the upstream firm. Rey and Vergé (2004) show that, under quantity competition with interim observability, or price competition, profitable multilateral deviations may indeed exist, which prevent the existence of PBEs with passive beliefs. This allows them to conclude (p. 732) that "any passive-beliefs equilibrium is a contract equilibrium, but a contract equilibrium is not a passive-beliefs equilibrium if it does not survive multilateral deviations."

In this note, we consider an upstream supplier and two downstream retailers and show that the problem of the non-existence of a PBE with passive beliefs can be alleviated by addressing the problem from a different perspective. In particular, we exploit the relationship between the contractual terms at a PBE with passive beliefs and TIOLI offers and the ones obtained from "Nash-in-Nash" bargaining.[2] The relationship between these two approaches was first pointed out in the seminal paper by O'Brien and Shaffer (1992).[3] We argue that, with this shift in perspective, the non-existence problem vanishes.

In a nutshell, in the cooperative approach, the "out-of equilibrium event" within a negoti-

[1] Note that the "wary beliefs" may be consistent with the "full capacity beliefs" defined by Avenel (2012).

[2] As (Collard-Wexler *et al.*, 2019, p. 165) put it, the Nash-in-Nash approach is a "Nash equilibrium in Nash bargains" that is, separate bilateral Nash bargaining problems within a Nash equilibrium to a game played among all pairs of firms".

[3] Note that O'Brien and Shaffer (1992) adopt, as equilibrium concept, that of contract equilibrium. This choice avoids the problem of non-existence by not allowing multilateral deviations altogether.

ation is a failure to reach the agreement, Accordingly all downstream firms off the equilibrium path are not operating in the final market, thus their beliefs about the contract offered to the *other* firms are immaterial for the out-of-equilibrium continuation game. Similarly, the out-of-equilibrium pricing behavior of the "surviving" downstream firm would be optimal, *given* any contract signed with the upstream producer.[4] In the TIOLI setup, by contrast, an out-of-equilibrium offer may be *any* of the contracts that satisfy (with equality) the participation constraint of the downstream firms. This implies that here the downstream firms are always active *also* in the out-of-equilibrium path, which entails that the beliefs about the contract offered to the rivals are consequential for their pricing behavior. Under passive beliefs, the incentive by the upstream firm to multilaterally renegotiate its offers may backfire, and destroy equilibrium behavior.

Still, existence problems could in principle arise within the Nash Bargaining approach, because of a possible non-concavity of the Nash products. In the next Section we argue that under the usual assumption on the concavity of the revenue functions, this is not the case.

2 The formal framework

We consider the setup with one upstream ($\mathcal{M}$) and two downstream $\mathcal{D}_1$ and $\mathcal{D}_2$ firms as in Rey and Vergé (2004). The trade between the upstream and downstream firms is governed by contracts based on two-part tariffs $T_i \equiv \{w_i, f_i\}, i = 1, 2$, with w_i being the wholesale price and f_i being a monetary transfer (positive or negative) from $\mathcal{D}_i$ to $\mathcal{M}$. Negotiations are simultaneous and secret.

We assume that contracts are *interim unobservable*: downstream firms never observe the rival's contractual terms. This implies that, f_i cannot depend on T_j. Furthermore, when setting its optimal price, firm i cannot condition it on w_j, and thus needs to form beliefs about it. In the following, let $\Pi_{\mathcal{M}}(w_1, w_2)$ be the overall profit of the upstream firm, which is made of both the wholesale profit and the revenue from the fixed payments. Let $\pi_i(w_i)$

[4] The usual narrative behind the Nash Bargaining Solution is that once agents agree on some suitable properties of the solution of their bargaining problem (i.e. the Nash axioms), then the only allocation that satisfies them is that provided by the Nash Bargaining Solution itself. Needless to say, the interpretation of the interaction changes in this case, we will come back on this point in Section 3.

be the profit of downstream firm i.

Under TIOLI offers we assume that the downstream firms have passive beliefs. The upstream firm uses the fixed fee to extract all the surplus of the downstream firms: $(f_i = \pi_i)$ and chooses the optimal wholesale prices $\{w_1^*, w_2^*\}$ to maximize its profit[5]

$$\begin{aligned}\Pi_{\mathcal{M}}(w_1, w_2) \equiv \pi_{\mathcal{M}}(w_1, w_2) + \pi_1(w_1) + \pi_2(w_2) = \\ &[(w_1 - c)D_1(P_1(w_1), P_2(w_2)) + (w_2 - c)D_2(P_2(w_2), P_1(w_1))] \\ &+ (P_1(w_1) - w_1)D_1(P_1(w_1), P_2(w_2^*)) + (P_2(w_2) - w_2)D_2(P_2(w_2), P_1(w_1^*)), \quad (1)\end{aligned}$$

where $\pi_{\mathcal{M}}$ is the wholesale revenue of the upstream firm, $D_i(p_1, p_2)$ is the demand for good $i = 1, 2$ and $P_i(w_i) = \arg\max_{p_i}(p_i - w_i)D_i(p_i, p_j^*)$, with p_j^* being the equilibrium price. Rey and Vergé (2004) show that in such a framework, no equilibrium exists when the two goods are "close" substitutes (Proposition 2, p. 733). The intuition is that, although the contracts that maximize (1), namely $T_i^* = \{w_i^*, \pi_i(w_i^*)\}$, are by construction immune to unilateral deviations, they are not immune to multilateral deviations of the upstream firm with *both* firms simultaneously.

Given the passive beliefs assumption, the problem to maximize (1) coincides with the problem of finding a contract equilibrium in the equivalent framework of O'Brien and Shaffer (1992), which is, therefore, affected by the non-existence problems of PBE due to the profitability of multilateral deviations. In their paper, however, the authors show that the concept of contract equilibrium may be applied also to the case where the downstream firms retain some bargaining power. Clearly, in this alternative framework, the idea of TIOLI offers made by the upstream firm is no longer satisfactory and should be replaced by a more suitable concept. O'Brien and Shaffer (1992) introduce the idea "bargaining equilibrium" by applying the generalized Nash Bargaining Solution to each upstream-downstream negotiation, and argue that (p. 305) "a bargaining equilibrium is a contract equilibrium with a particular distribution of rents." This is in line with the idea of Nash-in-Nash bargaining, which is the workhorse for modelling simultaneous negotiations among parties endowed with bargaining power. In particular, under this approach, with secret negotiations and interim

[5]See Appendix A for details

unobservable contracts, the upstream firm $\mathcal{M}$ and each of the downstream firms i jointly and simultaneously set the contractual terms by maximizing the following Nash products with respect to $\{w_i, f_i\}, i \in \{1, 2\}$,

$$\phi_i(w_1, w_2, f_1, f_2) = (\pi_{\mathcal{M}}(w_1, w_2) + f_1 + f_2 - d_{\mathcal{M}i})^{\alpha}(\pi_i(w_i) - f_i)^{1-\alpha}, \tag{2}$$

with $i \in \{1, 2\}$ and $\pi_{\mathcal{M}}(\cdot)$ and $\pi_i(\cdot)$ being the same as in (1).

In the foregoing equation, α (res. $1 - \alpha$) $\in]0, 1[$ is the bargaining power of the upstream (res. downstream) firm in the bargaining, and $d_{\mathcal{M}i} \geq 0$ is the outside option of the upstream firm in the negotiation with downstream firm i. Each maximization problem in (2) can be conceptually split in two steps: first the firms agree on a transfer prices that maximize their joint surplus, second they share the maximized surplus according to their bargaining weights. This can be visualized by inspecting the first-order conditions for the maximization of the Nash products. The optimal transfers solve $\frac{\partial \phi(\cdot)}{\partial f_i} = 0$, which lead to

$$f_i(w_1, w_2, f_j) = \alpha\pi_i(w_i) - (1 - \alpha)(\pi_{\mathcal{M}}(w_1, w_2) + f_j - d_{\mathcal{M}i}), \quad i, j \in \{1, 2\}, i \neq j. \tag{3}$$

It is apparent that the optimal transfer to the upstream firm, as in (3), is increasing in its bargaining weight α. If α is small enough such a transfer is actually *negative*, i.e. a transfer from the upstream to the downstream firm. As α increases this f_i decreases in absolute value and becomes eventually positive.[6]

By plugging (3) back into (2) it is immediate to obtain that the Nash products reduce to

$$\phi_i(w_1, w_2, f_j) = \alpha^{\alpha}(1 - \alpha)^{1-\alpha}(\pi_{\mathcal{M}}(w_1, w_2) + f_j - d_{\mathcal{M}i} + \pi_i(w_i)), \quad i, j \in \{1, 2\}, i \neq j. \tag{4}$$

It is worth noticing here that the term in the rightmost bracket is the sum of the joint profit of firms $\mathcal{M}$ and i, which depend on the wholesale price w_i, and of the term $f_j - d_{\mathcal{M}i}$, which does not. As a consequence, the w_1 and w_2 that maximize (4) are the same that maximize (1). Thus the optimal wholesale prices in the take-it-or-leave-it contract (w_i^* for $i \in \{1, 2\}$),

[6] This is intuitive. As will be clearer later, the optimal wholesale prices are always positive in this setup. Accordingly, the transfer of surplus from $\mathcal{D}_i$ to $\mathcal{M}$ in terms of wholesale revenue is positive for any positive input purchase. If α is close enough to zero, almost all the surplus of the relationship must be captured by firm $\mathcal{D}_i$. This is only possible if the fixed transfer is *negative*, i.e. from the upstream to the downstream firm.

coincide with the ones in the Nash-in-Nash bargaining.[7]

Now, remember that the distribution of bargaining powers at the contracting stage determines the apportioning of the surplus within the pair, net of the outside options (see 3). In particular, when α tends to one, the upstream firm has "almost all" the bargaining power in the negotiations, which entails that in each bargaining the upstream monopolist obtains "almost all" the surplus generated in the relationship. More important, in this case, the limit of the cooperative equilibrium contractual terms $T_i^C = \{w_i^*, f_i(w_1^*, w_2^*, f_j^*)\}$ tend to those of the (candidate) take-it-or-leave it equilibrium contract, $\lim_{\alpha \to 1} T_i^C = T_i^*$. The reason is that the maximizer of (4) does not depend on α (this parameter only enters the first two factors of the product, which do not include w_i), and, by (3), $\lim_{\alpha \to 1} f_i(w_1, w_2, f_j) = \pi_i(w_i)$. It is easy to show that the assumption of concavity of the revenue functions (as in Rey and Vergé, 2004), namely $2\partial_1 D(\cdot) + p_i \partial_{11}^2 D(\cdot) < 0$, guarantees that the objective functions in (2) are concave at the (candidate) optimal contracts, which technically defuses the potential non-existence problem of each Nash bargaining solution.[8]

3 Discussion

Intuitively, the crucial difference between the TIOLI and the Nash-in-Nash bargaining approaches is in the out-of-equilibrium behavior of the firms. In particular, with take-it-or-leave-it offers, firm $\mathcal{M}$, both along and off the equilibrium path, offers to each downstream firm $\mathcal{D}_i$ a contract such that the participation constraint of each firm is always satisfied. This implies that the downstream firms will always be active both in equilibrium and out of equilibrium. Clearly, in order to evaluate the profitability of a deviation from a candidate equilibrium contract, firm $\mathcal{D}_i$ needs to form beliefs about the other contract, beliefs which will be used to optimally set its off-equilibrium price. Because the upstream firm is entitled to offer the contractual terms, it can also propose simultaneous deviations, whence the threat to equilibrium existence in the case of passive beliefs, as above discussed.

In the cooperative approach, by contrast, all out-of-equilibrium contracts, i.e. contracts

[7] This is pointed out by O'Brien and Shaffer (1992). See Appendix A for a formal proof.

[8] Here we adopt Rey and Vergé (2004) notation, whereby, e.g., $\partial_{12}^2 g(\cdot)$ is the second-order partial derivative of the function g with respect to its first and second argument. See Appendix B for the details.

which violate at least one of the Nash Axioms, can be interpreted as possible deviations and are rejected by at least one of the parties involved in the negotiation. In this occurrence, at least one firm $\mathcal{D}_i$ is inactive, and firm $\mathcal{M}$ cannot receive any payment from that firm. This has two implications. The first is that all non-equilibrium contracts reduce the profits of both the upstream and downstream firms because they prevent firms from beneficially trade with one another. The second is that, because any out-of-equilibrium contract leads to disagreement, and thus no production by the downstream firm concerned, beliefs about the *other* contract are immaterial for out-of equilibrium downstream competition.[9]

This standpoint is consistent with the "bargaining equilibrium" for multilateral vertical contracting recently introduced by Rey and Vergé (2017), which "discards the possibility of multi-sided deviations" (Rey and Vergé, 2017, p.7). Indeed, Rey and Vergé's Bargaining Equilibrium is defined as a vector of (non-linear) tariffs, each of which "(i) maximizes the joint profit of the two partners, given the contracts negotiated by each firm with its other partners as well as downstream rivals' equilibrium prices, and taking into account the impact of the negotiated contract on the downstream firm's own prices; and (ii) shares the surplus from a successful negotiation according to some pre-determined sharing-rule." (p.3). This definition is clearly consistent with the optimal tariffs found through the Nash-in-Nash Bargaining in our setup.

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References

Avenel, E. (2012). "Upstream capacity constraint and the preservation of monopoly power in private bilateral contracting". *The Journal of Industrial Economics*, **60**(4):578–598.

[9]Stated differently, with TIOLI contracts any downstream firm receiving an out-of-equilibrium offer by $\mathcal{M}$ must set up a pricing strategy along the deviation path, based on the beliefs about the other contract and knowing that both itself and the rival will be active in the final market. Under bargained contracts, instead, all firms know that any off-equilibrium contract will not allow for input trade and thus production of the final good.

Collard-Wexler, A., Gowrinsankaran, G., and Lee, R. S. (2019). "Nash-in-Nash bargaining: A microfundation for applied work". *Journal of Political Econonomy*, **1**:163–195.

Crémer, J. and Riordan, M. H. (1987). "On governing multilateral transactions with bilateral contracts". *The RAND Journal of Economics*, **18**(3):436–451.

de Fontenay, C. C. and Gans, J. S. (2005). "Vertical integration in the presence of upstream competition". *The RAND Journal of Economics*, **36**(3):544–572.

Hart, O. and Tirole, J. (1990). "Vertical integration and market foreclosure". *Brookings Papers on Economic Activity. Microeconomics*, **21**:205–286.

McAfee, R. P. and Schwartz, M. (1994). "Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity". *The American Economic Review*, **84**(1):210–230.

O'Brien, D. P. and Shaffer, G. (1992). "Vertical control with bilateral contracts". *The RAND Journal of Economics*, **23**(3):299–308.

Rey, P. and Vergé, T. (2004). "Bilateral control with vertical contracts". *The RAND Journal of Economics*, **35**(4):728–746.

Rey, P. and Vergé, T. (2017). "Secret contracting in multilateral relations". Working Paper 2017-44, CREST.

Appendix A TIOLI and Nash-in-Nash bargaining contracts

TIOLI. The TIOLI problem of the upstream firm $\mathcal{M}$ is

$$\max_{w_1,w_2,f_1,f_2} \pi_{\mathcal{M}}(w_1,w_2) + f_1 + f_2, \tag{1}$$

$$\text{s.t. } f_1 \leq \pi_1(w_1) \text{ and } f_2 \leq \pi_2(w_2). \tag{2}$$

It should be noted here that because of the assumption of passive beliefs, the profit of each downstream firm i depends on own wholesale price w_i and on the *expected* equilibrium price p_j^*, which in turn depends on the expected wholesale price w_j^*. By contrast, the profit of firm $\mathcal{M}$ depends on both wholesale price w_1 and w_2. The Lagrangian of this problem is

$$\mathcal{L}(w_1,w_2,f_1,f_2) = \pi_{\mathcal{M}}(\cdot) + f_1 + f_2 - \lambda_1(f_1 - \pi_1(\cdot)) - \lambda_2(f_2 - \pi_2(\cdot)). \tag{3}$$

The set of first-order conditions is

$$\begin{cases} \frac{\partial \mathcal{L}(\cdot)}{\partial w_1} = \frac{\partial \pi_{\mathcal{M}}(\cdot)}{\partial w_1} + \lambda_1 \frac{\partial \pi_1(\cdot)}{\partial w_1} = 0, \\ \frac{\partial \mathcal{L}(\cdot)}{\partial w_2} = \frac{\partial \pi_{\mathcal{M}}(\cdot)}{\partial w_2} + \lambda_2 \frac{\partial \pi_2(\cdot)}{\partial w_2} = 0, \\ \frac{\partial \mathcal{L}(\cdot)}{\partial f_1} = 1 - \lambda_1 = 0, \\ \frac{\partial \mathcal{L}(\cdot)}{\partial f_2} = 1 - \lambda_2 = 0, \\ \frac{\partial \mathcal{L}(\cdot)}{\partial \lambda_1} = f_1 - \pi_1(\cdot) = 0, \\ \frac{\partial \mathcal{L}(\cdot)}{\partial \lambda_2} = f_2 - \pi_2(\cdot) = 0. \end{cases} \tag{4}$$

System (4) implies (i) that the optimal fixed fee is equal to the whole profit of the downstream firm: $f_i = \pi_i(w_i)$. In addition (ii) the optimal wholesale prices are implicitly and uniquely defined by the system of first-order conditions, which do not include f_i:

$$\frac{\partial \pi_{\mathcal{M}}(w_1,w_2)}{\partial w_i} = -\frac{\partial \pi_i(w_i)}{\partial w_i} \quad i = 1,2. \tag{5}$$

Equations (5) show that firm $\mathcal{M}$ sets each wholesale price w_i so as to maximize the sum of its wholesale revenue and the profit of firm i: $\pi_{\mathcal{M}}(\cdot) + \pi_i(\cdot)$. Importantly, the passive beliefs assumption guarantees that, when setting w_i, firm $\mathcal{M}$ disregards the effect this choice has on $\pi_j(\cdot)$ and thus on f_j^*, although it accounts for the effect on the revenue reaped from firm j through the wholesale price, $(w_j - c)D_j(P_1(w_1,w_2), P_2(w_1,w_2))$, as included in $\pi_{\mathcal{M}}(w_1,w_2)$. The solution to each of the equations in (5) is thus $w_i = g_i(w_j); i,j = 1,2, i \neq j$. By simultaneously solving these two equations one obtains the equilibrium wholesale prices

(w_1^*, w_2^*) under TIOLI.

Nash-in-Nash bargaining. The Nash Bargaining Solution to each negotiation is found by solving the problem

$$\max_{w_i, f_i} \phi_i(w_1, w_2, f_1, f_2), \quad i = 1, 2. \tag{6}$$

Function $\phi_i(\cdot)$ (see eq. **??**) is the so-called Generalized Nash Product. Its terms are the profits minus the outside options of the firms involved in the negotiation. The power of each factor represents the bargaining weight of the corresponding firm.

When bargaining over two-part tariffs, firm $\mathcal{M}$ and firm i use the fixed fee to apportion that surplus according to the bargaining power distribution, as pointed out by equation (**??**). The wholesale price, instead, serves to achieve Pareto-optimality, namely to maximize the surplus generated by their specific relationship. Here the surplus of the relationship is the sum of the profits of the two firms, minus the sum of their outside options.[1] This is the term in the rightmost brackets in equation (**??**). It should be noticed that, when negotiating over the contractual terms (w_i, f_i), firm $\mathcal{M}$ and firm i consider both their outside options and the *other* contractual terms (w_j, f_j) as given.[2] Within each negotiation, the optimal wholesale price w_i is found by solving the of first-order conditions

$$\frac{\partial \phi_i(\cdot)}{\partial w_i} = 0 \Leftrightarrow \frac{\partial \pi_{\mathcal{M}}(w_1, w_2)}{\partial w_i} = -\frac{\partial \pi_i(w_i)}{\partial w_i}, \quad i = 1, 2. \tag{7}$$

It is immediate that equations (5) and (7) coincide. This entails that the solution to each of the two above first-order conditions is $w_i = g_i(w_j); i, j = 1, 2, i \neq j$. Now, in order to make the choices in the two negotiations "*a Nash Equilibrium in Nash bargains*", i.e. mutually optimal (see footnote **??**) one has to solve the system of these two FOCs. Clearly, therefore, the resulting wholesale prices are (w_1^*, w_2^*), the same obtained under TIOLI.

Needless to say, because wholesale prices are the same across the two approaches, the surplus generated by each product is the same. However, the sharing of that surplus differs. Under TIOLI offers it is completely captured by firm $\mathcal{M}$. Under Nash bargaining it is shared according to the Bargaining power distribution.

[1] The outside option of firm i is normalized to zero.

[2] This is consistent with the Nash-in-Nash approach.

Appendix B Concavity of the Nash products

To ascertain that the concavity of the revenue functions is sufficient for the concavity of the Nash products, start by considering the second-order conditions for the optimal transfers $f_i, i = 1, 2$, namely

$$-\alpha(1-\alpha)[\pi_M(\cdot) + f_1 + f_2 - d_M]^{-2+\alpha}[\pi_i(\cdot) - f_1]^{-(1+\alpha)}[\pi_M(\cdot) + \pi_i(\cdot) + f_2 - d_M]^2 < 0, \quad (8)$$

which are clearly satisfied as long as the (expected) profits of the firms in pair i exceed their outside options. Now turn to the condition that guarantees the concavity of the problem of finding the optimal input price w_i, namely the negativity of the second-order derivative of $\phi_i(\cdot)$:

$$\alpha^\alpha(1-\alpha)^{1-\alpha}\left[\partial^2_{11}\pi_M(\cdot) + \partial^2_{11}\pi_i(\cdot)\right] < 0. \quad (9)$$

The sign of inequality (9) depends on the sign of the last term. As a consequence, the problem of finding a maximizer for (**??**) requires that $\partial^2_{11}\pi_M(\cdot) + \partial^2_{11}\pi_i(\cdot) < 0$. This condition can be re-written as:

$$[\partial_1 P_i(\cdot)]^2\left\{2\partial_1 D(\cdot) + [P_i(\cdot) - c]\partial^2_{11} D(\cdot)\right\} + \partial^2_{11} P_i(\cdot)\left\{D_i(\cdot) + [P_i(\cdot) - c]\partial_1 D_i(\cdot)\right\} < 0. \quad (10)$$

In the foregoing inequality, the first addend is negative because of the assumption on the concavity of the revenue functions, and the second non positive as long as $\partial^2_{11} P_i(w_i) \geq 0$, which is a common assumption in the literature.[3]

[3]Typically, this derivative is nil.