Accuracy of Verdicts under Different Jury Sizes and Voting Rules*

Alice Guerra[†], Barbara Luppi[‡], and Francesco Parisi[§]

February 25, 2020

Abstract

Juries are a fundamental element of the criminal justice system. In this paper, we model jury decision-making as a function of two institutional variables: jury size and voting requirement. We expose the critical interdependence of these two elements in minimizing the probabilities of wrongful convictions, of wrongful acquittals, and of hung juries. We find that the use of either large non-unanimous juries or small unanimous juries are alternative ways to maximize the accuracy of verdicts while preserving the functionality of juries. Our framework – which lends support to the elimination of the unanimity requirement in the presence of large juries – helps appraise U.S. Supreme Court decisions and state legal reforms that have transformed the structure of American juries.

Keywords: Jury Size, Voting Requirement, Criminal Trial
JEL Codes: K0, K4

1 Introduction

Jury design is a critical element of criminal adjudication. After more than six centuries without change, the structure and functioning of juries have recently undergone several significant transformations regarding jury size and voting requirements. Juries in a criminal case were traditionally composed of 12 members, who needed to reach a unanimous agreement to render a decision.[1] Although most Americans view the 12-member jury as a fixture of American legal procedure, several U.S. Supreme Court decisions have affirmed the constitutionality

*A previous version of this paper was circulated under the title "Designing Optimal Juries." Two self-standing papers were created out of the original manuscript, one focusing on jury structure and standard of proof (Guerra and Parisi, 2019), and the current paper, which focuses on the optimal combination of jury size and voting requirements to maximize the accuracy of verdicts. The authors are grateful to Murat Mungan, William S. Neilson, Harold Winter, and an anonymous referee for their helpful feedback, and Eamonn Gallagher for his editorial assistance.

†Department of Economics, University of Bologna, Strada Maggiore 45, Bologna, Italy; Rimini Campus, via Angherà 22, Rimini, Italy. ORCID: 0000-0003-1956-0270. E-mail: alice.guerra3@unibo.it.

‡Department of Economics, University of Modena and Reggio Emilia. E-mail: barbara.luppi@unimore.it.

§School of Law, University of Minnesota; Department of Economics, University of Bologna. E-mail: parisi@umn.edu.

[1] In the leading 1898 case *Thompson v. Utah*, the Court construed the Sixth Amendment to require that in all criminal cases, a jury must be comprised of exactly 12 persons.

of juries with fewer than 12 members, as well as juries operating under a voting requirement less stringent than unanimity. This paper seeks to evaluate the desirability – or lack thereof – of these institutional transformations by analyzing the impact of changes in jury size and voting requirements on the probability of wrongful convictions, wrongful acquittals (i.e. convicting the innocent and acquitting the guilty, respectively) and hung juries.

Prior literature on jury design has investigated jury size and voting requirement as independent policy variables or in pairwise choice frameworks, but this literature often neglected the critical interdependence of jury size and voting requirements in maximizing the accuracy of verdicts. Prior contributions have separately investigated how large a jury should be (Paroush, 1997; Ben-Yashar and Paroush, 2000; Dharmapala and McAdams, 2003; Helland and Raviv, 2008; Luppi and Parisi, 2013), and how juries should vote to reach an accurate verdict (Klevorick and Rothschild, 1979; Klevorick et al., 1984; Ladha, 1995; Young, 1995; Neilson and Winter, 2005).

Our paper contributes to the existing literature by exposing the critical interplay between jury size and voting requirement in criminal adjudication. We extend the criminal trial model developed in Neilson and Winter (2000, 2005) by *both* relaxing the unanimity requirement *and* varying the jury size. We investigate how different combinations of these two institutional variables affect the probabilities of accurate verdicts, wrongful verdicts, and hung juries. Our results reveal that jury size and voting requirements should inversely depend on one another: large non-unanimous juries or small unanimous juries are alternative solutions to maximize the accuracy of verdicts. We discuss these findings in the light of recent legal transformations to jury structure, and we offer insights for policy analysis.

The paper is organized as follows. Section 2 briefly reviews the legal and economic backgrounds on jury design. Section 3 presents the criminal trial model. Section 4 introduces a numerical example to investigate how different combinations of a jury's institutional characteristics affect the probability of wrongful convictions and wrongful acquittals, as well as the ability for the jury to reach a deliberation. Section 5 concludes with a discussion of our results and their relevance for policy purposes.

2 Related Literature

For the last six centuries, criminal verdicts have been rendered by juries composed of 12 members, deliberating unanimously. In recent years, the U.S. Supreme Court has granted states the freedom to reduce the size of juries and to relax the jury's voting requirement, allowing non-unanimous verdicts. The changes have taken place through a series of cases decided by the U.S. Supreme Court between 1968 and 1979. In one of these cases, the well-known *Williams v. Florida*,[2] the Supreme Court recognized that a verdict rendered unanimously by fewer than 12 jurors was not inconsistent with the Constitutional right to have a trial by jury. In a subsequent decision, *Ballew v. Georgia*,[3] the Supreme Court set a lower limit on jury size, affirming that any jury with fewer than six members would be unconstitutional because it would be too small to be representative of the relevant community.

[2] *Williams v. Florida*, 3399 U.S. 78. (1970).

[3] *Ballew v. Georgia*, 435 U.S. 223 (1978).

Other important changes took place with respect to the jury's voting requirement. Unanimity for criminal verdicts has generally been viewed as an important requirement to preserve the public confidence in the criminal justice system, since wrongful convictions of innocent defendants are less likely under unanimity (Coughlan, 2000).[4] However, unanimity allows any single juror to veto a proposed verdict and single-handedly lead to a mistrial. The increasing administrative and financial cost of mistrials led some states to consider criminal justice reforms that relaxed the unanimity requirement.[5] These state reforms were challenged at the federal level.

In the leading cases — *Duncan v. Louisiana*, *Johnson v. Louisiana*, and *Apodaca v. Oregon* — the U.S. Supreme Court ruled that verdicts reached under a qualified majority rule do not violate the U.S. Constitution.[6] This gave states the flexibility to pursue criminal justice reforms by allowing verdicts to be reached under a qualified majority rule. In 1979, the Court in *Burch v. Louisiana* held that states could either reduce jury size or lessen the voting requirement but not both simultaneously: non-unanimous verdicts could only be rendered by juries of 12, and smaller juries could only deliberate unanimously.[7] As of today, only Oklahoma, Oregon, and Louisiana allow non-unanimous verdicts in misdemeanor cases; Oregon and Louisiana allow them also in felony cases.[8]

The abolition of the unanimity requirement for criminal verdicts was met with a mixture of approval and skepticism. Supporters viewed non-unanimous decision-making as a possible solution to the hung-jury problem (e.g., Amar, 1994; Glasser, 1996; Morehead, 1997). Opponents viewed the abolition of the unanimity requirement as a violation of a fundamental principle of criminal justice for the protection of innocent defendants (e.g., Kachmar, 1996; Smith, 1996; Klein and Klastorin, 1999).[9] The views in the literature are split, revealing an objective difficulty in balancing the policy goals of accuracy in adjudication and reduction of the costs of criminal justice.

Several law and economics contributions have investigated the effects of changing jury size on the expected trial outcomes. A central argument in the literature on juries and jury decision-making is that a group will make a better decision than an individual (Condorcet's Jury Theorem). Some contributions refined the Condorcet's Jury Theorem and demonstrated that, under certain conditions, this theorem does not hold (e.g., in the presence of strategic voting, as shown by Feddersen and Pesendorfer, 1998). For example, larger, unanimous juries may

[4] See Rule 31 of the Federal Rules of Criminal Procedure.

[5] See, for example, the multi-phased Hannaford-Agor et al.'s (2002) NCSC research on mistrials, motivated by the concern that mistrials were reaching unacceptably high levels in some jurisdictions. See also Kalven and Zeisel's (1966) study of the American jury, which briefly discussed the phenomenon of mistrials in criminal cases.

[6] See *Duncan v. Louisiana*, 391 U.S. 145 (1968); *Apodaca v. Oregon*, 406 U.S. 404 (1972); *Johnson v. Louisiana*, 406 U.S. 356 (1972).

[7] *Burch v. Louisiana*, 441 U.S. 130 (1979).

[8] See Oregon Revised Statutes §136.450, and Louisiana Laws Code of Criminal Procedure 782. Several states permit non-unanimous verdicts in civil trials. See State Court Organization, 1998, Figure 42 (Trial Juries: Size and Verdict Rules). For a more extensive discussions on these state regulations and mistrials, see Hannaford-Agor et al. (2002) and Luppi and Parisi (2013).

[9] See also Ben-Yashar and Nitzan (1997), proving that the optimal rule for fixed-size committee in dichotomous choice situations is the qualified weighted majority. Feddersen and Pesendorfer (1998) showed that, when jurors behave strategically, the probability of convicting the innocent in large juries is higher under the unanimity rule than under qualified majority rules. When there is uncertainty about jurors' preferences, in the presence of strategic jurors with private information, the unanimity rule may still be preferable to protect innocent defendants against wrongful convictions (Luppi and Parisi, 2013).

be more likely to reach an accurate verdict, but may fail to reach any decision at all. Hence, a tradeoff emerges between accuracy and decisiveness (Luppi and Parisi, 2013). Notwithstanding the widespread adoption of smaller juries in state criminal courts, statistics indicate that overall mistrial rates have not declined (Kalven and Zeisel, 1966; Hannaford-Agor et al., 2002). A few empirical studies have attempted to evaluate how jury size affects trial results. Most of them concluded that there is no detectable difference between six-member and 12-member juries with respect to mistrial rates (e.g., Hannaford-Agor et al., 2002; Eisenberg et al., 2005). By contrast, experimental studies and statistical models on jury size found that jury size does affect trial outcomes and jurors' behavior (e.g., Mukhopadhaya, 2003; Helland and Raviv, 2008; Guarnaschelli et al., 2000). For example, Guarnaschelli et al. (2000) revealed that larger juries may convict fewer innocent defendants than smaller juries under unanimity.

Our key original contribution to the literature is the specification of a different objective function that should guide the design of juries. While previous studies focused on either jury size or voting requirement, in this paper, we reveal the crucial interdependence of these two variables, and we analyze their optimal combination in minimizing the probabilities of wrongful convictions and hung juries.

3 Criminal Trial Model

In this section, we construct a simple model of the criminal trial process to analyze how varying jury size and voting requirement affects different expected trial outcomes.[10]

We follow the classical jury model (e.g., Miceli, 1990; Feddersen and Pesendorfer, 1998; Coughlan, 2000; Duggan and Martinelli, 2001; Persico, 2004; Neilson and Winter, 2000, 2005). There are two states of the world, I and G (Innocent and Guilty). Let $P(G)$ denote the prior probability that the defendant is guilty and $1-P(G)$ the prior probability that the defendant is innocent. Let s be the strength of evidence found against the defendant, whereby stronger evidence is associated with a higher probability of guilt. Let $f(s|G)$ and $f(s|I)$ be the probability density functions of the strength of evidence given that the defendant is guilty or innocent, respectively, and let $F(s|G)$ and $F(s|I)$ be the corresponding cumulative functions. The two density functions are represented in Figure 1.

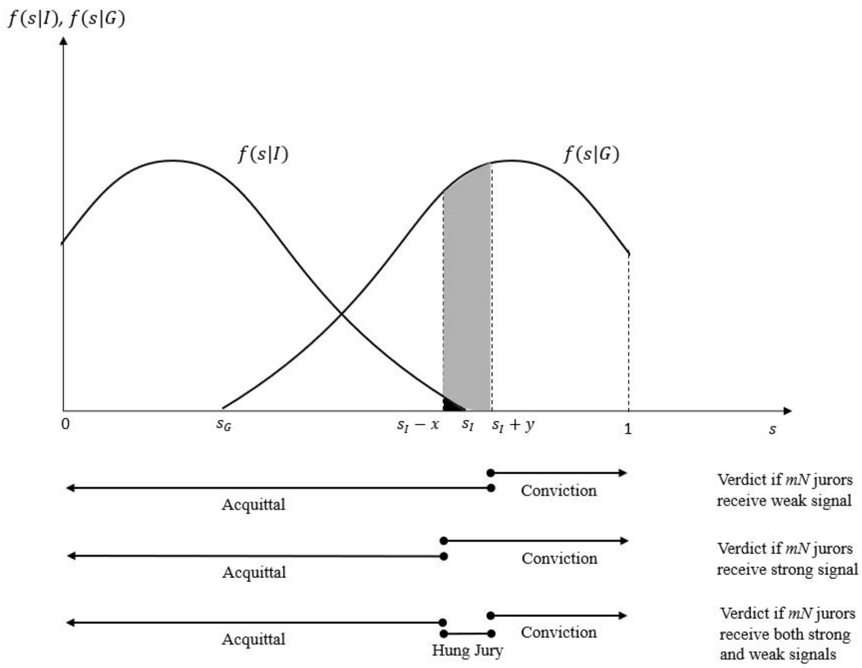
The traditional standard of proof in criminal trials in the United States is *proof beyond a reasonable doubt*, where each juror must *individually* believe in the guilt of the accused beyond any reasonable doubt.[11] As in Neilson and Winter (2000, 2005), to model the reasonable-doubt standard we assume that some evidence is inconsistent with an innocent defendant. Specifically, an innocent defendant can generate evidence in the interval $[0, s_I]$, whereas a guilty defendant can generate evidence in the interval $[s_G, 1]$, with $0 \leq s_G < s_I < 1$. If a juror observes $s \geq s_I$, that juror can state that the defendant is guilty beyond a reasonable doubt. The opposite holds when $s < s_I$. Put another way, s_I represents the reasonable-doubt standard threshold.[12]

[10]Our model relies upon Neilson and Winter's (2005) theoretical setup, with the main difference that we vary not only voting requirements, but also jury size. For a similar formulation of the court's problem, see also Rubinfeld and Sappington (1987).

[11]The beyond a reasonable-doubt standard has been used in criminal trials since at least the 1700s. It was adopted by most jurisdictions even before the case *In re Winship* (397 U.S. 358, 1970) and recognized it as a constitutional requirement.

[12]The reasonable-doubt standard threshold follows from Judge Blackstone's dictum, that it is "better that ten

Figure 1: The distribution of evidence (Neilson and Winter, 2000, 2005)



Analytically, this is equivalent to assuming that the probability density function for a guilty defendant first-order stochastically dominates the probability distribution function of an innocent defendant. Thus, under first-order stochastic dominance, it is more likely to find incriminating evidence for a guilty defendant than an innocent defendant.[13] Graphically, the first-order stochastic dominance is represented by the fact that the $f(s|G)$ distribution is shifted further to the right than the $f(s|I)$ distribution.[14]

As in Neilson and Winter (2005), we introduce juror heterogeneity by assuming that they do not directly observe the true evidence s, but they rather observe signals of varying strength related to the evidence.[15] Juror heterogeneity is a necessary assumption: if all individual jurors

guilty persons escape than that one innocent suffer" (Blackstone, 1769). Blackstone's ratio of 10 to 1 – or any variation of such ratio in state case law (Rizzolli and Saraceno, 2013; Pi et al., 2020) – follows from the fact that a wrongful conviction in criminal adjudication (convicting the innocent) is perceived to be worse than a wrongful acquittal (acquitting the guilty). For a discussion on the standard of proof in criminal law, see, e.g., Garoupa (2017) and Wickelgren (2017). Variations in the standard of proof may impact the optimal combination of jury size and voting requirement, and vice-versa. For an analysis on the optimal standard of proof in conjunction with alternative combinations of jury size and voting requirements, see Guerra and Parisi (2019).

[13]The assumption of first-order stochastic dominance has also been used by Rubinfeld and Sappington (1987); Miceli (1990); Miceli (2009, p.125); and Feess and Wohlschlegel (2009).

[14]Formally, for any evidence level s, $f(s|G) \geq f(s|I)$. Equivalently, in terms of cumulative distribution function, $F(s|G) \leq F(s|I)$. Note, in the following analysis, as jury design changes the shapes of the functions, $f(s|I)$ and $f(s|G)$ remain unchanged. For a similar setting, see Miceli (1990), Neilson and Winter (2005), and Rizzolli and Saraceno (2013).

[15]This is equivalent to assuming that jurors observe s with bias, as in Neilson and Winter (2000), or that jurors hold different beliefs about the true strength of evidence, as in Feddersen and Pesendorfer (1998) and Guarnaschelli et al. (2000). For other forms of juror heterogeneity, see, e.g., Arce et al. (1996); Alpern and Chen (2017).

were perfectly able to observe the true strength of evidence, juries would always reach unanimous verdicts. However, this is not the case in real-world criminal trials, as the actual rates of hung juries and judicial errors show (e.g., Hannaford-Agor et al., 2002).

Each juror assesses evidence differently and, as a result, can express different opinions when deliberating for a verdict. Another interpretation of the weak/strong signal is that jurors are heterogeneously informed. This may be driven by differing levels of juror sensitivity to the arguments presented by the prosecutor or defense counsel, or it may be driven by other factors that affect the persuasion of relevant facts or evidence presented at trial.

Specifically, each juror has a probability $\pi \in (0,1)$ of receiving a strong signal of incriminating evidence, $s_S = s + x$, with $x \geq 0$, and a probability $1-\pi$ of receiving a weak signal of incriminating evidence, $s_W = s - y < s_S$, with $y \geq 0$.[16] A juror who receives the strong signal votes to convict if $s_S \geq s_I$, that is, if $s \geq s_I - x$, as represented in Figure 1. A juror who receives the weak signal votes to convict if $s_W \geq s_I$, that is, if $s \geq s_I + y$, as represented in Figure 1. In a nutshell, a juror receiving the strong signal is more likely to believe that the defendant is guilty beyond any reasonable doubt than is a juror receiving the weak signal.

Let $N \in [1, 2, \ldots, n]$ denote the size of a jury, and $m \in [0,1]$ denote the majority to reach a verdict. For the majority rule case, mN is the smallest integer greater than $N/2$; for the unanimity case, $m = 1$.[17]

Let X be a random variable which follows the binomial distribution $X \sim B(N, \pi)$. The probability that mN jurors receive the strong signal (or, equivalently, vote to convict) is $P_C = Pr(X = mN) = \binom{N}{mN}\pi^{mN}(1-\pi)^{N-mN}$. Similarly, let Y be a random variable which follows the binomial distribution $Y \sim B(N, 1-\pi)$. The probability that mN jurors receive the weak signal (or, equivalently, vote to acquit) is $P_A = Pr(Y = mN) = \binom{N}{mN}(1-\pi)^{mN}\pi^{N-mN}$. The probability that a jury is neither prone to convict nor prone to acquit is $P_B = 1 - P_A - P_C$.

We can now derive the probabilities of a wrongful conviction, a wrongful acquittal, and a hung jury in a single trial.[18]

A wrongful conviction occurs when: (a) the defendant is innocent, which occurs with probability $1 - P(G)$; (b) the jury is likely to convict (or, equivalently, mN jurors receive the strong signal), which occurs with probability P_C; and (c) the evidence is sufficiently strong to meet the reasonable-doubt standard, that is $s \geq s_I - x$. Putting this all together, the probability of a wrongful conviction P_{WC} is given as:

$$P_{WC} = [1 - P(G)][1 - F(s_I - x|I)]P_C \tag{3.1}$$

A wrongful acquittal occurs when: (a) the defendant is guilty, which occurs with probability

[16] We follow Neilson and Winter (2000, 2005) in assuming that the probability distribution f of s given the state of the world G or I does not depend on the probability that a juror receives a strong or weak signal (π). Exploring different systems of signals could represent an interesting extension to our jury model. We thank an anonymous referee for this suggestion.

[17] For the purpose of our analysis, we assume that a jury reaches a decision by taking a simultaneous vote, that is, jurors ignore any group strategy aspects and decide independently from other jurors. This means that jurors do not vote against their signal: if a juror receives a guilty (innocent) signal, he votes to convict (acquit). This assumption – which is the behavior assumed by Condorcet – allows us to isolate the role of our two institutional variables from the possible effects of signaling and informational cascades (e.g., Luppi and Parisi, 2013), and the possibility of strategic voting of jurors (e.g., Ladha, 1992; Feddersen and Pesendorfer, 1998; Kaniovski and Zaigraev, 2011).

[18] For the purpose of the present analysis, we focus on the outcome of a single trial. Our basic framework can be extended to consider appeals and retrials. See Neilson and Winter (2005).

$P(G)$, but (b) the evidence is not strong enough to convict him. Formally, the probability of a wrongful acquittal P_{WA} is given as:

$$P_{WA} = P(G)[P_A F(s_I + y|G) + (1 - P_A) F(s_I - x|G)] \tag{3.2}$$

The first term within the squared brackets is the probability that at least mN jurors receive the weak signal (P_A), and the evidence is not sufficiently strong to convict ($s < s_I + y$). The second term is the probability that at least mN jurors receive the strong signal (P_C) or receive both the strong and weak signals ($1 - P_C - P_A$), but the evidence is not sufficiently strong to convict ($s < s_I - x$).

The probability of a wrongful verdict is given by $P_W = P_{WC} + P_{WA}$.

A hung jury occurs (a) if the jury is neither prone to convict nor prone to acquit, which happens with probability $P_B = 1 - P_A - P_C$, and (b) if the true strength of evidence is sufficiently close to the reasonable-doubt standard, i.e., it ranges between $s_I - x$ and $s_I + y$ (the hung jury range, as shown in Figure 1). In this range, jurors who receive the strong signal vote to convict, and those who receive the weak signal vote to acquit, resulting in a mistrial. Formally, the probability of a hung jury P_H is given as:

$$P_H = [1 - P(G)][1 - F(s_I - x|I)]P_B + P(G)[F(s_I + y|G) - F(s_I - x|G)]P_B \tag{3.3}$$

where the first term is the probability of a mistrial when the defendant is innocent, and the second term is the probability of a mistrial when the defendant is guilty.

From the equations above, it is straightforward to derive the probability of an accurate verdict, that is $P_V = 1 - P_W - P_H$.

The social loss function – which depends on the social costs of a wrongful conviction, of a wrongful acquittal, and of a mistrial – can be expressed as following:

$$\min_{N,m} L(N,m) = P_{WC}C_{WC} + P_{WA}C_{WA} + P_H C_H \tag{3.4}$$

where C_H, C_{WA}, and C_{WC} are the monetary social costs for a hung jury, wrongful acquittal, and wrongful conviction, respectively.[19]

The objective of the social planner is to minimize the social loss, as expressed in Equation (3.4), by optimally choosing jury size and voting requirement. In the next section, we analyze how different combinations of the two aforementioned institutional variables affect accuracy and the social cost of criminal adjudication through changes in P_{WC}, P_{WA}, and P_H.

[19] As in Neilson and Winter (2000), the administrative costs of increasing N are omitted. The social function in Equation (3.4) is similar to the social loss function considered by Miceli (1990). The main differences are that Miceli (1990) did not analyze jury size and voting requirement as factors influencing accuracy of decisions, and failed to consider the costs associated with mistrials. Our social function is also comparable to the social loss function considered by Neilson and Winter (2000). The main differences are that Neilson and Winter (2000) did not analyze how different combinations of jury size and voting requirement affect the accuracy of adjudication and the probability of mistrials.

4 Optimal Jury Size and Voting Requirement

In this section, we analyze how different combinations of jury size and voting requirement affects trial outcomes.

Let us start by discussing the benchmark case of varying jury size under unanimous verdicts. By restating the Condorcet's jury theorem under unanimity, we obtain the following lemma:

Lemma 4.1 (Jury-Size Effect)**.** *The probability of a wrong verdict decreases in jury size at a decreasing rate. However, given the greater incidence of mistrials, the probability of reaching an accurate unanimous verdict decreases in jury size at a decreasing rate.*

Proof. See Appendix A. □

Lemma 4.1 unveils an interesting tradeoff. Larger juries are less likely to be wrong but are also less likely to reach an accurate verdict because of the greater difficulty in deliberating unanimously.

Next, let us discuss the implications of varying the voting requirement under a given jury size.

Lemma 4.2 (Voting Requirement Effect)**.** *For any given jury size, if $\pi = .5$ the probability of a wrong verdict decreases with the voting requirement. However, given the greater incidence of mistrials, the probability of reaching an accurate verdict decreases with the voting requirement. Each probability decreases at an increasing rate for $m \in [.5, \bar{m})$ if the voting requirement tends to the majority rule and at a decreasing rate for $m \in (\bar{m}, 1]$, where $\bar{m} \in (.5, 1)$.*

Proof. See Appendix A. □

Similar to what we observed in Lemma 4.1, we can see that changes in the voting requirement have a double-edged effect. Relaxing a jury's voting requirement (i.e., allowing non-unanimous verdicts) facilitates the reaching of a verdict, but at the same time, it increases the probability of adjudication errors. As the majority requirement is reduced, more verdicts will be reached, but wrongful convictions and wrongful acquittals will also increase.

If each juror has the same the probability of receiving a strong signal or weak signal (i.e., $\pi = .5$), Lemma 4.2 reveals the desirability of relaxing the unanimity requirement over relaxing jury size to increase the accuracy of verdicts. Analytically, this result is explained by the fact that P_V is concave with respect to N and convex with respect to m (when m tends to the majority rule). Relaxing the unanimity requirement (under a fixed jury size) generates a "fast" marginal increase in the probability of accurate verdicts, whereas restricting jury size (under unanimity) generates a "slow" marginal increase in the probability of accurate verdicts.

These results can be summarized in the following proposition:

Proposition 4.3. *Given Lemmas 4.1 and 4.2, to increase accuracy of verdicts, relaxing the unanimity requirement down to a majority rule (for any given jury size) is a more effective alternative to restricting jury size (under unanimity) if $\pi = .5$.*

Proof. See Appendix A. □

Under the constraints on jury size and voting requirement set out by the U.S. Supreme Court in *Burch v. Louisiana*, state courts are not allowed to modify jury size and voting requirements simultaneously.

Relaxing the unanimity requirement in favor of a less demanding majority rule yields better results in terms of accuracy of verdicts than a reduction in jury size. Relaxing unanimity increases accuracy at a faster rate than the corresponding change obtainable with a reduction in jury size.

Consistent with *Burch v. Louisiana*, Proposition 4.3 implies that jury size and voting requirements inversely depend on one another. The accuracy of verdicts is maximized when requiring unanimous verdicts for small juries or allowing non-unanimous verdicts with large juries. These findings provides a rationale for the constraints introduced by the U.S. Supreme Court in *Burch v. Louisiana*: a combined use of small juries and non-unanimous verdicts would not be desirable in criminal adjudication.

5 Conclusion

Let us now step back to review the previously stated results from a bird's-eye perspective. Our findings help evaluate the effect of the changes to jury structure brought about by the U.S. Supreme Court and state legislation. The results on the capacity of a jury to reach an accurate verdict, taken in isolation, provides an economic rationale for the constraints introduced by the *Burch v. Louisiana* decision. Large non-unanimous juries or small unanimous juries are alternative ways to maximize the accuracy of verdicts while preserving the functionality of juries. In the choice between these alternatives, the vast majority of jurisdictions retained the unanimity rule, and there is near-universal acceptance to require it for capital murder cases given the severity of the consequences resulting from wrongful convictions. In these cases the probability of convicting an innocent should be kept to a minimum, avoiding as much error as possible. Notwithstanding the limited adoption of non-unanimous juries in U.S. state courts, our results lend support to the elimination of the unanimity requirement in the presence of large juries. As we move away from capital murder cases, combining a qualified majority rule with larger juries would seem desirable, inasmuch as the undesirability gap between wrongful convictions and wrongful acquittals narrows. Optimal jury size can even fall below the lower-limit of six members set by the U.S. Supreme Court in the *Burch v. Louisiana* case – inasmuch as juries are required to decide unanimously. Our result largely aligns with the conventional wisdom in existing literature. When unanimity is required, the use of smaller juries could reduce the probability of a single juror causing a deadlock and may be desirable to empower the jury with the capacity to reach a verdict.

Future research in this field should extend our analysis to investigate how optimal jury design would change when considering retrials (Neilson and Winter, 2005), correlated votes (Rubinfeld and Sappington, 1987), endogenous social values of adjudication errors (Miceli, 1990), behavioral cascades (Luppi and Parisi, 2013), and strategic voting by jurors (Feddersen and Pesendorfer, 1998). For all these extensions, our model could usefully serve as a building block for the understanding of more complex jury decision-making scenarios. Finally, as shown in Pi et al. (2020), the choice of different Blackstonian ratios by U.S. jurisdictions indirectly implies the jurisdiction's commitment to different "beyond a reasonable doubt" thresholds. Our next research objective is to explore how the jurisdictions' choices of different standards of proof influence their choices regarding jury size and voting requirements (Guerra and Parisi, 2019).

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Appendix A Proofs

Proof of Lemma 4.1. Given $m=1$, $\frac{\partial P_{WC}}{\partial N}=[1-P(G)][1-F(s_I-x|I)]\pi^N\ln\pi$, and $\frac{\partial P_{WA}}{\partial N}=[F(s_I+y|G)-F(s_I-x|G)](1-\pi)^N\ln(1-\pi)$, which are both negative since $\pi\in(0,1)$. Hence, $\frac{\partial P_W}{\partial N}<0$. The second-order derivatives are equal to $[1-P(G)][1-F(s_I-x|I)]\pi^N\ln^2\pi$, and $[F(s_I+y|G)-F(s_I-x|G)](1-\pi)^N\ln^2(1-\pi)$, respectively, which are both positive. Hence, $\frac{\partial^2 P_W}{\partial N^2}>0$.

Next, we prove that $\frac{\partial P_H}{\partial N}>0$. Given $m=1$, $\frac{\partial P_H}{\partial N}=\frac{\partial P_B}{\partial N}\{[1-P(G)][1-F(s_I-x|I)]+P(G)[F(s_I+y|G)-F(s_I-x|G)]\}$. Since $\frac{\partial P_B}{\partial N}=-[(1-\pi)^N ln(1-\pi)+\pi^N\ln\pi]$, which is positive since $\pi\in(0,1)$, it follows that $\frac{\partial P_H}{\partial N}>0$.

Finally, we prove that $\frac{\partial P_V}{\partial N}<0$, $\frac{\partial^2 P_V}{\partial N^2}>0$. Given $m=1$, $\frac{\partial P_V}{\partial N}=[1-P(G)][1-F(s_I-x|I)](1-\pi)^N\ln(1-\pi)+P(G)[F(s_I+y|G)-F(s_I-x|G)]\pi^N\ln\pi$, which is negative since $\pi\in(0,1)$. The second-order derivative is equal to $[1-P(G)][1-F(s_I-x|I)](1-\pi)^N\ln^2(1-\pi)+P(G)[F(s_I+y|G)-F(s_I-x|G)]\pi^N\ln^2\pi$, which is positive. □

Proof of Lemma 4.2. For a given N, $\frac{\partial P_{WC}}{\partial m}=[1-P(G)][1-F(s_I-x|I)]\frac{\partial P_C}{\partial m}$, where $\frac{\partial P_C}{\partial m}$ is equal to

$$\pi^{mN}(1-\pi)^{N-mN}\left\{\frac{N!}{mN!(N-mN)!}N\ln\left(\frac{\pi}{1-\pi}\right)+\frac{\partial}{\partial m}\left[\frac{N!}{mN!(N-mN)!}\right]\right\}$$

,
which is negative if $\pi\le .5$. For a given N, $\frac{\partial P_{WA}}{\partial m}=P(G)[F(s_I+y|G)-F(s_I-x|G)]\frac{\partial P_A}{\partial m}$, where $\frac{\partial P_A}{\partial m}$ is equal to

$$(1-\pi)^{mN}\pi^{N-mN}\left\{\frac{N!}{mN!(N-mN)!}N\ln\left(\frac{1-\pi}{\pi}\right)+\frac{\partial}{\partial m}\left[\frac{N!}{mN!(N-mN)!}\right]\right\}$$

,
which is negative if $\pi\ge .5$. Hence, $\frac{\partial P_W}{\partial m}<0$ if $\pi=.5$.

Next, we prove that $\frac{\partial P_H}{\partial m}>0$. For a given N, since $\frac{\partial P_B}{\partial m}=-\left[\frac{\partial P_C}{\partial m}+\frac{\partial P_A}{\partial m}\right]$, whereby $\frac{\partial P_C}{\partial m}<0$ if $\pi\le .5$ and $\frac{\partial P_A}{\partial m}<0$ if $\pi\ge .5$, it follows that $\frac{\partial P_H}{\partial m}>0$ if $\pi=.5$.

Finally, we prove that $\frac{\partial P_V}{\partial m}<0$. For a given N, $\frac{\partial P_V}{\partial m}=[1-P(G)][1-F(s_I-x|I)]\frac{\partial P_A}{\partial m}+P(G)[F(s_I+y|G)-F(s_I-x|G)]\frac{\partial P_C}{\partial m}$, which is negative if $\pi=.5$.

Finally, let us analyze second-order partial derivatives with respect to m. If $\pi=.5$, the second-order partial derivative of P_{WC} with respect to m is

$$[1-P(G)][1-F(s_I-x|I)](.5)^N\frac{\partial^2}{\partial m^2}\left[\frac{N!}{mN!(N-mN)!}\right]$$

To compute the second-order derivative of the factorial function, let us consider $N=6$ and marginal increases of .1 in m from $m=.5$ to $m=1$. The computations show that the factorial function is decreasing in m at an increasing rate for $m\in[.5,.8)$ and at a decreasing rate for $m\in(.8,1]$. A similar trend occurs if $N=12$, whereby the factorial function is decreasing in m at an increasing rate for $m\in[.5,.7)$ and at a decreasing rate for $m\in(.7,1]$. Let $\bar{m}$ denote the inflection point. Thus, $\frac{\partial^2 P_{WC}}{\partial m^2} < 0$ for $m \in [.5, \bar{m})$, and $\frac{\partial^2 P_{WC}}{\partial m^2} > 0$ for $m \in (\bar{m}), 1]$. The same holds for $\frac{\partial^2 P_{WA}}{\partial m^2}$, which is negative. Hence, $\frac{\partial^2 P_W}{\partial m^2} < 0$.

□

Proof of Proposition 4.3. This follows from Proofs of Lemmas 4.1 and 4.2, since $\frac{\partial^2 P_V}{\partial N^2} > 0$, whereas $\frac{\partial^2 P_V}{\partial m^2} < 0$ if $\pi = .5$ and $m \in [0.5, \bar{m})$. □