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Comparison of soil water content estimation equations using ground penetrating radar

This is the final peer-reviewed author's accepted manuscript (postprint) of the following publication:

Published Version:

Anbazhagan P., Bittelli M., Pallepati R.R., Mahajan P. (2020). Comparison of soil water content estimation equations using ground penetrating radar. JOURNAL OF HYDROLOGY, 588, 1-9 [10.1016/j.jhydrol.2020.125039].

Availability:

This version is available at: <https://hdl.handle.net/11585/765737> since: 2020-07-11

Published:

DOI: <http://doi.org/10.1016/j.jhydrol.2020.125039>

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(Article begins on next page)

This is the final manuscript draft of:

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Journal of Hydrology Volume 588, September 2020, 125039

The final published version is available online at:

<https://doi.org/10.1016/j.jhydrol.2020.125039>

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Abstract

Soil water content (SWC) has an important impact on many fundamental biophysical processes. The quantification of SWC is necessary for different applications, ranging from large-scale calibration of global-scale climate models to field and catchment scale monitoring in hydrology and agriculture. Many techniques are available today for measuring SWC, ranging from point scale soil water content sensors to global scale, active and passive, microwave satellites. Geophysical methods are important methods used for several decades to measure SWC at different scales. Among these methods, Ground Penetrating Radar has been shown to be one of the most reliable and promising ones. Soil water content measurement using Ground Penetrating Radar requires the applications of parametric equations that will convert the measured dielectric permittivity to water content. While several studies have been performed to test equations for soil water content sensors such as Time Domain Reflectometry, a few studies have been performed to test different formulae for application to Ground Penetrating Radar. In this study, we compare available formulae for converting dielectric permittivity obtained from detailed laboratory scale measurement of reflected waves using Ground Penetrating Radar. Four soils covering a wide range of textures were used and the measured soil water contents were compared with values obtained from gravimetric measurements. Results showed that the dielectric mixing model of Roth (1990) provided the best fit both for individual soil textural classes, except for sandy soils. However, for all data combined the dielectric mixing model performed much better with significant difference in coefficient and determination and Root Mean Square Error (RMSE = $0.028 \text{ m}^3 \text{ m}^{-3}$ and $R^2 = 0.888$). Empirical equations developed from calibration of TDR performed poorly when applied to estimation of soil water content obtained from GPR. Differences in sample volume, frequency of operation and data analysis between GPR and TDR, suggest to use more flexible and robust electromagnetic mixing formulae, allowing for incorporating the dielectric properties of constituents materials and geometrical features of the media. Sensitivity analysis was then performed to provide detailed information for the most accurate application of the selected dielectric model.

Keywords: Ground Penetrating Radar, Soil Water Content, Dielectric Mixing Models, Empirical Equations, Sensitivity Analysis

Highlights

- Accurate measurement of dielectric permittivity with reflected waves GPR was obtained
- Evaluation of Water Content Estimation Equations using wide range of textures
- Identification of best fit equation and critical analysis of current equations
- Sensitivity Analysis of the best fit model

Comparison of Soil Water Content Estimation Equations using Ground Penetrating Radar

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1 1. Introduction

2 Soil water content (SWC) is a fundamental property affecting a large variety of pro-
3 cesses relevant to hydrology, agricultural sciences, engineering and soil sciences. Over the last
4 decades many techniques have been developed to measure SWC at different temporal and spa-
5 tial scales. Bittelli (2011) provided a review describing the most common methods available
6 for measuring SWC. Among geophysical techniques, Ground Penetrating Radar (GPR) is a
7 powerful and promising one. GPR has the advantage of covering larger areas with respect to
8 point-based measurements typical of soil moisture sensors such as Time Domain Reflectom-
9 etry (TDR), filling the gap between point scale and large scale satellite-based measurement.
10 Soil water content can be obtained by performing different types of analysis and methods us-
11 ing GPR. Huisman et al. (2003) and Klotzsche et al. (2018) presented reviews about advances
12 in applications of GPR, for measurement of SWC. In their reviews they discuss available
13 methodologies, including continuous multi-offset measurements, off-ground measurements,
14 three-dimensional measurements, vertical radar profiling, modelling and inverse methods.

15 When the value of dielectric permittivity for the material under test is obtained from
16 GPR, relationships must be employed to convert permittivity to volumetric SWC. Commonly,
17 the relationships used for GPR are the ones derived from the calibration of TDR. Since both
18 TDR and GPR are volumetric measurements, during the calibration measurement of bulk
19 density is necessary to convert the mass-based gravimetric measurement to volume-based
20 soil water content. Many equations were derived over the years.

21 One of the most widely used is the one by Topp et al. (1980), which is a third order poly-
22 nomial. The authors used TDR to measure the dielectric permittivity for a range of granular

23 samples placed in a coaxial transmission line and they estimated an error of $0.013 \text{ m}^3 \text{ m}^{-3}$.
24 Ledieu et al. (1986) proposed an equation where the calibration of TDR was performed against
25 gamma-ray attenuation, an accurate technique used for measuring water content. The cali-
26 bration equation accounted for the change in bulk density of the specimen. Later, Roth et al.
27 (1992) proposed calibration functions for mineral, organic and magnetic soils. Malicki et al.
28 (1996) also presented a formulation accounting for bulk density. These are empirical equations.

29 Roth et al. (1990) proposed a dielectric mixing model based on theoretical considerations.
30 This model includes: 1) the effect of bulk density (by accounting for soil porosity), 2) a
31 geometrical parameter describing the orientation of soil particles with respect to the electric
32 field and 3) the values of dielectric permittivity for the solid, liquid and gas phase. While the
33 gas phase permittivity is constant, the solid phase permittivity changes with soil minerals,
34 while the liquid phase permittivity is temperature dependent (assuming constant or narrow-
35 band frequency).

36 The dielectric mixing model of Roth et al. (1990) belongs to the family of the electro-
37 magnetic mixing models, which are applied to a large variety of media including snow, ice,
38 emulsions and biological materials. One of the most exhaustive description and review about
39 the theory of electromagnetic mixing formula was presented by Sihvola (1999). As pointed out
40 by Sihvola (1999), heterogeneous mixtures (such as a soil) have properties that depend upon
41 its constituents but differ from the original components. Although the dielectric properties of
42 a mixture are an average of the components permittivities, often the whole character of the
43 dielectric is changed by the mixing process. An important aspect of the effect of the mixing
44 process is the geometrical orientation of the inclusions (particles) with respect to the electric

45 field and their depolarisation factors, which depend on the shape of the inclusions.

46 The relationships currently used for GPR applications were derived from experiments
47 performed with TDR and applied to various studies. Weihermuller et al. (2007) used the Topp
48 et al. (1980) formula to derive water content from GPR. Gerhards et al. (2008) and Steelman
49 et al. (2012) derived SWC from multiple transmitter and receiver GPR, employing the Roth
50 et al. (1990) dielectric mixing model. However, there are many differences between TDR and
51 GPR, in terms of frequency of operation, sampling volume, data analysis and interpretation.
52 Therefore there is the need to test the current equations applied to GPR. Only a few studies
53 have been performed.

54 Lambot et al. (2004) estimated SWC directly from GPR, using a soil-specific empirical
55 model (third order polynomial) similar to Topp's equation. However, their experiment was
56 limited to a sand box with only a sandy sample as testing material. Steelman and Endres
57 (2011) presented a comparison among petrophysical relationships for application to GPR.
58 They concluded that the general empirical equation by Roth et al. (1992) provided the best
59 fit for the sandy loam soil. When the entire data set was analyzed, they found that the Topp
60 et al. (1980) and Roth et al. (1992) relationships provided the most accurate estimates. When
61 the dielectric mixing and effective media models were tested, the Roth et al. (1990) equation
62 provided the best fit, but with small improvement with respect to the empirical equations.

63 However, Steelman and Endres (2011) used permittivity data, to test the equations,
64 obtained from GPR using the Common Midpoint (CMP) sounding method. With this method,
65 stacking velocity fields are extracted from multi-offset radar soundings at a fixed central
66 location. Yet, CMP-derived velocity estimates are generally characterized by low resolution

67 and high uncertainty (Tillard and Dubois, 1995; Lambot et al., 2004). The success of the
68 measurements depends on the presence of clearly reflecting layers in the soil. For this reason
69 the calibration equations derived from dielectric permittivity obtained from CMP may be
70 affected by low resolution and high uncertainty.

71 The travel time of the reflected GPR wave depends on the depth of the reflector and the
72 mean dielectric permittivity above the reflector. In general, in field applications the reflectors
73 depth are unknown, requiring the use of techniques to derive the dielectric permittivity, de-
74 scribed in the review by Klotzsche et al. (2018). However, for controlled studies on calibration
75 equations, it is more accurate to perform GPR measurements with a strong reflector installed
76 at a known depth to derive an accurate travel time, as performed by Lambot et al. (2004).
77 The authors performed detailed radar measurements carried out in controlled laboratory con-
78 ditions on a tank filled with a disturbed sandy soil. The purpose of their paper was to test
79 forward GPR modelling, therefore to test the modelling analysis they selected an accurate and
80 robust approach to obtain travel time in controlled laboratory conditions.

81 In this study, the performance of various published physical relationships used to obtain
82 soil water content estimates from GPR, were evaluated. Dielectric permittivities of the ma-
83 terials under test were obtained by using a tank filled with disturbed soil samples and with a
84 metal reflector installed at a known depth. Four different materials were tested ranging from
85 sand to kaolinite clay, to obtain a broad range of textures. Variations in water content and
86 density were independently measured for comparison. Sensitivity analysis of the best fit model
87 was then performed.

88 2. Theory

89 Ground Penetrating Radar reflections occur when there are significant changes in dielec-
90 tric permittivity. In natural conditions they can be sedimentation layers, groundwater tables,
91 rocks stratification. In man-made structures they can be archaeological remains, pipes used
92 for utilities, cavities, roads layering. Since SWC strongly affects the dielectric permittivity
93 of porous media, GPR is an effective technique to measure SWC. One of the most common
94 techniques for measuring SWC is based on derivation of dielectric permittivity from travel
95 time analysis.

96 The velocity v (m s^{-1}) of an electromagnetic wave, is affected by the dielectric permittivity
97 ϵ , and the magnetic permeability μ , as:

$$v = \frac{c}{\sqrt{\mu\epsilon}} \quad (1)$$

98 where c is the speed of light, 2.997×10^8 (m s^{-1}). From a mechanical standpoint, the velocity
99 v of an electromagnetic wave traveling through a space of length d (m), is given by:

$$v = \frac{2d}{t} \quad (2)$$

100 where t is time (s). For a reflected wave, the number 2 in front of the length is included
101 because the wave is reflected back to the receiving antenna. For most soils μ_r is equal to 1
102 (Roth et al., 1992), therefore Eqn. 1 can be written as:

$$v = \frac{c}{\sqrt{\epsilon}} \quad (3)$$

By equating the definitions of velocity:

$$\frac{c}{\sqrt{\epsilon}} = \frac{2d}{t} \quad (4)$$

103 and solving for ϵ :

$$\epsilon = \left(\frac{ct}{2d} \right)^2 \quad (5)$$

104 Equation 5 allows for obtaining the dielectric permittivity by measuring the travel time t , since
105 the position of the reflecting plane d and the speed of light c are known. When the material is
106 a composite mixture such a soil, we refer it as bulk dielectric permittivity (ϵ_b). Knowledge of
107 the distance between the antenna and the reflector d , allows for obtaining the travel time and
108 the dielectric permittivity, this method is usually called the *two – way* travel times analysis
109 (Pereira et al., 2005).

110 2.1. Soil Water Content relationships

111 2.1.1. Empirical Equations

112 The empirical relationship by Topp et al. (1980) is:

$$\theta = -5.3 \times 10^{-2} + 2.92 \times 10^{-2} \epsilon_b - 5.55 \times 10^{-4} \epsilon_b^2 + 4.3 \times 10^{-6} \epsilon_b^3 \quad (6)$$

113 where θ is the volumetric water content ($\text{m}^3 \text{ m}^{-3}$) and ϵ_b is soil bulk dielectric permittivity.
114 The authors fitted the third order polynomial to TDR data collected in a coaxial transmission
115 line for four soils.

116 Ledieu et al. (1986) developed an equation obtained from calibrating TDR against SWC
 117 data obtained from gamma-ray attenuation. Since dielectric permittivity is density dependent
 118 they also included the bulk density. They stated that their procedure and calibration equation
 119 had accuracy of less than 1 %. However the experiment was performed only on one sample of
 120 sand. The equation proposed is:

$$\theta = 0.1138\sqrt{\epsilon_b} - 0.1758 \quad (7)$$

121 Roth et al. (1992) proposed three different empirical equations for mineral, organic and mag-
 122 netic soils. The equation for mineral soil is also a third-order polynomial similar to Topp's
 123 equation, but with different coefficients and a prediction error of $0.015 \text{ m}^3 \text{ m}^{-3}$:

$$\theta = -7.28 \times 10^{-2} + 4.48 \times 10^{-2}\epsilon_b - 19.5 \times 10^{-4}\epsilon_b^2 + 36.1 \times 10^{-6}\epsilon_b^3 \quad (8)$$

124 2.1.2. Electromagnetic Mixing Formulas

125 Electromagnetic mixing formulae relate the value of the individual permittivities of the
 126 mixture components to their volumetric fractions. A widely used class of mixing models
 127 are called power-law models (see Sihvola (1999), page 166), where a certain power of the
 128 permittivity is averaged over volume weights:

$$\epsilon_b^\beta = f\epsilon_i^\beta + (1 - f)\epsilon_j^\beta \quad (9)$$

129 where ϵ_i and ϵ_j are the generic dielectric permittivities of a two phase systems. In the Birchak

130 et al. (1974) equation, the parameter β is equal to 1/2. Another known model is the Looyenga
 131 (1965) formula, where β is equal to 1/3. Later, Roth et al. (1990), extended the power-law
 132 model to compute the bulk dielectric permittivity as a weighted sum of the dielectric permit-
 133 tivity of each soil phase:

$$\epsilon_b = (\phi_s \epsilon_s^\alpha + \theta \epsilon_l^\alpha + \phi_g \epsilon_g^\alpha)^{1/\alpha} \quad (10)$$

134 where ϕ_s , θ and ϕ_g are the solid, liquid and gas phase volumetric fractions. The corresponding
 135 dielectric permittivities are ϵ_s , ϵ_l and ϵ_g , while α is the parameter describing the geometry of
 136 the medium with relation to the applied electric field. The volumetric solid fraction can be
 137 also written as $\phi_s = (1 - \phi_f)$, where ϕ_f is the porosity and the volumetric fraction of the gas
 138 phase as $\phi_g = (\phi_f - \theta)$. Using these relationships and substituting into Eqn.10, leads to:

$$\theta = \frac{\epsilon_b^\alpha - [(1 - \phi_f)\epsilon_s^\alpha + \phi_f \epsilon_g^\alpha]}{\epsilon_l^\alpha - \epsilon_g^\alpha} \quad (11)$$

139 The liquid phase dielectric permittivity is temperature dependent with:

$$\epsilon_l = 78.54 \times (1 - (4.579 \times 10^{-3} \times \Delta T)) \quad (12)$$

140 where T is temperature in Celsius and $\Delta T = T - 25$. To use this equations, knowledge of
 141 porosity (which can be obtained from measurement of bulk density) and dielectric permittivity
 142 of the solid phase is needed. Porosity is obtained from measured bulk density by:

$$\phi_f = 1 - \frac{\rho_b}{\rho_s} \quad (13)$$

143 where the density of the solid phase (ρ_s) was assumed to be equal to 2.65 g cm^{-3} .

144 The sum of the different volume-weighted permittivities can be extended to include the
145 contribution of organic matter in organic soils, or ice in partially frozen soils (Bittelli et al.,
146 2004). Table 1 provides dielectric permittivity values for different materials (Daniels, 2004).
147 In this study we used the following values: $\epsilon_s = 4$, ϵ_l was computed with Eqn. 12 at $25 \text{ }^\circ\text{C}$, ϵ_g
148 $= 1.005$ and $\alpha = 0.5$.

Table 1: Dielectric permittivity of materials at 100 MHz. From Daniels (2004)

Material	Dielectric permittivity
Vacuum	1
Air	1.0005
Fresh water	$78.54 \times (1 - 4.579 \times 10^{-3}(T - 25))$
Fresh water ice	3.2
Quartz	4–6
Concrete dry	4–10
Sand Dry	2–6
Sandstone dry	2–5
Soil Dry Clay	4–10
Granite Dry	5
Limestone dry	7

149 The selection of these four models was also based on previous results obtained by Steelman
150 and Endres (2011) as discussed above.

151 The disadvantages of using empirical models are manyfolds: a) the models are not derived
152 from theoretical considerations regarding the interactions between the electric field and the
153 media therefore they are theoretically less robust, b) the parameters are obtained from fitting
154 the equations over a given data set, therefore if the model is applied to other materials they may
155 fail and c) the coefficients of empirical models are commonly fixed values. The opposite is true
156 for dielectric mixing models where: a) the model are based on theoretical considerations where

157 the parameters represent measurable physical properties, b) the parameters can be selected
158 based on measured properties of the media (for instance the mineralogical composition of the
159 solid phase) or from existing tables, and c) the parameters can be used as fitting parameters
160 for a specific study, allowing for flexibility in the equation form.

161 3. Material and Methods

162 Four different soils were used in this study, namely sand, sandy loam, loamy sand and
163 kaolinite clay. Samples were collected from the Tumkur district, Karnataka, India. The
164 soil samples were collected from the top 25 cm of soil. The experiments were conducted at
165 laboratory temperature of 25 °C. This value was used for correcting the dielectric permittivity
166 of the liquid phase in the dielectric mixing models (Eqn.12), which provided a value of $\epsilon_l =$
167 78.54. The tested soils were cleaned for presence of organic material like grass, leaves etc. and
168 sieved with a 2.5 mm size sieve. Figure 1 shows a schematic of the experimental setup, while
169 Figure 2 shows two photographs of the experimental setup.

170 FIGURE 1

171 FIGURE 2

172 The soil was placed into a plastic tank (with base 0.6 m \times 0.4 and 0.3 m height) for a total
173 volume of 0.072 m³, with a reflecting metal plate at the bottom. The distance for travel time
174 calculation between the antennas and the reflecting metal plate was $d \simeq 0.3$ m. According to
175 the manufacturer (Mala Inc.) the antennas are positioned at the bottom of the GPR, where
176 a plastic lower case of a few mm thickness separate the antennas from the soil. Therefore a
177 value of $d \simeq 0.3$ is the correct physical distance between the antennas and the metal reflector.

178 The distance between the transmitting and receiving antennas is 0.1 m. Materials underneath
179 the metal sheet have no influence on the measured backscattered signal (Lambot et al., 2004).

180 The soil was prepared by following the ASTM D1557 12 standard for laboratory com-
181 paction (ASTM, 2015). A fixed amount of water was added to a specific mass of soil, and
182 mixed to obtain uniform distribution of water. Specifically: a) the sand was packed into the
183 tank volume of 0.072 m^3 , at an average dry bulk density of 1593 kg m^{-3} , corresponding to
184 a dry mass of 114.7 kg; b) the sandy loam at an average bulk density of 1561 kg m^{-3} , cor-
185 responding to a dry mass of 112.4 kg; c) the loamy sand at an average bulk density of 1571
186 kg m^{-3} , corresponding to a dry mass of 113.1 kg and d) the kaolinite clay was packed at an
187 average bulk density of 1175 kg m^{-3} , corresponding to a dry mass of 84.6 kg.

188 The soil samples were then placed into the tank and packed to the densities described
189 above, in three incremental layers of 0.1 meters each, of equal mass. The layers in the figure
190 do not represent different soil types, but the layers used for packing. The packing was done by
191 layering to achieve a uniform density. GPR antenna was then placed on the top of the plastic
192 box and readings were taken in time-triggering mode.

193 Subsequently, the soil was removed from the tank and fixed amounts of water were added
194 to increase water content. The same packing procedure was then repeated, therefore everytime
195 the soil was prepared and repacked into the tank, for each SWC measurement. The mixing of
196 soil and water was done by hand, with scoops and shovels, into a separate larger open container.
197 The soil was then enclosed into the container to avoid evaporation and let equilibrate for 24
198 hrs, to allow for water redistribution and equilibration within the sample. Mixing was then
199 performed again. To test the effectiveness of the mixing, periodically gravimetric tests of the

200 mixtures during the mixing and equilibration processes were also performed (ASTM, 2015).

201 This procedure was followed, not only because it is a standard ASTM, but also because it
202 was not possible to increment water content within the tank by either percolation or capillary
203 rise. At the bottom of the tank a reflecting metal plate was positioned for GPR analysis,
204 therefore we could not control the lower boundary condition for percolation or capillary rise
205 with installation of either ceramic or porous plates. Moreover, percolation of water into a tank
206 often results in preferential flows of water along the walls and preferential pathways, resulting
207 in non-homogeneous distributions. For these reasons, the soil was repacked each time for
208 each individual SWC measurement. Sometime, after adding water to the target amounts, the
209 densities underwent some variations during packing.

210 However, to verify water content and bulk density values and to test SWC equations, after
211 the GPR measurement was performed, three soil samples were collected in metal rings from
212 the center of the tank (below the positions of the GPR antennas) and independent gravimetric
213 SWC and bulk densities were measured. Although special care was paid to pack the soil at
214 the same density, since the volume of the tank and the soil mass were relatively large, it
215 was not possible to repack the soil exactly at the same densities, therefore variations in bulk
216 densities were recorded during the measurement. These variations were included for each
217 measurements into the SWC models that allowed to include porosity, namely the empirical
218 model of Ledieu et al. (1986) and the dielectric mixing model of Roth et al. (1990).

219 *3.1. Ground Penetrating Radar measurements*

220 The GPR was a Mala Inc., with a 800 MHz shielded antennas. The setup was the
221 following: time window = 38.8 ns, depth = 0.3 m, sampling frequency = 8230.951172 MHz

222 and antenna separation = 0.1 m. The data were analysed using the software Prism 2 (Radar
223 System Inc.) and Reflex (Sandmeier, 2019). The acquisition was performed in time-based
224 trace triggering mode.

225 Figure 3 shows an example of radargrams showing the reflector depth. The reflection in
226 the upper part of the signal are the typical air and ground wave. The transmitting antenna
227 propagates waves giving rise to an air wave that travel directly from the transmitting to the
228 receiving antenna. Similarly, the propagating wave give rise to the ground wave. The upper
229 part of the radargram shows the air and ground wave. Figure 3 shows the strong reflection of
230 the metal reflector. The graph does not display any unit on the x-axis since the measurement
231 is performed in time-triggering mode. Detailed analysis on single trace analysis (A-scan) is
232 explained below.

233 FIGURE 3

234 Depth penetration is controlled by the dielectric permittivity and electrical conductivity of
235 the sample. In fine textured soils, in particular in clay soils, the signal can be highly attenuated.
236 Moreover, in fine textured samples relaxation processes (such as Maxwell-Wagner or double
237 layer polarization) may determine additional dissipation processes and further attenuation of
238 the signal (Schwing et al., 2013).

239 The procedure to identify the reflections was based on the calibration procedure presented
240 by Pereira et al. (2005). The authors pointed out that one of the main problems related to GPR
241 technology is that the technical information provided by the different companies is practically
242 inexistent. The lack of information for the different parameters for antenna emissions and
243 emitted signal is a serious difficulty for data interpretation. For instance, the authors showed

244 that the rate of drift of the signal was not exactly the same for the three antennas under test,
245 operating at 500, 800 and 1000 MHz. Indeed, the time base of GPR measurements is also
246 not exactly determined and it may shows a significant drift due to a temperature difference
247 between the instrument electronics and the ambient temperature. Accordingly, as suggested
248 by the authors we increased the warming time of the GPR to 30 minutes to equilibrate with
249 the laboratory temperature. Since the authors used the same GPR manufacturer used in this
250 study (Mala Inc.), we employ their procedure to identify the time zero parameter.

251 An exact definition of time zero in field conditions is very difficult if not impossible, since
252 it is not a constant value but depends on the investigated material and the antenna set up
253 configuration (Sandmeier, 2019). However, when the physical distance of the reflector and
254 the distance between the antenna are known, it is possible to determine the time zero for the
255 investigated material. An automatic and stable static correction (definition of time zero) may
256 be done either on the first negative, first zero crossing or first positive peak (Sandmeier, 2019).
257 Pereira et al. (2005) suggested to use the first positive peak (Fig. 4 in (Pereira et al., 2005))
258 for the 800 MHz antenna.

259 Figure 4 shows an example of a trace and identification of the reflection for computation
260 of travel time. The lower plate shows the complete trace acquired during the experiment and
261 the upper plate a zoom over the relevant section. The origin was fixed by starting off at the
262 greatest amplitude value from the first positive semiperiod peak. After obtaining the travel
263 time, the bulk dielectric permittivity was then computed as detailed above.

264 FIGURE 4.

265 *3.2. Error Analysis*

266 The accuracy of the volumetric soil water content estimates was estimated using the Root
267 Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (\theta_{meas} - \theta_{pred})^2}{N}} \quad (14)$$

268 where N is the total number of samples, θ_{meas} ($\text{m}^3 \text{m}^{-3}$) is the volumetric water content
269 obtained from gravimetric measurements and θ_{pred} ($\text{m}^3 \text{m}^{-3}$) is the volumetric water content
270 predicted by the different equations, and obtained from GPR measurement of bulk dielectric
271 permittivity. The coefficient of determination R^2 was also used to evaluate regression equation
272 in the scatter plot analysis.

273 **4. Results and Discussion**

274 The estimated volumetric water contents, θ , obtained from the different equations are
275 presented in Figure 5 for the four different textural classes and the RMSE results are presented
276 in Table 2. The dielectric mixing model of Roth et al. (1990) provided the best fit for the
277 tested soils, except for the sandy loam where the Topp's and Ledieu's equations provided the
278 best fit. However, when all the data were combined the dielectric mixing model of Roth's
279 provided the best fit, with RMSE of $0.028 \text{ m}^3 \text{m}^{-3}$.

280 As confirmed by the values of RMSE, it is also possible to visually see the best fitting of the
281 dielectric mixing model for the indicated textures. Considering the experimental difficulties
282 in achieving uniform packing of wetted soil into a large tank with large amount of soils, the
283 scatter of the experimental data is fairly small, confirming the accuracy of the experimental

Table 2: Root Mean Square Error (RMSE) ($\text{m}^3 \text{ m}^{-3}$) for the four different soil types and all data. DMM stands for dielectric mixing model.

Relationships	sand	sandy loam	loamy sand	kaolinite clay	all data
Topp et al. (1980)	0.024	0.035	0.022	0.033	0.051
Ledieu et al., (1986)	0.023	0.035	0.025	0.030	0.052
Roth et al. (1992)	0.049	0.054	0.015	0.012	0.051
Roth et al. (1990)-DMM	0.022	0.040	0.010	0.010	0.028

284 procedure.

285 FIGURE 5

286 Figure 6 shows a scatter plot between the measured volumetric water contents (θ_{meas})
 287 and the estimated corresponding values (θ_{est}). Regression equations and the coefficient of
 288 determination (R^2) are also listed in the graph for each equation. The coefficients of deter-
 289 mination showed that the best fitting was obtained with the dielectric mixing model of Roth
 290 et al. (1990), followed by the Roth et al. (1992), the Ledieu et al. (1986) and the Topp et al.
 291 (1980), with R^2 respectively of 0.888, 0.790, 0.795 and 0.731.

292 With respect to the 1:1 line in the scatter plot, all models slightly overestimated lower
 293 water contents and underestimate higher water contents. It seems that SWC data obtained
 294 from GPR display a different relationship with permittivity, with respect to the TDR (Fig
 295 6.). Somehow, this is expected since the GPR measurements explore a much larger volume of
 296 soil, with respect to the TDR probes, with larger variations in both water content and bulk
 297 density. Moreover, the GPR operates at different frequencies. The common antennas used in
 298 the field operates at lower frequency with respect to the operational frequency of the TDR.

299 Indeed, for all the samples combined the empirical equations obtained from TDR per-
 300 formed poorly, with low R^2 . The better performance of the dielectric mixing model is due to

301 its ability to incorporate the effect of porosity and dielectric permittivities of the individual
302 phases. Moreover, its mathematical form allows for more flexibility in describing the data, as
303 described in the next section (see Fig. 7 and 8).

304 In this study the dielectric mixing model was employed by using fixed parameters obtained
305 from the literature. If the model was fitted to the experimental data, further improvement in
306 SWC estimation would have been achieved.

307 FIGURE 6.

308 Indeed, note that the equations that use the value of porosity (or bulk density), such as
309 the dielectric mixing model of Roth et al. (1990) are not always smooth lines (Fig. 5) and
310 in particular for sand. This is due to the varying values of bulk density measured for each
311 independent measurement of gravimetric SWC. As described above, experimentally was not
312 possible to repack the soil at the exact same values of bulk density, therefore bulk density
313 was measured every time the soil was repacked. The ability of estimating SWC as function
314 of porosity is one of the reasons the dielectric mixing model performed better than the other
315 models. Moreover, the varying bulk densities stress the experimental difficulties of preparing
316 large amount of soil material at uniform water content and density.

317 Using empirical equations, such as the Topp's equation, where estimation of SWC is
318 not density dependent, will lead to inaccurate estimation of SWC since density, in natural
319 conditions, usually changes with depth. In agricultural conditions, where soil is subject to
320 compaction and softening due to machines and tillage, the changes in bulk density over the
321 growing season are significant, requiring equations that include the possibility of time and
322 space dependent bulk density. For these reasons, there are active lines of research, where

323 direct measurements of both SWC and bulk density are derived from TDR waveforms (Jung
324 et al., 2013a,b; Curioni et al., 2018).

325 These results are consistent with the work of Gerhards et al. (2008), where they derived
326 accurate SWC from GPR using a multiple transmitter and receiver setup, and employing the
327 dielectric mixing model of Roth et al. (1990). As pointed out by Sihvola (1999) the use of
328 dielectric mixing models is preferable with respect to the use of empirical equations since they
329 allow for incorporating dielectric properties of constituent materials and their temperature and
330 frequency dependence. While the major dipole relaxation for water occurs at higher frequency
331 (19 GHz), additional relaxations in soils, such as double layer or Maxwell-Wagner relaxations,
332 may occur in the operational frequencies of GPR, depending on the selected antenna Olmi
333 and Bittelli (2015).

334 Another parameter that significantly changes soil water content estimation is the param-
335 eter α , which is discussed in the next section.

336 *4.1. Sensitivity Analysis of the dielectric mixing model*

337 To employ the dielectric mixing model for different media it is important to quantify
338 the effect of the individual parameters on the estimation of water content. As described
339 above, the permittivity of the gas phase is constant, the porosity depends on bulk density,
340 the permittivity of the liquid phase is temperature dependent (assuming a constant or narrow
341 band frequency) and the permittivity of the solid phase depends on mineralogy.

342 Figure 7 depicts the variations of volumetric water content as function of permittivity for
343 different values of α . The other parameters are kept fixed with $\epsilon_s = 2$, $\epsilon_l = 78,54$ (at 25 °C),
344 $\epsilon_g = 1.005$ and $\phi_f = 0.547$ (with $\rho_b = 1.2 \text{ g cm}^{-3}$).

345 FIGURE 7.

346 The parameter α depends on the shape and orientation of the inclusions affecting the
347 depolarisation factors, as detailed by Sihvola, (1999). Values of 1/2 was used by Birchak
348 et al. (1974) or 1/3 by Looyenga (1965). Other values can also be selected for the power-law
349 relationship. The domain is $-1 \leq \alpha \leq 1$, where $\alpha = 1$ for plates or other inclusions for which
350 no depolarisation is induced, or when the electric field is parallel to the layering. $\alpha = -1$ if
351 the field is perpendicular to the layering and $\alpha = 0.5$ for isotropic two-phase medium. Using a
352 non-linear least square minimization algorithm, Roth et al. (1990) found an optimal value of
353 $\alpha = 0.46$ for their experimental data, which is close to 0.5, the value obtained by Birchak et al.
354 (1974) from theoretical reasons. While in this study the dielectric model was not calibrated
355 and a fixed value of 0.5 was used, α can be modified if information about the soil layering is
356 available, such as stratifications, sedimentation layers and others. Alternatively, α can also be
357 used as fitting parameter. At decreasing values of α corresponds significantly increasing values
358 of θ . Being the relationship non linear the variation depends on the corresponding values of
359 permittivity.

360 The effect of the solid phase permittivity was also evaluated (Fig. 8). The parameters
361 were kept fixed as for the previous analysis, with $\alpha = 0.5$, and ϵ_s was changed from 2 to 10.
362 These values are the ones reported in Table 1, for different earth materials. Lower values
363 are associated to dry sandstone and sand, while higher values are associated to dry clay. The
364 increase of the solid phase dielectric permittivity determines a decrease in the estimated SWC.
365 For this parameters set, a change from 2 to 10, determines a decrease in θ of $0.1 \text{ m}^3 \text{ m}^{-3}$.
366 This behavior is due to the higher weight given to the the solid phase by an increased ϵ_s in

367 the weighted volumetric sum, and therefore less weight to the volumetric contribution of the
368 liquid phase. Also in this case, information regarding the mineralogical composition of the
369 analyzed media allows for modification of this parameter.

370 FIGURE 8.

371 The effect of temperature on the liquid phase permittivity, and therefore on θ , is fairly
372 small with estimated variations in volumetric water contents of about $0.03 \text{ m}^3 \text{ m}^{-3}$ over a
373 temperature range between 4 and 20 °C. Finally, the effect of porosity on soil water content
374 is about $0.06 \text{ m}^3 \text{ m}^{-3}$ over a variation of ϕ_f between 0.7 and 0.1, with increasing θ with
375 increasing porosity. Considering that in field conditions bulk density can easily range, for
376 instance, between 0.8 and 2.4 g cm^{-3} (corresponding to variations in porosity between 0.7 to
377 $0.09 \text{ m}^3 \text{ m}^{-3}$), the effect of bulk density is significant on SWC estimation.

378 Overall, the parameters that have a larger effect on estimated SWC with the dielectric
379 mixing model are the exponent α , the solid phase permittivity ϵ_s and porosity ϕ_f . The first
380 two can be used as fixed parameters with values of 0.5 and 4 respectively or used as fitting
381 parameters. Porosity should be measured or obtained from bulk density. In absence of porosity
382 or bulk density data, density can be obtained from TDR waveforms (Jung et al., 2013a,b;
383 Curioni et al., 2018) or from pedotranfer functions by knowledge of textural composition
384 (Rodriguez-Lado et al., 2015).

385 5. Conclusions

386 Different relationships to estimate SWC derived from soil permittivities obtained from a
387 two-way GPR analysis data were compared. The GPR data were obtained in a controlled
388 laboratory setting using a soil tank with a metal reflector positioned at a known depth, allowing
389 for accurate determination of the soil bulk dielectric permittivity as function of varying water
390 contents. The data were obtained for four distinct soil textural classes (sand, sandy loam,
391 loamy sand and kaolinite clay) covering a wide range of soil moisture conditions. The physical
392 relationships under test were empirical formulae and dielectric mixing models. Results showed
393 that the dielectric mixing model of Roth et al. (1990) provided the most accurate estimate of
394 volumetric soil water content for all soils, except for sandy loam. However, for all the data
395 combined the dielectric mixing model performed much better with significant differences in the
396 coefficient of determination and Root Mean Square Error. The performance of the dielectric
397 mixing model could have been further improved by using the geometric parameter and the
398 dielectric permittivity of the solid phase as fitting parameters.

399 Sensitivity analysis of the dielectric mixing model was performed showing that the ge-
400 ometric parameter α and the dielectric permittivity of the solid phase ϵ_s are the two most
401 sensitive parameters, determining important variations in the estimation of SWC. Based on
402 these results, these two parameters are suggested as fitting parameters to be selected if the
403 model is fitted to data. Otherwise, the model can be successfully used without calibration,
404 as presented in this study, by using $\alpha = 0.5$ (as also suggested by the authors) and $\epsilon_s = 4$,
405 which is an average value for soil minerals. Overall, we suggest to employ the dielectric mixing
406 model for estimation of SWC from dielectric permittivity obtained with GPR.

407 6. Acknowledgment

408 This study was funded by: Project Scheme for Promotion of Academic and Research Col-
409 laboration (SPARC). A Government of India Initiative. Project Title: Prediction of Soil Hydro
410 Agricultural Properties using Ground Penetrating Radar for Improving Agricultural Practice
411 (Proposal ID: 375). Responsible: Prof. Anbazhagan P., Department of Civil Engineering,
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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Figure Captions

1. Schematic of the plastic tank.
2. Picture of the soil within the plastic tank and GPR (Mala Inc., 800 MHz).
3. Example of radargram indicating the strong reflection from the metal plate.
4. Example of travel time determination on a representative trace.
5. Ground penetrating radar (GPR) permittivity with corresponding volumetric water contents collected for the sand, sandy loam, loamy sand and kaolinite clay textural classes. Points are gravimetric water contents and lines are estimated values for the four different models.
6. Scatter plot of measured and estimated data for the four soil types with linear relationships. Regression equations and coefficient of determinations are reported for the four equations.
7. Sensitivity analysis for the parameter α .
8. Sensitivity analysis for the parameter ϵ_g .

Figure1

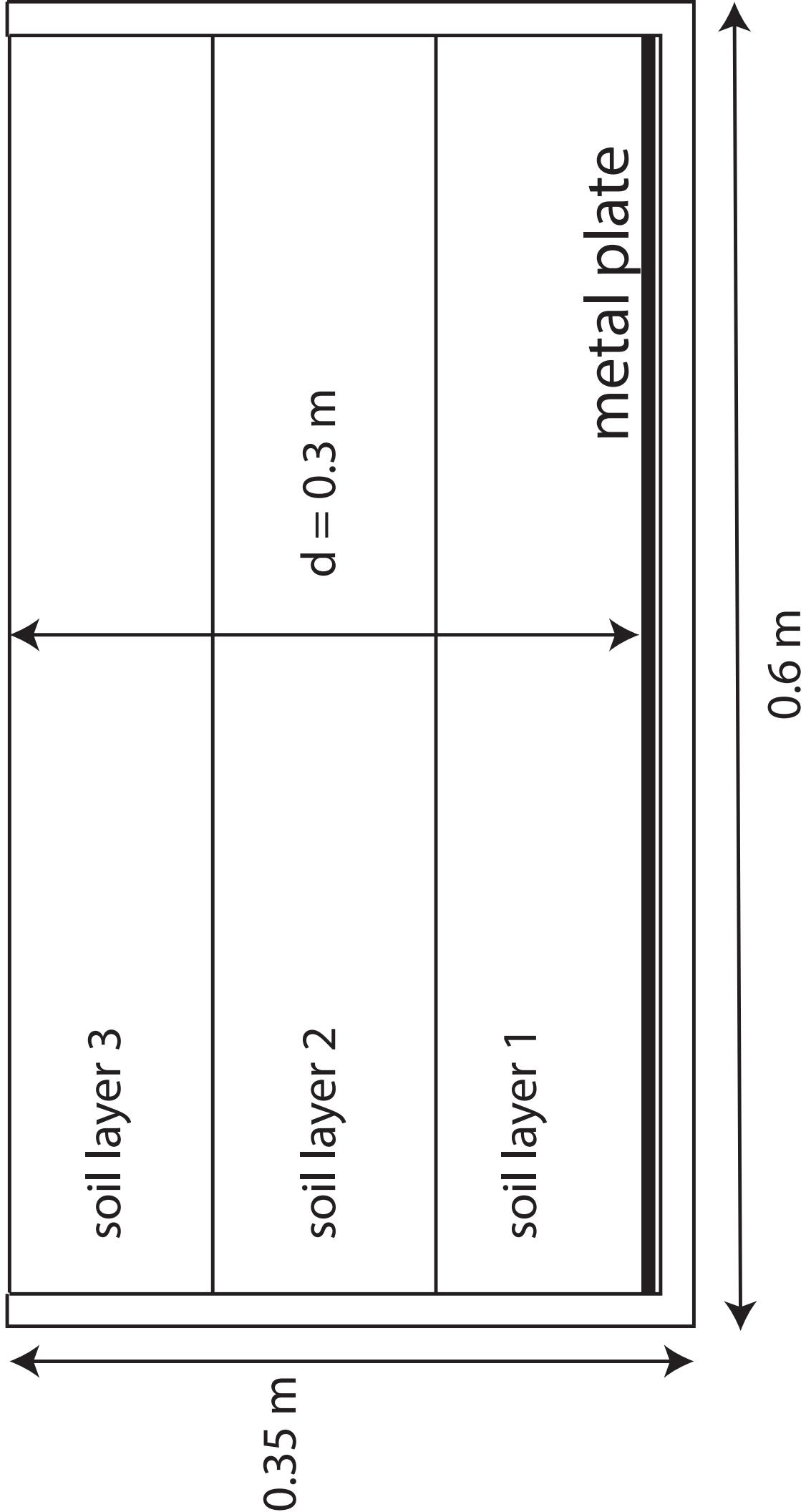
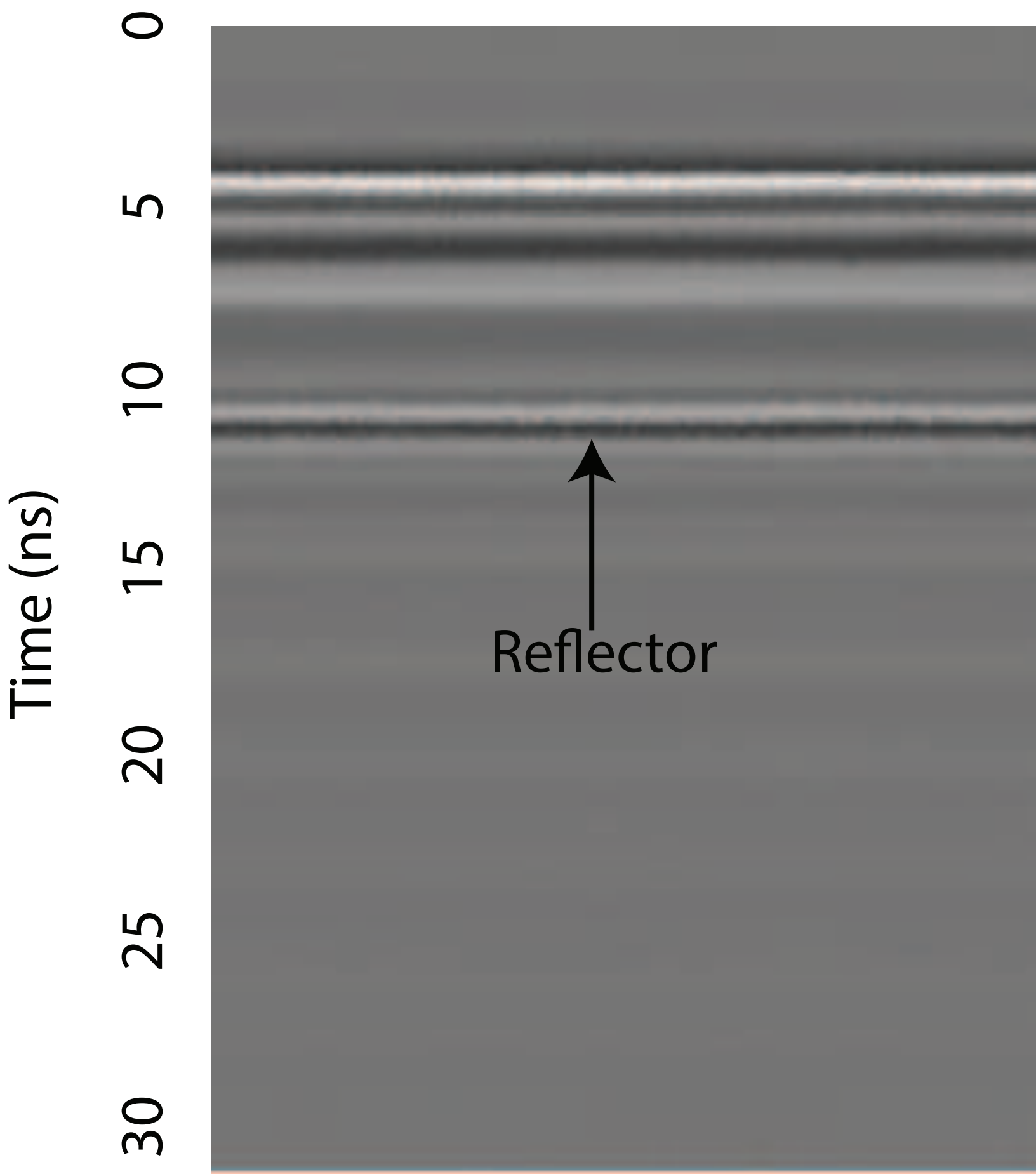


Figure 2



Figure3



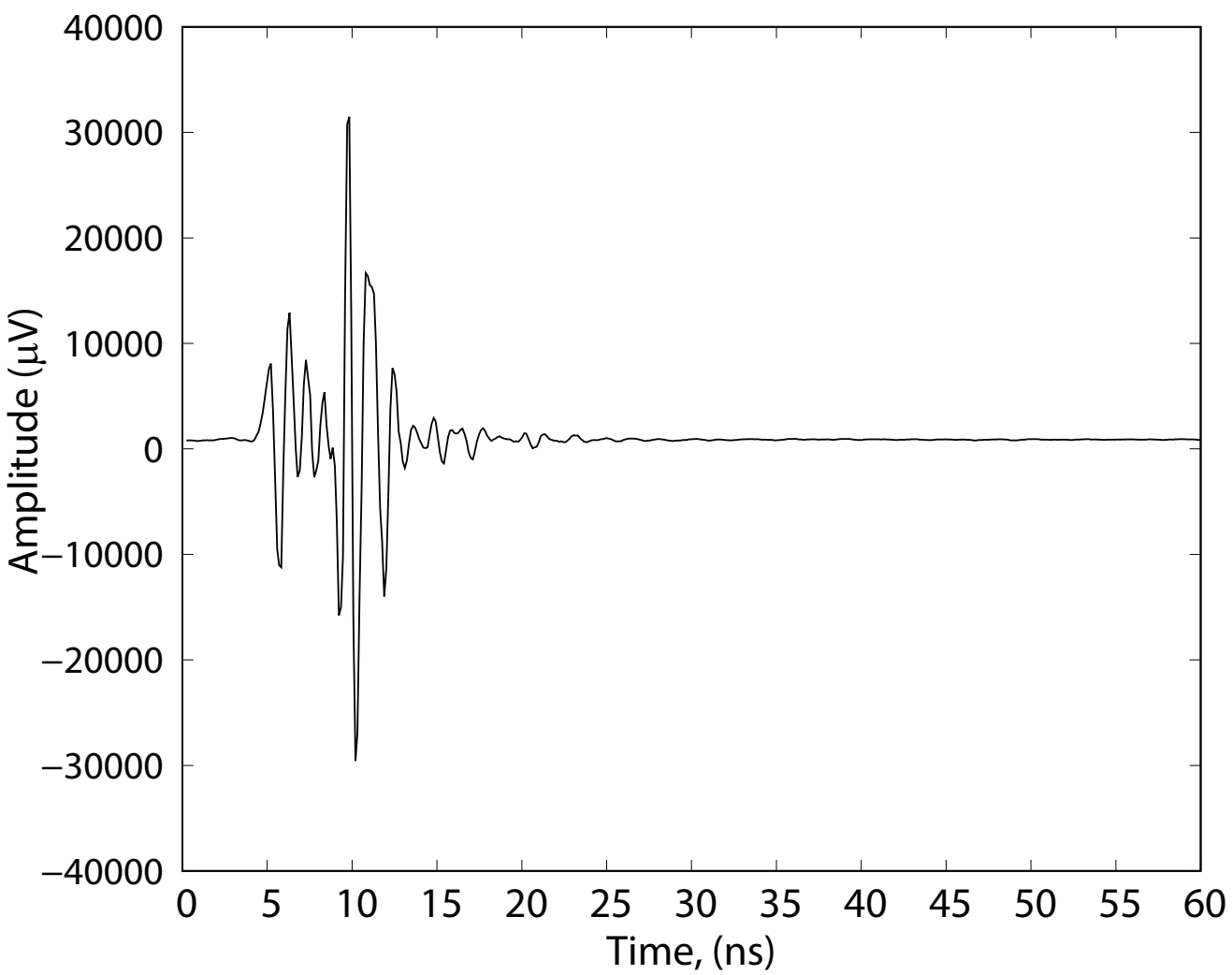
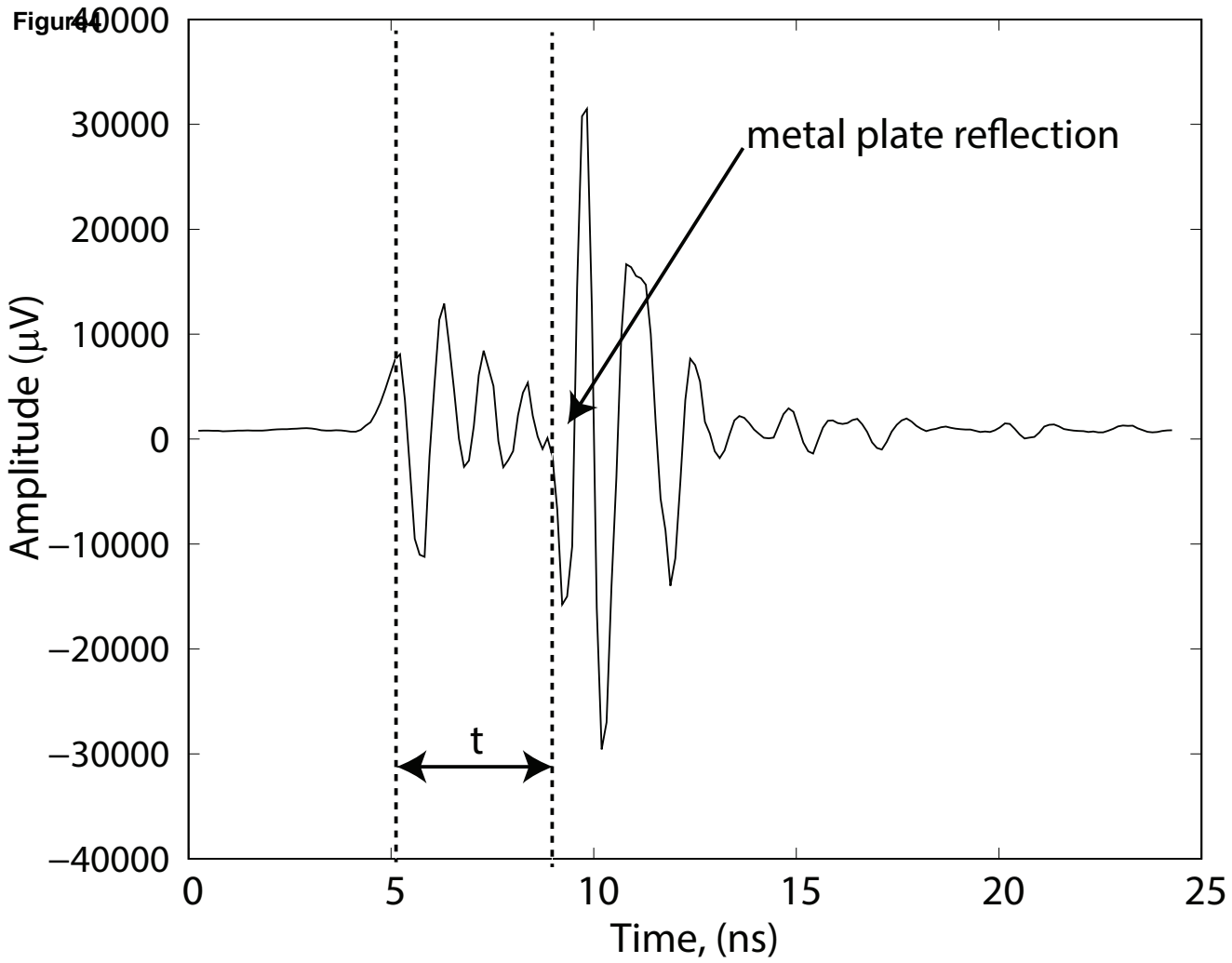


Figure 5

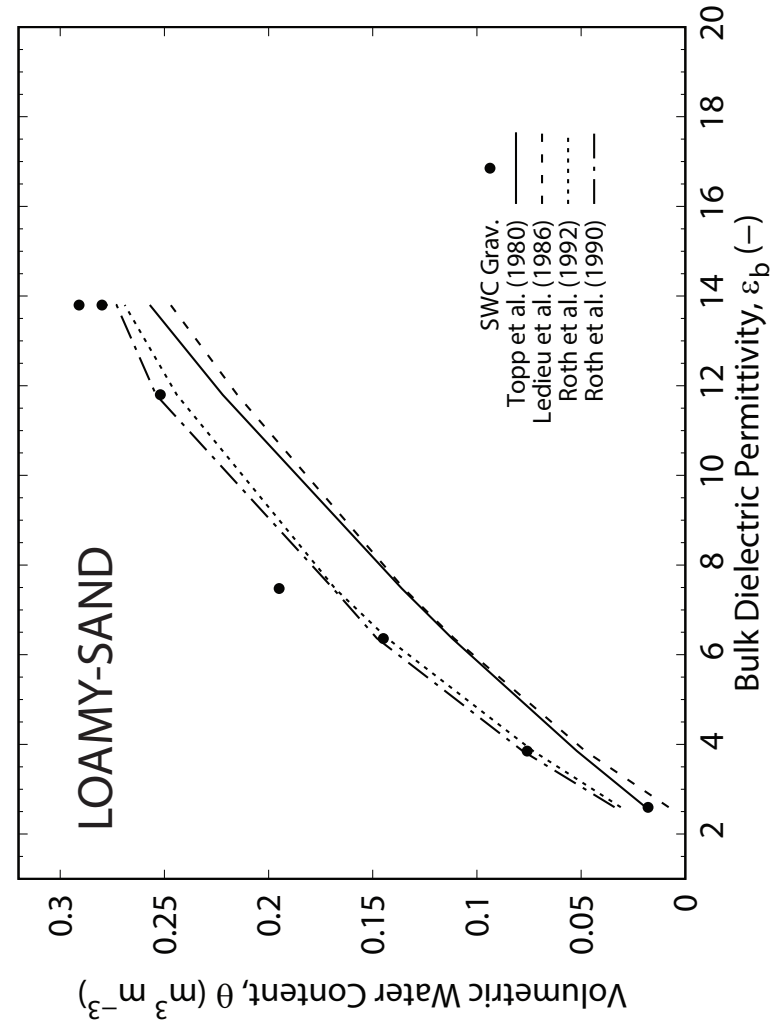
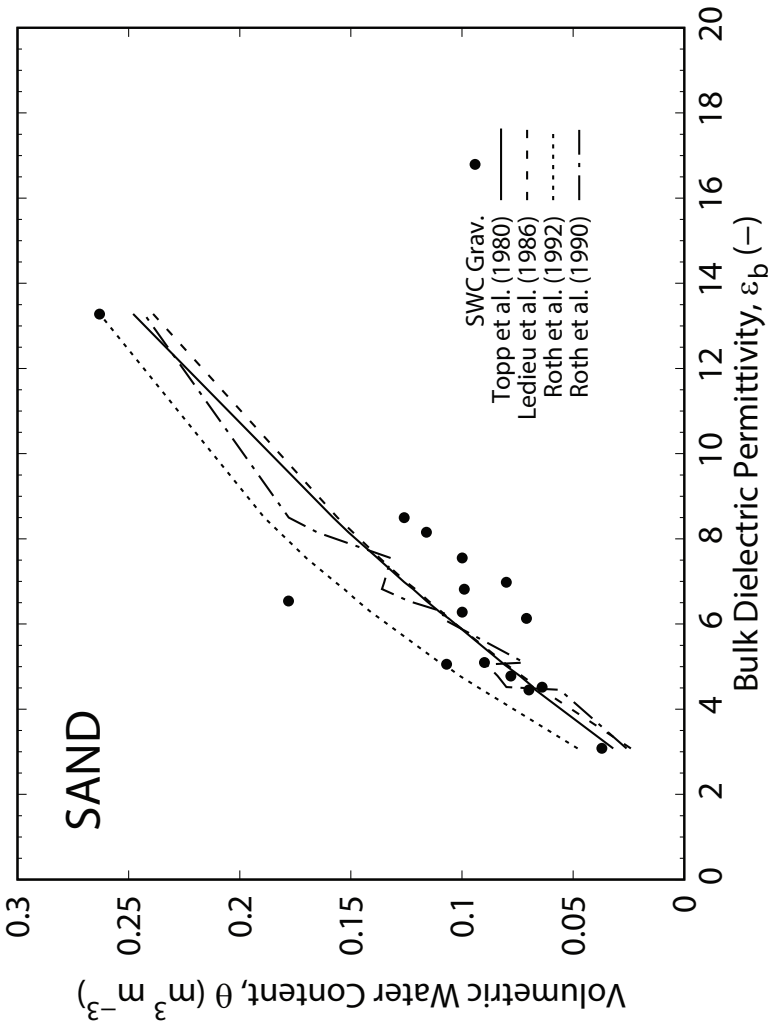
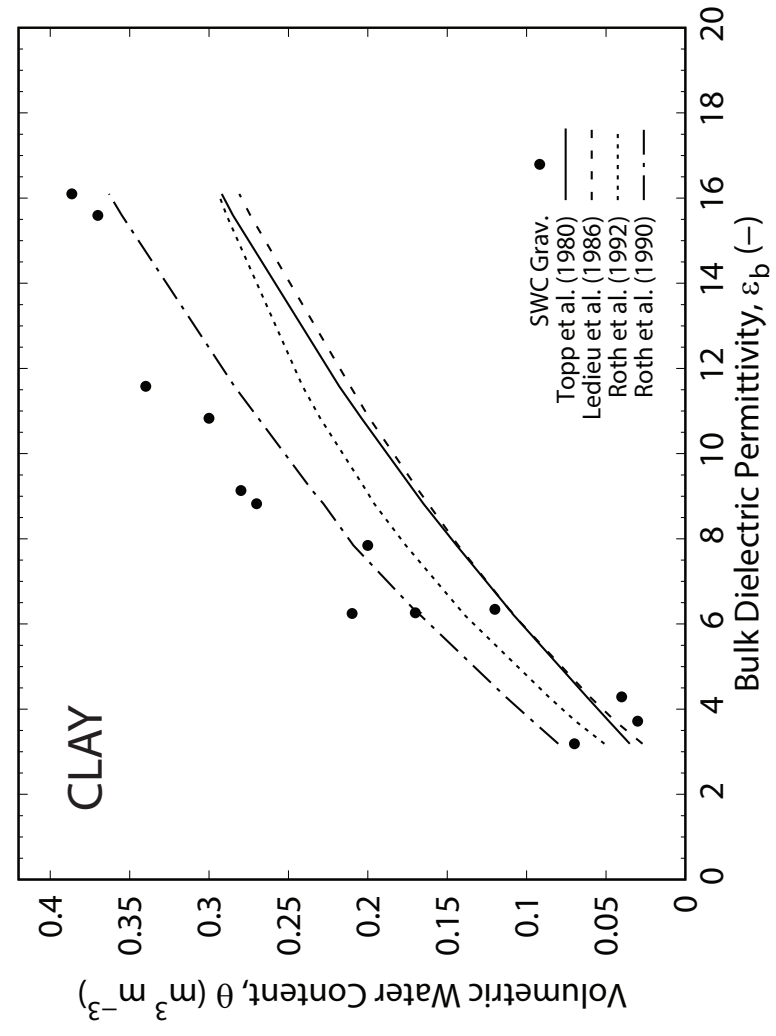
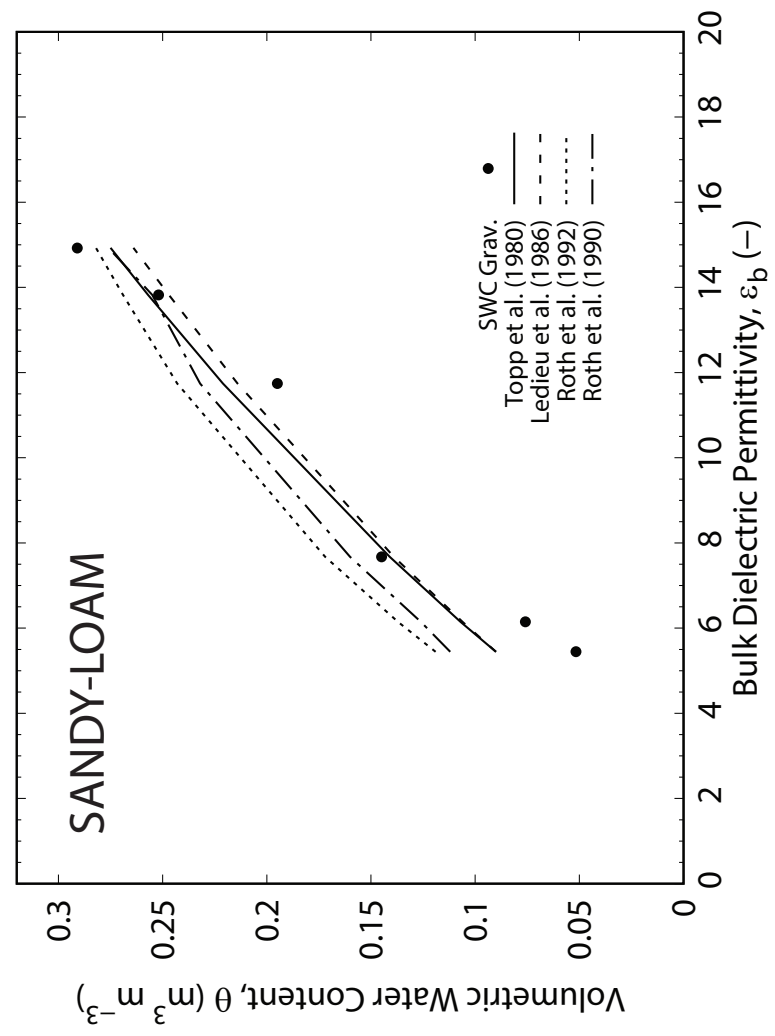


Figure 6

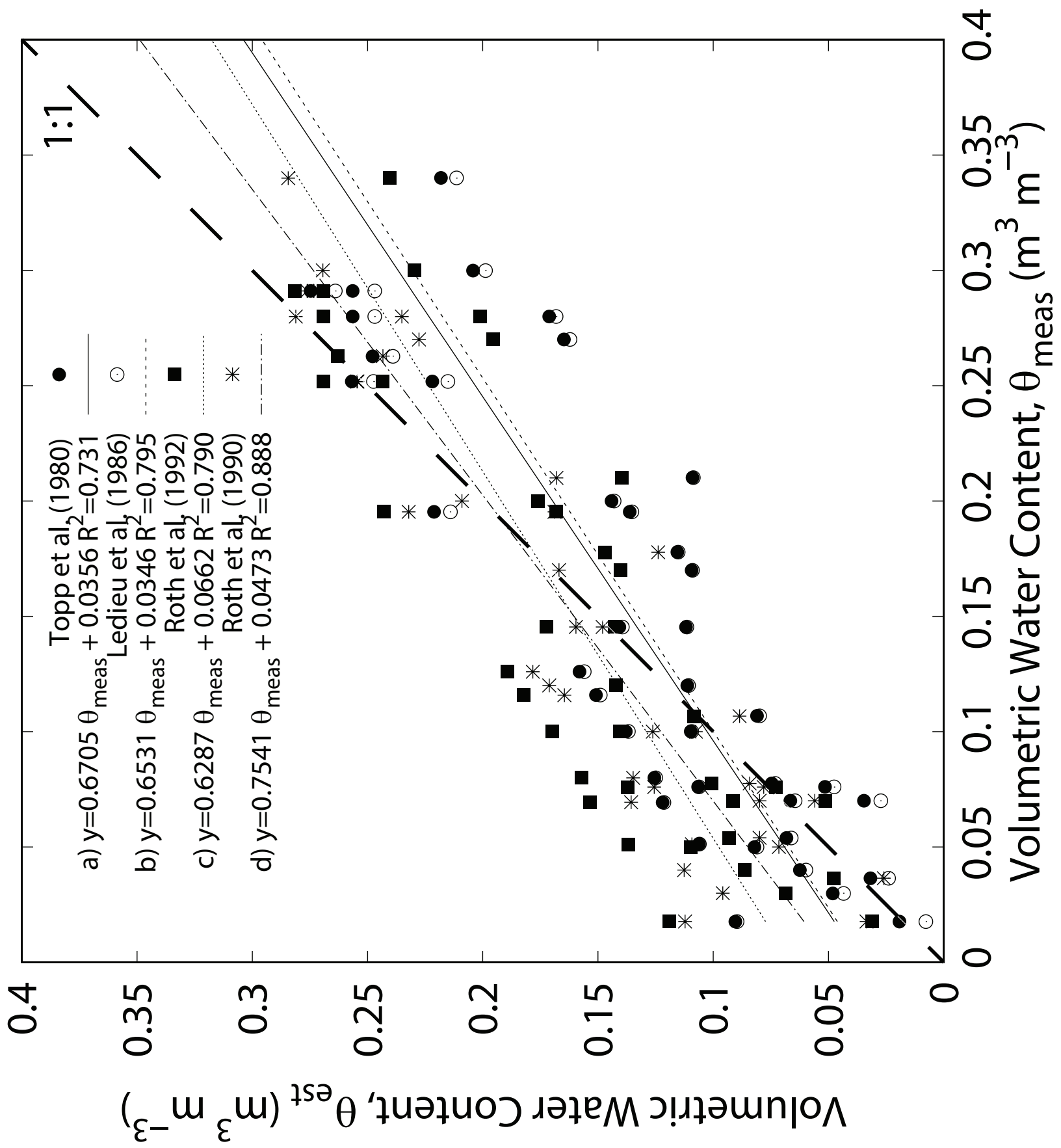


Figure 7

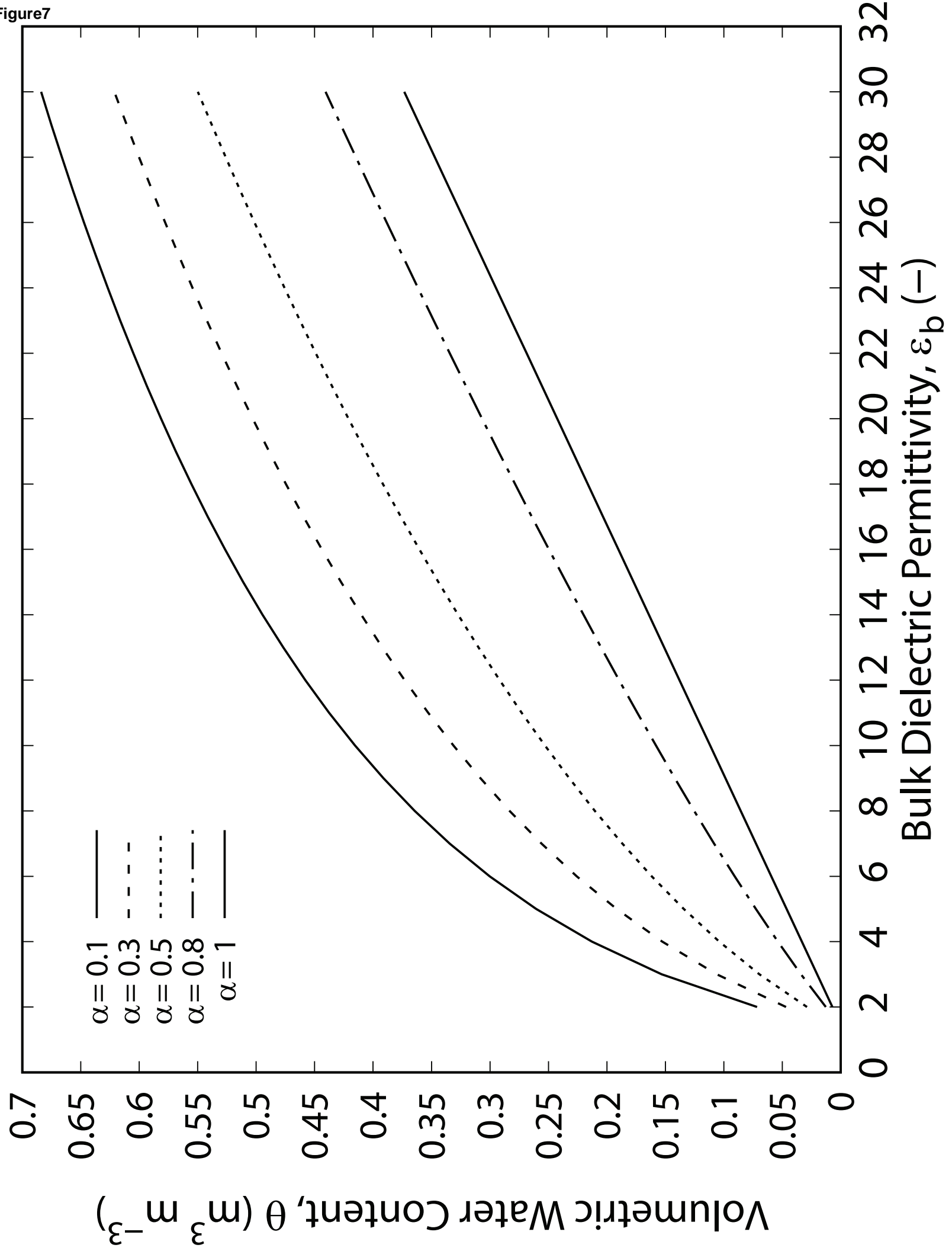


Figure8

