Supplementary appendix not intended for publication

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In this appendix we explore the robustness of our results under the alternative scheme of linear tariff contracts. In the first three sections, we derive the profit of the upstream firm $\mathcal{U}$ under an auction and negotiation(s), and we determine its optimal choice among these selling mechanisms. In section 4 we compare the profit of the upstream firm using a linear tariff with its profit under two-part tariffs and argue that, in our setup, the upstream supplier would always choose selling mechanisms based on two-part tariffs.

The assumptions about firms, demands and timing are the same as in the analysis in the paper, only the contractual structure is different. Before proceeding, it is worth to mention that the cases of an auction and an exclusive negotiation deliver closed-form solutions. Unfortunately, under non-exclusive negotiations the calculations become overly cumbersome, which prevents us to obtain intelligible analytical results. To cope with this issue, we perform simulations to determine the optimal choices of the upstream firm.

1 Auction

As in the paper, we assume that the winning bid is the one that maximizes the upstream firm's profit upon execution of the contractual terms. This clarified, if $\mathcal{U}$ auctions off the terms of an exclusive linear tariff contract, $\mathcal{D}_h$ wins the auction with a bid that gives $\mathcal{U}$ a profit

$$\tilde{\Pi}^a_m = \frac{u_l}{8}. \tag{1}$$

The proof is as follows. If $\mathcal{U}$ were entitled to make a take-it-or-leave-it offer for a linear contract to $\mathcal{D}_l$, the offer would be $w_l = \frac{u_l}{2}$, which is the input price that maximizes $\mathcal{U}$'s profit given the contractual form. $\mathcal{U}'s$ profit level at that input price would be $\frac{u_l}{8}$. This is, by construction, the maximum bid firm $\mathcal{D}_l$ can submit. Clearly $\mathcal{D}_h$ can outbid $\mathcal{D}_l$ by submitting a w_h such that $D_h(p_h^*(w_h)) \times w_h > \frac{u_l}{8}$. Intuitively, the revenue to firm $\mathcal{U}$ from the winning bid (demand at the optimal price of firm $\mathcal{D}_h$, as induced by w_h, multiplied by the input price itself) is larger than that it would obtain should $\mathcal{D}_l$ win the auction.[1] It is straightforward that $\mathcal{D}_l$ cannot submit a higher o lower bid, because by construction $\frac{u_l}{4}$ maximizes $\mathcal{U}$'s profit conditional on $\mathcal{D}_l$ winning the auction. Similarly, $\mathcal{D}_h$ cannot lower the bid, because it would lose the auction, and raising it would lower its profit, without affecting the outcome of the auction. As a consequence, the equilibrium bids are $\tilde{w}_l^a = \frac{u_l}{2}$, $\tilde{w}_h^a = \frac{1}{2}\left(u_h - \sqrt{u_h}\sqrt{u_h - u_l}\right) + \eta$. At the winning bid, $\mathcal{U}$'s profit is (1).[2]

It is worth mentioning that the message conveyed by Remark 1 in the paper applies here as well: the value of the auction for $\mathcal{U}$ is larger the more homogeneous the goods are. This is easily seen from (1): for given u_h, the larger u_l is, the larger the profit from the auction. This is also seen from the winning bid: The closer u_l is to u_h the closer the winning bis is to $\frac{u_h}{2}$, which is the input price that firm $\mathcal{U}$ would impose to firm $\mathcal{D}_h$ if it were entitled to make take-it-or-leave-it offers.

2 Negotiation(s)

Let w_i be the wholesale price negotiated by $\mathcal{U}$ and $\mathcal{D}_i$, $i = h, l$. In the following we present only the negotiation stage of the game, the downstream prices setting stages being the same as those presented in Bacchiega *et al.* (2018).

2.1 Exclusivity

If the upstream supplier opts for an exclusive contract, at the negotiation stage the generalized Nash product is:

$$NP_i^e(w_i) = \hat{\Pi}^e(w_i)^{\mu}\hat{\pi}_i^e(w_i)^{1-\mu}. \tag{2}$$

with $\hat{\Pi}^e(w_i) = \frac{(u_i - w_i)w_i}{2u_i}$ and $\hat{\pi}_i(w_i) = \frac{(u_i - w_i)^2}{4u_i}$ being the profits of $\mathcal{U}$ and $\mathcal{D}_i$ respectively, evaluated at the optimal price of firm i. The maximization of (2) with respect to w_i yields the equilibrium per-unit input price: $\tilde{w}_i^e = \frac{u_i}{2}\mu$. Therefore, the equilibrium downstream price is $\tilde{p}_i^e = \frac{u_i}{4}(2+\mu)$ and the equilibrium demand is $\tilde{D}_i^e = \frac{(2-\mu)}{4}$. The equilibrium profits of the upstream and downstream firms are, respectively:

$$\tilde{\Pi}_i^e = \frac{u_i}{8}(2-\mu)\mu \quad \text{and} \quad \tilde{\pi}_i^e = \frac{u_i}{16}(\mu - 2)^2 \tag{3}$$

[1] Easy computations return the optimal bid $w_h = \frac{1}{2}\left(u_h - \sqrt{u_h}\sqrt{u_h - u_l}\right) + \eta$, with $\eta > 0$ and arbitrarily small.

[2] It is easy to show that, omitting η, at the equilibrium bids, the price of the good is $\tilde{p}_h^a = \frac{1}{4}\left(3u_h - \sqrt{u_h}\sqrt{u_h - u_l}\right)$, the demand for the good is $\frac{1}{4}$ and firm $\mathcal{D}_h$'s profit is $\tilde{\pi}_h^a = \frac{1}{16}\left(\sqrt{u_h - u_l} + \sqrt{u_h}\right)^2$

It is immediate to ascertain that the the profit to firm $\mathcal{U}$ is larger when the exclusive contract is signed with firm $\mathcal{D}_h$, which entails that if $\mathcal{U}$ goes for an exclusive contract, it offers it to that firm.

2.2 Non-exclusivity

If $\mathcal{U}$ enters simultaneous and separate negotiations with the downstream firms, the profits functions at the negotiation stage are given by:

$$
\begin{aligned}
\hat{\pi}_h(w_h, w_l) &= \frac{\left[2u_h^2 + u_h(w_l - 2(u_l + w_h)) + u_l w_h\right]^2}{(u_h - u_l)(4u_h - u_l)^2}, \\
\hat{\pi}_l(w_l, w_h) &= \frac{u_h\left[u_h(u_l - 2w_l) + u_l(w_h + w_l - u_l)\right]^2}{u_l(u_h - u_l)(4u_h - u_l)^2}, \\
\hat{\Pi}(w_h, w_l) &= \frac{u_l(w_h^2(u_l - 2u_h) + 2u_h w_h(u_h - u_l)) + u_h u_l w_l(u_h - u_l + 2w_h) + u_h w_l^2(u_l - 2u_h)}{u_l(u_h - u_l)(4u_h - u_l)},
\end{aligned}
$$

and the generalized Nash products are as follows.[3]

$$
\begin{aligned}
NP_h(w_h, w_l^n) &= \left[\hat{\Pi}(w_h, w_l^n) - \frac{u_l}{8}(2-\mu)\mu\right]^{\mu} \hat{\pi}_h(w_h, w_l^n)^{1-\mu}, &(4)\\
NP_l(w_h^n, w_l) &= \left[\hat{\Pi}(w_h^n, w_l) - \frac{u_h}{8}(2-\mu)\mu\right]^{\mu} \hat{\pi}_l(w_l, w_h^n)^{1-\mu}. &(5)
\end{aligned}
$$

The equilibrium negotiated input prices are obtained by simultaneously maximizing the generalized Nash products, but although they can be obtained analytically, their cumbersomeness prevents us to draw any insight from them. One immediate consequence is that the equilibrium profits are non-tractable too. To bypass this problem and compare the profits of firm $\mathcal{U}$ under exclusive and non-exclusive negotiations we revert to numerical simulations; this way, we can determine the optimal choice between them. Figure 1 depicts the equilibrium profit of $\mathcal{U}$ if it enters an exclusive negotiation ($\tilde{\Pi}_m^e$) and non-exclusive negotiations ($\tilde{\Pi}^n$), for $\mu = (0.1, 0.5, 0.9)$, $u_h = 1$, and $u_l \in (0, 1)$.

The clear message drawn from Figure 1 is that, conversely to the two-part tariff case, an exclusive negotiation on the per-unit price is dominated by non-exclusive negotiations ($\tilde{\Pi}^n > \tilde{\Pi}_m^e$), therefore it cannot be an equilibrium outcome.

[3]The Nash products are defined under the assumption of contingent contracts.

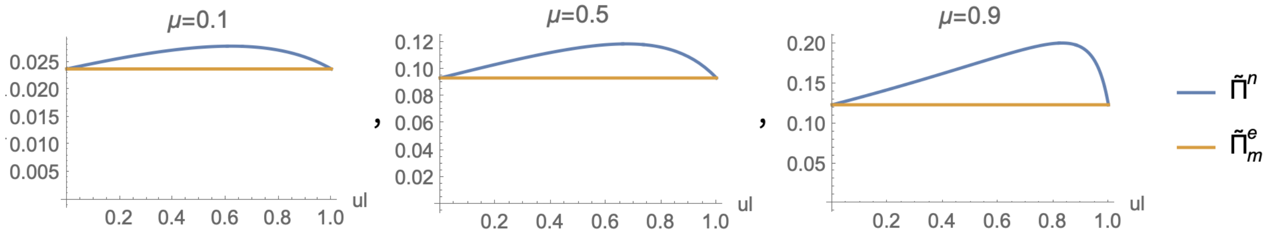

Figure 1: $\mathcal{U}$ profits under exclusive negotiation and non-exclusive negotiations

3 Auction vs. negotiations

We are now in a position to compare the auction and non-exclusive negotiations as selling mechanisms from the standpoint of firm $\mathcal{U}$. Figure 2 reports the equilibrium profit of $\mathcal{U}$ in the case of an auction ($\tilde{\Pi}^a_m$), and of non-exclusive negotiations ($\tilde{\Pi}^n$), for $\mu = (0.1, 0.5, 0.9)$, $u_h = 1$, and $u_l \in (0, 1)$.

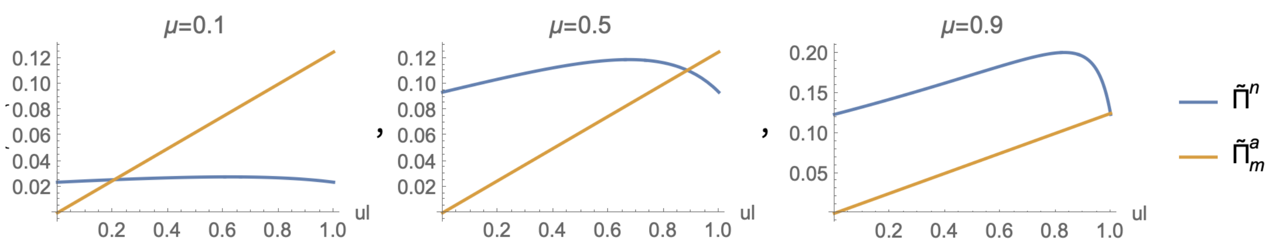

Figure 2: $\mathcal{U}$ profit under negotiation and auction

This figure shows that the bargaining power (μ) and the degree of product differentiation ($\frac{1}{ul}$) are still the main factors determining the upstream supplier's decision about which selling mechanism to implement.[4] As under two-part tariffs, the auction is the optimal choice when bargaining power is low and the products are homogeneous enough. The intuition is the same of that provided in the case of two-part tariffs.

4 Linear tariff vs. two-part tariff

In the following we compare the profit of $\mathcal{U}$ under linear and two-part tariffs and suggest that profitwise, the latter dominate the former. We build our argument on two observations.

[4] As we perform the simulations for $u_h = 1$, the differentiation degree is given by $\frac{1}{ul}$.

(i) First, the profit reaped by $\mathcal{U}$ with an auction with a linear tariff, $\tilde{\Pi}_m^a = \frac{u_l}{8}$ (see 1), is always lower than that under from an auction with a two-part tariff $\Pi_m^a = \frac{u_l}{4}$ (see Lemma 2). This implies that, whenever the upstream supplier finds it optimal to set-up an auction, it prefers to run it with two-part tariffs.

(ii) The comparison of non-exclusive negotiations under linear and non-linear contacts has to be performed through numerical simulations. However, before proceeding, it should be noticed that Lemma 1 in the paper points out that non-exclusive negotiations are an optimal choice, with non-linear contracts, only as long as $\mu < \frac{3}{4}$, and for μ exceeding that threshold, an exclusive negotiation with $\mathcal{D}_h$ is $\mathcal{U}$'s optimal choice. Keeping this in mind, the first two panels of Figure 1 below (cases $\mu = .1$ and $\mu = .5$) report $\mathcal{U}$'s profit under non exclusive negotiations with two-part tariffs (green curves) and with linear tariffs (blue curves). The third panel ($\mu = .9$) compares the profits from an exclusive negotiation over two-part tariffs with $\mathcal{D}_h$ (yellow line), with the profit from non-exclusive negotiations over linear tariffs (blue curve).

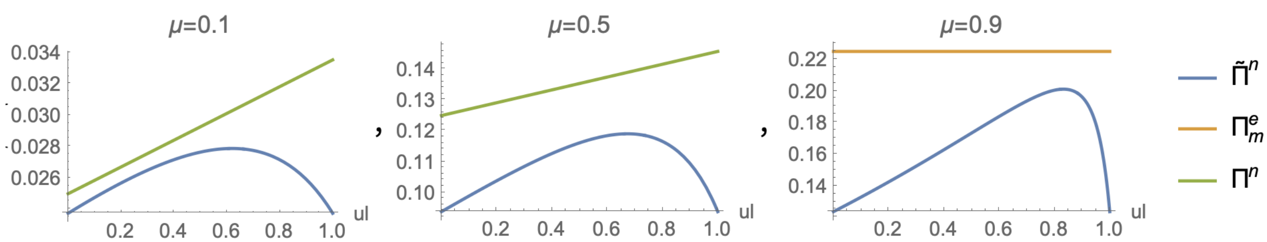

Figure 3: Profit of $\mathcal{U}$ according the negotiated contract form (linear tariff or two-part-tariff)

Figure 3 indicates that under negotiations linear contracts result in lower profits than non-linear (exclusive or non-exclusive) ones.

Points (i) and (ii), taken together, imply that, unless somehow restricted in its choices, firm $\mathcal{U}$ always wants to implement selling mechanisms based on two-part tariffs.

References

Bacchiega, E., Bonroy, O., and Petrakis, E. (2018). "Contract contingency in vertically related markets". *Journal of Economics & Management Strategy*, **27**(4):772–791.