Loyalty discounts and price-cost tests*

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Abstract

We analyze, by means of a formal economic model, the use of the discount-attribution test to assess the competitive effects of loyalty discounts. (The discount-attribution test is a variant of the price-cost test, where the discount is attributed only to the share of total demand that is regarded as effectively contestable.) In the model, a dominant firm enjoys a competitive advantage over its rivals and uses market-share discounts to boost the demand for its own products. In this framework, we show that the attribution test is misleading or, at best, completely uninformative. Our results cast doubts on the applicability of price-cost tests to loyalty discount cases.

Keywords: Loyalty discounts; As-efficient competitor; Price-cost test; Contestable share; Discount-attribution test.

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1 Introduction

Loyalty discounts are a highly controversial issue in competition policy.[1] There is no consensus on the rationale for these practices, their competitive effects, and the appropriate antitrust treatment. Much of the policy debate centers around the problem of whether the legality of loyalty discounts should be assessed by means of some sort of price-cost test. It is this last question, in particular, that we address in this paper.

Price-cost tests come in different varieties. All of them compare the dominant firm's relevant price to its own cost, on the grounds that only those practices that are capable of foreclosing an as-efficient competitor ought to be regarded as anticompetitive. But different versions of the test use different prices for the comparison.

The traditional test takes the relevant price to be simply the average price, which is obtained by spreading the total rebate over all of the dominant firm's sales. Critics however argue that this inflates the relevant price, making the test too easy to pass. In this form, in fact, the price-cost test had almost become a synonym for tolerant policy in loyalty discounts cases.

But a new test has recently been introduced by antitrust authorities and the courts, which does not attribute the total rebate to the entire demand but only to the part of it that is regarded as effectively contestable. This new test is often referred to as the *incremental test*, or the *discount-attribution test*. Since the rebate is attributed to a lower volume, the price used for the comparison is lower. This implies that the new test is generally more demanding than the traditional one.

We contend, however, that neither variant of the test really helps screen out cases where loyalty discounts are anti-competitive. For the traditional variant, our analysis simply confirms arguments already made in the liter-

[1] The term "loyalty discounts" is variously interpreted to refer to market-share discounts only, also include bundled discounts, or even include some forms of volume discounts. To fix ideas, in this paper we restrict the analysis to market-share discounts, i.e., discounts conditional on purchasing from the seller at least a certain percentage of one's total requirements.

ature. The more original contribution of the paper is the analysis of the discount-attribution test. This test, too, has been vigorously criticized but early critics (with the exception of Greenlee et al. 2008) have focused on the practical difficulties of implementing the test, and on antitrust agencies' tendency to underestimate the contestable demand. If this were the only problem, though, the solution would simply be to use more precise estimates. We argue, in contrast, that the problem is not only that of measurement errors. Indeed, we show that even if contestable demand were measured perfectly, the discount-attribution test would produce many false positives and false negatives. This means that the test is in itself misleading or, at best, completely uninformative.

The paper closest to ours is Greenlee et al. (2008). These authors analyze the case of bundled discounts and show, by numerical examples, that the incremental test may fail when the discounts increase social welfare (false positives), and pass when they decrease it (false negatives). We focus instead on the case of market-share discounts, and show that type I and type II errors are not only possible, but actually prevalent. Our results thus complement and reinforce those obtained by Greenlee et al. (2008). Taken together, they cast doubts on the usefulness of price-cost tests in loyalty discount cases.

The rest of the paper is organized as follows. Section 2 provides a more extensive discussion of the scholarly and policy debate. The paper then proceeds to the formal analysis. Section 3 sets out a simple model of two firms, a dominant firm and its rival, which supply differentiated products. The model's equilibrium is characterized in section 4, while in section 5 we analyze the profitability and welfare effects of the discounts. Section 6 analyzes the application of the discount-attribution test and shows that it systematically produces many type I and type II errors. Section 7 discusses how the attribution test might be modified so as to better account for product differentiation but shows that even the proposed variant of the test suffers from the same drawbacks. Section 8 offers some concluding remarks.

2 The policy debate

In this section, we briefly discuss the relevant case law and the main theories of harm, and we then present in more detail the discount-attribution test.

2.1 Case law

After the recent decision of the European Court of Justice in the *Intel* case,[2] loyalty discounts are subject to the rule of reason on both sides of the Atlantic. But there is a heated debate on what factors agencies and the courts should consider in their assessment. In particular, should they use some variant of the price-cost tests originally devised for predatory pricing cases?[3]

On this issue, the case law is split both in the US and in Europe. In the US, price-cost tests have been adopted by several Circuit Courts of Appeals,[4] with the notable exception, however, of the Third Circuit.[5] The Supreme Court has not taken a stance on the matter yet. In Europe, the Commission has endorsed the use of price-cost tests in its *Article 102 Guidance Paper* and has applied the discount-attribution test in the *Intel* decision.[6] But the European Court of Justice has taken a more cautious approach so far, stating that, in any case, price-cost tests cannot be dispositive.[7]

[2] This decision overturned previous case law, according to which loyalty discounts were regarded as presumptively illegal when used by dominant firms.

[3] This test was advocated by Areeda and Turner (1975) and adopted by the US Supreme Court in the *Brooke Group* case: see *Brooke Grp. Ltd. v. Brown & Williamson Tobacco Corp.*, 509 U.S. 209 (1993).

[4] See e.g. *Concord Boat Corp. v. Brunswick Corp.*, 207 F.3d 1039, 1062 (8th Cir. 2000)), *Cascade Health Solutions v. PeaceHealth*, 515 F.3d 883 (9th Cir. 2008), and *Ortho Diagnostic Systems v. Abbott Laboratories*, 920 F. Supp. 455 (S.D.N.Y. 1996).

[5] See *Le Page's Inc. v. 3M*, 324 F 3d 141 (3rd Cir 2003) (en banc), *ZF Meritor LLC and Meritor Transmission Corporation v. Eaton Corporation*, 696 F.3d 254 (3d Cir.2012), *Eisai Inc. v. Sanofi Aventis U.S., LLC*, No. 14-2017.

[6] The Commission's *Guidance Paper* and its decision on the Intel case may be found at European Commission (2009) and Case C-413/14 P Intel Corp. v. European Commission EU:C:2017:632, respectively.

[7] See the recent decision on the *Intel* case at Case C-413/14 P Intel Corp. v. European Commission EU:C:2017:632, and the previous decision on *PostDanmark* at Case C-209/10 Post Danmark A/S v. Konkurrencerådet ("Post Danmark I") EU:C:2012:172.

2.2 Theories of harm

The use of price-cost tests in loyalty discount cases is rooted in the view that these practices are similar to predatory pricing. According to this view, market-share discounts entail a sacrifice of profit and thus can be profitable (and produce anticompetitive effects) only *indirectly*, by weakening the dominant firm's rivals and reducing their ability to compete in the future, or in adjacent markets. This argument is articulated, for instance, in Bernheim and Heeb (2015) and indeed reflects the conclusions of several prominent theories.[8] The sacrifice-recoupment logic is clearly reminiscent of the mechanism of predatory pricing and thus invites adoption of the same sort of policies. Hence the emphasis on price-cost tests.

The test applied in predation cases must however be adapted to the case of market-share discounts, where the dominant firm offers in fact a non-linear price schedule comprising both a reference price and a discounted price. The most natural way to proceed is to take as the relevant price simply the discounted price, which applies if the target market share is reached. Some authors have argued that this "traditional" price-cost test should actually be dispositive: see, for instance, Amici Curiae (2013).

This approach, which would tend to be rather tolerant in practice, has been criticized from two different perspectives. On the one hand, the economics literature has developed other theories of harm, in which loyalty discounts are profitable *directly* rather than indirectly, i.e., contemporaneously and in the very same market. One example is the demand-boost theory adopted in this paper and further discussed below, but other explanations

[8] These theories share the view that loyalty discounts entail no direct gain but differ as for the nature of the indirect gain. This may be, for instance, entry deterrence (e.g. Rasmusen et al., 1991), the exploitation of a future entrant (e.g. Aghion and Bolton, 1987), increased market power in an adjacent market (e.g. Bernheim and Whinston, 1998), or the protection of non-contractible investments (e.g. Marvel, 1982). See Fumagalli, Motta and Calcagno (2018) for an excellent survey of this literature. In fact, most of these theories have been developed for the case of exclusive dealing. As argued by Ide, Montero and Figueroa (2016), however, loyalty discounts may be markedly different from exclusive dealing in some frameworks. If this is so, then the relevance of the profit sacrifice/recoupment approach to loyalty discounts ought to be reconsidered.

for loyalty discounts share the same feature.[9] In all of these theories, the dominant firm's price always exceeds its unit cost, so the traditional price-cost test would imply that the discounts should always be legal. But in fact, in these models loyalty discounts may well be anti-competitive.

On the other hand, even some scholars who share the profit-sacrifice view call the traditional price-cost test into question. These scholars argue that in loyalty discount cases the sacrifice and recoupment phases may be intertwined and thus may not be easy to discern. They also argue that loyalty discounts have a greater anticompetitive potential than predatory pricing, implying that false negatives should be given a greater weight, and false positives a lower one, than in predation cases. From this, these authors conclude that loyalty discounts should be governed by more stringent legal standards.[10]

2.3 The discount-attribution test

In response to these concerns, antitrust authorities and the courts have developed a new variant of the price-cost test, the *discount-attribution test*. The attribution test assumes that the hypothetical as-efficient competitor in fact cannot serve the entire demand, and thus cannot perfectly replace the dominant firm as the buyer's sole supplier. Before engaging in the price-cost comparison, then, the discount is attributed only to that part of total demand that is regarded as effectively contestable. This inflates the discount and thus deflates the price used for the comparison, making the test more difficult to pass.[11] In particular, the test may now fail (and hence the discount may be

[9] These include raising rivals' costs theories (Salop, 2017), the "downstream competition" theory, in which downstream firms coordinate on an equilibrium in which they obtain lump-sum subsidies from the upstream incumbent in exchange for keeping a potential entrant out (Asker and Bar-Isaac, 2014, DeGraba, 2013 and DeGraba and Simpson, 2014), the screening theory of Majumdar and Shaffer (2009), or the dampening of inter-brand competition theory of Inderst and Shaffer (2010).

[10] See for instance Bernheim and Heeb (2015) and Moore and Wright (2015).

[11] The attribution test was originally applied by US lower courts in cases involving bundled discounts. It was later endorsed by the Antitrust Modernization Commission (2007) in the US, and by the European Commission in its *Article 102 Guidance Paper*: see the "Guidance on the Commission Enforcement Priorities in Applying Article 82 of the EC Treaty to Abusive Exclusionanry Conduct by Dominant Undertakings" at 2009 O.J. (C45) 7. The attribution test has been applied, for instance, by US Ninth Circuit

regarded as potentially anti-competitive) even if the actual price is greater than the unit cost. In principle, this might accommodate theories of harm where loyalty discounts are directly profitable.

The discount-attribution test however raises the issue of the measurement of the share of contestable demand. In cases of bundled discounts, contestable demand is taken to be the demand for the tied products only. But in cases involving market-share discounts, contestable demand is conceptually more difficult to identify, and even more difficult to measure with any precision.[12]

Here, however, we abstract from measurement issues and propose a more radical critique of the discount-attribution test. That is, we show that even if contestable demand were perfectly measurable, the discount-attribution test would be of no help in separating the pro-competitive from the anti-competitive practices.

3 The model

In this section, we present a simple model of market-share discounts where the rationale for the practice is to boost the demand for the dominant firm's products. According to this "demand-boost" theory, the upside of market-share discounts is that they increase the demand for the dominant firm's product, at the expenses of the demand for rival products. The downside is the price reduction that is necessary to entice the buyer to reach the target market share. This creates a price-volume trade off. The theory argues that the trade-off may be favorable, and hence the discounts may be profitable, if marginal prices are distorted upwards. Such distortions are often observed in practice and may indeed be optimal for a variety of reasons, such as for instance adverse selection, moral hazard, the presence of downstream

Court in the *PeaceHealth* case, and by the European Commission in the *Intel* case (Case T-457/08, *Intel v Commission*, [2009] E.C.R. II-12*).

[12] What is perhaps most worrisome is the vagueness of the notion of contestability, which implies that the quantitative assessment of the share of contestable demand may be quite arbitrary: see for instance Steuer (2017) and the literature cited therein.

competition, or other factors.[13]

The model is intentionally built so that market-share discounts can be used only to boost the demand for the dominant firm's product. To this end, we assume that all firms are already active and price simultaneously, and that the firms interact only once, and only in one market. Furthermore, marginal costs are taken to be constant and predetermined. We also abstract from fixed costs, so firms always remain active, at least potentially. This rules out more roundabout mechanisms involving profit sacrifice and recoupment, as well as raising rivals' costs strategies.

3.1 Demand and cost

We consider a market where a dominant firm and its rival supply substitute products. We denote the goods by $i = 1, 2$; the same indexes are used also for the firms. We refer to firm 1 as the dominant firm, and firm 2 as its competitor.

Marginal costs are denoted by c_i. With no loss of generality, we normalize the dominant firm's marginal cost to 0 and denote the cost gap $c_2 - c_1$ by c. We mostly focus on the case $c \geq 0$; however, the analysis below still applies if c is negative but not too large in absolute value.

Buyers are retailers or, more generally, downstream firms that do not interact strategically with each other.[14] Thus, we can focus on the firms' relationships with a single retailer. The retailer's demand is derived from

[13] The demand-boost theory was first proposed by Mathewson and Winter (1987) in a model of exclusive dealing. Mathewson and Winter simply assume linear pricing, so distorting the marginal price upwards is the only way, in their model, in which firms can exploit their market power. Bernheim and Whinston (1998, section V) and Calzolari and Denicolò (2015) extend the theory to the case of non-linear pricing, respectively in models of moral hazard and adverse selection. Calzolari et al. (2019) show that these three models, which are sometimes regarded as competing or, at best, unrelated, in fact share the same mechanism and lead to the same predictions. The demand-boost theory has also been applied to all-units quantity discounts (Chao et al. 2018, 2019), and to bundled discounts (Greenlee et al., 2008).

[14] In principle, buyers could also be final consumers. In practice, however, the enforcement of contracts that reference rivals' volumes is easier when buyers are sufficiently large that it is possible to observe their purchases from rivals.

the symmetric payoff function $v(q_1, q_2)$, which represents the gross profit of a retailer who buys q_1 units of good 1 and q_2 units of good 2.

We make standard regularity assumptions on the payoff function. In particular, we assume that it is increasing and concave so that the goods are imperfect substitutes:

$$v_{q_i q_i}(q_i, q_j) \leq v_{q_i q_j}(q_i, q_j) \leq 0. \tag{1}$$

Furthermore, we assume that the function $v(q_i, q_j)$ vanishes when $q_1 = q_2 = 0$, has finite satiation points $\bar{q}$ implicitly defined by $v_{q_i}(\bar{q}, 0) = 0$, and finite choke prices $\bar{p}_i = v_{q_i}(0, 0) > c_i$. Further regularity assumptions will be introduced as needed. To obtain closed-form solutions, at times we shall use a quadratic specification:

$$v(q_1, q_2) = q_1 + q_2 - \frac{1}{2}(q_1^2 + q_2^2) - \gamma q_1 q_2, \tag{2}$$

which implies that demand is linear:

$$q_i = \frac{1 - \gamma - p_i + \gamma p_j.}{1 - \gamma^2}. \tag{3}$$

In this specification, the parameter γ captures the degree of substitutability between the products: it ranges from 1 (perfect substitutes) to 0 (independent goods).

3.2 Contestable demand

Taken at face value, the as-efficient-competitor principle envisions a hypothetical scenario where the two firms have the same costs (so that $c = 0$) and face symmetric demands. However, in such a hypothetical scenario, where the competitor is as efficient as the dominant firm in all relevant respects, the competitor could replicate exactly any strategy of the dominant firm and thus could not be foreclosed. Explicitly or implicitly, antitrust concerns arise when the "as efficient" competitor is in fact, in some sense, weaker than the dominant firm.

In particular, one can distinguish between the ability to compete at the margin, i.e., for supplying one additional unit of the product, or for the entire market. In loyalty discounts cases, the antitrust concern seems to be precisely that superior efficiency in the competition for the market could allow a dominant firm to foreclose a competitor that ought not to be excluded, as it is equally (if not more) efficient at the margin.

To capture this notion, we assume that firm 2 is capacity constrained, whereas the dominant firm is not.[15] We denote firm 2's capacity by K, and we assume that firm 2 can sell more than K units only by incurring an extra cost θ.[16] Thus, firm 2's marginal cost function is in fact

$$MC_2 = \begin{cases} c & \text{if } q_2 \leq K \\ \\ c + \theta & \text{if } q_2 > K. \end{cases} \tag{4}$$

We further assume that θ is sufficiently large that it is never profitable to produce beyond capacity.

Obviously, capacity K must not be so large that the constraint becomes irrelevant. We therefore assume that $K < \tilde{K}$, where

$$\tilde{K} = \arg\max[v(0, q_2) - cq_2]. \tag{5}$$

This guarantees that even if $c = 0$, the dominant firm is more efficient than the rival as the retailer's sole supplier.[17]

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[15] This approach has been followed, among others, by Yong (1996) and Chao et al. (2018, 2019). An alternative approach is to posit demand asymmetries, so that a fraction of total demand can be served only by the dominant firm, as in DeGraba (2013) and Ide, Montero and Figueroa (2016). Ide and Montero (2019) propose a general model of "must have," where a non-contestable demand may arise because of shopping costs and competition among the retailers.

[16] This assumption was first proposed by Dixit (1980) and later used by Bulow et al. (1985) and Maggi (1996), among others. It serves to guarantee the existence of a pure strategy equilibrium, as discussed more extensively in footnote 20 below.

[17] In fact, this must be true even if c is negative (otherwise, the two firms would switch their roles). For the quadratic payoff function (2), for instance, this requires that

$$c \geq \tilde{c} = -\frac{(1-K)^2}{2K}.$$

3.3 Pricing

Firms compete in two-part tariffs, $P_i = p_i q_i + F_i$. When market-share discounts are prohibited, each firm names a tariff that must apply irrespective of the quantity the retailer purchases from the rival. When market-share discounts are permitted, by contrast, the firm may condition its price on the rival's volume. Specifically, we assume that firms may offer contracts of the type:

$$P_i = \begin{cases} p_i^H q_i + F_i^H & \text{if } \frac{q_i}{q_i+q_j} < s_i \\ \\ p_i^L q_i + F_i^L & \text{if } \frac{q_i}{q_i+q_j} \geq s_i \end{cases} \tag{6}$$

with $0 \leq s_i \leq 1$. That is, the firm names both a reference tariff $\left(p_i^H, F_i^H\right)$, which applies if the retailer buys less than a prescribed share s_i of his total purchases, and a reduced tariff $\left(p_i^L, F_i^L\right)$, which applies if the retailer buys at least the prescribed share. The difference is a proper *market-share discount* if $s_i < 1$, an *exclusivity discount* if $s_i = 1$.[18]

With constant marginal costs, two-part tariffs in principle allow for efficient profit extraction. In fact, with complete information and absent any kind of market imperfections, firms would set marginal prices at cost and extract the profit only by means of fixed fees. But this pattern of pricing is no longer optimal in the presence of moral hazard, adverse selection, competition among the retailers, or other contracting externalities. In these cases, upstream firms may optimally choose to reduce the fixed fees and raise marginal prices above marginal costs.

These price distortions are crucial for the demand-boost theory of market-share discounts. If a firm priced efficiently, setting marginal prices at cost and extracting the surplus by means of lump-sum payments, the increase in volumes entailed by market-share discounts would not improve profitability. In fact, the firm might have to reduce the fixed payment in order to compensate

[18] The discounts are assumed to be all-units, as they often are in reality. Notice also that both prongs of the tariffs are set simultaneoulsy. Elhauge (2009), in contrast, models loyalty discounts assuming that the firm first commits to a given discount, and then sets the price after buyers have signed the contract. See also Elhauge and Wickelgren (2015).

the retailer for the loss of the option of buying more of the rival's product. But when the price-cost gap is strictly positive at the margin, any increase in sales translates into higher profits. If the compensation required by the retailer is not too large, market-share discounts may then be profitable.

Following Calzolari et al. (2019), we capture any possible market imperfection that may create price distortions in a reduced-form way. That is, we directly assume that extracting rents by means of fixed fees creates deadweight losses: with a lump-sum payment of F_i, the firm earns F_i but the retailer loses $(1+\mu)F_i$, with $\mu \geq 0$. It is assumed that the cost μ appears only when $F_i > 0$. This guarantees that whereas fixed fees are costly, lump-sum subsidies do not entail any special benefit.

Firms set tariffs simultaneously to maximize their respective payoffs

$$\pi_i = (p_i - c_i)q_i + \mathbf{1}_i F_i, \tag{7}$$

where $\mathbf{1}_i$ is and indicator function which is 1 when $q_i > 0$ and 0 when $q_i = 0$. The retailer then chooses the quantities q_1 and q_2 so as to maximize his net payoff

$$\pi_R = v\left(q_1, q_2\right) - p_1 q_1 - p_2 q_2 - \mathbf{1}_1(1+\mu)F_1 - \mathbf{1}_2(1+\mu)F_2. \tag{8}$$

4 Equilibrium

In this section, we first derive the equilibrium when market-share discounts are prohibited and then when they are permitted. We focus on the limiting case $\mu \to \infty$, where the fixed fees vanish, and $s = 1$, where firms offer exclusivity discounts. The analysis can be extended to the case where μ is finite, so that fixed fees are positive, and $s < 1$; these extensions are developed in the working paper version of this paper. The concluding section briefly reports on how results would change.

We shall refer to the case in which the retailer purchases only one product as *exclusive representation*, and to the case where he buys both products as *common representation*.

4.1 Common representation

We start from the case where exclusivity discounts are prohibited, so each firm is restricted to offer tariffs that must apply irrespective of whether the retailer purchases also from the rival or not.

Provided that c is not too large, common representation will then prevail in equilibrium. The equilibrium is given by the intersection of the best-response functions. For the dominant firm, the best response is entirely standard. For firm 2, which is "capacity constrained," the best response has three branches, as shown in Figure 1.[19] The lower branch applies when the implied output q_2 is lower than K and is the same as if there were no capacity constraint. Along the middle branch, by contrast, firm 2 prices in such a way that the demand for product 2 is exactly equal to K. Firm 2 would like to produce more if it could do so at cost c, but is careful not to attract any demand in excess of K as serving such demand would be too costly.[20] Finally, the upper branch corresponds to prices so high that it would be profitable even to serve the demand in excess of K at the higher marginal cost $c + \theta$. As said, however, we assume that θ is so large that the equilibrium never lies on this branch of firm 2's best response.

Existence and uniqueness of the common representation equilibrium may be guaranteed by standard regularity conditions. These conditions hold, in particular, in the case of the linear demand functions (3), where the equilib-

[19] For a formal proof, see Maggi (1996).

[20] The assumption here is that the firm cannot ration demand (see Bulow et al., 1985 and Maggi, 1996 for a discussion). If instead demand could be turned down at no cost, the dominant firm would enjoy positive demand spillovers, which would create Edgeworth price cycles. The equilibrium would then involve mixed strategies. The working paper version of this paper analyzes the mixed-strategy equilibrium for the case of linear demand (3). The equilibrium can be characterized fully when the degree of product differentiation γ is small, as in this case the support of the mixed strategies is finite and small. The results are then qualitatively similar to the pure-strategy equilibrium considered here. When γ increases, however, the support of the equilibrium mixed strategies gets larger and larger, becoming a continuum for $\gamma = 1$ (Boccard and Wauthy, 2009). As a result, the characterization of the mixed-strategy equilibrium becomes very cumbersome.

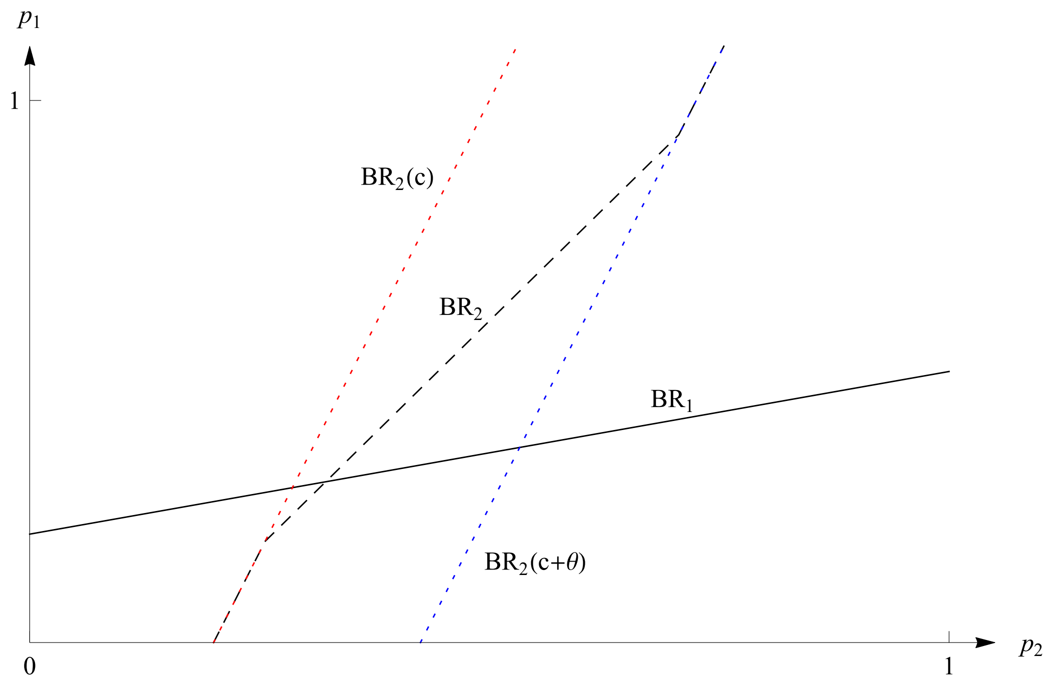

Figure 1: Best-response functions.

rium can be calculated explicitly. For $K \geq \hat{K}$, where

$$\hat{K} = \frac{1}{(2-\gamma)(1+\gamma)} - \frac{2-\gamma^2}{4-5\gamma^2+\gamma^4}c, \tag{9}$$

the equilibrium lies on the lower branch of firm 2's best-response curve and is therefore the standard Bertrand equilibrium

$$\begin{aligned} p_1 &= \frac{1-\gamma}{2-\gamma} + \frac{\gamma}{4-\gamma^2}c \\ p_2 &= \frac{1-\gamma}{2-\gamma} + \frac{2}{4-\gamma^2}c. \end{aligned} \tag{10}$$

If instead $K < \hat{K}$, the equilibrium lies on the middle branch of firm 2's best-response curve and is

$$\begin{aligned} p_1 &= \frac{(1-\gamma K)(1-\gamma^2)}{2-\gamma^2} \\ p_2 &= 1 - 2K + \frac{2K-\gamma}{2-\gamma^2}. \end{aligned} \tag{11}$$

The qualitative properties of the equilibrium are simple and intuitive. Each firm exploits the market power it enjoys thanks to product differentiation to extract a positive profit. Due to its competitive advantage, the dominant firm sets a lower price, and sells a greater output, than its rival.

Even if exclusivity discounts are prohibited, an exclusive representation equilibrium will emerge when the cost gap c is sufficiently large, or the goods are sufficiently close substitutes. In particular, under our regularity conditions there exists a threshold $c_{\lim}$ such that the common representation outcome derived above prevails when $c < c_{\lim}$.[21] When instead $c > c_{\lim}$, we have either a limit pricing equilibrium or a monopoly equilibrium. Under limit pricing, the dominant firm prices at $p_1^{\lim} = v_{q_1}(q_1^{\lim}, 0)$, where the limit quantity $q_1^{\lim}$ is implicitly defined by $v_{q_2}(q_1^{\lim}, 0) = c$. That is, the dominant firm prices in such a way that the residual demand for product 2 lies entirely below the marginal cost c. In the monopoly equilibrium, instead, the monopoly output is greater than $q_1^{\lim}$ and hence the rival does not really exert any competitive pressure on the dominant firm. The monopoly output is $q_1^M = \arg\max\left[v_{q_1}(q_1, 0)q_1\right]$, and the associated price is $p_1^M = v_{q_1}(q_1^M, 0)$. The monopoly equilibrium is obtained when $c > c_M$, where $c_M = v_{q_2}(q_1^M, 0)$. For example, with the linear demand functions (3) we have $c_{\lim} = \frac{1-\gamma}{2-\gamma}$, $p_1^{\lim} = 1 - \frac{1-c}{\gamma}$, $c_M = 1 - \frac{\gamma}{2}$, and $p_1^M = \frac{1}{2}$.

Clearly, when $c > c_M$ there is nothing to gain by offering market-share discounts. The analysis of market-share discounts is therefore interesting only when $c \leq c_M$.

4.2 Exclusive representation

If exclusivity discounts are permitted, firms can effectively choose whether to compete for each marginal unit or for the entire market.[22] Plainly, firm 2

[21] For formal proofs, see Calzolari et al. (2019).

[22] In our setting, where firms price simultaneously, an exclusivity discount with a sufficiently high list price p_1^H is effectively equivalent to an exclusive dealing arrangement. As argued by Ide, Montero and Figueroa (2016), however, this equivalence no longer holds in models where the firms move sequentially.

never has an incentive to compete for the entire market, as it would obtain zero profits, as we shall see presently. But things are different for the dominant firm. As long as c is positive, the dominant firm always has an incentive to offer a market-share discount.

The reason for this is as follows. If firms do not offer any market-share discounts, the equilibrium must be the one characterized in the previous subsection. Starting from this equilibrium, the dominant firm always has a profitable deviation. For example, the dominant firm might offer an exclusivity discount with the "reduced" price p_1^L set at the level of the common-representation equilibrium (eq. (10) or (11) in the linear demand case), and p_1^H arbitrarily large. The retailer would then be restricted to buy from either firm, but not both. Faced with the choice of which firm to buy from, the retailer would choose the dominant firm, which, as noted, charges the lower price. Since the products are substitutes, this would increase the demand for the dominant firm's product, and hence its profit. The deviation from the original equilibrium is therefore profitable.[23]

The above argument implies that the equilibrium necessarily entails exclusive representation.[24] Under exclusive representation, firms compete in *profit space*, where their products are effectively homogeneous. In other words, firms may be thought of as offering to the retailer not differentiated products, but levels of net profit. The standard Bertrand logic then implies that the dominant firm wins the competition for the market by undercutting its rival in profit space.

[23] If the original equilibrium is a limit pricing equilibrium, the dominant firm could set p_1^L at the monopoly level p_1^M. Volumes would then decrease, but the profit would still increase.

[24] In fact, this is true only if each firm can offer only one tariff. If both firms offer two tariffs, one involving exclusivity discounts and one not, there might exist multiple equilibria. One of these is always the exclusive representation equilibrium characterized below. This equilibrium is unique if it is profitable for the dominant firm. But when the dominant firm, while having a unilateral incentive to offer an exclusivity discount, eventually loses from the fiercer competition it thereby engenders, there might exist also common representation equilibria, where the dominant firm offers a discount that is not accepted by the buyer (Ramezzana, 2016). Here however we focus only on the equilibrium where exclusivity discounts are not only offered but also accepted, on the grounds that antitrust cases can be brought only if the discounts are observed in practice.

The weaker firm, which is foreclosed, must stand ready to supply its product at competitive terms. In other words, it must offer a contract that maximizes the retailer's net surplus under the constraint that its own profit is non-negative. In view of inequality (5), this entails supplying K units and results in a reservation payoff for the retailer of $v(0,K)-cK$.[25]

To undercut the rival in profit space, the dominant firm must guarantee at least this net payoff to the retailer. Subject to this "participation" constraint, the discounted price must maximize the dominant firm's profit. The solution to this problem is[26]

$$p_1^L = \min\left[p_1^M, \tilde{p}_1\right] \tag{12}$$

where p_1^M is the monopoly price, and $\tilde{p}_1$ is implicitly defined by the condition

$$\max_{q_1}[v(q_1,0)-\tilde{p}_1 q_1] = v(0,K)-cK. \tag{13}$$

The corresponding equilibrium outputs are $q_2^* = 0$ and (with obvious notation)

$$q_1^* = \max\left[q_1^M, \tilde{q}_1\right]. \tag{14}$$

For example, with the quadratic payoff (2) the dominant firm charges $p_1^M = \frac{1}{2}$ when $K \leq 1-\frac{\sqrt{3}}{2}$ and $\tilde{p}_1 = 1-\sqrt{2K-K^2}$ otherwise. Intuitively, when K is small firm 2 does not really exert any competitive pressure as the monopoly price in itself leaves to the retailer a rent greater than $v(0,K)-cK$. If instead K is large, the competitive pressure is stronger, and the dominant firm must reduce the price to guarantee participation.

While the exclusive representation outcome is pinned down fully, the off-path reference price p_1^H is not. The condition that the retailer prefers exclusive to common representation implies a lower bound on p_1^H, but apart from this, the reference price may be set arbitrarily and does not affect the equilibrium outcome.[27]

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[25] For example, one way to guarantee this reservation payoff to the retailer is to set $p_2 = v_{q_2}(0,K)$ and $F = -\left[v_{q_2}(0,K)-c\right]K$ (remember that $\mu = 0$ for lump-sum subsidies).

[26] To avoid well known issues of equilibrium existence, we assume that ties are broken in favour of the dominant firm.

[27] One way to pin down p_1^H is to modify the model so as to allow for the possibility

5 Competitive effects

Having characterized the equilibrium when market-share discounts are permitted or prohibited, it is now possible to make a comparison of the two. We consider both the effect on firms' profits and on welfare.

5.1 Profits

Firm 2 always loses from exclusivity discounts, which invariably lead to its foreclosure. As for the dominant firm, the profitability of exclusivity discounts depends crucially on the size of its "competitive advantage." Specifically, exclusivity discounts are profitable when the dominant firm's competitive advantage is big, unprofitable when it is small. By competitive advantage we mean the dominant firm's superior ability to compete for the entire market. It is bigger, the greater is c and the lower is K: indeed, both factors combine to decrease the retailer's reservation payoff $v(0, K) - cK$.

Figure 2 shows the *profit frontier*, c_{PROF}, demarcating the region where exclusivity discounts are profitable or unprofitable, for the case of linear demand functions (3).[28] Below the frontier, where the competitive advantage is small (c small and K large), the dominant firm is caught in a sort of prisoner dilemma: it has a unilateral incentive to offer exclusivity discounts, but is eventually harmed by the more intense competition it thereby engenders. Above the frontier, the competitive advantage is big (c large and K small), and the dominant firm benefits from exclusivity discounts.

To see intuitively why a larger competitive advantage makes exclusivity discounts more likely to be profitable, remember that the upside of the discounts is that they boost the demand for the dominant firm's product. The downside is that profit extraction is constrained by the retailer's par-

that some sales occur at the reference price p_1^H, as for instance in Greenlee and Reitman (2006).

[28] Even if the explicit solutions for the linear demand case are relatively simple, the equation of the profit frontier is very cumbersome. It is therefore relegated to an online appendix, which is available at Harvard Dataverse, https://doi.org/10.7910/DVN/JQQQ9V. The same is true of the welfare frontier and the test frontier introduced below.

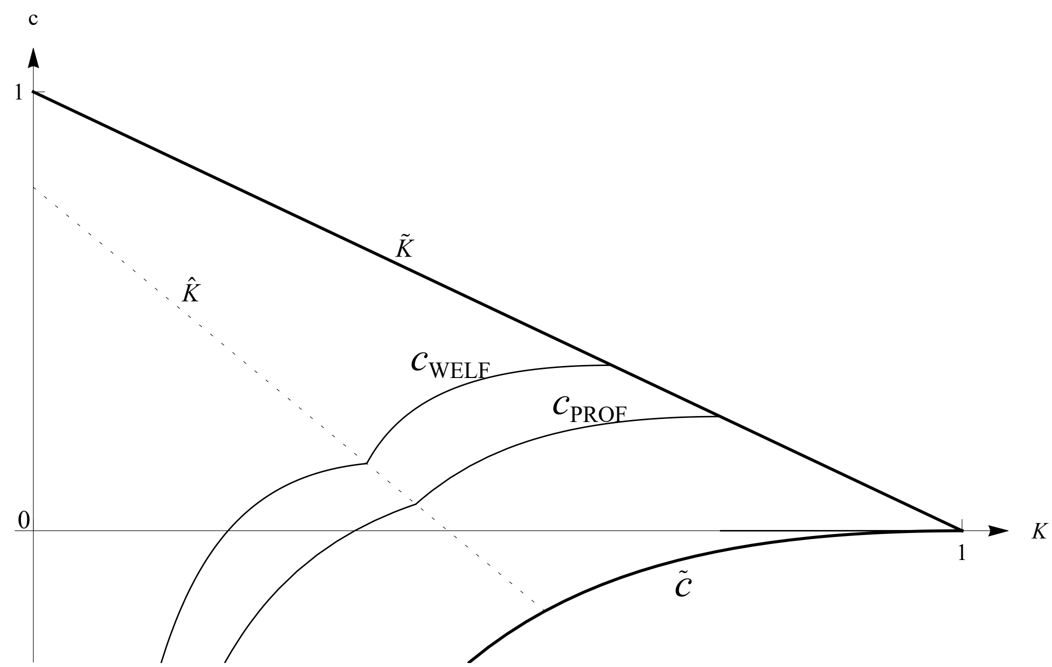

Figure 2: Exclusivity discounts are profitable above the c_{PROF} curve, and welfare decreasing above the c_{WELF} curve. The upper and lower bounds $\tilde{K}$ and $\tilde{c}$ are defined by (5) and footnote 17, respectively, whereas $\hat{K}$ is given by (9). The figure is drawn for $\gamma = 0.6$.

ticipation constraint, which requires that he obtains at least the reservation payoff. This condition imposes an upper bound on the dominant firm's price, $\tilde{p}_1$. When the competitive advantage is small, this upper bound is tight, so there is little to be gained from the increase in volumes. For example, when c is close to zero and K is close to $\bar{q}$, so that inequality (5) barely holds, the dominant firm's profits under exclusivity discounts tend to vanish. Under common representation, by contrast, the dominant firm could take advantage of product differentiation to obtain positive profits even in those circumstances. If instead c is large and K is small, there is more room for taking advantage of the boost in demand. In fact, the dominant firm may be able to engage in monopoly pricing even if $c < c_M$, in which case exclusivity discounts are definitely profitable.

5.2 Consumer welfare

To assess the welfare impact of market-share discounts, we focus on consumer surplus. In our model, consumer surplus can be proxied by the retailer's payoff, $v(q_1, q_2) - p_1 q_1 - p_2 q_2$.[29] Using this criterion, it is easy to see that the impact of market-share discounts on welfare is ambiguous.

Intuitively, there are two effects. On the one hand, market-share discounts modify the equilibrium prices. On the other hand, they reduce product variety (which is, in fact, completely lost under exclusive representation).

When the dominant firm's competitive advantage is big, exclusivity discounts do not entail a significant reduction in the dominant firm's price. As noted, the price could in fact even raise up to the monopoly level. And since product variety is completely wiped out, the discounts are definitely anti-competitive.

When instead the dominant firm's competitive advantage is small, the price reduction is more substantial and may more than compensate the buyer for the loss of product variety. In this case, exclusivity discounts may become pro-competitive.

This possibility is illustrated, for the case of linear demand, in Figure 2. Along with the profit frontier discussed above, Figure 2 shows also the *welfare frontier*, c_{WELF}, demarcating the regions where exclusivity discounts decrease or increase consumer surplus. The figure is drawn for a value of γ such that the products are already fairly good substitutes. As γ increases further, however, the region where exclusivity discounts are pro-competitive shrinks and disappears altogether when $\gamma = 1$: in our model, exclusivity discounts are always anti-competitive when the product is homogeneous.

The reason for this is two-fold. First, when the product is homogeneous exclusive representation does not entail any actual loss of product variety. Second, under common representation the equilibrium is always one of limit pricing, where the dominant firm undercuts the rival by pricing just below c.

[29] Even if the fixed fees were positive, one ought to abstract from them as they are a fixed cost for the retailer, which as such may not be passed on to consumer prices.

With exclusivity discounts, in contrast, the dominant firm can leverage on the second source of its competitive advantage (namely, the fact that firm 2 is capacity constraint) to raise its price above c. This increases its profits, but decreases social welfare.

6 The discount attribution test

In the preceding section, we have seen that exclusivity discounts may be either pro- or anti-competitive, depending on the circumstances. We now ask whether price-cost tests may help screen out anti-competitive cases.

Essentially, price-cost tests ask the following question. Suppose that an as-efficient competitor prices at cost; given the dominant firm's price schedule, can the retailer reduce his total expenditure by diverting his purchases away from the dominant firm and towards the rival? If the answer is yes, then the test is passed. Competition is viable, and the discounts are not regarded as anti-competitive. Should foreclosure nevertheless be observed, it must be because the excluded firm is in fact less efficient. If the answer is no, then even an equally efficient competitor would be foreclosed by the dominant firm's pricing strategy. The test then fails, and the discounts are presumed to be anti-competitive.

The traditional version of the price-cost test assumes that for each unit diverted towards to rival, the buyer would save the discounted price charged by the dominant firm. Under this assumption, the test reduces to a comparison between the dominant firm's discounted price, p_1^L, and its unit cost. Therefore, the test would always pass when loyalty discounts are directly profitable and do not entail a sacrifice of profit, as is the case in the demand-boost theory.

But the discount-attribution test that is increasingly being applied in loyalty discount cases is subtler. It accounts for the possibility that output diversion may not be complete because the competitor may not be able to perfectly replace the dominant firm as the buyer' sole supplier.

In our model, output diversion has indeed an upper bound of K. Accounting for this, the discount attribution test becomes (reverting, for better clarity, to the general notation where marginal costs are c_i)

$$c_1 K + p_1^H (q_1^* - K) \leq p_1^L q_1^*. \tag{15}$$

The right-hand side of the inequality is the actual expenditure, given that in equilibrium the retailer purchases only from the dominant firm at the discounted price p_1^L. The left-hand side is what the retailer would spend in the counterfactual where he purchases as much as he can from an as-efficient competitor that prices at cost ($p_2 = c_2 = c_1$), and the rest from the dominant firm at the reference price p_1^H. The test is passed if and only if the inequality holds.

From (15), it follows that the price effectively used for the comparison is no longer p_1^L. Rewriting inequality (15) as

$$p_1^H - \frac{p_1^H - p_1^L}{\frac{K}{q_1^*}} \geq c_1, \tag{16}$$

it appears that the relevant price, i.e., the left-hand side of (16), is lower than p_1^L as the denominator is less than 1. Essentially, for the purposes of the test the discount $(p_1^H - p_1^L)$ is attributed only to the contestable share of the market,

$$s_C = \frac{K}{q_1^*}.\text{[30]} \tag{17}$$

6.1 The reference price

In the attribution test, five variables are involved: the dominant firm's volume, the contestable share, the dominant firm's marginal cost, the actual

[30] Another way to restate inequality (15) is to compare the contestable share to the so-called "required" share

$$s_R = \frac{p_1^H - p_1^L}{p_1^H - c_1}.$$

The test passes if and only if $s_C \geq s_R$. This is the formula applied, for instance, by the European Commission in the *Intel* case (Decision C(2009) 3726 of 13 May 2009).

(discounted) price, and the hypothetical price that would apply if the buyer did not qualify for the discount.

In practice, some of these variables may be observed but others must be estimated.[31] As discussed above, however, here we abstract from measurement issues. In our model, the dominant firm's marginal cost, c_1, and the contestable output, K, are just parameters. The actual quantity, q_1^*, and the discounted price, p_1^L, are fully pinned down in equilibrium (eq. (12) and (14)).

What is not pinned down uniquely is the reference price p_1^H.[32] However, we can determine a lower bound for p_1^H as this price must be sufficiently high that the retailer would prefer the reduced price p_1^L, in spite of the loss of product variety. To prevent profitable deviations by firm 2, the payoff obtained by the retailer by accepting the reduced price must in fact exceed the largest joint payoff obtainable by the retailer and firm 2. The condition is

$$\max_{q_1}[v(q_1, K) - p_1^H q_1 - c_2 K] \leq \max_{q_1}[v(q_1, 0) - p_1^L q_1]. \tag{18}$$

This condition uniquely determines a lower bound for p_1^H, $\underline{p_1^H}$.[33] It may be

[31] Actual volume and price are typically easy to observe. The dominant firm's cost must be estimated, but this poses the same problems as in standard predatory pricing tests. Over the years, antitrust authorities and the courts have come to cope with these problems. The estimation of the contestable demand, by contrast, is a very difficult problem that arises specifically when the attribution test is applied, as discussed in more detail in Section 2 above.

[32] In actual practice, the reference price is often taken to be the list price. But this may overestimate the size of the discount. In fact, list prices are often inflated, and even buyers who do not reach the target may often get a discount over the list price. The *Intel* case is a relevant example. Intel's informal contract with Dell specified the amount of the discount Dell was entitled to in case of exclusivity, but there was substantial uncertainty about what price would apply if Dell breached the exclusivity clause. Dell was confident that it would still obtain a discount over the list price, but the exact amount of the discount was a matter of considerable speculation. See the European Commission's Decision C(2009) 3726 of 13 May 2009, paras 256-61.

[33] The lower bound may take on two values. When $p_1^L = \tilde{p}_1$, it is

$$\underline{p_1^H} = v_{q_1}(0, K).$$

Intuitively, off-path the retailer buys K units of product 2 only, so the price p_1^H must be so high that he does not gain any extra surplus by purchasing, on top of that, some positive amount of product 1. When instead $p_1^L = p_1^M$, the price p_1^H can be somewhat reduced.

argued that p_1^H should indeed be set at this lower bound.[34] However, all we need for our purposes is that in any exclusive representation equilibrium $p_1^H \geq \underline{p}_1^H$.

6.2 Symmetric costs

With all the necessary ingredients at hand, we can now analyze the results that would be produced by the test in an industry that is represented exactly by our model. We start, in this subsection, by assuming that $c_2 = c_1$ not only hypothetically but also in reality. In this case, the only source of the dominant firm's competitive advantage is that firm 2 is capacity constrained.

For this case, one can show that the discount-attribution test is never passed, for any $p_1^H \geq \underline{p}_1^H$. In other words, the test would indicate that exclusivity discounts are always anti-competitive, even if we know that they are pro-competitive under certain conditions.

The failure of the discount-attribution test is diametrically opposed to that of the traditional price-cost test: the former says that loyalty discounts are always anti-competitive, the latter that they are always pro-competitive. This may help explain why price-cost tests tend to be a synonym of lenient antitrust policy when the traditional test is applied, and instead of toughness when the discount-attribution variant is applied.

The proof that the attribution test always fails when $c_2 = c_1$ is very simple. For the test to pass, the retailer should be able to purchase $q_1^* - K$ units of product 1 and K units of product 2 at a lower total cost than q_1^*

The lower bound is implicitly defined by the condition

$$\max_{q_1}[v(q_1, K) - p_1^H q_1 - c_2 K] = v(q_1^M, 0) - p_1^M q_1^M.$$

[34]The justification is twofold. First, it is easy to confirm that $\underline{p}_1^H$ is always greater than the profit-maximizing price under common representation. Assuming that the profit function is quasi-concave, this implies that $\underline{p}_1^H$ minimizes the profit loss that the dominant firm would suffer if the retailer inadvertently chose the reference price, p_1^H, rather than the reduced one, p_1^L. Second, if the dominant firm is concerned about the possibility of antitrust intervention, it must choose the price that keeps the size of the discount to a minimum as this minimizes the risk that the test is failed and the discount is found anticompetitive. The risk-minimizing price is precisely $\underline{p}_1^H$.

units of product 1 only. But if this were so, then one should observe common rather than exclusive representation in equilibrium. The reason for this is that symmetry and concavity of the payoff function imply that the retailer has a preference for variety: $v(q_1^* - K, K) > v(q_1^*, 0)$. If the total quantity were the same and total expenditure not greater, he would then prefer to buy from both firms rather than from only one. However, in equilibrium the retailer accepts the exclusivity discount offered by the dominant firm. This contradiction proves the result by *reductio ad absurdum*.

6.3 Asymmetric costs

While the test refers to a *hypothetical* as efficient competitor, it may of course be applied even if the *actual* competitor is less (or more) efficient than the dominant firm. (Note that exclusion is not necessarily efficient even if $c_2 > c_1$, as the products are differentiated.) The above argument implies that the attribution test would obviously continue to fail when $c_2 < c_1$, as in this case firm 2 could profitably attract the retailer if the test were passed. However, the discount-attribution test may be passed if $c_2 > c_1$.

This is illustrated in Figure 3 for the case of linear demand. The figure depicts the frontier between the regions where the exclusivity discount would pass or fail the attribution test.[35] To facilitate the comparison, the figure reproduces also the frontier between the regions where the discount is pro- or anti-competitive. Ideally, the two frontiers should coincide. In practice, small differences could be tolerated, but in fact the welfare frontier is increasing while the *test frontier* is mostly decreasing,[36] and the two frontiers are nearly orthogonal to each other. In the areas of disagreement, which are depicted in grey in Figure 3, the test delivers either type I or type II errors. Evidently, errors are systematic. In fact, they are so prevalent that the price-cost test

[35] For the derivation of the test frontier, we have set $p_1^H = \underline{p_1^H}$.

[36] The test frontier can be non-monotone only when $p_1^L = \overline{p_1^M}$ (a case where exclusivity discounts are always anticompetitive). The reason for the possible non-monotonicity is that the frontier assumes that p_1^H is set at the lower bound $\underline{p_1^H}$, and this depends on the cost gap. If p_1^H were constant, the test frontier would always be decreasing.

is completely uninformative.

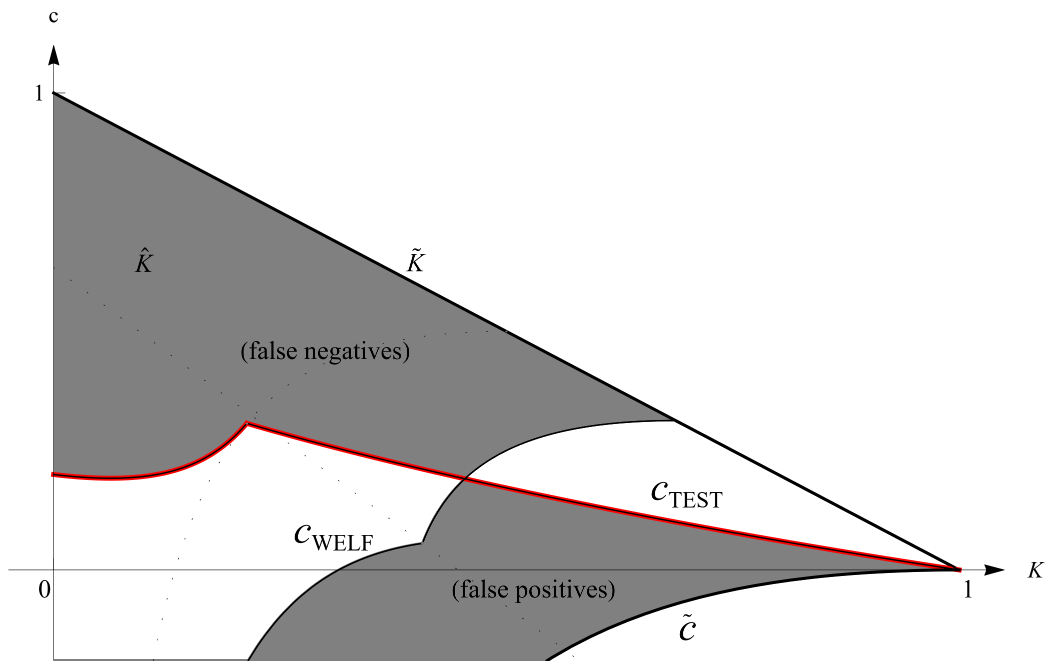

Figure 3: The test is correct in the white areas, mistaken in the grey ones. More precisely, the test fails below the test frontier and exclusivity discounts are anti-competitive above the welfare frontier. Therefore, in the upper grey region the test delivers false negatives (discounts are anti-competitive but pass the test), in the lower one false positives (discounts are pro-competitive but the test fails). The figure is drawn for $\gamma = 0.6$.

While Figure 3 represents the case of linear demand, the discount attribution test is destined to remain uninformative even with more general demand functions. Indeed, for any specification of demand, (i) the welfare frontier is always increasing, and (ii) the test frontier is always decreasing when the welfare effects are potentially ambiguous. Claim (i) follows from the fact that the welfare effects of exclusivity discounts depend on the overall size of the competitive advantage, and the two sources of competitive advantage substitute for each other. An increase in the cost gap, for instance, may be compensated by an increase in K (the capacity constraint becomes looser), and vice versa. Claim (ii) follows from the fact that exclusivity discounts are always anticompetitive when $p_1^L = p_1^M$, and the test frontier is always decreasing when $p_1^L = \tilde{p}_1$. The reason for this latter property is that reducing the cost gap c makes the test harder to pass (the reduced price p_1^L must fall as the competitive pressure from the rival gets stronger), whereas increasing K makes the test easier to pass (the contestable share increases). For example, when $c = 0$ the test always fails, and when $K = \tilde{K}$, so that the competitive advantage is due only to cost gap, it always passes, as it then boils down to the traditional test.

7 A modified test

In the previous section, we have analyzed the discount attribution test as it is actually applied in practice. As noted, however, the test implicitly treats the products of the two firms as if they were homogeneous, whereas in the model (and, often, in the real world) they are differentiated. One may wonder that this is the reason why the test is flawed.

To address this concern, this section proposes a variant of the attribution test that accounts for product differentiation. This variant requires the estimation of one more variable and thus may be impractical, but, once again, we abstract from measurement issues. The analysis shows that even this variant of the test, albeit conceptually better grounded, fails to screen out anti-competitive cases.

7.1 Local as efficiency

Remember that the antitrust concern is that a dominant firm, which enjoys a competitive advantage in the competition for the market, may use market-share discounts to foreclose a competitor that ought not to be excluded being equally, if not more efficient at the margin. Now, if the products are differentiated, the competitor may be equally efficient at the margin even if $c_2 > c_1$ provided that the marginal value of product 2 is greater than that of product 1. With symmetric demand and diminishing marginal willingness to pay, this is precisely what happens when q_1 is larger than q_2, and hence, in particular, when firm 2 is foreclosed.

To capture this idea formally, we say that firm 1 and 2 are *locally* as efficient at $(\bar{q}_1, \bar{q}_2)$ if and only if

$$v_{q_1}(\bar{q}_1, \bar{q}_2) - c_1 = v_{q_2}(\bar{q}_1, \bar{q}_2) - c_2. \tag{19}$$

In other words, local-as-efficient means that the marginal value of product 1, net of the production cost, is as large as that of product 2.

One can therefore think of another version of the test, which assumes that the hypothetical competitor is locally as efficient as the dominant firm at (q_1^*, q_2^*) in the sense of condition (19). Since in a foreclosure equilibrium $q_2^* = 0$, the hypothetical as-efficient competitor must have a cost $\tilde{c}_2$ implicitly given by

$$\tilde{c}_2 = c_1 + \left[v_{q_2}(q_1^*, 0) - v_{q_1}(q_1^*, 0)\right]. \tag{20}$$

Since the term inside square brackets is positive, firm 2 can be locally as efficient as the dominant firm even if its unit cost is substantially higher.

7.2 The test

With this new notion of equal efficiency, noting that $v_{q_1}(q_1^*, 0) = p_1^L$ the test becomes

$$\left\{c_1 + \left[v_{q_2}(q_1^*, 0) - p_1^L\right]\right\} K + p_1^H(q_1^* - K) \leq p_1^L q_1^*. \tag{21}$$

Plainly, this modified test is more difficult to pass than the standard attribution test. The implication of this is twofold. First, the test always fails, *a fortiori*, when $c_2 \leq c_1$. Second, for $c_2 > c_1$ the likelihood of false negatives decreases but that of false positives increases, as shown in Figure 4. Overall, the precision of the test does not seem to improve.

We therefore conclude that accounting for product differentiation does not fix the problems we have highlighted. The discount-attribution test remains uninformative; if anything, it actually tends to become even more misleading.

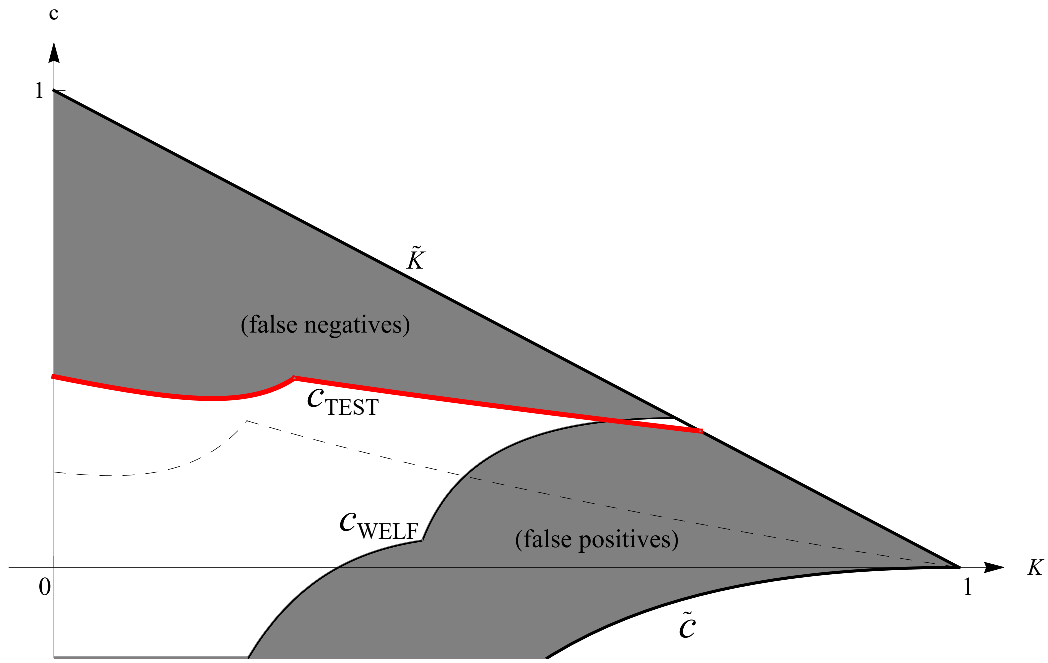

Figure 4: The modified test for local-as-efficiency: the test is correct in the white areas, mistaken in the grey ones. To facilitate the comparison with the previous section, the frontier for the standard incremental test is reproduced here as the dashed curve. The figure is drawn for $\gamma = 0.6$.

8 Conclusions

In this paper, we have analyzed by means of a formal economic model the use of the discount-attribution price-cost test to assess the competitive effects of market-share discounts. In the model, a dominant firm uses market-share discounts as a means to boost the demand for its product. In this framework, market-share discounts may be profitable directly, without necessarily entailing an immediate sacrifice of profit, and hence the need of recoupment. The discounts are pro-competitive if the competitive advantage of the dominant firm is big, anti-competitive if it is small.

Our analysis has shown that if the competitor is actually as efficient, at the margin, as the dominant firm, then the discount-attribution test always fails and thus is of no help in screening out the anticompetitive cases. When the dominant firm is more efficient not only in the competition for the market but also at the margin, the attribution test can be passed. However, even in this case the test is misleading or, at best, uninformative.

For ease of exposition, we have presented our results for the limiting case $\mu \to \infty$, where the fixed fees vanish, and $s = 1$, where firms offer exclusivity discounts. The working paper version of this paper shows that the qualitative results do not change when μ is finite and hence the fixed fees are positive. When instead $s < 1$, the test frontier shifts down whereas the welfare frontier stays unchanged. This implies that there are fewer false positives, but also more false negatives. Overall, the test remains largely uninformative.

The general message of this paper is that the application of price-cost tests to loyalty discount cases is problematic. Our analysis cannot rule out the possibility that price-cost tests may be informative in different models. However, their drastic failure in sensible and realistic settings suggests that they ought perhaps to be simply dismissed. With what should they be replaced, is a problem that we address in another paper.

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