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Understanding the acoustics of St. John's Baptistery in Pisa through a virtual approach

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Abstract

The virtual acoustics provides an important tool in the comprehension of the architectural evolution of a historical place, offering the possibility to consciously operate for its preservation. St. John's Baptistery represents an outstanding example of medieval architecture, recognised as UNESCO world heritage and popular for its huge reverberation. When compared to other similar central-symmetry buildings, the Baptistery proves to be an outlier, enough to raise the interest of scholars. A model of the building was calibrated considering in-situ acoustic measurements, the material properties and the liturgical use. The numerical simulations of the current geometry and the early design hypotheses are used to understand the factors affecting the peculiar sound behaviour. The temporal analysis of a Gregorian chant - ad hoc recorded - seems to confirm the needing of large reverberation.

Keywords Acoustic simulation, Archaeoacoustics, Worship acoustics, Gregorian chants, Intangible cultural heritage.

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1 Introduction

Considering the worship space as one of the places of the cultural birth and growth for the western society [1], several works about ruins and destroyed buildings used virtual acoustics as tool for supporting the archaeological research [2, 3]. By means of virtual acoustics techniques, others works investigated acoustic peculiarities of ancient places of our cultural heritage, which had been refurbished in the past [4, 5] or are used in alternative ways at the present time [6, 7]. Indeed, the historical analysis about the relationship between the building and its acoustics highlights the importance to consider the "intangible" cultural heritage [8]. The so-called archaeoacoustics involves historical research about the construction process, an expertise of numerical simulations techniques including the calibration [9], and the knowledge of uses and rituals [10]. Several studies carried out simulation results of acoustic parameters in worship places by comparing the calibration techniques [11, 12], or by evaluating the influence of source placements [13], of furnitures [14], or occupancy conditions [15].

The interest of acousticians was often focused on a central-plan worship spaces. This kind of architectures include *rotundae*[16], churches [17, 18, 19] and mosques [20, 21]. The symmetry of these spaces, which recalls symbolic religious elements, presents also peculiarities from the acoustic point of view. The focusing effects are due to the time-aligned reflections and the so-colled "whispering gallery" are due to the wave-guide effect of domes. These aspects are usually studied as acoustics problems [22, 23]. Furnishings or carpets on the floor de-emphasize these effects in mosques [24].

Desarnaulds [25] and Meyer [26] pointed out the crucial issue in the more general topic of worship acoustics: on one hand these spaces require enough reverberation to keep the faithful enevloped by choir and musics; on the other hand a too high level of reverberation "unfocused" the liturgical narrative, that requires sound clarity to be understood. Some scholars attempted to compensate the lack of a specific method which would allow to assess the acoustic quality of this typology of interiors. Their study lead to a single index of acoustic quality [27], or to a double index that takes into account both speech and music conditions [28]. Further effort was made to define the optimal reverberation time as preferred subjective listening condition in case of Byzantine chants [29]. The effect of different musical motifs (spanning from Gregorian chants to symphonic music) was also investigated in a survey carried out in Catholic churches [30], through a regression analysis was evaluated in order to point out the relationship between subjective judgements and acoustical parameters. Main results of the latter research show that preferred listening conditions vary as a function of the musical motif, depending on early decay time for choirs and on lateral energy for instruments. Temporal properties of the sound signal were used in more rigorous way by Ando [31], and by Vitale et al. [32].

Taking into account all the findings resumed in this introduction, the aim of the present work is to explore a number of open research questions on the Baptistery. Why the acoustics of St. John Baptistery has been interesting for both scholars and common people? Can the measurement results confirm the subjective perceptions? In the present study numerical techniques aid in answering of these questions and better understanding the peculiarities of this acoustic space, the factors which influence the huge reverberation.

2 History and liturgy in the Baptistery

2.1 The early vicissitudes of St. John's Baptistery

At the time of the construction of St. John's Baptistery, in 1153, Pisa was at the height of its importance and it came to contact with the vast Islamic world, from which it drew typological and decorative references, and with the Greek world, from which derived the metaphysical approach to sacred architecture. The vicissitudes of construction and the authorship of

Figure 1: *Piazza del Duomo* in Pisa, UNESCO World heritage site [33]: the Baptistery, the Cathedral and the Tower.

several parts of the architectural complex of the Baptistery are the subject of different and discordant theories. There was a stratification of interventions, solutions started and then abandoned, transformations and operations of union of style. The main development of this project can still be organized into four main periods differentiated from the figures who led the construction and who gave their imprint to a part of the building.

12th C. Diotisalvi, a medieval architect and sculptor, was responsible for the fundamental features of the building: he set the project combining formal elements of the facade and Romanesque architectural culture, inspired also by the church of Holy Sepulcre in Jerusalem. The construction of the Baptistery had a first phase of great fervour, but subsequently, the site suffered a progressive slowdown, until it came to a complete stop.

13th C. In 1260 it is testified the installation of the pulpit made by the sculptor Nicola Pisano (1223–1281). From this moment on, he took on the role of magister and the Baptistery's decorative style changes profoundly.

late 13th C. – early 14th C. In the years following 1278, Giovanni Pisano (1248–1315), son of Nicola, intervened in the Baptistery.

14th C. Numerous workers took part in the final phase of the work, which saw the final arrangement of the roof. In 1395 the construction of the Baptistery ended.

A construction carried out in stages, moreover not consequent, led to an autonomous and incongruous design of the baptistery compared to the harmonious continuity of the complex of *Piazza del Duomo* [34]. The perceptible difference between the Baptistery and the external context gave rise to a long debate on what was the form originally thought by the designer Diotisalvi. Speiser conducted an in-depth historical and architectural research of the Baptistery, parallel to a comparison with the other buildings of *Piazza del Duomo*: he made important considerations about internal and external spatial geometry, basing on the geometry and the proportions of the buildings [35]. Some authors accept the hypothesis of a project designed by a single mind, even though it was carried out at different times; others perceive the typological discontinuity between the plan and the elevation. De Fleury's [36] and Boeck's later [37], focused on the figure of a cusped dome open on the top with heights corresponding to the current building. The geometrical differences they found respect the actual Baptistery focus on the *matroneum*: observing the historical drawing it is possible to note that there are no arches in the vaulted ceiling and according to De Fleury the height was lower than the actual one (figure 2 (b), 2(c)). Another proposal, which deviate from the actual design, is conducted by Manenti Valli [38]. The study is based on the analysis of symbols and numerical proportions of the whole complex of Piazza del Duomo. In the Manenti Valli's hypotesis the height of the building appears considerably reduced compared to the current aspect and changes the internal proportions: the *matroneum* has a much lower height and a hemispherical dome is placed on the inner ring, instead of the conical dome. A comparison between the actual Baptistery and the different proposals is shown in figure 2.

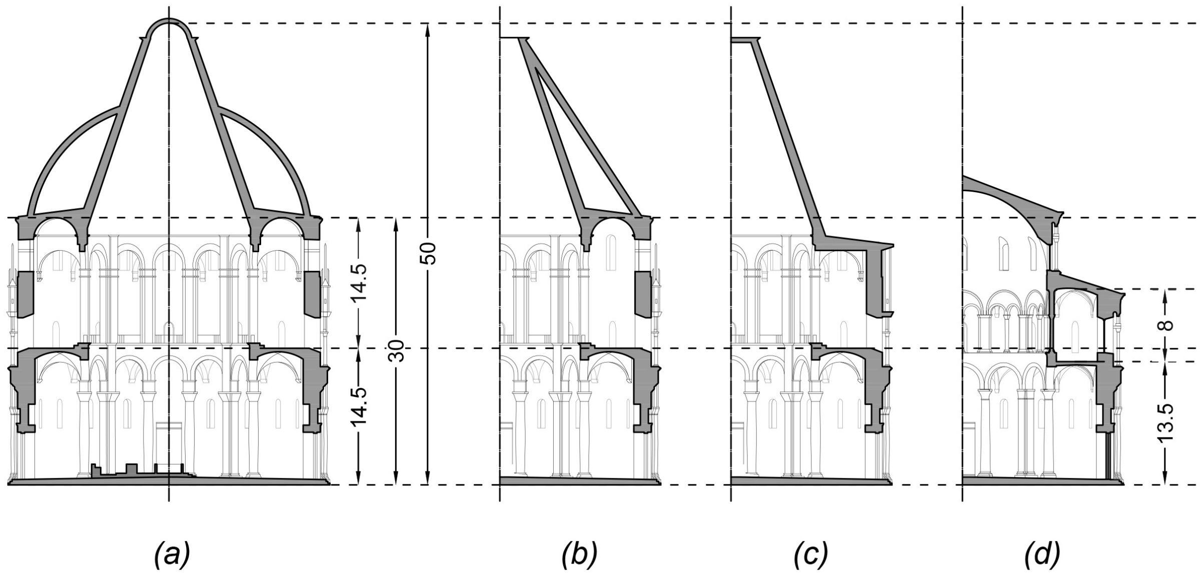


Figure 2: Comparison between the current Baptistery *(a)* and the three historical hypotheses about the early design of the building: Boeck's *(b)*, De Fleury's *(c)* and Manenti Valli's *(d)*.

2.2 The liturgical use

The baptisteries were built with the aim of celebrate the ceremony during which people were welcomed into the Christian community. At the time of the building, the rite of baptism involved a group of novices who accessed the baptistery from the portal facing west. The novices could access the baptismal font where they entered and immersed themselves. A reference to this baptismal rite of the 13$^{\text{th}}$ Century by immersion, may be found in the XIX *canto* of Hell of Dante's *Divine Comedy.* Once the ceremony was over, the group of novices left the baptistery and headed towards the east door and the church where the celebration was held with all the faithful [39].

Christian lyturgical tradition, given its purely oral diffusion, remained for a long time the domain of a narrow circle of people and it is only at the end of the 16th Century that the prayers and liturgical chants began to be written. A real musical theory began to spread in Christian countries and this process is commonly attributed to Pope Gregory the Great (540–604, Papacy 590–604), hence the name of *Gregorian Chants*. Religious buildings thus became the privileged places for the composition and reproduction of songs. From the 13^{th} Century onwards, with the spread of Gregorian chants and the birth of polyphony, the link between music and architecture was consolidated: the great reverberation of the churches led to the composition of music. St. John's Baptistery was active from the 13^{th} Century to nowadays, hence different chant and musical writing typologies were succeeded inside. It is however conceivable that the acoustics of the Baptistery could be direct to specific chant typology, as studied for St. Vitale in Ravenna basing on two anechoic parts of chant [40]. A 19^{th} Century Massenet's oratorio – recorded in near field during a performance of the Conservatoire de Paris – was used to study the acoustics of Notre-Dame de Paris [41].

There are no direct documents that prove that the baptisteries were used for music, as well as there are few information on the performance of Gregorian chants. However, there are several Gregorian chants written for the Easter rites, during which baptismals were celebrated.

2.3 The building

The St. John's Baptistery consists in an iposing building with a circular plan surmounted by a high conical dome (see fig. 3). The interior of the ground floor is subdivided by columns into two areas: the first is the inner ring, which delimits the central nucleus where the octagonal baptismal font, the altar and the pulpit are located, and the second is the ambulatory, accessible by four doors from the outside. This latter, consisting of marble blocks, has sixteen small windows and a series of steps along the perimeter that form a system of seating. Columns and pillars, that saperate the areas, are linked by round arches and they are connected to the perimetral wall by plastered cross vaulted. On the marble floor are present some ancient tombs and several sculptural elements are present on the baptismal font and on the hexagonal Nicola Pisano's pulpit, built and carved in marble. The altar, slightly smaller than the font and located behind it, is surrounded by a wooden bench. The capitals of the pillars and columns are decorated with floral and zoomorphic motifs. Along the perimeter wall of the ambulatory there are two small doors that give access to narrow staircases, developed inside the thick septum of the walls, that lead to the upper floors. The first floor consists in the so-called *matronuem*, similar to the ambulatory for its size, but much more bare in the ornaments. The capitals of columns and pillars are reduced to simple frames, and the plastered vaults are barrel shaped. The gallery is paved in terracotta and has a large double step that allows to look down to the ground floor and up to the large dome. To access the vault system tunnel from the second level there is only one staircase. The room is accessible by an annular gallery, the resulting space between the intrados of the outer dome and the extrados of the vault covering the central nucleus.

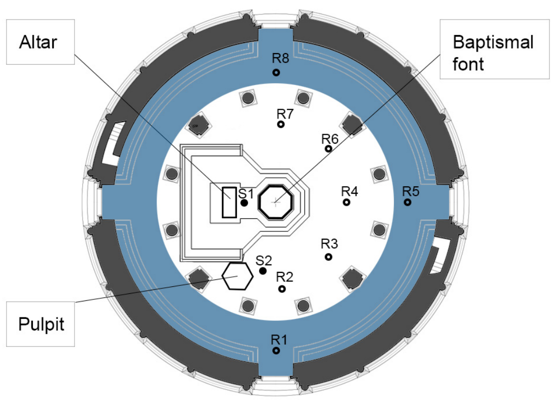
3 Method and materials

Following the state-of-the-art procedures the work involved several steps. A preliminary campaign of *in situ* measurements was carried out in order to calibrate a numerical model. The calibrated model was then used to study the sound behaviour in the Baptistery in the current and in the past configurations. Liturgy and chants were taken into account both in the simulation process and in the discussions, exploiting historical resources and technical research.

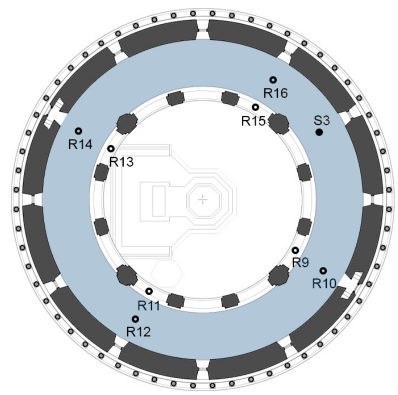
3.1 Measuring acoustic properties

The acoustic measurements of the Baptistery was performed in April 2019 in order to collect a set of objective room criteria as reference point during the calibration of the virtual model. The Exponential Sine Sweep (ESS) technique (512k at 48 kHz) allowed a proper signal-to-noise ratio ($> 45\,\text{dB}$) in each octave band, according to ISO 3382 requirements [42]. Two high-SPL dodecahedrons [43] and six monoaural microphones were used simultaneously in order to carry out the whole survey during one night of measurements.

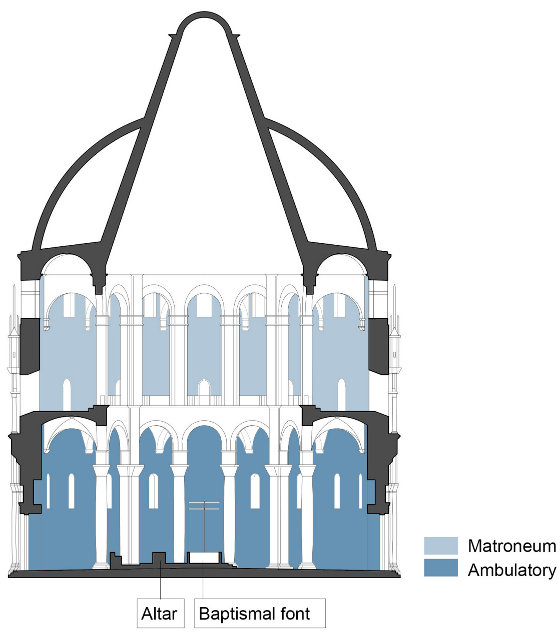
The placement of the sound sources and the receivers was determined in view of the particular spatial distribution of the volumes within the Baptistery (see fig. 3). The goal was to investigate the mutual acoustic effects between the main volume and the coupled volume of *matroneum* at the first floor. The first two sound source positions at the ground floor - on the altar (S1) and on the pulpit (S2) - are in line with the liturgical use [44, 13]. A third sound source (S3) was placed in the *matroneum* to understand the effect of this area on the whole sound field behaviour. Three monoaural receivers were placed in the ambulatory (R1, R5, R8), five around the baptismal font (R2, R3, R4, R6, R7) and eight



(a) Plan of the ground floor



(b) Plan of the first floor



(c) Cross section

Figure 3: Plans of ground floor and first floor (above) and cross section (below) of the current geomety of the Baptistery. The sound sources (S) and the receivers (R) are spread in the ambulatory, in the central area around the baptismal font and in the *matroneum*.

in the *matroneum* (R9, R11, R13, R15 between the columns and R10, R12, R14, R16 most backward in the coupled volume).

3.2 Creating the virtual models

Four distinct 3D virtual models were built using a modelling software: one corresponding to the current geometry of the Baptistery and the other three models corresponding to the early design hypotheses previously mentioned (see fig 2). During the realisation of all the models the reduction of the architectural complexity was followed as general guideline to not increase the computational effort [45]. Elements that are small compared with the size of the whole geometry, e.g. most of the ornaments and decorations, were not modelled at all. The baptismal font and the pulpit were built as simplified objects with equivalent overall dimensions, as shown in fig. 4. The curves of elements as the dome and the vaults were

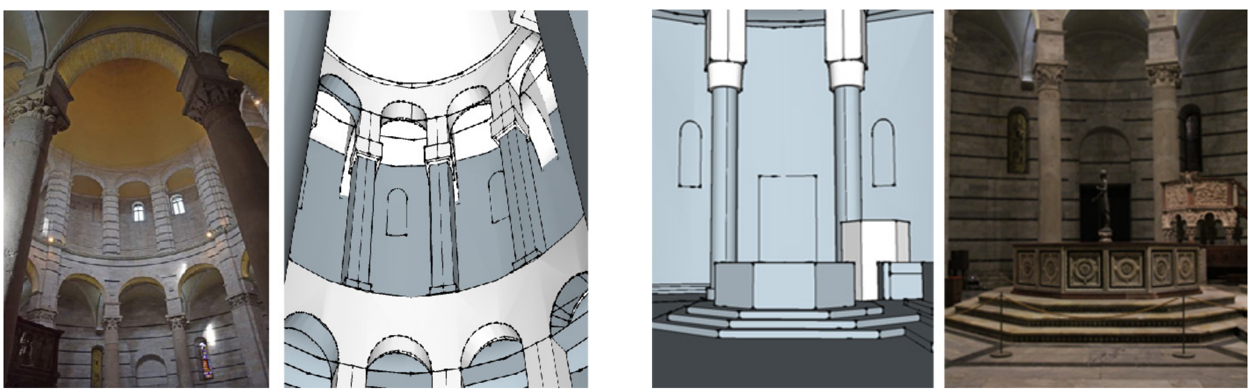


Figure 4: Comparison between actual architectural elements and the corresponding simplified object in the 3D virtual model. On the left a view of the *matroneum*, on the right view of the pulpit.

approximated with a discretization process. As a consequence of all these approximations, the outcoming geometry of all the models became considerably simplified and made up only of flat surfaces. As a common practice [45], each layer in the models corresponds to a specific material in the acoustic simulation phase. A small number of layers allows to reduce the uncertainty connected to the assignment of material properties. Therefore, each model involves five layers, representing the main materials: marble, plaster, terracotta, glass and wood.

3.3 Recording anechoic music materials

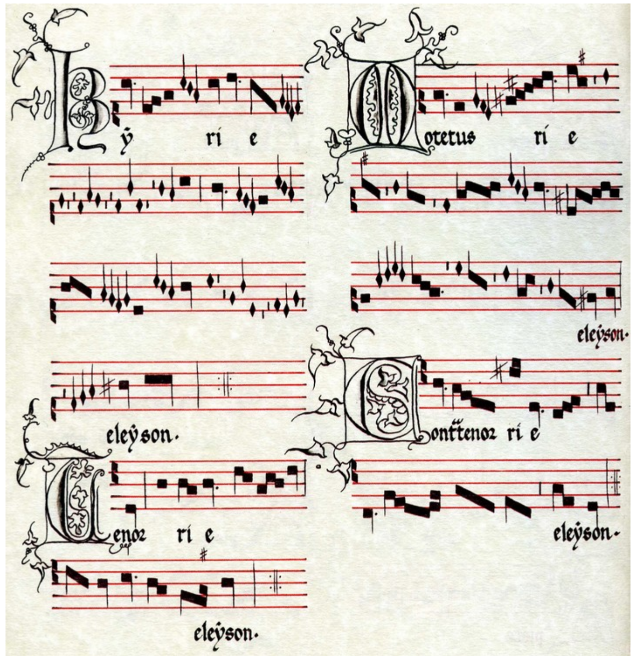


Figure 5: Score of the *missa de Notre Dame* recorded in anechoic conditions for the present work: parts of tenor, motetus, triplum and contratenor.

A multi-track anechoic motif was recorded following the same workflow of previous research works [46]. Each singer at a time played his part, following a reference video of the conductor. During the recording takes, the singers heard the sync track and the previous tracks through closed headphones. The four professional singers of Gregorian Choir involved in this study already had experiences with anechoic recordings. They are resident singers in the Cathedral of St. Petronius in Bologna and thus, they are used to sing in a church with more than 10 s of reverberation time [14].

The *missa de Notre Dame* di Guillaume de Machaut (1300–1377) was chosen as example of musical composition from the same years of the Baptistery completion. It is defined as a *cantus firmus*, that is the evolution of the monodic approach of the early Gregorian chant. The piece is strongly based on a tenor voice and then enriched by a contratenor, a triplum and a contrapuntal voice (motetus) that do not affect the principal temporal behaviour of the tenor (*cantus*). Figure 5) shows the score in a four lines staff with square notation.

4 Results

The sound field behaviour inside the Baptistery was investigated through the analysis of the following room criteria, according to previous works [11, 6, 13].

The reverberation time T_{30} in s, as the 60 dB decay of the integrated squared impulse response, fitted between -5 dB and -35 dB thresholds. It can be assumed as a property of the space because it is generally not affected by the source-receiver position. The early decay time EDT in s, as the 60 dB decay of the integrated squared impulse response, fitted between 0 dB and -10 dB thresholds. According to ISO 3382-1 this criterion corresponds to the perceived reverberation of the space (reverberance). It depends on the source-receiver position. The center time T_s in ms, which is the first-order momentum of the squared impulse response, expressed as:

$$T_s(\mathbf{r}, \mathbf{r_0}) = \frac{\int_0^\infty t h^2(\mathbf{r}, \mathbf{r_0}, t)\,\mathrm{d}t}{\int_0^\infty h^2(\mathbf{r}, \mathbf{r_0}, t)\mathrm{d}t} \qquad \text{(ms)} \tag{1}$$

where $h(\mathbf{r}, \mathbf{r_0}, t)$ is the measured impulse response with the sound source position $\mathbf{r_0}$ and the receiver position $\mathbf{r}$. The early lateral energy fraction LFC_{80}, that quantifies the subjective listener perception of the source width (i.e. the Apparent Source Width - ASW) [42] defined as:

$$LFC_{80}(\mathbf{r}, \mathbf{r_0}) = \frac{\int_{5\,\mathrm{ms}}^{80\,\mathrm{ms}} |h_L(\mathbf{r}, \mathbf{r_0}, t) h(\mathbf{r}, \mathbf{r_0}, t)|\ \mathrm{d}t}{\int_{0\,\mathrm{ms}}^{80\,\mathrm{ms}} h^2(\mathbf{r}, \mathbf{r_0}, t)\mathrm{d}t} \tag{2}$$

where the pedix L means a figure-of-eight receiver.

The subjective perception was studied in terms of further parameters, concerning the characteristics of the sound signal and described hereafter. The amplitude of the modulation spectrum, defined as:

$$M(f_m) = \left| \int_0^\infty |H\{p(t)\}| e^{j\omega f_m t} \mathrm{d}t \right| \tag{3}$$

where $H\{p(t)\}$ is the Hilbert-transform of the chant $p(t)$, assumed as anechoic. The modulation frequency f_m spans from 0.06 Hz to 12.5 Hz in the case under study, according to [32] instead of the range of IEC 60268-16 (focused on speech) that spans from 0.63 Hz to 12.5 Hz. Indeed, the temporal fluctuations of the Gregorian chant are slower than the speech and requires more low frequencies analysis. The amplitude of modulation spectrum is related to the ability of human listeners to identify consonants on the basis of primarily temporal information [47]. The effective duration of the autocorrelation (τ_e) which measures the degree of temporal similarities of sound during a certain time interval [31]. The moving autocorrelation function is defined as:

$$\Phi(\tau,t) = \int_{t-T}^{t+T} W(\xi)p(\xi)p(\xi+\tau)\mathrm{d}\xi \tag{4}$$

where t is the temporal axis of the time-windowing, τ is the time-delay of the autocorrelation function, W is the windowing function. When analysing music signals, scholars set a rectangular windowing ($W = 1$) with a width $2T = 2\,s$. The $\tau_e(t)$ is the time-delay of 10 dB decay related of each short-time autocorrelation function $\Phi(\tau, t)$ [48]. Its value can spans from few milliseconds to hundred of milliseconds for organ music [49]. The τ_e definition is based on neurophysiological path of perception and it was proved that τ_e is involved in the musician performance feedback processes, in the subjective perception of the reverberance and more [49].

4.1 Calibration

The model corresponding to the current geometry of the Baptistery was calibrated through a hybrid geometrical acoustic (GA) software [50], achieving the match between the measured and simulated room criteria (see fig. 6). An initial data set of reliable absorption coefficients was provided by existing databases [51, 52] and previous works [11] and then applied to all the surfaces. Some of these values include considerations concerning the building technologies and the historical context [6, 53]. During the iterative process of calibration, some absorption coefficients were slightly modified depending on the specific case and conditions (e.g. elements which were significantly simplified or even not modeled), in line with the procedures of the state-of-the-art [12]. In comparison to the typical values of absorption coefficients associated to the plaster material [53, 12], the selection of high values at the mid-low frequencies (125 - 250 Hz) in the present case study is justified considering the absence of architectural details and the limits of GA methods at low frequencies.

In view of the fact that GA algorithms assume that sound propagates along rays, the wave nature of sound has to be introduced by assigning scattering properties to each surface. The total scattering s_r, spanning from 0 (specular reflection) to 1 (ideal diffusivity), is expressed as:

$$s_r = 1 - (1 - s_d)(1 - s_s) \tag{5}$$

where the surface roughness s_s is set by the user and diffraction due to the edges s_d is calculated by the algorithm according to the geometry. The lack of small details in the modeling phase was compensated assigning higher scattering properties to the approximated surfaces. For instance, a scattering value equal to 0.70 was assigned to the baptismal font and the pulpit, that also represent the only elements of discontinuities of the whole geometry in terms of shape and materials. The single scattering value assigned to each surface of the model is referred to the medium frequency of 707 Hz and then extended to all the octave bands with a transfer function included in the algorithm of the software. The final scattering and absorption coefficients involved in the simulations are reported in table 1.

In order to have as more accurate results as possible, the number of late rays was chosen as 190,000 according to the highest level of precision suggested. In view of the reverberant space under analysis, 20,000 ms were considered a suitable length of the impulse response in the calculation setup. Since the transition order (TO) defines the threshold between the early and late reflections, this parameter was set equal to 2. Therefore, the first two orders of reflections are handled by the deterministic image source method (ISM) whilst all the following orders of reflections are handled with the statistic ray-tracing approach.

Since the lowest frequency ($\simeq 80\,\mathrm{Hz}$) of 125 Hz octave band is enough higher than the Schroeder frequency of the building ($2000\sqrt{T/V} \simeq 40\,\mathrm{Hz}$), the GA model provides reliable results above this octave band. According to ISO 3382 recommendations [42] the frequency range involved in this study spans from 125 Hz to 4000 Hz.

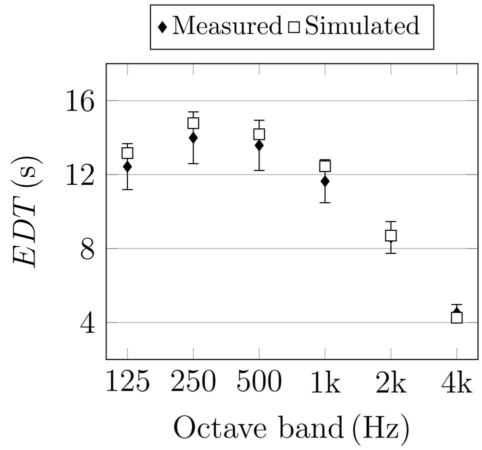
4.2 Simulation

Once calibrated, the numerical model was used to better understand the acoustics of the Baptistery during its liturgical use in occupied conditions. The presence of 150 people, involving clergy and faithful, was supposed to be spread around the font according to the historical liturgy of the Baptism. No further occupancy was considered in the ambulatory and the matroneum. The occupancy was simulated through a layer at 1.50 m above the floor, with the possibility to switch it on or off during simulations. Only two sound sources were considered in this step: the first one on the altar, the second one on the pulpit, simulated one at a time as in the ritual of Catholic mass. Each simulation was run keeping the same set up of the calibration process and repeated three times in order to improve the accuracy by averaging the results.

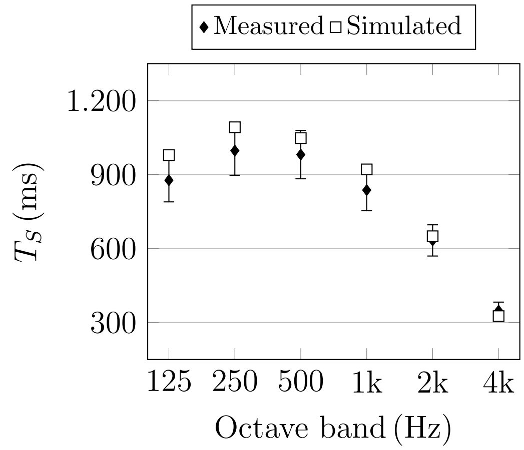
Furthermore, some factors should be taken into account when an historical configuration is simulated. Indeed, in the past the churches were not as unadorned as they are at the

Table 1: Absorption (α) and scattering (s) coefficients for all the materials involved in the simulation.

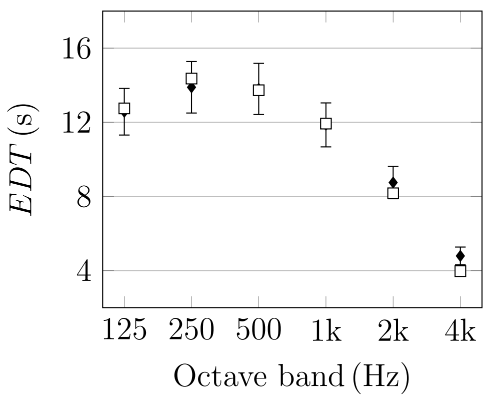
Materials	Surface		Absorption/Scattering coefficient						Ref.
	%		125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz	
Marble	73	α	0.01	0.01	0.01	0.02	0.02	0.02	[52]
		s	0.05	0.10	0.20	0.35	0.45	0.50	
Plaster	20	α	0.07	0.07	0.05	0.03	0.04	0.05	adapted from [52]
		s	0.05	0.05	0.10	0.15	0.25	0.30	
Terracotta (*Matroneum* floor)	4	α	0.05	0.05	0.03	0.03	0.03	0.03	[52]
		s	0.05	0.10	0.20	0.35	0.45	0.50	
Windows	2	α	0.35	0.25	0.18	0.12	0.07	0.04	[26]
		s	0.01	0.01	0.05	0.10	0.20	0.25	
Wood	1	α	0.10	0.15	0.18	0.20	0.20	0.20	[52]
		s	0.05	0.10	0.20	0.35	0.45	0.50	
Occupancy (1 person/m^2)	1.5	α	0.16	0.29	0.55	0.80	0.92	0.90	[53]
		s	0.15	0.20	0.65	0.80	0.82	0.84	



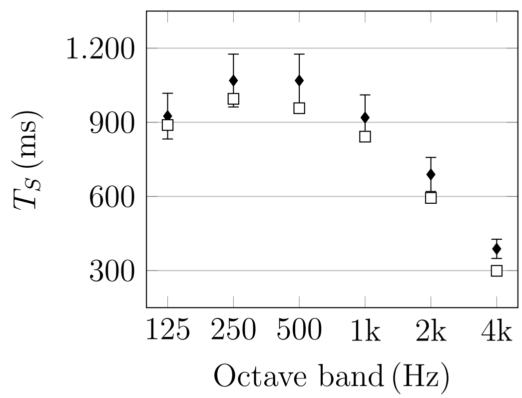
(a) Sound source in the altar



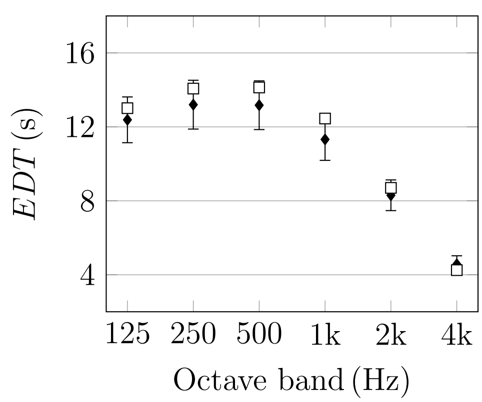
(b) Sound source in the altar



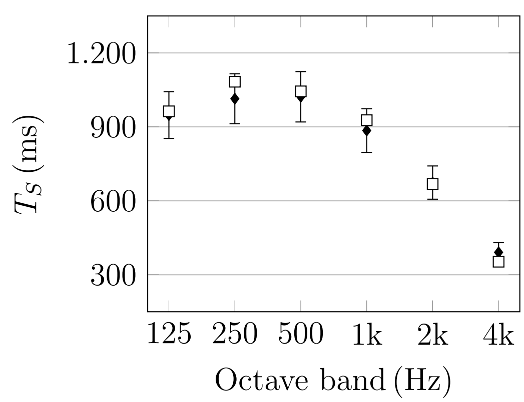
(c) Sound source in the pulpit



(d) Sound source in the pulpit



(e) Sound source in the *matroneum*



(f) Sound source in the *matroneum*

Figure 6: Comparison between measured and simulated values of EDT and T_S for the three significant sound sources. The tolerance range was chosen equal to the 10% of the relative values for both the criteria assessed [54].

Table 2: Simulated *EDT*, T_s, LFC_{80} values in the actual model, referred to the central octave bands ($500 - 1000$ Hz), in unoccupied conditions and occupied conditions considering a standing audience of 1 person/m^2 spread over the central area of the floor around the baptismal font. Mean values are provided for the two main source positions, the altar and the pulpit (S1 and S2 in figure 3) for the three main groups of receivers: central area, ambulatory and *matroneum*.

Sound Source	Receiver	**Unoccupied condition**		
		EDT (s)	T_s (ms)	LFC_{80}
Altar	Central area	12.7	828	0.40
	Ambulatory	12.6	842	0.41
	Matroneum	14.0	1136	0.34
Pulpit	Central area	11.3	605	0.34
	Ambulatory	12.4	717	0.32
	Matroneum	14.0	1152	0.40
		Occupied condition		
		EDT (s)	T_s (ms)	LFC_{80}
Altar	Central area	9.3	580	0.40
	Ambulatory	9.2	615	0.41
	Matroneum	11.0	902	0.34
Pulpit	Central area	7.2	378	0.22
	Ambulatory	8.8	488	0.32
	Matroneum	11.1	928	0.38

present time: draperies were exposed during the masses [55] and historical clothes have more acoustic absorption than the present ones. Due to the absence of documents on these aspects, the present study concerns the simulation of the hall as it appears at the present time without further adornments, but taking into account the occupation only.

The results of the simulations are resumed in table 2 by collecting the receivers on three groups: in the central area (around the baptismal font and among the faithful), in the ambulatory (far from the sound sources) and in the *matroneum* (far from the sound sources and from the faithful). Some of these last receivers are not directly visible from the sound sources meaning that the relative IRs do not have the direct sound contribution.

Results confirm how the EDT values can depend on the source-receiver mutual position. The reverberation perceived by faithful, at mid frequency, during the mass spans from 7.2 s (when the priest is on the pulpit) to 9.3 s (when the priest is on the altar). It is interesting to note that these values are lower than the reverberation time, whose value in occupied condition is around 10 s (see figure 7) and its behaviour is constant over the whole space. The criterion centre time T_S can be assumed, in this case, as a metrics of intelligibility [54]; the results show how the faithful understand the speaker on the pulpit (378 ms) much better than the speaker on the altar (580 ms). Moreover, the spatial impression LFC_{80} is higher when the speaker is on the altar (0.40 instead of 0.22). These results are in line with previous similar studies [13] and with the liturgy needings: on one hand the rites celebrated on the altar (prays, litany) need more envelopment and less intelligibility; on the other hand the sermon from the pulpit needs more intelligibility and less envelopment.

5 Discussions

5.1 The Baptistery as an outlier in the worship acoustics

Worship spaces were often built around a central symmetry for symbolic reasons. Central plan buildings have a lot of acoustic peculiarities and raised the interest of scholars from the birth of the architectural acoustics as a science - which is conventionally moved back to the Sabine's work [56] in the early 1900. In the Western Christianity, central symmetry was used until the Romanesque cathedrals, whose plan was inspired by latin cross. In these cathedrals the acoustic effects due to the central symmetry was preserved in the dome - Sabine himself and scholars of the same period were interested in the dome of St. Paul's

Cathedral. Progressively from 14th to the 16th century, baptismal font was moved inside of the church and there was no more baptisteries built after the Trento Council (1545-1563). Outside of the western Christianity the central symmetry remains still nowadays, in the Orthodox churches and in the mosques.

It is interesting to compare the acoustics of the Baptistery with the ones of other central symmetry worship spaces, whose buildings span from the 4th - late Roman buildings converted into churches [16] - to the 20th century. Table 3 collects a dataset of central simmetry worship spaces, with volume spanning from about 10000 m^3 to about 100000 m^3. The data include mosques, early Christian and Orthodox churches from previous researches [3, 16, 17, 18, 19, 20, 21, 40].

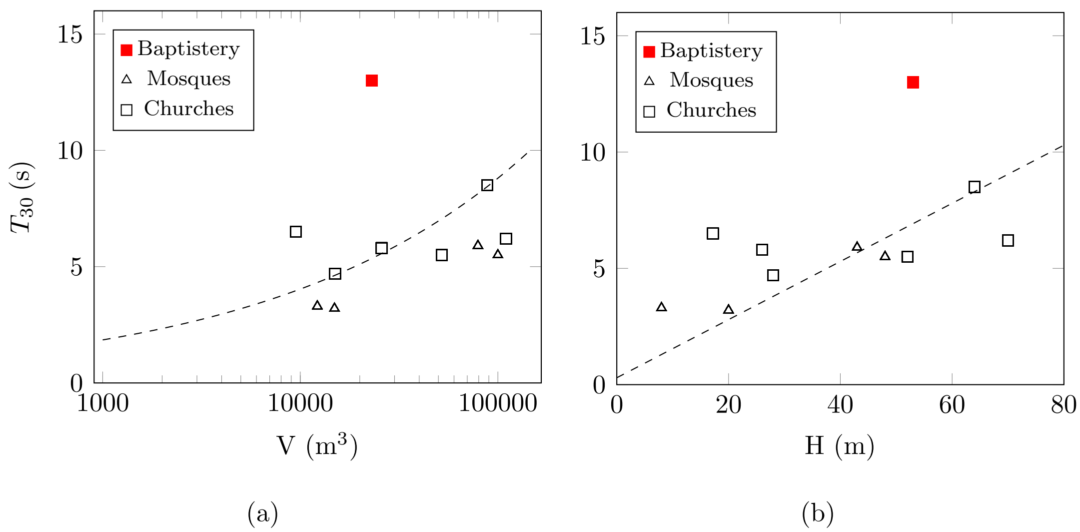
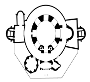
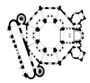
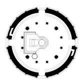
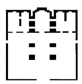
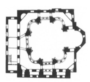
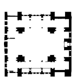
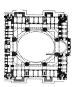
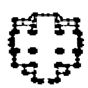
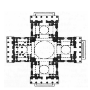
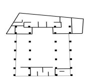
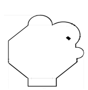


Figure 7: Reverberation time at mid frequencies (500−1000 Hz) as a function of room volume (V) and room maximum height (H). Dashed regression was based on orthodox churches only [18].

In figure 7(a) the reverberation time value at mid frequencies measured in the Baptistery is compared with the dataset of table 3. Orthodox churches show a trend in which the values of reverberation time T_{30} should be, in first approximation, proportional to $V^{1/3}$ (see the dashed regression curve [18]). On the same figure, the mosques show lower values of reverberation time. This is probably due to carpets on the floor, which have good properties of acoustic absorption at mid-high frequencies. On the contrary, Christian buildings benefit of the use of low-absorption materials, such the marble or the stone. Starting on the Roman basilica, the early Christian buildings show a trend in which the plaster surfaces decrease

Table 3: Comparison between the Baptistery and a data set of central symmetry worship spaces. V is the volume, S_f is the area of the ground floor, H is the maximum height, $T_{30,M}$ is the reverberation time at mid frequencies.

Plan	Church name	Century	Denomination	V (m^3)	S_f (m^2)	H (m)	$T_{30,M}$ (s)	Ref.
	Rotunda of St. George, *Thessaloniki (GR)*	4^{th}	Orthodox church	15000	830	28	4.7	[16]
	St. Vitale, *Ravenna (IT)*	6^{th}	Early Christian church	25800	980	26	5.8	[40]
	**St. John's Baptistery**, ***Pisa (IT)***	12^{th}-14^{th}	Catholic church	23000	720	53	13	
	Dormition Cathedral, *Moscow (RU)*	15^{th}	Orthodox church	11500	450	20	4.5	[17]
	St. Sergio and Bacchus, *Istanbul (TR)*	16^{th}	Mosque	14900	750	20	3.2	[21]
	Slëymaniye Mosque, *Istanbul (TR)*	16^{th}	Mosque	100000	4350	48	5.5	[3]
	Selimiye Mosque, *Edirne (TR)*	16^{th}	Mosque	79300	1840	43	5.9	[21]
	Smolny Cathedral, *St. Petersburg (RU)*	18^{th}	Orthodox church	110000	3650	70	6.2	[17]
	Holy Trinity, *St. Petersburg (RU)*	19^{th}	Orthodox church	52000	1930	52	5.5	[18]
	Christ the Saviour, *Moscow (RU)*	20^{th}	Orthodox church	88400	3425	64	8.5	[18]
	Masjid Jamek, *Kuala Lumpur (MY)*	20^{th}	Mosque	12200	1600	8	3.3	[20]
	Holy Spirit, *Białystok (PO)*	20^{th}	Orthodox church	9500	560	17.2	6.5	[19]

and the marble/stone surfaces increase. The St John's Baptistery design was at the end of this temporal trend – focused by [57] – which began in the Roman age with the catacombs [58] and Domus Ecclesiae and came up to the medieval age. Indeed the ratio of the marble surfaces is much higher in the Baptistery (about 73% of total surface) than the other worship spaces of the early Christianity (about 35-40%) [57], justifying the building as an outlier in fig. 7(a).

It was proved that, in Orthodox churches, the reverberation time could be related to the height of the dome [18]. It means that there is a sort of preferred way of the sound propagation, involving the floor and the dome, which contributes to the reverberation more than other factors. The values in fig. 7(b) confirm this tendency for the whole dataset of central symmetry churches and mosques. The Baptistery is still an outlier, meaning that the huge reverberation does not depend on the height of the superior conical dome. The floor area and the height-to-base ratio of the worship spaces dataset are similar to the Baptistery, but they differ in the presence of the *matroneum*. Therefore, it is more likely that this coupled volume is the reason of the acoustic peculiarities of the Baptistery.

5.2 Archaeoacoustics helps understanding the coupling effects

Archaeoacoustic research usually aims at returning information on the past. Conversely, in this work, the hypotheses of the past configurations of the Baptistery help to better understand the present through numerical simulations.

Assuming the hypotheses discussed in the section 3.2, early designs for St. John's Baptistery were simulated using the methodology described in the previous section. Three different GA models were shaped according to the respective historical studies [36, 37, 38] and to the modeling method described before (see fig. 2). De Fleury (DF) and Boeck's (BOE) models preserve the geometry of the actual Baptistery, but they are different for the top of the dome (hypotised to be open) and for the *matroneum*: it has no arches and in the DF model the ceiling is lower.

Concerning materials properties, the roughness of marble and plasters was quite different in the past but not enough to influence the results of acoustic simulations. The numerical simulations were run using the same source-receivers couples and the same setup of the calibrated model.

The model by Manenti Valli (MV) differs from the actual model for the height of the

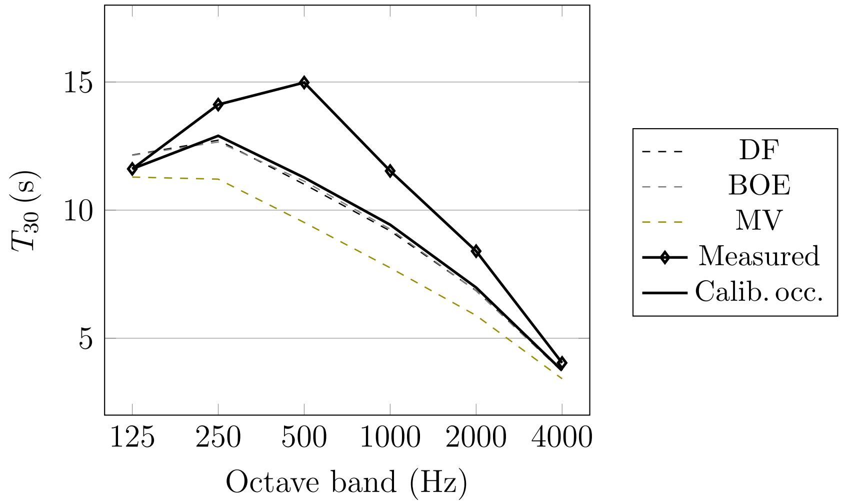


Figure 8: Simulated T_{30} values in octave bands: the actual geometry of the Baptistery (calibrated model) and restitutions of the early design hypotheses (DF, BOE, MV). Mean values are provided in unoccupied (unocc.) and occupied (occ.) conditions, averaged over all the receivers positions.

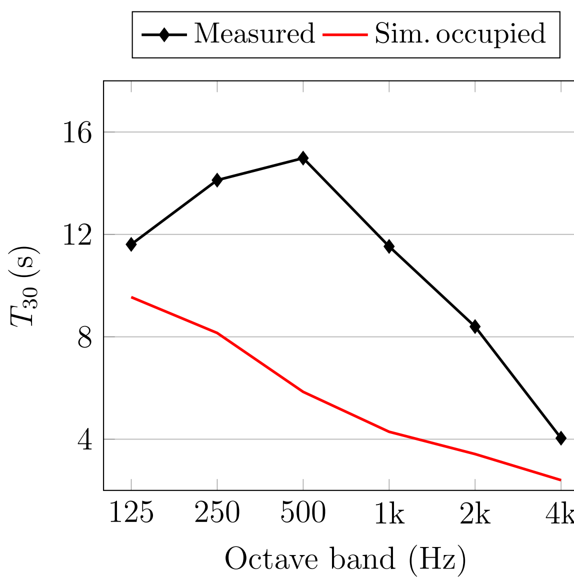


Figure 9: Simulated T_{30} values in octave bands: the actual geometry of the Baptistery (calibrated model) and restitutions of the early design hypotheses (DF, BOE, MV). Mean values are provided in unoccupied (unocc.) and occupied (occ.) conditions, averaged over all the receivers positions.

building and the shape of the dome. It leads to a lower values of reverberation time, both in the unoccupied and occupied conditions . In the remaining two models (DF and BOE), the effect of open conical ceiling seems to influence only the low frequencies: above the 500 Hz octave band the reverberation time values correspond to the present ones. It is interesting to notice the shape of reverberation time in occupied conditions: those values decrease with frequency. The frequency peak at mid frequency, typical of large volumes with low absorption, is flattened. This may be due to several factors, which can involve the acoustic coupling effects, the changes in radial modes, the statistic distribution of path lengths. There are different ways to treat the same acoustic problem: the acoustic coupling was focused on energy assumptions, the modal approach involves the symmetry of the space and the boundary conditions, the distribution of path lengths concerns the statistic hypotheses of the reverberation. The first two aspects are discussed in several works [59, 60], while the last one is less treated. Numerical software are useful tools to understand the reverberation behaviour through the free path analysis.

In the reverberation process, a spheric wave can be decomposed in a sum of geometric rays, which propagate in the space bringing a fraction of the energy of the sound source. Each wall reflection decreases the energy of the ray by a factor $(1 - \alpha)$, when α is the absorption coefficient. A free path is the distance between two subsequent reflections, and the mean value of free path distribution plays a key role in the reverberation theory. The mean free path value was proved to be equal to $4V/S$, where V is the volume and S the surface, even if the geometry is complex. This result was confirmed in the case of concert halls [61] and Serbian churches [62].

Mean free path values are calculated as $4V/S$ for the actual and the hypothised model, basing on 3D models. Similar values are found (see fig. 9): the MFP of actual geometry is around 11,6 m and it decreases by 0.5 m without the conical dome, while the smallest model (MV) analysed returns a value equal to 10.5 m. This short range of values may suggest similar reverberation time values - assuming the same material properties - but the simulations show a large spread of reverberation time values (see fig. 8).

This may be due the coupling effects in the Baptistery, which corresponds to a multimodal behaviour the statistical distribution of free paths. Figure 9 shows the hits of free paths in case of the actual configuration (used for the calibration process), the actual configuration without considering the conical dome and the early restitution proposed by Manenti Valli [38]. Free paths were simulated for the first two seconds of the reverberation, corresponding

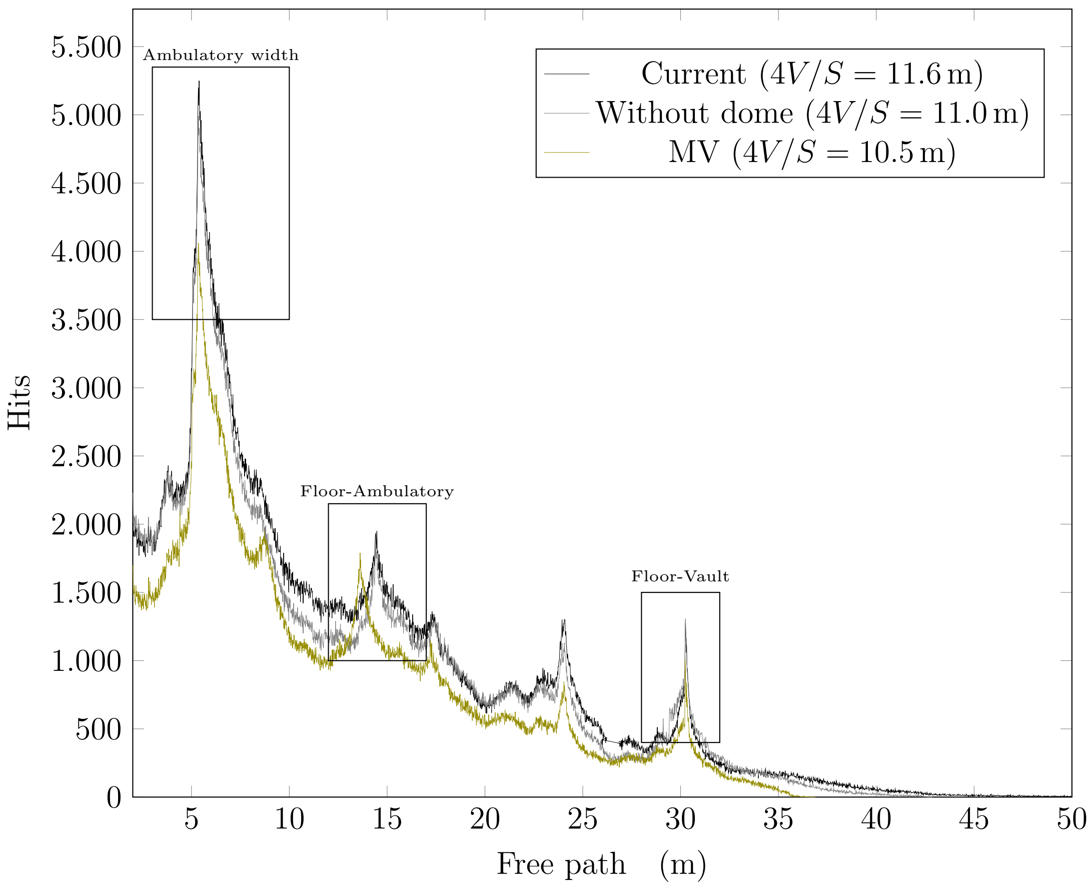


Figure 10: Statistical distribution of free paths in case of the current geometry, the same model without considering the conical dome and the MV model. The most relevant paths correspond to the main distances of the three geometries, as highlighted in frames.

to the first 10 dB of the energy decay which are involved in the EDT calculation.

The first consideration concerns the maximum length of the rays: it is about 30 m, corresponding to the vault of the Baptistery (see fig. 2). It means that there are no rays coming from the conical dome to the floor, whose length could be larger than 30 m, and thus there are no focusing effects.

The local maxima between the three models match on the same values, except for the maximum around 14 m. This value corresponds to height of the ambulatory, which differs from actual model (14.5 m) to MV restitution (13.5 m).

The different height of *matroneum* may be seen in the peak around 8 m (MV model) and the peak at 14.5 m (actual configuration).

Finally, the peaks of hits around 5 m correspond to the width of the coupled volumes of *matroneum* and the ambulatory: the distance between the columns and the external walls. In the MV model, the absolute number of hits is lower, meaning that the two sub-volumes and the main volume act as one single volume. It could be also the reason of the lower reverberation time of MV model with respect to other models (see fig. 8).

5.3 Should Gregorian chants need huge reverberation?

The optimal range of reverberation time for worship places [25, 26] or index for music [27] or speech [28] might reveal that the Baptistery is inadequate to liturgical use. Nevertheless, the analysis of the evolution of choir music could allow to understand the actual suitability of this particular worship space for its use over the centuries. The medieval liturgy was not based on the speech intelligibility, but on the perception of listening envelopment: the liturgy in the Western Catholic churches was celebrated in Latin until the mid 20th century (2nd Vatican Council, 1962-1965); Gregorian chant was performed only by clergy following macro-sequences of vocal emissions which corresponded to syllables of Psalm verses; there was not time indication and the performance was no note-based until the 13th century (see fig. 5). On this basis, the reverberation of the Baptistery proves to be an optimal support to the listening of Gregorian chant. In order to demonstrate this assumption two methods will be used: the first one, based on spectral amplitude of modulation (see eq. 3); the second one, based on effective duration of the autocorrelation function (see eq. 4).

The first method considers the choir music as a combination of a modulated carrier (the sung note) and the modulation signal that takes into account the temporal behaviour of the singing process. The reverberation deteriorates the information if its value is higher than a certain limit, but at the same time the reverberation enriches the voices and increase the chant loudness. The spectral amplitude of modulation (eq. 3) shows how many information is carried by which modulation frequency. For this kind of analysis anechoic recordings are needed as signal $p(t)$, because they are not deteriorated by reverberation. In the present section three anechoic excerpts are analysed (see fig. 11):

1. The cantus firmus excerpt “Kyrie” by Guillaume de Machaut (1300-1377), recorded ad hoc (see section 3.3) and freely available [63];

2. The baroque piece “Almighty and Everlasting God” by Orlando Gibbons (1532-1625), recorded by Freheit [64];

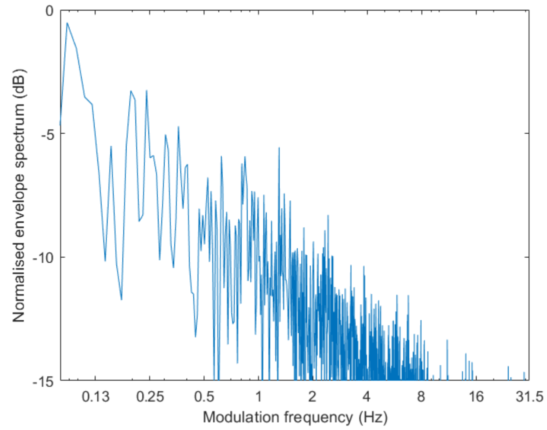
3. A speech radio recording, which can be assumed as anechoic.

Spectral amplitude of modulation was extracted from each motif with Matlab software, then normalized and plotted in logarithmic scale (see fig. 11). The *Kyrie* shows the maximum at low modulation frequencies - around 0.2 s - while the baroque excerpt shows its maximum at higher modulation frequencies, around 2.0 s. The lower is the modulation frequency, the higher will be the reverberation time necessary to improve the intelligibility of the excerpt. Moreover, in the *Kyrie* from the modulation frequency $f_m = 2\,\text{Hz}$ onwards the values are lower than 10 dB compared to the maximum, meaning that the corresponding information content is negligible. If the function is flat in frequency, as the case of *Speech* and (*Almighty*) the two signals carry information at high modulation frequencies that need lower reverberation time to be properly ‘demodulated’. That is the reason why the Speech Transmission Index [65] could result meaningless when used to analyse Gregorian chant.

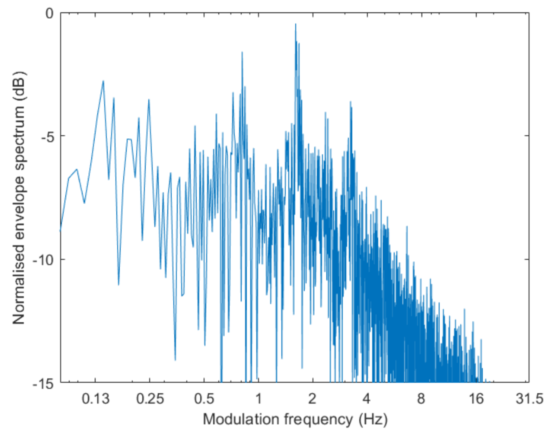
The second analysis is based on the temporal similarities. Using the same anechoic excerpts, the information carried is investigated through a short-time analysis, instead of the whole signal length. The metric used is the effective duration of the autocorrelation (τ_ϵ) which can span from 10 ms (impulsive consonants) to more than 200 ms (organ music). Ando showed that the preferred reverberation time is equal to:

$$[T]_p = 23\tau_e \tag{6}$$

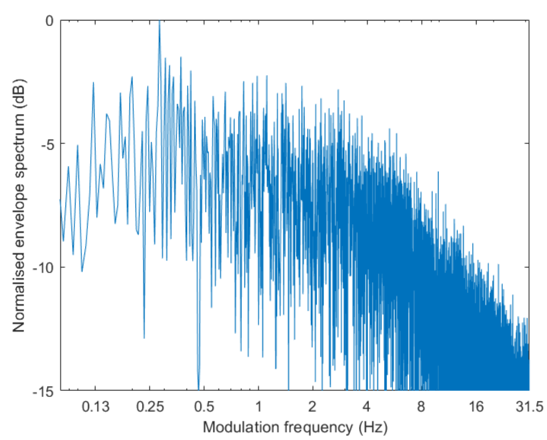
where the pedix ’p’ means ’preferred’ [31]. In the present paper the reverberation time is considered at mid-frequencies (mean of the values at 500-1000 Hz octave bands) and τ_e value



(a) Kyrie (14th C.)

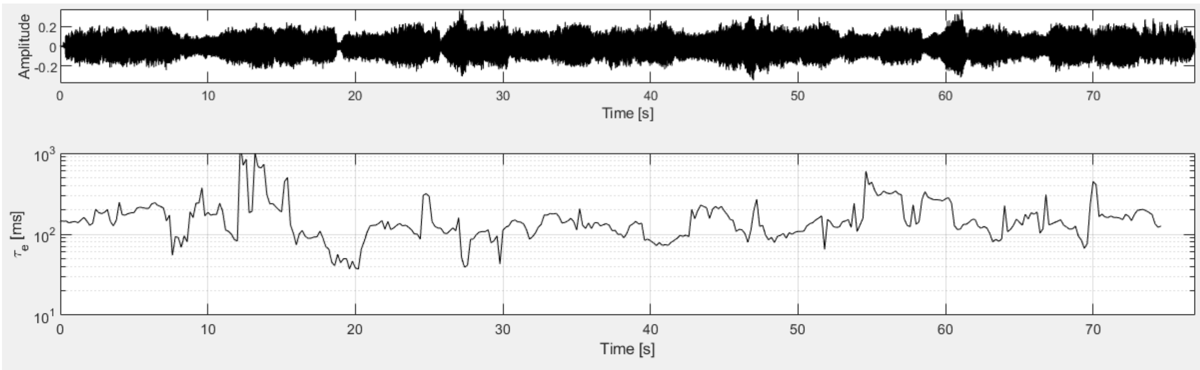


(b) Almighty (17th C.)

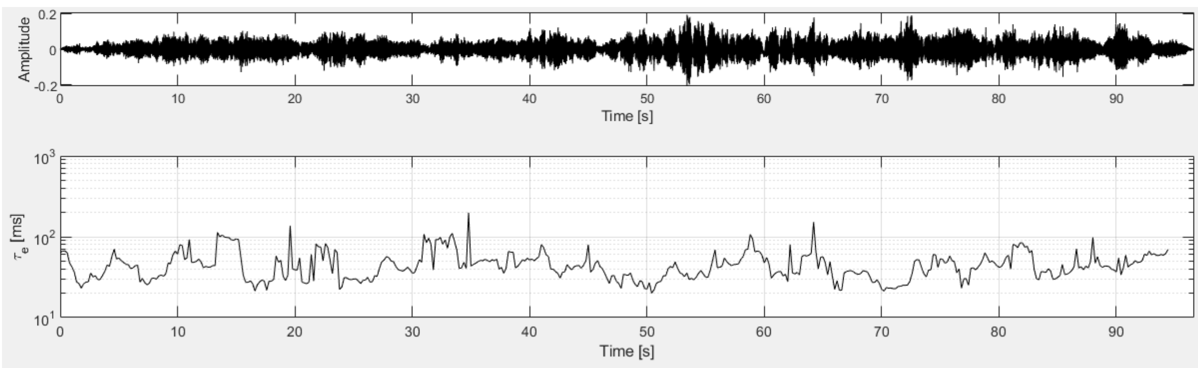


(c) Speech

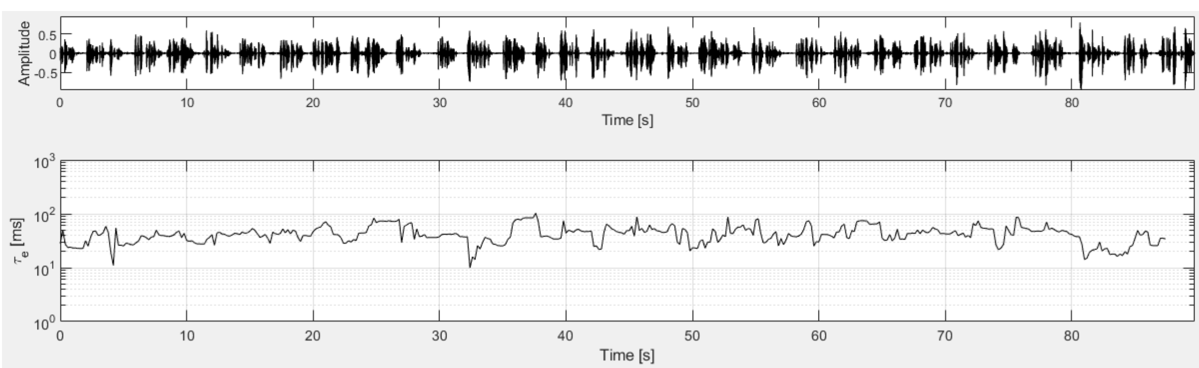
Figure 11: Normalised envelope spectra as functions of modulation frequency.



(a) Kyrie (14^{th} C.) $\tau_e \simeq$100-200 ms



(b) Almighty (17^{th} C.) $\tau_e \simeq$40-80 ms



(c) Speech $\tau_e \simeq$20-40 ms

Figure 12: Temporal behavior of the effective duration of the autocorrelation function for each anechoic excerpt under study (integration time 2T=2 s over rectangular time windowing, step=0.2 s)

(which is a function of time) is averaged over time, returning a single value. The analyses of $\tau_e(t)$ for each anechoic motif are calculated by Matlab code [66] and are shown in the figure 12. Due to logarithmic nature of the parameter τ_e and due to the high sensitivity to errors [48], it could be assumed an approximated range, keeping a rigorous method. In the case of *Kyrie*, values span from 100 to 200 ms, while for *Almighty* the averaged value is around 60 ms. *Speech* shows lower values, meaning that there are faster variations in each short-time window with respect to the previous two. The behaviour of *Speech* radio signal is quite regular, due to several potential conditions: the speaker's voice is controlled in dynamics, emission, distance to the mic, etc.. If the results of the τ_e analysis are applied to the eq. 6, it follows that the preferred reverberation time are around 5 s for Gregorian chant, 1.5 s for baroque chant, 700 ms for contemporary radio speech. These values may be read as an increasing influence of the text in the choir music. For all these reasons, St John's Baptistery could be considered an optimal room for Gregorian chant, even if it is an outlier from other worship places and its criteria exceed from the recommended ranges. The results of the present section show how ancient music and acoustics of historical places were mutually influenced, as the case of pre-Sabinian opera houses [67].

6 Conclusions

The present paper investigates the sound of St. John's Baptistery in Pisa, which is included in a site of UNESCO World Heritage. The high reverberation of the Baptistery make the building an outlier if compared with similar central symmetry worship spaces. A comparison between the hypoteses on early design of the architecture helped to highlight the influence of the ambulatory and the *matroneum*. The free path distribution – extracted by numerical simulation – confirmed that the behaviour of sound energy is concentrated in these areas and then released with some time-delay through the apertures between the columns.

According to recent trends of archaeoacoustics, the role of the reverberation on the music of 14^{th} century is investigated. Temporal analyses proved that the modulation frequencies and the temporal similarities of the Gregorian chant matched the acoustic peculiarities of this space. The multi-track anechoic Gregorian chant, recorded ad hoc and used in the analyses, is freely available in a repository.

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List of Tables

List of Figures