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Cocktails Done Right: Price Competition and Welfare When Substitutes Become Complements

Abstract

In this paper we analyze the effects of the introduction (by either firms or authorities) of a composite good consisting of a fixed proportion of two imperfectly substitutable stand-alone products. First, we find that such a "cocktail" rises the Bertrand equilibrium prices as it introduces a certain degree of complementarity. It also creates incentives to price discriminate and products can be sold at a discount or at a premium (depending on their degree of substitutability) when they are used as part of the composite good. We consider two distinct forms of price discrimination: a traditional one, in which producers set their prices independently of each other and a coordinated one, in which producers cooperate (collude) when setting the price of the composite good. Composite goods might have either a positive or a negative impact on consumer surplus. The sign of the impact depends on the form of price discrimination and consumers tend to be better off if producers coordinate. The impact is also more likely to be positive if "cocktails are done right", i.e., if their quality is high compared to the quality of the stand-alone products.

Keywords: complements, vertical differentiation, price discounts and premia, price discrimination, excessive pricing, pharmaceutical markets.

JEL classification: C7, D42, D43, K21, L11, L12, L13, L40, M21

1. Introduction

In pharmaceutical markets, patients very often take combinations of different drugs to improve the efficacy of a particular treatment or to weaken collateral effects. For instance, half of the new cholesterol reducing treatments entering phase 3 of clinical trials in 2007 were "cocktails" of drugs that had already been approved as single products to treat the same symptoms (Blume-Kohout and Sood, 2013). Similarly, in 2008, more than one-third of US colorectal cancer patients under chemotherapy were under cocktail regime, and nowadays most HIV/AIDS patients are cured with a combination of two or more drugs (Lucarelli et al., 2017).Three alternative drug combinations are common in the treatment of primary open angle glaucoma: Dorzolamide + Timolol; Brimonidine + Timolol and Latanoprost + Timolol (Kumbar et al., 2015). Rituximab plus lenalidomide is used in patients with previously untreated follicular lymphoma (Morschhauser et al., 2018).

In other terms, in a pharmaceutical market with a given number of stand-alone (imperfectly) substitutable drugs, a new "rival" treatment may become available, consisting of a precise combination of existing products that are complementary components of the treatment (cocktails). Health authorities generally "approve" a cocktail when it demonstrates superior efficacy, fewer side-effects or greater convenience.[1]

Often, the components of the composite good are produced by separate, competing companies. The introduction of the composite good affects price competition, but the direction of such influence is not obvious a priori. Do "cocktails" soften competition or increase it? Do they enhance consumer surplus? At this purpose, the paper studies how the presence of a new "bundle" consisting of a fixed proportion of two existing stand-alone products influences the competition in a Bertrand duopoly with imperfectly substitutable goods and affects the incentives to sell products at a discount or at a premium (if that is allowed).

Our results will be illustrated using pharmaceutical markets as a primary reference, but other examples of such "bundles" can be found quite easily. For instance, tourist attractions (such as

[1] For instance, the Food and Drug Administration follows this procedure. In the market for colorectal cancer chemotherapy drugs, cocktails of two or more substitute drugs are often approved and used in order to treat patients who suffer strong side - effects when treated with one single drug. Organizations like the National Comprehensive Cancer Network (NCCN) also recommend the amount of each drug that doctors should use in each cocktail/regimen, based on the dosages used in clinical trials or in actual practice (Lucarelli et al., 2017).

the museums of a city) can be substitutes for some consumers but complements for others. In fact, they are often explicitly offered in a "cocktail" through a "tourist card" that might be offered at a discount. Airline competition in interline city-pair markets (that is markets where traveling on more than one carrier is necessary) is often characterized by this phenomenon, as well. Consider, for example, the air routes from Indianapolis to Denver and from Denver to Amsterdam and assume that only two airlines serve both routes. For all passengers flying either from Indianapolis to Denver or from Denver to Amsterdam, flights operated by the two airlines are substitutes to some degree. For all passengers flying from Indianapolis to Amsterdam the two routes are instead perfect complements. This aspect will clearly affect market competition, possibly giving incentives to airlines to form alliances and engage in price discrimination, as it actually happened in the case of Northwest Airlines and KLM on the two mentioned routes (Brueckner, 2001).[2]

In the first part of the paper, we find that the introduction of a "cocktail" always rises the components' prices but might decrease the duopolists' profits, so that it is not obvious that firms would be in favor of new bundles consisting of combinations of existing products. In particular, when the cocktail does not provide a substantial quality improvement on the existing stand-alone goods, we have a decrease in both profits and consumer surplus. This result indicates that the introduction of the cocktail generates a particular version of the market distortion known as "Cournot effect" or "complementary oligopoly" (Sonnenschein, 1968), which always appears when two perfectly complementary goods are sold by two different firms. In fact, prices are higher and profits are lower than those earned by a monopolist selling both goods. Each firm does not consider the negative impact that an increase in its price exerts on demand (exactly as in a Cournot duopoly it does not consider the negative impact of an increase in its quantity on the price) and sets a price too high. As a consequence, a monopoly would be both profit-enhancing and yield higher consumer surplus.

[2]In Brueckner's model, the benefits of alliances arise because cooperative pricing of trips by the partners puts downward pressure on fares in the interline city-pair markets. The loss of competition in the interhub market, which connects the hub cites of the partners, however, generates a countervailing effect, tending to raise the fare in that market. While the presence of economies of traffic density complicates these impacts by generating cost links across markets, his simulation shows that the first tendencies typically prevail. Moreover, welfare analysis shows that both consumer and total surplus typically rise following formation of an alliance despite the harm to interhub passengers, suggesting that the positive effects of alliances may outweigh any negative impacts.

This is especially important in pharmaceutical markets, when the cocktail does not significantly improve the efficacy of the existing drugs, so that the increase in prices unambiguously decreases consumer surplus.[3]

In the second part of the paper, we investigate the effects of a cocktail on price competition and welfare when firms are allowed to price discriminate. We analyze both an uncoordinated and a coordinated (possibly collusive) form of price discrimination. With the former, producers set their prices independently of each other. With the latter, producers cooperate (maximizing the sum of their profits) when setting the price of the composite good (while still competing in the markets for stand-alone products). We find that a multi-product monopoly price discriminates when cocktails have a better quality than stand-alone goods. In a duopolistic setting, instead, firms always price discriminate. When they set prices without coordinating, selling the separate components of the cocktail at a premium is a dominant strategy. However, higher prices are more than counterbalanced by fiercer competition in the stand-alone markets, so that the Bertrand equilibrium might not be Pareto-efficient (profits would be higher with uniform pricing) and firms may be caught in a prisoner's dilemma: they get inefficiently aggressive in the market for stand-alone treatments, recovering only part of their lost earnings with the premium on the price of the cocktail's components.

This problem disappears if firms are able to coordinate when setting the price of the cocktail. We find that they always price discriminate but their profits are always higher than with uniform pricing. We also find that forms do not always sell the cocktail at a premium. They might apply a discount to the cocktail price when the degree of substitutability between the stand-alone products is low, while they apply a premium when substitutability is high.

Moreover, this form of "partial collusion" softens the negative impact of the Cournot effect, so that consumers are actually better-off compared to the uncoordinated price discrimination case. Partial collusion might yield a higher consumer surplus even compared to uniform pricing.

Finally, it is important to stress the role played by the quality of the cocktail. In general, cocktails benefit consumers when they are "done right", i.e., when their quality is high and they

[3] This also explains the attention that the Food and Drug Administration pays when approving new therapies in the US.

represent a significative improvement upon stand-alone goods. Cocktails might impact negatively consumer surplus if their quality is too close to the already available one and price discrimination is all the more harmful for consumers the lower the cocktail's quality.

1.1. Related literature

Of course, firms might have various reasons to bundle or tie their own products and lots of them have already been investigated by a vast economic literature: to price discriminate (MacAfee et al., 1989), to leverage monopoly power in one market by foreclosing sales and discouraging entry in another market (Whinston, 1990; Chen, 1997; Nalebuff, 2004).[4] However, little is known from a theoretical standpoint about price changes when a firm's product is bundled with those of its rivals and about the welfare effects of such practice.

Gans and King (2006) analyze the optimal discounting strategy of two rivals bundling two independent products as a way to relax price competition against a set of stand-alone competitors. Specifically, they prove that such firms can profit from offering a bundled discount to the detriment of other firms and of consumers. With respect to such results, we find that, when goods are initially not independent but imperfect substitutes, the creation of a cocktail might prompt firms to sell their components at a premium rather than at a discount and such practice might not necessarily be welfare-reducing. Brito and Vasconcelos (2015) show that when firms decide whether to participate in a discounting scheme before prices are set, all pairs of firms producing goods of the same quality level offer bundled discounts, but all headlines prices rise. As a result, both consumer and social welfare decrease.

Similarly to our setting, Armstrong (2013) extends the standard model of bundling to allow for substitutable products and finds that firms have an incentive to introduce bundling discounts when the demand for a bundle is elastic relative to the demand for stand-alone products. In particular, he finds that separate firms often have a unilateral incentive to offer inter-firm bundle discounts when products are substitutes, although this depends on the detailed form of substitutability. Bundle discounts mitigate the innate substitutability of products, which can relax

[4] Adams and Yellen (1976) and Lewbel (1985) show that a monopolist might find profitable to engage in bundling thanks to its ability to sort customers into groups with different reservation prices characteristics, and hence to extract consumer surplus.

competition between firms. Our model focuses less on market elasticity and directly relates the incentives to discount or to sell at a premium to the degree of substitutability. Also, Armstrong's results are here generalized to market structures in which products might exhibit different effectiveness (an element of vertical differentiation), firms might engage in different forms of price discrimination and cocktail might call for different (possibly asymmetric) weights in the complementary relationship between its components.

Lucarelli et al. (2017), empirically analyze the welfare effects of cross-firm bundling in the pharmaceutical industry and their counterfactual analysis is consistent with the main results of our model.[5] Specifically, they study the economic effects of cocktails by performing a series of counterfactual exercises on the estimated demand and profit-maximizing condition. First, they find that cocktail regimens increase profits for all firms involved in the cocktail (and also for entrants producing new drugs) but harm consumers. Second, they find that the incremental profits from creating a bundle are sometimes as large as the incremental profits from a merger of the same two firms. Our results are richer than theirs: firms in our model can both increase or decrease profits and they can be good or detrimental for consumers. We lay down the conditions under which each of these results holds.

The welfare effects of a complementary oligopoly is the focus of Alvisi and Carbonara (2013), that studies the case of a composite good consisting of two perfectly - complementary components and considers the possibility to introduce competition in the market for each component. If one component is still produced by a monopolist, introducing competition in the other market may reduce welfare, unless competitors differ in the quality they supply. For the problem of complementary oligopoly to be solved, then, competition has to be introduced in each sector and the number of competing firms has to be sufficiently high.[6] Alvisi, Carbonara, and Parisi (2011) further explore this issue, showing that the presence of a quality leader (i.e., a firm manufacturing a superior version of both components) may change the nature of the complementary oligopoly problem, rendering competition in the markets for the perfectly complementary goods always

[5] Their paper clearly inspired ours, as the sale of cocktails at a discount or at a premium is indeed equivalent to the sale of bundles of substitute goods supplied by independent competitors. In their data-set, firms cannot price discriminate because each drug is produced by a different firm and it is a physician who creates the bundle in her practice from the component drugs.

[6] Alvisi and Carbonara (2013) then go on to argue that regulatory policies aimed at introducing asymmetric competition have to be carefully evaluated before being implemented.

preferable, in terms of consumer surplus, to a situation in which both components are produced by a monopolist.

Both papers are related to the model presented here. The creation of a "cocktail", a bundle of otherwise substitute goods that some consumers have to consume together in fixed proportions, generates a problem of complementary monopoly, at least to some extent. Competition therefore can impact consumer surplus negatively, especially when the cocktail is not of a sufficiently higher quality than the goods that compose it.

Our paper also provides one new possible theoretical foundation to the recent worries about sudden and huge price increases in pharmaceutical markets. Indeed, "excessive pricing" is nowadays a major concern not only for medias, but also for lots of competition authorities that have recently ruled against substantial price increases of pharmaceutical drugs.

One common explanation for excessive pricing is the abuse of a dominant position in the relevant market. A recent example regards the Italian Market Competition Authority (AGCM), which fined the multinational pharmaceutical company Aspen near 5.2 million Euros on 14 October 2016, following its finding that Aspen abused its dominance to artificially inflate the price of four of its cancer drugs (AGCM (2017)). (Veiga, 2018) models price differentiation with endogenously differentiated goods, showing that profit maximization may entail excessively high prices.

This paper clarifies that there might be a different motivation for "excessive pricing": the introduction of new therapies made of combinations of different drugs ("cocktails") generates an intrinsic incentive to increase the prices of the component drugs. Indeed, the desired ("profit-maximizing") price of drugs will tend to increase significantly after the introduction of new cocktail treatments, especially when the single components are produced by different firms and when firms can engage in price discrimination (that is, pricing a drug differently depending on whether it is sold as a stand-alone product or as a component of a cocktail).[7]

[7] A "less obvious" reason for "excessive pricing" could be the existence of fixed-dose combination therapies, which involve combining two or more pharmaceutical drugs in a single tablet. Clarke and Avery (2014) make this important point highlighting the substantial costs arising from a loophole in the Australian price-setting mechanism, allowing multi-brand fixed-dose combinations (FDCs) listed on the Pharmaceutical Benefits Scheme (PBS) to retain price premiums long after premiums on their individual components have eroded. In Australia, when there are multiple brands of the same combination, even if supplied by the same manufacturer, the drug costs are subject to a mechanism known as "price disclosure". This mechanism bases future PBS subsidies on the average wholesale cost to pharmacies of individual drugs, so that, over time, the government pays a cost that

Finally, while our analysis is particularly suitable for countries in which drug prices are mostly "market-based", it also sheds some light on the incentives that firms might have in requesting price increases in those situations in which drug prices are negotiated with NH authorities and/or are regulated. At the same time, it confirms the necessity of adopting great care when approving new therapies composed of cocktails: while they are best considered as treatment options to overcome treatment inertia and poor adherence, they often tend to increase the manufacturers' "desired" price for its single components. Such effect could be so overwhelming as to darken the benefits that the patients obtain with a more effective therapy.

The paper is organized as follows. Section 2 introduces the theoretical model, deriving demand functions when a cocktail of imperfect substitute goods is introduced in the market. Section 3 determines the effect of such newly available product on the Bertrand-Nash equilibrium prices, quantities and profits. In Section 4, we study the incentives to price discriminate by a multi-product monopolist and by two firms in a duopoly when the drugs they produce can be used in a cocktail. Section 5 extends the analysis of price discrimination in duopoly assuming that firms can coordinate on a bilateral discount or premium whenever the two goods are purchased to be used in a cocktail. Section 6 discusses the positive welfare effects of duopolistic price discrimination and shows how the latter could offset the negative impact on the consumer surplus of the "Cournot effect" better than an integrated market structure. Section 7 presents two extensions. In the first, we generalize our analysis to the case of "asymmetric cocktails", in which the duopolists supply different percentages of the cocktail. In the second, we discuss the impact on equilibrium prices and welfare of a cocktail whose efficacy is so high that no firm would find profitable to supply its components as stand-alone products. Section 8 concludes and pinpoints some promising research developments. Proofs of Propositions and Lemmas can be found in the Appendix.

reflects the market price. When multiple brands are available, price disclosure only takes account the wholesale costs of these drugs, and there is no link to the cost of the separate components.

2. The Model

Consider a standard model in which two substitute treatments, 1 and 2, are sold by two independent firms (e.g., two drugs to cure HIV/AIDS) . The market demands for the two goods are derived from the following social welfare function (Dixit (1979)

$$U(q_1, q_2, M) = M + (\alpha_1 q_1 + \alpha_2 q_2) - \frac{\beta}{2}\left(q_1^2 + q_2^2\right) - \gamma q_1 q_2. \tag{1}$$

where M is the total expenditure on other goods (drugs) different from 1 and 2 and γ measures the degree of substitutability between the two therapies ($\gamma \in [0, \beta)$).[8] The function in (1) represents the aggregate utility of all patients suffering from a disease that can be cured using drugs 1 and 2. This approach is compatible with the idea of a "benevolent physician", who chooses the welfare-maximizing quantity (number of doses) of drugs 1 and 2, and then, indirectly, the number of patients to cure with a given therapy (or the number of prescriptions). Such interpretation fits also with the practice of centralized drug purchases by hospitals.

Parameters α_1 and α_2 represent the quality of drugs 1 and 2 respectively, i.e., their efficacy in the cure of a disease and their side effects. We assume $\alpha_1 = \alpha_2 = 1$, since we want to focus on the impact that the introduction of a cocktail has on prices and not on the asymmetric distribution of drugs' efficacy.

To prevent changes in the number of products n (here $n = 2$) and γ to affect total market demand, we set[9]

$$\beta = n - (n-1)\gamma = 2 - \gamma \tag{2}$$

Maximizing (1) with respect to quantities, we obtain the demands for the two drugs:

$$q_1 = \frac{2(1-\gamma) - p_1(2-\gamma) + \gamma p_2}{4(1-\gamma)} \tag{3}$$

$$q_2 = \frac{2(1-\gamma) - p_2(2-\gamma) + \gamma p_1}{4(1-\gamma)} \tag{4}$$

We assume that there are no fixed costs and that marginal costs are constant, common to

[8] Second order conditions for utility maximization require $\gamma < \beta$.

[9] Schubik and Levitan (1980). With this normalization, γ varies in the interval $[0, 1)$.

both products and normalized to zero.[10]

Monopoly. If drugs 1 and 2 are produced and marketed by a single monopolist, the profit function is $\Pi = p_1 q_1 + p_2 q_2$, whereas the profit-maximizing price and output levels are $p_1^M = p_2^M = \frac{1}{2}$ and $q_1^M = q_2^M = \frac{1}{4}$, respectively. Profits equal $\Pi_M^* = \frac{1}{4}$.

Duopoly. In a duopoly, each firm's profits are $(i, j = 1, 2,\ i \neq j)$:

$$\Pi_i = p_i\, q_i = p_i \frac{2(1-\gamma) - p_i(2-\gamma) + \gamma p_j}{4(1-\gamma)} \tag{5}$$

Firms compete *à la* Bertrand and the Nash equilibrium prices are:

$$p_i^{nc} = \frac{2(1-\gamma)}{4-3\gamma} \tag{6}$$

Equilibrium quantities and profits are:

$$q_i^{nc} = \tfrac{2-\gamma}{2(4-3\gamma)} \quad \text{and} \quad \Pi_i^{nc} = \tfrac{(1-\gamma)(2-\gamma)}{(4-3\gamma)^2} \tag{7}$$

where the superscript "nc" stands for "no cocktail".

2.1. Introducing the Cocktail

Consider now this same market when some consumers, as before, are treated by their physician with a single drug, whereas some others are now prescribed a cocktail of the two substitute drugs. For the latter type of consumers, then, the two drugs work as complements.

Physicians prescribe either of the two single drugs or the cocktail, according to the patient's characteristics. Define q_3 the consumption of the drug cocktail. The social welfare function becomes

$$U(q_1, q_2, q_3, M) = M + (q_1 + q_2) + \alpha_3 q_3 - \frac{\beta}{2}\left(q_1^2 + q_2^2 + q_3^2\right) + \frac{\gamma}{2}\sum_{i=1}^{3}\sum_{j\neq i} q_i q_j \tag{8}$$

[10]The latter assumption is with no loss of generality. In case of positive, constant marginal production costs, prices can be interpreted as per-unit margins.

where α_3 is the quality of the cocktail and $\alpha_3 \geq 1$, so that we allow the cocktail to represent a more effective therapy than the existing ones.[11] The parameter β is still given by (2), with $n = 3$.

Maximizing $U(\cdot)$ with respect to q_1, q_2, q_3, the demands for the three products are

$$q_1 = \frac{[3 - (2 + \alpha_3)\gamma] - p_1(3 - \gamma) + \gamma(p_2 + p_3)}{9(1 - \gamma)} \tag{9}$$

$$q_2 = \frac{[3 - (2 + \alpha_3)\gamma] - p_2(3 - \gamma) + \gamma(p_1 + p_3)}{9(1 - \gamma)} \tag{10}$$

$$q_3 = \frac{[\alpha_3(3 - \gamma) - 2\gamma] - p_3(3 - \gamma) + \gamma(p_1 + p_2)}{9(1 - \gamma)} \tag{11}$$

We assume that the newly developed and approved cocktail combines goods 1 and 2 in proportions r_1 and r_2, r_1, $r_2 > 0$ and $r_1 + r_2 = 1$.[12] Moreover, in order to separate the impact on prices of the creation of a cocktail from the possible asymmetry in the dosage of the two drugs (i.e., $r_1 \neq r_2$), we will assume from now on that $r_1 = r_2 = \frac{1}{2}$, leaving the asymmetric case to a later extension.

As for pricing, we consider two possible strategies. They can apply the same price no matter whether the drug is sold to be used in a cocktail or as a standalone drug. In such case, the cost

[11] When $\alpha_3 = 1$, our normalization $\beta = n - (n-1)\gamma = 3 - 2\gamma$ implies that, at the same prices, the demand size, and then the number of cured patients, remains the same. What is new is simply that a fraction of patients previously cured with either drug 1 or 2 is now treated with the cocktail. When instead $\alpha_3 \geq 1$, the cocktail's superior efficacy increases demand size, in addition to redistributing patients among the three products. For instance, a set of patients could not be cured with the two existing therapies because of strong negative side effects but can now be treated with the cocktail.

[12] In pharmaceutical markets, the dosage of each single component of a cocktail is not a strategic choice of the firms, but it is established exogenously by researchers and certified , in the US, by the FDA. Our results can however be easily generalized to a different setting in which two producers of imperfectly substitutable goods can strategically coordinate and introduce a new product in the market composed of a fraction r_1 of good 1 and a fraction r_2 of good 2, $r_1 + r_2 = 1$, with the goal of maximizing joint profits. The stages of the problem would then become:

1. Firms decide whether to supply a cocktail;
2. They choose $r_i, i = 1, 2, r_1 + r_2 = 1$;
3. Firms compete à la Bertrand, setting prices.

In such a more general environment, in which "complementarity" is created artificially, it can be easily proven that when $0.175 < \gamma < 1$, the cocktail is profitable for both firms and the joint profit-maximizing fractions of the cocktail are indeed $r_1^* = r_2^* = \frac{1}{2}$, that is the ones used in our paper. When instead $0 < \gamma < 0.175$, consistently with Proposition 1 below, cocktails decrease both firms' profits, so that they would not be created in stage 1.

of one dose of the cocktail would be

$$p_3 = r_1\, p_1 + r_2\, p_2. \tag{12}$$

Alternatively, producers can engage in price discrimination. They can charge a price that depends on whether a drug is prescribed as a stand-alone treatment or is used in a cocktail. Clearly, the choice between these two strategies depends *in primis* on their availability (i.e., price discrimination must be feasible and legal) and then on their profitability.

2.2. Crowding Out Standalone Products

It should be noted that (9) and (10) are negative if $\alpha_3 > \alpha_3^*(\gamma) > 1$, where

$$\alpha_3(\gamma) = \frac{3 - 2\gamma - (3-\gamma)p_i + \gamma(p_j + p_3)}{\gamma}, i, j = 1, 2, i \neq j. \tag{13}$$

In such case we have a corner solution and $q_1 = q_2 = 0$. Substituting these values into the expression for q_3 in (8) and maximizing with respect to q_3, we get

$$\hat{q}_3 = \frac{\alpha_3 - p_3}{3 - 2\gamma} \tag{14}$$

Therefore, when α_3 is particularly high, it might happen that the introduction of a cocktail crowds out standalone treatments. This occurs when firms find it profitable to charge high prices, so that the demand of their drugs as stand-alone products is equal to zero and they sell them only as components of the cocktail (consumers are willing to pay a high price for the cocktail, due to its high quality).

We want to exclude this possibility in the main sections of our paper, assuming that $1 \leq \alpha_3 < \alpha_3(\gamma)$. However, $\alpha_3(\gamma)$ is a function of the prices and changes according to the market structure and the pricing strategies adopted by firms. To make sure that our assumption is satisfied, we then choose the minimum out of all the values that $\alpha_3(\gamma)$ takes in a duopoly.[13] In particular,

[13]See Section 7.2.

this value is

$$\alpha_3^*(\gamma) = \frac{6\gamma^2 - 25\gamma + 21}{6 - \gamma - 3\gamma^2} > 1 \tag{15}$$

and setting $\alpha_3 \in [1, \alpha_3^*(\gamma))$ guarantees that in all the Bertrand equilibria we will analyze, both the stand-alone products and the cocktails are purchased in positive amounts.[14]

3. The Effects of the Cocktail

In this section, we study the effect of the introduction of a cocktail on prices, quantities and profits absent price discrimination. A drug is sold at the same price, irrespective of it being part of a cocktail or not.

We analyze both a monopoly (where either good is produced by the same company) and a duopoly (where two distinct firms produce a good each). Substituting p_3 from (12) into (9) - (11), given our $r_1 = r_2 = \frac{1}{2}$, we obtain the demands of the standalone treatments and of their cocktail:

$$q_i^c = \frac{6 - 3p_1(2-\gamma) - (4 + 2\alpha_3 - 3p_2)\gamma}{18(1-\gamma)}, \; i = 1, 2 \tag{16}$$

$$q_3^c = \frac{2\alpha_3(3-\gamma) - 4\gamma - 3(1-\gamma)(p_1 + p_2)}{18(1-\gamma)}. \tag{17}$$

In what follows, we separate the event $\alpha_3 = 1$ from that with $\alpha_3 > 1$. While the model with $\alpha_3 = 1$ already offers clear-cut results about the effects of a cocktail on price competition and welfare, the case with $\alpha_3 > 1$ is particularly relevant, since the introduction of a cocktail by the authority in charge is often motivated by its superior efficacy.

3.1. The Multiproduct Monopolist

Starting with the multiproduct monopolist, when $\alpha_3 = 1$ and the cocktail has the same efficacy as the standalone treatments, its introduction does not affect the monopolist's behavior

[14] As we will see in the next section, however, $\alpha_3^*(\gamma)$ in expression (15) is not the smallest value that $\alpha_3(\gamma)$ can take in our model. Specifically, for given values of γ, $\alpha_3(\gamma)$ can be lower in a monopoly. The welfare impact of corner solutions appearing when one assumes greater values for α_3 will be studied in one of our extensions of Section 7.

nor its profits. This follows from the normalization imposed in (2), so that total demand size does not change with the number of available products/therapies. When $\alpha_3 > 1$, however, the monopolist has the opportunity to exploit the larger average efficacy of the therapies and will set higher prices. This yields higher profits but also higher consumer surplus, as the following Proposition proves.

Proposition 1. *When $\alpha_3 = 1$, the introduction of a cocktail does not affect products' prices, profits and consumer surplus. The total number of treated patients is still $\frac{1}{2}$, but $\frac{1}{3}$ of it is now cured with the cocktail.*

When $\alpha_3 > 1$ prices, profits and consumer surplus are higher than in the absence of the cocktail and are increasing in α_3.

Before moving on to the analysis of cocktails in duopoly, we should mention that this is the only market structure (multi-product monopoly with uniform pricing) in which, in our assumed parameters range, the cocktail may crowd out standalone treatments, as the proof of Proposition 1 points out. The value of α_3 beyond which q_1 and q_2, α_{3i}^{*M} lies below $\alpha_3^*(\gamma)$ for $\gamma > \frac{1}{3}$.[15] Cocktails crowd out standalone products if $\gamma > \frac{1}{3}$ and $\alpha_3 \in [\alpha_{3i}^{*M}, \alpha_3(\gamma)]$. This has no effect on the results in Proposition 1, as shown. It may however affect the results in Section 4, as we shall discuss.

3.2. Duopoly

If products 1 and 2 are supplied by two independent firms, we can use demands from (16) and (17) to compute their profits. We can thus write:

$$\Pi_i = \frac{p_i\left[2(2+\alpha_3)(1-\gamma) - p_i(5-3\gamma) + p_j(1-3\gamma)\right]}{12(1-\gamma)},\ i,j = 1,2,\ i \neq j \tag{18}$$

and each firm's reaction function can be determined as:

$$p_i = \frac{2(2+\alpha_3)(1-\gamma) - (1-3\gamma)p_j}{2(5-3\gamma)}, i = 1,2 \tag{19}$$

From (19) we can prove the following.

Lemma 1. *The slope of the reaction functions change its sign as γ increases. In particular, $\frac{dp_i}{dp_j} \gtreqless 0$ iff $\gamma \gtreqless \frac{1}{3}$.*

[15]The value α_{3i}^{*M} is found using expression (13).

Lemma 1 shows that the degree of substitutability across products influences the nature of competition. The creation of the cocktail introduces a certain degree of complementarity between the two substitute goods. In particular, when $\gamma < \frac{1}{3}$, reaction functions are negatively sloped. The degree of substitutability is very low and the complementarity introduced in the market by the cocktail dominates. Firms react to their rivals' price reduction with a price increase.[16] On the other hand, when $\gamma > \frac{1}{3}$, substitutability prevails and reaction functions are positively sloped, as it is usual in Bertrand games. This suggests that a particular version of the so called "Cournot effect"(Sonnenschein, 1968) might emerge here. The Cournot effect (also known as "complementary monopoly") occurs when two goods are complements and they must be purchased together in some fixed proportion (as in the case of a patient cured with the cocktail). Then, the sum of the two prices set in a duopoly is higher than the sum of the two prices chosen by a monopolist selling both goods. This clearly reduces profits (and consumer surplus) but, when two goods are complements, each firm does not consider the negative impact that an increase in its price exerts on demand (exactly as in a Cournot duopoly it does not consider the negative impact of an increase in its quantity on the price), thus setting a too high price. Lemma 1 hints that the complementarity brought in by the cocktail might increase drug prices.

Other factors have been often held responsible for price increases in drug markets, especially collusion and price discrimination. With price discrimination, drugs might be sold at different prices depending on whether they are bought for stand-alone use or in a cocktail. In addition, firms producing cocktail components might collude and sell their drugs at a discount or at a premium. Thus, both price discrimination and collusion might justify an increase in drug prices following the introduction of a cocktail, as we will prove in later sections. Our first result, however, is that cocktails may entail a price increase even in the absence of price discrimination and/or collusive practices, as a direct consequence of simple profit maximization.

We now compare prices and profits in a duopoly with and without the cocktail. As before, we separate the analysis of the case with $\alpha_3 = 1$ from that with $\alpha_3 > 1$.

[16]Price competition with complementary goods is characterized by downward sloping reaction functions (Alvisi and Carbonara, 2013).

When $\alpha_3 = 1$ (the cocktail is of the same quality as the stand-alone drugs), the following results hold.

Proposition 2. *When $\alpha_3 = 1$, the introduction of a cocktail increases equilibrium prices for the two drugs ($p_i^c > p_i^{nc}$), so that $q_i^* < q_i^{nc}$, ($i = 1, 2$), and it decreases consumer surplus. Profits increase ($\Pi_i^c > \Pi_i^{nc}$) if and only if $\gamma > 0.175$.*

According to Proposition 2, prices are always higher with the cocktail. The pricing externality (the "Cournot effect") generated by the cocktail dominates over the pro-competitive effect of an increasing number of substitute products. This impacts consumer surplus negatively.

Also, while cocktails - ceteris paribus - soften competition, their presence does not necessarily benefit firms. In fact, when γ is very low (in our setup, when $\gamma < 0.175$), the Cournot effect is particularly severe, so that the demand for the two goods drops significantly, reducing each firm's profit. Thus, the introduction of a cocktail, absent any improvement in the efficacy of existing drugs (i.e., with $\alpha_3 = 1$), does not simply reduce consumer surplus, but may also reduce each firm's profits.

In conclusion, the introduction of cocktails, , not only reduces consumer surplus but it may also penalize firms.

This is a particularly relevant conclusion for the market of drugs and justifies the attention that the Food and Drug Administration (FDA) pays when approving new pharmaceutical cocktails in the US. In fact, cocktails are authorized only if they exhibit superior efficacy, fewer side effects or greater "convenience" relative to existing drugs (Lucarelli et al., 2017).

We now extend Proposition 2 to the case in which the cocktail is more effective than its single components, that is, when $\alpha_3 > 1$.

Proposition 3. *When $\alpha_3 > 1$:*

a. *A cocktail increases the prices for both goods ($p_i^c > p_i^{nc}$).*

b. *When $\gamma > 0.175$, a cocktail always increases profits ($\Pi_i^c > \Pi_i^{nc}$), $i = 1, 2$. If $\gamma < 0.175$, there exists a value $\bar{\alpha}_3 \in (1, \alpha_3(\gamma))$, such that $\Pi_i^c > \Pi_i^{nc}$ if and only if $\alpha_3 > \bar{\alpha}_3$, so that, even when the cocktail exhibits superior efficacy, its introduction might reduce profits (although for a smaller range of values of γ compared to the $\alpha_3 = 1$ case).*

c. *There exists a value $\tilde{\alpha}_3 \in (1, \alpha_3(\gamma))$ such that the cocktail increases consumer surplus when $\alpha_3 \in (\tilde{\alpha}_3, \alpha_3(\gamma)]$*

Similarly to Proposition 2, once again, if $\gamma > 0.175$ the introduction of a cocktail increases profits. Differently from Proposition 2, when $\gamma < 0.175$, profits may increase, notwithstanding a low γ, if the cocktail is sufficiently more effective that standalone drugs. This is because high quality counteracts the decrease in demand due to the higher prices generated by the Cournot effect. Also, cocktails increases consumer surplus when α_3 is high enough.

However, the introduction of a cocktail can decrease both profits and consumer surplus. This occurs when γ is very low, so that the three treatments are hardly substitutable and complementarity dominates.

Moreover, when $\gamma > 0.175$, cocktails might at the same time increase profits but decrease consumer surplus and are somehow detrimental for welfare. This occurs for an intermediate range of values $\alpha_3 \in (1, \tilde{\alpha}_3)$. In such range, while firms benefit from higher prices, consumers are harmed by both the higher prices and the lower quantities resulting from the introduction of the cocktail, so that they would need a very high cocktail quality to compensate, which explains the existence of the threshold $\tilde{\alpha}_3$.

Intuitively, we have seen that the introduction of the cocktail raises the prices of the standalone treatments with uniform pricing. As we will see, this result will hold also when firms engage in price discrimination.

Finally, it is worth pointing out that the cocktail's price is higher with a duopoly than with a multi - product monopoly if $\gamma < \frac{1}{3}$, as expected in a complementary monopoly.

Lemma 2. *The price for a unit of the cocktail treatment is higher in a duopoly when $\gamma < \frac{1}{3}$ and higher in a multi-product monopoly if $\gamma > \frac{1}{3}$.*

4. Cocktails and Uncoordinated Price Discrimination

We now introduce the possibility for the producers to price discriminate, that is to charge a different price when a drug is sold as a cocktail component. In this section we focus on non-coordinated price discrimination. This is the typical form of price discrimination, with firms setting their prices (for the drug sold as a standalone treatment and as a component of the cocktail) independently. Section 5 will consider coordinated price discrimination (a strategy that can be assimilated to collusion) and assume that firms cooperate when setting the price of cocktails (but not when setting the price of the standalone treatment).[17]

[17] One obvious question about price-discriminating practices under a cocktail regimen concerns its feasibility, especially in the presence of a premium. Consumers might have no incentive to reveal that the product they are

We will conduct our analysis in two steps, so as to disentangle the specific impact of competition. We consider the optimal pricing strategy of a multiproduct monopolist first and then we study the incentives to price discriminate in a duopoly. In this second step, our focus will on non-coordinated price discrimination here and on coordinated price discrimination in Section 5.

Specifically, we first analyze whether price discrimination is per se profitable and, if it is, whether producers would sell their cocktail component at a discount or at a premium compared to the price of the stand-alone product. For all subcases, a particular attention will be devoted to the impact of price discrimination on consumer surplus

4.1. The Multiproduct Monopolist

A price-discriminating, multi-product monopolist producing drugs 1 and 2 charges prices p_1 and p_2 when the two drugs are sold as stand alone therapies and p_3 for a cocktail unit, consisting of r_1 units of drug 1 and r_2 units of drug 2.

The monopolist chooses prices to maximize profits $\Pi_M = p_1\, q_1 + p_2\, q_2 + p_3\, q_3$. Substituting the expressions for demands q_1 - q_3 from (9) - (11), setting $\alpha_3 = 1$ and differentiating with respect to p_1, p_2 and p_3, we find equilibrium prices and quantities: $p^*_{M1} = p^*_{M2} = p^*_{M3} = \frac{1}{2}$ and $q^*_{M1} = q^*_{M2} = q^*_{M3} = \frac{1}{6}$. These are the same prices and quantities of a non-discriminating monopolist (see Section 3.1). Thus, no matter whether the good is sold separately or as a cocktail component, its price is the same and no price discrimination occurs. Profits are $\Pi^*_M = \frac{1}{4}$.

However, this result is not robust to an increase in the efficacy of the cocktail, i.e. when

purchasing will be used as a component of a cocktail and firms might not be able to extract this information. Although this might render price discrimination difficult, it is often possible to adopt second - degree price - discrimination mechanisms. For instance, premiums are often imposed indirectly, either through packaging or through "strategic dosage". To illustrate these practice, consider the following numerical example. Therapy 1 and therapy 2 consist of a single 10mg-tablet per day of drug 1 and drug 2, respectively, produced by two independent firms and to be consumed for N days. Therapy 3 consists of 5mg-per day of drug 1 and 5mg-per day of drug 2, also for N days. Suppose that a box of drug 1 and 2 contains N/3 10mg-tablets. Then, both therapies 1 and 2 need three boxes of the corresponding drug. However, in order to consume N doses of the cocktail, that is N/2 10mg-tablets of drug 1 and N/2 10mg-tablets of drug 2, 2 boxes of drug 1 and two boxes of drug 2 would need to be purchased, so that the effective per-dose price would be higher under a cocktail therapy. The same logic would also apply if each firm could sell two different packages in the market, one containing 10mg-tablets and the other 5mg-tables, choosing the number of tablets per box appropriately, while keeping the per-tablet price the same.

Even if the 5mg-tablet boxes contained N/2 tablets, premiums might arise if the 5mg boxes cost more that half of the price of the 10mg boxes and a group of consumers treated with the cocktail exhibits some aversion to the risk of preparing wrong dosages on their own or of wasting some tablets in the attempt of dividing them in two exact parts. In fact, it often happens that firms charge the same price per tablet, irrespective of the quantity of the active substance. So, a box with N 10mg-tablets could cost the same as a box with N 5mg-tablets.

$\alpha_3 \in [1, \alpha_3(\gamma)]$, as illustrated in the following Proposition.

Proposition 4. *When $\alpha_3 \in [1, \alpha_3(\gamma)]$, the price-discriminating monopolist sells the cocktail at a premium equal to $\delta_M = \frac{\alpha_3 - 1}{2}$. Prices and profits are $p^d_{Mi} = \frac{1}{2}$ $(i = 1, 2)$, $p^d_{M3} = \frac{\alpha_3}{2}$, and $\Pi^d_M = \frac{6+(3-\gamma)\alpha_3^2 - 4\gamma(1+\alpha_3}{36(1-\gamma)}$.*

Profits are always higher than under uniform pricing.

Price discrimination decreases consumer surplus compared to uniform pricing both when $\gamma < \frac{1}{3}$ and when $\gamma \geq \frac{1}{3}$ but $\alpha_3 \in (1, \alpha_3^{*M})$. When instead $\gamma \geq \frac{1}{3}$ and $\alpha_3 \in [\alpha_3^{*M}, \alpha_3(\gamma)]$, price discrimination increases consumer surplus.

In conclusion then, the possibility of engaging in price discriminatory practices by a multi-product monopolist might either lower the positive impact that the introduction of an effective cocktail per se has on consumers (see Proposition 1) or reinforce it, depending on the parameters' values. In particular, price discrimination improves consumer welfare when both the degree of substitutability and the improved efficacy of the cocktail are relatively high. When this happens, the monopolist would sell the cocktail together with the stand-alone treatments under price discrimination, but would only sell the cocktail under uniform pricing.

In any case, it can be easily verified that, in the presence of price discrimination, consumer surplus remains higher than without the cocktail. The presence of a new, more effective treatment would never decrease consumer surplus and, a fortiori, total surplus.

4.2. Uncoordinated Price Discrimination in Duopoly

We now assume that the two firms can engage in price discrimination, setting two different prices simultaneously and non-cooperatively: one for the good sold as a stand-alone product and one for the same good sold as a component of the cocktail.

Define p^d_i the equilibrium price of the stand-alone product i $(i = 1, 2)$ and p^d_{ic} the equilibrium price of the good sold as a cocktail component $(i = 1, 2)$.

The timing is as follows. Firms decide first whether to engage in price discrimination or not, then they compete in prices. The following results hold.

Proposition 5. *When $\alpha_3 \geq 1$ and firms engage in price discrimination non-cooperatively, selling the cocktail at a premium is a dominant strategy. Moreover, when $\gamma < 0.92$, there exists a value $\bar{\bar{\alpha}}_3 > 1$ such that price discrimination is not a Pareto efficient equilibrium when $\alpha_3 < \bar{\bar{\alpha}}_3$: both firms would get higher profits through uniform pricing, and price competition leads to a prisoner's dilemma. When $\gamma > 0.92$, price discrimination is Pareto efficient for any $\alpha_3 > 1$.*

Then, for $\gamma < 0.92$ there is a parameters' range in which price competition might lead to a prisoner's dilemma. This result can be explained as follows.

Selling their cocktail component at a premium lowers firms' profits if their opponent does the same. This is because high prices on the cocktail components are counterbalanced by fiercer price competition in the stand-alone markets.[18] Thus, firms get inefficiently aggressive in the market for stand-alone treatments, recovering only part of their lost earnings with a premium on the price of their cocktail component. At the same time, uniform pricing is not an equilibrium strategy, since each firm has the incentive to price discriminate.

As for consumer surplus, we get a result that is different from the one obtained in a multi-product monopoly:

Proposition 6. *In a duopoly, non-coordinated price discrimination decreases consumer surplus compared to uniform pricing.*

In a duopoly, price discrimination always harms consumers when firms sell both the stand-alone treatments and the cocktail. This result contrasts with that obtained in the case of a multi-product monopolist. In Section 7 we show that this result is not confirmed when we release our assumption about the range of values that α_3 can take. Particularly, when the duopolists sell only the cocktail under uniform pricing and the stand-alone treatments in addition to the cocktail under price discrimination, the latter yields higher consumer surplus, even if the cocktail is sold at a premium.[19]

What happens however if we compare the surplus consumer enjoy with price discrimination to consumer surplus before the introduction of the cocktail? In Proposition 3 we have shown that the cocktail may decrease consumer surplus when firms apply a uniform price. The next Lemma extends Proposition 3 to the case of price discrimination, finding qualitatively similar results.

Lemma 3. *When firms engage in price discrimination, if $\gamma < 0.69$, there exists $\hat{\alpha}_3 \in (1, \alpha_3(\gamma))$ such that the introduction of the cocktail raises consumer surplus if $\alpha_3 > \hat{\alpha}_3$. If $\gamma \geq 0.69$, consumer surplus is always lower with the cocktail.*

[18]This can be proven comparing the prices without price discrimination (p_i^c, found in Proposition 3) with those charged with price discrimination (p_i^d, found in the proof of Proposition 5).

[19]This case occurs when $\alpha_3 * (\gamma) < \alpha_3 < \alpha_{3i}^{*Dd}$. See the proof of Proposition 5.

5. Bundling Among Rivals: Coordinated Price Discrimination

In this section, we consider the incentives to coordinate on a bilateral discount (or premium) by two independent firms producing the two separate drugs. Following Gans and King (2006), the cocktail will be offered at a per-unit price $p_3 = r_1 p_1 + r_2 p_2 - \delta$, where firm 1 bears a portion k of the discount (or enjoys a portion k of the premium when $\delta < 0$), whereas firm 2 contributes for the remaining portion $1 - k$.

Given that the two firms are symmetric and they contribute to the cocktail with the same proportion of their respective drugs, we will assume that $k = \frac{1}{2}$.

The effective per-unit revenue of the cocktail are $r_1\, p_1 - \frac{\delta}{2}$ and $r_2\, p_2 - \frac{\delta}{2}$ for firms 1 and 2, respectively. Profits are

$$\Pi_1^\delta = p_1\, q_1 + (r_1\, p_1 - \frac{\delta}{2})\, q_3 \qquad (20)$$

$$\Pi_2^\delta = p_2\, q_2 + (r_2\, p_2 - \frac{\delta}{2})\, q_3 \qquad (21)$$

The timing of the game is as follows: given $k = \frac{1}{2}$, in the first stage firms choose δ to maximize joint profits $\Pi_1^\delta + \Pi_2^\delta$. In the second stage, they compete la Bertrand, setting the prices of their respective drugs non-cooperatively.

We can prove the following result:

Proposition 7. *When $\alpha_3 = 1$, under the sharing rule $k = \frac{1}{2}$, firms coordinate on a discount when $0 < \gamma < \frac{1}{3}$, while they coordinate on a premium when $\frac{1}{3} < \gamma < 1$.*

Thus, the cocktail is sold at a discount when the complementarity between the drugs dominates (and reaction functions are negatively sloped, recall Lemma 1). A premium is applied when the substitutability between the drugs in the cocktail dominates (and reaction functions are positively sloped).

This result is consistent with Gans and King (2006), who establish that discounts can be used as a device to soften competition in the prices for the stand-alone treatments, as shown in the following Lemma.

Lemma 4. *a. Under coordinated discrimination, the prices for stand-alone treatments are higher than without price discrimination ($p_i^{\delta *} > p_i^c$, $i = 1, 2$) and the price for the cocktail is lower ($p_3^{\delta *} > r_1 p_1^c + r_2 p_2^c$) if and only if $\gamma < \frac{1}{3}$.*

b. *The prices for stand-alone treatments are higher and the price of the cocktail is lower under coordinated than under non-coordinated price discrimination ($p_i^{\delta *} > p_i^d$, $i = 1, 2$, and $p_3^{\delta *} < r_1 p_{1c}^d + r_2 p_{2c}^d$) for any $\gamma \in [0, 1)$.*

c. *Profits are higher under coordinated price discrimination than under non-coordinated price discrimination and uniform pricing for any $\gamma \in [0, 1)$: $\Pi_i^{\delta *} > max\{\Pi_i^{cd}, \Pi_i^c\}$.*

The fact that coordination yields the highest profits and no price discrimination the lowest is rather intuitive.

The results relative to prices are however quite interesting : compared to uniform pricing, when substitutability is low ($\gamma < \frac{1}{3}$) and complementarity dominates, firms find it optimal to sell the cocktail at a discount, recouping the lost profits with an increase in the stand-alone prices. Vice-versa, when $\gamma > \frac{1}{3}$, firms would find it profitable to sell the cocktail at a premium, and to make competition on single products fiercer, again compared to uniform pricing.

As a result, coordinated price discrimination may have a positive impact on consumer surplus relative to uniform pricing, even when the cocktail does not exhibit superior efficacy, because firms sell either the cocktail or the stand-alone treatments at a lower price.

The following Proposition indeed establishes that this intuition is true for any value γ suche that the cocktail is offered at a discount.

Proposition 8. *When $\alpha_3 = 1$:*

a. *consumer surplus is higher under coordinated price discrimination than under uniform pricing if and only if $\frac{1}{3} < \gamma < 0.78$.*

b. *consumer surplus is always higher under coordinated than under non-coordinated price discrimination.*

When comparing coordinated and non-coordinated price discrimination, we should first notice that in both cases the duopolists use cocktail prices to reduce price competition on the stand-alone drugs and this irrespectively of the degree of substitutability γ. Morevover, as indicated in Proposition 8, the prices of the stand-alone treatments are always higher when firms coordinate, whilst the price of the cocktail is always smaller. This second effect seems to dominate, so that consumer surplus is always larger with coordination, as indicated by Proposition 8. This result looks interesting, because it indicates that, even when the three regimen are perceived to be of the same quality ($\alpha_3 = 1$) so that, as established earlier, the introduction of the cocktail without price discrimination always damages consumers (Proposition 2), there is a rationale to allow firms to coordinate and price discriminate. The results in Lemma 4 and Proposition 8 indeed indicate that consumer surplus is enhanced when the price of the cocktail decreases and that of the stand-alone treatments increases.

In equilibrium, firms leverage the price of the cocktail, reducing it in order to increase the demand for it more than the increase in the price of the stand-alone treatments reduces their demand.

We now generalize Proposition 7 to the case $\alpha_3 > 1$. We notice that firms have an increasing incentive to coordinate on a premium rather than on a discount.

The results on welfare are qualitatively similar to those obtained in Proposition 8 and are therefore omitted.

Proposition 9. *When $\alpha_3 > 1$, the firms would coordinate on a discount iff $\gamma < \widetilde{\gamma}(\alpha_3)$, while they coordinate on a premium iff $\gamma > \widetilde{\gamma}(\alpha_3)$, where $\frac{d\widetilde{\gamma}(\alpha_3)}{d\alpha_3} < 0$, so that the incentives to coordinate on a premium increase with α_3. In particular, if $\alpha_3 > 1.025$ firms will always coordinate on a premium for any $\gamma \in [0, 1)$.*

6. Cocktails: Monopoly or Competition?

In this section we compare the performance in terms of welfare of a multi-product monopoly and a duopoly in the presence of a cocktail.[20]

We know that in both market structures, firms would always engage in price discrimination, if feasible. Proposition 4 proves that profits are always higher with discrimination in a monopoly when $\alpha_3 > 1$. Proposition 5 shows that price discrimination is a dominant strategy in a duopoly and Lemma 4 proves that colluding on the price of the cocktail grants even higher profits than uncoordinated price discrimination. The highest producer surplus is obtained under a monopolistic market structure.[21]

[20] Computations are tedious and lengthy and follow the same procedures illustrated in the proofs of the propositions. They are therefore omitted for the sake of brevity, but are available upon request.

[21] With price discrimination, the duopolists' aggregate profit is lower than the one obtained by a monopolist operating in the same conditions. When $\alpha_3 = 1$, a monopolist has no incentive to price discriminate and its profits are equal to $\Pi_M = \frac{1}{4}$, while in a duopoly each firm obtains a profit equal to $\Pi_i^d = ((1-\gamma)(39-25\gamma))/(9(6-5\gamma)^2)$ (where Π_i^d has been obtained substituting $\alpha_3 = 1$ into the expressions provided in the proof of Proposition 5). It can be checked that $\Pi_M > \Pi_1^d + \Pi_2^d$. When $\alpha_3 > 1$, results are qualitatively the same. Finally, and quite obviously, the monopolist profit is higher than the aggregate profit in duopoly also when price discrimination is not feasible.

Interestingly, even when the duopolists manage to collude in setting the cocktail price, their aggregate profit is still strictly lower than the multi-product monopolist's profit, with the exception of $\gamma = \frac{1}{3}$, when the two profits are equal.[22] This is because coordinating on the cocktail price is not equivalent to a fully collusive setting: the duopolists maximize their joint profit only with respect to the cocktail price, whereas they set the stand-alone drug prices noncooperatively.

Turning to consumer surplus, we first notice that the introduction of the cocktail always increases consumer surplus under monopoly, whereas it may harm consumers under duopoly, both with uniform pricing and discrimination.

Moreover, absent price discrimination, it can be proven that consumer surplus is higher under monopoly when $\gamma < \frac{1}{3}$ and higher under duopoly if $\gamma > \frac{1}{3}$. When $\gamma < \frac{1}{3}$, the complementarity between the drugs is high and the prices set in duopoly are higher than the monopoly prices: in welfare terms, the possible benefits to consumers of a more competitive setting are overshadowed by the elimination of the "Cournot effect" thanks to a monopolistic structure.

Under uncoordinated price discrimination, it is useful to compare monopoly and duopoly for different values of α_3. When $\alpha_3 = 1$, we know that the monopolist has no incentive to price discriminate and in such case we are going to prove that consumer surplus is higher with a monopolistic setting if $\gamma < 0.359$. Indeed, price discrimination increases the range of the values of γ such that monopoly is preferable in terms of consumer welfare (it was $\gamma < \frac{1}{3}$ without discrimination). The unsurprising reason for that is the further increase of the cocktail price implied by price discrimination in a duopoly. In our assumed range of parameters $\alpha_3 \in [1, \alpha_3^*(\gamma))$, we can expand on this result, getting some further interesting insights, summarized by the following Lemma (the proof consists in a straightforward comparison of consumer surplus in the two market structures):

Lemma 5. *If firms engage in uncoordinated price discrimination,*

a. *If* $\alpha_3 = 1$, *consumer surplus is higher under monopoly if* $\gamma < 0.359$.
b. *If* $\alpha_3 \in (1, \alpha_3^*(\gamma)]$, *there exists* $0.359 < \bar{\gamma}(\alpha_3) \leq 0.697$, *increasing in* α_3, *such that consumer surplus is higher under monopoly if* $\gamma < \bar{\gamma}(\alpha_3)$.

Lemma 5 shows that consumers would prefer competition (a duopoly) when goods are highly substitutable (high γ) and monopoly when goods are complements (low γ). The cocktail quality

[22]When $\gamma = \frac{1}{3}$ the duopolists' reaction functions do not depend on the competitor's price.

α_3 determines the exact value of the cutoff $\bar{\gamma}(\alpha_3)$. In particular, the difference between the cocktail price in duopoly and in monopoly is increasing in α_3, so that, the higher α_3, the more likely it is that consumers prefer monopoly.

Finally, monopoly yields a higher consumer surplus than under coordinated price discrimination if $\gamma < \frac{1}{3}$ and a lower one otherwise. Recall that the duopolists coordinate on a discount when $\gamma < \frac{1}{3}$. The result we find could therefore look puzzling. Notice, however, that the duopoly prices (for the stand-lone treatments and for the cocktail) are higher than the monopoly ones if $\gamma < \frac{1}{3}$ and lower otherwise.

7. Extensions

7.1. Asymmetric Cocktails

In this section we verify how and in which direction the incentives to discriminate change when firms contribute asymmetrically to the cocktail. We assume now that r_1 can take any value in $[0, \frac{1}{2}]$ (the two firms are symmetric but for r_1 and r_2, thus when $r_1 > \frac{1}{2}$, firm 1 and firm 2 simply switch roles.

As a matter of fact, when firms engage in uncoordinated price discrimination, our generalization on the value of r_1 is not particularly relevant. In fact, it can be shown that the impact of cocktails in terms of profits and consumer surplus is qualitatively the same obtained in the case $r_1 = \frac{1}{2}$.

Things change substantially both when the duopolists charge uniform prices and when they engage in coordinated price discrimination.

Performing numerical simulations, we obtain that under uniform pricing the lower r_1 (and therefore the higher r_2, since $r_2 = 1 - r_1$) the lower the prices, the higher the difference between the profits of the two firms (clearly, higher profits accrue to the firm with the highest share of the cocktail). Moreover, the lower r_1, the higher consumer surplus. In summary, then, the bigger the asymmetry in the shares of the cocktail, the fiercer the competition between firms.

These results are confirmed when firms coordinate on the cocktail price. Particularly, the higher r_1, the higher the value of γ below which the firms apply a discount on the cocktail price. This seems to suggest that discounting will always occur when the degree of competition is relatively low. In fact, no matter the value of r_1, a discount is always applied when $\gamma < \frac{1}{3}$.

7.2. The Crowding-out Effect

In our analysis of the various market structures and pricing strategies, we have seen that cocktails crowds out standalone treatments when the prices producers set for their drugs push the demands of q_1 and q_2 to zero and leave only the cocktail in the market. This happens for high values of α_3, that is for $\alpha_3 > \alpha_3(\gamma) > 1$, where $\alpha_3(\gamma)$ is given by (13). Given that $\alpha_3(\gamma)$ depends on prices, each market structure has a different $\alpha_3(\gamma)$, that may vary also with respect to pricing strategy (uniform pricing vs. discrimination). We have computed the values of $\alpha_3(\gamma)$ pertaining to each market configuration in the proofs to Propositions. Comparing the various values of $\alpha_3(\gamma)$ under duopoly (given by α_{3i}^{*D} in (.10) for uniform pricing, by α_{3i}^{*Dd} in (.13) for uncoordinated discrimination and by $\alpha_{3i}^{*Dcoord}$ in (.32) for coordinated discrimination), we verify that $\alpha_{3i}^{*D} < \alpha_{3i}^{*Dd} < \alpha_{3i}^{*Dcoord}$ for all values of $\gamma \in [0, 1]$.

If we thus set $\alpha_3^*(\gamma) = \alpha_{3i}^{*D}$, and consider values of $\alpha_3 \in [1, \alpha_3^*(\gamma))$, we ensure that no crowding out occurs in duopoly.

Moreover, comparing the values of $\alpha_3(\gamma)$ under monopoly, we see that α_{3i}^{*M}, the cutoff value in a monopoly with uniform pricing (given by expression (.1)), is always smaller than α_{3i}^{*Md}, the cutoff value in a monopoly with discrimination (given by expression (.11)) and that $\alpha_{3i}^{*Md} > \alpha_3^*(\gamma)$.

However, as pointed out in Section 3.1, $\alpha_{3i}^{*M} < \alpha_{3i}^{*D}$ for $\gamma > \frac{1}{3}$.[23] For $\gamma > \frac{1}{3}$ we might have that $\alpha_{3i}^{*M} < \alpha_3 < \alpha_3^*(\gamma)$. In that range of parameters, a multi-product, non-discriminating monopolist would sell the cocktail only, whereas a monopolist applying price discrimination would sell both the standalone and the cocktail treatments. We have seen in Proposition 4 that while profits are always higher under discrimination, in that range price discrimination increases consumer surplus if α_3 is sufficiently high.

What happens when we release our assumption $\alpha_3 < \alpha_3^*(\gamma)$ and we look instead at higher values of $\alpha_3 > \alpha_3^*(\gamma)$? When we consider a duopoly with uniform prices and we compare it to a duopoly without cocktails, we have that the former sells the cocktail only, whereas the latter sells the standalone treatments only. The introduction of the cocktail raises consumer surplus if $\gamma < 0.31$. If $\gamma > 0.31$, there exists $\hat{\hat{\alpha}}_3 > \alpha_3^*(\gamma)$ such that the introduction of the cocktail increases

[23]We have dealt with this case in Proposition 4.

consumer surplus if $\alpha_3 > \hat{\hat{\alpha}}_3$ and lowers it otherwise.[24] This result is interesting, as it points out that a cocktail might have a negative effect on consumers when treatments are relatively close substitutes and the cocktail's quality is relatively low.

If we then introduce the possibility of (non-coordinated) price discrimination, if $\alpha_3^*(\gamma) < \alpha_3 < \alpha_{3i}^{*Dd}$, the cocktail is the only treatment sold with uniform pricing, whereas the standalone treatments in addition to the cocktail are sold under discrimination. This ensures that consumer surplus is higher in the latter case. Thus, price discrimination guarantees higher consumer surplus.

Finally, we compare non-coordinated price discrimination with a situation in which the cocktail is not available. If $\alpha_3^*(\gamma) < \alpha_3 < \alpha_{3i}^{*Dd}$, the introduction of the cocktail raises consumer surplus. If $\alpha_3 > \alpha_{3i}^{*Dd}$, the introduction of the cocktail raises consumer surplus if $\gamma < 0.82$. If $\gamma > 0.82$, there exists $\hat{\hat{\alpha}}_3 > \alpha_{3i}^{*Dd}$ such that the introduction of the cocktail raises consumer surplus if $\alpha_3 > \hat{\hat{\alpha}}_3$. This latter result is similar to that obtained when comparing a duopoly without cocktails to a non-discriminating monopoly with cocktails.

8. Conclusions

We have analyzed the effects of the introduction of a cocktail composed of a fixed proportion of two existing stand-alone products in a Bertrand duopoly with imperfectly substitutable goods. We have shown that a cocktail rises the Bertrand equilibrium prices as it introduces a certain degree of complementarity between previously substitute products. As a consequence, the improvement of the efficacy of the cocktail compared to stand-alone products must be large enough for consumers to be better-off with the introduction of the cocktail. In our model, the welfare-reducing impact of the "Cournot effect" might be mitigated by price discrimination, especially when it is coordinated among firms. In such case, firms sell the separate components at a discount or at premium depending on the degree of substitutability, so that the negative externalities associated to the "complementary oligopoly" are partially internalized and consumer surplus is greater than under uniform pricing, even when the cocktail's efficacy is not significantly higher. Instead, uncoordinated price discrimination has a detrimental effect on consumer

[24]The proofs of the results illustrated in this Section are available upon request.

surplus, unless the increase in the efficacy of the new treatment is large enough. These results might represent the benchmark for future research that investigates whether cocktails favor or rather deter the introduction of new therapies in pharmaceutical markets. Preliminary results in this direction show that cocktails reduce the opportunity to earn positive profits with a new stand-alone product if the producer is a new entrant, but it might enhance it if the producer is an incumbent supplying a component of the cocktail. In the latter case, the new product will be a substitute of both the existing drugs and of the cocktail but will not participate to the latter, so that it can be interpreted as an "innovative", possibly more effective, product. We find that the innovator raises the price of its old product in order to divert part of the demand to the new product. Interestingly, such innovation can be at the same time profitable and welfare reducing, so that its introduction might harm consumers. On the other hand, it is never the case that an innovation that increases consumer surplus is not profitable.

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Appendix

Proof of Proposition 1

From straightforward profit maximization, we have the following:

- **If** $\boldsymbol{\alpha_3 = 1}$, the monopolist chooses in equilibrium the same prices it would choose without a cocktail, namely $p^*_{Mi} = \frac{1}{2}$. Each treatment serves a demand equal to $q^*_{M1} = q^*_{M2} = q^*_{M3} = \frac{1}{6}$. Thus, the total quantity sold of good i is $q^*_i = \frac{1}{6} + r_i \frac{1}{6} = \frac{1}{4}$, $i = 1, 2$, whereas the total number of treated patients is $q^*_{M1} + q^*_{M2} + q^*_{M3} = \frac{1}{2}$, $\frac{1}{3}$ of it is cured with the cocktail. Quantities thus are $q^*_{Mi} = \frac{1}{4}$, $i = 1, 2$, while profits equal $\Pi^*_M = \frac{1}{4}$.

- **If** $\boldsymbol{\alpha_3 > 1}$, according to equation (13), q_1 and q_2 are sold both as stand-alone drugs and in a cocktail if $\alpha_3 < \alpha_3(\gamma)$. Define α^{*M}_{3i} the value taken by $\alpha_3(\gamma)$ in this case, where the superscript M stands for "monopolist". We shall determine the exact value of α^{*M}_{3i} below. Assume first that $\alpha_3 < \alpha^{*M}_{3i}$.

  The two prices are set at $p^{\alpha<\alpha^*_{3i}}_{M1} = p^{\alpha<\alpha^*_{3i}}_{M2} = \frac{2+\alpha_3}{6}$ (where $p^{\alpha<\alpha^*_{3i}}_{Mi} > p^*_{Mi}$: the prices the monopolist charges are higher with the cocktail). Monopoly profits are $\Pi^{\alpha<\alpha^{*M}_3}_M = \frac{(2+\alpha_3)^2}{36}$, which are always greater than Π^*_M, the profits obtained in the absence of the cocktail.

Substituting $p_{M1}^{\alpha<\alpha_{3i}^*}$ and $p_{M2}^{\alpha<\alpha_{3i}^*}$ into either the expression for q_1 in(9) or for q_2 in (10)

$$\alpha_3(\gamma) = \frac{2(2-\gamma)}{(1+\gamma)} = \alpha_{3i}^{*M} \tag{.1}$$

Comparing α_{3i}^{*M} with $\alpha_3^*(\gamma)$ in equation (15), $\alpha_{3i}^{*M} \geq \alpha_3^*(\gamma)$ if $\gamma \leq \frac{1}{3}$. In such a case, our restriction $\alpha_3 \in [1, \alpha_3^*(\gamma))$ ensures that q_1 and q_2 are always sold both as stand-alone drug and in the cocktail.

If $\gamma \geq \frac{1}{3}$ and $\alpha_3 \in [\alpha_3^M, \alpha_3^*(\gamma)]$, we have a corner solution where $q_1 = q_2 = 0$ and q_3 is given by (14).

Maximizing profits in such case, the equilibrium cocktail price is $p_{M3}^{\alpha_3>\alpha_3^*M} = \frac{\alpha_3}{2}$, so that $q_{M3}^{\alpha_3>\alpha_3^{*M}} = \frac{\alpha_3}{2(3-2\gamma)}$ and $\Pi_M^{\alpha_3>\alpha_3^{*M}} = \frac{\alpha_3^2}{4(3-2\gamma)}$. Prices are higher than in the case with $\boldsymbol{\alpha_3 < \alpha_{3i}^*}$ and without a cocktail. Profits are also higher.

We now consider the impact of a cocktail on consumer surplus. Consumer surplus is defined as

$$CS = U(q_1, q_2, q_3, M) - \left\{ \sum_{i=1}^{3} p_i q_i - M \right\} \tag{.2}$$

Substituting equilibrium quantities and prices, in a multi-product monopoly, consumer surplus when the cocktail is not sold is

$$CS_M^{nc} = \frac{1}{8}. \tag{.3}$$

When $\alpha_3 = 1$, the introduction of the cocktail has no effect on consumer surplus, so that

$$CS_M^{\alpha_3=1} = \frac{1}{8}. \tag{.4}$$

Both when $\gamma < \frac{1}{3}$ and when $\gamma \geq \frac{1}{3}$ but $\alpha_3 \in (1, \alpha_3^{*M})$ consumer surplus is

$$CS_M^{\alpha_3<\alpha_3^{*M}} = \frac{\alpha_3^2(9-\gamma) - 4\alpha_3(3+\gamma) + 4(3-\gamma)}{72(1-\gamma)} \tag{.5}$$

which is always greater than $\frac{1}{8}$.

Finally, when $\gamma \geq \frac{1}{3}$ and $\alpha_3 \in [\alpha_3^{*M}, \alpha_3^*(\gamma)]$, consumer surplus is

$$CS_M^{\alpha_3>\alpha_3^{*M}} = \frac{\alpha_3^2}{8(3-2\gamma)} \tag{.6}$$

Notice that in this case $CS_M^{\alpha_3>\alpha_3^{*M}} > \frac{1}{8}$, as well. Quite intuitively, when α_3 is high, consumers benefit from the presence of the cocktail. □

Proof of Proposition 2

Without the cocktail, the two firms charge prices $p_i^{nc} = \frac{2(1-\gamma)}{4-3\gamma}$, $i = 1, 2$, while the demands of the two drugs are $q_i^{nc} = \frac{(2-\gamma)}{8-6\gamma}$, $i = 1, 2$. This yields profits $\Pi_i^{nc} = \frac{(2-\gamma)(1-\gamma)}{(4-3\gamma)^2}$. Using the general expression in (.2), consumer surplus in this case

$$CS^{nc} = \frac{(2-\gamma)^2}{2(4-3\gamma)^2} \tag{.7}$$

With the cocktail, the equilibrium prices are $p_1^c = p_2^c = \frac{6(1-\gamma)}{11-9\gamma}$, while the quantities sold of each regimen (i.e., the demands of the two separate drugs, q_1^c, q_2^c and that of the cocktail q_3^c) are $q_i^c = \frac{(5-3\gamma)}{33-27\gamma}$, $i = 1, 2, 3$. Profits are $\Pi_i^c = \frac{3(1-\gamma)(5-3\gamma)}{(11-9\gamma)^2}$, $i = 1, 2$. Consumer surplus is

$$CS_{\alpha_3=1}^c = \frac{(5-3\gamma)^2}{2(11-9\gamma)^2} \tag{.8}$$

Comparison between prices, quantities, profits and consumer surplus yields the results.□

Proof of Proposition 3

Assume that $\alpha_3 \in (1, \alpha_3^*(\gamma))$, so that all demands in (9) - (11) are positive.

a. In such case, the Bertrand equilibrium prices are: $p_i^c = \frac{2(2+\alpha_3)(1-\gamma)}{11-9\gamma}$, $q_i^c = \frac{\alpha_3(20-27\gamma+9\gamma^2)-26+33\gamma-9\gamma^2}{6(44-69\gamma+27\gamma^2)}$, $\Pi_i^c = \frac{(2+\alpha_3)^2(1-\gamma)(5-3\gamma)}{3(11-9\gamma)^2}$, $i = 1, 2$. It is straightforward to check that $p_i^c > p_i^{nc}$ and $q_i^c < q_i^{nc}$, for any $\gamma \in [0, 1)$.

b. Profits Π_i^c are increasing in α_3, whereas Π_i^{nc} is obviously invariant with respect to α_3. It is immediate to show that $\Pi_i^c > \Pi_i^{nc}$ if $\alpha_3 = 1$ and $\gamma \geq 0.175$. Being Π_i^c increasing in α_3, this means that, for $\gamma \geq 0.175$, $\Pi_i^c \geq \Pi_i^{nc}$ $\forall \alpha_3 \in (1, \alpha_3(\gamma))$.

   If $\gamma < 0.175$, $\Pi_i^c < \Pi_i^{nc}$ at $\alpha_3 = 1$, whereas $\Pi_i^c > \Pi_i^{nc}$ at $\alpha_3 = \alpha_3(\gamma)$. Hence, there exists $\bar{\alpha}_3 \in (1, \alpha_3^*(\gamma))$ such that $\Pi_i^c \gtrless \Pi_i^{nc}$ if $\alpha_3 \gtrless \bar{\alpha}_3$.

c. We turn now to consumer surplus, again defined by equation (.2). Substituting prices and quantities, consumer surplus in a duopoly without a cocktail is given by equation (.7), whereas consumer surplus with the cocktail is

$$\begin{aligned} CS_d^{\alpha_3} &= \frac{\alpha_3^2\left(9\gamma^3 - 201\gamma^2 + 451\gamma - 267\right) + 4\alpha_3\left(9\gamma^3 + 42\gamma^2 - 143\gamma + 96\right)}{18(11-9\gamma)^2(\gamma-1)} + \\ &+ \frac{36\gamma^3 - 318\gamma^2 + 616\gamma - 342}{18(11-9\gamma)^2(\gamma-1)} \end{aligned} \tag{.9}$$

   Given $\alpha_3 \in (1, \alpha_3(\gamma))$, there exists $1 < \tilde{\alpha}_3 < \alpha_3(\gamma)$ such that $CS_d^{\alpha_3} \gtrless CS_d^{nc}$ if $\alpha_3 \gtrless \tilde{\alpha}_3$.

   In fact, $CS_d^{\alpha_3}$ is increasing in α_3 and CS_d^{nc} does not depend on α_3. $CS_d^{\alpha_3} < CS_d^{nc}$ at $\alpha_3 = 1$, whereas $CS_d^{\alpha_3} > CS_d^{nc}$ at $\alpha_3 = \alpha_{3i}^{*D}$.

   It is worth mentioning that numerical simulations indicate that, when $\gamma < 0.175$, $\tilde{\alpha}_3 > \bar{\alpha}_3$. The cutoff $\tilde{\alpha}_3$ is decreasing in γ.

   Finally, at the equilibrium prices

$$\alpha_3(\gamma) = \frac{6\gamma^2 - 25\gamma + 21}{6 - \gamma - 3\gamma^2} = \alpha_{3i}^{*D} \tag{.10}$$

   which is exactly the value chosen for our cutoff $\alpha_3^*(\gamma)$ in equation (15). We will show in Appendix 2 that α_{3i}^{*D} is the value that guarantees that the standalone treatments are always sold in duopoly.

Proof of Proposition 4

If the monopolist price discriminates, it maximizes profits with respect to p_1, p_2 and p_3. Substituting the expressions for demands q_1, q_2 and q_3 from (9), (10) and (11), equilibrium prices are $p^d_{Mi} = \frac{1}{2}$ ($i = 1, 2$, where the superscript d stands for "discrimination "), and $p^d_{M3} = \frac{\alpha_3}{2}$. Equilibrium quantities are $q^d_{Mi} = \frac{3-(2+\alpha_3)\gamma}{18(1-\gamma)}$, $i = 1, 2$ and $q^d_{M3} = \frac{(3-\gamma)\alpha_3-2\gamma}{18(1-\gamma)}$. Equilibrium profits are $\Pi^d_M = \frac{6+(3-\gamma)\alpha_3^2-4\gamma(1+\alpha_3)}{36(1-\gamma)}$.

Notice first that, given the equilibrium prices, q_1 and q_2 are positive iff

$$\alpha_3 < \alpha_{3i}^{*Md} = \frac{3-2\gamma}{\gamma} \tag{.11}$$

However, it is easy to check that $\alpha_{3i}^{*Md} > \alpha_3(\gamma)$. Differently from uniform pricing, then, the possibility to price discriminate always implies that goods 1 and 2 will be sold both as stand-alone treatments and as parts of the cocktail. In such parameter's range the monopolist charges a premium on the goods sold in the cocktail, since $p^d_{M3} = \frac{\alpha_3}{2} > r_1 p^d_{M1} + r_2 p^d_{M2} = \frac{1}{2}$. Particularly, the premium is equal to $\delta_M = p^d_{M3} - \frac{1}{2} = \frac{\alpha_3-1}{2}$.

Finally, it can be proven both that $\Pi^d_M > \Pi_M^{\alpha_3<\alpha_3^{*M}}$ and that $\Pi^d_M > \Pi_M^{\alpha_3>\alpha_3^{*M}}$. Thus, the monopolist always finds it profitable to price discriminate.

Using expression (.2), consumer surplus with price discrimination when $\alpha_3 < \alpha_3(\gamma)$ and goods are sold both as stand alone and in the cocktail is

$$CS^d_M = \frac{\alpha_3^2(3-\gamma) - 4\alpha_3\gamma + 2(3-2\gamma)}{72(1-\gamma)} \tag{.12}$$

Under uniform pricing, both when $\gamma < \frac{1}{3}$ and when $\gamma \geq \frac{1}{3}$ but $\alpha_3 \in (1, \alpha_3^{*M})$, consumer surplus is given by $CS_M^{\alpha_3<\alpha_3^{*M}}$ in expression (.5) in the proof of Proposition 1, while when $\gamma \geq \frac{1}{3}$ and $\alpha_3 \in [\alpha_3^{*M}, \alpha_3(\gamma)]$ the monopolist sells the cocktail only and consumer surplus is given by the expression (.6)

Comparing consumer surplus with and without discrimination when $\gamma < \frac{1}{3}$ and when $\gamma \geq \frac{1}{3}$ but $\alpha_3 \in (1, \alpha_3^{*M})$ so that in both cases drugs are sold as stand-alone treatments and in a cocktail, we see that

$$CS_M^{\alpha_3<\alpha_3^{*M}} - CS^d_M = \frac{(\alpha_3-1)^2}{12(1-\gamma)} > 0$$

Thus, in this case discrimination entails a reduction in consumer surplus when $\gamma < \frac{1}{3}$ and $1 < \alpha_3 < \alpha_3^{*M}$.

Comparing consumer surplus with and without discrimination when $\gamma \geq \frac{1}{3}$ and $\alpha_3 \in [\alpha_3^{*M}, \alpha_3(\gamma)]$ entails comparing a situation in which the consumer sells the cocktail only (with uniform pricing) and a situation in which the consumer sells the stand-alone treatments and the cocktail (with discrimination). We thus need to compare expression (**??**) to expression (.12).

We see that

$$CS^d_M > CS_M^{\alpha_3>\alpha_3^{*M}}$$

since

$$CS^d_M - CS_M^{\alpha_3>\alpha_{3i}^{*M}} = \frac{((\alpha_3+2)\gamma-3)^2}{36(1-\gamma)(3-2\gamma)} > 0$$

□

Thusm discrimination entails a reduction in consumer surplus, for all $\gamma \in [0,1]$.

Proof of Proposition 5

Define p_i^d and p_{ic}^d $(i = 1,2)$ the price of drug i when it is sold as a stand-alone treatment and when it is a cocktail component, respectively. Conditional on both firms deciding to price discriminate, profits in the second stage would be $\Pi_i = p_i^d\, q_i + \frac{1}{2}\, p_{ic}^d\, q_c$, $i = 1,2$.

Bertrand equilibrium prices are $p_i^d = \frac{3(1-\gamma)}{(6-5\gamma)}$, $p_{ic}^d = \frac{2}{3}\left(\alpha_3 - \frac{\gamma}{6-5\gamma}\right)$. Quantities are $q_i^d = \frac{5(\alpha_3+2)\gamma^2-6(\alpha_3+6)\gamma+27}{27(\gamma-1)(5\gamma-6)}$, $i = 1,2$, $q_c^d = \frac{\alpha_3(\gamma-3)+2\gamma}{27(\gamma-1)}$.

Demanded quantities of the stand-alone treatments are positive ($q_i^d > 0$) if

$$\alpha_3 < \alpha_{3i}^{*Dd} = \frac{27 - 36\gamma + 10\gamma^2}{\gamma(6-5\gamma)} \tag{.13}$$

However, it is easy to check that $\alpha_{3i}^{*Dd} > \alpha_3^*(\gamma)$, so that such quantities are always positive in our parameters' range. Moreover, the demanded quantity of the cocktail is always positive under this pricing scheme, as well. In fact, we have $q_3^d > 0$ if $\alpha_3 > \frac{2\gamma}{3-\gamma} \leq 1$ for all $\gamma \in [0,1]$.

Comparing prices, it can be seen that the cocktail will be sold at a premium δ^d, where $\delta^d = p_{ic}^d - \{r_1 p_1^d + r_2 p_1\} = \frac{2}{3}\,\alpha_3 - \frac{9-7\gamma}{3(6-5\gamma)} > 0$.

Equilibrium profits are $\Pi_i^d = \frac{1}{81}\left(\frac{9\left(5(\alpha_3+2)\gamma^2-6(\alpha_3+6)\gamma+27\right)}{(6-5\gamma)^2} + \frac{(\alpha_3(\gamma-3)+2\gamma)\left(\alpha_3+\frac{\gamma}{5\gamma-6}\right)}{\gamma-1}\right)$, $i = 1,2$.

We now compare Π_i^c, the profits without price discrimination we found in the proof of Proposition 3, with Π_i^d.

Particularly, we notice that $\Pi_i^d - \Pi_i^c > 0$ if $\alpha_3 > \bar{\alpha}_3(\gamma) = \frac{-738\gamma^3+2527\gamma^2-2892\gamma+1107}{-360\gamma^3+1342\gamma^2-1662\gamma+684} < \alpha_3(\gamma)$ so that both firms would engage in price discrimination if α_3 is sufficiently high.

Also, $\bar{\alpha}_3(\gamma) \leq 1$ if and only if $\gamma \geq 0.92$, which implies that, when the two stand-alone products are highly substitutable, price discrimination is profitable for any $\alpha_3 \geq 1$.

However, when $\gamma < 0.92$, $\frac{d\bar{\alpha}_3}{d\gamma} < 0$, meaning that the less substitutable products are, the more likely it is that uniform pricing yields higher profits than price discrimination.

We now show that the ability to price discriminate might lead to a Prisoner's dilemma, that is, to a situation in which both firms price discriminate even if they would be better off if they both did not and this because price discrimination is a dominant strategy.

If firm 1 decides to engage in price discrimination while firm 2 chooses uniform pricing, profits would be

$$\begin{aligned} \Pi_1 &= p_1\, q_1 + \frac{1}{2}\, p_{1c}\, q_c \\ \Pi_2 &= p_2\, (q_2 + \frac{1}{2}\, q_c) \end{aligned} \tag{.14}$$

Bertrand equilibrium prices are $p_1^{d\prime} = \frac{(1-\gamma)((2\alpha_3-23)\gamma+57)}{6(6\gamma^2-23\gamma+19)}$, $p_2^{d\prime} = \frac{2\alpha_3\left(7\gamma^2-31\gamma+27\right)+\gamma^2+5\gamma-12}{18\gamma^2-69\gamma+57}$ and $p_{1c}^{d\prime} = \frac{2(1-\gamma)(\alpha_3(3-2\gamma)+4(3-\gamma))}{18\gamma^2-69\gamma+57}$.

Equilibrium quantities are $q_1^{d\prime} = \frac{-4(\alpha_3+2)\gamma^3+6(3a3+8)\gamma^2-(16a3+95)\gamma+57}{18(1-\gamma)(6\gamma^2-23\gamma+19)}$, $q_2^{d\prime} = \frac{(2-\gamma)\left(\alpha_3\left(4\gamma^2-6\right)+8\gamma^2-39\gamma+33\right)}{18(1-\gamma)(6\gamma^2-23\gamma+19)}$ for the stand-alone treatments and $q_3^{d\prime} = \frac{\alpha_3\left(-2\gamma^3+17\gamma^2-40\gamma+27\right)-4\gamma^3+13\gamma^2-5\gamma-6}{9(1-\gamma)(6\gamma^2-23\gamma+19)}$ for the cocktails.

It can be checked that $p_i^{d'} > p_i^d$, $(i = 1, 2)$ and $p_{1c}^{d'} > p_{ic}^d$, indicating that firm 1 would charge higher prices on both its stand-alone treatment and its cocktail component compared to the case in which both competitors price discriminate.

Substituting equilibrium prices and quantities into profits in (.14), we can see that[25]

$$\Pi_1^{d'} > max\left\{\Pi_i^c, \Pi_i^d\right\} > \Pi_2^{d'} \tag{.15}$$

When $\gamma < 0.92$ and $\alpha_3 < \bar{\alpha}_3(\gamma)$, we know that $\Pi_i^c > \Pi_i^d$, so that the following holds:

$$\Pi_1^{d'} > \Pi_i^c > \Pi_i^d > \Pi_2^{d'} \tag{.16}$$

Choosing to price discriminate in the first stage is a dominant strategy for both firms, given the symmetry of the game. The resulting equilibrium outcome is not Pareto-Efficient, though, given that $\Pi_1^c > \Pi_i^d, i = 1, 2$. $\square$

Proof of Proposition 6

Consider first the case $\alpha_3 = 1$. Without price discrimination, consumer surplus is given by expression (.8) in the proof of Proposition 2.

With non-coordinated price discrimination, using the definition of consumer surplus in expression (.2), we find:

$$CS_{\alpha_3=1}^d = \frac{25\gamma^2 - 82\gamma + 66}{18(6 - 5\gamma)^2} \tag{.17}$$

A direct comparison of expressions (.8) and (.17) shows that $CS_{\alpha_3=1}^c > CS_{\alpha_3=1}^d$ for all $\gamma \in [0, 1)$. Surplus with and without price discrimination is the same only if $\gamma = 1$, that is, when the two treatments are perfect substitutes. That is, however, a limit case, in which prices are set equal to marginal costs (here zero).

We then turn to the case in which $\alpha_3 > 1$.

Using again expression (.2), when $1 < \alpha_3 < \alpha_3^*(\gamma)$, consumer surplus under uniform pricing equals expression (.9) in the proof of Proposition 3. With price discrimination, consumer surplus equals

$$CS_{\alpha_3>1}^d = \frac{\alpha_3^2(6 - 5\gamma)^2(\gamma - 3) + 4\alpha_3(6 - 5\gamma)^2\gamma + 100\gamma^3 - 588\gamma^2 + 972\gamma - 486}{162(6 - 5\gamma)^2(\gamma - 1)} \tag{.18}$$

Computing the difference between (.9) and (.18) we find

$$\begin{aligned} CS_{\alpha_3>1}^c - CS_{\alpha_3>1}^d \quad &= \quad \frac{4\alpha_3^2(6 - 5\gamma)^2\left(171\gamma^2 - 418\gamma + 255\right) - 64\alpha_3(6 - 5\gamma)^2\left(18\gamma^2 - 44\gamma + 27\right)}{81(11 - 9\gamma)^2(6 - 5\gamma)^2(1 - \gamma)} + \\ &+ \quad \frac{11781\gamma^4 - 57364\gamma^3 + 104826\gamma^2 - 85212\gamma + 26001}{81(11 - 9\gamma)^2(6 - 5\gamma)^2(1 - \gamma)} \end{aligned} \tag{.19}$$

The denominator in expression (.19) is always positive. We thus focus on the sign of the numerator. First of all, notice that (.19) is increasing in α_3 iff:

$$\alpha_3 > \frac{8\left(18\gamma^2 - 44\gamma + 27\right)}{171\gamma^2 - 418\gamma + 255} \tag{.20}$$

[25] Profits are computed using Mathematica and are very long polynomials in γ and α_3. We omit their algebraic expressions here, since they do not add much to the argument. Full calculations are available upon request.

Since the right-hand-side of (.20) is always smaller than 1, this condition is always satisfied, given our assumptions ($\alpha_3 > 1$). Thus, the difference $CS^c_{\alpha_3>1} - CS^d_{\alpha_3>1}$ is always increasing in α_3 and to prove that $CS^c_{\alpha_3>1} - CS^d_{\alpha_3>1} > 0$ it suffices that $CS^c_{\alpha_3>1} - CS^d_{\alpha_3>1} > 0$ for $\alpha_3 = 1$. In fact, we have shown before that, for $\alpha_3 = 1$, $CS^c_{\alpha_3=1} > CS^d_{\alpha_3=1}$ always. Thus, $CS^c_{\alpha_3>1} > CS^d_{\alpha_3>1}$ for all $\gamma \in [0, 1)$ and for all $1 < \alpha_3 < \alpha_3^*(\gamma)$.

Proof of Lemma 3

When $1 < \alpha_3 < \alpha_(\gamma)$, we need to compare expression (.18) with the amount of consumer surplus without the cocktail ((.7)).

$CS^d_{\alpha_3>1}$ is an increasing function of α_3, whereas CS^{nc} is invariant with respect to α_3.[26]

At $\alpha_3 = 1$, $CS^d_{\alpha_3>1} < CS^{nc}$.

At $\alpha_3 = \alpha_3^*(\gamma)$, $CS^d_{\alpha_3>1} > CS^{nc}$ if $\gamma < 0.69$.

Thus, if $\gamma \geq 0.69$, $CS^d_{\alpha_3>1} \leq CS^{nc}$ at $\alpha_3 = \alpha_3^*(\gamma)$.

If $\gamma < 0.69$, there exists $\hat{\alpha}_3$, such that $CS^d_{\alpha_3>1} > CS^{nc}$ if and only if $\alpha_3 > \hat{\alpha}_3$ and $CS^d_{\alpha_3>1} < CS^{nc}$ if and only if $\alpha_3 < \hat{\alpha}_3$.$\square$

Proof of Proposition 7

We solve the game by backward induction. Given δ and k, each firm maximizes Π_i $(i = 1, 2)$ after substituting demand expressions from (9), (10) and (11). We thus obtain the Bertand equilibrium prices:

$$p_1^\delta = \frac{2(1-\gamma)(9(3-\gamma)+\delta(k(11-9\gamma)+8))}{3\left(33-38\gamma+9\gamma^2\right)} \qquad (.21)$$

$$p_2^\delta = \frac{2(1-\gamma)(9(3-\gamma)+\delta(19-9\gamma-k(11-9\gamma)))}{3\left(33-38\gamma+9\gamma^2\right)} \qquad (.22)$$

Using these equilibrium prices, and assuming $k = \frac{1}{2}$ we obtain the equilibrium quantities:

$$q_i^\delta = \frac{15+9\gamma^2-24\gamma+4\delta(6-5\gamma)}{81\gamma^2-180\gamma+99}, \quad i = 1, 2, 3 \qquad (.23)$$

Finally, substituting these equilibrium values in the profit functions (20) and (21), we obtain the equilibrium profits as a function of δ:

$$\Pi_i^\delta = \frac{9\gamma^2\left(-9\delta^2+\delta+33\right)-\gamma\left(-200\delta^2+12\delta+351\right)+3\left(-41\delta^2+\delta+45\right)-81\gamma^3}{9(11-9\gamma)^2(1-\gamma)}, \quad i = 1, 2 \qquad (.24)$$

Maximizing joint profits $\Pi_1^\delta + \Pi_2^\delta$ with respect to δ, we obtain

$$\delta^* = \frac{3(1-3\gamma)(1-\gamma)}{162\gamma^2-400\gamma+246} \qquad (.25)$$

from which it is immediate to see that $\delta^* > 0$ when $\gamma < \frac{1}{3}$ and $\delta^* < 0$ when $\gamma > \frac{1}{3}$.$\square$

[26]Now, $CS^d_{\alpha_3>1}$ is increasing in α_3 if $\alpha_3 > \frac{2\gamma}{3-\gamma}$, which is always true since $\frac{2\gamma}{3-\gamma} < 1$.

Proof of Lemma 4

Substituting δ^* in the Bertrand-Nash equilibrium prices, quantities and profits we obtain

$$p_i^{\delta*} = \frac{3(45-37\gamma)(1-\gamma)}{162\gamma^2-400\gamma+246}, \quad i=1,2 \tag{.26}$$

$$p_3^{\delta*} = \frac{51\gamma^2-117\gamma+66}{81\gamma^2-200\gamma+123} \tag{.27}$$

$$q_i^{\delta*} = \frac{54\gamma^2-155\gamma+111}{486\gamma^2-1200\gamma+738}, \quad i=1,2 \tag{.28}$$

$$q_3^{\delta*} = \frac{27\gamma^2-82\gamma+57}{243\gamma^2-600\gamma+369} \tag{.29}$$

and

$$\Pi_i^{\delta*} = \frac{(61-36\gamma)(1-\gamma)}{324\gamma^2-800\gamma+492}, \quad i=1,2 \tag{.30}$$

a. Comparing the price in (.26) to the price set absent price discrimination (obtained in Proposition 2), $p_i^c = \frac{6(1-\gamma)}{11-9\gamma}$, we can see that $p_i^{\delta*} > p_i^c$ if and only if $\gamma < \frac{1}{3}$.

   Comparing the price of the cocktail with coordinated price discrimination (in expression (.27)), with the cocktail price without discrimination (obtained in Proposition 2 and equal to $p_3^c = r_1 p_1^c + r_2 p_2^c$), we can see that $p_3^{\delta*} < p_3^c$ if and only if $\gamma < \frac{1}{3}$.

b. Similarly, comparing (.26) and (.27) to the prices set with non-cooperative price discrimination (obtained in Proposition 5), $p_i^d = \frac{3(1-\gamma)}{6-5\gamma}$ $(i=1,2)$, and $p_{3c}^d = \frac{2}{3}\left(\alpha_3 - \frac{\gamma}{6-5\gamma}\right)$ it is possible to prove that $p_i^{\delta*} > p_i^d$ $(i=1,2)$ and $p_3^{\delta*} < p_3^d$ for all $\gamma \in [0,1)$.

c. Comparing profits, it is possible to check that $\Pi_i^{\delta*} > \Pi_i^{cd} > \Pi_i^c$, that is, the profits under coordinated price discrimination are always higher than those under non-coordinated price discrimination, which, in turn, are higher than profits without discrimination, and this for all $\gamma \in [0,1)$ □

Proof of Proposition 8

Substituting equilibrium prices and quantities into the general expression for consumer surplus (.2), consumer surplus with coordinated price discrimination is

$$CS^{\delta*} = \frac{18\gamma^2-61\gamma+51}{324\gamma^2-800\gamma+492} \tag{.31}$$

a. Comparing equation (.31) with the expression for consumer surplus under uniform pricing, CS^c (obtained in Proposition 2) we find that $CS^{\delta*} \geq CS^c$ if $\gamma \leq \frac{1}{3}$.

b. Similarly, comparing equation (.31) with the expression for consumer surplus under non-coordinated price discrimination $CS^c_{\alpha_3=1}$ found in expression (.8) in the proof of Proposition 6, we find that $CS^{\delta*} > CS^c_{\alpha_3=1}$ for all $\gamma \in [0,1)$.

□

Proof of Proposition 9

Following the same steps described in the proof of Proposition 7, the value of δ that maximizes joint profits is $\delta^* = \frac{12-206\gamma+87\gamma^2-2a_3(60-97\gamma+39\gamma^2)}{(246-400\gamma+162\gamma^2)}$. Notice that $\frac{\partial\delta^*}{\partial\alpha_3} < 0$ for any $\gamma \in [0,1)$, and that $\delta^* \lesseqgtr 0$ iff $\gamma \gtreqless \widetilde{\gamma}(\alpha_3) = \frac{97a_3-103+\sqrt{49a_3^2+52a_3-92}}{3(26a_3-29)}$, where $\frac{d\widetilde{\gamma}(\alpha_3)}{d\alpha_3} < 0$. Finally, $\widetilde{\gamma}(\alpha_3) = 0$ when $\alpha_3 = 1.025$, so that for any $\alpha_3 \geq 1.025$, firms would only coordinate on a premium no matter the degree of substitutability across single products.$\square$

Notice that, this case,

$$\alpha_{3i}^{*Dcoord} = \frac{369 - 108\gamma^3 + 517\gamma^2 - 774\gamma}{54\gamma^3 - 110\gamma^2 + 24\gamma + 36} \qquad (.32)$$