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Published Version:
Moguillansky M.O., Rotolo A., Simari G.R. (2019). Hypotheses and their dynamics in legal argumentation. EXPERT SYSTEMS WITH APPLICATIONS, 129, 37-55 [10.1016/j.eswa.2019.03.047].

Availability:
This version is available at: https://hdl.handle.net/11585/711688 since: 2020-01-06

Published:

DOI: http://doi.org/10.1016/j.eswa.2019.03.047

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Hypotheses and Their Dynamics in Legal Argumentation

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Abstract

We investigate some legal interpretation techniques from the viewpoint of the Argentinian jurisprudence. This allows the proposal of a logical framework –from a computer science perspective– for modeling such specific reasoning techniques towards an appropriate construction of legal arguments. Afterwards, we study the usage of assumptions towards construction of hypotheses. This is proposed in the dynamic context of legal procedures, where the referred argumentation framework evolves as part of the investigation instance prior to the trial. We propose belief revision operators to handle such dynamics, preserving a coherent behavior with regards to the legal interpretation used. Abduction is finally proposed to construct systematic hypothesization, with the objective to bring semi-automatic recommendations to push forward the investigation of a legal case.

Keywords: Legal Reasoning, Argumentation, Hypothetical Reasoning, Fact Finding, Argumentation Theory, Belief Revision.

1. Introduction

Reasoning about evidence is central in legal argumentation. This also involves the generation and finding of additional probatory elements towards constructing the *rules of evidence* upon which the definite argumentation will arise. Some of those elements are a result of inquiries and analysis of, e.g., testimonies of the parties in dispute about their view of the facts. However, additional elements may arise by posing hypotheticals, for example, to critique proposed facts. This implies that the process of fact investigation is dynamic by nature. The investigator develops the inquiry in unique episodes and each time hypotheses will arise for finding new evidence.

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March 7, 2019

Indeed, reasoning about evidence in formal legal procedure is a complex multi-stage process where different pieces of evidence are added and different hypotheticals can be considered and revised. This paper addresses two research issues

- the possibility of incorporating new hypotheses to a legal instance, which corresponds to a possibly iterated process of belief revision;
- the process of incorporation of hypothetical interplays with an argumentation setting where the interpretation of law is modelled.

Our claim is that, the investigator or the decision-maker can be assisted through *specialised information systems* in her dynamic process of fact investigation. This would involve the consideration of arguments on available evidence as well as hypotheticals about missing or incomplete evidence connecting yet available pieces of evidence. This latter alternative brings not only the possibility of a better understanding of the matters of facts, but also on the consideration of new assumptive elements, upon admitted levels of probative force, for advancing in the reasoning process.

In this article we present a fresh logical formalisation about the mentioned dynamic aspect of legal arguments on evidence (Ashley, 1991; Walton, 2003, 2004; Walton et al., 2008; Walton, 2008; Kaptein et al., 2009; Bex, 2011) and a legal framework in which such legal arguments interact. Upon such legal framework, we provide the possibility of making assumptions for constructing hypotheses. This sort of special arguments will allow us to push forward the analysis of a legal case. Based on the theory of belief revision (Alchourrón et al., 1985; Hansson, 1999; Moguillansky et al., 2012), we discuss a method for including such special kind of arguments in the legal framework which would also incorporate the necessary assumptions for their construction. This process will ultimately impact on the resulting argumentation contributing to the investigation of the legal case in the preliminary stages before the trial. Therefore, the logical formalisation presented here proposes the foundations for a further implementation of software recommenders dedicated to assist the construction of proofs previous to the trial.

The main contributions of this article can be summarized as:

1. A mathematical model for the formalization of
   (a) legal interpretations
   (b) legal arguments
   (c) hypotheses
   (d) change operations for dealing with the dynamics of legal cases
2. Legal practicioners would benefit from future implementations with the automatic verification for
   (a) The constructive correctness of arguments and hypotheses
   (b) The completeness of relevant jurisprudence to refer
   (c) Contributing with procedural economy
   (d) Measuring legal certainty

Figure 1 summarizes the different classification of arguments handled in this article. This brings a general overview of the expressivity that the theory here proposed is capable of reaching. Observe that a *hypothetical argument* can be sub-classified according to the sort of information it assumes: novel norms (*lex ferenda*) or assumptions (*lex lata*). Similarly, an *hypothetical lex lata* can be considered *internal* or *external* depending if its assumptions are already included in the case's investigation.

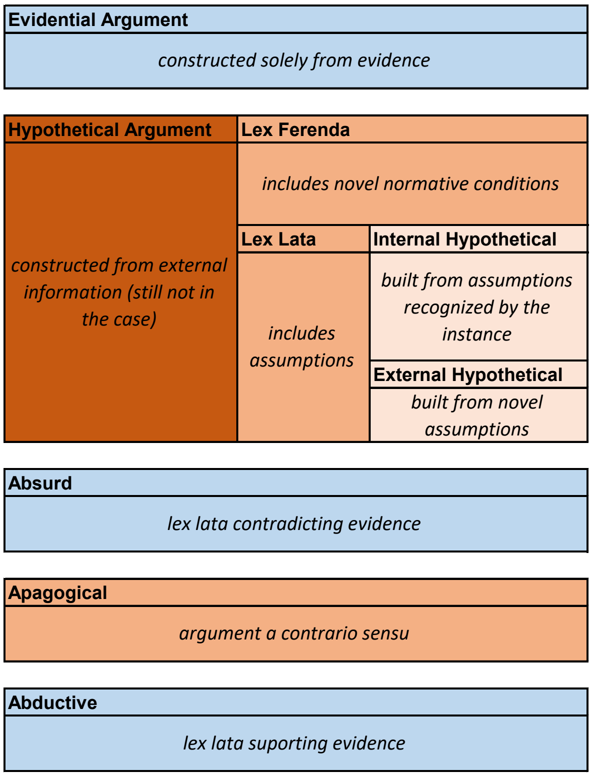

Figure 1: Arguments Summarized

2. A Theoretical Proposal Towards Specialized Information Systems

Before introducing the formal details of our proposal, we would like to introduce the interested reader to the underlying practical perspective. Although this article does not propose any particular software implementation, a brief discussion of our standpoint on the matter could be valuable for understanding some particularities of this theoretical proposal. In particular, we do not introduce any natural language processing (NLP) algorithm to obtain the fundamental knowledge upon which the argumentation will be constructed. Throughout the article, the reader will find some extensive discussions about the way to represent propositional knowledge in the light of some specific legal interpretation.

One of the purposes of this paper is to propose a model for legal interpretation in a way that they could bring a sort of flexibility to the knowledge stated by propositions and the way such propositions interact with each other. The focus is concentrated on legal interpretations in a twofold manner: how to formally model them and how to bring them the possibility to change. The formal model for legal interpretation we propose, would be capable of evolving according to the specific viewpoint of the legal actor and the jurisprudence considered at each time.

Upon a set of propositional knowledge we can construct a legal interpretation to investigate a case. The legal interpretation will determine the argumentation. As arguments will be built upon such propositional knowledge, the underlying legal interpretation will be the responsible for the logical "glue" within arguments. In other words, a legal interpretation will be the responsible for the linkage among propositions inside arguments and hypotheses.

With the focus on legal interpretation, we now can get back our attention to propositional knowledge. The reader will note that we extract propositions from legal texts as if it were a handmade job. We do not observe any specific theory to recognize propositions from legal texts. We assume that the propositional knowledge is somehow obtained from legal texts. In this sense, an assumption of our theory is that, in a further implementation, there should be an underlying strata focused on NLP algorithms. Such a strata would simply recognize propositional knowledge from legal texts. Afterwards, the outcome of such NLP strata would serve as the entries for the strata implementing the theory we propose here.

On the other hand, the construction of legal interpretations is assumed to be a semiautomatic process. The strata implementing our theory would give the user the possibility to incorporate logical details for refining the interaction of propositions. This will make explicit the personal subjectivities that are present in every legal decision. Moreover, having specified the legal interpretation of each legal case allows to compare the resulting argumentation for analogous cases and to propose different manners of measuring interpretation distance for controlling arbitrariness in legal decisions. Arbitrariness control through legal interpretation measures is part of our ongoing work.

3. A Legal Argumentation Framework

We will develop our proposal of a *legal argumentation framework* by relying upon the definition of *legal interpretations* (Section 3.1) which will be the cornerstone for the construction of *legal arguments* and the recognition of conflicts between pairs of arguments (Section 3.2). Afterwards, we define a *legal knowledge base* (Section 3.3) which will deliver the sources –both from the law and from an actual legal case– for the construction of the *legal argumentation instance*. From such an instance, the theory will obtain the different types of legal arguments of a specific legal case.

3.1. Modelling Legal Interpretations

The interpretation process in the legal domain has been widely investigated in legal philosophy, legal theory (Tarello, 1980; Guastini, 2003; MacCormick and Summers, 1991; Brozek, 2013) and in the AI&Law literature (Prakken and Sartor, 2013; Macagno et al., 2012; Rotolo et al., 2015; Malerba et al., 2016; Araszkiewicz and Zurek, 2015). For the objectives of our proposal, a formal definition of a structure for keeping account of legal interpretations is given. This is crucial to consider previous legal cases for performing significant recommendations in the construction of the rules of evidence, and towards an appropriate conclusion for a case under study. This is of utmost relevance for case-based reasoning, both, in the common law, where legal decisions are achieved upon precedents, but also in the continental law, where case-based reasoning is the way the legal system has for justifying, or enforcing certainty of, the legal decisions achieved through application of positive (written) norms.

Legal interpretations are the key of the judge's decision. Any judicial decision relies upon the criterion of a judge for defining a number of situations through a legal interpretation from which the case is resolved. Norms usually appear as a result of interpreting the natural language in which a legal text is written. But also, as legal texts pretend to model families of different situations in a general way, it is usually necessary to interpret whether a given fact is really subsumed by the premise of a norm (or a legal text). Similarly, a subsumption-like operation may determine the way different norms can be chained one each other to reach a decision. Moreover, there exist several different and more complex operations defined as legal interpretations which may finally trigger new norms out from the particular understanding of a legal reasoning previously performed in some specific precedent. For a better understanding of the sort of situations that may appear from legal interpretations, let us develop a brief analysis of a recent case in the Argentinean law.

Legal Case 1 (Schiffrin (Corte Suprema de Justicia de la Nación (CSJN), 2017)). *On March of 2017, the Argentinean Supreme Court decided on a long-term dispute which started in 1994, when the National Constitution (NC) was reformed. A constitutional reform according to the Argentinean legal system can only take place after an* ad-hoc *Reform Commission (RC) is conformed by the legislature along with a corresponding necessity-reform law (NL) which serves as guide for the reform, where the congressmen detail the different parts of the constitution to be revised. Afterwards, the RC reforms the constitution according to the faculties that were granted to her by the NL. Of course, any excess on such faculties may be at risk of unconstitutional declaration. The reform in 1994 was the last modernization of the Argentinean constitution which consisted in several points of revision among which it authorized the revision of specific articles dedicated to the appointment of federal judges.*

In particular, the third paragraph of the 99th article, inc. 4th (from now on the paragraph *or* parr.*) posed an age limit of 75 years old for federal judges to keep their tenures. A 75 year old judge may ask for a special reappointment in his office to the Executive Power –i.e., the presidency.*

Carlos Fayt held at that moment of the reform a Supreme Court's judge tenure, appointed for his office in 1983. He was 76 at that time. Shortly afterwards, he demanded the nullity of the paragraph stating that the age limit promulgated by the RC was a sort of removal from office and thus a clear interference to article 96 NC (currently 110 NC) which was not included among the authorized articles listed by the NL.

In 1999, the Supreme Court decided in his favor, interpreting that the faculties of a CR are literally restricted to the specific articles listed in the corresponding NL. This was the unique constitutional norm that was declared unconstitutional in the Argentinean history.

On March of 2017, the Supreme Court delivered his decision on case Schiffrin returning the validity of the paragraph. The argument delivered for that purpose stated that the 75 year old age limit cannot be interpreted as a sort of removal from office since it would be violating the independence nature of the Judicial Power which still makes possible the removal from office only through an extraordinary destitution trial. In turn, the age limit should be interpreted as a modification to the lifetime condition of the tenure requiring the intervention of the Executive Power for the reappointment in case of a federal judge exceeding the age limit. Afterwards, and since the NL authorized the reform of faculties of the Executive Power regarding the process of appointment of federal judges, it was clear that the RC decided in full observance of the NL. However, admitting that this could not be the unique interpretation, stated that in case of doubt, it should be decided in favor of the RC (p. 5) through an extensive interpretation by considering the general subject of reform, in the contrary position to the restrictive interpretation referred in Fayt (p. 14).

Let us review both cases through informal schemes of their reasoning steps:

> ***Fayt 1999:*** *age limit* [is interpreted as] *removal from office* [which implies] *modifying art. 96 (irremovable tenure)* [which is interpreted as] *a reform not authorized by the NL (given that it does not include art. 96)* [which implies] *RC exceeded its faculties* [which implies] *that the paragraph is unconstitutional.*

There are two key points in this line of reasoning, on one hand, the first interpretation in which the age limit is understood as a sort of removal from office, and on the other hand, the second interpretation in which it is assumed that the RC faculties are restricted to the specific articles listed in the NL –independently of the general subject that motivated the constitutional reform. Both interpretations were abandoned in Schiffrin's since the age limit is not interpreted as a sort of removal from office and the faculties of the RC can be extended beyond the suggested list of articles as long as the reform observes the general subject of constitutional reform.

> ***Schiffrin 2017:*** *age limit* [is interpreted as] *alteration of the lifetime condition of the tenure* [which implies] *requiring a special reappointment to the Executive Power* [which is interpreted as] *contained in the general subject of reform in the NL meaning that the RC acted in accordance to it* [which implies] *the paragraph is constitutional.*

Of course, according to both new interpretations, the precedent stated in Fayt needed also to be abandoned in order to protect the sovereign will of the people. In turn, in Schiffrin the Court gave its new interpretation to Fayt's which ends up coinciding with the 1994 Camera's claim: Fayt could not be removed from office not because of an unconstitutional norm, but because he held a position appointed beforehand. This is necessary to preserve the general principle of legal certainty *which prohibits* ex post facto laws*–i.e., laws that affect acts committed before their promulgation. This means that Fayt could not had been removed from office anyway, but according to a very different argumentation.*

It is clear that legal interpretations affect the construction of the argumentation for a decision, and that following a precedent implies necessarily the observation of its inner interpretations. This is a keystone in our theory which we will recall afterwards. Meanwhile, we will analyze the intuitions behind legal interpretations for constructing a formal definition for them.

We rely upon propositions which are bearers by themselves of truth and falsity. Roughly speaking, a proposition can be seen as a sentence which affects a subject through a predicate. For instance, in a proposition like "*the victim's body was found with gunshot wounds*", we have a subject *victim's body* which is affected by the predicate *was found with gunshot wounds*. For identifying propositions in an abstract manner, we will rely upon (possibly sub-indexed) predicative letters like $\alpha, \beta, \gamma, \delta$ and φ. Without further formalization, we will refer to an unspecified language $\mathcal{L}$ which will be assumed to define the accepted formulæ[3]. Thus, we will write $\alpha \in \mathcal{L}$ to ensure that the proposition represented by the predicative letter α corresponds to a well-formed formula accepted by the language $\mathcal{L}$. Deduction in $\mathcal{L}$ will be represented through the operator $\vdash$ which will rely upon *modus ponens* as its inference rule. Thus, given $\Gamma = \{\alpha, \alpha \to \beta\}$, we have that $\Gamma \vdash \beta$ holds. Next we define legal interpretations $\mathcal{I}$ through a special structure containing the language $\mathcal{L}$, a set $\Pi^{\mathcal{I}}$ registering predicative letters standing for $\mathcal{L}$-propositions considered by the interpretation, and a special operator which is capable of interrelating propositions in a way that the truth of a proposition enforces, by means of a *subsumption* operator $\sqsubseteq$, the truth of a second one. That is, $\alpha \sqsubseteq \beta$ states that β's truth is supported (or enforced) through α's. It is important to note however that this sort of truth-enforcing operator is subjective in the sense that it depends on the criterion of the judge who adheres to such an interpretation. Therefore, a different judge may disagree and thus, a subsumption may be seen as a sort of defeasible implication. We will retake this discussion afterwards.

The idea behind truth-enforcing subsumption relations is to allow a way of representing subjective inference for reasoning as it is usually done in legal argumentation. That is, having a legal interpretation $\mathcal{I}$ where $\alpha \sqsubseteq \beta$ holds would allow the inference of γ from a set like $\{\alpha, \beta \to \gamma\}$. For instance, recalling part of the argumentation in Fayt 1999 (see Legal Case 1), we may have the predicate letter α standing for the proposition *A paragraph reformed an art. out from NL list*. According to the interpretation of the judges, reforming an article out from the NL list should be understood as an excess of the RC faculties. Hence, the proposition β could stand for *A paragraph exceeds RC faculties*, and the subsumption $\alpha \sqsubseteq \beta$ would allow the representation of the subjective criterion adopted by the judges. Of course, if we know that the RC faculties were violated according to some specific paragraph of the reform, then the law states that the paragraph leads to a constitutional violation. This allows the use of the classical entailment $\beta \to \gamma$, where γ, standing for *A paragraph is unconstitutional*, would be inferred in $\mathcal{I}$ through the use of the subsumption enforcing the truth of β through α's.

In addition, we would allow the use of two special symbols. On one hand, the use of negated symbols in subsumptions will allow to state that the truth of a proposition can imply the preclusion of truth of a second one. Thus, $\alpha \sqsubseteq \neg\beta$ would serve for stating that α precludes β's truth given that α enforces the truth of $\neg\beta$. On the other hand, we will allow the use of the *top symbol* $\top$ for representing "subjective tautologies", that is *assumptions*. Thus, a subsumption like $\top \sqsubseteq \alpha$ will allow to assume the proposition behind the predicative letter α to be truth according to the subjective criterion of the judge. Assumptions will be useful to hypothesize, as we will see later on.

It is important to keep in mind that legal interpretations will be constructed from the interaction with the user. This is the keystone of our proposed theory, which pretends to state the formal conditions for implementing a software recommender for reasoning in a semi-automatic manner. This means that a human-machine interaction will be necessarily expected.

Definition 1 (Legal Interpretation). *A tuple $\langle \mathcal{L}, \Pi^{\mathcal{I}}, \underline{\cdot}^{\mathcal{I}} \rangle$ is a **legal interpretation** (or simply, interpretation) $\mathcal{I}$ iff $\mathcal{L}$ is the logic for representing formulæ, $\Pi^{\mathcal{I}} \subseteq \mathcal{L}$ is the finite **predicate domain** containing (possibly negated) predicate letters standing for atomic formulæ in $\mathcal{L}$, and a **subsumption (predicate) function** $\underline{\cdot}^{\mathcal{I}} : \Pi^{\mathcal{I}} \longrightarrow \wp(\Pi^{\mathcal{I}}) \cup \{\top\}$ interrelating predicates such that for any pair of predicate letters $\alpha, \beta \in \Pi^{\mathcal{I}}$, $\beta \in \underline{\alpha}^{\mathcal{I}}$ stands for β is subsumed by (or is more concrete than a more general proposition) α.*

[3]Note that propositions can be represented in different ways, from reductions of natural language, to propositional or classical logic, or to more expressive logics like first order's fragments, Deontic Logics or even Description Logics used for the construction of ontologies. For simplicity, we will only refer to predicative letters as explained before and will occasionally –and only if necessary– make explicit reference to a concrete logic.

Subsumtions may capture complex instantiations. In a well known example, Hart[4] proposed the analysis of application of a general rule like *it is forbidden to take a vehicle into the public park* to different instances of it like taking a bike, a skate or an ambulance into the public park. We will consider a pair of propositions, α for *it is forbidden to take an ambulance into the public park* and β for *it is forbidden to take a vehicle into the public park.* Note that β is more general than α since ambulances are a particular type of vehicle. In order to determine the interrelation of both predicates, it will be necessary the user interaction. In case we state the subsumption $\alpha \in \underline{\beta}^{\mathcal{I}}$ or equivalently $\alpha \sqsubseteq \beta$ in the context of a legal interpretation $\mathcal{I}$, we will be clearly defending the position in which the entrance of an ambulance is not allowed given that it is a special kind of forbidden vehicle. Of course, the interpretation of the judge can state that the intention behind the norm under β, *i.e.*, its *ratio legis*, was to protect pedestrians and therefore an emergency would allow the entrance taking a regular ambulance. Similarly, it would be allowed the entrance with a bike or a skate since that would carry no major risk for pedestrians. In such a case, a different legal interpretation $\mathcal{I}'$ will state $\alpha \notin \underline{\beta}^{\mathcal{I}'}$. In this manner, it is possible to keep track of the criterion considered for resolving a legal case. Now suppose α' states that *A civilian cannot take an ambulance into the public park.* It is clear that $\alpha' \sqsubseteq \alpha$ but also that $\alpha' \sqsubseteq \beta$ which can serve for modeling a situation in which *John* owns an old ambulance as his bizarre weekend vehicle. This shows the potential of relying upon legal interpretation structures according to Definition 1 for representing the judge's criterion. Moreover, this will be very useful when comparing the interpretation that has been considered by a judge for a jurisprudence's case and the interpretation being currently considered for a new case sharing some common characteristics with the jurisprudence's one. This is the milestone of our proposal for studying recommender prototypes of case-based reasoning. We will refer to this usage afterwards.

For the right understanding of the meaning behind subsumptions within the context of a legal interpretation $\mathcal{I}$, we will formalize the notion of *interpretation semantics*, modeled through the satisfaction relation $\models^{\mathcal{I}}$ as follows.

Definition 2 (Interpretation Semantics). *Let $\mathcal{I} = \langle \mathcal{L}, \Pi^{\mathcal{I}}, \underline{\cdot}^{\mathcal{I}} \rangle$ be a legal interpretation and $\alpha, \beta, \beta_1, \ldots, \beta_n, \gamma \in \Pi^{\mathcal{I}}$ be predicate letters, the **interpretation semantics** is given by a satisfaction relation $\models^{\mathcal{I}}$ such that:*

- $\beta \models^{\mathcal{I}} \alpha$ *iff* $\mathcal{I} \models [\Delta]\beta \sqsubseteq \alpha$, *with* $\Delta \subseteq \Pi^{\mathcal{I}}$, *where* $|\Delta| = n \geq 0$ *iff either*
  - $n = 0$ *and* $[\Delta] = []$. *Hence,* $\beta \in \underline{\alpha}^{\mathcal{I}}$, *or equivalently* $\mathcal{I} \models \beta \sqsubseteq \alpha$.
  - $n > 0$ *and* $[\Delta] = [\beta_n, \ldots, \beta_1]$. *Hence,* $\beta \in \underline{\beta_1}^{\mathcal{I}}$ *and* $\beta_1 \in \underline{\beta_2}^{\mathcal{I}}$ *and* $\ldots$ *and* $\beta_n \in \underline{\alpha}^{\mathcal{I}}$, *or equivalently,* $\mathcal{I} \models \beta \sqsubseteq \beta_1, \mathcal{I} \models \beta_1 \sqsubseteq \beta_2, \ldots, \mathcal{I} \models \beta_n \sqsubseteq \alpha$.

  Intuitively, β is subsumed by (or is more concrete than a more general proposition) α. We say also that α's certainty is enforced or supported by β's certainty, through a subsumptive sequence *(or just* s-sequence*)* $[\Delta]$.

- $\models^{\mathcal{I}} \alpha$ *iff* $\beta \models^{\mathcal{I}} \alpha$, *with* $\beta = \top$, *or equivalently* $\mathcal{I} \models \alpha$.

  Intuitively, α's truth is assumed or is inferred from an assumption (and reached through an s-sequence).

- $\beta \models^{\mathcal{I}} \neg\gamma$ *iff* $\beta \models^{\mathcal{I}} \alpha$, *with* $\alpha = \neg\gamma$.

  Intuitively, β precludes γ's certainty.

- $\neg\alpha \models^{\mathcal{I}} \neg\beta$ *iff* $\beta \models^{\mathcal{I}} \alpha$ *holds.*

  Intuitively, if α's truth is denied that implies that all β that enforces α's truth is necessarily denied as well.

- $\beta \not\models^{\mathcal{I}} \alpha$ *iff neither* $\beta \models^{\mathcal{I}} \alpha$ *nor* $\beta \models^{\mathcal{I}} \neg\alpha$, *hold.*

  Intuitively, β is not related to α.

[4]This example is taken from a well known hypothetical proposed by (Hart, 1958).

- $\beta \models^{\mathcal{I}} \bot$ *iff both* $\beta \models^{\mathcal{I}} \alpha$ *and* $\beta \models^{\mathcal{I}} \neg\alpha$*, hold.*

  Intuitively, β *is contradictory and thus, the interpretation* $\mathcal{I}$ *is incoherent.*

- $\models^{\mathcal{I}} \bot$ *iff there is some* α *such that* $\models^{\mathcal{I}} \alpha$ *and* $\models^{\mathcal{I}} \neg\alpha$*, or equivalently* $\mathcal{I} \models \bot$*.*

  Intuitively, the interpretation $\mathcal{I}$ *has contradictory assumptions and thus,* $\mathcal{I}$ *is inconsistent. Whenever no such* α *exists, we will write* $\not\models^{\mathcal{I}} \bot$*.*

- $\Gamma \models^{\mathcal{I}} \Upsilon$ *iff for every* $\alpha \in \Upsilon$ *there is some* $\beta \in \Gamma$ *such that* $\beta \models^{\mathcal{I}} \alpha$*.*

A short motivation behind the construction of transitive subsumptions through the use of s-sequences $[\Delta]$. As aforementioned, two propositions interrelated through a subsumption operator may be seen as a sort of defeasible implication, given that it represents a partial or subjective viewpoint in accordance to a personal criterion followed by the judge who adhered to such interpretation. This means that, having $\alpha \sqsubseteq \beta$ is a way to ensure that knowing α's truth is a pathway to enforce the certainty about β's truth. However, such an asseveration may have detractors. Let us assume a second subsumption like $\beta \sqsubseteq \gamma$. In general, we need to accept that there could be opposed viewpoints for this particular subsumption as well. Therefore, it seems to be inappropriate to propose transitivity for the subsumption operator since it would automatically trigger the subsumption $\alpha \sqsubseteq \gamma$ which would be hiding an intermediate point of discussion from where the truth-enforcing relation from α to γ appeared. In turn, we will allow a transitive construction through the notation $[\beta]\alpha \sqsubseteq \gamma$. Nevertheless, the subsumption $\alpha \sqsubseteq \gamma$ could be proposed as part of the interpretation, however, as said before, it cannot be expected to be inferred transitively and thus it should necessarily be made explicit by adding α to $\underline{\gamma}^{\mathcal{I}}$. This also means that the subsumption operator could not be proposed anti-transitive but intransitive, since it could occasionally verify transitivity.

For the legal reasoning, such flexible construction can be very useful. For instance, the following example shows its utility for interpreting the regulation of contracts for adjudicating service contracts.

Example 1. *The Argentinean Civil and Commercial Code (CCCN) that was in force until 2014 had no positive articles regulating service contracts. Such a situation was captured by the new CCCN whose promulgation in 2014 overrode the elder one. Nevertheless, during the old CCCN service contracts where recognized, in particular for interpreting consumer protection regulations. Concretely, art. 1138 (CCCN - Law No. 340)*[5] *stated that* contracts are referred as bilateral whenever both parts accept a mutual obligation one with regards to the other. *That can be modeled as follows:*

β_1 : *There is a mutual obligation between two parties, A and B.*

β_2 : *There is a bilateral contract between A and B.*

According to art. 1138 (CCCN), an entailment like

$\mathcal{A}_1 : \beta_1 \rightarrow \beta_2$*, would appear. However, more specific variations of bilateral contracts were regularly recognized through the judge's criterion:*

γ_1 : *There is a mutual obligation between an adherent and a proponent.*

α_1 : *There is a mutual obligation between a consumer and a service provider.*

α_2' : *There is a bilateral contract between a consumer and a service provider.*

α_2 : *There is a public service contract between a consumer and a service provider.*

Considering subsumptions like:

$\alpha_1 \in \underline{\gamma_1}^{\mathcal{I}}$, $\gamma_1 \in \underline{\beta_1}^{\mathcal{I}}$ *and* $\alpha_2' \in \underline{\beta_2}^{\mathcal{I}}$*, and a common sense entailment like* $\mathcal{A}_2 : \alpha_2' \rightarrow \alpha_2$*, we could sketch an informal proof like:*

> GIVEN $\mathcal{I} \models [\gamma_1]\alpha_1 \sqsubseteq \beta_1$, WE KNOW α_1 SUPPORTS $\mathcal{A}_1$'S PREMISE β_1 THROUGH γ_1. AS A RESULT, THE GENERAL INDIVIDUALS A AND B, APPEARING IN BOTH β_1 AND β_2, END UP CONCRETIZED AS *consumer* AND *service provider*, RESPECTIVELY. AFTERWARDS, A CONCRETIZED VERSION

[5] From the original text in Law No. 340: *Art. 1138. Los contratos se denominan en este código unilaterales, o bilaterales. Los primeros son aquellos en que una sola de las partes se obliga hacia la otra sin que ésta le quede obligada. Los segundos, cuando las partes se obligan recíprocamente la una hacia la otra.*

of β_2 is entailed, which ends up being α'_2. This is so, given that α'_2 is subsumed in β_2 since $\mathcal{I} \models \alpha'_2 \sqsubseteq \beta_2$ holds. Also, since α'_2 is $\mathcal{A}_2$'s premise, the right-hand side α_2 is entailed. Hence, there is a proof for α_2 if it happens that α_1 is certain.

For identifying conflicting propositions, *i.e.*, propositions which could not be simultaneously verified, the interpretation relies upon deduction in $\mathcal{L}$ in conjunction with the deduction derived from the subsumption operator. This renders the definition of *(in)consistency.*

Definition 3 (Consistency). *Given an interpretation $\mathcal{I}$ and any pair of predicates $\alpha, \beta \in \Pi^{\mathcal{I}}$, the set $\{\alpha, \beta\}$ is said **consistent**, noted $\{\alpha, \beta\} \not\models^{\mathcal{I}} \bot$ iff there are no pair of propositions $\gamma_1, \gamma_2 \in \Pi^{\mathcal{I}}$, such that $\alpha \models^{\mathcal{I}} \gamma_1$ and $\beta \models^{\mathcal{I}} \gamma_2$ allows the satisfaction of $\{\gamma_1, \gamma_2\} \vdash \bot$. In such a case, α and β are said **pairwise consistent**.*

For instance, according to the aforementioned Hart's example, where α stands for *it is forbidden to take an ambulance into the public park* and β for *it is forbidden to take a vehicle into the public park*, if we assume γ as *the safety of pedestrians is guaranteed* standing for the *ratio legis* of β, we will have $\beta \in \underline{\gamma}^{\mathcal{I}}$. But then, since an ambulance may be required in the park for emergencies, we may assume that prohibiting its entrance will preclude the truth of γ, which implies $\alpha \in \underline{\neg\gamma}^{\mathcal{I}}$. Afterwards, $\{\alpha, \beta\} \models^{\mathcal{I}} \{\gamma, \neg\gamma\}$, and since $\{\gamma, \neg\gamma\} \vdash \bot$ we have that $\{\alpha, \beta\} \models^{\mathcal{I}} \bot$, which means that according to the legal interpretation $\mathcal{I}$, α and β represent contradictory propositions, or equivalently, $\{\alpha, \beta\}$ is an inconsistent set. This shows the risk of allowing to model $\alpha \in \underline{\beta}^{\mathcal{I}}$ in such circumstances, thus rendering the need for controlling the construction of rational interpretations avoiding inconsistencies and incoherences. Hence, the reasoning process of a legal system will rely upon an interpretation which necessarily verifies *admissibility* conditions for ensuring rational consequences. Admissibility will require a reflexive, anti-symmetric and intransitive subsumption operator, which cannot trigger inconsistency nor incoherency, and which will avoid circularity as a way to ensure the progression of a reasoning process. The intention is to model a sort of flexible operation capable of breaking free from the restrictions of a usual conditional in classical logic.

Definition 4 (Admissible Interpretation). *An interpretation $\mathcal{I}$ is **admissible** iff*

(Consistency) $\not\models^{\mathcal{I}} \bot$,

(Coherency) *there is no $\alpha \in \Pi^{\mathcal{I}}$, $\alpha \models^{\mathcal{I}} \bot$,*

(Reflexivity) *for any $\alpha \in \Pi^{\mathcal{I}}$, $\alpha \in \underline{\alpha}^{\mathcal{I}}$,*

(Anti-symmetry) *for any $\alpha, \beta \in \Pi^{\mathcal{I}}$, if $\alpha \in \underline{\beta}^{\mathcal{I}}$ and $\beta \in \underline{\alpha}^{\mathcal{I}}$ then $\alpha = \beta$, and*

(Intransitivity) *for any $\alpha, \beta, \gamma \in \Pi^{\mathcal{I}}$, if $\alpha \in \underline{\beta}^{\mathcal{I}}$ and $\beta \in \underline{\gamma}^{\mathcal{I}}$ then $\alpha \in \underline{\gamma}^{\mathcal{I}}$ does not necessarily hold.*

(Non-circularity) *for any $\alpha, \beta_1, \dots, \beta_n \in \Pi^{\mathcal{I}}$, if $\alpha \in \underline{\beta_1}^{\mathcal{I}}$ and $\beta_i \in \underline{\beta_{i+1}}^{\mathcal{I}}$ (with $1 \leq i < n$) then $\beta_n \notin \underline{\alpha}^{\mathcal{I}}$ necessarily hold. Circularity may be seen as a sort of* indirect reflexivity *like $\mathcal{I} \models [\beta_1, \dots, \beta_n]\alpha \sqsubseteq \alpha$, which will be inadmissible.*

The following example retakes the Legal Case 1 presented before for proposing a way to model it through an interpretation structure.

Example 2. *According to the Legal Case 1, let us determine a possible interpretation representing the line of reasoning for Fayt 1999. We will assume a predicate domain $\Pi^{\mathcal{I}_f}$ containing predicates like:*

α_1 : *The paragraph introduced the age limit reform.*

α_2 : *A paragraph reformed Art. 96.*

α_3 : *Art. 96 is not included in the NL list.*

Regarding subsumption, it is clear that α_1 is a particular case of α_2 given that, as being interpreted in the judicial sentence, the age limit reform is understood as a sort of removal from office which is claimed by art. 96 NC. Thus:

$\underline{\alpha_2}^{\mathcal{I}_f} = \{\alpha_1, \alpha_2\}$

Note that propositions α_1 and α_3 are facts of the legal case. That is, propositions whose truth is indisputable –unlike the interpretation about $\alpha_1 \sqsubseteq \alpha_2$ which can be questioned through a different interpretation. Afterwards, given that art. 96 is not included in the NL list (α_3), if it happens that there is some paragraph reforming art. 96 (α_2) then it is trivial to entail a general proposition like:

α_4 : *A paragraph reformed an art. out from NL list.*

Intuitively, we can assume the existence of an entailment like $\mathcal{A}_1 : \alpha_2 \wedge \alpha_3 \rightarrow \alpha_4$. Note that the judges' decision states that according to their interpretation, reforming an article out from the NL list should be understood as an excess of the RC faculties. This leads to a new and more general proposition:

α_5 : *A paragraph exceeds RC faculties,*

where $\underline{\alpha_5}^{\mathcal{I}_f} = \{\alpha_4, \alpha_5\}$. Of course, if we know that the RC faculties were violated according to some specific paragraph of the reform, then it is natural to think that the paragraph leads to a constitutional violation. That is, an entailment like $\mathcal{A}_2 : \alpha_5 \rightarrow \alpha_6$, where:

α_6 : *A paragraph is unconstitutional.*

Observe that we can construct the court's argument as a sort of proof:

> Given $\mathcal{I}_f \models \alpha_1 \sqsubseteq \alpha_2$ and since α_1 is a fact, we know that α_2 is true. Besides, since α_3 is a fact, we satisfy the premises of rule $\mathcal{A}_1$. This allows to verify the truth of α_4 and given that $\alpha_4 \in \underline{\alpha_5}^{\mathcal{I}_f}$, we also have that α_5 is true. Finally, by relying upon rule $\mathcal{A}_2$, we verify α_6 truth. This allows us to satisfy a proposition like *The paragraph is unconstitutional*, which appears through the concretization of the general individual in α_6 with the concrete one in α_1.

A short comment on the modeling of the legal case in the example above. Note that it can also be possible to state a subsumption of α_1 in α_4. However, although this will correctly imply that the age limit affects some implicit article which is not included in the NL list, it will not appropriately represent the fact that the affected article is –with an unquestionable character– out of the NL list. The problem here is subjective. Subsumptions are a way to state a legal interpretation as a manner to present a point of view, and therefore, disputable. The representation of facts should be left out from the interpretation structure. We will get back to this matter in Section 3.3. In what follows, we will concentrate in a formalization for the logical construction of an argument structure like the one given in Example 2. Firstly, we will concentrate on the formal construction of rules like $\mathcal{A}_1$ and $\mathcal{A}_2$.

3.2. Construction of the Legal Argument

There are two main sorts of conditional rules for the construction of legal arguments. Such a distinction is relevant given that, in terms of Toulmin (Toulmin, 1958), their kind of *backing*[6] may differ and that would ultimately impact over the conclusive force of the argument. On one hand, we refer to *legal rules* as those standing for norms derived from legal codes, jurisprudence, precedents and doctrine. They are inferred from the positive (written) law. On the other hand, we refer to *generalization rules* as those which are more related to a particular vision or criterion adopted in order to push forward the reasoning process towards a conclusion. The introduction of generalization rules for legal arguments on evidence is responsibility of the judge, who –in the continental law– will rely upon his *sound criticism*[7] for their construction.

> *"In arguments about evidence, several different kinds of general propositions play an important role both as discrete steps in an argument and as background knowledge. At this stage it is useful to make elementary distinctions between scientific truths (such as the law of gravity, that eyewitness identification evidence is often unreliable), common sense generalizations (such as that running away is indicative of a sense of guilt), commonly held beliefs (such as national or*

[6]A backing is the referred source upon which the usage of the rule is justified.

[7]Sound criticism, or *sana crítica*, is a technical resource that the judge is obliged to use for justifying a decision, only when the legislation does not impose a specific criterion to do so. Sound criticism is part of a probatory system.

ethnic stereotypes, including prejudices, that suggest that a person of such origins has certain characteristics), and general background information bearing on the present case (case specific generalizations, such as a generalization about X's habits or Y's character)" (Anderson et al., 2005)

It is clear that the backing for a legal rule will have more strength than the one referred for a generalization rule. This means that an argument from evidence which is solely constructed upon legal rules should have more certainty than another argument containing generalization rules. In this article, we will not differentiate the construction of arguments in that sense. However, the reader should keep in mind that, eventually, concepts like strength, force or certainty, should be considered for the implementation of an expert system.

For representing legal and generalization rules, we rely upon *atoms*, a structure which abstracts away from any concrete representation of rules. We will refer here to the simplest form of atoms in which a *set of premises* requires support for achieving a given *conclusion* or *claim*. However, the structure of atoms may easily evolve from a pair to a triple, or even higher order structures, in order to allow, if necessary, the incorporation of further elements like conclusive force, strength, credibility or plausibility, among others.

Definition 5 (Atom). *A pair $\mathcal{A} = \langle \Gamma, \alpha \rangle$ is an **atom** iff $\Gamma \subseteq \mathcal{L}$ is a finite (non-empty) set of premises and $\alpha \in \mathcal{L}$ its claim. The set $\mathfrak{source}(\mathcal{A}) = \Gamma \cup \{\alpha\}$ represents the **predicate source** of the atom's construction under $\mathcal{L}$.*

The interaction of an interpretation and an atom is defined through the definition of *model of an atom*.

Definition 6 (Model of an Atom). *Given an admissible interpretation $\mathcal{I}$ and an atom $\mathcal{A} = \langle \Gamma, \alpha \rangle$, we say $\mathcal{I}$ is a **model** of $\mathcal{A}$, writing $\mathcal{I} \models \mathcal{A}$ iff $\mathfrak{source}(\mathcal{A}) \subseteq \Pi^{\mathcal{I}}$ and it holds $\mathfrak{source}(\mathcal{A}) \not\models^{\mathcal{I}} \bot$ (consistency).*

For instance, if a fact tells that *the victim's body was found with gunshot wounds* (α) it is easy to see it as an instance of a more general proposition like *(it was found) a body with gunshot wounds* (β). This is represented through a legal interpretation $\mathcal{I}$ specifying $\alpha \in \underline{\beta}^{\mathcal{I}}$, or equivalently, $\alpha \sqsubseteq \beta$. The consideration of atoms triggers a new and more complex process for resolving the instantiation of propositions. For instance, if there is an atom $\mathcal{A} = \langle \{\beta\}, \beta' \rangle$, the premise β may be instantiated through the proposition α. This will necessarily affect the resulting instance of the atom $\mathcal{A}$ by determining a unique proposition α' subsumed by the atom's conclusion β', *i.e.*, $\alpha' \sqsubseteq \beta'$. However, since several propositions may appear in $\underline{\beta'}^{\mathcal{I}}$ we need to specify a special notation like $\alpha' = \beta'[\mathcal{A}, \Upsilon]$, which ends up verifying $\beta'[\mathcal{A}, \Upsilon] \sqsubseteq \beta'$, for identifying such circumstance in which α' appears as a functional consequence of instantiating the set of premises from the atom $\mathcal{A}$ with a set of propositions $\Upsilon \subseteq \Pi^{\mathcal{I}}$. To that end, we propose the definition of *instance of an atom* which will rely upon the notion of *set of support*, a subsumption function overloaded for applying over sets of predicates.

Definition 7 (Set of Support). *Given an admissible interpretation $\mathcal{I}$ and two sets of propositions $\Upsilon, \Gamma \subseteq \Pi^{\mathcal{I}}$, we say Υ is a **support set for** Γ, writing $\mathcal{I} \models \Upsilon \sqsubseteq \Gamma$ iff $|\Upsilon| = |\Gamma|$ and for every $\alpha \in \Upsilon$ there is a single $\rho \in \Gamma$ such that $\alpha \models^{\mathcal{I}} \rho$.*

Before formalizing the instance of an atom it is fair to recall the well known process of *substitution* used in mathematical-logic. A substitution $v = \bar{x}/\bar{y} = \{x_1/y_1, \ldots, x_n/y_n\}$ is referred for mapping variables (or *subjects*, for remarking the perspective from natural language) x_i in $\bar{x} = \langle x_1, \ldots, x_n \rangle$ to other variables (or constants) y_i in $\bar{y} = \langle y_1, \ldots, y_n \rangle$. Thus, $\alpha(\bar{x})[v] = \alpha(\bar{y})$ holds whenever vector $\bar{x}$ is substituted through $v = \bar{x}/\bar{y}$ by vector $\bar{y}$. For instance, $(p_1(x_1) \wedge p_2(x_2) \to r(x_1, y_2))[v] = p_1(y_1) \wedge p_2(y_2) \to r(y_1, y_2)$ given the substitution $v = \{x_1/y_1, x_2/y_2\}$. Substitutions may also be applied over sets Γ of formulæ. For instance, $\Gamma(\bar{x})[v] = \Gamma(\bar{y})$ holds by substituting each the vector $\bar{x}$ in each formula in Γ by vector $\bar{y}$ through a substitution $v = \bar{x}/\bar{y}$. Moreover, whenever an explicit reference to variables and/or constants is not necessary, we may write $\Gamma[v]$ for identifying a set whose formulæ are those in Γ substituted through v. Next, we define the *instance of an atom* by overloading the usual notation of substitutions to identify the proposition α' resulting from the substitution of a proposition α by effect of a set of support Υ for the set of premises of an atom $\mathcal{A}$ claiming α, *i.e.*, $\alpha' = \alpha[\mathcal{A}, \Upsilon]$.

Definition 8 (Instance of an Atom). *Given a model $\mathcal{I}$ of an atom $\mathcal{A} = \langle \Gamma, \alpha \rangle$, a set of propositions $\Upsilon \subseteq \Pi^{\mathcal{I}}$, and a proposition $\alpha' \in \Pi^{\mathcal{I}}$, we say α' is the* ***instance of*** *$\mathcal{A}$ according to Υ in $\mathcal{I}$, writing $\mathcal{I} \models \alpha' = \alpha[\mathcal{A}, \Upsilon]$ iff it holds:*

1. $\mathcal{I} \models \Upsilon \sqsubseteq \Gamma$,
2. $\mathcal{I} \models \alpha' \sqsubseteq \alpha$,
3. *the* subjects *of α' are contained in the* subjects *of Υ, i.e., if $\Upsilon(\bar{y})$ and $\alpha'(\bar{x})$ then $\bar{x} = \langle x_1, \dots, x_n \rangle$ and $\bar{y} = \langle y_1, \dots, y_m, x_1, \dots, x_n \rangle$, and*
4. *the substitution of* subjects *in α according to those in Υ is equivalent to α', i.e., $\alpha(\bar{z})[v] = \alpha'(\bar{x})$ holds, where $v = \{z_1/x_1, \dots, z_n/x_n\}$.*

Observe that the instance of an atom can be constructed upon a set of support relying on s-sequences. That is, $\alpha' = \alpha[\langle \{\rho_1, \dots, \rho_n\}, \alpha \rangle, \{\beta_1, \dots, \beta_n\}]$, where the n atom's premises are supported one to one by each of the propositions of the set of support, such that $\beta_i \models^{\mathcal{I}} \rho_i$ (with $1 \leq i \leq n$). However, s-sequences $[\Delta_i]$ could be needed in order to make concrete the support, *i.e.*, $\mathcal{I} \models [\Delta_i]\beta_i \sqsubseteq \rho_i$, where β_i ends up being the first link in the reasoning chain determined by the s-sequence $[\Delta_i]$ which ends up enforcing the truth of the premise ρ_i. Of course, in those cases where an empty s-sequence is enough, the support turns out being straightforward from a single subsumption like $\mathcal{I} \models \beta_i \sqsubseteq \rho_i$.

For instance, in Example 1, we have that $\alpha_2 = \beta_2[\langle \{\beta_1\}, \beta_2 \rangle, \{\alpha_1\}]$ given that $\mathcal{I} \models [\gamma_1]\alpha_1 \sqsubseteq \beta_1$ and that the concretization of general individuals triggers the proposition α_2 given that $\mathcal{I} \models \alpha_2 \sqsubseteq \beta_2$. This is the first approach to the construction of arguments. A generalization of such a construction will rely upon sets of interrelated atoms. Let us assume a sequence of n atoms $\mathcal{A}_1 = \langle \Gamma_1, \alpha_1 \rangle$, $\mathcal{A}_2 = \langle \Gamma_2, \alpha_2 \rangle, \dots, \mathcal{A}_n = \langle \Gamma_n, \alpha_n \rangle$, where atoms $\mathcal{A}_2, \dots, \mathcal{A}_n$ could be supporters of atom $\mathcal{A}_1$. For that to happen, we need to verify that for each $\mathcal{A}_i$ (with $2 \leq i \leq n$) there is an instance $\alpha'_i = \alpha_i[\mathcal{A}_i, \Upsilon_i]$ according to some set of support $\Upsilon_i \subseteq \Pi^{\mathcal{I}}$, with $\mathcal{I} \models \Upsilon_i \sqsubseteq \Gamma_i$, such that $\alpha_i \models^{\mathcal{I}} \rho_i$, with $\Gamma_1 = \{\rho_2, \dots, \rho_n\}$, and thus $\mathcal{I} \models [\Delta_i]\alpha'_i \sqsubseteq \rho_i$, where Δ_i an s-sequence making possible the support of the premise ρ_i. This means that each conclusion α_i of atom $\mathcal{A}_i$ ends up concretized as a result of the concretization of its premises in Γ_i by the facts in Υ_i, leading to a proposition α'_i which, along with an s-sequence Δ_i, will enforce the truth of the premise ρ_i, supporting it. Since this happens for each of the $\mathcal{A}_1$'s premises, the atom ends up supported, and whose corresponding set of support ends up constructed by the set of instances of the atoms $\mathcal{A}_2 \dots \mathcal{A}_n$, *i.e.*, $\mathcal{I} \models \{\alpha_2[\mathcal{A}_2, \Upsilon_2], \dots, \alpha_n[\mathcal{A}_n, \Upsilon_n]\} \sqsubseteq \Gamma_1$.

Such a support process will clearly affect the instance of $\mathcal{A}_1$ whose conclusion will end up being some specific $\alpha \in \underline{\alpha_1}^{\mathcal{I}}$. The way to univocally determine such proposition is through the recursive application of the formal notation given before. That is, $\alpha = \alpha_1[\mathcal{A}_1, \{\alpha_2[\mathcal{A}_2, \Upsilon_2], \dots, \alpha_n[\mathcal{A}_n, \Upsilon_n]\}]$. Afterwards, with a slight overload of the notation, by allowing the inclusion of sets of atoms, the expression can be rewritten in an *unfolded* manner, as $\alpha = \alpha_1[\{\mathcal{A}_1, \dots \mathcal{A}_n\}, \Upsilon_2 \cup \dots \cup \Upsilon_n]$. Relying upon such kind of expressions allows to determine a specific concrete individual which appears as a conclusion of an inter-chaining of atoms through a support relation. That is the cornerstone for the formalization of legal arguments.

From a philosophical viewpoint, we will structure the construction of legal arguments by relying upon the Aristotelian notions of major and minor premises as a way to organize in general statements and specific statements, respectively, the contributions in an argument towards its conclusion. For legal arguments the specific statements can be seen as the available "trifles" of the legal case that may derive in pieces of evidence. Such elements make possible the construction of reasoning chains upon their linkage with general statements or atoms. In consequence, we will adhere to the idea of two different sort of premises. Firstly, considering as *minor premise* when there is no need to instantiate the proposition since it is already a statement about a specific fact. To the contrary, *major premises* would be those stating general conceptualizations which could apply to different situations and, indeed, we would need to instantiate them –through the usage of the subsumption operator– to facts of the current case in order to achieve a specific conclusion. Technically speaking, the formal construction of our legal argument relies upon recursion sharing the spirit of well known argumentation systems like ASPIC[8] among others, where arguments are defined upon subarguments (argu-

[8]Throughout the article, it will be clear the need for a new specialized representation of arguments for the legal reasoning.

ments contained in an argument) towards an ultimate and primitive subargument whose only subargument is itself.

Definition 9 (Argument). *A triple $\mathcal{S} = \langle \mathcal{P}_M, \mathcal{P}_m, \alpha \rangle$ is an* ***argument*** *for α in an admissible interpretation $\mathcal{I}$ iff the conclusion or* ***ultimate probandum*** *is represented through a proposition $\alpha \in \Pi^{\mathcal{I}}$, the* ***minor premise*** *through a (non-empty) set $\mathcal{P}_m \subseteq \Pi^{\mathcal{I}}$ of case data expressed through $\mathcal{L}$-ground predicates, and the* ***major premise*** *through a (non-empty) set $\mathcal{P}_M$ of atoms modelled by $\mathcal{I}$, such that:*

1. $(\mathcal{P}_m \cup \{\alpha\} \cup \bigcup_{\mathcal{A} \in \mathcal{P}_M} \mathfrak{source}(\mathcal{A})) \not\models^{\mathcal{I}} \bot$*, and*
2. $\mathtt{top}(\mathcal{S}) = \langle \Gamma, \beta \rangle \in \mathcal{P}_M$ *is the* ***top atom*** *with $\mathcal{I} \models \alpha = \beta[\mathcal{P}_M, \mathcal{P}_m]$, and either*
   - *$\mathcal{S}$ is a* ***primitive argument*** *iff:*
     (a) *($\mathcal{P}_M = \{\langle \Gamma, \beta \rangle\}$ and $\mathcal{I} \models \mathcal{P}_m \sqsubseteq \Gamma$) iff $\mathfrak{subs}(\mathcal{S}) = \{\mathcal{S}\}$, or else*
   - *$\mathcal{S}$ is a* ***composite argument*** *iff:*
     (a) *each top atom's premise $\rho_i \in \Gamma$ ($1 \leq i \leq n = |\Gamma|$), namely a* ***penultimate probandum****, is supported by a proposition $\alpha_i \in \Pi^{\mathcal{I}}$, where $\alpha_i \models^{\mathcal{I}} \rho_i$, and α_i is either:*
        i. *a minor premise $\alpha_i \in \mathcal{P}_m$, or*
        ii. *the claim of a sub-argument $\mathcal{S}_i = \langle \mathcal{P}_{Mi}, \mathcal{P}_{mi}, \alpha_i \rangle$, where*
           A. $\bigcup_{i=1}^{n} \mathcal{P}_{Mi} \cup \{\mathtt{top}(\mathcal{S})\} = \mathcal{P}_M$,
           B. $\bigcup_{i=1}^{n} \mathcal{P}_{mi} \subseteq \mathcal{P}_m$*, and*
           C. $\mathfrak{subs}(\mathcal{S}) = \{\mathcal{S}\} \cup \bigcup_{i=1}^{n} \mathfrak{subs}(\mathcal{S}_i)$.

The set $\mathbb{A}_{\mathcal{I}}$ stands for the ***domain*** *of $\mathcal{I}$-arguments. Thus, $\mathcal{S}$ is an $\mathcal{I}$-argument iff $\mathcal{S} \in \mathbb{A}_{\mathcal{I}}$. In that case, we say that $\mathcal{I}$ is a model of $\mathcal{S}$, writing $\mathcal{I} \models \mathcal{S}$, or just $\mathcal{S}_{\mathcal{I}}$.*

The top of an argument follows the construction presented in (Anderson et al., 2005) where an argument's conclusion, as the *ultimate probandum*, is assumed to be split in (probably several) *penultimate probanda* for constructing reasoning chains standing for the branches in a tree structure. Observe that inner-consistency of arguments is covered by condition 1 while minimality is defined through conditions 2(a)iiA and 2(a)iiB. Sub-argumentation is defined by condition 2(a)iiC.

Example 3. *Recalling the example given before, the atom $\mathcal{A} = \langle \{\beta\}, \beta' \rangle$ stands for a* CSG *claiming that* a body with gunshot wounds *(β) is presumably the effect of* a living person being shot to death *(β'), and assuming a fact like* the victim's body was found with gunshot wounds *(α), we have that $\alpha \sqsubseteq \beta$ holds. Afterwards, we know that there exists a formula $\alpha' \in \Pi^{\mathcal{I}}$ such that $\mathcal{I} \models \alpha' = \beta'[\{\mathcal{A}\}, \{\alpha\}]$ holds, where α' claims that* the victim was shot to death. *That explains the construction of a primitive argument $\mathcal{S} \in \mathbb{A}_{\mathcal{I}}$ where $\mathcal{S} = \langle \{\mathcal{A}\}, \{\alpha\}, \alpha' \rangle$.*

It is easy to see that those arguments whose claims are in opposition counter-argue one each other. This describes a sort of *conflict* which can be defined recursively to determine oppositions with regards to the subargumentation function.

Definition 10 (Conflict). *Given two arguments $\mathcal{S}, \mathcal{S}' \in \mathbb{A}_{\mathcal{I}}$, we say that they constitute a* ***conflicting pair*** *where $\mathcal{S}$* ***counterargues*** *$\mathcal{S}'$ iff $\mathcal{S} = \langle \mathcal{P}_M, \mathcal{P}_m, \alpha \rangle$ and $\mathcal{S}'' = \langle \mathcal{P}_M{}'', \mathcal{P}_m{}'', \beta \rangle$, for some $\mathcal{S}'' \in \mathfrak{subs}(\mathcal{S}')$ such that $\{\alpha, \beta\} \models^{\mathcal{I}} \bot$.*

An *attack relation* will be determined from conflicts adjudicated in accordance to some *preference criterion* $>$. Different alternatives can be taken in consideration for concretizing such a criterion. For simplicity we will abstract away from any such concretization. Then, we will overview an example where the criterion is concretized in terms of *trust* parameters. Afterwards, in a subsequent example, we will analyze the conflict between a legal case and a legal precedent in which the case is founded.

Definition 11 (Attack). *Given a conflicting pair of arguments $\mathcal{S}, \mathcal{S}' \in \mathbb{A}_{\mathcal{I}}$, we say that $\mathcal{S}$* ***attacks*** *(or* ***defeats****) $\mathcal{S}'$, noted $\mathcal{S} \hookrightarrow \mathcal{S}'$ iff $\mathcal{S}$ counterargues $\mathcal{S}'$ and the conflict is adjudicated in favor of $\mathcal{S}$ in the light of some preference criterion $> \in \mathbb{A}_{\mathcal{I}} \times \mathbb{A}_{\mathcal{I}}$ such that $\mathcal{S} > \mathcal{S}'$.*

Example 4. *Let us recall the construction of argument $\mathcal{S} = \langle \{\mathcal{A}\}, \{\alpha\}, \alpha' \rangle$ from example 3 claiming that* the victim was shot to death *by relying upon an atom $\mathcal{A}$ standing for a* CSG *from experience claiming that* a body with gunshot wounds *is presumably the effect of* a living person being shot to death. *Let us assume now that* there is a forensic inform concluding that the gunshots impacted on the yet lifeless victim *(α''). Consider now an atom $\mathcal{A}' = \langle \{\beta''\}, \beta''' \rangle$ that stands for a* CSG *claiming that* an inform from expert opinion inferring a conclusion *(β'') is more likely to be* trusted as a backing for the consideration of such a conclusion as a piece of evidence for the case *(β'''). Therefore, we have that $\alpha'' \sqsubseteq \beta''$ holds. Afterwards, we know that there exists a formula $\alpha''' \in \mathcal{L}$ such that $\alpha''' = \beta'''[\{\mathcal{A}'\}, \{\alpha''\}]$ holds, where α''' claims that* the gunshots impacted on the yet lifeless victim. *This means that we have a second primitive argument $\mathcal{S}' \in \mathbb{A}_{\mathcal{I}}$ where $\mathcal{S}' = \langle \{\mathcal{A}'\}, \{\alpha''\}, \alpha''' \rangle$.*

It is easy to see that both arguments $\mathcal{S}$ and $\mathcal{S}'$ constitute a conflicting pair given that α' stands for the victim was shot to death *which is in the opposite direction of α'''. It is easy to see that both α' and α''' are mutually exclusive, i.e., it is impossible to satisfy both simultaneously. Thus, both $\alpha' \sqsubseteq \neg\alpha'''$ and $\alpha''' \sqsubseteq \neg\alpha'$ hold. This means that both arguments counterargue each other. Afterwards, since the conclusion of an inform from expert opinion, like that inferred through atom $\mathcal{A}'$, will usually be more trustful than a conclusion derived from a* CSG *from experience, like the one formalized trough atom $\mathcal{A}$, we may assume $\mathcal{S}' > \mathcal{S}$ which allows to adjudicate the attack in favor of $\mathcal{S}'$, implying $\mathcal{S}' \hookrightarrow \mathcal{S}$.*

Example 5. *According to the Legal Case 1 and the modelation given in Example 2, we have the interpretation for Fayt 1999 as:*

$$\Pi^{\mathcal{I}_f} = \left\{ \begin{array}{ll} \alpha_1: & \textit{The paragraph introduced the age limit reform.} \\ \alpha_2: & \textit{A paragraph reformed Art. 96.} \\ \alpha_3: & \textit{Art. 96 is not included in the NL list.} \\ \alpha_4: & \textit{A paragraph reformed an art. out from NL list.} \\ \alpha_5: & \textit{A paragraph exceeds RC faculties.} \\ \alpha_5': & \textit{The paragraph exceeds RC faculties.} \\ \alpha_6: & \textit{A paragraph is unconstitutional.} \\ \beta_f: & \textit{The paragraph is unconstitutional.} \end{array} \right\}$$

$\underline{\alpha_2}^{\mathcal{I}_f} = \{\alpha_1, \alpha_2\}$
$\underline{\alpha_5}^{\mathcal{I}_f} = \{\alpha_4, \alpha_5, \alpha_5'\}$
$\underline{\alpha_6}^{\mathcal{I}_f} = \{\beta_f, \alpha_6\}$

Note that propositions α_1 and α_3 are facts of the case. Besides, we have the construction of atoms:
$\mathcal{A}_1 = \langle \{\alpha_2, \alpha_3\}, \alpha_4 \rangle$, *and*
$\mathcal{A}_2 = \langle \{\alpha_5\}, \alpha_6 \rangle$.

The next two arguments can be constructed:
$\mathcal{S}_1 = \langle \{\mathcal{A}_1\}, \{\alpha_1, \alpha_3\}, \alpha_5' \rangle$, *and*
$\mathcal{S}_2 = \langle \{\mathcal{A}_1, \mathcal{A}_2\}, \{\alpha_1, \alpha_3\}, \beta_f \rangle$,
where $\mathcal{S}_1 \in \mathfrak{subs}(\mathcal{S}_2)$.

Now, let us propose an interpretation for Schiffrin 2017:

$$\Pi^{\mathcal{I}_s} = \Pi^{\mathcal{I}_f} \cup \left\{ \begin{array}{ll} \alpha_7: & \textit{The paragraph requires reappointment by Executive Power.} \\ \alpha_8: & \textit{Inclusion of the Executive Power in tenure nominations is} \\ & \textit{part of the general subject in NL.} \\ \alpha_9: & \textit{A paragraph includes a reform under the scope of general} \\ & \textit{subject in NL.} \end{array} \right\}$$

$\underline{\alpha_2}^{\mathcal{I}_s} = \{\alpha_2\}$
$\underline{\alpha_5}^{\mathcal{I}_s} = \{\alpha_5, \neg\alpha_9\}$
$\underline{\alpha_9}^{\mathcal{I}_s} = \{\alpha_9, \neg\alpha_5\}$
$\underline{\neg\alpha_5}^{\mathcal{I}_s} = \{\neg\alpha_5, \alpha_9\}$
$\underline{\neg\alpha_9}^{\mathcal{I}_s} = \{\neg\alpha_9, \alpha_5\}$

Note that in this case, propositions α_7 and α_8 are facts. Assuming a pair of new atoms:
$\mathcal{A}_3 = \langle\{\neg\alpha_5\}, \neg\alpha_6\rangle$*, and*
$\mathcal{A}_4 = \langle\{\alpha_7, \alpha_8\}, \alpha_9\rangle$*,*
we can construct an argument:
$\mathcal{S}_3 = \langle\{\mathcal{A}_3, \mathcal{A}_4\}, \{\alpha_7, \alpha_8\}, \neg\beta_f\rangle$*, which contains a subargument*
$\mathcal{S}_4 = \langle\{\mathcal{A}_4\}, \{\alpha_7, \alpha_8\}, \alpha_9\rangle$*.*

Note that if we consider both interpretations $\mathcal{I}_f$ and $\mathcal{I}_s$ together, both arguments $\mathcal{S}_2$ and $\mathcal{S}_3$ would be conflicting. This shows the need for proposing a new standard to interpret the result in Fayt 1999 in order to avoid a conflict with a precedent which would be precluding the observance of the general principle of legal certainty.

3.3. Recognition of the Legal Instance of a Case

Next we define the *knowledge base* of a case as a composite construction of atoms and formulæ. We refer as *rule base* to a base that includes atoms standing for norms derived from legal codes, rules derived from jurisprudence and/or precedents, and generalizations describing *common sense reasoning* referred by judges. On the other hand, a *case base* includes the knowledge that is related to a specific case/s under analysis. Here what we may have is not only the available evidence, but also the assumptions that have been taken for granted for practical purposes. Assumptions are a kind of ground formulæ that allows to move forward in a reasoning process under insufficient or absent evidence. The consideration of assumptions allows the construction of hypotheses for analyzing the evolution of the reasoning process under contingent situations. A technique for contrasting hypotheses allows discarding unfeasible assumptions while keeping –ideally– only a small set of assumptions that could not be coherently denied. Of course, assumptions should not be against any known evidence datum, and thus, it seems natural to require its consistency regarding both the set of evidence and the set of assumptions itself.

Definition 12 (Knowledge Base). *A tuple $\Sigma = \langle\mathcal{R}, \mathcal{C}\rangle$ is the* ***knowledge base (KB)*** *of a legal case iff the* ***rule base*** *is represented through a set $\mathcal{R}$ of atoms $\mathcal{A} \in \wp(\mathcal{L}) \times \mathcal{L}$ and the* ***case base*** *or data of the legal case through a set $\mathcal{C}$ of formuæ $\varphi \in \mathcal{L}$. The rule base is internally organized through a pair $\mathcal{R} = \langle\mathcal{N}, \mathcal{G}\rangle$ where $\mathcal{N}$ is the set of atoms standing for legal norms stemming from the underlying legal system and $\mathcal{G}$ the set of atoms standing for common sense generalizations referred by judges. On the other hand, the case base is internally organized through a pair $\mathcal{C} = \langle\mathcal{E}, \mathcal{H}\rangle$ where $\mathcal{E}$ is the set of formulæ standing for evidence data and $\mathcal{H}$ the set of formulæ standing for assumptions or hypotheticals.*

For simplicity we will write $\varphi \in \mathcal{R}$ or $\Gamma \subseteq \mathcal{R}$ implying that either $\varphi \in (\mathcal{N} \cup \mathcal{G})$ or $\Gamma \subseteq (\mathcal{N} \cup \mathcal{G})$, holds. Similarly, we will write $\varphi \in \mathcal{C}$ or $\Gamma \subseteq \mathcal{C}$ implying that either $\varphi \in (\mathcal{E} \cup \mathcal{H})$ or $\Gamma \subseteq (\mathcal{E} \cup \mathcal{H})$, holds.

Let $\Sigma = \langle\mathcal{R}, \mathcal{C}\rangle$ be a knowledge base of a legal case and $\mathcal{I}$, a legal interpretation for it. The well formation of Σ and its interaction with $\mathcal{I}$ is regulated through a set of *K-validity rules*. For specifying the validity rules, we will rely upon an *overloaded in operator*, namely *subsumptive in* (or just, *s-in*), $\in_{\mathcal{I}}$: $\Pi^{\mathcal{I}} \times \wp(\Pi^{\mathcal{I}}) \longrightarrow \{\mathtt{true}, \mathtt{false}\}$ such that $\varphi \in_{\mathcal{I}} X$ *iff* there is some $\varphi' \in X$ where $\varphi' \models^{\mathcal{I}} \varphi$.

We will rely upon the s-in operator for recognizing when a proposition's truth is enforced from a set of given propositions through the subsumption relation as specified in an interpretation $\mathcal{I}$. For instance, if $X = \{\mathtt{mustang}\}$[9] and $\mathtt{mustang} \in \underline{\mathtt{vehicle}}^{\mathcal{I}}$ then $\mathtt{vehicle} \in_{\mathcal{I}} X$. This will be very useful for avoiding

[9]Assuming that, `vehicle` stands for *a vehicle was seen by eye witnesses*, and `mustang` for *a mustang was seen by eye witnesses*.

redundant assumptions (see K2 below). That is, if we have that `mustang` was taken as evidence by the judge, we have `mustang` $\in \mathcal{E}$, and thus, there would be no novelty to assume the truth of `vehicle` as part of $\mathcal{H}$ given that we could infer it from evidence since `vehicle` $\in_{\mathcal{I}} \mathcal{E}$. Observe that the other way around would not be redundant, that is, if there is evidence enforcing the truth of `vehicle` then assuming `mustang` is just a rational hypothesization for assuming a concrete predicate subsumed by `vehicle`. Besides, a function $\mathfrak{h}(\mathcal{I}) \subseteq \Pi^{\mathcal{I}}$ stands for the *set of assumptions* from $\mathcal{I}$ *iff* $(\alpha \in \mathfrak{h}(\mathcal{I})$ *iff* $\mathcal{I} \models \top \sqsubseteq \alpha)$.

(K1) Normative Split: $\mathcal{N} \cap \mathcal{G} = \emptyset$.

(K2) Non-redundancy: for any $\beta \in \mathcal{H}$ it does not hold $\beta \in_{\mathcal{I}} \mathcal{E}$.

(K3) Assertive Consistency: $(\mathcal{E} \cup \mathcal{H}) \not\models^{\mathcal{I}} \bot$.

(K4) Interpretative Correlation: $\mathcal{H} = \mathfrak{h}(\mathcal{I})$ and for every $\mathcal{A} \in \mathcal{R}$, $\mathcal{I} \models \mathcal{A}$.

The *K-validity rules* state conditions upon a legal knowledge base with the intention to control its rationality through the interaction with a legal interpretation. It is natural to maintain separated legal norms from common sense generalizations (K1) as well as assumptions from facts that has been accepted as evidence for founding the legal case (K2) –this includes the s-in operation. Consistency of assertions is ensured through (K3). Of course, evidence cannot be contradictory, otherwise, a trivial mistake while constructing the interpretation is assumed, but also (K3) ensures consistency among assumptions and among both sets of assertions altogether –that is, evidence $\mathcal{E}$ and assumptions $\mathcal{H}$. That is, no assumption can be acceptable if it is contradicting some piece of evidence, but also (K3) allows for avoiding the possibility of building trivially opposite hypotheses through contradictory assumptions. Finally, (K4) ensures firstly that the set of assumptions is exactly the one derived from the interpretation and also that every atom in the KB is modeled by the interpretation of the case. That ensures a rationale interaction between subjective considerations contained in the interpretation regarding the knowledge registered by the KB.

We will recognize the set of arguments constructed from an underlying knowledge base through the notion of *legal argumentation instance*.

Definition 13 (Legal Argumentation Instance). *Given a knowledge base* $\Sigma = \langle \mathcal{R}, \mathcal{C} \rangle$*, where* $\mathcal{C} = \langle \mathcal{E}, \mathfrak{h}(\mathcal{I}) \rangle$*, and* $\mathcal{I}$ *is a legal interpretation, a set* $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$ *is the* ***(legal) argumentation instance*** *of* Σ *in the light of* $\mathcal{I}$ *iff it follows:*

- $\langle \mathcal{P}_M, \mathcal{P}_m, \alpha \rangle \in \mathbf{A}_{\mathcal{I}}(\Sigma)$ *iff* $\langle \mathcal{P}_M, \mathcal{P}_m, \alpha \rangle \in \mathbb{A}_{\mathcal{I}}$*,* $\mathcal{P}_m \subseteq \mathcal{C}$ *and* $\mathcal{P}_M \subseteq \mathcal{R}$*.*

Next we specify the different constructions for arguments according to their contents. The first classification relies upon arguments whose minor premises are solely composed by evidential data.

Definition 14 (Evidential Argument). *Given a KB* Σ *and its instance* $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$*, an argument* $\langle \mathcal{P}_M, \mathcal{P}_m, \alpha \rangle \in \mathbf{A}_{\mathcal{I}}(\Sigma)$ *is an* ***evidential argument*** *iff* $\mathcal{P}_m \subseteq \mathcal{E}$*.*

In addition, a different sort of arguments, referred as *hypotheticals*, can appear from the consideration of assumptions or even from other type of belief that can be currently external to the legal base. Following the legal doctrine, we will name to external arguments that provide new general statements as *lex ferenda arguments* since they can be understood as proposals to incorporate new norms or even to reform elder ones. On the other hand, arguments that are constructed from minor premises that are not evidence are referred as *hypothetical lex lata arguments*: arguments constructed upon the current positive base and certain assumptions. Finally, we refer to those hypothetical lex lata arguments based upon assumptions that are considered in the case base, *i.e.*, inside $\mathcal{H}$, as *internal hypothetical arguments* whereas for arguments upon assumptions not accepted in the case base, we speak about *external hypothetical arguments*.

Definition 15 (Hypothetical Argument). *Given an argument* $\mathcal{S}' \in \mathbb{A}_{\mathcal{I}}$*, a KB* Σ *and its instance* $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$*, we say that* $\mathcal{S}' = \langle \mathcal{P}_M{}', \mathcal{P}_m{}', \alpha' \rangle$ *is a* ***hypothetical argument*** *iff* $\mathcal{S}'$ *is non-evidential and either:*

1. *$\mathcal{S}'$ is a **Lex Ferenda Argument** iff ${\mathcal{P}_M}' \not\subseteq \mathcal{R}$, and $\mathcal{S}' \notin \mathbf{A}_{\mathcal{I}}(\Sigma)$, or else*
2. *$\mathcal{S}'$ is a **Hypothetical Lex Lata** iff there is some sub-argument $\mathcal{S} \in \mathfrak{subs}(\mathcal{S}')$ which is primitive and non-evidential, i.e., $\mathcal{S} = \langle\{\langle\Gamma, \beta\rangle\}, \Gamma', \alpha\rangle$ but $\Gamma' \not\subseteq \mathcal{E}$. Two alternatives arise, either:*
   - *$\mathcal{S}$ (and $\mathcal{S}'$) is an **Internal Hypothetical** iff $\Gamma' \subseteq \mathcal{H}$, and $\mathcal{S} \in \mathbf{A}_{\mathcal{I}}(\Sigma)$, or*
   - *$\mathcal{S}$ (and $\mathcal{S}'$) is an **External Hypothetical** iff $\Gamma' \not\subseteq \mathcal{C}$, and $\mathcal{S} \notin \mathbf{A}_{\mathcal{I}}(\Sigma)$.*

Observe that there is a missing representation that we may refer as *external evidential argument.* That is, an argument that is built from an apparent new piece of evidence that one would seek to incorporate to the reasoning process. Observe that this representation is hidden behind the definition of the external hypothetical, where $\mathcal{S} \notin \mathbf{A}_{\mathcal{I}}(\Sigma)$ because $\Gamma' \not\subseteq \mathcal{C}$ but assuming that Γ' contains facts that should be considered "new evidence" of the case. This means that the facts considered are still not officially incorporated to the case. A situation in which an external evidential is incorporated to the instance can be simulated through the incorporation of an external hypothetical. Such hypothetical is considered external given that the pieces of evidence are still not accepted by the judge. Afterwards, the new hypothetical is expected to evolve towards an (internal) evidential argument by the time its assumptions become new pieces of evidence. We will provide afterwards some formal operations for modeling such sort of dynamics.

Remark 1. *Given the instance $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$, an argument $\mathcal{S} \in \mathbb{A}_{\mathcal{I}}$ is either:*

Internal: *$\mathcal{S} \in \mathbf{A}_{\mathcal{I}}(\Sigma)$ iff $\mathcal{S}$ is either evidential or internal hypothetical.*

External: *$\mathcal{S} \notin \mathbf{A}_{\mathcal{I}}(\Sigma)$ iff $\mathcal{S}$ is lex ferenda or external hypothetical.*

It is important to note that for those arguments $\mathcal{S} \in \mathbb{A}_{\mathcal{I}}$ that are external, *i.e.*, $\mathcal{S} \notin \mathbf{A}_{\mathcal{I}}(\Sigma)$, the legal interpretation $\mathcal{I}$ from which $\mathcal{S}$ can be constructed will necessarily register in $\Pi^{\mathcal{I}}$ every predicate referred by the argument construction either through its conclusion, the minor, or the mayor premises. This includes those predicates which are not part of the KB and which are therefore responsible for the argument to be external to the base.

4. Admission of Hypothesis and its Dynamics

An external hypothetical is an hypothetical argument which is constructed upon some elements which are external to the knowledge base. They may include assumptions and even new atoms, that still have not been considered for the case. In addition, an external hypothetical may also include new pieces of evidence that still are not seen as such by the judge given that it is still not considered as part of the KB. In such a case, the expectation is to incorporate a new piece of evidence assuming it will be accepted to the case. That is a good reason for understanding such an external argument as an external hypothetical, given that the pieces of evidence it brings are still somehow based on the assumption that our support to that evidence is unquestionable and thus, they will be accepted by the case.

Introducing a new hypothetical to the KB is intended to push forward the analysis of the legal case in the primary instances of the legal process which are previous to the legal debate, *i.e.*, the trial. This allows not only the introduction of assumptions, but also the introduction of a whole argument constructed upon new assumptions, giving in this manner a specific context of usage of the new hypothetical. Somehow, it resembles the usage of rhetoric explanations. This is a very common practice in advocacy, where hypothesization allows the analysis of alternative situations with the intention to reconstruct the social context which led to the current state of affairs motivating the legal process.

For providing a theory capable of dealing with the needs for analyzing and preparing a legal case in its primary instances, we need to study how to incorporate external hypotheticals to the legal instance built so far. Given an admissible interpretation $\mathcal{I}$, the instance $\mathbf{A}_{\mathcal{I}}(\Sigma)$ and an argument $\mathcal{S}$ which is external to the instance, *i.e.*, $\mathcal{S} \notin \mathbf{A}_{\mathcal{I}}(\Sigma)$, the interaction between the KB Σ and $\mathcal{I}$ may either:

- Satisfy K4 which implies an argument $\mathcal{S}$ with no new assumptions. This means that $\mathcal{S}$ is considering some piece of evidence which is still outsider. Clearly, the argument is modeled by $\mathcal{I}$, *i.e.*, $\mathcal{S} \in \mathbb{A}_{\mathcal{I}}$.
- Violate K4 which necessarily implies an argument $\mathcal{S}$ considering some assumption still not incorporated to the KB. In this case, the assumption is not considered as such by the legal interpretation $\mathcal{I}$ and therefore, we might consider a different interpretation $\mathcal{I}_s$ such that $\mathcal{S} \in \mathbb{A}_{\mathcal{I}_s}$, given that $\mathcal{S} \notin \mathbb{A}_{\mathcal{I}}$.
- Alternatively, an argument may include new atoms which will imply dealing with a lex ferenda. However, a lex ferenda may also include new assumptions or even new pieces of evidence leading to a sort of hybrid external hypothetical which would be captured by some of the previous alternatives.

This analysis shows the need for specifying some sort of operation allowing us to manage different interpretations by mixing them up into a single one. We will refer to such operation as *interpretation append.*

Definition 16 (Interpretation Append). *Given two admissible interpretations* $\mathcal{I}_1 = \langle \mathcal{L}, \Pi^{\mathcal{I}_1}, \underline{\cdot}^{\mathcal{I}_1} \rangle$ *and* $\mathcal{I}_2 = \langle \mathcal{L}, \Pi^{\mathcal{I}_2}, \underline{\cdot}^{\mathcal{I}_2} \rangle$*, an operator* $\circledcirc$ *is an* ***interpretation append*** *iff* $\mathcal{I} = \mathcal{I}_1 \circledcirc \mathcal{I}_2 = \langle \mathcal{L}, \Pi^{\mathcal{I}_1} \cup \Pi^{\mathcal{I}_2}, \underline{\cdot}^{\mathcal{I}} \rangle$*, and for every* $\beta \in \Pi^{\mathcal{I}_1} \cup \Pi^{\mathcal{I}_2}$ *either:*

1. $\underline{\beta}^{\mathcal{I}} = \underline{\beta}^{\mathcal{I}_1}$ *iff* $\beta \in \Pi^{\mathcal{I}_1}$ *and* $\beta \notin \Pi^{\mathcal{I}_2}$*, or else*
2. $\underline{\beta}^{\mathcal{I}} = \underline{\beta}^{\mathcal{I}_1} \cup \underline{\beta}^{\mathcal{I}_2}$ *iff* $\beta \in \Pi^{\mathcal{I}_1}$ *and* $\beta \in \Pi^{\mathcal{I}_2}$*.*

Interpretation appending may not necessarily result in an admissible appended interpretation. This means that appended subsumptions may not necessarily end up in a consistent state. An appropriate merge-like operation between pairs of interpretations should guarantee a consistent result possibly by getting rid of, or contracting, those subsumptions that may lead to interpretative conflicts. For understanding the reasons by which some specific subsumption needs to be changed, it would require a deep philosophical analysis which falls beyond the scope of this article. We will just adopt an append operation for merging interpretations and a pair of *assumption contraction* and *multiple assumption contraction* operations to make the necessary corrections to avoid clashes between pairs of assumptions in the resulting interpretation. Such sort of corrections, along with the new assumptions to be incorporated, will be managed as a consequence of the application of the *hypothesis revision operation* (see Definition25).

As aforementioned, we will abstract away from the problem of dealing with clashes between general subsumptions. Hence, we will not be interested in the question of how to deal with clashes like $\alpha \sqsubseteq \beta$ and $\alpha \sqsubseteq \neg\beta$, since we will avoid appending such sort of pairs by controlling the interpretations that would be *pairwise compatible* for being appended. In other words, we will restrict the application of an interpretation append to pairs of admissible interpretations that will not trigger subsumption clashes. This makes sense since the objective is to incorporate to the argumentation instance only those external hypotheticals which would serve for pushing forward the preparation of the legal case before reaching an eventual trial. To that reason, we will be looking for a specific construction of interpretation which would lead to the smallest sort of model of the external hypothetical to incorporate afterwards to the legal instance. We will call such sort of interpretation as *minimal model.*

Definition 17 (Minimal Model). *Given an admissible interpretation* $\mathcal{I}_s$ *and an argument* $\mathcal{S} \in \mathbb{A}_{\mathcal{I}_s}$*, where* $\mathcal{S} = \langle \mathcal{P}_M, \mathcal{P}_m, \alpha \rangle$*, we say* $\mathcal{I}_s$ *is the* ***minimal model*** *of* $\mathcal{S}$ *iff it follows:*

1. $\mathcal{I}_s \models \mathcal{S}$*,*
2. $\Pi^{\mathcal{I}_s} = (\mathcal{P}_m \cup \{\alpha\} \cup \bigcup_{\mathcal{A} \in \mathcal{P}_M} \mathfrak{source}(\mathcal{A}))$*,*
3. $\mathfrak{h}(\mathcal{I}_s) \subseteq \mathcal{P}_m$*, and*
4. *for any pair* $\beta, \gamma \in \Pi^{\mathcal{I}_s}$ *such that* $\beta \in \underline{\gamma}^{\mathcal{I}_s}$ *and any admissible interpretation* $\mathcal{I}$ *such that* $\Pi^{\mathcal{I}} = \Pi^{\mathcal{I}_s}$*, if* $\beta \notin \underline{\gamma}^{\mathcal{I}}$ *then* $\mathcal{I} \not\models \mathcal{S}$*.*

Observe that there is a family of minimal models of an argument $\mathcal{S} = \langle \mathcal{P}_M, \mathcal{P}_m, \alpha \rangle$ which are those interpretations $\mathcal{I} \models \mathcal{S}$ whose assumptions are contained in $\mathcal{P}_m$. For instance, let $\mathcal{S} = \langle \mathcal{P}_M, \{\beta_1, \ldots, \beta_n\}, \alpha \rangle$ be an argument, two interpretations $\mathcal{I}_1$ and $\mathcal{I}_2$, such that $\mathfrak{h}(\mathcal{I}_1) = \{\beta_1, \ldots, \beta_n\}$ and $\mathfrak{h}(\mathcal{I}_2) = \{\}$, are both minimal models of $\mathcal{S}$, *i.e.*, $\mathcal{I}_1 \models \mathcal{S}$ and $\mathcal{I}_2 \models \mathcal{S}$. The difference between both interpretations is the consideration of what is accepted as pieces of evidence for the construction of the argument. That is, in the light of $\mathcal{I}_1$, $\mathcal{S}$ is a hypothetical argument given that all its minor premises are assumed by $\mathcal{I}_1$, on the other hand, $\mathcal{S}$ is assumed to be evidential according to $\mathcal{I}_2$ given that otherwise, the argument could not be built. For this latter case to hold, for any KB Σ, if $\mathcal{S} \in \mathbf{A}_{\mathcal{I}_2}(\Sigma)$ then $\mathcal{E} = \{\beta_1, \ldots, \beta_n\}$ necessarily holds. Such examples make appropriate the specification of two functions, $\mathfrak{e} : \mathbb{A}_{\mathcal{I}} \longrightarrow \Pi^{\mathcal{I}}$ and $\mathfrak{h} : \mathbb{A}_{\mathcal{I}} \longrightarrow \Pi^{\mathcal{I}}$ for an interpretation $\mathcal{I}$, such that $\mathfrak{e}(\mathcal{S}) = \mathcal{P}_m \setminus \mathfrak{h}(\mathcal{S})$ and $\mathfrak{h}(\mathcal{S}) = \mathcal{P}_m \cap \mathfrak{h}(\mathcal{I})$.

Now we are able to identify those external hypotheticals which would be appropriate for being incorporated to the legal case. Those will be external hypotheticals which preserve the admissibility of the resulting appended interpretation and, of course, which will not be conflicting with the previous pieces of evidence. Such sort of external hypotheticals will be referred simply as *hypothesis*.

Definition 18 (Hypothesis). *Given an instance $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$ satisfying K1…K4, and an external hypothetical $\mathcal{S}_{\mathcal{I}_s} \in \mathbb{A}_{\mathcal{I}_s}$ based upon a minimal model $\mathcal{I}_s$, we say $\mathcal{S}_{\mathcal{I}_s}$ (i.e., $\mathcal{S}$ along with its minimal model $\mathcal{I}_s$) is a **hypothesis for** $\mathbf{A}_{\mathcal{I}}(\Sigma)$ iff the following conditions are met:*

1. *$\mathcal{I} \circledcirc \mathcal{I}_s$ is admissible,*
2. *for every $\mathcal{A} \in \mathcal{R}$, where $\Sigma = \langle \mathcal{R}, \mathcal{C} \rangle$, it holds $\mathcal{I} \models \mathcal{A}$ iff $\mathcal{I} \circledcirc \mathcal{I}_s \models \mathcal{A}$,*
3. *$\mathcal{S}_{\mathcal{I}_s} \in \mathbb{A}_{\mathcal{I} \circledcirc \mathcal{I}_s}$, and*
4. *$\mathcal{E} \cup \mathfrak{e}(\mathcal{S}) \not\models^{\mathcal{I} \circledcirc \mathcal{I}_s} \bot$.*

The conditions for an external hypothetical (and its corresponding minimal model) to be accepted as a hypothesis may be intuitively seen as those which ensure that the incorporation of the external hypothetical to the legal instance would not imply giving up believing in any atom nor any piece of evidence. This is as natural as requiring an appended resulting interpretation to be admissible. For a further illustration, please refer to Example 6.

In what follows we will propose alternatives for dealing with the possibility of incorporating a hypothesis to a legal instance. To such end, we will follow the theory of belief revision (Alchourrón et al., 1985; Hansson, 1999), however, the reader should be aware that the usual principles of belief revision will not be appropriate for our theory. For instance, consistency restoration has a very different meaning since arguments are pairwise potentially inconsistent. Let us propose a *hypothesis expansion* which will be the simplest form of hypothesis incorporation.

Definition 19 (Hypothesis Expansion). *Given an instance $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$ satisfying K1…K4, and a hypothesis $\mathcal{S}_{\mathcal{I}_s} \in \mathbb{A}_{\mathcal{I}_s}$ where $\mathcal{S}_{\mathcal{I}_s} = \langle \mathcal{P}_M, \mathcal{P}_m, \alpha \rangle$, an operator $+$ is a **hypothesis expansion** iff $\mathbf{A}_{\mathcal{I}}(\Sigma) + \mathcal{S}_{\mathcal{I}_s} = \mathbf{A}_{\mathcal{I} \circledcirc \mathcal{I}_s}(\Sigma^{\mathcal{S}})$, where $\Sigma^{\mathcal{S}} = \langle \mathcal{R} \cup \mathcal{P}_M, \mathcal{C}' \rangle$ and $\mathcal{C}' = \langle \mathcal{E} \cup \mathfrak{e}(\mathcal{S}_{\mathcal{I}_s}), \mathfrak{h}(\mathcal{I} \circledcirc \mathcal{I}_s) \rangle$.*

The resulting instance of an hypothesis expansion incorporates the hypothesis. However, that does not ensure triggering a fully rationale instance: the hypothesis by which the instance is expanded could be claiming a proposition that clashes with a piece of evidence, or even the same hypothesis could be introducing a new piece of evidence which would clash with the claim of a previous internal hypothetical. Such situations could be controlled by requiring the verification of the general principle that we called *hypothesis rationality.*

(R1) Hypothesis Rationality if $\mathcal{S}$ is an hypothetical for α then there is no piece of evidence $\beta \in \mathcal{E}$ such that $\{\alpha, \beta\} \models^{\mathcal{I}} \bot$, for any $\mathcal{S} \in \mathbf{A}_{\mathcal{I}}(\Sigma)$.

The objective for observing R1 is to ensure that every assumption has a rational purpose for being considered in the legal case and so it is the hypothesis it serves to be built. On the other hand, violation of

R1 proposes an interesting situation in which an assumption could not be rationally supported given that it allows the construction of an internal hypothetical claiming an *absurdity* given that its claim clashes a piece of evidence. Such absurdity can only appear from a mistaken *assumption* and therefore, the absurdity could be avoided if we have a way to roll back from the construction of such a hypothetical by denying the mistaken assumption. Let us first identify those hypotheticals that lead to an absurdity through the notion of *absurd hypothetical*.

Definition 20 (Absurd Hypothetical). *An argument $\mathcal{S} \in \mathbf{A}_{\mathcal{I}}(\Sigma)$ for α is an **absurd hypothetical** iff there is a piece of evidence $\beta \in \mathcal{E}$ such that $\{\alpha, \beta\} \models^{\mathcal{I}} \bot$. An argument $\mathcal{S}^{\bot}$ is a **minimal absurd hypothetical** iff $\mathcal{S}^{\bot} \in \mathbf{A}_{\mathcal{I}}(\Sigma)$ is an absurd hypothetical and for any absurd hypothetical $\mathcal{S}' \in \mathfrak{subargs}(\mathcal{S}^{\bot})$ it holds $\mathcal{S}^{\bot} = \mathcal{S}'$.*

To roll back the construction of an absurd hypothetical, we need to specify a contraction operation for getting rid of specific assumptions within an interpretation. As said before, since an absurdity is claimed through an absurd hypothetical, we would need not only to give up believing in some of its assumptions but also, that such an assumption cannot be taken, and therefore, we would also need to start believing the contrary assumption is certain. That is, accepting that an assumption β leads to an absurd hypothetical implies not only giving up believing in β but also believing in $\neg\beta$. An operation for contracting an assumption in such manner will be called *assumption contraction*.

Definition 21 (Assumption Contraction). *Given an admissible interpretation $\mathcal{I} = \langle \mathcal{L}, \Pi^{\mathcal{I}}, \underline{\cdot}^{\mathcal{I}} \rangle$ and an assumption $\beta \in \Pi^{\mathcal{I}}$, i.e., $\mathcal{I} \models \top \sqsubseteq \beta$, an operator $\ominus$ is an **assumption contraction** iff $\mathcal{I} \ominus \beta = \langle \mathcal{L}, \Pi^{\mathcal{I}}, \underline{\cdot}^{\mathcal{I}\ominus\beta} \rangle$ such that:*

- $(\mathcal{I} \ominus \beta) \models \top \sqsubseteq \neg\beta$
- *for every $\gamma \in \Pi^{\mathcal{I}}$ such that $\gamma \neq \beta$, it holds $\underline{\gamma}^{\mathcal{I}} = \underline{\gamma}^{\mathcal{I}\ominus\beta}$*
- $\underline{\beta}^{\mathcal{I}\ominus\beta} = \underline{\beta}^{\mathcal{I}} \setminus \{\top\}$

We are able now to introduce a simple *hypothetical contraction operation* which will allow rolling back from the construction of an internal hypothetical that has turned into absurd.

Definition 22 (Hypothetical Contraction). *Given an instance $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$ satisfying K1...K4 and a hypothetical $\mathcal{S} \in \mathbf{A}_{\mathcal{I}}(\Sigma)$, an operator $-$ is a **hypothetical contraction** iff $\mathbf{A}_{\mathcal{I}}(\Sigma) - \mathcal{S} = \mathbf{A}_{\mathcal{I}\ominus\beta}(\Sigma)$, for some $\beta \in \mathfrak{h}(\mathcal{S})$.*

Observe that, similarly to the conclusions obtained from mathematical proofs by *reductio ad absurdum*, we can accept that whenever an absurd hypothetical is recognized, some of its assumptions can only be taken *a contrario sensu*. The effect of such change would result in the impossibility to construct the absurd hypothetical given that some of its premises can no longer be supported through the mistaken assumption that now has been denied. This brings about the necessity of formalizing a generalized operation capable of denying assumptions by *reductio ad absurdum* with the objective of guaranteeing R1. We will refer to such an operation as *reductio ad absurdum contraction* which will rely upon a *multiple assumption contraction*: a sort of generalization of the operation of assumption contraction which is capable of contracting a set of assumptions.

Definition 23 (Multiple Assumption Contraction). *Given an admissible interpretation $\mathcal{I} = \langle \mathcal{L}, \Pi^{\mathcal{I}}, \underline{\cdot}^{\mathcal{I}} \rangle$ and a set $\mathcal{H} \subseteq \Pi^{\mathcal{I}}$ of assumptions $\beta \in \mathcal{H}$, i.e., $\mathcal{I} \models \top \sqsubseteq \beta$, an operator $\ominus$ is a **multiple assumption contraction** iff $\mathcal{I} \ominus \mathcal{H} = \langle \mathcal{L}, \Pi^{\mathcal{I}}, \underline{\cdot}^{\mathcal{I}\ominus\mathcal{H}} \rangle$ such that:*

- *for every $\beta \in \mathcal{H}$, $(\mathcal{I} \ominus \mathcal{H}) \models \top \sqsubseteq \neg\beta$*
- *for every $\gamma \in \Pi^{\mathcal{I}}$ such that $\gamma \notin \mathcal{H}$, it holds $\underline{\gamma}^{\mathcal{I}} = \underline{\gamma}^{\mathcal{I}\ominus\mathcal{H}}$*
- *for every $\beta \in \mathcal{H}$, $\underline{\beta}^{\mathcal{I}\ominus\mathcal{H}} = \underline{\beta}^{\mathcal{I}} \setminus \{\top\}$*

We are able now to define the operation of *reductio ad absurdum contraction*, which will be capable of applying a sort of R1-closure over an instance violating R1. As we will see afterwards, this generalized contraction –which can also be seen as a kind of *consolidation operation* (Hansson, 1999)– will serve as a sub-operation for the *hypothesis revision operation* that we propose in Definition 25.

Definition 24 (Contraction by Reductio ad Absurdum). *Given an instance $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$ satisfying K1...K4, the operator $-$ is overloaded as the **contraction by reductio ad absurdum** iff $\mathbf{A}_{\mathcal{I}}(\Sigma) - \bot = \mathbf{A}_{\mathcal{I}\ominus\mathcal{H}^{\bot}}(\Sigma)$, where $\mathcal{H}^{\bot} \subseteq \Pi^{\mathcal{I}}$ is a set of assumptions, such that:*

1. *$\mathfrak{h}(\mathcal{S}^{\bot}) \cap \mathcal{H}^{\bot} \neq \emptyset$ holds for every $\mathcal{S}^{\bot} \in \mathbf{A}_{\mathcal{I}}(\Sigma)$, and*
2. *there is no $\mathcal{H}' \subset \mathcal{H}^{\bot}$ verifying condition 1.*

The legal doctrine usually refers as *apagogical argument*(Guastini, 2003; Rodríguez-Toubes Muñiz, 2012) to an argument constructed upon an assumption –a hypothetical in our terms– that will ultimately lead to a contradiction. On such basis, the assumption is considered an absurd and therefore, its denial is proven. Also, the apagogical argument is understood as a complex sort of legal interpretation from which a hypothesis can be certified and even a new norm can be derived. The consideration of this sort of arguments is a well known practice for handling both the dynamics of legal systems and for representing the *rules of evidence*. This shows the need for proposing change operations capable of including an external hypothetical while preserving the resulting instance from building absurd hypotheticals. That is, if an apagogical appears after an instance expansion, a revision operation would need to accommodate the assumptions of the case in accordance to a *reductio ad absurdum*-like reasoning behavior.

Revision operations are referred in the theory of belief revision (Alchourrón et al., 1985) as a way to model dynamics of knowledge by relying usually upon two sub-operations: an expansion to incorporate the new beliefs followed by a contraction which is responsible for discarding beliefs which would be triggering inconsistency along with the new incorporated beliefs. Belief revision has been applied to formal argumentation through different perspectives, like is done in the ATC[10] approaches(Moguillansky et al., 2012), where an external argument is incorporated to the framework through a revision operation which also discards that information which would serve for constructing defeaters of the inserted argument. Observe that, in such cases, the revision –through its inner contraction sub-operation– does not restore consistency but ensures the new argument will end up undefeated according to the corresponding argumentation semantics.

In our theory, a similar behavior occurs with regards to the R1-principle. The inclusion of an external hypothetical may imply the violation of its rationality. It is necessary to formalize an operation capable of restoring such rationale state, or capable of avoiding breaking such previous rationality. Next we define the *hypothesis revision* operation by relying upon the contraction by *reductio ad absurdum* for restoring the rationality observed through the R1-principle.

Definition 25 (Hypothesis Revision). *Given an instance $\mathbf{A}_{\mathcal{I}}(\Sigma) \subseteq \mathbb{A}_{\mathcal{I}}$ satisfying K1...K4 and a hypothesis $\mathcal{S}_{\mathcal{I}_s} \in \mathbb{A}_{\mathcal{I}_s}$, an operator $*$ is a **hypothesis revision** iff $\mathbf{A}_{\mathcal{I}}(\Sigma) * \mathcal{S}_{\mathcal{I}_s} = (\mathbf{A}_{\mathcal{I}}(\Sigma) + \mathcal{S}_{\mathcal{I}_s}) - \bot$.*

5. Practical Applications of the Hypothesis Revision

The evidential revision will clearly impact over the legal argumentation instance. As a result of handling that kind of dynamics upon assumptions, while new arguments could appear, as a result of preserving R1, some other arguments could no longer be constructed. That describes a regular non-monotonic behavior given that the inclusion of a new argument to the instance will not necessarily imply a bigger instance as a result.

It is interesting to note that this sort of operation can be referred as a tool for *hypothesis confirmation*: introducing a new assumption which is complementary to a pre-existing one, as a way to provoke a suspected

[10]The acronym ATC stands for Argument Theory Change.

absurdity, can be used as a methodology for semi-automatic proof generation. This can be very useful in the state of argumentation construction previous to the trial in a legal process.

The next example presents an *apagogical argument* from a Spanish civil case.

Example 6. *The company was covered by an all risk insurance policy but not by the complementary machinery-damage policy. An all risk policy handles damages caused by sudden, accidental and unforeseeable acts, on the other hand, the complementary machinery-damage policy handles damages caused by sudden, accidental and unforeseeable acts which can arise due to negligence.*

After a damage caused to the machinery by a negligent act of its driver, the company claimed to the insurance company who alleged that the damage should be handled by the complementary machinery-damage policy and not by the all-risk policy. The case was taken by the Audiencia Provincial de Madrid, *from now on, the audience, who defended the position in which the damage under analysis cannot be covered given that the all-risk policy only considers unforeseeable acts, while the audience interpreted that the accident occurred due to a human error which could not be seen as unforeseeable if we consider that it could be avoided if a due care would had been taken.*

The case reached the Supreme (Civil) Tribunal of Spain who delivered –in sentence 464/2010– the following apagogical argument*:*

> "[I]t is absurd to consider foreseeable to every damage caused by negligence, because otherwise it would make no sense for the complementary insurance policy of machinery damage to exist, given that it handles damages caused by unforeseeable negligence. [...] The possibility of damages qualified as negligent and unforeseeable covered by an insurance policy makes absurd the claim *negligent damages are always foreseeable* and serves for proving the contrary claim: *not every negligent damage is foreseeable.*[11]" *(Rodríguez-Toubes Muñiz, 2012)*

Let us reconstruct the case in the terms of our theory. The following propositions appear:

$\texttt{allRisk}$*:* a damage is covered by the all-risk policy

$\texttt{machineryDamage}$*:* a damage is covered by the machinery-damage complementary all-risk policy

$\texttt{suddenAccidental}$*:* an act is sudden and accidental

$\texttt{foreseeable}$*:* an act is foreseeable

$\texttt{negligent}$*:* an act is negligent

Now we can propose two atoms standing for each policy, atom $\mathcal{A}_a$ for the all-risk policy and $\mathcal{A}_b$ for the complementary machinery-damage policy:

$\mathcal{A}_a = \langle\{\texttt{suddenAccidental}, \neg\texttt{foreseeable}\}, \texttt{allRisk}\rangle$

$\mathcal{A}_b = \langle\{\texttt{negligent}, \texttt{suddenAccidental}, \neg\texttt{foreseeable}\}, \texttt{machineryDamage}\rangle$

The audience interpreted that negligent acts are always foreseeable. This can be handled in our theory through an interpretation:

$\mathcal{I}_s \models \texttt{negligent} \sqsubseteq \texttt{foreseeable}$*.*

Afterwards, by reasoning a contrario sensu *from the all-risk policy conditions, i.e., from $\mathcal{A}_a$, the audience considered a non-covering all-risk policy situation which can be formalized through an atom $\mathcal{A}'_a$:*

$\mathcal{A}'_a = \langle\{\neg\texttt{suddenAccidental} \vee \texttt{foreseeable}\}, \neg\texttt{allRisk}\rangle$

Finally, the intention of the audience, was to incorporate an argument like:

$\mathcal{S}_{\mathcal{I}_s} = \langle\{\mathcal{A}'_a\}, \{\texttt{negligent}_C\}, \neg\texttt{allRisk}_C\rangle$

However, the Supreme Tribunal detected the absurd which would be also detected in our theory. That is, the revision $\mathbf{A}_\mathcal{I}(\Sigma) * \mathcal{S}_{\mathcal{I}_s}$ cannot be appllied given that $\mathcal{S}_{\mathcal{I}_s}$ does not fulfill the requirements for being accepted as a hypothesis for $\mathbf{A}_\mathcal{I}(\Sigma)$ given that, $\mathcal{I} \circledcirc \mathcal{I}_s \not\models \mathcal{A}_b$ but $\mathcal{I} \models \mathcal{A}_b$. Observe that $\mathcal{I} \circledcirc \mathcal{I}_s \not\models \mathcal{A}_b$

[11] From the original text: *"es absurdo que todo daño causado por negligencia se considere previsible, porque en otro caso carecería de sentido la póliza opcional de avería de maquinaria, la cual cubre daños causados por negligencia siempre que sean imprevisibles. [...] La posibilidad de que haya daños negligentes e imprevisibles cubiertos por un seguro hace efectivamente absurdo afirmar que los daños negligentes son siempre previsibles y prueba la tesis contraria, según la cual no todos los daños negligentes son previsibles"* (Rodríguez-Toubes Muñiz, 2012, p. 7)

arises given that the preconditions of the complementary insurance policy $\mathcal{A}_b$ would be inconsistent, i.e., $\{\texttt{negligent}, \texttt{suddenAccidental}, \neg\texttt{foreseeable}\} \models^{\mathcal{I} \circledcirc \mathcal{I}_s} \bot$.

This example shows a sort of apagogical argument which is handled in our theory simply at a validation stage.

The previous example shows the need to detect cases in which an interpretation append cease modeling some previous atoms, not because this could not be possible, but because this should be treated accordingly. In our theory, while such situation is detected, the revision operator we propose does not admit its application. A different, and more complex revision operator could be proposed for dealing with such situations in order to let the user consider if he specifically accepts a sort of collateral contraction of the instance by letting some atom to be lost. This shows an interesting sort of change operation to be studied. This is part of our ongoing work.

5.1. An Example of Procedure Dynamics through Appellate Instances

Under certain circumstances a court can let a criminal defendant off on probation[12]. However, this special benefit may not be granted for a defendant involved in a gender-based violence case, since probation may end up dismissing the criminal case and thus, no effective legal procedure would be granted for the woman[13], contradicting multilateral treaties on the matter like the *Vienna Convention on the law of treaties* (or *Vienna*, for short) and the *Convention of Belem do Pará on the prevention, punishment, and eradication of violence against women* (or *BdP*, for short).

That exceptional case was not considered by the Argentinean Law until the Supreme Court dictated the definitive sentence on *Góngora's case*, on April 23rd, 2013 (Corte Suprema de Justicia de la Nación (CSJN), 2013). In this example, we analyze such a sentence which revoked the prior sentence dictated by the lower instance court, referred as *Cámara Federal de Casación Penal*, from now on, *Casación*. In short, Casación's sentence referred to the Criminal Code or *Código Penal* (CP) for conceding the benefit of probation for the case, despite the state of affairs included gender violence evidence as shown in the first instance court. We will summarize the case, by developing the argumentation provided by the Supreme Court, without specific details about Casación's arguments given in the lower instance court.

The argumentation presented through the sentence dictated by the Supreme Court relies mainly upon Vienna and BdP treaties for reinterpreting Art.76 bis and Art.76 ter from CP, specifically:

> ***Article 31(1) (Vienna).*** *"A treaty shall be interpreted in good faith in accordance with the ordinary meaning to be given to the terms of the treaty in their context and in the light of its object and purpose."*
>
> ***Article 7.b (BdP).*** *"Apply due diligence to prevent, investigate and impose penalties for violence against women."*
>
> ***Article 7.f (BdP).*** *"Establish fair and effective legal procedures for women who have been subjected to violence which include, among others, protective measures, a timely hearing and effective access to such procedures."*

According to the Supreme Court, although Casación was aware of the existence of evidence of gender violence in Góngora's and of the BdP treaty binding the state to sanction cases with evidence of violence against women, according to Casación's interpretation that does not preclude the application of 76bis CP for granting probation:

> *To Casación's chamber the fact that the Argentine State has the obligation to sanction those illegal facts reveling* the existence of violence specially directed over women as a reason of gender*, in the light of "Convención de Belem do Pará" (cfr. art. 7, inc. 1st of this legal text),* does not preclude judges the possibility for them to grant

[12]Probation is a humanitarian effort to give second-chance to minor offenders instead of serving time in prison, being also a valid resource towards procedural economy.

[13]This is a posture assuming that compensation –as one of the conditions of probation– cannot be considered an admissible legal procedure's outcome in gender-violence cases since it would be a way to interrumpt (or simplify) the conventional legal procedure.

probation to the accused of committing such facts *as being referred by art. 76 bis of the Argentinean Criminal Code.*[14]

Casación's argument seems to be constructed through the usage of an atom like $\mathcal{A}_c = \langle\{\texttt{lessThan3Years}\}, \texttt{probation}\rangle$[15], and knowing that the characteristics of the case would not deliver a sentence mandating an imprisonment time above three years, Casación assumes $\texttt{lessThan3Years}_G$. This would allow the construction of a hypothetical $\mathcal{S}^c = \langle\{\mathcal{A}_c\}, \{\texttt{lessThan3Years}_G\}, \texttt{probation}_G\rangle$ whose claim is that granting probation for Góngora is admissible. Note that, this is possible from considering that $\texttt{lessThan3Years}_G \sqsubseteq \texttt{lessThan3Years}$ and $\texttt{probation}_G = \texttt{probation}[\{\mathcal{A}_c\}\{\texttt{lessThan3Years}_G\}]$.

Casación also admits that there are particular factors in the case that may qualify as gender violence, which means that there should be a piece of evidence like $\texttt{genderViolence}_G$. According to Alchourrón and Bulygin (Alchourron and Bulygin, 1971), a norm is admissible if it survives a process of closure which involves the verification of inexistence of factors that could serve as exceptions for the application of such norm. That means that Casación should verify if a piece of evidence like $\texttt{genderViolence}_G$ could preclude the application of art. 76 bis CP for granting probation. This is exactly the interpretation that Casación defends by claiming that having evidence of gender violence would be irrelevant for the case[16].

According to the Supreme Court, Casación's argument was a clear misapplication of art. 76 bis CP against BdP. To that end, the Supreme Court's arguments include the following main argument:

Supreme Court's main argument.

Minor Premise: Góngora's case present evidence of gender violence.
Major Premise: Probation suspends the legal procedure, frustrating the possibility to prove its facts and to determine the accused responsibility upon such facts and the appropriate sanction that would correspond according to the proven facts.
Major Premise: Interpreting BdP –according to Art. 31(1) (Vienna)– in the light of its objective and purpose –to eradicate violence against women– for gender violence cases, a legal procedure should be ensured (Art. 7.f) without delay (Art. 7.b).
Conclusion: probation cannot be granted for Góngora.

The first major premise arises from the interpretation made by the Supreme Court about probation as a way to suspend a legal process given that it could ultimately preclude the possibility to clarify facts that has been qualified as "violence against women" as well as the responsibility of the accused about such facts and the appropriate sanction that could be applied. Concretely:

> *Particularly, regarding this case, conceding the suspension of the procedure [i.e., granting probation] in favor of the accused* might frustrate the possibility of clarifying*, in such procedural state, the existence of facts that* prima facie *have been qualified as* violence against women*, along with the determination of the responsibility of whom has been indicted of committing such facts and the sanction that in such case could correspond.* [17]

[14]From the original text: *"Para la cámara de casación, la obligación de sancionar aquéllos ilicitos que revelen* la existencia de violencia especialmente dirigida contra la mujer en razón de su condición*, que en virtud de la "Convención de Belem do Pará" ha asumido el Estado Argentino (cfr. artículo 7, inciso primero de ese texto legal),* no impide a los jueces la posibilidad de conceder al imputado de haberlos cometido la suspensión del juicio a prueba *prevista en el artículo 76 bis del Código Penal."* (Corte Suprema de Justicia de la Nación (CSJN), 2013, p. 4)

[15]For a matter of simplicity and legibility, we identify propositions through the usage of single words, for instance `probation` refers to the proposition *probation is granted.*

[16]The *property of relevance* for qualifying factors has been proposed in Walker et al. (Walker et al., ress). There, the authors identify an *irrelevant factor* through the usage of a label like `IRF` before the proposition takes place. The theory here proposed could take into consideration such sort of propositions by adding a new parameter to the atom's tuple from a domain of labels $\{\texttt{IRF}, \texttt{RF}\}$ –being `RF` the label corresponding to a *relevant factor.* That would allow the construction of an atom like $\langle\texttt{IRF}, \{\texttt{genderViolenceEvidence}\}, \texttt{probation}\rangle$. This shows the flexibility of the proposed theory to be adjusted in accordance to the desired domain of reasoning. However, the consideration of such a parameter may introduce a new dimension of reasoning for building legal arguments. For simplicity, in this article we will skip such expanded representation for atoms.

[17]From the original text: *"Particularmente, en lo que a esta causa respecta, la concesión de la suspensión del proceso a prueba al imputado* frustraría la posibilidad de dilucidar *en aquél estadio procesal la existencia de hechos que prima facie han sido calificados como de violencia contra la mujer, junto con la determinación de la responsabilidad de quien ha sido imputado de cometerlos y de la sanción que, en su caso, podria corresponderle."* (Corte Suprema de Justicia de la Nación (CSJN), 2013, p. 6)

The Supreme Court enforces this interpretation by providing a hypothetical:

> *If we analyze the conditions by which that benefit is regulated* [probation] (...) *the main consequence of its application is the* suspension of the debate *[process]. Afterwards, if it is the case that the accused observes the conditions imposed by the norm during the time of such suspension fixed by the corresponding tribunal,* the possibility of developing it *[process]* is definitively canceled given the extinction of the corresponding penal action *(cfr. art. 76 bis and art. 76 ter. of the cited code).* [18]

There the court ensures that the consequence of probation is the suspension of the "debate" –in reference to the legal process– and the subsequent implication, if the accused observes the conditions imposed by the tribunal, is the extinction of the criminal process and any criminal action against the accused.

The court analyzes the concept of *sanction* referred by the international treaties and relates it to the domestic concept of *judgment*:

> *This impediment arises, in first term, by considering that the meaning of the term "process" as referred in the analyzed clause coincides with the meaning given by the procedural codes to the final stage of the criminal process (cf. Third Book, Title 1 of the Criminal Procedural Code of the Argentinean Nation* [for short, 3rd-I CPP]*), being that uniquely from there can be derived a definitive pronouncement about guilty or innocence of the accused, for verifying the possibility of sanctioning such sort of facts as being mandated by the "Convención".* [19]

In short, according to CSJN, the possibility to determine the culpability or innocence of the accused is only through a (complete) criminal procedure which can be derived from 3rd-I CPP. Thus, a complete criminal procedure allows to pronounce judgment whose meaning can be comparable to the objective underlying the concept of sanction, referred in BdP.

The first major premise constitutes an interpretation of a concept of the Argentinean law, where probation, from a view point, is a possible outcome of the criminal procedure, whereas according to the supreme court's argument, probation should be considered as an interruption of the legal procedure given that it provides a shortcut for achieving a final decision towards procedural economy and in benefit of the human rights of the accused that is not considered a menace to society and whom may, under the observation of certain requirements, re-accommodate himself in a common welfare society. Although that viewpoint has been mostly followed by further legal cases including gender-violence evidence, this is still a doctrinal discussion that is being developed in Argentina.

For bringing some light to the supreme court's viewpoint, we refer to the following passage of the aforementioned Art. 76 bis, keeping in mind that the usage of the legal concept of *probation* in the Argentinean law can be literally translated as *conditional suspension of the legal process*:

> *(...) the accused could also ask for a* conditional suspension of the legal process *in the case where the maximum applicable sentence would not exceed three years imprisonment time. The formal solicitation presented by the accused should offer a compensation for damages (...)* with no implications about confession nor recognition of any presumed civil responsibility.

In terms of the formal theory proposed in this article, the main supreme court's argument is including a new viewpoint for a legal concept. This constitutes an example of external argument. A dynamic process is underway.

The following propositions can be obtained:

[18] From the original text: *"Si examináramos las condiciones en las que se encuentra regulado ese beneficio* [probation] (...) *la principal consecuencia de su concesión es la de* suspender la realización del debate. *Posteriormente, en caso de cumplir el imputado con las exigencias que impone la norma durante el tiempo de suspensión fijado por el tribunal correspondiente,* la posibilidad de desarrollarlo se cancela definitivamente al extinguirse la acción penal a su respecto *(cfr. artículo 76 bis y artículo 76 ter. del citado ordenamiento)."* (Corte Suprema de Justicia de la Nación (CSJN), 2013, p. 4)

[19] From the original text: *"Este impedimento surge, en primer lugar, de considerar que el sentido del término juicio expresado en la cláusula en examen resulta congruente con el significado que en los ordenamientos procesales se otorga a la etapa final del procedimiento criminal (así, cf. Libro Tercero, Título 1 del Código Procesal Penal de la Nación), en tanto únicamente de allí puede derivar el pronunciamiento definitivo sobre la culpabilidad o inocencia del imputado, es decir, verificarse la posibilidad de sancionar esta clase de hechos exigida por la Convención."* (Corte Suprema de Justicia de la Nación (CSJN), 2013, p. 5)

1. The first consequence of granting probation is the suspension of the criminal procedure. That is, *if probation is granted then the criminal procedure is suspended.* This can be modeled through an atom $\mathcal{A}_1 = \langle\{\texttt{probation}\}, \neg\texttt{procedure}\rangle$. Observe that the decision of modeling the suspended procedure through a complementary concept like $\neg\texttt{procedure}$ has to do with the representation of a state in which no procedure is available, as opposed to an ongoing procedure.

2. The criminal procedure is the unique legal resource by which it is possible to pronounce judgment about culpability of the indicted. That implies, to prove the alleged facts, to determine the responsibility of the accused about such facts and to specify an appropriate sentence for the indicted in accordance to such proven facts.

   It is clear that having a sanction implies completing the criminal procedure, and since the sentence is given through a necessary condition, we also have that completing a criminal procedure implies the delivery of a corresponding sanction. Moreover, in logical terms, the counter-positive can also be satisfied, that is, a stage in which the criminal procedure is incomplete necessarily implies that a sanction will be missing. We can formalize this rule through an atom $\mathcal{A}_2 = \langle\{\neg\texttt{procedure}\}, \neg\texttt{sanction}\rangle$.

It is clear that having a specific case like Góngora, if we assume that probation is granted, let us write it as $\texttt{probation}_G$, then an *external hypothetical argument* can be constructed claiming for the absence of sanction for Góngora. That is:

$$\mathcal{S}^a = \langle\{\mathcal{A}_1, \mathcal{A}_2\}, \{\texttt{probation}_G\}, \neg\texttt{sanction}_G\rangle,$$

where $\texttt{probation}_G \sqsubseteq \texttt{probation}$ and $\neg\texttt{sanction}_G = \neg\texttt{sanction}[\{\mathcal{A}_1, \mathcal{A}_2\}, \{\texttt{probation}_G\}]$. Observe that the court seems to be implying a general rule like *If probation would be granted then no sanction could be delivered.*

Regarding the second major premise, the CSJN interprets that Casación misapplied probation given that he unconsidered the contextual evidence (gender violence) and the compromise contracted by the state about sanctioning these sort of cases, so contradicting Art. 31(1) (Vienna):

> *Taking into account the admission conceded to judges by the domestic law about the possibility of dispense with the debate* [process]*, Casación's decision ignores the context of the article in which the Argentinean State has been compromised to sanction this sort of facts, thus contradicting the interpretation of art. 31, inc. 1st, of the "Convención de Viena" about the rights of the treaties [...] This is so given that according to the fundamentals of the questioned resolution,* the aforementioned conventional obligation is left absolutely isolated from the rest of the particular obligations assigned to the associated states *towards accomplishing the general objectives proposed in the "Convención de Belem do Pará":* prevent, sanction and eradicate any kind of violence against the women *(cfr. art. 7, 1st paragraph).*[20]

It is interesting to remark that the court resorts to the word "isolation" for implying that Casación's decision was taken by unconsidering the application of BdP which would trigger otherwise a clash between the objectives of such treaty and the consequences of granting probation in favor of the indicted. This is can be understood as an indirect reference to principles like *law-as-integrity* and *coherence* proposed by Ronald Dworkin (Dworkin, 1978, 1986). Finally, this supports the conclusion of the Supreme Court's argument where probation is stated as an inadmissible resource for gender violence cases.

> *To the contrary, this Court understands that following an interpretation relating the aforementioned objectives to the need of developing a "fair and effective legal procedure for women", including a "timely hearing" (cfr. inc. "f", cited article), the mentioned norm obliges to consider that the system of a juridical ordering that has*

[20]From the original text: *"Teniendo en cuenta la prerrogativa que el derecho interno concede a los jueces respecto de la posibilidad de prescindir de la realización del debate, la decisión de la casación desatiende el contexto del artículo en el que ha sido incluído el compromiso del Estado de sancionar esta clase de hechos, contrariando así las pautas de interpretación del artículo 31, inciso primero, de la Convención de Viena sobre el Derecho de los Tratados [...] Esto resulta así pues, conforme a la exégesis que fundamenta la resolución cuestionada,* la mencionada obligación convencional queda absolutamente aislada del resto de los deberes particulares asignados a los estados parte *en pos del cumplimiento de las finalidades generales propuestas en la "Convención de Belem do Pará", a saber:* prevenir, sancionar y erradicar todas las formas de violencia contra la mujer *(cfr. artículo 7, primer párrafo)."* (Corte Suprema de Justicia de la Nación (CSJN), 2013, pp.4–5)

incorporated such international treaty, as is the case of our country, adopting different alternatives for achieving a sentence for the case, in the instance of the oral debate, is inadmissible.[21]

From the final sentence (text emphasized), it is clear that to the CSJN, probation is a different alternative for a criminal case of such characteristics which does not achieve the formal definition of the legal procedure as is required by BdP.

In short, the court refers to Vienna, which states that adhering to an international treaty implies an obligation to observe its objectives. Also, Argentina adheres to BdP for which sanctioning gender violence cases can be identified as one of its main objectives. Thus, it is clear that according to Vienna we are obliged to observe BdP's objectives, which in particular means to sanction any gender violence case. Thus, an atom $\mathcal{A}_3 = \langle\{\texttt{genderViolenceCase}\}, \texttt{sanction}\rangle$ can be constructed. Afterwards, by considering the evidence of gender violence in Góngora's we have a sentence like *Góngora is a gender violence case*, noted as $\texttt{genderViolenceCase}_G$. It is easy to see that an evidential argument

$$\mathcal{S}^b = \langle\{\mathcal{A}_3\}, \{\texttt{genderViolenceCase}_G\}, \texttt{sanction}_G\rangle,$$

can be built which is a counterargument for the hypothetical $\mathcal{S}^a$ which necessarily precludes the validity of the assumption made in $\mathcal{S}^a$: to grant probation for Góngora.

Observe, however, that having $\mathcal{A}_3$ implies also its counterpositive, $\mathcal{A}_4 = \langle\{\neg\texttt{sanction}\}, \neg\texttt{genderViolenceCase}\rangle$, from which it is possible to construct a hypothetical containing $\mathcal{S}^a$. That is:

$$\mathcal{S}'^a = \langle\{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_4\}, \{\texttt{probation}_G\}, \neg\texttt{genderViolenceCase}_G\rangle,$$

Let us assume that we have an instance $\mathbf{A}_\mathcal{I}(\Sigma)$ where $\Sigma = \langle\mathcal{R}, \langle\mathcal{E}, \mathcal{H}\rangle\rangle$, $\mathcal{R} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}$ and $\mathcal{E} = \{\}$ and $\mathcal{H} = \{\texttt{probation}_G\}$. Clearly, $\mathcal{S}'^a \in \mathbf{A}_\mathcal{I}(\Sigma)$. Now, suppose we want to incorporate the external hypothetical argument $\mathcal{S}^b$ through a hypothesis revision $\mathbf{A}_\mathcal{I}(\Sigma) * \mathcal{S}^b_{\mathcal{I}_b}$, where $\mathcal{I}_b \not\models^{\mathcal{I}_b} \texttt{genderViolenceCase}_G$ which implies that $\texttt{genderViolenceCase}_G$ will end up being a new piece of evidence in the resulting revised instance. This means that $\mathcal{S}^a$ will end up being an absurd hypothetical in the intermediate state $\mathbf{A}_\mathcal{I}(\Sigma) + \mathcal{S}^b_{\mathcal{I}_b}$. As a result, if $\mathbf{A}_{\mathcal{I}_r}(\Sigma^S) = \mathbf{A}_\mathcal{I}(\Sigma) * \mathcal{S}^b_{\mathcal{I}_b}$, we have that $\mathcal{I}_r \models \top \sqsubseteq \neg\texttt{probation}_G$ and thus, $\mathcal{S}'^a \notin \mathbf{A}_{\mathcal{I}_r}(\Sigma^S)$, where $\Sigma^S = \langle\mathcal{R}, \langle\mathcal{E}^S, \mathcal{H}^S\rangle\rangle$, $\mathcal{E}^S = \{\texttt{genderViolenceCase}_G\}$ and $\mathcal{H}^S = \{\neg\texttt{probation}_G\}$.

6. Future Work on Abductive Hypotheses for Fact Finding

We will propose some ideas towards a special sort of hypothetical argument dedicated to fact investigation. Our objective is to formalize a construction for hypothesizing about the possible affairs that could have occurred as an explanation for the surprising availability of a piece of evidence.

Abductive reasoning is a reasoning process that involves creativity. It is usually stated that the abductive process is more likely to be understood as a way of backward reasoning. That is, instead of beginning with the reasoning process from premises based upon evidence to reach a conclusion, it involves reasoning from evidence to a hypothesis that could explain that evidence datum. An informal model of abductive argument is commonly presented as follows:

Minor Premise: Surprising event E occurred.
Major Premise: If H were true then E would follow as a matter of consequence.
Conclusion: No other hypothesis explains E as well as H does, thus H should be true.

[21] From the original text: *"En sentido contrario, esta Corte entiende que siguiendo una interpretación que vincula a los objetivos mencionados con la necesidad de establecer un "procedimiento legal justo y eficaz para la mujer", que incluya "un juicio oportuno" (cfr. el inciso "f", del artículo citado), la norma en cuestión impone considerar que en el marco de un ordenamiento jurídico que ha incorporado al referido instrumento internacional, tal el caso de nuestro país,* la adopción de alternativas distintas a la definición del caso en la instancia del debate oral es improcedente.*"* (Corte Suprema de Justicia de la Nación (CSJN), 2013, p. 5)

In law this reasoning is known as *argumentum ad ignorantiam* given that the conclusion is only reachable because *there is no other plausible explanation for it*(Walton, 2004). More interestingly, is the observation that abduction is a process of putting together pieces of information that we already know. In this sense, Pierce talked about flashes of insight (Peirce, C.S. and Turrisi, P.A., 1997)

> *The abductive suggestion comes to us like a flash. It is an act of insight, although of extremely fallible insight. It is true that the different elements of the hypothesis were in our minds before; but it is the idea of putting together what we had never before dreamed of putting together which flashes the new suggestion before our contemplation.*

More recently, the case of abduction has been also studied as a special sort of argument which relies upon a causal rule as its major premise (Bex, 2011; Prakken, 2002). This inverts the structure of the argument, which now proposes the evidence as its conclusion in order to verify the hypothesis.

Minor Premise: There is a reason to believe in H.
Major Premise: H is a plausible cause for E.
Conclusion: Event E turns predictable.

This sort of argument structure applies well for analyzing the motives and credibility of testimonies. For instance, is there a motive for the supposed witness to lie as a justification for his testimony? This form of argumentation has been referred in the literature as *causal story.*

This new argument is a kind of *abductive hypothesis* whose conclusion is an evidence datum and whose premises cannot be completely supported through the evidence data. Somehow, an abductive hypothetical is similar to an absurd hypothetical, which in turn of clashing a piece of evidence with its claim, it supports it. That is, an hypothetical $\mathcal{S}$ for α where $\alpha \in \mathcal{E}$.

An interesting question would be: is it possible to find new abductive hypotheses through an automated process that could propose new valid assumptions either for fact-finding or for pushing further a reasoning process? The idea is to take advantage of an automatic process of legal argumentation that allows us to hypothesize about the current state of affairs in the light of the reasoning process that was already developed in a previous similar case.

To that end, suppose we have the precedent of a previous legal case whose instance is $\mathbf{A}_{\mathcal{I}_2}(\Sigma_2)$, sharing some common characteristics with the case under analysis, $\mathbf{A}_{\mathcal{I}_1}(\Sigma_1)$. It would be possible to recognize all small arguments from the previous case that provide support for a claim that could be useful in the current case. However, we can assume that somehow we cannot construct those arguments from the current case's base given that there is some missing evidence (and possibly some missing *CSG*). That is, arguments contained in $\mathbf{A}_{\mathcal{I}_2}(\Sigma_2)$ which are external abductive hypotheticals in $\mathbf{A}_{\mathcal{I}_1}(\Sigma_1)$.

For instance, let us assume an argument from a jurisprudence's previous case:

- Edith and Freddie were in an intimate relationship
- Edith is older than Freddie
- In intimate relationships the older partner tends to be dominant
- There is a reason to believe that Edith dominated Freddie's will
- There is a reason to believe that Edith incited Freddie to murder her husband

Assume that such an argument served as a backing for the available evidence: *The lifeless body of Edith's husband was found.* Now, assuming we have a new case with the following facts:

- Rosie's husband was murdered
- The main suspect is Rosie's young colleague

Let us assume we are pursuing the investigation of the case as public prosecutors, we may ask ourselves: is it relevant to investigate whether Rosie and his colleague were involved in an affair? Would it be relevant towards probing the defendant, Rosie's colleague, guilty of Rosie's husband murdering? Should an automated legal reasoner recommend about this possibility for further fact-finding?

An external abductive hypothesis could be constructed as follows:

- Rosie and her colleague were in an intimate relationship [Assumption]
- Rosie is older than her colleague
- In intimate relationships the older partner tends to be dominant
- There is a reason to believe that Rosie dominated her colleague's will
- There is a reason to believe that Rosie incited her colleague to murder her husband

Then, it would be necessary to investigate the assumption's truthfulness given that such an assumption could be relevant for achieving a decision for the case.

The objective of constructing abductive hypothesis is to redirect the investigations towards finding new evidence for the case which allows to achieve the necessary proof towards finding the defendant guilty, in the case of being prosecutors, or to the contrary, if we are part of the defense, the objective would be to find new hypothesis for constructing new defeaters for the prosecutors' arguments towards maintaining the innocence of the defendant. This means that the construction of relevant abductive hypotheses is understood as part of a bigger operation of warrant pursuing of the main argument of the case.

We believe the theory here proposed brings the necessary theoretical elements to formalize an advanced automatic process to build hypotheses through abduction which will afterwards be incorporated to the legal case under analysis. Thus, an external hypothetical can be built from an analogous precedent, constructing its minimal model from the legal interpretation of the corresponding precedent. Afterwards, the hypothesis revision given on Definition 25 would make possible the incorporation of the external hypothetical to the current case ensuring the assumptions it brings to the case do not clash with other pieces of evidence yet available.

This is the projection we envision for this theory, being part of our ongoing work on this direction. Software recommenders dealing with this sort of automatic process would be really useful in assisting the fact investigation of a legal case before the eventual instance of a trial.

7. Related Work

There is a large literature in AI & Law have addressed on legal interpretation. For an overview, see (Prakken and Sartor, 2015).

For example, several works focused on the role of teleological reasoning or legal doctrine (see e.g., (Araszkiewicz, 2013, 2014)). Indeed, this idea is standard in legal theory and the goals of legal rules are recognised by jurists as decisive in clarifying the scope of the legal concepts that qualify the applicability conditions for those rules (Bench-Capon, 2002; Prakken and Sartor, 2015; Skalak and Rissland, 1992; Hage, 1997). (Bench-Capon, 2002; Prakken and Sartor, 2015) use goals and values in frameworks of case based reasoning for modelling precedents mainly in a common law context. (Skalak and Rissland, 1992) analyses a number of legal arguments even in statutory law, which include cases close to the ones discussed here. The proposal which is closer to our contribution is (Hage, 1997). In (Hage, 1997) Jaap Hage addresses, among others, the problem of reconstructing extensive and restrictive interpretation. This is done in Reason-Based Logic, a logical formalism that can deal with rules and reasons: the idea is that the satisfaction of rules' applicability conditions is usually a reason for application of these rules, but there can also be other (and possibly competing) reasons, among which we have the goals that led the legislator to make the rules.

All these approaches in AI & Law highlight the importance of legal goals, and (Hage, 1997), in particular, follows this idea to formalise extensive and restrictive interpretation. However, it seems that no work so far has attempted to couple this view with a framework for reasoning with counts-as rules and their dynamics. In this perspective, we believe that this paper may contribute to fill a gap in the literature.

More recently, works on legal interpretation have focused on modelling legal arguments and canons. In particular, following (Prakken and Sartor, 2013; Macagno et al., 2012), (Rotolo et al., 2015) have devised a defeasible formalism for modelling reasoning about interpretive canons and thus for justifying the choice of a certain canon and the resulting legal outcome over competing interpretations.

Following some doctrinal and judicial practice, (Sartor et al., 2014) argued that interpretive canons are defeasible rules licensing deontic interpretive claims, namely, the claim that a certain expression in a statute ought, ought not, may or may not be interpreted in a certain way. (Sartor et al., 2014) also argued that an interpretive canon for statutory law can be expressed as follows: if provision n occurs in document D, n has a setting of S, and n would fit this setting of S by having interpretation a, then, n ought to be interpreted as a. Further extension works for handing interpretation across different legal systems—something happening, e.g., in private international law—is presented in (Malerba et al., 2016).

On the other hand, in the area of knowledge representation and reasoning, there is a large literature in formal argumentation dealing with the construction of structured arguments and with problems related to the dynamics of arguments. We will firstly give a brief overview of some previous work which constitute the fundamentals underlying the theory proposed here, and will thereafter relate it to other well known argumentation frameworks.

The framework presented in (Moguillansky and Simari, 2016) defines an abstract structure of argumentation that motivates the argumentation defined in the present article. There, the argument structure is constructed upon an argument-language which brings the possibility to define the structure of atoms and therefore, the way atoms can be interrelated through support to finally compose the arguments. Similarly, the theory proposed in the present article relies upon atoms and, although we do not define an argument-language, the flexibility for inter-chaining atoms is achieved through an interpretation structure which fits better the needs for legal reasoning.

Some specific problems of the dynamics of arguments whose reasoning services rely upon dialectical semantics is tackled in (Moguillansky et al., 2012). Alteration of dialectical trees gives the possibility to understand which arguments should "disappear" in order to ensure warrant of a new argument to incorporate. A similar proposal was defined in (Moguillansky and Simari, 2017), although in that case, the idea was not to erase, but to include new arguments as protectors of a main argument to be incorporated, pursuing its warrant state through the dialectical tree. In this latter paper, the theory proposed is more focused to legal trials since dialectical trees stand for the argumentation proposed by both the prosecutor and defender. Such interchange is proposed as a simulation which would apply well for the legal case preparation before the trial. The incorporation of new arguments stand for automatic recommendations towards achieving warrant of the main argument to be included.

In the present article, dynamics is handled with a slight different objective. Although we pursue a theory for automatic assistance of legal reasoning that may serve for a case preparation before trial, we only investigate the implications of including new assumptions that would be useful for preparation of hypotheses. In this article, we do not work dynamics at an argumentation semantics level like we did before. We do not care of the warrant state of an argument, but on the acceptability of a new hypothesis whose inclusion would serve to push forward the investigation of the legal case. Belief revision techniques like (Alchourrón et al., 1985; Hansson, 1999) are always present in these sort of theories for handling dynamics of arguments.

The recursive definition of arguments formalised here resembles, for instance, to the original proposal of the ASPIC+ framework (Modgil and Prakken, 2014). Arguments in ASPIC+ are built with both defeasible and strict inference rules. The latter allows to guarantee the argument's claim whereas the former only create a presumption in its favor. Similarly, the theory we propose handles hypotheses through assumptions, which can be understood as a way to rely upon defeasible assertions, and arguments relying upon evidence, which can be seen as strict or unquestionable assertions. Therefore, it is natural to have semantics assessing a different treatment to evidential arguments with respect to hypotheses. In ASPIC+ this is present, for instance, in the ways arguments can be attacked on their premises, defeasible inferences or conclusions. In this sense, our approach is more similar to ABA (assumption-based argumentation) (Bondarenko et al., 1997; Dung et al., 2009), where no distinction is made for rules, since all of them are deductive. In ABA the defeasibility is determined through the use of assumptions for the construction of arguments. This is different from ASPIC+ where defeasibility relies directly on the usage of defeasible rules.

In our approach, we have different levels of defeasibility depending on the source from where the rule (or atom) has been extracted. A highest level rule would be one corresponding to the constitutional hierarchy, whereas a *common sense generalization* (*CSG*) would be the lowest possible hierarchy of a rule, being more similar to a classical defeasible rule. However, here defeasibility can be somehow "tricky". A *CSG* might not correspond to the lowest hierarchy of a rule when it is considered under its original legal interpretation. That is, as a *CSG* is proposed by a judge under a legal interpretation, it will be considered by that judge right above other *CSG*'s proposed in a different legal interpretation (possibly learned from a precedent). Hence, not all *CSG*'s will correspond to the lowest hierarchy of rules.

Nevertheless, in this article we are more concentrated on analysing new assumptions to build hypotheses. Naturally, this prohibits a new assumption to contradict any piece of evidence, but also, makes necessary to observe that no hypothesis should claim in opposition to evidence. This brings about a new sort of change operation, named *contraction by reductio ad absurdum*, which allows the readjustment of assumptions to avoid the construction of absurd hypotheses in a legal reasoning process. That novel operation ends up as a sub-operation of the *hypothesis revision*, which allows the incorporation of new hypotheses to push forward the investigation of a legal case in advance to the trial.

8. Conclusions

We proposed a theory capable of bringing flexibility to the support of premises for modeling legal interpretations. The usage of such theorization allows to set up the final link between pairs of rules standing for the logical support. The construction of arguments –which will justify a legal decision– is possible upon such linkage. Saving the register of those specific details that conformed the interpretation of a legal case allows to project legal reasoning considering precedents in a more concrete manner, not only by referring to the rules utilized in such previous case, but also by considering the subjectivities that triggered that decision. This is of utmost importance to keep coherency with regards to the new subjectivities related to the current legal case. But also, keeping track of previous interpretations allows to investigate the use of discretion in the legal decision. This would be an interesting tool for preventing arbitrariness.

Upon a theory capable of handling legal interpretations, we proposed a new legal argumentation framework able to construct arguments and hypotheses. Hypothesization is essential to the preparation of a legal case prior to the trial. To this end, we proposed a formal theory based upon belief revision techniques providing operators to handle the dynamics of legal reasoning. The construction of a new hypothesis has to consider specific restrictions related to the legal interpretation that is governing the support relations between rules, and thus, the current argumentation. Therefore, an hypothesis can only be accepted by the legal argumentation framework according to the conditions that are observed by the *hypothesis revision operator* here proposed, like the impossibility to hypothesize against evidence. This seems to be an obvious restriction, however, its discovery could not be so trivial given that this could appear from an indirect construction of a new hypothesis based upon assumptions made for a different purpose in a different hypothesis.

In this article, we make deep analysis of some legal cases that are cornerstones for the recent Argentinian jurisprudence. This allows us to give a formal context of application, showing the behavior of our proposal to deal with a convoluted reasoning in legal interpretation.

Finally, abductive argumentation is proposed to incorporate the consideration of a new form of hypothesization which could be interesting for semi-automatic recommendation during the investigation of a legal case prior to its trial instance. This is proposed as part of the ongoing work.

Acknowledgments

Dr. Moguillansky thanks to Fulbright and CELFI programs for their research scholarships granted in 2017 and 2018, respectively, and to professors Vern Walker (Hofstra University), Kevin D. Ashley (University of Pittsburgh) and Hernán Bouvier (University of Córdoba), for the many fruitful discussions on previous draft versions of this article.

This work was partially supported by UNS (PGI 24/NO40), Fulbright Scholar Program and CONICET, and by EU H2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 690974 for the project MIREL: *MIning and REasoning with Legal texts*.

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