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Original article

Monitoring of the Garisenda Tower through GNSS using advanced approaches toward the frame of reference stations

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ABSTRACT

The Garisenda tower in Bologna is a symbol of the city and one of the most valuable heritages of the medieval age. The tower is leaning markedly since the XIV century because of a foundation failure and its stability is nowadays under constant monitoring through many sensors. In 2013 a GNSS permanent station was installed on the top of the tower with the aim to test the satellite technology for this particular kind of structural monitoring. Being the leaning of the Garisenda the subject of the investigation and being the sensor placed on its top, one fundamental hypothesis is the stationarity of the ground under the tower with respect to the reference system used for the GNSS measures. This hypothesis has demonstrated to be unreliable considered the high precision of the survey and the Earth crust dynamics, therefore opening interesting issues concerning the reference to be used in such kind of monitoring. The proposed solution rely on a strain model of the area surrounding the Garisenda tower, estimated using data from four other GNSS permanent stations already present in Bologna. The method is described and results are shown in terms of trend over time of the Garisenda's leaning. Nevertheless, the methodology can be generalized for every kind of structural monitoring based on GNSS data for which millimetre level of precision is needed.

1. Introduction

Italian cultural heritages are mostly located in historical city centers and need for a careful monitoring of their stability in order to plan preventive actions both for their maintenance and public safeguard. Asinelli and Garisenda towers are one of the traditional symbols of Bologna (Italy), which is also called the "city of the two towers". These are also famous because of the very impressive leaning of the smaller one, which is the Garisenda. The two towers were built during the medieval age in the hearth of the city and nowadays are still the most known cultural heritage of the City and an attraction for the tourists. Therefore it is mandatory for the municipality[1] to preserve them over time and numerous studies have been developed from both historical and scientific point of view [1–4].

Continuous monitoring of structures is possible thanks to the GNSS technology and it has become a very active focus for both the research community and the commercial sector. Generally, structures movements are very small and therefore require for very high precision in the measures. GNSS technology is nowadays one of the most interesting and growing technology for precise positioning and monitoring purposes. In literature there are several examples demonstrating that this technology can be applied on the monitoring of the structures [5–9] thanks to its precision and the capability to continuously provide data through automated procedures.

The traditional approach toward the use of GNSS for monitoring purposes is based on relative positioning between a reference station (RS) and one, or more, monitoring stations (MS). The RS has to be placed in a stable area, as close as possible to the MS in order to improve the performance of the system. Assuming the RS as stable, the movement of the MS can be evaluated estimating the change of its coordinates in time. While looking for quite big displacements, meaning several centimeters, the assumption of stability of the RS can be definitely true just choosing a proper site. Differently, if the investigated movements are very small, this hypothesis should be carefully considered and further observations have to be taken into account to reach reliable results.

In 2013, the Department of Civil, Environmental and Materials Engineering (DICAM) of Bologna University installed a GNSS permanent station on the top of the Garisenda tower with the aim of testing this technology in a complex and remarkable case study. In

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[1] http://www.comune.bologna.it/duetorri/contenuti/146:10442/.

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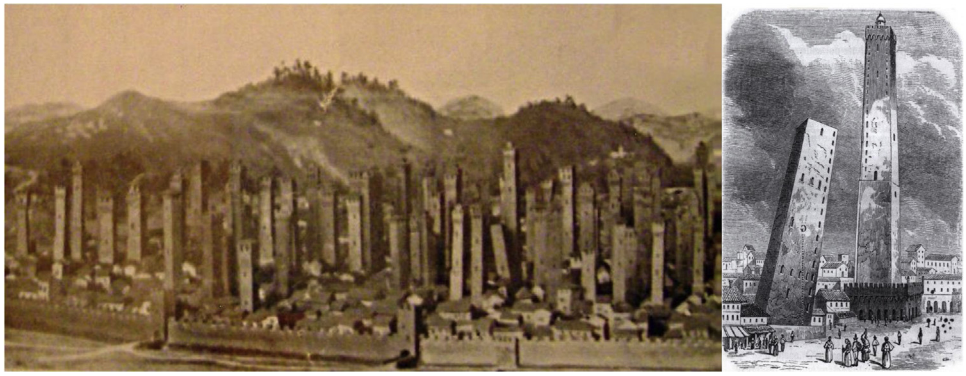

Fig. 1. Portray of the Bologna skyline in XII century and the two towers Asinelli and Garisenda.

this paper, we investigate the stability of the RSs in the monitoring of the Garisenda tower. Fortunately, 4 GNSS permanent stations are located within about 2 km from the tower and these particular boundary conditions allowed a deep investigation about the addressed issue. Four years of data acquired by the GNSS station located on the Garisenda roof and the neighbouring ones were used for the tests. The impact of the choice of a particular RS instead of another one will be discussed together with the impact of using a more sophisticated approach that is here described and applied. All the results will be presented in terms of velocity vector indicating direction and magnitude of the average movement related to the MS.

2. Research aim

The monitoring of cultural heritages is a fundamental practice for their maintenance. In case of monitoring the deformation of a structure, modern satellite techniques can be used. In particular, this paper relates on the GNSS technique applied on the Garisenda tower in Bologna (Italy), which needs for a careful monitoring due to its leaning. Nevertheless, the proposed approach is suitable for every tall structure located in urban areas, which is the case of many historical towers.

The proposed method can be applied whenever a cluster of more than three GNSS permanent station is available in the area around the monitored structure. The aim of the method is to improve the reliability (in term of accuracy) in the determination of the trend of the tower leaning, meaning the mean variation of the leaning over time. Such parameter is mostly related to foundations stability and constitute a fundamental indicator for the prediction of a possible failure of the structure.

3. Case of study: the Garisenda tower in Bologna (Italy)

The Bologna two towers, Asinelli and Garisenda, belong to the about 180 towers actually existing during the XII century (Fig. 1)[2] that were symbol of power for the rich families of the city, besides to be a defensive system. Nowadays only 22 of these towers still exist.

The Garisenda tower takes name from the Garisendi family that ordered its construction in the beginning of the XII century, the same epoch in which also the Asinelli tower were built. The two initially had to reach the same height, but in 1351 the Garisenda started to tilt because of a foundation failure [10]. The tower had reached the height of 61 m at the time, but was reduced of about 13 meters in order to avoid its collapse. For such reason the Garisenda have been also named “the truncated tower”.

Fig. 2. Present picture of the “Two Towers” of Bologna. The Garisenda tower is the lowest one, on the left, while the tallest on the right is the Asinelli.

Leaned to the base of the two towers were built structures for commercial activities. A church named for “Santa Maria delle Grazie” were also built nearby, but in 1804 the ownership of the Garisenda tower passed to the Ranuzzi family and all the buildings at its base were substituted by selenite blocs. Nowadays, after several passages, the two towers are property of the municipality of Bologna that provides for their maintenance.

Today the Garisenda tower (Fig. 2) is 48 meters tall, with a square section having 7.5 m sides and is leaning for about 3.22 m to the East. Since the beginning of the XX century Prof. F. Cavani did several studies concerning the leaning and the stability of the tower [11–13]. In the eighties a technical commission has been established with the aim to study and monitoring the structure. This commission involved also the DICAM, which performed several tests and surveys such as: spring levelling, GPR, photogrammetry, coring and mechanical tests. Many studies and topographical surveys have been performed in order to investigate about the tower stability [14,15]. Starting from these studies several projects have been undertaken to reinforce and maintain the structure.

[2] https://www.vanillamagazine.it/le-100-torri-della-bologna-medioevale-un-fascino-che-supera-la-moderna-new-york/.

Fig. 3. The BOGA GNSS antenna installed on the top of Garisenda tower by the DICAM department since 2013.

Between years 1998 and 2000 works of restoration and consolidation were performed on the Garisenda walls and later, in 2009, a complex monitoring system was installed including:

- four invar strand deformometers on the walls;
- five deformometers straddling already present cracks;
- four extensometers on the metal hoops;
- six biaxial inclinometers;
- three laser distance meters pointing to Prediparte tower, Asinelli tower and the dome of the "S. Maria della Vita" church respectively;
- one gonio-anemometer station;
- one meteorological monitoring station.

Besides, in 2012 the INGV (Italian National Geophysical Institute) was requested to provide a dynamic monitoring and thus installed a couple of continuously acquiring tri-axial seismometers. In the area surrounding the tower there is also a classical topographical network that is surveyed every four months through high accurate spirit levelling technique by the Engineering Faculty of the University of Bologna since around 1960. Finally, in October 2013 the DICAM has installed a GNSS permanent station (Fig. 3) on the roof of the Garisenda, named BOGA, with the aim to test the effectiveness of such technology for structural monitoring purposes. The GNSS station provides raw data at 1 Hz frequency allowing also a dynamic monitoring. For this purpose a sequential filtering method was developed so improving the accuracy of the kinematic solutions [16].

Nevertheless, one of the most important aspects that should be monitored is the trend of the coordinates in time, meaning in this case the averaged movement of the roof of the tower, which is linked to the leaning of the Garisenda itself. The GNSS technology is capable to provide time series of coordinates and it allows the estimation of the linear trend of a monitoring station with few mm accuracy. Therefore, this technique should be suitable for the monitoring of structures such as the Garisenda tower.

4. Data set

In order to obtain a precise determination of the linear trend for a GNSS permanent station at least three years of daily data are necessary [17,18]. For this reason, the four years between October 2013 and October 2017 have been considered in this work using the data acquired by BOGA station during this time span.

One fundamental aspect that should be pointed out is that the BOGA station, being located on the roof of the Garisenda tower, can be used to monitoring the displacements of this particular point, but such displacement can be directly linked to the tilt of the structure only if the eventual displacement of the tower base is monitored too. Unfortunately, it is not possible to place a GNSS permanent station at the bottom of the tower, and in general of a building. Therefore the hypothesis that the ground beneath the Garisenda is fixed has to be carefully verified in order to consider the trend of the BOGA station as the real variation in the tilting of the tower.

In this scenario another hypothesis can be done: the base ground around the Garisenda has the same trend that other permanent stations located around Bologna have. Fortunately, in this particular case four other GNSS permanent stations are actually active in the area, that are BOL1, BO01, BOLG and BLGN as shown in Fig. 4. All these stations are quite close to BOGA, meaning about 2 km or less. Therefore, also data acquired by these four stations in the considered time span were included in the dataset.

The whole dataset is constituted by daily RINEX (Receiver Independent Exchange Format) files [19], each containing observations acquired with a sampling rate of 30 seconds.

5. Data processing and preliminary results

The most used approach for GNSS data processing is the one defined "differenced approach" [20], that provide accurate estimations of the three-dimensional vectors, called baselines, linking two or more GNSS antennas. Therefore, the differenced approach does not provide the coordinate of a monitoring station but only its position relatively to one, or more, reference station of which coordinates are supposed to be known. Nevertheless, in the last decade a completely different approach toward the GNSS observables has become widely used for long term monitoring based on daily data, which is the Precise Point Positioning (PPP) [21]. In this approach the coordinates of the stations are obtained directly from the data processing and are inherently referred to the same reference system used to define the GNSS ephemerids. PPP has proved to provide coordinates having the same accuracies, or even better, of the ones given by the differenced approach. This is true in particular considering long time series based on daily 24 hours data acquired by permanent stations [22,23].

For this reasons, in this work the PPP approach was used for the estimation of the coordinates of the GNSS stations, doing so by means of the GIPSY-OASIS II software package [24], version 6.4. Standard data processing parameters were used and coordinates were aligned to the ITRS reference system [25] according to [26]. Being this reference system a global one, it evidences the tectonic plate motions, therefore the coordinates of each station, although stable, are moving with respect to the ITRS. In particular, the Bologna area is moving toward North-East with about 2 cm/year velocity with respect to the global reference system [27].

For this reason a change in the reference system has been performed and the coordinates were transformed into the ETRS89 (European Terrestrial Reference System) [28], which is linked to the stable area of the Eurasian plate. In particular, the 14 Helmert parameters given by Boucher and Altamimi in [29] were applied

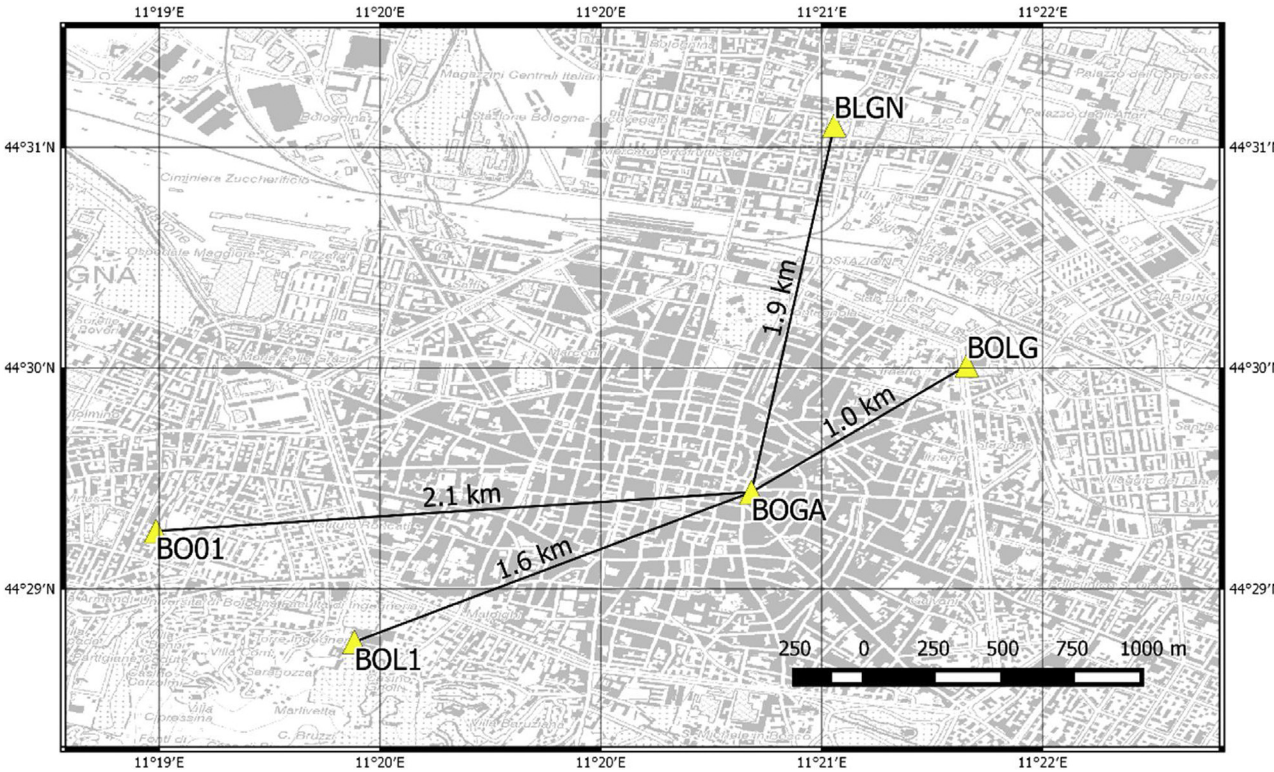

Fig. 4. Map showing the positions of the Garisenda tower monitoring station (BOGA) and the four reference stations (BOL1, BOLG, BLGN, BO01) together with the relative distances.

Table 1
Mean velocities related to ETRF2000 reference frame for the five GNSS permanent stations considered in the test. Values are expressed in mm/y.

Site	V_{e_i}	V_{n_i}	V_{u_i}
BLGN	0.1	4.5	−0.8
BO01	−0.0	4.0	−3.4
BOL1	−0.2	3.5	0.4
BOLG	0.4	3.1	0.8
BOGA	2.3	4.4	0.9

so that the new coordinates are aligned to the ETRF2000 reference frame.

Fig. 5 shows the time series of the topocentric coordinates, which are northing (n), easting (e) and the height, otherwise called up (u) component, calculated for the BOGA MS and expressed in the ETRF2000. Besides a periodical recursive effect having one-year periodicity, a trend of the positions toward north east is quite evident. It should be pointed out that such trends would be considered representative of the real displacement of the Garisenda top without further considerations on the reference system.

In Fig. 6 are shown the time series of the four RS surrounding the Garisenda tower, expressed using ad hoc local topocentric reference system for each station for representation purposes. Although these stations are located few kilometres far from each other, these show quite different periodical signals on one hand, but also slightly different trends on the other hand.

and up components are reported from top to bottom. Coordinates are aligned to the ETRF2000 reference frame.

The regression straight lines of the time series expressed in the ETRF2000 were calculated. Their slopes can be considered the averaged linear velocities of the monitored points or, in other words, the trends of the coordinates. These are reported in Table 1 and confirm that the BOGA station has a higher velocity with respect to the others, but also that the other stations are moving with respect to the reference system too.

One could estimate the difference between the trends of BOGA station and another one among the others nearby with the aim to remove the motion of the area with respect to the ETRF2000. In this case a new residual velocity can be estimated for the BOGA station for each of the other stations used as reference. Fig. 7 shows what can be found in the plan by subtracting the trends of BOL1, BOLG, BLGN or BO01 respectively to the trend of BOGA.

All the resulting velocity vectors for BOGA station point substantially toward the East-northeast directions. Nevertheless, considered the high precision of the estimated values, these are significantly different depending on the choice of the station assumed as fixed. Similar considerations can be done for what concerns the height component just looking at Table 1 and considering the marked differences between the vertical velocities of the stations surrounding BOGA. It is worth noting that results shown in Fig. 7 are the same that would be found performing the GNSS data processing using the differenced approach, rather than PPP, and calculating the baselines between BOGA and each of the other station.

These considerations mean that the whole area around the two towers should not be considered as completely fixed with respect to the ETRF2000 nor that one of the neighbour stations could be used as a stable reference. For these reasons, the estimation of the tilting of the tower may be inaccurate if only the displacement of its top is considered without taking into account how the base ground coordinates are actually changing in time.

The coordinates that form the five sets of time series shown in Fig. 5 and Fig. 6 constitute the dataset starting from which the proposed method will be applied. Together with the coordinates, also the formal covariance matrix given by the GIPSY software package for each set of coordinate will be taken into account.

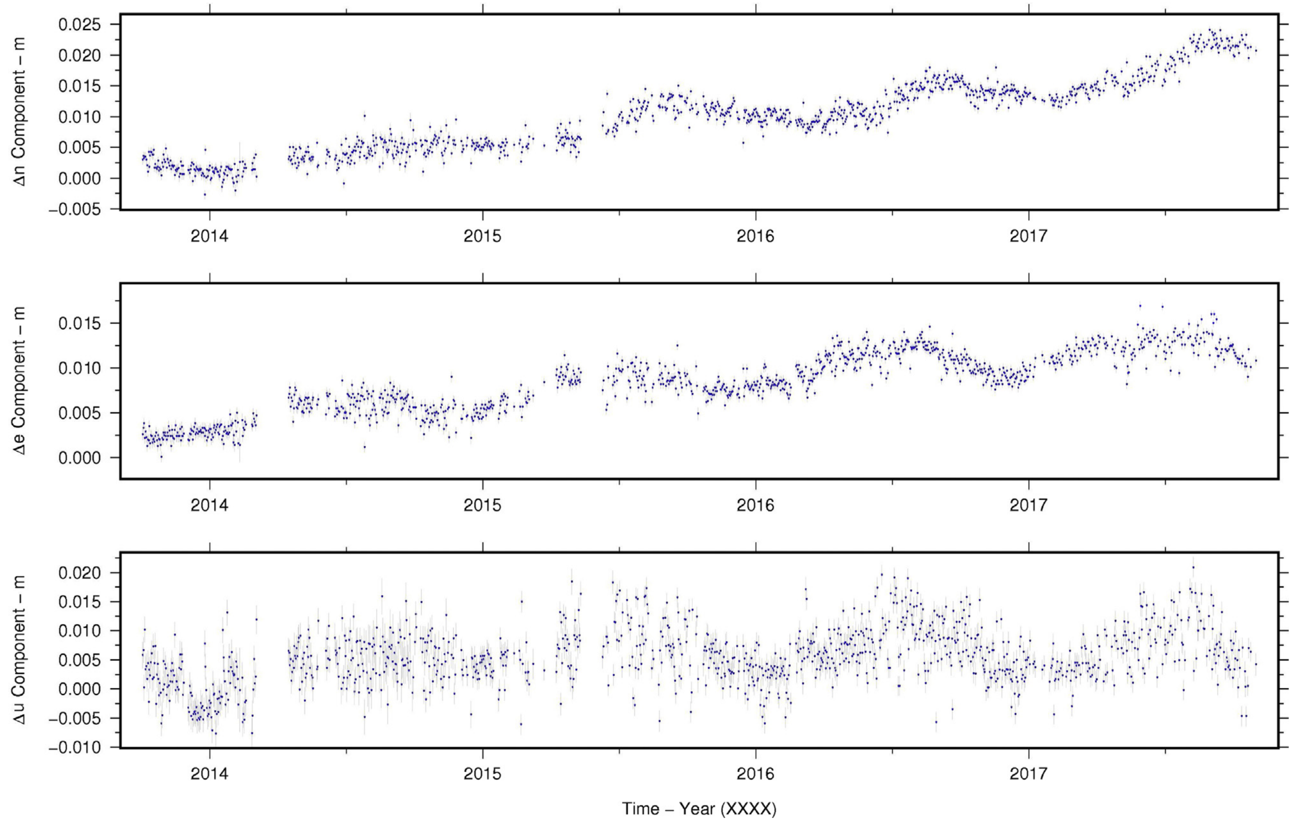

Fig. 5. Time series of the topocentric coordinates for the BOGA station. Northing, easting.

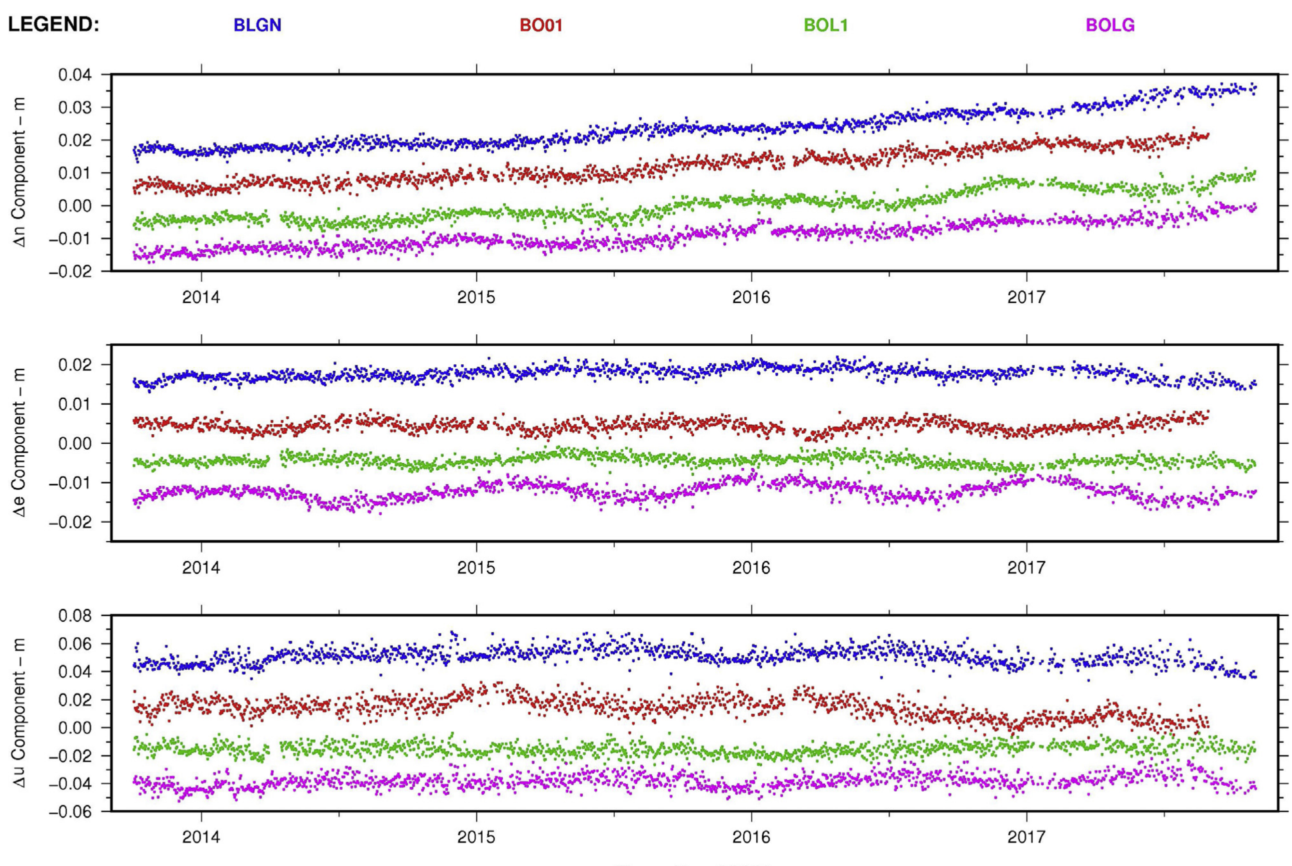

Fig. 6. Time series of the topocentric coordinates for the reference stations of BLGN, BO01, BOL1 and BOLG. The northing component is represented on the top while easting and up components are in the middle and at the bottom respectively. Coordinates are aligned to the ETRF2000 reference frame.

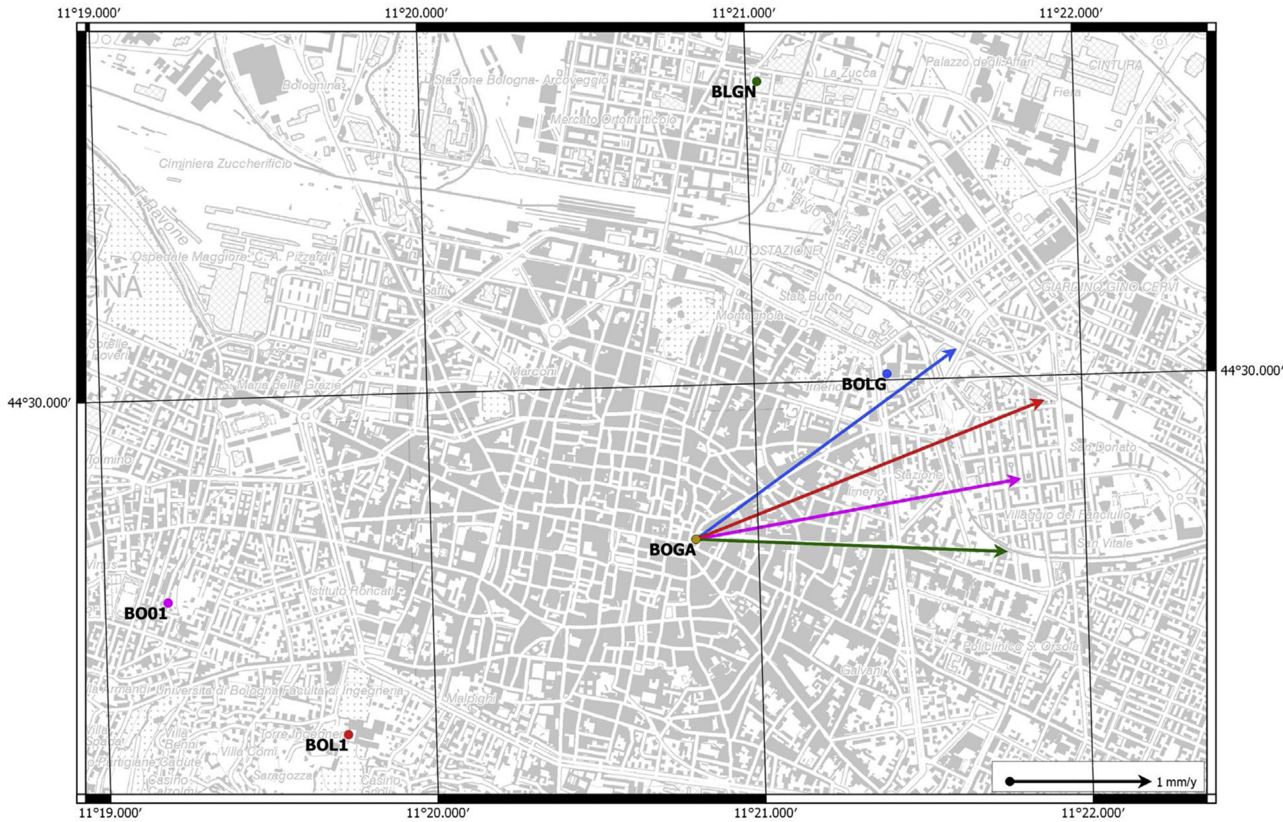

Fig. 7. Vectors representing the displacement velocities for the BOGA station with respect to the four surrounding reference stations. Colors are linking each vector to the related reference station assumed as stable.

6. Method

The ETRF2000 has proven not to be the best choice in order to estimate the real trend of the top of the Garisenda tower with respect to the base ground. Therefore, the problem addressed in this section concerns the estimation of a trend that can be attributed to the bottom of the tower in order to estimate the relative displacements at its top using GNSS. This can be done through the estimation of a deformation model representing the displacements in the area surrounding the point of interest, i.e. the Garisenda. The model can be defined using data given by the GNSS stations located in the same area and estimating their velocity vectors, together with the related full covariance matrix. Therefore, the main steps hereafter described are: the estimation of the trends for each GNSS station together with their full covariance matrix, the estimation of a deformation model suitable in the considered area and, finally, the application of such model in the position of the Garisenda.

6.1. Estimation of the three-dimensional linear trend

For each of the considered GNSS stations we have a three-dimensional time series of coordinates expressed in a topocentric reference system which axes are orthogonal e, n, u. Each point of the time series has coordinate vector $\boldsymbol{x}_i = [e_i, n_i, u_i]^T$ at time t_i, where $i = 1, \ldots, n$ being n the number of daily solutions. We also consider that the estimated accuracy of such coordinates is expressed through the covariance matrix Σ_{xx_i} given by the GIPSY software.

These coordinates may have a linear trend, which is what we want to estimate. Although a line in the space is defined through the intersection of two planes, we can also represent it by means of the decomposition along the three components of the topocentric coordinate system:

$$\begin{cases} e_i = v_e(t_i - t_1) + b_e \\ n_i = v_n(t_i - t_1) + b_n \\ u_i = v_u(t_i - t_1) + b_u \end{cases} \tag{1}$$

where v_e, v_n, v_u are the slopes and b_e, b_n, b_U are the intercepts. These parameters can be estimated at once through a weighted Gauss-Markov model that consider the measures $\boldsymbol{x}_i$ together with their full covariance matrix Σ_{xx_i}. The observables and the unknowns can be included in vectors like $\boldsymbol{f} = [\ldots, e_i, n_i, u_i, \ldots]^T$ and $\boldsymbol{m} = [v_e, v_n, v_u, b_e, b_n, b_u]^T$ respectively, whereas the vector $\boldsymbol{r} = [\ldots, r_{e_i}, r_{n_i}, r_{u_i}, \ldots]^T$ contains the residuals of each coordinate with respect to the regression line. The design matrix of the model, which has 6 columns and $3n$ rows, will be:

$$A = \begin{bmatrix} t_1 & 0 & 0 & 1 & 0 & 0 \\ 0 & t_1 & 0 & 0 & 1 & 0 \\ 0 & 0 & t_1 & 0 & 0 & 1 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ t_n & 0 & 0 & 1 & 0 & 0 \\ 0 & t_n & 0 & 0 & 1 & 0 \\ 0 & 0 & t_n & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The Gauss-Markov model can be written as:

$$A\boldsymbol{m} = \boldsymbol{f} + \boldsymbol{r} \tag{3}$$

Without considering the correlation over time of the measures, we can also define the weight matrix as:

$$W \propto \left[\mathrm{d}iag\left(\Sigma_{xx_i}, \ldots, \Sigma_{xx_n}\right)\right]^{-1} \tag{4}$$

The unknown can be estimated trough a classical least square approach by calculating:

$$\boldsymbol{m} = (A^T W A)^{-1} A^T W \boldsymbol{f} \tag{5}$$

and the residual $\boldsymbol{r} = A\,\boldsymbol{m} - \boldsymbol{f}$ allow to evaluate the variance covariance matrix associated to the solution.

$$\Sigma_m = \frac{\boldsymbol{r}^T W \boldsymbol{r}}{3(n-2)} (A^T W A)^{-1} \tag{6}$$

The first three components of the vector $\boldsymbol{m}$ represent the slopes and the first block (3×3) of Σ_m contains the associated variance covariance matrix. These are the input parameters in the definition of the strain models presented in the next sections.

Although GNSS provide three-dimensional positions and the model could be defined for the three components at once, it is worth keep separating the two aspects because this way the models are easier to be defined. Moreover vertical and plan displacements are usually representing different phenomena and/or effects.

6.2. *Definition of a plane strain model for velocity field estimation using GNSS reference stations*

In this section a method for the estimation of the linear trend of a generic point within a specific area will be described. The starting point are the mean velocities of a certain number of sites around the area estimated applying what is described in the previous section. We consider for each GNSS stations (i) the position vector of the plan components $\boldsymbol{p}_i^0 = [e_i, n_i]^T$ expressed in a topocentric reference system and referred to a temporal origin (t_0). We also define a spatial origin $\boldsymbol{p}_C^0$ in the centroid of the GNSS stations at the initial epoch. The relative position of each station with respect to the centroid is:

$$\boldsymbol{\Delta p}_i^0 = \left[\Delta e_i^0, \Delta n_i^0\right]^T = \boldsymbol{p}_i^0 - \boldsymbol{p}_C^0 \tag{7}$$

The position of the i-*th* station at a generic time (t_j) can be calculated using the vector of the mean velocities $\boldsymbol{v}_i = \left[v_{e_i}, v_{n_i}\right]^T$:

$$\boldsymbol{\Delta p_i^j} = \boldsymbol{\Delta p}_i^0 + (t_j - t_0)(\boldsymbol{v}_i - \boldsymbol{v}_c) \tag{8}$$

We use a uniform strain model [30] to parameterize the velocity within the considered area. The L matrix containing the velocity gradients has form:

$$L = \begin{bmatrix} \dfrac{\partial v_e}{\partial e} & \dfrac{\partial v_e}{\partial n} \\ \dfrac{\partial v_n}{\partial e} & \dfrac{\partial v_n}{\partial n} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \tag{9}$$

Therefore, the velocity field in the area can be described by:

$$\boldsymbol{v}_i = \boldsymbol{v}_c + L\left(\boldsymbol{p}_i^0 - \boldsymbol{p}_C^0\right) = \boldsymbol{v}_c + L\boldsymbol{\Delta p}_i^0 \tag{10}$$

Since the mean velocities of a number of stations $i \geq 3$ are known, it's possible to estimate the six remaining unknowns characterizing the strain model $\boldsymbol{s} = [v_{e_c}, v_{n_c}, L_{11}, L_{12}, L_{21}, L_{22}]^T$. Also in this case the a weighted Gauss-Markov model can be defined, in which the vector of the input data $\boldsymbol{j} = \left[v_{e_1}, v_{n_1}, \ldots, v_{e_k}, v_{e_k}\right]^T$ contains the mean velocities of the k GNSS stations. The design matrix of the system B takes form:

$$B = \begin{bmatrix} 1 & 0 & \Delta e_1^0 & \Delta n_1^0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta e_1^0 & \Delta n_1^0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & 0 & \Delta e_k^0 & \Delta n_k^0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta e_k^0 & \Delta n_k^0 \end{bmatrix} \tag{11}$$

Considering that the vector $\boldsymbol{j}$ contains the measured quantities, therefore affected by errors, a vector of the residuals $\bar{\boldsymbol{r}}$ must be introduced too and the model becomes:

$$B\,\boldsymbol{s} = \boldsymbol{j} + \bar{\boldsymbol{r}} \tag{12}$$

The velocities of the GNSS stations can be calculated as shown in the previous section. For each station the full covariance matrix of the plan topocentric components is also available and is:

$$\Sigma_i = \begin{bmatrix} \sigma_{v_{e_i}}^2 & \sigma_{v_{e_i} v_{n_i}} \\ \sigma_{v_{n_i} v_{e_i}} & \sigma_{v_{e_i}}^2 \end{bmatrix} \tag{13}$$

the whole covariance matrix of the known terms $(\Sigma = \mathrm{d}iag(\Sigma_1, \ldots, \Sigma_k))$ is used to define the weight matrix $(W = \sigma_0^2 \Sigma^{-1})$, where σ_0^2 is the a priori variance per unit weight. According to the Least Squares criterion the problem can be solved as:

$$\boldsymbol{s} = \left(B^T\,W\,B\right)^{-1} B^T\,W\boldsymbol{j} \tag{14}$$

considering residuals $(\bar{\boldsymbol{r}} = B\,\boldsymbol{s} - \boldsymbol{j})$ the estimated a posteriori covariance matrix of the unknowns can be estimated as:

$$\Sigma_{ss} = \frac{\bar{\boldsymbol{r}}^T\,W\bar{\boldsymbol{r}}}{2k-6} \left(B^T\,W\,B\right)^{-1} \tag{15}$$

finally, the covariance matrix of the residuals is:

$$\Sigma_{\bar{r}\bar{r}} = \frac{\bar{\boldsymbol{r}}^T\,W\bar{\boldsymbol{r}}}{2k-6} [W^{-1} - B\left(B^T\,W\,B\right)^{-1} B^T] \tag{16}$$

Basing on the $\Sigma_{\bar{r}\bar{r}}$ matrix is possible to perform statistical tests that allow point out any GNSS station having an inconsistent behaviour, in terms of plan velocities, with respect to the others. Such incoherence could affect the effectiveness of the estimated model, thus the eventual incoherent station should be removed from the dataset and the model recalculated. The matrix $\Sigma_{\bar{r}\bar{r}}$ also permit to represent the error ellipses related to the residual velocities $(\bar{\boldsymbol{r}})$ of the GNSS stations with respect to the estimated model.

Once the model has been defined, the aim is to apply it on a generic site located amid the stations that contributed to estimate the model itself, so defining a new velocity vector which simulates the behaviour of the ground in the chosen point. We can consider G as the generic site having known position at the initial epoch $\boldsymbol{p}_G^0 = [e_G, n_G]^T$. The deformation model can be applied through:

$$\boldsymbol{v}_G = \boldsymbol{v}_c + L\,\boldsymbol{\Delta p}_G^0 \tag{17}$$

with $\boldsymbol{\Delta p}_G^0 = \left[\Delta e_G^0, \Delta n_G\right]^T = \boldsymbol{p}_G^0 - \boldsymbol{p}_C^0$ representing the relative position between the centroid and G. The system can be explicated in the as:

$$\begin{bmatrix} v_{e_G} \\ v_{n_G} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta e_G^0 & \Delta n_G^0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta e_G^0 & \Delta n_G^0 \end{bmatrix} \begin{bmatrix} v_{e_C} \\ v_{n_C} \\ L_{11} \\ L_{12} \\ L_{21} \\ L_{22} \end{bmatrix} \equiv B_G\,\boldsymbol{s} \tag{18}$$

The uncertainty associated to the estimate of the velocity vector in G can be estimated as:

$$\Sigma_{vv_G} = B_G \Sigma_{ss} B_G{}^T \tag{19}$$

The covariance matrix (Σ_{vv_G}) allows the definition of the error ellipse related to the velocity vector estimated in G.

6.3. Implementation of the model for vertical displacements

This section describes how the presented method can be modified in order to obtain a deformation model that takes into account the vertical movements in the considered area. Without further explanations, it is possible to modify previous equations as we describe below.

The vector $\boldsymbol{v}_i$ becomes a scalar v_{u_i} meaning the vertical component of the velocity vector related to the i-th permanent station. Then eq. 10 becomes:

$$v_i = v_c + \boldsymbol{l}\,\left(\boldsymbol{p}_i^0 - \boldsymbol{p}_C^0\right) = v_c + \boldsymbol{l}\boldsymbol{\Delta p}_i^0 \tag{20}$$

Where the matrix L is now a vector $\boldsymbol{l}$ which is:

$$\boldsymbol{l} = \left[\frac{\partial v_u}{\partial e}\;\frac{\partial v_u}{\partial n}\right] = [l_{11}\, l_{12}] \tag{21}$$

The vector of the unknowns in this case takes form:

$$\boldsymbol{s} = [v_{u_C}, l_{11}, l_{12}]^T \tag{22}$$

The vector of the measured quantities become:

$$\boldsymbol{j} = \left[v_{u_1}, \ldots, v_{u_k}\right]^T \tag{23}$$

The design matrix of the system B takes form:

$$B = \begin{bmatrix} 1\ \Delta e_1^0\ \Delta n_1^0 \\ .\quad .\quad . \\ 1\ \Delta e_k^0\ \Delta n_k^0 \end{bmatrix} \tag{24}$$

The covariance matrix (13) changes into a scalar ($\Sigma_i \rightarrow \sigma_{v_{ui}}^2$) and the weight matrix becomes diagonal. In this case eq. (15) changes into:

$$\Sigma_{ss} = \frac{\boldsymbol{r}^T W \bar{\boldsymbol{r}}}{k-3}\left(B^T W B\right)^{-1} \tag{25}$$

Finally, the vertical deformation model can be applied considering that (17) changes into:

$$v_G = v_c + \boldsymbol{l}\,\boldsymbol{\Delta p}_G^0 \tag{26}$$

that in another form is:

$$v_G = \boldsymbol{b}_G\,\boldsymbol{s} \tag{27}$$

where:

$$\boldsymbol{b}_G = \left[1\ \Delta e_G^0\ \Delta n_G^0\right] \tag{28}$$

Table 2
Plan velocities of the five GNSS stations estimated trough eq. 5 and the related uncertainties.

Site	v_e (mm/y)	v_n (mm/y)	σ_{v_e} (mm/y)	σ_{v_n} (mm/y)	$\sigma_{v_e v_n}$ (mm^2/y^2)
BLGN	0.08	4.52	0.04	0.05	0.00001
BO01	−0.03	4.01	0.04	0.05	−0.00008
BOL1	−0.21	3.46	0.04	0.05	0.00003
BOLG	0.40	3.07	0.04	0.05	0.00009
BOGA	2.27	4.37	0.04	0.06	0.00014

Table 3
Values calculated for the model parameters applying eq. 14 and the related uncertainties estimated through eq. 15.

Parameter	Value	Dev. St.
v_{e_c} (mm/y)	0.04	0.12
v_{n_c} (mm/y)	3.79	0.15
L_{11}	1.62 E-07	1.6 E-07
L_{12}	−1.61 E-08	1.8 E-07
L_{21}	−5.72 E-07	2.0 E-07
L_{22}	8.10 E-07	2.2 E-07

Table 4
Residual velocities of each GNSS station with respect to the estimated model of displacements (columns 2 and 3) and the associated uncertainties (columns 4-5). The uncertainties related to the first four stations are given by eq. 16 whereas the uncertainty for BOGA is calculated combining values given by eq. 19 and Table 2.

Site	Δv_e (mm/y)	Δv_n (mm/y)	$\sigma_{\Delta v_e}$ (mm/y)	$\sigma_{\Delta v_n}$ (mm/y)
BLGN	−0.09	0.08	0.08	0.09
BO01	0.17	−0.17	0.14	0.20
BOL1	−0.14	0.13	0.12	0.16
BOLG	0.14	−0.11	0.12	0.14
BOGA	2.12	1.25	0.20	0.25

and the uncertainty associated to the vertical velocity in G can be estimated, as for (19), by:

$$\sigma_{v_G}^2 = \boldsymbol{b}_G \Sigma_{ss} \boldsymbol{b}_G{}^T \tag{29}$$

7. Results

A deformation model of the area around the Garisenda Tower was calculated through the procedure described in the section 6 starting from the time series of the BOL1, BOLG, BLGN and BO01 stations. Table 2 reports the linear velocities of the plan components for the five stations, including BOGA, considered in the test. Velocities are expressed with respect to the ETRF2000 and were estimated applying eq. 5 to the time series given by each station.

The vectors representing the linear trends with respect to ETRF2000 and the related standard error ellipses are shown in Fig. 8. This shows that all the stations have significant velocities, which are comparable in magnitude but have different directions despite the closeness between the sites. The fact that the Italian territory is affected by deformations with respect to the Eurasian plate, and thus to ETRS89, has already been shown in [31,32]. Nevertheless, the inhomogeneity of the ETRF2000 velocities of the four stations surrounding the Garisenda imposes an ad hoc modelling of the area. The model parameters $\boldsymbol{s}$ obtained by applying eq. 14 are reported in Table 3 together with the associated uncertainties. The fact that the latters have quite high magnitudes if compared to the ones of the parameters ought to be due to the poor redundancy of the system and the hypothesis of uniform deformations implicit in the model, which is too restrictive to comply perfectly with the input data.

The model parameters were then applied for estimating the velocities in the positions corresponding to the five considered GNSS stations. These velocities were subtracted to those in Table 2 and these differences are reported in Table 4. Note that the four lines referred to the stations surrounding Garisenda tower represent

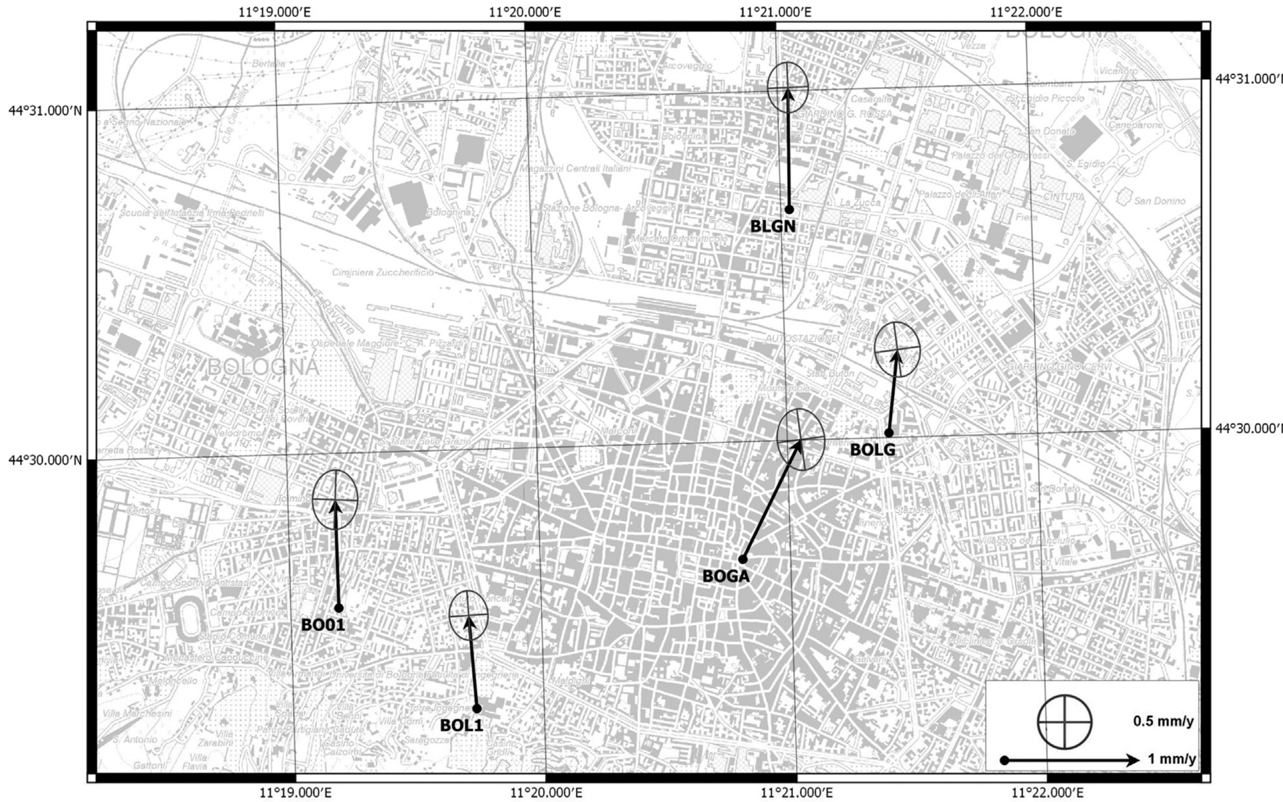

Fig. 8. Map representing the velocity vectors of the GNSS stations with respect to ETRF2000 reference frame, together with the related standard error ellipses.

Table 5
Vertical velocities of the five GNSS stations estimated trough eq. 5 and the related uncertainties.

Site	$v_u\,(mm/y)$	$\sigma_{v_u}\,(mm/y)$
BLGN	−0.81	0.14
BO01	−3.41	0.18
BOL1	0.37	0.14
BOLG	0.77	0.16
BOGA	0.85	0.17

Table 6
Values of the parameters estimated by applying eq. 14 for the model describing vertical displacements. The third column reports the related uncertainties estimated through eq. 25.

Parameter	Value	Dev. St.
$v_{u_c}\,(mm/y)$	−0.64	0.69
l_{11}	1.72 E-06	9.21 E-07
l_{12}	−1.48 E-06	1.02 E-06

Table 7
Residual velocities of each GNSS station with respect to the estimated model of displacements (columns 2) and the associated uncertainties (columns 3). The uncertainties related to the first four stations are given by eq. 16 whereas the uncertainty for BOGA is calculated combining values given by eq. 29 and Table 5.

Site	$\Delta v_u\,(mm/y)$	$\sigma_{\Delta v_u}\,(mm/y)$
BLGN	−0.41	0.01
BO01	1.02	0.03
BOL1	−0.72	0.02
BOLG	0.63	0.02
BOGA	−0.15	1.12

nothing but the residuals used in eq. 15. Differently, the line which refers to BOGA includes the discards between the linear velocity on the top of the tower (BOGA antenna) and the trend at its bottom, which is represented by the model velocity in such position. This difference can actually be considered the most reliable estimation of the trend of the leaning of Garisenda tower obtainable starting from GNSS data, at least under the assumption of uniform strain model.

All the residual velocities with respect to the deformation model are also represented in Fig. 9 through vectors, with the related uncertainties expressed in terms of standard error ellipses. It is interesting to compare the vector on BOGA station shown in Fig. 9 with the ones reported in Fig. 7. The leaning trend of the tower estimated through the model is mostly similar to the one estimated assuming BOLG as stable reference, with a slightly higher magnitude of the vector in the former case. The higher impact of BOLG with respect to the other stations in the definition of the model velocity for the Garisenda position can be explained looking at the distances reported in Fig. 4. In fact, BOLG is the GNSS station closest to BOGA while the others are quite farther, especially BLGN and BO01, and thus these have less impact on that specific position.

As for the vertical displacements, the model presented in section 6.3 was calculated too, in this case basing on values reported in Table 5 for the first four stations. The table also reports the vertical velocity of BOGA station estimated through eq. 5 and the related uncertainty.

The parameters (22) estimated through eq. 14 are reported in Table 6. Note that the input velocities of the four stations surrounding BOGA are quite different with respect to each other, in particular for BO01, and the system redundancy is minimum in this case. For these reasons the relatively high uncertainties in Table 6 are not surprising.

The estimated model was applied through eq. 26 in the position of the Garisenda tower. The discard between the BOGA velocity reported in Table 5 and the one given by the model in the corresponding position was calculated. This discard is reported in Table 7

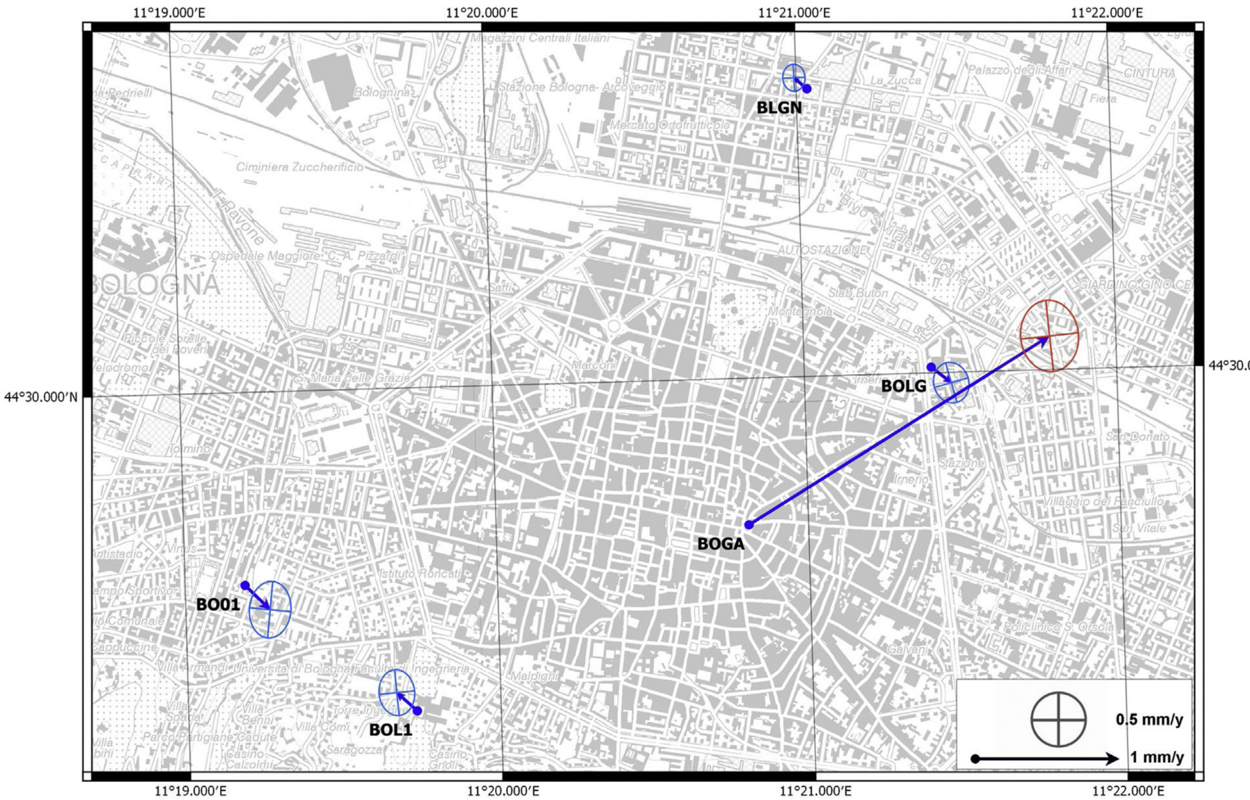

Fig. 9. Map of residual velocities of the GNSS stations with respect to the estimated deformation model together with the related standard error ellipses. The blue ellipses are calculated through eq. 16 whereas the red ellipse is calculated combining values given by eq. 19 and 6.

together with the residuals relating to the other four stations and the uncertainties estimated through eq. 16 and eq. 29.

The vertical velocity on the top of the Garisenda with respect to its bottom is negative, that is compliant with an increasing leaning of the tower. Nevertheless, this result has not to be considered highly reliable because of the high uncertainty shown in Table 7, which can be due to several facts: vertical displacements caused by the leaning are lower than horizontal ones on the top of the tower and the GNSS time series are inherently less precise on the vertical component. Moreover, the redundancy in this application is only one and the model cannot work at its best. Nevertheless, the model can be applied also for vertical movements in applications where this component is of major interest and suitable data are available.

8. Conclusions

The focus of this work is the monitoring of the Garisenda tower, one of the most important cultural heritages in Bologna city. The monitoring has been performed basing on GNSS data only, that were processed through the Precise Point Positioning approach.

The main issue that was addressed is the definition of a reference position of the base ground under the tower to which the coordinates measured on the top of the tower has to be referred. In other words, the problem is that the velocity vector that can be estimated by processing data from BOGA GNSS station, which is the one placed on the top of the Garisenda, includes both the effect of the leaning of the tower and the displacements of the base ground at its bottom.

It has been demonstrated that the arbitrary choice of a single reference stations, even if few km close to the tower, may lead to significant differences in the estimation of the trend of the tower leaning. Being available four other permanent stations close to the Garisenda, it was possible to estimate a model of movement representing the deformations of the area surrounding the tower. This allowed to define a reference velocity referred to the ground in the location of the Garisenda. Such one has been subtracted to the velocity directly measured for BOGA station, therefore acting as reference for the monitoring of the changes in the leaning of the tower over time.

The presented method is particularly effective for the specific applications on the Garisenda tower in Bologna. Nevertheless, it worth noting that this method can be used in every monitoring application, even not based on GNSS data, in which the stability and/or coherence of the reference system is not guaranteed.

Note that the same methodology can be also applied to GNSS data calculated through the differenced approach, instead of using the PPP, by estimating the baselines between the RS. Once fixed the position of one of these is possible to estimate the deformation model by using the relative velocities of the other stations. It can be applied on the BOGA time series of the baselines calculated between the MS and the fixed RS. In summary, whenever the reference points have relative movements of a magnitude that is not negligible with respect to the monitored phenomenon, then become important to apply methodologies such as the one here discussed.

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