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A 3D optimization algorithm for sustainable cutting of slabs from ornamental stone blocks

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(Article begins on next page)

1 **Abstract**

2 Ornamental stones are natural building materials, extracted from quarries, which need to be cut
3 and processed sustainably. Natural discontinuities adversely affect the sawing/cutting of blocks
4 into commercial-size slabs. This work presents a 3D optimization algorithm for the sawing/cutting
5 of ornamental stone blocks. The developed algorithm is based on 3D modeling of discontinuities
6 as data input. The algorithm search for the intersection between a 3D cutting grid formed of a
7 determined size of slabs and the model of discontinuities leading to calculate the recovery ratio
8 considering several cutting orientations and displacements of the 3D cutting grid. The algorithm
9 was coded in a program named SlabCutOpt that allows speed problem solving. SlabCutOpt was
10 implemented on a real case study of a commercial-size limestone block extracted from a quarry
11 in Italy. A number of 37 different commercial-sizes of slabs forming 37 cutting grids were tested
12 to investigate the optimum results in geo-environmental direction (recovery ratio) and economic
13 directions (revenue). The findings revealed that a certain slab size gave the optimum recovery
14 ratio, whilst another slab size provided the optimum revenue.

15 **Keywords:**

16 Sustainable quarrying; 3D optimization algorithm; Dimension stone; Revenue optimization;
17 Waste minimization

18 **1. Introduction**

19 Ornamental stones are natural non-renewable resources that have to be exploited and processed
20 in an optimized way to minimize waste production and maximize revenue. It is just a third of the
21 extracted stone raw material arrives to the global market as a finished product, whilst the two-

22 thirds remaining are waste, considering the worldwide average in the ornamental stones
23 extraction [1]. Ornamental stones are considered as main economic resources for many countries
24 overall the world [2]. Therefore, and since ornamental stones are natural non-renewable
25 resources, they have to be extracted and processed sustainably for environmental and economic
26 reasons.

27 The right exploitation of ornamental stone is crucial for any competitive economic growth
28 roadmaps of countries aiming the sustainable development [3]. Environmental and economic
29 sustainability strategies for ornamental stone industry is being studied in literature [4–7]. There
30 are recent trends for stone waste recycling [8–11], but minimization of waste production during
31 exploitation and processing is with more preferable impact environmentally and economically.
32 Ornamental stones blocks are extracted from quarries. Blocks are cut to obtain slabs and tiles.
33 The size of tiles and slabs depends on building and construction final applications. Slabs and tiles
34 can be subjected to several types of surface treatment. From a commercial point of view, the
35 main factor that define the commercial price is the dimension of slabs. In General, the larger the
36 size of slabs, the selling price is higher.

37 Blocks are cut/sawed in processing plants using diamond disc saws, diamond blade saws, or gang-
38 saws. Currently, the most used is multi-blades gang-saws that can cut a block to slabs with certain
39 and specific thickness. Several previous work studied the cutting process of ornamental stones
40 considering the sawability [12], the cutting energy consumption [13], and the sawing
41 performance [14]. In this paper, we consider a further aspect that is needed to fulfil the
42 sustainable development of the cutting/sawing process of ornamental stone blocks.

43 Discontinuities, fractures and bedding planes define the exploitability of commercial-size
44 ornamental stone blocks from quarries [15]. Discontinuities not detected on the outcrops
45 adversely affect the quality of ornamental stone blocks. The recovery ratio of the stone deposit
46 depends strongly on these parameters. The possibility to detect structural weaknesses regions
47 of ornamental stone blocks before cutting it is an environmental and economic important phase.
48 The most common methods used in practice are the traditional observation based methods,
49 which can lead to erroneous or non-reliable results. Nowadays, there is an unsatisfactory lack in
50 the use of non-destructive methods for fracture detection in ornamental stone blocks.

51 A survey on fracture detection methods is provided in [16]. Among the used methods, the Ground
52 Penetrating Radar (GPR), appeared to be a valid non-destructive method for discontinuities
53 detection in ornamental stone quarries of different rock types [17–28].

54 With reference to the use of GPR in order to detect and model discontinuities and assessing of
55 rock blocks, it worth to mention the works in [24,29–33]. However, in [32], it is possible to analyze
56 the application of a 3D deterministic discontinuities model, to ornamental stone blocks,
57 significant for the topic of this paper.

58 Previous works presented stochastic or geometric algorithms, aiming to analyze the fracture
59 geometry and quantify the volume of the so called natural blocks, that are defined by the natural
60 discontinuities planes [34–36]. The natural rock blocks geometry identification can be used for a
61 preliminary reserves estimation in quarries, as in the case of the calculation of the maximum
62 largest cuboid [37] or of the marketable block size [38] that fit into the natural rock blocks. The
63 previously listed methods were based on the manual survey method of out-cropping

64 discontinuities. The target of this paper can be achievable when hidden discontinuities can be
65 detected and modeled as well. Identification of natural blocks and the maximum largest cuboid
66 algorithms may not work well on the ornamental stone blocks scale, considering that on that
67 scale planes of discontinuities may or mayn't be intersected at this small scale.

68 A number of works have considered the production optimization at quarry or bench scale [39–
69 42], based on the mapping of fractures through the manual survey method. However, this paper
70 considers the production optimization at a block scale through a new methodological approach,
71 on the basis of a 3D deterministic modeling approach-based GPR that models fractures as 3D
72 surfaces, not planes as typically modeled in literature. The method presented in this paper does
73 not have a limitation to the number of fracture families and their restricted input parameters
74 (dip angle and dip direction, spacing). It is worth mentioning the work of [43] that provided a
75 geometrical design computation of optimum cutting shapes from polygonal boundary
76 ornamental stone slabs.

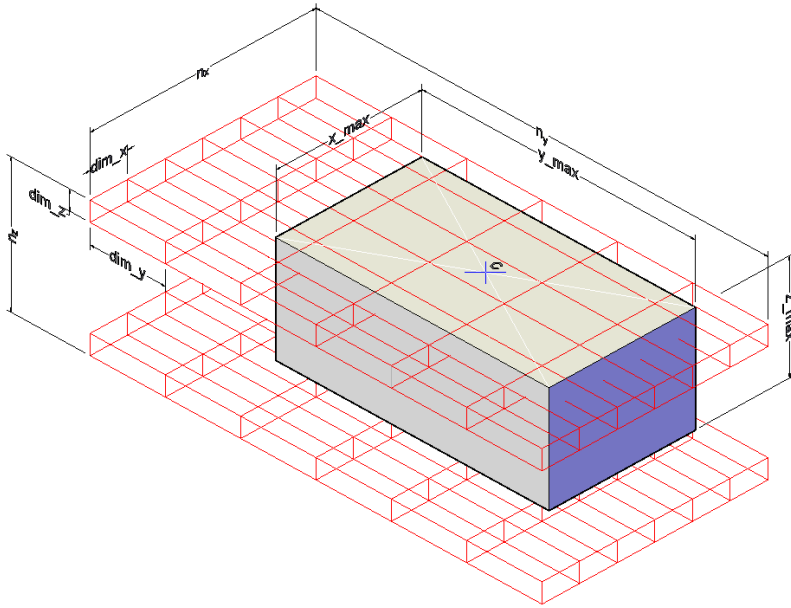
77 Several open questions on production optimization during the cutting phase of slabs from a block
78 are: (i) which dimensions of slabs/tiles to be cut? (ii) Which is the best orientation of block cutting
79 (parallel to x-plane, y-plane, or z-plane)? and (iii) What is the optimum angle of slabs cut from a
80 block? In order to answer these questions, this paper presents a 3D computational algorithm
81 developed to optimize the cutting grid pattern of a block, starting from a 3D fracture modeling
82 method presented in [17]. The developed algorithm was coded using the C++ programming
83 language in a software package named SlabCutOpt. The software code SlabCutOpt allows the
84 computation of the problem and the visualization of the results. Indeed, the latter atomizes the
85 finding of the optimum design related to the cutting grid pattern of a block for each commercial

86 slab size tested, providing the optimum recovery ratio. The paper presents the application of the
87 SlabCutOpt code to a case study of a limestone block and discusses the optimization results. To
88 the best of the authors' knowledge, this is the first attempt of a production optimization model
89 at the block scale.

90 **2. Method**

91 **2.1. The optimization algorithm**

92 The developed 3D optimization algorithm generates a 3D cutting grid for a block. The 3D cutting
93 grid represents the slabs with equal dimensions (dim_x , dim_y , dim_z), as shown in Fig. 1. The
94 3D cutting grid was built starting from the center of the block (C) with a specified number of cuts
95 in three orthogonal directions: n_x , n_y , n_z . The specified dimensions cover the 3D domain of the
96 block, during the whole run of the algorithm, using a number of slabs in a way that $n_i * dim_{i>$
97 block dimension l , with $i=x,y,z$. The block is described by the minimum and maximum Cartesian
98 coordinates (x_{min} , x_{max} , y_{min} , y_{max} , and z_{min} : z_{max}).



99

100 Fig. 1. Illustrative sketch showing the 3D cutting grid of the slabs (colored in red)
 101 body.

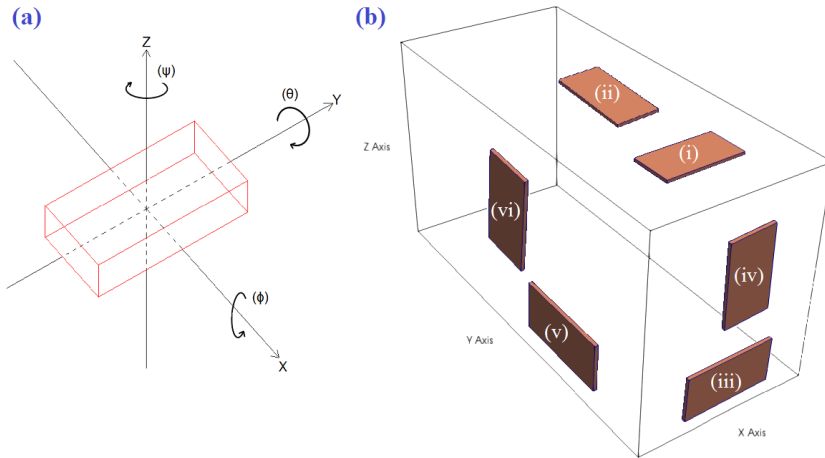
102 The algorithm detects the intersection between the slabs (cutting grid) and the fractures
 103 or the six faces of the block body. In this paper, the discontinuities model represents a model of
 104 fractures, joints, and voids. Iteratively, the cutting grid is rotated using a 3D rotation mode. The
 105 cutting orientations of the slabs can be parallel to the XY plane, XZ plane, or YZ plane of the block
 106 (see Fig. 2 and Table 1). The cutting grid is also subjected to several 3D displacements (dx, dy, dz),
 107 defined by Eq. (1), Eq. (2), and Eq. (3).

108 $(-dim_x)/2 \leq dx < (+dim_x)/2$, with a step of dx_step , for dx (1)

109 $(-dim_y)/2 \leq dy < (+dim_y)/2$, with a step of dy_step , for dy (2)

Commentato [SB1]: I addes plural. Is correct?

110 $(-\text{dim}_z)/2 \leq dz < (+\text{dim}_z)/2$, with a step of dz_step , for dz (3)



111
 112 Fig. 2. (a) The possible orientation-rotation angles of the 3D cutting grid of the slabs; (b) A sketch
 113 showing the possible orientation-rotations within a block (details of the sub-figures numbering
 114 are given in Table 1).

115 **Table 1**
 116 The possible orientations-rotations of the cutting grid.

| cutting orientation of slabs | rotation of slabs | the relative orientation-rotation | | | figure |
|------------------------------|-------------------|-----------------------------------|------------|------------|--------------|
| | | theta θ | phi ϕ | psi ψ | |
| XY plane | 0.0° | 0.0° | 0.0° | 0.0° | Fig. 2b(i) |
| XY plane | 90.0° | 0.0° | 0.0° | 90.0° | Fig. 2b(ii) |
| XZ plane | 0.0° | 90.0° | 0.0° | 0.0° | Fig. 2b(iii) |
| XZ plane | 90.0° | 90.0° | 90.0° | 0.0° | Fig. 2b(iv) |
| YZ plane | 0.0° | 90.0° | 0.0° | 90.0° | Fig. 2b(v) |
| YZ plane | 90.0° | 90.0° | 90.0° | 90.0° | Fig. 2b(vi) |

117
 118
 119 The intersection between a slab and a discontinuity (or with the faces of the block body) is
 120 detected by using a segment/triangle intersection algorithm [44]. The segment/triangle
 121 intersection algorithm [44] compute the signs of 4 determinants and don't require any explicit

122 constructions that can be subject to numerical errors. Each discontinuity and the faces of the
123 block body are geometrically defined by 3D triangles. For each 3D cutting grid patterns, each slab
124 is checked if it intersects a discontinuity or a face of the block. This implies that a slab that is
125 composed by 12 edge segments, the algorithm checks the intersection between the 12 edge
126 segments and all the triangles that represent the discontinuities model. The algorithm computes
127 the number of non-intersected slabs and total number of slabs (intersected + non-intersected)
128 within the block body. The recovery ratio is calculated for each displacement and for each
129 orientation-rotation to determine the optimum cutting grid pattern using Eq. (4). The recovery
130 ratio is also called coefficient of utilization as given and defined in [45]. The recovery ratio
131 described in Eq. (4) is limited to a fixed slab size as defined by the user. The SlabCutOpt algorithm
132 consider a regular cutting grid, because the use of several slab sizes at the same time (irregular
133 cutting grid), can theoretically provide higher recovery ratio, but a grid of several different sizes
134 of slabs cut from a block can be hardly/practically carried out within the typical cutting process
135 of ornamental stone blocks. Considering also that small non-commercial sizes of slabs may be
136 included in the cutting grids. In most cases, the quarrymen cut a block to produce a specific size
137 of slabs which the markets already need. However, in this paper, we aimed at comparing the
138 results of several cutting grids comparing several slab size.

$$139 \text{ recovery (\%)} = \frac{\text{volume of non-intersected slabs with full dimension inside the block body} \times 100}{\text{total volume of the block body}} \quad (4)$$

140 In the processing plants, the cutting saws work with a continuous water flow that is required for
141 cooling and reducing the dust emissions caused by sawing. The dust is the material lost during
142 sawing which can be then reused for further applications [46–49]. The algorithm presented in

143 this paper takes into account the thickness of the cutting saw and the volume of material
144 lost/wasted due to sawing.

145 **2.2. Software development**

146 The developed algorithm was coded using the C++ programming language and makes use of the
147 OpenMP library for multithreading computations. The software package developed was named
148 “SlabCutOpt” on the basis of the algorithm aim: Slab Cutting Optimization.

149 **2.2.1. Input data files**

150 The input data of SlabCutOpt is composed of several files:

- 151 • SlabCutOpt.par: ASCII file containing the input parameters of cutting grid. It has to contain the
152 geometric dimensions of the block body, the parameters about orientation-rotation and
153 displacement and options concerning the operating mode and the output mode, etc. The
154 algorithm can also work only in 2D, avoiding testing for the Z direction. See Appendix A for details;
- 155 • PLY files: each discontinuity or set of discontinuities has to modelled in one or more file using
156 the polygon file format (ply) [50]. See Appendix B for details;
- 157 • PLY_FileList.dat: this is the ASCII file with the list of the ply files. See Appendix C for details.
- 158 • slab_dimensions.dat: this is the ASCII file contains the list of the several sizes of slabs to be
159 tested. See Appendix D for details.

160

161

162 **2.2.2. Files of results**

163 The results files of SlabCutOpt are ASCII files with detailed information on the computations of
164 the grid cutting optimization search, in addition to files needed for the 3D visualization. The
165 results files of SlabCutOpt are:

166 •Results.log file: this is an ASCII file, where each row contains, for each slab size, the orientation
167 of the 3D cutting grid, the applied displacements, the number of non-intersected slabs inside the
168 block body, and the number of intersected slabs inside the block body. See Appendix E for details.

169 •slabs vtu files: optionally, a set of vtu files are generated for the 3D visualization of the cutting
170 grid. The vtu file generation can be the whole set of cutting grids scenarios or just for the best
171 solution (maximum number of non-intersected slabs). A slab type code is assigned to each slab,
172 in the vtu files, to allow an easy visual perception of the kind of slab: out bounding of the block,
173 intersected and non-intersected slabs. The vtu files can be then visualized using, for example, the
174 free and open source visualization software ParaView [51]. Fig. 3 shows a schematic diagram of
175 the software package structure.

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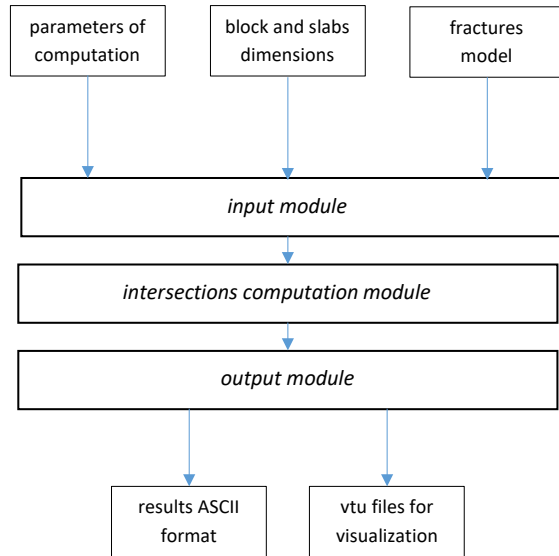
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191 **Fig. 3.** A schematic diagram of SlabCutOpt software structure.

192 **3. Case study**

193 SlabCutOpt was applied to a real case study. The rock type of the commercial size block under
194 study (1.55 m x 2.9 m x 1.10 m) was compacted limestone with a creamy-white color. It is a highly
195 fractured limestone block whose out-cropping fractures and furthermore discontinuities could
196 be detected and deterministically modeled as 3D surfaces based on GPR survey, this model of
197 discontinuities and description of the block under study is presented in [32].

198 The revenue estimation of the block was carried out on the basis of a Relative Money Value (RMV)
199 for each slab size, calculated by Eq. (5) (given that the real selling prices are confidential):

200
$$\text{RMV for a slab} = \frac{\text{the slab selling price (€)}}{\text{the maximum unit price of all the slabs (€)}} \quad (5)$$

201 Table 2 presents the selected commercial sizes of the slabs with their RMV and surface area. A
202 quasi linear correlation between the size (area) of the slabs and the commercial price was clearly
203 found (Figure 4). The optimization solutions were investigated by SlabCutOpt for the whole
204 commercial-sizes of the slabs listed in Table 2.

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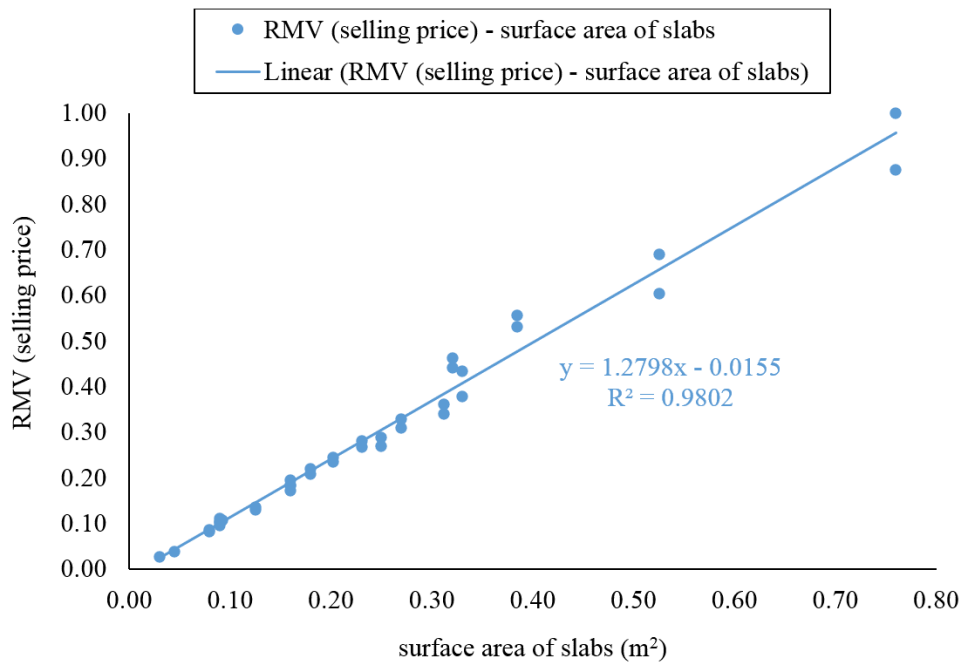
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216

217 **Table 2**

218 The RMV of the commercial-sizes of slabs tested.

| size No. | dimensions of slabs and tiles | | | RMV per unit | surface area (m ²) |
|----------|-------------------------------|--------|--------|--------------|-----------------------------------|
| | x (cm) | y (cm) | z (cm) | | |
| size 1 | 40.00 | 20.00 | 1.50 | 0.082 | 0.08 |
| size 2 | 50.00 | 25.00 | 1.50 | 0.127 | 0.13 |
| size 3 | 40.00 | 20.00 | 2.00 | 0.087 | 0.08 |
| size 4 | 50.00 | 25.00 | 2.00 | 0.136 | 0.13 |
| size 5 | 60.00 | 30.00 | 1.50 | 0.207 | 0.18 |
| size 6 | 30.00 | 30.00 | 1.50 | 0.104 | 0.09 |
| size 7 | 40.00 | 40.00 | 1.50 | 0.184 | 0.16 |
| size 8 | 60.00 | 45.00 | 1.50 | 0.311 | 0.27 |
| size 9 | 45.00 | 45.00 | 1.50 | 0.233 | 0.20 |
| size 10 | 48.00 | 48.00 | 1.50 | 0.265 | 0.23 |
| size 11 | 60.00 | 30.00 | 2.00 | 0.219 | 0.18 |
| size 12 | 30.00 | 30.00 | 2.00 | 0.110 | 0.09 |
| size 13 | 40.00 | 40.00 | 2.00 | 0.195 | 0.16 |
| size 14 | 60.00 | 45.00 | 2.00 | 0.329 | 0.27 |
| size 15 | 45.00 | 45.00 | 2.00 | 0.246 | 0.20 |
| size 16 | 48.00 | 48.00 | 2.00 | 0.280 | 0.23 |
| size 17 | 80.00 | 40.00 | 1.50 | 0.442 | 0.32 |
| size 18 | 80.00 | 48.00 | 1.50 | 0.531 | 0.38 |
| size 19 | 80.00 | 40.00 | 2.00 | 0.463 | 0.32 |
| size 20 | 80.00 | 48.00 | 2.00 | 0.556 | 0.38 |
| size 21 | 30.50 | 30.50 | 1.00 | 0.107 | 0.09 |
| size 22 | 30.00 | 15.00 | 3.00 | 0.040 | 0.05 |
| size 23 | 30.00 | 10.00 | 3.00 | 0.027 | 0.03 |
| size 24 | 30.00 | 110.00 | 2.00 | 0.380 | 0.33 |
| size 25 | 35.00 | 150.00 | 2.00 | 0.604 | 0.53 |
| size 26 | 40.00 | 190.00 | 2.00 | 0.875 | 0.76 |
| size 27 | 30.00 | 110.00 | 3.00 | 0.434 | 0.33 |
| size 28 | 35.00 | 150.00 | 3.00 | 0.691 | 0.53 |
| size 29 | 40.00 | 190.00 | 3.00 | 1.000 | 0.76 |
| size 30 | 15.00 | 60.00 | 2.00 | 0.098 | 0.09 |
| size 31 | 20.00 | 80.00 | 2.00 | 0.174 | 0.16 |
| size 32 | 25.00 | 100.00 | 2.00 | 0.271 | 0.25 |
| size 33 | 25.00 | 125.00 | 2.00 | 0.339 | 0.31 |
| size 34 | 15.00 | 60.00 | 3.00 | 0.104 | 0.09 |
| size 35 | 20.00 | 80.00 | 3.00 | 0.184 | 0.16 |
| size 36 | 25.00 | 100.00 | 3.00 | 0.288 | 0.25 |
| size 37 | 25.00 | 125.00 | 3.00 | 0.360 | 0.31 |



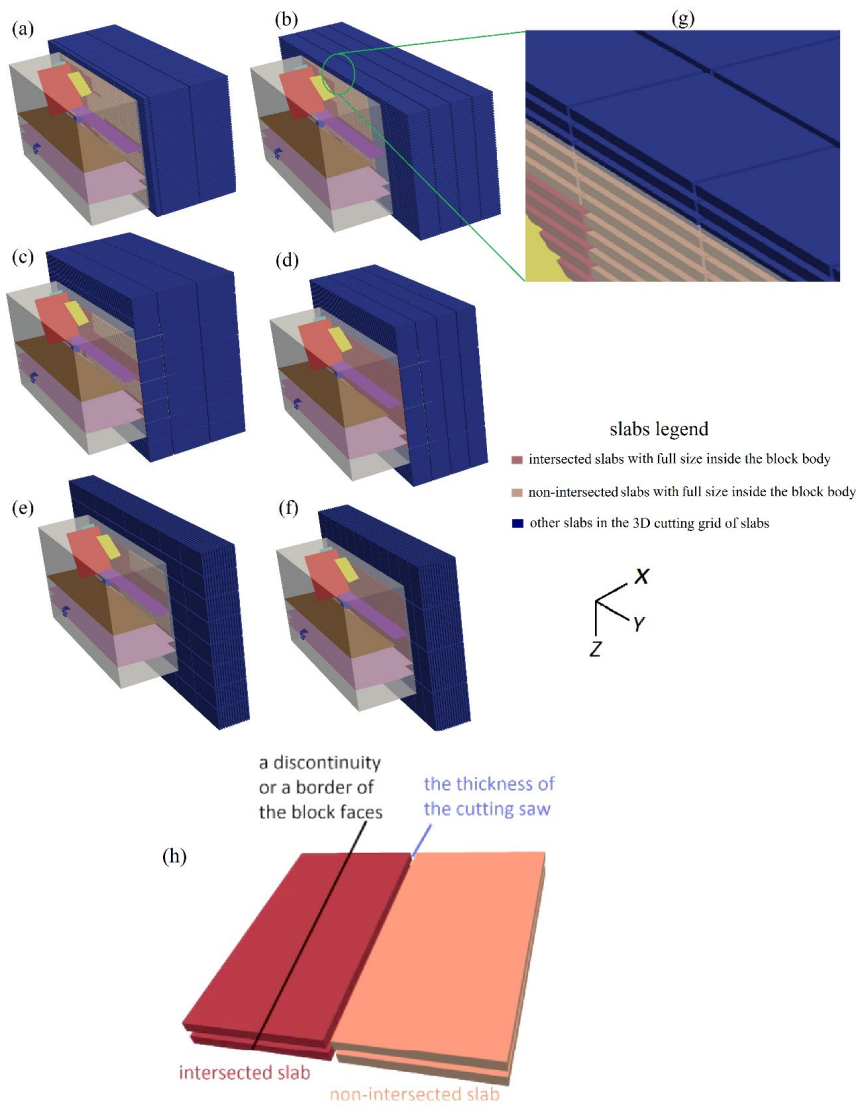
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221 Fig. 4. Correlation between the sizes of the slabs and the RMV.

222 **4. Results**

223 For each slab size of Table 2, a 3D cutting grid was used to compute the non-intersected slabs,
 224 using the six orientations-rotations and several displacements of the cutting grid. The algorithm
 225 parameters used in this study (SlabCutOpt.par file) are presented in Appendix A. Fig. 5 shows, for
 226 slab size No. 1 of Table 2, the computation results of several simulated 3D cutting grids. For
 227 clarity, the discontinuities inside the limestone block body are represented using the same colors
 228 as in the model presented in [32]. The cutting saw thickness is clearly visible in Fig. 5g and Fig 5h,
 229 as the slabs are separated from each others. Fig. 5h simplifies graphically the intersection and
 230 calculation methodology within a single discontinuity.

231 The results of the slab No. 1 (size 40.0 cm x 20.0 cm x 1.50 cm) are given in Fig. 6. For this cutting
232 grid scenario, the optimum recovery ratio was found to be 35.7 %. For this size, the optimum
233 cutting orientation is parallel to the plane XY – slab rotation 0.0° and with a displacement of (dx
234 = -0.01 m, dy = 0.1 m, dz = -0.01 m). From Fig. 6, it can be observed that the recovery ratio varies
235 within each single orientation-rotation as well as between the six orientations-rotations.



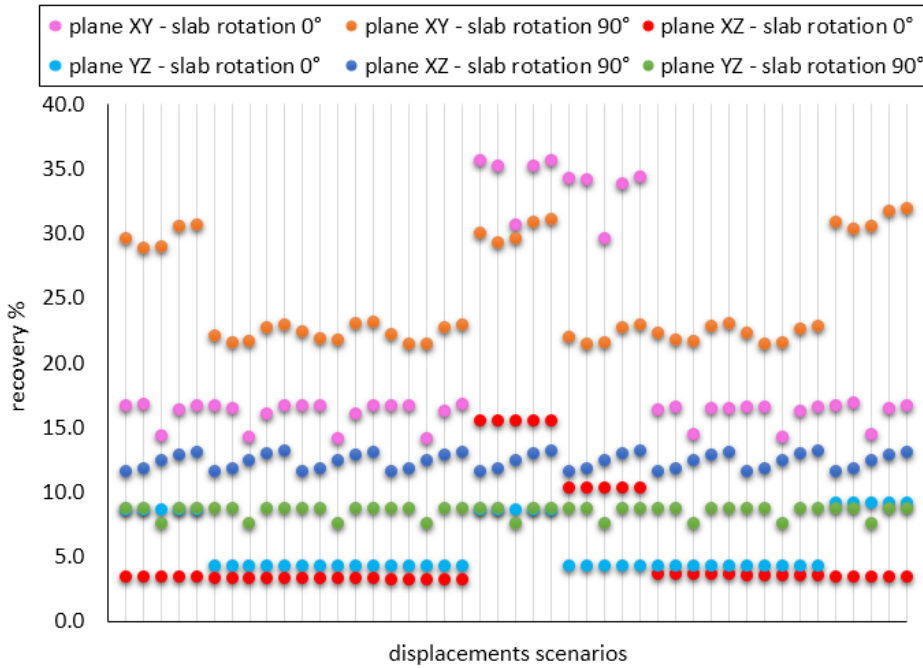
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237 Fig. 5. The 6 orientations/rotations (sub-figures a, b, c, d, e, and f) of the cutting grid scenario
 238 using ParaView. This figure refers to the results of slab size No. 1. Details of the sub-figures a, b,
 239 c, d, e, and f are provided in Table 3. Sub-figures g and h are illustrative figures.

240 **Table 3**
 241 The sub-figures details presented in Fig. 5.

| sub-figure | cutting orientation of the slabs | rotation of the slabs |
|------------|----------------------------------|-----------------------|
| Fig. 4a | XY plane | 0.0° |
| Fig. 4b | XY plane | 90.0° |
| Fig. 4c | XZ plane | 0.0° |
| Fig. 4d | XZ plane | 90.0° |
| Fig. 4e | YZ plane | 0.0° |
| Fig. 4f | YZ plane | 90.0° |

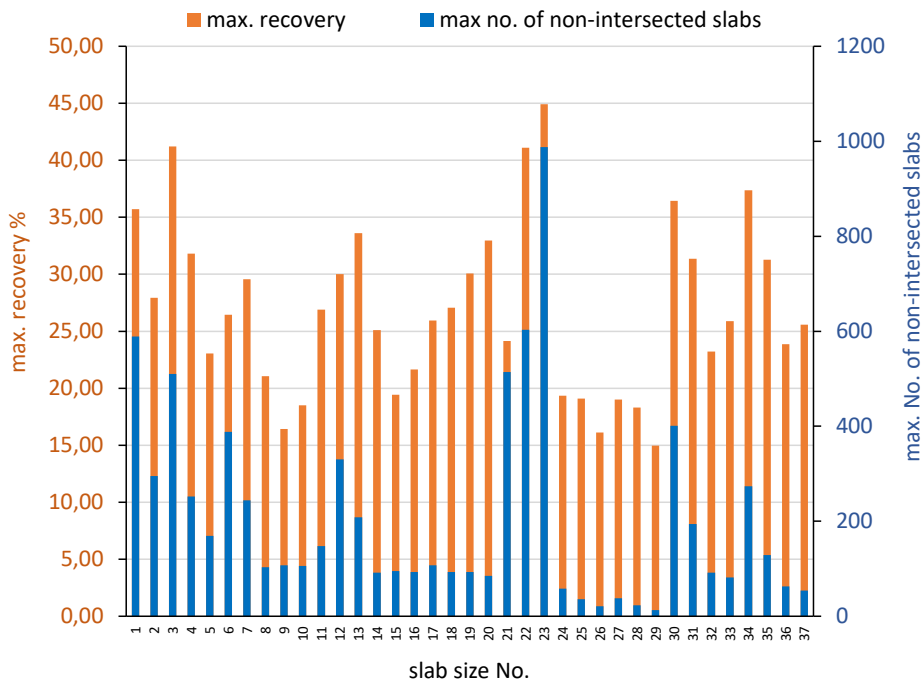
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244
 245 Fig. 6. Graphical representation of the optimization results for the slab size No. 1.

246 The maximum number of non-intersected slabs and the maximum recovery ratio for each tested
 247 slab size are represented in Fig. 7. The best recovery ratio of 44.91 % was found for the cutting
 248 grid of the slab size No. 23. The cutting orientation is parallel to plane XY – slab rotation 90.0°,

249 with a displacement of (dx = -0.01 m, dy = 0.05 m, dz = -0.02 m), which can be taken as the most
 250 environmental friendly solution. From Fig. 7, the histogram of the maximum recovery is not in
 251 agreement with the maximum number of non-intersected slabs. In fact, for example, the
 252 maximum recovery ratio for the slab size No. 13 is higher than for slab size No. 12. However, the
 253 maximum number of non-intersected slabs for the cutting grid of slab size No. 13 is lower than
 254 for slab size No. 12. This is because the volumes of slabs are different and the volume controls
 255 the recovery ratio.

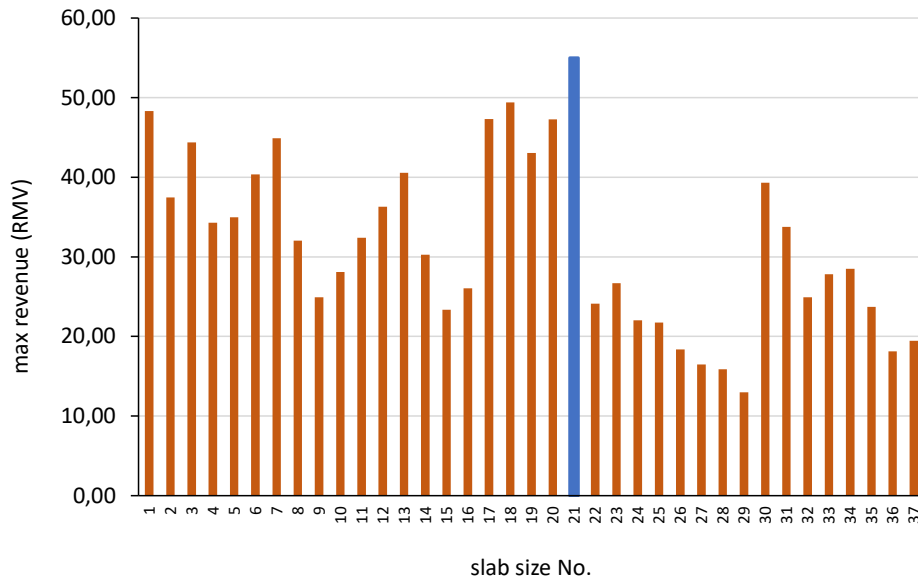


256
 257 Fig. 7. Maximum recovery ratio and maximum number of non-intersected slabs for all sizes of
 258 slabs .

259 From an economic point of view, neither the maximum recovery ratio nor the maximum number
260 of non-intersected slabs can be used as comparison indicator for the cutting grid optimization
261 within a range of different slab sizes. When several sizes are considered, the total revenue
262 calculated by Eq. (6) can be a more effective comparison indicator to assess the optimum
263 solution.

$$264 \text{ revenue} = \text{number of non intersected slabs} \times \text{RMV per unit} \quad (6)$$

265 Interestingly, as shown in Fig. 8, the optimal (maximum) revenue is then obtained from the
266 cutting grid scenario No. 21 with a total revenue value of 55.0 RMV. Neither the maximum
267 number of non-intersected slabs nor the maximum recovery ratio could be found in this slab size.
268 This imply that, the final economical solution (maximum RMV), given by SlabCutOpt, for this
269 block, was identified in slab size No. 21, at the cutting orientation parallel to plane XY – slab
270 rotation 0° with a displacement: (dx = -0.11 m, dy = 0.04 m, dz = -0.01), and equally at the cutting
271 orientation parallel to plane XY – slab rotation 90° with a displacement: (dx = -0.11 m, dy = 0.04
272 m, dz = -0.01). These results were expected since the slab is square-shaped (30.5 cm x 30.5 cm),
273 therefore, the slab rotation had no effect on the final results. Fig. 9 shows a 3D clip visualization
274 of the final optimal economical solution (slab size No. 21).

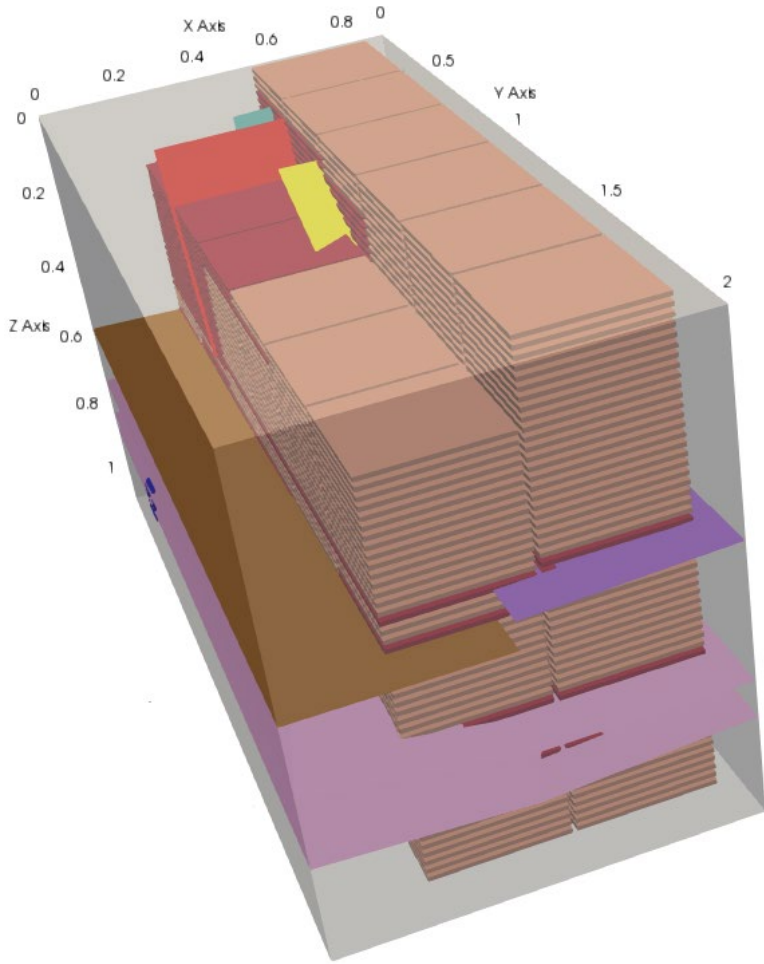


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276 Fig. 8. The maximum revenue obtained for each slab size.

slabs legend

- intersected slabs with full size inside the block body
- non-intersected slabs with full size inside the block body

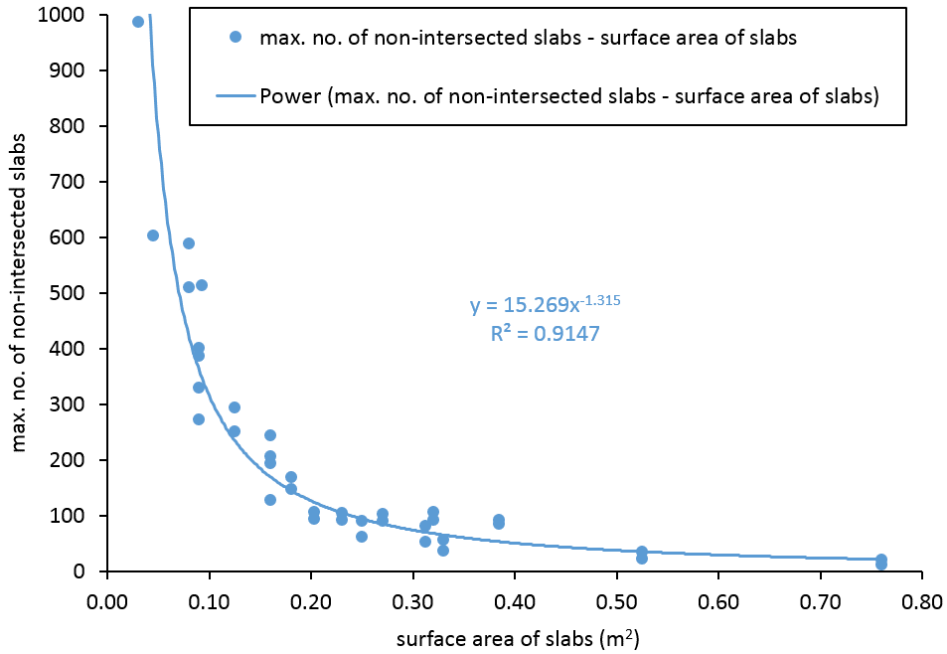


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278 Fig. 9. A 3D clipped view of the optimal economical solution visualized using ParaView.

279 The grain texture or pattern of an ornamental stone product is an important commercial
280 characteristic. The final product aesthetic pleasure is taken in account when the block is cut for
281 applications of prestigious and decoration purposes. The rotation of the cutting grid may cause
282 a change of the final texture of the ornamental stone product in case of stones with veins or color
283 gradation patterns. The presented algorithm can be used for stones which have a uniform color
284 with a texture that is insensitive to the grid rotation and for materials used for building and
285 construction applications that do not concern with texture. It was referred to such these cases of
286 possible rotation of the cutting pattern also by [43]. If the commercial value of the slabs is known
287 (or can be estimated) for different cutting orientation/rotation, then it is still possible to estimate
288 the total revenue for each cutting grid scenario.

289 The surface area of the slabs tested, in the case study, played a main role in the run of SlabCutOpt
290 software. The maximum number of non-intersected slabs, is strongly correlated in a negative
291 power relation with the surface area value of the slabs (Fig. 10). When information about the
292 commercial price for other non-tested slab sizes in SlabCutOpt is available, the use of the
293 correlation formula (in Fig. 10) can be used to estimate their relative revenue. However,
294 SlabCutOpt will still be needed in order to know the relative geometric design of the optimum
295 cutting orientation-rotation of slabs.



296

297 Fig. 10. The relation between the surface area of slabs and the number of non-intersected slabs

298 obtained for each size.

299 The computation time of the SlabOptCut run (considering parameter write_vtu=1, see Appendix

300 A), on a personal computer equipped with a 64-bit operative system Windows 10, a processor

301 Intel i7-3770K CPU @ 3.5 GHz , and an installed memory (RAM) of 8.00 GB, was 23.0 minutes.

302 Depending by the kind of application, the computation time strongly depends on Input/Output

303 files operations. Avoiding time consuming operation of printing out vtu files for each simulated

304 cutting grid, the computation time was reduced up to to 37.0 seconds (considering parameter

305 write_vtu=2, see Appendix A). The SlabCutOpt algorithm used the OpenMP[52] library to

306 compute the number of no-intersected slab for different scenarios simultaneously using several
307 threads for fast computation.

308 **5. Conclusions**

309 Cutting scenarios (3D cutting grid patterns) of a stone block could be tested using the presented
310 software code SlabCutOpt to optimize the recovery ratio or the revenue of the block. We
311 developed a 3D optimization algorithm to simulate several 3D cutting scenarios of slabs.
312 Discontinuities need to be modeled for the whole volume of the ornamental stone block. For
313 each 3D cutting scenario, the algorithm calculates the number of non-intersected slabs - with
314 discontinuities or the block body borders - which allowed calculating the recovery ratio. The
315 optimization algorithm SlabCutOpt was successfully coded in C++ and allowed to visualize the
316 optimized result in 3D using a data visualization software package, such as ParaView.

317 The presented algorithm can work with discontinuities data described in PLY files whatever the
318 detection method is. In the presented case study, the results was based on modeling
319 discontinuities, as 3D surfaces, detected by GPR survey, as recommended by the authors.
320 However, the obtained results are subjected to the limitations of the GPR method. If a tinny
321 hidden fracture could not be detected, it may lead to have unconsidered fractured slab,
322 particularly when it is open fracture. Therefore, using more than a method for fracture detection
323 or more than one GPR frequency (higher) can increase the accuracy of the discontinuities model.

324 For the presented case study of a limestone block, among 37 several -sizes of slabs, the cutting
325 grid of slab size No. 23 of dimensions 30.0 cm x 10.0 cm x 3.0 cm provided the optimum
326 production recovery ratio. Furthermore, economic factors have to be taken into account to

327 correctly evaluate the optimal solution of the cutting grid scenario. Slab size No. 21 of dimensions
328 30.5 cm x 30.5 cm x 1.0 cm provided the final optimal economical solution.

329 A negative power correlation was found between the maximum number of the non-intersected
330 slabs and the surface area of slabs. This correlation can be used, for example, to estimate the
331 maximum number of non-intersected slabs for other cutting grid sizes.

332 SlabCutOpt is recommended to be used in quarrying companies to maximize recovery and
333 revenue, it can provide stone processing factories with the slab sizes optimize the revenue and
334 minimize the waste. Moreover, SlabCutOpt allows quarrying companies to study the potential
335 revenue value of various slab sizes products enabling quarrying companies to estimate the
336 revenue and recovery when commitment with customers exists regarding certain slab sizes.
337 Waste material quantity and size can be estimated using SlabCutOpt allowing further studying of
338 waste re-processing and recycling potential.

339 Future works include the considerations of the cutting cost, energy consumption, and material
340 texture preference in the algorithm. A comparison study between the computed cutting patterns
341 by SlabCutOpt and the actual cutting results of a stone block at a processing plant is
342 recommended. A combination of non-destructive fracture detection and modeling techniques
343 and SlabCutOpt is recommended for environmental and economic sustainable reasons.

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502 **Appendix A**

503 The directives of the working mode, the bounding domain, output options of SlabCutOpt are
504 included in the SlabCutOpt.par file. Keywords are briefly resumed in the given example file below.

```

#SlabCutOpt parameters file
# Block domain definition
x_max=+0.9 #
x_min=+0.0 #
y_max=+2.0 #
y_min=+0.0 #
z_max=+1.1 #
z_min=+0.0 #
#Cutting slab dimensions
dim_x=0.41 # Distance between cutting planes in x direction
dim_y=0.21 # Distance between cutting planes in y direction
dim_z=0.04 # Distance between cutting planes in z direction
cut_saw_thickness=0.01 # Saw thickness (meter)
#Space of solutions
#angles (radian)
theta_step=1.570796327 #
theta_max=1.570796328 #
phi_step=1.570796327 #
phi_max=1.570796328 #
psi_step=1.570796327 #
psi_max=1.570796328 #
#displacement (meters)
dx_step=0.05 #
dy_step=0.05 #
dz_step=0.05 #
#Options
read_slab_dimensions=1 #0: read only the dimensions of the single slab indicated here;
1: consider also a list of different dimensions of slabs
read_bound=0 #0: the ply file describing the faces of the block in a form of triangles
rotation_method=1 #0: Euler rotation; 1: Cartesian axis rotation
BiDimensional=0 # #0: 3D operation; 1: 2D operation
write_vtu=1 #0: no vtu file in output; #1: for each slab size within all the tested
displacements, a slab.vtu file is generated; #2: only the solution of the maximum
number of non-intersected slabs for each slab size
read_PLY_FileList=1 #0: Read the fracture set from "test_fractures.ply"; 1 read the
PLY_FileList.dat for using several PLY files of discontinuities
end=end

```

505

506 **Appendix B**

507 The PLY file format readable by SlabCutOpt is a simplified version of the more general PLY file

508 format, such as the example below. Each element must have 3 vertices (a triangle in the 3D

509 space). The fixed characters are presented in *italic*, while the geometrical parameters for a
510 discontinuity description are presented in **bold**. After the header, a number specified in [*element*
511 *vertex*] indicates the coordinates of the vertices, then the following [*element face*] lines define
512 the number of vertices (3) and the vertices index of each triangle. It is worth mentioning that, in
513 the more general PLY file format, the number of vertices can be higher for polygonal elements.

```
ply
format ascii 1.0
comment example of a fracture represented by 2 triangles
element vertex 4
property float x
property float y
property float z
element face 2
property list uchar int vertex_index
end_header
0.000 2.000 0.553
0.450 2.000 0.553
0.000 0.000 0.583
0.450 0.000 0.599
3 0 1 2
3 1 2 3
```

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518 **Appendix C**

519 The PLY_FileList.dat input file must contain the list of the ply files of the modelled discontinuities.

520 No header is allowed. Below is an example of PLY_FileList.dat.

block_fracturepink_g.ply
block_fracturepink_h.ply
block_fracturepink_k.ply
brown_frac.ply
extended_void_blue.ply
purple_frac.ply
red_frac.ply
single_void.ply
turquoise_frac.ply
yellow_frac.ply

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532 **Appendix D**

533 The slab_dimensions.dat input file must contain the list of the different sizes of slabs that the
534 algorithm must test. Below is an example of slab_dimensions.dat. The order of the columns, from
535 left to right, is dim_x, dim_y, dim_z. The dimensions of the slabs must be increased by the
536 cut_saw_thickness for geometrical considerations in the algorithm.

| | | |
|------|------|-------|
| 0.41 | 0.21 | 0.025 |
| 0.51 | 0.26 | 0.025 |
| 0.41 | 0.21 | 0.03 |
| 0.51 | 0.26 | 0.03 |
| 0.61 | 0.31 | 0.025 |
| 0.31 | 0.31 | 0.025 |
| 0.41 | 0.41 | 0.025 |
| 0.61 | 0.46 | 0.025 |
| 0.46 | 0.46 | 0.025 |
| 0.49 | 0.49 | 0.025 |
| 0.61 | 0.31 | 0.03 |
| 0.31 | 0.31 | 0.03 |
| 0.41 | 0.41 | 0.03 |
| 0.61 | 0.46 | 0.03 |
| 0.46 | 0.46 | 0.03 |
| 0.49 | 0.49 | 0.03 |

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542 **Appendix E**

543 The results file is an ASCII file containing the optimization results. The first part lists the dimension
544 of the slabs tested, as shown in the example below. Following, a list of solutions is provided for
545 each size. The list consists of 12 columns, from left to right there are: the order number of
546 iteration, dim_x, dim_y, dim_z, theta, phi, psi, the total number of slabs inside the block, and the
547 number of intersected slabs.

548 Optimization results:

```
Number of tested dimensions of slabs: 37
[0] dim_x= 0.410000 dim_y= 0.210000 dim_z= 0.025000
[1] dim_x= 0.510000 dim_y= 0.260000 dim_z= 0.025000
.. .. .. .. ..
[n] dim_x= 0.260000 dim_y= 1.260000 dim_z= 0.040000

Results_for_slab_[0]
.. .. .. .. ..
.. .. .. .. ..
.. .. .. .. ..
Results_for_slab_[1]
.. .. .. .. ..
.. .. .. .. ..
.. .. .. .. ..

764 0.510000 0.260000 0.025000 1.570796 1.570796 0.000000 -0.205000 -0.030000 -0.012500 426 121
765 0.510000 0.260000 0.025000 1.570796 1.570796 0.000000 -0.205000 0.020000 -0.012500 422 70
766 0.510000 0.260000 0.025000 1.570796 1.570796 0.000000 -0.205000 0.070000 -0.012500 422 75
767 0.510000 0.260000 0.025000 1.570796 1.570796 0.000000 -0.205000 0.120000 -0.012500 422 88
768 0.510000 0.260000 0.025000 1.570796 1.570796 0.000000 -0.155000 -0.130000 -0.012500 425 106
769 0.510000 0.260000 0.025000 1.570796 1.570796 0.000000 -0.155000 -0.080000 -0.012500 425 113

.. .. .. .. ..
.. .. .. .. ..
.. .. .. .. ..
Results_for_slab_[n]
.. .. .. .. ..
```

549