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Reconfigurability in cellular manufacturing systems: a design model and multi-scenario analysis

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Abstract

Within Cellular Manufacturing Systems (CMSs), families of parts are assigned to manufacturing cells, composed by homogeneous sets of machines. In conventional CMSs, each cell is devoted to the production of a specific part family, reducing material handling and work-in-process. Despite their flexibility, such systems still suffer from coping with the present market challenges asking for dynamic part mix and the need of agility in manufacturing. To meet these challenges, the recent literature explores the idea of including elements of the emerging reconfigurable manufacturing paradigm in the design and management of CMSs, leading to the Cellular Reconfigurable Manufacturing System (CRMS) concept. The aim of this paper is to propose an original linear programming optimization model for the design of CRMSs with alternative part routing and multiple time periods. The production environment consists of multiple cells equipped with Reconfigurable Machine Tools (RMTs) made of basic and auxiliary custom modules. By changing the auxiliary modules, different operations become available on the same RMT. The proposed approach determines the part routing mix and the auxiliary module allocation best balancing the part flows among RMTs and the effort to install the modules on the machines. The approach discussion is supported by a literature case study, while a multi-scenario analysis is performed to assess the impact of different CMS configurations on the system performances, varying both the number of cells and the RMT assignment to each of them. [A benchmarking concludes the paper comparing the proposed CRMS against a conventional CMS configuration. The analysis shows relevant benefits in terms of reduction of the intercellular travel time \(-58.6%\) getting a global time saving of about 53.3%. Results prove that reconfigurability is an opportunity for industries to face the dynamics of global markets.](#)

Keywords: cellular manufacturing; reconfigurable manufacturing systems; reconfigurability; modularity; optimization

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Abstract

Within Cellular Manufacturing Systems (CMSs), families of parts are assigned to manufacturing cells, composed by homogeneous sets of machines. In conventional CMSs, each cell is devoted to the production of a specific part family, reducing material handling and work-in-process. Despite their flexibility, such systems still suffer from coping with the present market challenges asking for dynamic part mix and the need of agility in manufacturing. To meet these challenges, the recent literature explores the idea of including elements of the emerging reconfigurable manufacturing paradigm in the design and management of CMSs, leading to the Cellular Reconfigurable Manufacturing System (CRMS) concept. The aim of this paper is to propose an original linear programming optimization model for the design of CRMSs with alternative part routing and multiple time periods. The production environment consists of multiple cells equipped with Reconfigurable Machine Tools (RMTs) made of basic and auxiliary custom modules. By changing the auxiliary modules, different operations become available on the same RMT. The proposed approach determines the part routing mix and the auxiliary module allocation best balancing the part flows among RMTs and the effort to install the modules on the machines. The approach discussion is supported by a literature case study, while a multi-scenario analysis is performed to assess the impact of different CMS configurations on the system performances, varying both the number of cells and the RMT assignment to each of them. [A benchmarking concludes the paper comparing the proposed CRMS against a conventional CMS configuration. The analysis shows relevant benefits in terms of reduction of the intercellular travel time \(-58.6%\) getting a global time saving of about 53.3%. Results prove that reconfigurability is an opportunity for industries to face the dynamics of global markets.](#)

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1 Introduction

In conventional Cellular Manufacturing Systems (CMSs) similar parts or products are grouped to create families, while the required working machines compose manufacturing cells with the aim of reducing production time, setups, work-in-process, increasing quality and the system productivity [1-3]. This production philosophy integrates the benefits of flexible and mass production systems cutting down the system operation costs [4-8]. However, in the last few years, an increasing number of factors such as short lead times, dynamic market demand, fluctuating volumes and high-customized variants drive the transition from traditional manufacturing systems toward the so-called Next Generation Manufacturing Systems (NGMSs) [9-12]. In this context, traditional systems as Dedicated Manufacturing Systems (DMSs), Flexible Manufacturing Systems (FMSs) and CMSs show increasing limits in adapting themselves to the recent industrial and market trends

1 [13]. Focusing on CMSs, once machine cells are designed, the physical relocation of the facilities included in
2 each cell in response to new production requirements becomes difficult. To overcome such and other
3 weaknesses, the recent research focuses on modularity to designing manufacturing cells using modular
4 machines achieving reconfigurability in manufacturing [14, 15]. According to the original definition, a
5 Reconfigurable Manufacturing System (RMS) is *'designed at the outset for rapid change in structure as well*
6 *as in hardware and software components to quickly adjust production capacity and functionality within a part*
7 *family in response to sudden changes in market or in regulatory requirements'* [16, 17]. Such systems include
8 the so-called Reconfigurable Machine Tools (RMTs) characterized by an adjustable and modular structure that
9 enables machine scalability and convertibility using basic and auxiliary manufacturing modules [18-21]
10 increasing the set of the feasible operations [15]. Typically, basic modules are structural in nature, while
11 auxiliary modules are kinematical or motion-giving. A combination of these modules provides the operational
12 capability to the RMT. In the recent years, numerous attempts are made to merge CMSs and RMSs to overcome
13 the main shortcomings of cellular manufacturing. The concept of Cellular Reconfigurable Manufacturing
14 System (CRMS) is introduced. CRMSs are made of a set of Reconfigurable Machine Cells (RMCs) in which
15 machines are logically, instead of physically, organized [22]. This means that RMCs can change during the
16 production plan horizon by changing the auxiliary custom modules on the RMTs.

17 Starting from this scenario, this paper presents an original procedure, based on a linear programming model,
18 to optimally design and manage CRMSs from a multi-product and multi-period perspective, exploring how to
19 best-balance the part flows and the effort to install the modules on the machine on which the part is located.
20 To the Authors' knowledge, the trade-off analysis between inbound logistics and machine reconfiguration is
21 new and it has never been explored by the literature. The proposed model minimizes the inter-cell parts travel
22 time and the setup time to assemble and disassemble the auxiliary modules. In addition, a multi-scenario
23 analysis studies the effect of different machine-cell configurations on these system performances.

24 According to the introduced background and the outlined goals, the reminder of this paper is organized as
25 follows: Section 2 revises the literature on the topic. Section 3 states the problem and describes the linear
26 programming model supporting the design and management of CRMSs while Section 4 applies the model to
27 a representative case study frequently adopted in the CM literature to benchmark the proposed algorithms. The
28 multi-scenario analysis is in Section 5 before concluding the paper in the last Section 6.

29 **2 Literature review**

30 Experiences of Cellular Manufacturing (CM) implementation in industry and performance improvements are
31 studied widely by the scientific literature [2, 23-25]. Particularly, several approaches are discussed by
32 researchers in the last decades to increase the performances of CM systems. **Optimal, heuristic and meta-**
33 **heuristic procedures are used [26].** As example, Ateme-Nguema and Dao [27] proposed a hybrid approach to
34 solve the cellular systems design problem for large industrial data sets introducing an ant colony optimization
35 and tabu search procedure. The goal is to minimize the dissimilarities among machines or parts. Luo and Tang
36 [28] presented a model combining ordinal optimization and iterated local search to maximize the grouping

1 efficacy index. Ghezavati and Saidi-Mehrabad [29] introduced a mathematical model for the cell formation
2 problem integrated with group scheduling decisions with the aim of minimizing the total costs. Yilmaz and
3 Durmusoglu [30] and Yilmaz et al. [31] considered the batch scheduling problem in a multi-hybrid cellular
4 manufacturing system (MHCMS) taking into account worker resources. To reach this goal, the Authors defined
5 mathematical models supporting the batch scheduling problem and developed heuristic methods, e.g. genetic
6 algorithms, simulated annealing and artificial bee colony, to apply the model to large sized problems. However,
7 as stated in Section 1, factors recently affecting industrial companies as short lead times, dynamic market
8 demand, fluctuating volumes and high-customized variants as well as the need to overcome the main
9 weaknesses of CMSs drive the transition toward the NGMSs and, particularly, toward CRMSs. In this field,
10 methods and models for the cell formation problem using reconfigurable machines are in Pattanaik et al. [14]
11 and Pattanaik and Kumar [32]. The Authors proposed a clustering-based approach supporting the design of
12 RMCs using modular machines. Xing et al. [22] introduced an approach to design and control CRMSs by using
13 artificial intelligence, focusing on the formation of RMCs coming from the dynamic and logical clustering of
14 subsets of manufacturing resources. Bai et al. [33] introduced an approach for the formation of virtual
15 manufacturing cells in a reconfigurable manufacturing environment characterized by multiple product orders.
16 Javadian et al. [34] presented a multi-objective dynamic cell formation model, minimizing the total cell load
17 variation and the sum of the miscellaneous costs. Ossama et al. [35] defined a mathematical model to form
18 simultaneously the part families and the corresponding cell configurations in a dynamic reconfigurable
19 production environment. Eguia et al. [36] and Eguia et al. [37] faced the design and loading of CRMSs in the
20 presence of alternative routing and developed a mixer-integer linear programming model to determining the
21 routing mix and the tool and module allocation with the aim of minimizing the total intercellular movements
22 of the parts and the production costs. Yu et al. [38] defined an optimization model to integrate part grouping
23 and loading in such systems, minimizing the workload assigned to the machines. Eguia et al. [15] extended
24 the previous formulation by considering multiple process plans for each part and RMTs with a library of
25 auxiliary modules and introduced a mathematical model that minimizes the transportation and holding costs.
26 Aljuneidi and Bulgak [39] presented a mathematical model for the joint investigation of CRMSs and hybrid
27 manufacturing-remanufacturing systems. Such model considers a conventional cell formation problem in
28 CMSs bridged with a production planning problem addressing the ‘reconfiguration’ issues of CMSs for
29 different production periods. The analysis of the past and recent literature highlights that studies addressing
30 the CRMS design problem exploring the relevant trade-off between inbound logistics, i.e. intercellular part
31 flows, and machine reconfiguration, i.e. auxiliary module assembly/disassembly, are missing. To fill this gap,
32 in this paper, an original mathematical model is proposed to optimally design and manage CRMSs exploring
33 how to best-balance the part flows and the effort to install the modules on the machine on which the part is
34 located. The problem statement and analytic modelling are in next Section 3.

3 Problem statement and analytic modelling

CRMSs include multiple RMCs made of a set of machines, i.e. RMTs. Each RMT has a library of basic and auxiliary customized modules. The basic modules are structural elements permanently attached to the RMT while auxiliary modules are kinematical or motion-giving, e.g. spindles, and they can be assembled or disassembled to provide different operational capabilities. In this paper, according to recent literature [14, 15], the reconfigurability attribute is modeled in terms of modularity of the existing RMTs. Fig. 1 shows a schematic framework of the considered CRMS structure.

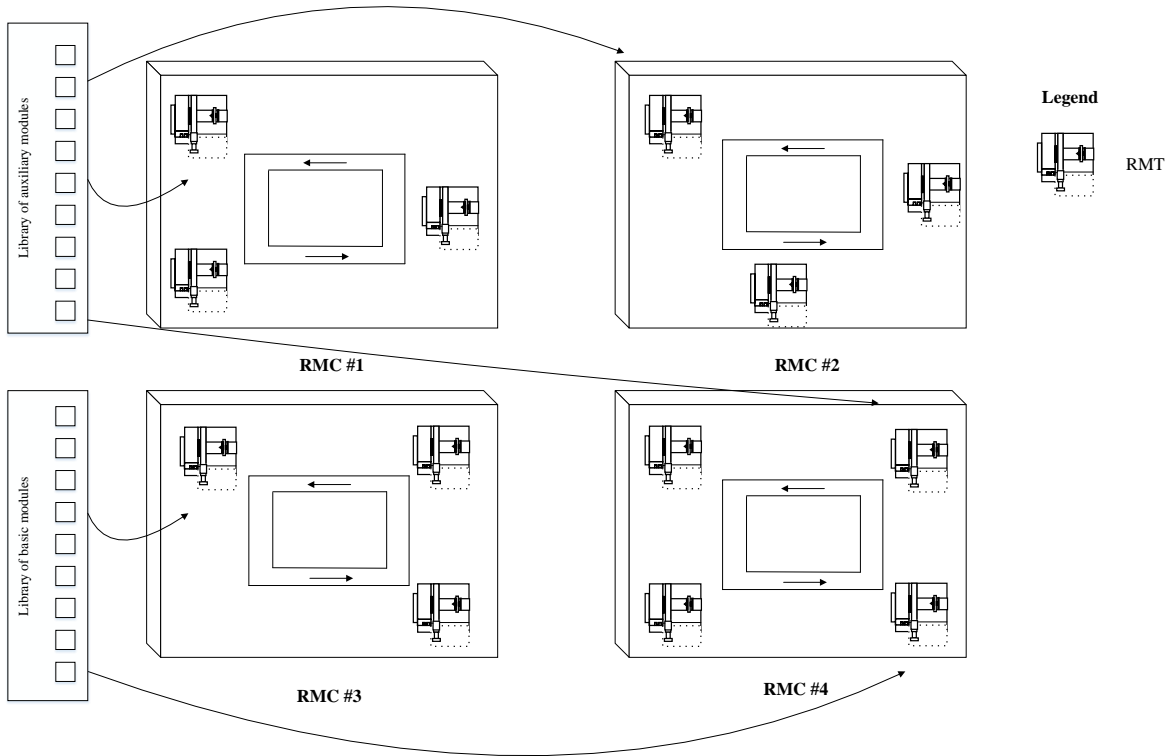


Fig. 1. Schematic framework of a CRMS structure

3.1 Problem description, assumptions and notations

The proposed CRMS design procedure starts from a given RMT-RMC assignment and, by using the information about the operation sequence and the compatibility among auxiliary modules, operations and RMTs, explores how to best-balance the part flows among RMCs and the effort to install the auxiliary modules on the RMT on which the part is located. To address this issue, the proposed optimization model minimizes the inter-cell parts travel time plus the setup time to assemble and disassemble the auxiliary modules, determining the product batch flows and the best allocation of the modules to the RMTs.

In the model development, the following assumptions are adopted, following the standard literature [14, 15]:

- The operation-based process plan for the parts is given;
- The requirement of modules and the RMT-module compatibilities are given;
- The auxiliary modules are available when needed;

- The reference RMT-RMC assignment is given, i.e. initial RMT-RMC layout. This condition is realistic because the existing industries have a defined layout and re-layout actions are time and cost consuming and may be assessed starting from the outcomes of the proposed model;
- Working and setup times, e.g. auxiliary modules assembly and disassembly times, together with part travel times are known and deterministic.

The following notations are introduced.

- Indices

i	parts $i = 1, \dots, M$
o	operations of the part work cycle $o = 1, \dots, O_i$
m	RMTs $m = 1, \dots, Z$
k	auxiliary module types $k = 1, \dots, L$
j	RMCs $j = 1, \dots, N$
t	time periods $t = 1, \dots, T$

- Parameters

G_{omk}	1 if operation o can be performed on RMT m using auxiliary module type k ; 0 otherwise [<i>binary</i>]
r_{it}	definition of the operation in which the batch of part i is in period t
t_{ijj_1}	travel time for batch of part i from cell j to cell j_1 [<i>min/batch</i>]
λ_{mk}	assembly time of module k on RMT m [<i>min/module</i>]
μ_{mk}	disassembly time of module k from RMT m [<i>min/module</i>]
τ_{om}	time to perform operation o on RMT m [<i>min/op</i>]
MAC_{mj}	1 if RMT m is assigned to cell j ; 0 otherwise [<i>binary</i>]
ξ	available time per RMT and time period [<i>min/machine</i>]
R	maximum number of modules per RMT and period [#]
δ_i	planned production volume per period of time for part i [<i>parts</i>]

- Decisional variables

F_{ijj_1t}	1 if batch of part i moves from cell j to cell in j_1 in period t ; 0 otherwise [<i>binary</i>]
W_{mit}	1 if batch of part i is processed by RMT m in period t ; 0 otherwise [<i>binary</i>]
σ_{mkt}	1 if module k is on RMT m in period t , 0 otherwise [<i>binary</i>]
X_{mkt}	1 if module k is assembled on RMT m in period t , 0 otherwise [<i>binary</i>]
Y_{mkt}	1 if module k is disassembled from RMT m in period t , 0 otherwise [<i>binary</i>]

- Objective function

ψ Total part travel time and module assembly/disassembly time [*min*]

3.2 Model formulation

The analytic formulation of the proposed CRMS design model is in the following.

$$\min \psi = \sum_{m=1}^Z \sum_{k=1}^L \sum_{t=1}^W X_{mkt} \cdot \lambda_{mk}$$

$$+ \sum_{m=1}^Z \sum_{k=1}^L \sum_{t=1}^W Y_{mkt} \cdot \mu_{mk} \quad (1)$$

$$+ \sum_{i=1}^M \sum_{j=1}^N \sum_{j_1=1}^N \sum_{t=1}^{W-1} F_{ijj_1t} \cdot t_{ijj_1}$$

(1) minimizes the sum of three relevant terms having opposite trends. i.e. the time to install the auxiliary modules on the RMTs, the time to disassemble the modules from the RMTs and the inter-cell part travel time.

The model is subject to the following feasibility constraints:

$$\sum_{m=1}^Z W_{mit} = 1 \quad \forall t, i \quad (2)$$

$$G_{omk} \cdot W_{mit} \leq \sigma_{mkt} \quad \forall m, i, k, t, o: r_{it} = o \quad (3)$$

$$W_{mit} \leq \sum_{k=1}^L \sum_{o:r_{it}=o} G_{omk} \quad \forall m, i, t \quad (4)$$

$$\sigma_{mkt} \leq \sum_{i=1}^M \sum_{o:r_{it}=o} G_{omk} \quad \forall m, k, t \quad (5)$$

$$\sum_{k=1}^L \sigma_{mkt} \leq R \quad \forall m, t \quad (6)$$

$$X_{mkt} \geq \sigma_{mkt} - \sigma_{mkt-1} \quad \forall m, k, t = 2, \dots, T \quad (7)$$

$$Y_{mkt} \geq \sigma_{mkt-1} - \sigma_{mkt} \quad \forall m, k, t = 2, \dots, T \quad (8)$$

$$F_{ijj_1t} \leq \sum_{m=1}^Z \sum_{k=1}^L \sum_{o:r_{it}=o} G_{omk} \cdot MAC_{mj} \quad \forall i, j, j_1, t = 1, \dots, T-1 \quad (9)$$

$$F_{ijj_1t} \leq \sum_{m=1}^Z \sum_{k=1}^L \sum_{o:r_{it+1}=o} G_{omk} \cdot MAC_{mj_1} \quad \forall i, j, j_1, t = 1, \dots, T-2 \quad (10)$$

$$W_{mit} \leq \sum_{j=1}^N \sum_{j_1=1}^N F_{ijj_1t} \cdot MAC_{mj} \quad \forall i, m, t = 1, \dots, T-1 \quad (11)$$

$$W_{miT} \leq \sum_{j=1}^N \sum_{j_1=1}^N F_{ijj_1T-1} \cdot MAC_{mj_1} \quad \forall i, m \quad (12)$$

$$\sum_{j=1}^N \sum_{j_1=1}^N F_{ijj_1t} = 1 \quad \forall i, t = 1, \dots, T-1 \quad (13)$$

$$\sum_{j_1=1}^N F_{ijj_1t} = \sum_{j_1=1}^N F_{ijj_1t+1} \quad \forall i, j, t = 1, \dots, T-2 \quad (14)$$

$$\sum_{k=1}^L (X_{mkt} \cdot \lambda_{mk} + Y_{mkt} \cdot \mu_{mk}) + \sum_{i=1}^M \sum_{o:r_{it}=0} (W_{mit} \cdot \tau_{om} \cdot \delta_i) \leq \xi \quad \forall m, t \quad (15)$$

$$F_{ijj_1t} \text{ binary} \quad \forall j, j_1, t \quad (16)$$

$$W_{imt} \text{ binary} \quad \forall i, m, t \quad (17)$$

$$\sigma_{mkt}, X_{mkt}, Y_{mkt} \text{ binary} \quad \forall m, k, t \quad (18)$$

(2) ensures that each part batch in each period is processed by only one RMT. (3) guarantees the presence of module k on RMT m in period t if the module is required to perform the current operation. (4) allows the presence of the batch of part i on RMT m in period t if the required module k is available on that RMT, while (5) forces the presence of module k on RMT m in period t if the batch to work requires the module. (6) sets the maximum number of auxiliary modules that can be simultaneously assembled per RMT and time period. (7)-(8) set the auxiliary modules assembly and disassembly processes on/from RMTs. (9)-(10) admit the existence of flows of part i from cell j to cell j_1 if the required RMTs and modules are present in the initial and final cell. (11)-(12) link the variables W_{mit} and F_{ijj_1t} , while (13)-(14) guarantee the continuity of part flow along their work cycle. (15) force not to exceed the available working time. Finally, (16)-(18) give consistence to the decisional variables.

4 Case study

The proposed model is applied to a relevant case study frequently adopted in the CM literature and representative of an operative industrial context [24, 40, 41]. The problem is based on a 43 x 16 incidence matrix (number of parts x number of operations) [24, 41]. The work cycles and the daily target production volumes, together with data concerning the auxiliary module assembly and disassembly times, are outlined in Appendix A. The RMCs are 5 and a library of 10 auxiliary modules is available. In this phase, each RMT is assigned to a RMC, i.e. one machine per cell. The effect of different RMT aggregations will be analysed and discussed in the next Section 4. Consequently, the initial RMT-RMC assignment is as follows: RMT #1 is in RMC #1, RMT #2 is in RMC #2, RMT #3 is in RMC #3, RMT #4 is in RMC #4 and RMT #5 is in RMC #5. In addition, it is supposed that each RMT has a specific level of reconfigurability which affects the number of modules technologically compatible with that RMT. Three classes are considered.

- High level of reconfigurability, i.e. all the available auxiliary modules (100%) can be assembled;
- Mid level of reconfigurability, i.e. up to 50% of the modules can be assembled;

- Low level of reconfigurability: i.e. up to 33% of the auxiliary modules can be assembled.

The RMT-module and the operation-RMT-module compatibility matrices are in Table 1 and Table 2. Table 1 specifies the reconfigurability level of each RMT together with the auxiliary modules that can be assembled on each RMT, e.g. RMT #4 is characterized by low level of reconfigurability and auxiliary modules 1, 7 and 10 can be assembled. Table 2 reports the set of RMTs suitable for the execution of each operation together with the required auxiliary modules, in round brackets, and the unitary processing times in seconds, in squared brackets, e.g. Op3 can be executed on RMT #1 equipped with auxiliary module 5 with an unitary processing time of 8 seconds.

Table 1. RMT-module compatibility matrix

RMT	Reconfigurability class	Auxiliary modules									
		$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$	$k = 10$
$m = 1$	High	1	1	1	1	1	1	1	1	1	1
$m = 2$	Medium	0	1	1	0	0	1	1	0	1	0
$m = 3$	High	1	1	1	1	1	1	1	1	1	1
$m = 4$	Low	1	0	0	0	0	0	1	0	0	1
$m = 5$	Medium	1	0	0	1	1	0	0	1	0	1

Table 2. Operation-RMT-module compatibility matrix

Operations (o)	(auxiliary modules) – [processing times in seconds]				
	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
Op1	(1) – [12]		(1, 10) – [7]		(10) – [10]
Op2		(3, 9) – [7]			(5) – [9]
Op3	(5) – [8]	(2) – [11]		(1, 10) – [12]	
Op4	(4) – [11]	(3, 6) – [6]		(1) – [6]	
Op5		(7) – [9]		(7, 10) – [8]	(4, 8) – [11]
Op6	(3, 4) – [7]			(7) – [7]	
Op7	(2, 4, 9) – [10]		(8, 9) – [11]		
Op8		(2, 3, 6) – [12]	(3, 5) – [10]		
Op9			(4) – [12]		(4, 8) – [7]
Op10	(8) – [8]			(1) – [10]	(4, 5) – [12]
Op11		(6) – [7]	(2, 6) – [6]		
Op12		(6, 9) – [6]	(6) – [8]		
Op13	(10) – [8]		(1, 6, 8, 10) – [16]		(1, 10) – [11]
Op14	(1, 9) – [6]			(1, 7) – [11]	
Op15	(4, 6, 8) – [9]	(2, 3, 6, 7, 9) – [19]			(4, 8) – [9]
Op16		(2, 3, 7) – [10]	(1, 5, 9) – [8]	(1, 7) – [7]	(1, 4, 5, 10) – [24]

Table 3 shows the matrix containing the intercellular travel times, i.e. the time move a batch of part i from RMC j to RMC j_1 , expressed in minutes.

Table 3. Intercellular travel time, minutes

Cell Id.	RMC #1	RMC #2	RMC #3	RMC #4	RMC #5
RMC #1	-	2	18	11	22
RMC #2	2	-	18	6	16
RMC #3	18	18	-	19	8
RMC #4	11	6	19	-	17
RMC #5	22	16	8	17	-

Finally, the available time per RMT and period, i.e. ξ , is of two shifts of 8 hours each and a maximum of 20 modules can be simultaneously assembled on each RMT. Given a planning horizon of about 840 periods, i.e. 840 working days, the set of the input data leads to 631,860 decisional variables and 31,648,242 constraints. The model is coded in AMPL language and processed adopting Gurobi Optimizer© v.4.0.1.0 solver on an Intel® Core™ i7 CPU @ 2.40GHz and 8.0GB RAM workstation. The global solving time is of about 50 seconds. The key results for the outlined scenario are summarized in the next Section 3.1.

4.1 Results and discussion

This paragraph proposes the main results obtained by adopting the proposed CRMS design model to the introduced industrial case study. At first, the minimization of the objective function ψ leads to an impact of the intercellular flows up to 86.7% (9589 flows equal to 908 hours) and of the auxiliary module installation, in terms of assembly and disassembly processes, up to 13.3% (138 hours). In particular, the auxiliary modules assembly time is 7.6% (79 hours) of the total time, while the disassembly time is the 5.7% (59 hours). Table 4 shows the intercellular flows among the five cells, i.e. RMCs. Each flow corresponds to the shipment of a batch of parts at the end of a working period.

Table 4. Number of intercellular flows

Cell Id.	RMC #1	RMC #2	RMC #3	RMC #4	RMC #5
RMC #1	-	3078	6	30	0
RMC #2	3059	-	0	99	189
RMC #3	39	0	-	73	735
RMC #4	16	152	83	-	550
RMC #5	4	115	762	599	-

Despite most of the flows are between near RMCs (see Table 3 for the unitary travelling times), e.g. RMC #1 and #2, the intercellular flows highly impact on the value of the objective function, i.e. 86.7%, stating the convenience to move the parts to RMCs already equipped with the required auxiliary modules rather than to remain on the same RMT changing its configuration.

Focusing on the five RMTs, Fig. 2 shows the frequency diagrams of their use. Particularly, each graph focuses on an RMT and it presents the percentage of time periods the RMT works a mix of part types with the size indicated on the x -axis. This analysis is conducted by post-processing the variables W_{mit} , which denote the RMTs on which part types are processed in each time period.

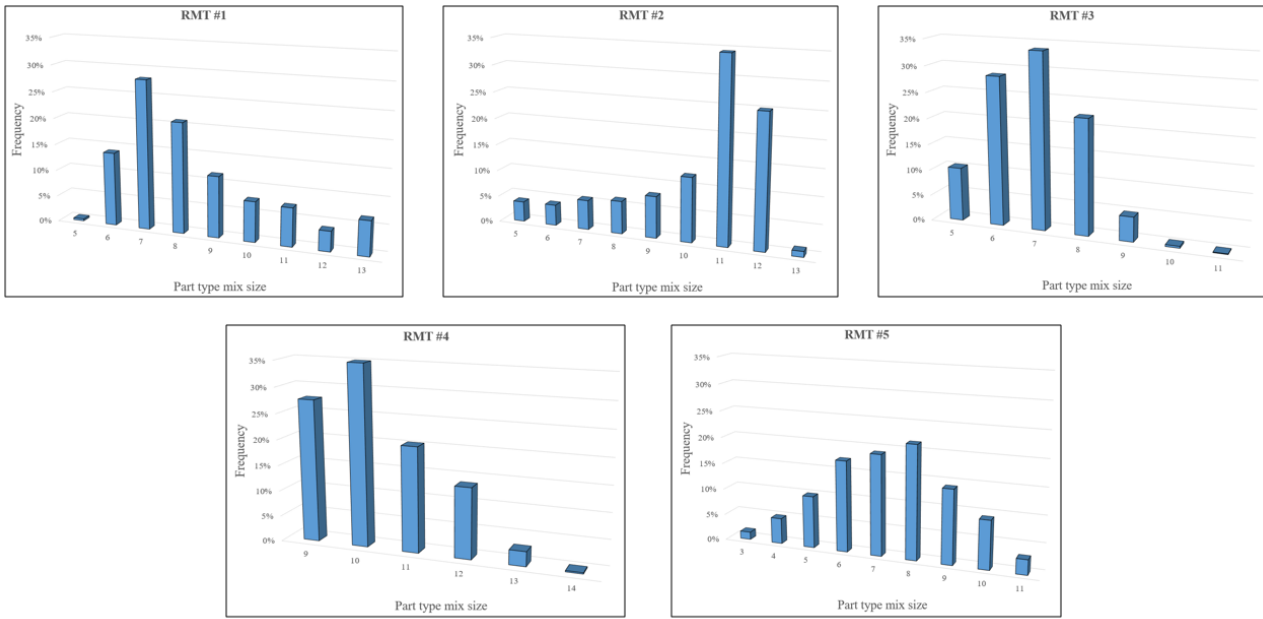


Fig. 2. RMT use frequency diagrams

As example, in most of the periods, seven part types are processed by RMT #1 and RMT #3, eleven part types on RMT #2, ten part types on RMT #4 and eight part types on RMT #5. Because of, within the same period, the RMT configuration remains the same, i.e. auxiliary modules are changed between each couple of consecutive periods, only, high frequency of mixes with big sizes means the effective management of the auxiliary modules to create useful RMT structures for a wide set of work phases to be done at that time.

Fig. 3 highlights the configuration of each RMT presenting the frequencies of installation of the number of auxiliary modules indicated on the x -axis. This analysis is conducted by post-processing the variables σ_{mkt} , which denote the RMTs on which auxiliary modules are located in each time period.

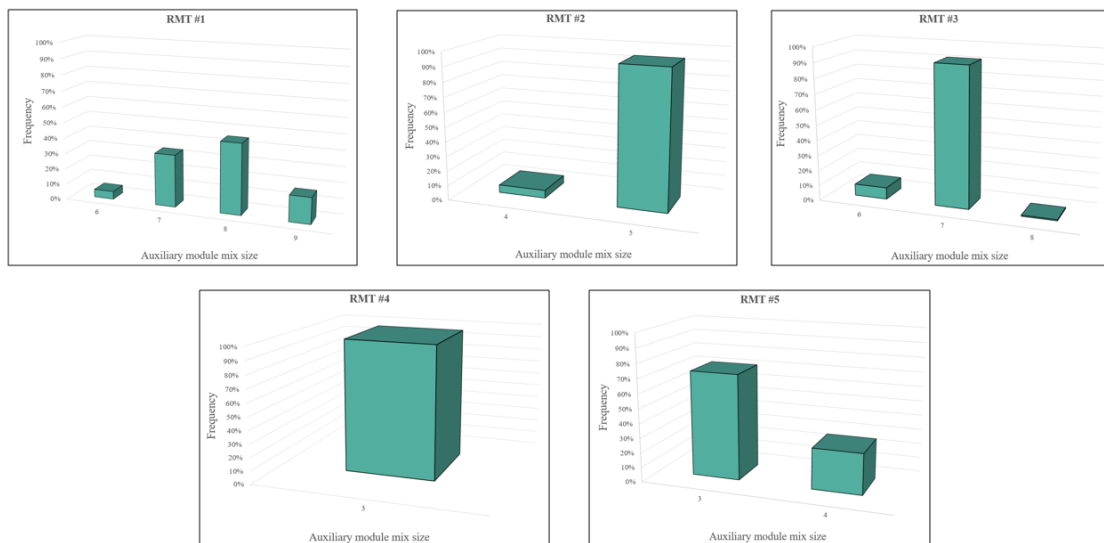


Fig. 3. Auxiliary modules-RMTs allocation over the considered time horizon

In most of the periods, eight auxiliary modules are assembled on RMT #1, five auxiliary modules on RMT #2, seven auxiliary modules on RMT #3 and three auxiliary modules on RMT #5. Finally, three auxiliary modules (not always the same ones) are always on RMT #4.

4.1.1 Benchmarking

To benchmark the results highlighting a global convenience, the CRMS is compared against a conventional CMS, not including the auxiliary modules as elements of reconfigurability. The benchmark solution is introduced by Bortolini et al. [24] considering the same input data. Key comparisons are in Table 5.

Table 5. Comparison between conventional CMS and CRMS

	Intercellular travel time [h]	Module assembly and disassembly time [h]	Total time [h]	Saving
Conventional CMS	2193.57	0	2193.57	
CRMS	908.68	49.15	957.83	56.33%

Compared to the rigid system, the implementation of the CRMS shows relevant benefits in terms of reduction of the global intercellular travel time (-58.6%) despite the rising time needed to assemble and disassemble the auxiliary modules. The global time saving is of about 56.33%.

To extend the results obtained by applying the model to the proposed case study, the next Section 4 presents a multi-scenario analysis varying the number of RMCs and the RMT-RMC assignment assessing the impact of these factors on the model outcomes.

5 Multi-scenario analysis

The multi-scenario analysis is performed to test the effect of different RMT-RMC configurations on the system performances, changing, in each scenario, both the number of the available RMCs, i.e. $j = 1, \dots, N$, and the RMT-RMC assignment, i.e. MAC_{mj} . Such inputs represent the most critical data in CRMS design and management.

The proposed model is solved considering a number of RMCs ranging from 1, i.e. all RMTs are in a unique cell, to the number of RMTs, i.e. one RMT per RMC. For each of these cases, the distances among the RMCs, affecting the intercellular travel time, are adapted to get an effective and continuous production system. To this purpose, the Stirling number of the 2nd kind in equation (19) [42] returns the number of ways in which a set of m elements, i.e. the RMTs, can be partitioned into n subsets, i.e. the RMCs.

$$S(m, n) = \frac{1}{n!} \cdot \sum_{\varphi=0}^n (-1)^\varphi \binom{n}{\varphi} (n - \varphi)^m \quad (19)$$

In the proposed case study, the number of RMTs is constant, i.e. $m = 5$, while the number of RMCs ranges from 1 to 5, i.e. $1 \leq n \leq 5$, leading to five Stirling numbers. Given each of them, the permutations of the RMTs within the set of the available RMCs are calculated to get the scenarios to test. Table 6 summarizes this phase leading to 541 scenarios.

Table 6. Partitions, permutations and number of scenarios for the multi-scenario analysis

Partition Id.	$S(m,n)$	Stirling numbers of the 2 nd kind	Permutations	# of scenarios
1	$S(5,1)$	1	1	1
2	$S(5,2)$	15	2	30
3	$S(5,3)$	25	6	150
4	$S(5,4)$	10	24	240
5	$S(5,5)$	1	120	120

5.1 Findings and comparison

For the sake of brevity, detailed results for each scenario are omitted. An example of the lists containing the objective function values for all scenarios of Partition 1 and 2 are in Appendix B. Table 7 focuses on the best and the worst scenario for each of the five partitions showing the incidence of the travel time and the module assembly and disassembly time on the objective function.

Table 7. Best and worst scenarios for each partition

Partition Id.	ψ	Best case configuration		ψ	Worst case configuration		Gap (%)
		Intercellular travel time (%)	Module assembly and disassembly time (%)		Intercellular travel time (%)	Module assembly and disassembly time (%)	
1	57	-	100	-	-	-	-
2	57	0	100	1563	95.86	4.14	-96.35
3	72	4.04	95.96	1728	94.80	5.20	-95.83
4	250	60.18	39.82	1853	91.67	8.33	-86.50
5	800	87.41	12.59	2030	95.22	4.78	-60.60

As example, the best configuration for Partition 3 corresponds to an impact of the intercellular flows on the objective function value up to 4.04% and of the module installation effort up to 95.96%. On the other side, the worst scenario stresses the intercellular flows (94.8%) toward the module installation effort (5.2%). Globally, the gap between these two opposite scenarios is of about 95.83%. Results allow concluding about the relevance of the problem addressed by the proposed model. For each partition, the relevant gap between the best and worst scenarios states the effect of wrong design choices in the RMT assignment to RMCs. In addition, the objective function values increase moving from Partition 1 to 5 guiding the designer in the case the number of RMCs becomes a free variable suitable to changes. Moreover, the proposed model considers more convenient the installation of the necessary auxiliary modules in presence of few RMCs, i.e. up to three. By increasing the RMCs, it becomes convenient mixing the module installation strategy and the part travel strategy. This is

1 because the global time needed to continuously assemble and disassemble the auxiliary modules overcomes
2 the time needed to move the part to a different RMT, located in another RMC, in which the required modules
3 are ready. As in multiple industrial problems, given the set of efficient solutions, the decision-makers are asked
4 to make the final choice best balancing the operative constraints and exogenous variables.
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8 **6 Conclusions and further research**

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10 Nowadays, achieving high level of flexibility in production system design and management is a critical asset
11 to compete. In Cellular Manufacturing Systems (CMSs) similar parts are grouped into families, and the
12 corresponding machines into cells, to reduce lead times, setup time and work-in-process maintaining good
13 levels of flexibility. Traditional CMSs show limits in adapting themselves to the emerging industrial and
14 market trends, i.e. dynamic demand, fluctuating volumes and high-customized variants. In particular, given
15 the cells, the physical relocation of the manufacturing tools to react to new production requirements becomes
16 difficult. To overcome such rigidity, an emerging research stream explores the integration between CMSs
17 and reconfigurable manufacturing paradigm leading to the concept of Cellular Reconfigurable Manufacturing
18 Systems (CRMSs).
19

20 This study presents and applies an original integer linear programming model to design and manage CRMSs
21 in a multi-product and multi-period environment best-balancing the part flows among machines ready to
22 process them and the effort to install the necessary modules on the current machine. This problem is new and,
23 to the Author's knowledge, it has never been explored by the literature. The proposed optimal procedure is
24 applied to a relevant case study, made of an instance inspired from the literature, while a multi-scenario
25 analysis widens the paper perspective assessing the impact of different machine-cell configurations on the
26 system performances. [Given the increasing customer request for different product variants in variable
27 production batch sizes, this model can be effectively used by industry and practitioners in CMS environments
28 to achieve reconfigurability and to support the decision-makers in defining the number of cells and their
29 configurations.](#) Future research deals with the extension of the model to include relevant issues not considered
30 at this stage, e.g. auxiliary module availability, economic assessment, etc., as well as the application to larger
31 industrial instances.
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48 **Conflict of Interest:** The authors declare that they have no conflict of interest.
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3 **Appendix A**

4 **Table A1.** Part work cycles and production volumes
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6	Part (<i>i</i>)	δ_i [pcs/period]	Work cycle
7	1	50	Op6-Op10-Op7-Op8-Op6
8	2	150	Op2-Op9-Op6-Op9-Op8-Op16-Op14-Op2
9	3	500	Op8-Op13-Op11-Op8
10	4	75	Op9
11	5	500	Op4-Op15-Op5-Op4
12	6	1200	Op6-Op14
13	7	1500	Op3-Op6-Op16-Op3
14	8	750	Op8-Op5-Op6
15	9	5000	Op4-Op11-Op5-Op8-Op4
16	10	1300	Op9-Op2-Op16
17	11	1239	Op8-Op12
18	12	575	Op8-Op6-Op10-Op8
19	13	1239	Op7-Op6-Op10
20	14	1500	Op4-Op6-Op5-Op6
21	15	14000	Op5-Op8
22	16	39	Op5
23	17	900	Op3-Op14-Op6-Op3
24	18	339	Op9-Op16
25	19	390	Op4-Op6-Op8-Op5-Op6-Op15
26	20	304	Op8-Op11
27	21	405	Op4-Op8-Op5-Op15-Op4
28	22	1200	Op5-Op12
29	23	5	Op4-Op6-Op5-Op8
30	24	35	Op8-Op11-Op13-Op12-Op8
31	25	390	Op7-Op10
32	26	750	Op10
33	27	39	Op11-Op12-Op8
34	28	320	Op2-Op9-Op8
35	29	1500	Op4-Op5
36	30	11300	Op11-Op12
37	31	310	Op8-Op10
38	32	430	Op2-Op9-Op6-Op16-Op9
39	33	500	Op5-Op15-Op6-Op5
40	34	275	Op3-Op6
41	35	500	Op14-Op3
42	36	600	Op3
43	37	1500	Op1-Op2-Op9-Op8-Op6-Op16-Op9
44	38	750	Op2-Op9-Op8-Op16-Op9
45	39	5000	Op6-Op10
46	40	1300	Op9-Op2-Op6-Op9
47	41	1239	Op5-Op8-Op15
48	42	575	Op1-Op2-Op9-Op6-Op2-Op16-Op1
49	43	1239	Op5-Op6-Op8-Op15-Op6

Table A2. Auxiliary module assembly and disassembly time (minutes)

Machine	Module	λ	μ
1	1	3.32	2.49
1	2	3.73	2.79
1	3	3.56	2.67
1	4	3.94	2.95
1	5	3.92	2.94
1	6	3.41	2.55
1	7	3.46	2.59
1	8	4.22	3.16
1	9	4.03	3.02
1	10	4.41	3.30
2	1	–	–
2	2	4.22	3.16
2	3	6.1	4.57
2	4	–	–
2	5	–	–
2	6	5	3.75
2	7	4.7	3.52
2	8	–	–
2	9	6.02	4.51
2	10	–	–
3	1	3.9	2.92
3	2	7.2	5.4
3	3	9	6.75
3	4	8.6	6.45
3	5	9.2	6.9
3	6	7.75	5.81
3	7	4.15	3.11
3	8	5.1	3.82
3	9	6.2	4.65
3	10	8.5	6.37
4	1	4.6	3.45
4	2	–	–
4	3	–	–
4	4	–	–
4	5	–	–
4	6	–	–
4	7	8.87	6.65
4	8	–	–
4	9	–	–
4	10	10.2	7.65
5	1	9.4	7.05
5	2	–	–
5	3	–	–
5	4	9.1	6.82
5	5	15	11.25
5	6	–	–
5	7	–	–
5	8	8.8	6.6
5	9	–	–
5	10	12	9

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Appendix B

Table B1. RMT assignment, in squared brackets, and objective function value for partition 1, i.e. one RMC

RMT-RMC assignment	
RMC#1	Objective function value [h]
[1 2 3 4 5]	57

Table B2. RMT assignment and objective function values for partition 2, i.e. two RMCs

RMT-RMC assignment		
RMC#1	RMC#2	Objective function value [h]
[5]	[1 2 3 4]	90
[1 2 3 4]	[5]	90
[4]	[1 2 3 5]	57
[1 2 3 5]	[4]	57
[4 5]	[1 2 3]	90
[1 2 3]	[4 5]	90
[3]	[1 2 4 5]	65
[1 2 4 5]	[3]	65
[3 5]	[1 2 4]	595
[1 2 4]	[3 5]	595
[3 4]	[1 2 5]	65
[1 2 5]	[3 4]	65
[3 4 5]	[1 2]	101
[1 2]	[3 4 5]	101
[2]	[1 3 4 5]	57
[1 3 4 5]	[2]	57
[2 5]	[1 3 4]	533
[1 3 4]	[2 5]	533
[2 4]	[1 3 5]	65
[1 3 5]	[2 4]	65
[2 4 5]	[1 3]	107
[1 3]	[2 4 5]	107
[2 3]	[1 4 5]	1563
[1 4 5]	[2 3]	1563
[2 3 5]	[1 4]	1329
[1 4]	[2 3 5]	1329
[2 3 4]	[1 5]	752
[1 5]	[2 3 4]	752
[2 3 4 5]	[1]	178
[1]	[2 3 4 5]	178