Exogenous uncertainty and the identification of Structural Vector Autoregressions with external instruments

Published Version:
Angelini, G., Fanelli, L. (2019). Exogenous uncertainty and the identification of Structural Vector Autoregressions with external instruments. JOURNAL OF APPLIED ECONOMETRICS, 34(6), 951-971 [10.1002/jae.2736].

Availability:
This version is available at: https://hdl.handle.net/11585/697266 since: 2022-04-12

Published:
DOI: http://doi.org/10.1002/jae.2736

Angelini, G., & Fanelli, L. (2019). Exogenous uncertainty and the identification of structural vector autoregressions with external instruments. Journal of Applied Econometrics, 34(6), 951-971.

https://doi.org/10.1002/jae.2736

Exogenous uncertainty and the identification of Structural Vector Autoregressions with external instruments

Giovanni Angelini
University of Bologna

Luca Fanelli*
University of Bologna

Last revision: July 2019

Abstract

We provide necessary and sufficient conditions for the identification of Structural Vector Autoregressions (SVARs) with external instruments considering the case in which r instruments are used to identify g structural shocks of interest, $r \geq g \geq 1$. Novel frequentist estimation methods are discussed by considering both a 'partial shocks' identification strategy, where only g structural shocks are of interest and are instrumented, and in a 'full shocks' identification strategy, where despite g structural shocks are instrumented, all $n = g+(n-g)$ structural shocks of the system can be identified under certain conditions. The suggested approach is applied to empirically investigate whether financial and macroeconomic uncertainty can be approximated as exogenous drivers of U.S. real economic activity, or rather as endogenous responses to first moment shocks, or both. We analyze whether the dynamic causal effects of non-uncertainty shocks on macroeconomic and financial uncertainty are significant in the period after the Global Financial Crisis.

Keywords: Exogenous Uncertainty, External Instruments, Identification, proxy-SVAR, SVAR.

J.E.L.: C32, C51, C52, E44.

1 Introduction

Structural Vector Autoregressions (SVARs) provide stylized and parsimonious characterizations of shock transmission mechanisms and allow to track dynamic causal effects in empirical macroeconomics. The identification of SVARs requires parameter restrictions on the matrix which maps the VAR disturbances to structural shocks, henceforth denoted with B, that are often implausible. The parameters in the matrix B capture the instantaneous impacts of the structural shocks on the variables and are crucial ingredients of the Impulse Response Functions (IRFs). One of the most interesting approaches developed in the recent literature to identify structural shocks by possibly avoiding recursive structures, or implausible assumptions on the elements of B is the so-called 'external instruments' or 'proxy-SVAR' (or 'SVAR-IV') approach, see Stock and Watson (2012, 2018) and Mertens and Ravn (2013, 2014). This method takes advantage of information developed from 'outside' the VAR in the form of variables which are correlated with the latent structural shocks of interest (relevance condition) and are uncorrelated with the other structural shocks of the system (exogeneity, or orthogonality condition).

The emerging literature on proxy-SVARs (throughout the paper we use the terms 'SVARs with external instruments' and 'proxy-SVARs' interchangeably) is mainly devoted to the use of one external instrument to identify a single structural shock of interest in isolation from all the other shocks of the system. For example, Stock and Watson (2012) identify six shocks (the oil shock, the monetary policy shock, the productivity shock, the uncertainty shock, the liquidity/financial risk shock and the fiscal policy shock) by exploiting many external instruments, but use them one at a time; see also Ramey (2016). Remarkable exceptions are Mertens and Ravn (2013) and Mertens and Montiel Olea (2018) who deal with the case of two instruments and two structural shocks ($r = g = 2$); see also Arias *et al.* (2018b).[1] Mertens and Ravn (2013) show that when $g > 1$, the restrictions provided by the external instruments do not suffice to identify the shocks and must be complemented with additional constraints. They obtain these constraints from a Choleski decomposition of a covariance matrix.

In general, there exists no result in the literature which provides a guidance for practitioners to address the following question: given $g \geq 1$ structural shocks of interest in a system of n variables and $r \geq g$ external instruments available for these shocks, how many restrictions do we need for the model to be identified and where do these restrictions need to be placed? One main contribution of this article is to provide such a general framework, i.e. we extend the identification analysis of proxy-SVARs to the case in which multiple instruments (r) are used to identify multiple shocks ($g \geq 1$). We show that when $g > 1$, the additional restrictions necessary to identify the shocks of interest (up to sign normalization) other the external instruments need not be Choleski-type constraints. We discuss novel frequentist estimation methods alternative to instrumental variables (IV) techniques: a classical minimum distance (CMD) approach and

Actually, Caldara and Kamps (2017) consider the case $r = g = 3$ and an identification strategy which can be classified as a 'full shocks' identification approach according to the characterization we provide in the paper.

maximum likelihood (ML) approach, respectively. Further, we argue that one of the advantages of covering the case $r > g$ (i.e. the proxy-SVAR features more instruments than shocks of interest) is that a practitioner can potentially include up to $r - g$ weakly relevant (or not relevant at all) external instruments in the proxy-SVAR without affecting the inference if at least g instruments are strongly correlated with the structural shocks of interest.

Our approach is based on the idea of augmenting the SVAR with an auxiliary model for the r external instruments. We obtain a 'larger' m-dimensional SVAR, with $m = n+r$, which conveys conveniently all the information relevant to identify the structural shocks of interest. This large system is called the AC-SVAR model, where 'AC' stands for 'augmented-constrained'. The AC-SVAR model is 'augmented' because it is obtained by appending the external instruments to the original SVAR equations. The AC-SVAR model is 'constrained' because it features a triangular structure in the autoregressive coefficients and a particularly constrained structure in the matrix that contains the structural parameters (the on-impact coefficients). We discuss two types of identification strategies which can be accommodated within the AC-SVAR framework depending on the information available to the practitioner: a 'partial shocks' identification approach, which is the typical case in the proxy-SVARs literature, and a 'full shocks' identification approach which occurs, under certain conditions, when all structural shocks of the system can be identified by using r external instruments for g structural shocks of primary interest.

In the 'partial shocks' identification approach, the objective is to identify the dynamic causal effects of $g \geq 1$ structural shocks of interest using $r \geq g$ valid external instruments, regardless of the other $n - g$ shocks of the system. When $r = g = 1$, the parameter which captures the correlation between the external instrument and the shock of interest is a scalar, say ϕ, and the parameters which are necessary to estimate the IRFs correspond to a column of the matrix B. When $g > 1$ and $r \geq g$ external instruments are used, $\phi = \Phi$ becomes a matrix with r rows and g columns and contains therefore more than one 'relevance' parameter. We propose a novel CMD estimation method for proxy-SVARs based on a set of moment conditions implied by the AC-SVAR model. In particular, we minimize the distance between a set of reduced form parameters, which can be easily and consistently estimated from the AC-SVAR model, and the parameters which capture the instantaneous impact of the instrumented shocks. The identification of the proxy-SVAR (up to sign) depends on a rank condition associated with the Jacobian matrix behind CMD estimation. We derive this Jacobian matrix analytically and discuss the necessary and sufficient condition and the necessary order condition for identification.

In the 'full shocks' identification approach, r valid external instruments are still used to identify (up to sign) $g \geq 1$ instrumented structural shocks, $r \geq g$, but it is now possible to infer, under some additional constraints, the dynamic causal effects of all n structural shocks of the system, including those associated with the $n - g$ non-instrumented shocks. The identification of the AC-SVAR model in the 'full shocks' approach amounts to the practice of identifying an enlarged 'B-model' using the terminology in Lütkepohl (2005) ('C-model' using the terminology in Amisano and Giannini, 1997). Estimation can be carried out by ML and requires minor adaptations to the algorithms discussed in e.g. Amisano and Giannini (1997) and Lütkepohl (2005) implemented in econometric packages.

Since we treat the proxy-SVAR as a 'large' SVAR, in our framework the issue of making bootstrap inference on the IRFs, discussed in Jentsch and Lunsford (2016) and Montiel Olea *et al.* (2018) and recently debated in Jentsch and Lunsford (2019) and Mertens and Ravn (2019), boils down to the problem of making bootstrap inference on the IRFs generated by SVARs, see e.g. Kilian and Lütkepohl (2017) for a review. For instance, in the empirical application of Section 8 we first check that the disturbances of the estimated AC-SVAR model are not characterized by conditional heteroskedasticity (because of the results in Brüggemann *et al.* (2016) on inference in SVARs with conditional volatility of unknown form), and then compute simultaneous confidence bands for the IRFs of interest by combining a standard residual-based recursive-design bootstrap algorithm with the 'sup-t' method discussed in Montiel Olea and Plagborg-Møller (2019).

Moreover, when the AC-SVAR system is overidentified, a convenient way to test the empirical validity of the proxy-SVAR model is to compute overidentification restrictions tests. Our analysis shows that these tests tend to reject the proxy-SVAR when the external instruments are erroneously assumed orthogonal to the non-instrumented shocks. Thus, we have analogs of the 'Sargan's specification test' in the instrumental variables framework, or the 'Hansen's J-test' in the generalized method of moments framework, and this appears a novelty in the literature on proxy-SVARs. Notably, in the full shocks identification approach, the quality of the identification can be evaluated not only by checking the relevance condition, but also the orthogonality between the external instruments and the non-instrumented shocks.

The second contribution of this article is empirical. We apply our methodology to address a recently debated issue of the uncertainty literature, i.e. whether uncertainty is a driver of the U.S. business cycle or rather a response to first moment shocks, or both. A well recognized fact in the literature is that uncertainty is recessionary in presence of real options effects (e.g. Bloom, 2009) or financial frictions (e.g. Christiano *et al.*, 2014). Instead, a less documented and controversial fact is whether uncertainty, a second moment variable, is also a response to first moment shocks, especially during periods of economic and financial turmoil. Indeed, uncertainty appears also to endogenously increase during recessions, as lower economic growth induces greater dispersion at the micro level and higher aggregate volatility.

Reverse causality between uncertainty and real economic activity using monthly or quarterly data can not be analyzed by recursive (triangular) SVARs which presume that some variables respond only with a lag to others. This issue has been analyzed in the recent literature by Ludvigson *et al.* (2018), Carriero *et al.* (2018) and Angelini *et al.* (2019). These authors use non-recursive SVARs and different identification methods and report mixed evidence. We focus on the U.S. economy after the Global Financial Crisis, in particular the 'Great Recession+Slow Recovery' period 2008-2015, and consider a small-scale monthly SVAR which includes measures of macroeconomic and financial uncertainty taken from Jurado *et al* (2015) and Ludvigson *et al.* (2018), respectively, and a measure of real economic activity, say the industrial production growth ($n = 3$). The scope of our analysis is to investigate whether the selected measures of macroeconomic and financial uncertainty respond on-impact (instantaneously) and/or with lags to a 'non-uncertainty' shock ($g = 1$). This requires a non-recursive (non-triangular) specification for the matrix B which makes our approach potentially attractive. The direction of causality we are primarily interested in runs from real economic activity to uncertainty, not the other way around, and this requires the use of valid external instruments for the variable of the system related to real economic activity. Thus, in our baseline specification we use two external instruments jointly ($r = 2$) to identify the 'non-uncertainty' shock of the system and track its dynamic impact on financial and macroeconomic uncertainty. This strategy differs from e.g. Stock and Watson (2012) who use valid external instruments (one at a time) to identify the effects of uncertainty shocks on macroeconomic variables. It differs also from Ludvigson *et al.* (2018)'s strategy, where two external instruments for financial and macroeconomic uncertainty shocks are employed to narrow the identification set obtained by directly restricting the structural shocks in correspondence of particular events (event constraints).

The external instruments we employ for the non-uncertainty shock are: (a) the time series innovations obtained from an auxiliary regression models for the changes in the log of new privately owned housing units started; (b) an oil supply shock identified along the lines of Kilian (2009); (c) the time series innovations obtained from an auxiliary regression model for the changes in the log of hours worked. The couples of external instruments (a,b) and (a,c) are used to identify the non-uncertainty shock of the system in a partial shocks identification strategy, but also in a full shocks identification strategy under an auxiliary hypothesis on the pass-through from financial to macroeconomic uncertainty. Empirical results show that macroeconomic and financial uncertainty do not respond significantly to the identified non-uncertainty shocks on-impact. Macroeconomic uncertainty does not respond at any lag to the identified real economic activity shocks while financial uncertainty displays a short-lived response after one period, so that the overall empirical evidence in favor of the hypothesis of 'endogenous uncertainty' appears scant. Notably, our analyses provide empirical support to the validity of the selected external instruments. Admittedly, however, our results can not be considered 'final' as they depend on the specific set of external instruments used to identify the non-uncertainty shock.

Our paper is naturally connected with the increasing strand of the macroeconometric literature which develops and applies estimation and inferential methods for SVARs with external instruments. We compare thoroughly our methodology with other approaches in Sections 7 and our empirical results with other works on exogenous/endogenous uncertainty in Section 8.3. Our approach is based on the maintained assumption that the external instruments are strongly correlated with the instrumented structural shocks, which might not be the case in applied work. Lunsford (2015) and Montiel Olea *et al.* (2018) discuss identification-robust inferential methods for weak external instruments. The extension of our approach to proxy-SVARs to weak instruments is left to future research.

The paper is organized as follows. Section 2 introduces the reference SVAR with external instruments and presents the main assumptions. Section 3 discusses the AC-SVAR representation of proxy-SVARs and Section 4 motivates two identification strategies featured by AC-SVAR models by considering an example centered of the concept of exogenous/endogenous uncertainty in SVAR models. Section 5 deals with the 'partial shocks' identification strategy and proposes a CMD estimation approach alternative to IV methods. Section 6 deals with the 'full shocks' identification strategy and discusses estimation through ML. Section 7 connects our approach to proxy-SVARs to the literature. Section 8 applies the suggested methodology to investigate the exogeneity/endogeneity of uncertainty in the U.S. in the period after the Global Financial Crisis. Section 9 contains some concluding remarks. Additional technical details, formal proofs, Monte Carlo experiments and further empirical results are confined in a Supplementary Appendix.

2 SVAR and the auxiliary model for the external instruments

We start from the SVAR system:

$$Y_t = \Pi X_t + \Upsilon_y D_{y,t} + u_t \ , \ \ u_t = B\varepsilon_t \ \ , \ \ t = 1, ..., T \tag{1}$$

where Y_t is the $n \times 1$ vector of endogenous variables, X_t:=$(Y'_{t-1}, ..., Y'_{t-k},)'$ is $nk \times 1$, $\Pi :=$ $(\Pi_1 : ... : \Pi_k)$ is the $n \times nk$ matrix containing the autoregressive (slope) parameters, $D_{y,t}$ is an d_y-dimensional vector containing deterministic components (constant, dummies, etc.) with parameters in the $n \times d_y$ matrix Υ_y; finally, u_t is the $n \times 1$ vector of *iid* reduced-form disturbances with positive definite covariance matrix $\Sigma_u := E(u_t u'_t)$. The initial conditions Y_{1-k}, ..., Y_0 are treated as given. The system of equations $u_t = B\varepsilon_t$ in eq. (1) maps the $n \times 1$ vector of *iid* structural shocks ε_t, which are assumed to have normalized unit covariance matrix $E(\varepsilon_t\varepsilon_t') := \Sigma_\varepsilon = I_n$, to the reduced form disturbances through the $n \times n$ matrix B.[2]

We call the elements in $(\Pi, \Upsilon_y, \Sigma_u)$ reduced-form parameters and the elements in B structural parameters or on-impact coefficients. Moreover, we use the terms 'identification of B', 'identification of the SVAR' and 'identification of the shocks' interchangeably. Let

$$A_y := \begin{pmatrix} \Pi_1 & \cdots & \Pi_k \\ I_{n(k-1)} & & 0_{n(k-1)\times n} \end{pmatrix} \tag{2}$$

be the VAR companion matrix. The responses of the variables in Y_{t+h} to one standard deviation structural shock ε_{jt} is captured by the IRFs:

$$IRF_j(h) := (J_n (A_y)^h J_n')b_j \ , \quad h = 0, 1, 2, ... \tag{3}$$

where b_j is the j-th column of B, $j = 1, ..., n$ and $J_n := (I_n : 0_{n\times n(k-1)})$ is a selection matrix such that $J_n J_n' = I_n$. Standard local and global identification results for the SVAR in eq. (1) are reviewed in the Supplementary Appendix A.2.

Our first assumption postulates the correct specification of the SVAR and the nonsingularity of the matrix of structural parameters B, the only formal requirement we place on this matrix, except where indicated.

Assumption 1 (DGP) *The data generating process belongs to the class of models in eq. (1) which satisfy the following conditions:*
(i) the companion matrix A_y in eq. (2) is stable, i.e. all of its eigenvalues lie inside the unit circle;
(ii) the matrix B is nonsingular.

Given Assumption 1 we consider, without loss of generality, the following partition of the vector of structural shocks:

$$\varepsilon_t := \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \quad \begin{matrix} g \times 1 \\ (n-g) \times 1 \end{matrix} \tag{4}$$

where $\varepsilon_{1,t}$ is the $g \times 1$ subvector of structural shocks henceforth denoted 'instrumented structural shocks', and $\varepsilon_{2,t}$ is the $(n-g)\times 1$ subvector of other structural shocks, denoted 'non-instrumented shocks'. The instrumented structural shocks are ordered first for notational convenience only:

[2] The structural shocks ε_t may also have diagonal covariance matrix $\Sigma_\varepsilon := diag(\sigma_1^2, ..., \sigma_n^2)$. In this case, the link between reduced form disturbances and structural shocks can be expressed in the form $u_t = B^*\Sigma_\varepsilon^{1/2}\varepsilon_t^*$, where $\varepsilon_t := \Sigma_\varepsilon^{-1/2}\varepsilon_t$ and B^* has exactly the same structure as B in eq. (1) except that the elements on the main diagonal are normalized to 1. Throughout the paper we follow the parameterization based on $u_t = B\varepsilon_t$ with $\Sigma_\varepsilon = I_n$, except where indicated.

the ordering of variables is irrelevant in our setup. Given the corresponding partition of reduced form VAR disturbances, $u_t := (u_{1,t}', u_{2,t}')$, where $u_{1,t}$ and $u_{2,t}$ have the same dimensions as $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, we partition the matrix of structural parameters B conformably with eq. (4):

$$B := \left(\underset{n\times g}{B_1} \;\vdots\; \underset{n\times(n-g)}{B_2} \right) := \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \quad \begin{array}{c|c} g\times g & g\times(n-g) \\ \hline (n-g)\times g & (n-g)\times(n-g) \end{array}. \tag{5}$$

In eq. (5), the dimensions of submatrices have been reported below and alongside blocks. B_1 is the submatrix containing the on-impact coefficients associated with the instrumented structural shocks $\varepsilon_{1,t}$, and B_2 is the submatrix containing the on-impact coefficients associated with the non-instrumented shocks $\varepsilon_{2,t}$; $rank(B_1) = g$ and $rank(B_2) = n-g$ because of Assumption 1(ii).

The external instruments approach postulates that given the partitions in eq.s (4)-(5), there are available $r \geq g$ observable 'external' (to the SVAR) variables called instruments, that we collect in the $r \times 1$ vector $v_{Z,t}$, which can be used to identify the dynamic causal effect of $\varepsilon_{1,t}$ on Y_{t+h}, $h = 0, 1, ...$, without the need to impose implausible assumptions on the elements of B. Thus, we can consider the instrumented structural shocks in $\varepsilon_{1,t}$ as the shocks of primary interest in the analysis and for which r valid external instruments are employed. However, as it will be shown below, there are cases in which despite $\varepsilon_{1,t}$ is instrumented, also the 'other' structural shocks in $\varepsilon_{2,t}$ might be of interest and identified under certain conditions. The key properties of $v_{Z,t}$ are formalized in the next assumption.

Assumption 2 (Relevance and orthogonality) *The $r \times 1$ vector $v_{Z,t}$ is generated by the system of equations:*

$$v_{Z,t} = R_\Phi \varepsilon_t + \omega_t = \Phi \varepsilon_{1,t} + \omega_t \tag{6}$$

where $R_\Phi := \left(\Phi : 0_{r\times(n-g)}\right)$, Φ is an $r \times g$ matrix of full column rank, and ω_t is a $r \times 1$ measurement error term uncorrelated with ε_t ($E(\varepsilon_t \omega_t') = 0_{n\times g}$) with positive definite covariance matrix $E(\omega_t \omega_t') := \Sigma_\omega < \infty$.

Assumption 2 ensures that the external instruments in $v_{Z,t}$ satisfy the conditions $E(v_{Z,t}\varepsilon_{1,t}') = \Phi \neq 0_{r\times n}$ ('relevance') and $E(v_{Z,t}\varepsilon_{2,t}') = 0_{r\times(n-g)}$ ('exogeneity', or 'orthogonality').[3] These conditions are typically presented in the proxy-SVAR literature under the setup $r = g$, see e.g. Stock and Watson (2012, 2018), Mertens and Ravn (2013, 2014), and require that the elements in $v_{Z,t}$ are correlated with the instrumented structural shocks $\varepsilon_{1,t}$ and are orthogonal to the other

Assumption 2 is consistent with a scenario in which the external instruments in $v_{Z,t}$ can potentially be correlated with past structural shocks, i.e. it may hold the condition $Cov(v_{Z,t}, \varepsilon_{t-i}) = E(v_{Z,t}\varepsilon_{t-i}') \neq 0_{r\times n}$, $i = 1, 2, ...$ which, because of eq. (6), requires $E(\omega_t \varepsilon_{t-i}') \neq 0_{r\times n}$, $i = 1, 2, ...$

shocks in $\varepsilon_{2,t}$.[4] The matrix $R_{\Phi} := \left(\Phi \,\vdots\, 0_{r\times(n-g)} \right)$ in eq. (6) characterizes the instruments validity as it collects the relevance and orthogonality conditions. We call Φ the 'relevance matrix' (or 'matrix of relevance parameters') thoughout the paper. The condition $rank(\Phi) = g$ in Assumption 2 ensures that each column of Φ carries independent - not redundant - information on the instrumented structural shocks. It will be shown in the next sections that $rank(\Phi) = g$ is a necessary condition for identification (up to sign normalization) which becomes also sufficient when $g = 1$. The additive measurement error ω_t in eq. (6) captures the idea that the external instruments are imperfectly correlated with the instrumented structural shocks; the covariance matrix Σ_ω can be possibly diagonal.

The 'one shock-one instrument' case mainly treated in the proxy-SVAR literature obtains for $r = g = 1$ and implies that $R_{\Phi} := \left(1 \,\vdots\, 0_{1\times(n-1)} \right)$ in eq. (6) is a row and $\Phi = \phi$ is a scalar. In this paper we focus on the general case $r \geq g \geq 1$ and explore the consequences of such generalization on the identification and (frequentist) estimation of proxy-SVARs. By considering the general case $r \geq g \geq 1$, we mimick the situation that occurs in the instrumental variable regressions when the number of instruments can be larger than the number of estimated parameters. As it will be shown next, when $r > g$, $(r - g)$ external instruments might be weakly correlated (or not correlated at all) with the g instrumented shocks without consequences on asymptotic inference if at least g external instruments are strongly correlated with the instrumented shocks.

To present our method it is convenient to generalize the auxiliary model for the external instruments postulated in eq. (6). Indeed, the data generating process specified for $v_{Z,t}$ in Assumption 2 maintains that the dynamics of the external instruments is expressed in 'innovation form, as $v_{Z,t}$ depends on the instrumented structural shocks $\varepsilon_{1,t}$ and the measurement error ω_t. In some situations, however, the practitioner might observe an $r\times 1$ vector of 'raw' (stationary) time dependent time series whose innovation part might potentially serve as external instruments. To account for these situations, we interpret $v_{Z,t}$ as the innovation part of Z_t, i.e. the quantity $v_{Z,t} := Z_t - E(Z_t \mid \mathcal{F}_{t-1})$, where $\mathcal{F}_{t-1}$ is the econometrician's information set available at time $t-1$. A specification consistent with the decomposition $Z_t = E(Z_t \mid \mathcal{F}_{t-1}) + v_{Z,t}$ is given by the dynamic system:

$$Z_t = \Theta(L) Z_{t-1} + \Gamma(L) Y_{t-1} + \Upsilon_z D_{z,t} + \Upsilon_{z,y} D_{y,t} + v_{Z,t} \tag{7}$$

where $\Theta(L) := \Theta_1 + ... + \Theta_p L^{p-1}$ is a matrix polynomial in the lag operator L, whose coefficients are in the $r\times r$ matrices Θ_i, $i = 1, ..., p$ (and can be possibly zero); $\Gamma(L) := \Gamma_1 + \Gamma_2 L + ... + \Gamma_q L^{q-1}$ is a matrix polynomial in the lag operator L whose coefficients are in the $r \times n$ matrices Γ_j,

Henceforth, the exogeneity of the external instruments with respect to the non-instrumented shocks will be denoted with the term 'ortogonality' in order to avoid ambiguities with the distinct concept of 'exogenous uncertainty' we discuss in the empirical section of the paper.

$j = 1, ..., q$ (and can be possibly zero); $D_{z,t}$ is an d_z-dimensional vector containing deterministic components (constant, dummies, etc.) specific to Z_t and not included in $D_{y,t}$; Υ_z and $\Upsilon_{z,y}$ are the $r \times d_z$ and $r \times d_y$ matrices of coefficients associated with $D_{z,t}$ and $D_{y,t}$, respectively (an can be possibly zero).

Equation (7) defines our auxiliary statistical model for the external instruments. It reads as a reduced form system where Z_t may depend on its own lags $Z_{t-1}, ..., Z_{t-p}$, the predetermined 'control' variables $Y_{t-1}, ..., Y_{t-k}$, and a set of deterministic terms. Obviously, $Z_t \equiv v_{Z,t}$ when all coefficient of the system in eq. (7) are zero, meaning that the elements in Z_t are already in innovations or *iid* shocks taken from other studies (see Section 8). When the Θ_i are non-zero, the stability requirement assumed for the SVAR (Assumption 1) is extended to Z_t and requires that the roots of $\det(I_r - \Theta_1 s - ... - \Theta_p s^p) = 0$ satisfy the condition $\mid s \mid > 1$. With this in mind, in the following we call 'external instruments' Z_t and $v_{Z,t}$ interchangeably.[5]

In the next section, the SVAR in eq. (1) will be combined with the auxiliary model in eq. (7) to form a 'larger' system which incorporates the dynamics of the external instruments in a coherent and efficient way.

3 The AC-SVAR representation

By coupling the SVAR in eq. (1) with the auxiliary model for the external instruments in eq. (7) we obtain the system:

$$\begin{pmatrix} Y_t \\ Z_t \end{pmatrix} = \sum_{j=1}^{\ell} \begin{pmatrix} \Pi_j & 0_{n\times r} \\ \Gamma_j & \Theta_j \end{pmatrix} \begin{pmatrix} Y_{t-j} \\ Z_{t-j} \end{pmatrix} + \begin{pmatrix} \Upsilon_y & 0_{n\times d_z} \\ \Upsilon_{z,y} & \Upsilon_z \end{pmatrix} \begin{pmatrix} D_{y,t} \\ D_{z,t} \end{pmatrix} + \begin{pmatrix} u_t \\ v_{Z,t} \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} u_t \\ v_{Z,t} \end{pmatrix} = \left(\begin{array}{c|c} B & 0_{n\times r} \\ \hline R_\Phi & \Sigma_\omega^{1/2} \end{array}\right) \begin{pmatrix} \varepsilon_t \\ \omega_t^{\circ} \end{pmatrix} \tag{9}$$

where $\ell := \max\{k, p, q\}$, $\Sigma_\omega^{1/2}$ denotes the symmetric square root of the matrix Σ_ω and the term $\omega_t^{\circ} := \Sigma_\omega^{-1/2}\omega_t$ can be here interpreted as a normalized measurement error.[6] System (8)

[5] Some of the external instruments in Z_t ($v_{Z,t}$) might be censored as in the case of narrative time series, see e.g. Mertens and Ravn (2013). The Supplementary Appendix A.11 summarizes how the approach presented in this paper can be amended to account for external instruments which are generated by censored autoregressive processes. A full treatment of this issue deserves a detailed analysis which goes beyond the scopes of the present article and is therefore postponed to future research. To our knowledge, Mertens and Raven (2013) and Jentsch and Lunsford (2019) are examples in which censoring is excplicitly accounted for in the current proxy-SVARs literature.

[6] Alternatively we might replace the square root matrix $\Sigma_\omega^{1/2}$ with e.g. the Choleski factor of Σ_ω, P_ω, and normalize the measurement error ω_t as $\omega_t^{\circ} := P_\omega^{-1}\omega_t$, without affecting results.

reads an m-dimensional VAR, $m := n + r$, of lag order ℓ which incorporates a constrained (triangular) autoregressive structure: the lags of Z_t and the deterministic variables in $D_{z,t}$ are not allowed to enter the Y_t-equations of the original SVAR. The matrices Γ_j and Θ_j, $j = 1, ..., \ell$ and $\Upsilon_{z,y}$ and Υ_z are restricted to zero in eq. (8) when $Z_t \equiv v_{Z,t}$. System (9) maps the term $\xi_t := (\varepsilon_t', \omega_t^{\circ\prime})'$ (which includes the structural shocks) onto $\eta_t := (u_t', v_{Z,t}')'$.

It is seen that the joint system (8)-(9) forms a large 'B-model' (Lütkepohl, 2005) which we call the 'augmented-constrained' SVAR (AC-SVAR) model. For future reference, we compact the AC-SVAR model in the expression:

$$W_t = \widetilde{\Psi} F_t + \widetilde{\Upsilon} D_t + \eta_t \quad , \quad \eta_t = \widetilde{G} \xi_t \tag{10}$$

where $W_t := (Y_t', Z_t')'$ and $\eta_t := (u_t', v_{Z,t}')'$ are $m \times 1$, the reduced form disturbance η_t has covariance matrix $\Sigma_\eta := E(\eta_t \eta_t')$, $F_t := (W_{t-1}', ..., W_{t-\ell}')'$ is $f \times 1$ ($f = m\ell$), $\widetilde{\Psi} := (\widetilde{\Psi}_1, ..., \widetilde{\Psi}_\ell)$ is $m \times f$, $D_t := (D_{y,t}', D_{z,t}')'$ is $d \times 1$ ($d := d_y + d_z$), $\widetilde{\Upsilon}$ is $m \times d$ and, finally, $\xi_t := (\varepsilon_t', \omega_t^{\circ\prime})'$ is $m \times 1$. We use the symbol '$\sim$' over the matrices Ψ and Υ and G to remark that these are restricted. The structure of the matrix $\widetilde{G}$ deserves special attention:

$$\widetilde{G} := \left(\begin{array}{c|c} B & 0_{n \times r} \\ \hline R_\Phi & \Sigma_\omega^{1/2} \end{array} \right) = \left(\begin{array}{c|c|c} B_1 & B_2 & 0_{n \times r} \\ \hline \Phi & 0_{r \times (n-g)} & \Sigma_\omega^{1/2} \end{array} \right). \tag{11}$$

It is seen that $\widetilde{G}$ contains the structural parameters in B, the relevance and (the zero) orthogonality conditions embedded in R_Φ and the parameters of the matrix $\Sigma_\omega^{1/2}$. The covariance restrictions implied by the AC-SVAR model (the ones stemming from $\Sigma_\eta = \widetilde{G}\widetilde{G}'$) boild down to

$$\Sigma_u = BB' \qquad \text{SVAR symmetry} \tag{12}$$

$$\Sigma_{v_Z,u} = \Phi B_1' \qquad \text{External instruments} \tag{13}$$

$$\Sigma_{v_Z} = \Phi\Phi' + \Sigma_\omega \qquad \text{External instruments.} \tag{14}$$

In the next sections we use the AC-SVAR model and the mapping in eq.s (12)-(14) to derive general necessary and sufficient conditions for identification and to put forth a novel estimation approach for proxy-SVARs.

4 Motivating example: exogenous/endogenous uncertainty in a small-scale SVAR

In this section we motivate empirically two types of identification strategies that the AC-SVAR model may feature. We discuss the exogeneity/endogeneity of measures of uncertainty with respect to the business cycle in a small-scale SVAR model, a topic which will be addressed empirically in Section 8 on U.S. monthly data.[7]

Consider a SVAR model for $Y_t{:=}(a_t, U_{F,t}, U_{M,t})'$ $(n = 3)$, where a_t is a measure of real economic activity, $U_{F,t}$ a measure of financial uncertainty and $U_{M,t}$ a measure of macroeconomic uncertainty. The relationship between the reduced form disturbances and the structural shocks is given by:

$$\underbrace{\begin{pmatrix} u_{a,t} \\ u_{F,t} \\ u_{M,t} \end{pmatrix}}_{u_t} = \underbrace{\begin{pmatrix} b_{a,a} & b_{a,F} & b_{a,M} \\ b_{F,a} & b_{F,F} & b_{F,M} \\ b_{M,a} & b_{M,F} & b_{M,M} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \varepsilon_{a,t} \\ \varepsilon_{F,t} \\ \varepsilon_{M,t} \end{pmatrix}}_{\varepsilon_t} \tag{15}$$

where $u_t{:=}(u_{a,t}, u_{F,t}, u_{M,t})'$ is the vector of VAR reduced form disturbances and $\varepsilon_t := (\varepsilon_{a,t}, \varepsilon_{F,t}, \varepsilon_{M,t})'$ is the vector of structural shocks. As is know, this SVAR is not identified in the absence of (at least three) restrictions on the matrix B. To inform the discussion, we temporarily label $\varepsilon_{a,t}$ as the 'non-uncertainty shock', $\varepsilon_{F,t}$ as the 'financial uncertainty shock' and $\varepsilon_{M,t}$ as the 'macroeconomic uncertainty shock'.

Since the seminal paper of Bloom (2009), attention in the empirical literature on uncertainty has been focused on measuring the impact of uncertainty shocks on real economic activity, which requires the identification of the parameters $b_{a,F}$ and $b_{a,M}$ in eq. (15). For instance, Stock and Watson (2012) use either stock market volatility or the economic policy uncertainty index of Baker *et al.* (2016) as external instruments to identify the effects of uncertainty shocks. In their framework, the parameters of interest are in the column $b_M{:=}(b_{a,M}, b_{F,M}, b_{M,M})'$ (or $b_F{:=}(b_{a,F}, b_{F,F}, b_{M,F})'$) of the matrix B. In this paper, we are interested in the other direction of causality, namely the impact of $\varepsilon_{a,t}$ on the variables $U_{F,t+h}$ and $U_{M,t+h}$ for $h = 0, 1, ...$ The on-impact responses $(h = 0)$ are captured by the parameters $b_{F,a}$ and $b_{M,a}$ contained in the first column $b_a{:=}(b_{a,a}, b_{F,a}, b_{M,a})'$ of B; the lagged responses $(h = 1, 2, ...)$ can be inferred from the IRFs in eq. (3) by setting $b_j = b_a$. As in Angelini *et al.* (2019), we consider financial and macroeconomic uncertainty 'exogenous' if $U_{F,t}$ and $U_{M,t}$ do not respond to $\varepsilon_{a,t}$ on-impact, which corresponds to the hypothesis $b_{F,a} = 0$ and $b_{M,a} = 0$ in eq. (15). Conversely, we consider financial and macroeconomic uncertainty 'endogenous' if $U_{F,t}$ and $U_{M,t}$ respond significantly on-impact to the non-uncertainty shock; see Carriero *et al.* (2018) for a similar characterization. Obviously, it may happen that $U_{F,t+h}$ and $U_{M,t+h}$ respond to $\varepsilon_{a,t}$ only after some periods the shock occurs, hence we distinguish between 'contemporaneous exogeneity' and lagged effects throughout the paper.

The reverse causality problem can not be addressed by placing 'conventional' restrictions

[7] As an exercise, the Supplementary Appendix A.4 applies the two identification strategies to a monetary SVAR model largely analyzed in the literature.

on the matrix B in eq. (15). Ludvigson *et al.* (2018), Angelini *et al.* (2018) and Carriero *et al.* (2018) face this issue by estimating non-recursive SVARs identified by different methods briefly reviewed in Section 8.3. In this paper, we argue that our proxy-SVAR approach can be a valuable alternative to the existing methods.

Partition the structural relationships in eq. (15) as follows:

$$\underset{u_t}{\begin{pmatrix} u_{a,t} \\ u_{F,t} \\ u_{M,t} \end{pmatrix}} = \underset{B_1 \equiv b_a}{\begin{pmatrix} b_{a,a} \\ b_{F,a} \\ b_{M,a} \end{pmatrix}} \underset{\varepsilon_{1,t}}{(\varepsilon_{a,t})} + \underset{B_2}{\begin{pmatrix} b_{a,F} & b_{a,M} \\ b_{F,F} & b_{F,M} \\ b_{M,F} & b_{M,M} \end{pmatrix}} \underset{\varepsilon_{2,t}}{\begin{pmatrix} \varepsilon_{F,t} \\ \varepsilon_{M,t} \end{pmatrix}} \tag{16}$$

which means that the structural shock of interest, and for which valid external instruments must be found is the non-uncertainty shock, i.e. $\varepsilon_{1,t} := (\varepsilon_{a,t})$ $(g = 1)$. Here $B_1 \equiv b_a$ coincides with the first column of B and contains the parameters of primary interest. Consider, as an example, the case in which $r = 2$ valid external instruments, collected in the vector $Z_t(v_{Z,t})$, are used for $\varepsilon_{a,t}$. The matrix $\widetilde{G}$ of the AC-SVAR model in eq. (11) reads:

$$:= \left(\begin{array}{c|c|c} B_1 & B_2 & 0_{3\times 2} \\ \hdashline \Phi & 0_{2\times 2} & \Sigma_{\varpi}^{1/2} \end{array}\right) = (\widetilde{G}_1 \vdots \widetilde{G}_2) := \left(\begin{array}{c|cc|cc} b_{a,a} & b_{a,F} & b_{a,M} & 0 & 0 \\ b_{F,a} & b_{F,F} & b_{F,M} & 0 & 0 \\ b_{M,a} & b_{M,F} & b_{M,M} & 0 & 0 \\ \hdashline \varphi_{1,a} & 0 & 0 & \varpi_{1,1} & \varpi_{2,1} \\ \varphi_{2,a} & 0 & 0 & \varpi_{2,1} & \varpi_{2,2} \end{array}\right) \tag{17}$$

where $\varphi_{1,a} := E(v_{Z_1,t}\varepsilon_{a,t})$ and $\varphi_{2,a} := E(v_{Z_2,t}\varepsilon_{a,t})$ are the relevance parameters contained in the 2×1 matrix Φ, and $\varpi_{1,1}$, $\varpi_{2,1}$, $\varpi_{2,2}$ are the 3 free elements of $\Sigma_{\varpi}^{1/2}$ (assumed here non-diagonal); recall from the previous section that Σ_{ϖ} is the covariance matrix of the measurement errors ω_t in eq. (6) and that with $\Sigma_{\varpi}^{1/2}$ we denote the 'square root' of Σ_{ϖ}. Observe that in eq. (17), $_1 := (b_a', \varphi')'$, where $b_a := (b_{a,a}, b_{F,a}, b_{M,a})'$ and $\varphi := (\varphi_{1,a}, \varphi_{2,a})'$.

The 'partial shocks' identification strategy identifies the parameters in the column $\widetilde{G}_1 := (b_a', \varphi')'$, hence the shock $\varepsilon_{a,t}$, regardless of the other shocks of the system. The idea is that $\widetilde{G}_1$ is the only ingredient (other than the reduced form parameters) necessary to track the dynamic causal effect of $\varepsilon_{a,t}$ on $U_{F,t+h}$ and $U_{M,t+h}$, $h = 0, 1, \ldots$. The other columns of $\widetilde{G}$, collected in $\widetilde{G}_2$, are not of interest. Since $Corr(v_{Z_1,t}, \varepsilon_{a,t}) = \varphi_{1,a}/\sigma_{v_{Z_1}}$ and $Corr(v_{Z_2,t}, \varepsilon_{a,t}) = \varphi_{2,a}/\sigma_{v_{Z2}}$, where $\sigma^2_{v_{Z_1}}$ and $\sigma^2_{v_{Z_2}}$ are the main diagonal elements of Σ_{v_Z}, the quality of the identification can be evaluated in this case by computing the measures $\hat{\varphi}_{1,a}/\hat{\sigma}_{v_{Z_1}}$ and $\hat{\varphi}_{2,a}/\hat{\sigma}_{v_{Z_2}}$, where $\hat{\varphi}_{1,a}$, $\hat{\varphi}_{2,a}$, $\hat{\sigma}_{v_{Z_1}}$ and $\hat{\sigma}_{v_{Z_2}}$ are consistent estimates of the parameters $\varphi_{1,a}$, $\varphi_{2,a}$, $\sigma_{v_{Z_1}}$ and $\sigma_{v_{Z_2}}$, respectively. In Section 5 we study the identification and estimation of the proxy-SVAR considering the case $r \geq g \geq 1$.

Suppose now that we have a some (limited) information on the non-instrumented structural shocks whose instantaneous effects are captured by the columns of the matrix B_2 in eq. (17). In particular, based on the results in Angelini *et al.* (2019), we claim that in the period after the Global Financial Crisis, the contemporaneous pass-trough between financial and macroeconomic uncertainty is one-way and runs from financial uncertainty to macroeconomic uncertainty, which implies $b_{F,M} = 0$ in eq. (17). While the original SVAR for $Y_t{:=}(a_t, U_{F,t}, U_{M,t})'$ is not identified with $b_{F,M} = 0$ in eq. (15), the AC-SVAR model based on $\widetilde{G}$ in eq. (17) and the additional restriction $b_{F,M} = 0$ is identified (see Section 8.2). The consequence of this result is that although the structural shock of primary interest is the non-uncertainty shock $\varepsilon_{1,t} := (\varepsilon_{a,t})$, we can also identify the financial and macroeconomic uncertainty shocks in $\varepsilon_{2,t} := (\varepsilon_{F,t}, \varepsilon_{M,t})'$ using instruments for $\varepsilon_{1,t}$ alone. We call this scenario the 'full shocks' identification strategy, which is studied in Section 6. Since both $\varepsilon_{1,t} := (\varepsilon_{a,t})$ and $\varepsilon_{2,t} := (\varepsilon_{F,t}, \varepsilon_{M,t})'$ can be identified, one can obtain $\hat{\varepsilon}_t := \hat{B}^{-1}\hat{u}_t$, $t = 1, ..., T$, where $\hat{u}_t$ are the reduced form VAR residuals and $\hat{B}$ is a consistent estimate of B recovered from $\widehat{\widetilde{G}}$. Accordingly, one way to evaluate the quality of the identification is computing the measures of strength $Corr(\hat{v}_{Z_1,t}, \hat{\varepsilon}_{a,t})$ and $Corr(\hat{v}_{Z_2,t}, \hat{\varepsilon}_{a,t})$ which should be significant with valid instruments, but also the correlations $Corr(\hat{v}_{Z_1,t}, \hat{\varepsilon}_{F,t})$, $Corr(\hat{v}_{Z_1,t}, \hat{\varepsilon}_{M,t})$, $Corr(\hat{v}_{Z_2,t}, \hat{\varepsilon}_{F,t})$ and $Corr(\hat{v}_{Z_2,t}, \hat{\varepsilon}_{M,t})$ which should not be statistically significant.[8] In this example, the 'price to pay' to move from a partial to a full shocks identification approach is given by the auxiliary restriction $b_{F,M} = 0$, which appears a modest cost relative to the benefits. In general, the investigator is required to take a (minimal) stand also on the structure of B_2 to identify all shocks.

5 Partial shocks identification and estimation strategy

In the partial shocks identification strategy, the objective of the analysis is to exploit the instruments in Z_t ($v_{Z,t}$) to solely identify the dynamic causal effects of the g instrumented shocks in $\varepsilon_{1,t}$, ignoring the other shocks collected in $\varepsilon_{2,t}$. This amounts to identify the submatrix B_1 in eq. (5), which in turn provides the IRFs in eq. (3) for $j = 1, ..., g$, $g < n$.

Our analysis starts from the AC-SVAR representation of the proxy-SVAR summarized in eq.s (10)-(11). We are interested in the first g columns of the matrix $\widetilde{G}$:

$$\widetilde{G}_1 := \begin{pmatrix} B_1 \\ \Phi \end{pmatrix}, \tag{18}$$

while the remaining $m - g$ columns in $\widetilde{G}_2$ are not of interest. The identification of the g columns $_1$ in eq. (18) reads as a partial identification exercise which requires imposing at least

Alternatively, one can compute F-type tests from regressions of the form $\hat{v}_{Z,t} = C_1\hat{\varepsilon}_{1,t} + C_2\hat{\varepsilon}_{2,t} + \epsilon_t$, $t = 1, ..., T$, where the rejection of $H_0^R : C_1 = 0_{r\times g}$ indicates that the relevance condition is supported by the data, and the non-rejection of $H_0^O : C_2 = 0_{r\times(n-g)}$ indicates that the orthogonality condition is supported by the data.

$1/2g(g-1)$ restrictions on $\widetilde{G}_1$ (i.e. on B_1 and Φ); obviously, no restriction is needed when $g = 1$. This necessary order condition for identification clearly shows that when $g > 1$, the r external instruments alone do not suffice to identify the shocks of interest, see e.g. Mertens and Ravn (2013), Mertens and Montiel Olea (2018) and Arias *et al.* (2018b). In principle, conditional on the validity of a rank condition we discuss below, the additional restrictions can be placed on the columns of B_1 leaving Φ free, or can be imposed on the columns of Φ (preserving the full column rank condition) leaving B_1 free, or possibly can be distributed on both B_1 and Φ.

It turns out that when the restrictions on $\widetilde{G}_1$ are homogenous (i.e. there are zero restrictions only) and separable across columns, one can check the identification of the proxy-SVAR by referring to the sufficient conditions for global identification established by Theorem 2 in Rubio-Ramirez *et al.* (2010). However, Theorem 2 in Rubio-Ramirez *et al.* (2010) provides only sufficient conditions for identification which are valid when the restrictions are homogeneous and separable across columns. To derive necessary and sufficient conditions for identification which are valid in more general situations, including the case of non-homogeneus, cross-columns restrictions, we find it convenient to exploit a set of moment conditions implied by the AC-SVAR model which pave the way for a CMD estimation approach.

The Supplementary Appendix A.5 shows that by using simple algebra, the moment conditions in eq.s (12)-(13) can be transformed into:

$$\Xi = \Phi\Phi' \quad , \quad \Sigma_{v_Z,u} = \Phi B_1' \tag{19}$$

where $\Xi := \Sigma_{v_Z,u}\Sigma_u^{-1}\Sigma_{u,v_Z}$ is an $r \times r$ symmetric matrix (of rank g) which is positive definite when $r = g$ and is positive semidefinite when $r > g$. The advantage of the representation in eq. (19), relative to that in eq.s (12)-(13) is that the nuisance parameters in B_2 have been marginalized out. The moment conditions in eq. (19) can be formally compacted in the expression:

$$\zeta = f(\vartheta) \tag{20}$$

where $\zeta := (vech(\Xi)', vec(\Sigma_{v_Z,u})')'$ is a vector whose elements depend on the reduced form parameters $\sigma_\eta^+ := vech(\Sigma_\eta)$ of the AC-SVAR model, ϑ is the vector which collects the unrestricted (free) elements in the matrix $\widetilde{G}_1$ in eq. (18) and $f(\vartheta) := (vech(\Phi\Phi')', vec(\Phi B_1')')'$ is a nonlinear differentiable vector function. The restrictions necessary to identify $\widetilde{G}_1$ are parameterized in explicit form by:

$$\beta_1 := vec(B_1) = S_{B_1}\alpha_1 + s_{B_1} \quad , \quad \phi := vec(\Phi) = S_\Phi \varphi \tag{21}$$

where α_1 is the $e_1 \times 1$ vector which collects the unrestricted (free) elements of β_1, $e_1 \leq ng$, $S_{B_1}$is an $ng \times e_1$ full column rank selection matrix and and s_{B_1} is an $ng \times 1$ vector containing zeros or known non-zero elements; φ is the $c \times 1$ vector which collects the unrestricted (free) elements of Φ, $c \leq rg$, and S_Φ is an $rg \times c$ full column-rank selection matrix. Obviously, when β_1 is unrestricted, $\beta_1 \equiv \alpha_1$, $e_1 \equiv ng$, $S_{B_1} \equiv I_{ng}$ and $s_{B_1} \equiv 0_{ng\times 1}$; when ϕ is unrestricted, $\phi \equiv \varphi$, $c = rg$ and $S_\Phi \equiv I_{rg}$. Because of the presence of the possibly non-zero term s_{B_1}, eq. (21) accommodates also non-homogeneous restrictions, which means that some elements of B_1 can be e.g. fixed to known non-zero constants. It is seen that $\vartheta := (\alpha_1', \varphi')'$ is $(e_1 + c) \times 1$.

Equation (20) defines a 'distance' between the $a \times 1$ ($a := 1/2r(r+1)+nr$) vector of reduced form parameters ζ and the $(e_1 + c) \times 1$ vector of parameters ϑ. From Rothenberg (1971) it follows that necessary and sufficient condition for ϑ being uniquely recovered from ζ is that the $a \times (e_1 + c)$ Jacobian matrix $\frac{\partial f}{\partial \vartheta'} := \digamma_\vartheta$ is regular and of full column rank in a neighborhood of the true value of ϑ.[9] We derive the Jacobian matrix $\digamma_\vartheta$ below.

Under Assumption 1, the estimator of the reduced form parameter σ_η^+ of the AC-SVAR model is consistent and asymptotically Gaussian, hence we have the result:

$$T^{1/2}(\hat{\zeta}_T - \zeta_0) \to_d N(0_{a\times 1}, \Omega_\zeta) \tag{22}$$

where ζ_0 denotes the true value of ζ, Ω_ζ is an $a \times a$ covariance matrix which can be estimated consistently (see Supplementary Appendix A.5 for details) and '$\to_d$' denotes convergence in distribution. The convergence in eq. (22) involves the estimator of the reduced form parameters of the AC-SVAR model and is valid also if Φ is zero, i.e. irrespective of whether the external instruments are strongly, weakly or not correlated at all with the structural shocks of interest. This result motivates a robust indirect test for the null hypothesis of 'no relevance' based on the idea that if in eq. (19) it is assumed that $B_1 \neq 0_{n\times g}$, the null hypothesis $H_0 : vec(\Phi) = 0_{rg\times 1}$ (no relevance) is equivalent to $H_0' : vec(\Sigma_{v_Z,u}) = 0_{rn\times 1}$ (no correlation between the external instruments and the VAR disturbances). We discuss a simple Wald-type test for H_0' in the Supplementary Appendix A.9.

Given eq.s (20) and the consistency of $\hat{\zeta}_T$, ϑ can be estimated by solving the CMD problem:

$$\min_{\vartheta}(\hat{\zeta}_T - f(\vartheta))'\widehat{\Omega}_\zeta^{-1}(\hat{\zeta}_T - f(\vartheta)) \tag{23}$$

where $\widehat{\Omega}_\zeta$ is a consistent estimate of Ω_ζ. The properties of the estimator $\hat{\vartheta}_T$ obtained from eq. (23) depend on the identification of the proxy-SVAR. The next proposition formalizes the necessary and sufficient rank conditions and the necessary order condition for the identification of the proxy-SVAR.

Proposition 1 [Partial shocks identification] Given the SVAR in eq. (1), a vector of r external instruments $v_{Z,t}$ for the $1 \leq g < n$ strucutral shocks in $\varepsilon_{1,t}$ and Assumptions 1-2,

[9] Let $M = M(\theta)$ be a matrix of rank m, whose elements depend on θ. M is 'regular' if $rank(M(\theta)) = m$ in a neighborhood of θ_0, where θ_0 is the true value of θ.

consider the identification of the g columns of $\widetilde{G}_1$ in eq. (18). Let $\vartheta_0 := (\alpha'_{1,0}, \varphi'_0)'$ be the true value of $\vartheta := (\alpha'_1, \varphi')'$. Then:

(a) necessary and sufficient rank condition for identification is

$$rank\{\mathcal{F}_{\vartheta_0}\} = e_1 + c$$

where $\mathcal{F}_{\vartheta_0}$ is the Jacobian matrix:

$$\mathcal{F}_{\vartheta} := \left(\begin{array}{c:c} 0_{1/2r(r+1)\times ng} & 2D_r^{+}(\Phi \otimes I_r) \\ \hdashline (I_n \otimes \Phi)K_{ng} & (B_1 \otimes I_r) \end{array}\right) \left(\begin{array}{c:c} S_{B_1} & 0_{ng\times c} \\ \hdashline 0_{rg\times e_1} & S_{\Phi} \end{array}\right) \tag{24}$$

evaluated in a neighborhood of ϑ_0 and is 'regular';[10]

(b) necessary order condition is that at least $1/2g(g-1)$ restrictions are placed on the g columns of $\widetilde{G}_1$, which is equivalent to the condition $e_1 + c \leq g(n+r) - 1/2g(g-1)$.

Proof: Supplementary Appendix A.3.

Some remarks are in order.

First, Proposition 1 provides an alternative to Mertens and Ravn's (2013) identification approach for proxy-SVARs with $g > 1$ multiple shocks. Mertens and Ravn (2013) show that when $g > 1$, the restrictions implied by the external instruments do not suffice alone to identify the g shocks of interest and must be complemented with additional constraints. In their setup, the $1/2g(g-1)$ additional constraints necessary to identify the proxy-SVAR stem from the mechanics of the IV approach and are obtained from a Choleski decomposition of a symmetric matrix. In our framework, the fact that it is necessary to impose at least $1/2g(g-1)$ restrictions to identify the g shocks of interest is a necessary order condition and the identification restrictions need not be Choleski-type constraints (the Supplementary Appendix A.7 compares in detailed Mertens and Ravn's (2013) identification approach with ours). Proposition 1 establishes necessary and sufficient conditions for the identification of the parameters in Φ and B_1 (i.e. of $\widetilde{G}_1$) which hold up to sign normalization, which means that if e.g. a given $\tilde{\Phi}$ satisfies the restriction $\Xi = \tilde{\Phi}\tilde{\Phi}'$ in eq. (19), also the matrix $\tilde{\Phi}^* \neq \tilde{\Phi}$, obtained from $\tilde{\Phi}$ by changing the sign of one or more than one of its columns, will satisfy eq. (19).

Given the $n \times g$ matrix M, K_{ng} denotes the $ng \times ng$ *commutation matrix* which satisfies $K_{ng}vec(M) = vec(M')$. $D_n^{+}{:=}(D_n'D_n)^{-1}D_n'$ denotes the Moore-Penrose inverse of D_n, where D_n is the $n^2 \times \frac{1}{2}n(n+1)$ *duplication matrix* such that $D_n vech(M) = vec(M)$, where $vech(M)$ is the column obtained from $vec(M)$ by eliminating all supra-diagonal elements. See Magnus and Neudecker (1999).

Second, according to Proposition 1(b), the proxy-SVAR is exactly identified when $e_1 + c = g(n+r) - 1/2g(g-1)$, i.e. when there are exactly $1/2g(g-1)$ restrictions on the elements of $\widetilde{G}_1$ in eq. (18), and is overidentified (and therefore testable) when $e_1 + c < g(n+r) - 1/2g(g-1)$.[11]

Third, Proposition 1 clarifies that in general, the full column rank condition of the matrix Φ (Assumption 2) is only necessary for identification. Indeed, the (2,1) block $(I_n \otimes \Phi)K_{ng}$ of the Jacobian matrix in eq. (24) suggests that $rank\{\Phi\} = g$ is necessary for $rank\{(I_n \otimes \Phi)K_{ng}\} = ng$, which in turn is a necessary condition for $rank\{\digamma_{\vartheta_0}\} = e_1 + c$. The structure of $\digamma_{\vartheta}$ also shows that when $g = 1$, the full column rank condition of Φ is also sufficient for the identification of the proxy-SVAR (whatever r). This is easily seen in the 'one shock-one instrument' case $r = g = 1$, where $\Phi = \phi = \varphi$ is a scalar ($c = 1$) and if β_1 is unrestricted ($\beta_1 \equiv \alpha_1$) the Jacobian $\digamma_{\vartheta}$ collapses to the $(n+1) \times (n+1)$ matrix:

$$\digamma_{\vartheta} := \left(\begin{array}{cccc|c} 0 & 0 & \cdots & 0 & 2\varphi \\ \hline & \varphi I_n & & & \alpha_1 \end{array} \right);$$

it is seen that $\varphi = \phi \neq 0$ is necessary and sufficient for identification.[12] The form of the Jacobian in eq. (24) also shows that one of the advantages of using more than one instrument to identify a single shock of interest, $r > g = 1$, is that $rank(\Phi) = 1$ if at least one component in $\Phi := (\varphi_1, ..., \varphi_r)'$ is different from zero, which means that provided at least one external instrument is strongly correlated with the shock of interest, $r - 1$ external instruments might potentially violate the relevance condition. This argument can be easily generalized to the case $r > g > 1$.

Coming back to the estimation problem (23), under Assumptions 1-2 and the conditions of Proposition 1 we have (Newey and McFadden, 1991):

$$T^{1/2}(\hat{\vartheta}_T - \vartheta_0) \to_d N(0_{(e_1+c)\times 1}, \Omega_{\vartheta}) \ , \ \ \Omega_{\vartheta} := \left(\digamma'_{\vartheta} \ \Omega_{\zeta}^{-1} \digamma_{\vartheta} \right)^{-1} \tag{25}$$

where the asymptotic covariance matrix Ω_{ϑ} can be estimated consistently by $\widehat{\Omega}_{\vartheta,T} := \left(\hat{\digamma}'_{\vartheta} \ \widehat{\Omega}_{\zeta}^{-1} \hat{\digamma}_{\vartheta} \right)^{-1}$ and $\hat{\digamma}_{\vartheta}$ is taken from eq. (24) by replacing the unconstrained (free) elements in B_1 and Φ with the corresponding elements in $\hat{\vartheta}_T := (\hat{\alpha}'_{1,T}, \hat{\varphi}'_T)'$.

[11] When $r = g$ one has $a = 1/2g(g+1) + ng$ and $e_1 + c = g(n+g) - 1/2g(g-1) = 1/2g(g+1) + gn$ under exact identification, hence the Jacobian matrix in eq. (24) is square. Instead, when $r > g$, the Jacobian matrix in eq. (24) is 'tall' (meaning that it has more rows than columns) even in the case of exact identification, i.e. $a = 1/2r(r+1) + nr = r^2 - 1/2r(r-1) + nr > g(n+r) - 1/2g(g-1) = e_1 + c$.

[12] The structure of this Jacobian matrix shows that if the true value of φ satisfies the local-to-zero embedding: $\varphi_0 := T^{1/2}\varrho$, $\varrho \neq 0$, the proxy-SVAR is not identified asymptotically and the estimator $\hat{\vartheta}_T$ discussed below is not consistent. We refer to Lunsford (2015) and Montiel Olea *et al.* (2018) for inference in proxy-SVARs in the presence of 'weak' instruments.

When according to Proposition 1 the proxy-SVAR model is overidentified, the CMD problem delivers a test of overidentifying restrictions because, under the null hypothesis $\zeta_0 = f(\vartheta_0)$ and Assumptions 1-2 the quantity $TQ(\hat{\vartheta}_T)$ converges asymptotically to a $\chi^2(l)$ random variable with $l := g(n+r) - 1/2g(g-1) - (e_1 + c)$ degree of freedoms. In the IV (GMM) framework, when the number of instruments (moment conditions) is larger than the number of estimated parameters, it is possible to compute Sargan's specification test (Hansen's J-test), which is typically interpreted as a specification test for the estimated model. The $TQ(\hat{\vartheta}_T)$ test can be used similarly. Our Monte Carlo experiments (summarized in the Supplementary Appendix A.8 to save space) show that the test $TQ(\hat{\vartheta}_T)$ rejects the overidentification restrictions when the external instruments are incorrectly assumed orthogonal to the non-instrumented structural shocks.

In case of exact identification, or when the overidentification restrictions are not rejected by the $TQ(\hat{\vartheta}_T)$ test, the IRFs of interest in eq. (3) can be estimated by replacing the companion matrix A_y with its consistent estimate $\widehat{A}_y$ derived from the AC-SVAR model and b_j with the j-th column of $\hat{B}_1$, for $j = 1, .., g$, where $\hat{B}_1$ is reconstructed from $\hat{\beta}_{1,T} := S_{B_1}\hat{\alpha}_{1,T} + s_{B_1}$. We refer to Jentsch and Lunsford (2019) and Mertens and Ravn (2019) for a recent debate on bootstrap inference for IRFs in proxy-SVARs; see also Montiel Olea *et al.* (2018). Since in our framework the proxy-SVAR is specified as a large (constrained) 'B-model', bootstrap confidence bands for the IRFs can be obtained by applying the methods currently available for SVARs reviewed e.g. in Kilian and Lütkepohl (2017, Ch. 12). In particular, when the disturbances $\eta_t := (u_t', v_{Z,t}')'$ in eq. (10) are conditionally homoskedastic, it is possible to combine residual-based bootstrap methods with the CMD approach by the algorithm summarized in the Supplementary Appendix A.10. Accordingly, the (innovations associated with the) external instruments are resampled jointly with the VAR residuals regardless of the number of instruments r and instrumented structural shocks g. For instance, in Section 8 we compute 90%-bootstrap simultaneous confidence bands for the IRFs by using the 'sup-t' bands discussed in Montiel Olea and Plagborg-Møller (2019). If instead the disturbances η_t in eq. (10) display conditional heteroskedasticity of unknown form, the results in Bruggemann *et al.* (2016) and Jentsch and Lunsford (2016, 2019) suggest that reliable inference must be based on the residual-based moving block bootstrap.

6 Full shocks identification and estimation strategy

In this case we investigate the conditions under which the instruments Z_t ($v_{Z,t}$) used to identify the structural shocks of interest $\varepsilon_{1,t}$ permit to identify the dynamic causal effects of all structural shocks in ε_t, including the non-instrumented ones in $\varepsilon_{2,t}$.[13] We formalize the identification analysis and estimation of proxy-SVARs in these situations and show that (frequentist) estimation of these models can be conveniently carried out by ML.

As in the partial shocks approach, our starting point is the AC-SVAR representation of the proxy-SVAR summarized in eq.s (10)-(11). This is a large 'B-model' whose identification depends on the (number and structure of) restrictions which characterize the matrix $\widetilde{G}$. Equation (11) shows that $\widetilde{G}$ incorporates by construnction $r(n-g)+nr$ zero restrictions plus the symmetry constraints stemming from the matrix $\Sigma_\omega^{1/2}$; however, these do not generally suffice to obtain at least $1/2m(m-1)$ restrictions necessary to identify the m columns of $\widetilde{G}$ (up to sign). It turns out that aside from special cases discussed below, a few restrictions on B_2, other than on B_1 and Φ, are necessary for identification. In the uncertainty example discussed in Section 4, if one adds the constraint $b_{F,M}=0$ in the sub-matrix B_2 of $\widetilde{G}$ in eq. (17), and if e.g. the covariance matrix of measurement errors Σ_ω is diagonal (which implies $\omega_{12}=\omega_{21}=0$), there are 13 homogeneous restrictions in $\widetilde{G}$ which are separable across columns (there are therefore 3 overidentification restrictions); it is possible to prove that in this case the model is identified.

When the restrictions on $\widetilde{G}$ are homogeneous and separable across columns, it is convenient to study the identification of the AC-SVAR model by checking whether the sufficient conditions for (global) identification in Theorem 1 of Rubio-Ramirez et al. (2010) are satisfied. We derive necessary and sufficient conditions for (local) identification which are valid in more general situations, including the case of non-homogeneus, cross-columns linear restrictions. To do so, we formalize the restrictions on B_1 and Φ as in eq. (21) but, in addition, we include also constraints on B_2 and $\Sigma_\omega^{1/2}$ as follows:

$$\beta_2 := vec(B_2) = S_{B_2}\alpha_2 \; + s_{B_2}, \quad \omega := vech(\Sigma_\omega^{1/2}) = S_{\Sigma_\omega}\varpi. \tag{26}$$

In eq. (26), α_2 is the vector collecting the e_2 unrestricted (free) elements of B_2 (if any), S_{B_2} is an $n(n-g)\times e_2$ full column rank selection matrix and $s_{B_2}$is an $n(n-g)\times 1$ vector containing zeros and known elements; obviously $S_{B_2}\equiv I_{n(n-g)}$, $\beta_2\equiv\alpha_2$ and $s_{B_2}=0_{n(n-g)\times 1}$ when β_2 is unrestricted; ϖ is the vector containing the s_ω unrestricted (free) non-zero elements of $\Sigma_\omega^{1/2}$ and S_{Σ_ω} is an $1/2r(r+1)\times s_\omega$ full column rank selection matrix, where s_ω:=$1/2r(r+1)$ when Σ_ω is full and s_ω:=r when Σ_ω is diagonal. Summing up, the identification restrictions featured by

In the current proxy-SVAR literature, a concrete example where an identification strategy based on external instruments identifies all shocks of the systen is Caldara and Kamps's (2017) fiscal framework. In a system of $n=5$ variables, they use $r=3$ non-fiscal instruments to identify $g=3$ non-fiscal shocks (output, inflation and monetary policy) but simultaneously they jointly identify tax and spending shocks ($n-g=2$) under the additional constraint that government spending does not respond contemporaneously to taxes.

the matrix $\widetilde{G}$ in eq. (11) can be represented (in explicit form) as:

$$vec(\widetilde{G}) = S_{\widetilde{G}}\theta + s_{\widetilde{G}} \tag{27}$$

where $\theta := (\alpha_1', \alpha_2', \varphi', \varpi')'$ has dimension $a_{\widetilde{G}} \times 1$, $a_{\widetilde{G}} := e_1 + e_2 + c + s_\omega$, S_G is an $m^2 \times a_{\widetilde{G}}$ full column rank selection matrix which depends on S_{B_1}, S_{B_2}, S_Φ and S_{Σ_ω}, respectively, and $s_{\widetilde{G}}$ is a known $m^2 \times 1$ vector. The next proposition provides the necessary and sufficient rank conditions for the (local) identification of the AC-SVAR model and the necessary order conditions.

Proposition 2 [Full shocks identification] Given the SVAR in eq. (1), a vector of r external instruments $v_{Z,t}$ for the $1 \leq g < n$ strucutral shocks in $\varepsilon_{1,t}$ and Assumptions 1-2, consider the identification of all shocks in $\varepsilon_t := (\varepsilon_{1,t}', \varepsilon_{2,t}')'$, i.e. the identification of the first n columns of $\widetilde{G}$ in eq. (11). Let $\theta_0 := (\alpha_{1,0}', \alpha_{2,0}', \varphi_0', \varpi_0')'$ be the true value of $\theta := (\alpha_1', \alpha_2', \varphi', \varpi')'$. Then:

(a) necessary and sufficient rank condition for identification is:

$$rank\left\{D_m^+(\widetilde{G}_0 \otimes I_m)S_{\widetilde{G}}\right\} = a_{\widetilde{G}} \tag{28}$$

where $\widetilde{G}_0$ is the matrix $\widetilde{G}$ evaluated in a neighborhood of θ_0 and is 'regular';

(b) necessary order condition for identification is: $a_{\widetilde{G}} \leq \frac{1}{2}m(m+1)$.

Proof: Supplementary Appendix A.3.

Some remarks are in order.

First, according to Proposition 2, the identification of the shocks in $\varepsilon_t := (\varepsilon_{1,t}', \varepsilon_{2,t}')'$ based on r instruments for $\varepsilon_{1,t}$ may occur in two situations: (i) when $g < (n-1)$ provided a few restrictions are also placed on B_2 (see the example discussed in Section 4); (ii) when $g = (n-1)$ (all structural shocks of the system are instrumented but one) provided the rank condition in eq. (28) holds.[14]

Second, when the AC-SVAR model is overidentified, the system features $l := \frac{1}{2}m(m+1) - a_{\widetilde{G}}$ testable restrictions which can be used to assess the empirical validity of the estimated proxy-SVAR, see the next section.

Third, since the analysis is based on the factorization $\Sigma_\eta = \widetilde{G}\widetilde{G}'$, also in this case identification holds up to sign normalization.

If the conditions of Proposition 2 are valid, the estimation of the AC-SVAR model in eq.s (10)- amounts to the estimation of a 'B-model' and can be carried out by ML by simply adapting

When $n = g - 1$ and $B_2 := b_2$ is left unrestricted, the total number of restrictions featured by the matrix $\widetilde{G}$ in eq. (11), denoted ϱ_1, is such that $\varrho_1 \geq 1/2m(m-1)$ for $r \geq g$, which means that the necessary order condition is always satisfied.

the algorithms reported in Amisano and Giannini (1997) and Lütkepohl (2005).[15] The Supplementary Appendix A.6 reviews the specification steps necessary to obtain the (concentrated) log-likelihood associated with the reduced form model, denoted $L_T(\sigma_\eta^+)$, and the (concentrated) log-likelihood associated with the structural form, denoted $L_T^s(\theta)$. Under Assumptions 1-2 and Proposition 2, the ML estimator $\hat{\theta}_T := \max_\theta L_T^s(\theta)$ is consistent and asymptotically Gaussian. Moreover, when the AC-SVAR model is overidentified, it is possible to compute the LR test $LR_T := -2(L_T^s(\hat{\theta}_T) - L_T(\hat{\sigma}_{\eta,T}^+))$ which under the null of correct specification is distributed asymptotically as a $\chi^2(l)$ variable with $l := \frac{1}{2}m(m+1) - a_{\widetilde{G}}$ degrees of freedom. This overidentification restrictions test can be interpreted similarly to the $TQ(\hat{\vartheta}_T)$ test discussed in Section 5, hence LR_T tends to reject the proxy-SVAR model when e.g. the external instruments are wrongly assumed orthogonal to the non-insrumented shocks, see the Monte Carlo resuls in the Supplementary Appendix A.8.

In case of exact identification, or if the overidentification restrictions are not rejected by the LR test, the IRFs are estimated from eq. (3) by replacing A_y with the consistent estimate $\hat{A}_y$ derived from the reduced form of the AC-SVAR model, and by replacing b_j with the j-th column of $\hat{\widetilde{G}} = \widetilde{G}(\hat{\theta}_T)$, $j = 1, ..., n$. In this case the computation of bootstrap confidence bands for the IRFs requires bootstrapping a SVAR model: see Section 5 and the Supplementary Appendix A.10.

7 Connections with the literature

The approach presented in the previous sections has several connections with the proxy-SVAR literature. Stock and Watson (2012, 2018), Mertens and Ravn (2013, 2014) and Montiel Olea *et al.* (2018) are seminal contributions in proxy-SVARs estimated by IV methods; see also Jentsch and Lunsford (2016).[16] Plagborg-Møller and Wolf (2018) is the only contribution in the frequentist framework (other than ours) where an auxiliary model for the external instruments plays an active role in the analysis. They consider a proxy-SVAR model similar to system (7) but with infinite lags for the variables. Plagborg-Møller and Wolf (2018) cover the case $r \geq g = 1$ and discuss inference on variance amd historical decompositions in a general semiparametric moving average model. Extending our approach to the infinite order case as in Plagborg-Møller and Wolf (2018) requires moving to a frequency domain approach.

Any econometric package which features the estimation of SVARs can be used or adapted to this scope. In practice, it is necessary to estimate a SVAR model for $W_t := (Y_t', Z_t')'$ by incorporating zero restrictions in the autoregressive coefficients.

Important applied developments based on IV methods include, among others, Gertler and Karadi (2015), Carriero *et al.* (2015) and Caldara and Kamps (2017).

In the Bayesian framework, the idea of appending the external instruments to the original SVAR model is not new. Caldara and Herbst (2019) consider the 'one shock-one instrument' case $r = g = 1$ and add an external instrument to the original SVAR system to identify a monetary policy shock (in their framework $Z_t \equiv v_{Z,t}$, hence the parameters Γ_js, Θ_js and Υ_z and $\Upsilon_{z,y}$ are zero in eq. (7)). Arias *et al.* (2018b) consider the case $r = g \geq 1$ and a dynamic representation of the proxy-SVAR similar to ours. However, while our AC-SVAR specification corresponds to a large (constrained) 'B-model', Arias *et al.* (2018b)'s parameterization reads as an large (constrained) 'A-model' (Lütkepohl, 2005). Their identification strategy features both zero and sign restrictions and modifies Arias *et al.* (2018a)'s algorithm to account for the highly constrained parametric structure of the proxy-SVAR model. In line with (and independently from) our analysis, Arias *et al.* (2018b) recognize that when $g > 1$, the additional (zero or sign) restrictions necessary to identify the structural shocks need not be Choleski-type constraints. Interestingly, these authors also observe that the additional identification restrictions that complement the restrictions implied by the external instruments can possibly be extended to the part of the system which pertains to the non-instrumented shocks, which fits with the logic of our identification strategy developed in Section 6.

Compared to the above mentioned contributions, we show that proxy-SVARs with multiple instruments and multiple instrumented shocks, $r \geq g \geq 1$, can be conveniently represented as 'B-models' with advantages in the identification and estimation. In our framework, the analysis of proxy-SVARs is not necessarily confined to partial identification strategies but depends on the overall information available to the practitioner. The suggested CMD and ML estimation approaches are novel in the proxy-SVAR literature and straighforward to implement in practice.

8 Empirical application

In this section we apply our methodology to investigate a recently debated issue of the empirical uncertainty literature, i.e. whether uncertainty is an exogenous source of business cycle fluctuations, or an endogenous response to first moment shocks, or both. In Section 4 we have already anticipated some of the technical challenges that the empirical investigation of this problem rises in the context of small-scale proxy-SVARs. Well documented facts suggest that heightened uncertainty triggers a contraction in real economic activity, and that uncertainty tends to be higher during economic recessions, see e.g. Bloom (2009), Stock and Watson (2012), Christiano *et al.* (2014), Jurado *et al* (2015), Carriero *et al.* (2015), Caggiano *et al.* (2017) and Angelini *et al.* (2019), just to mention a few. It is less clear, however, whether the higher uncertainty observed in correspondence of periods of high economic and financial turmoil is rather a consequence of first moment shocks hitting the business cycle, not the cause. The empirical assessment of the exogeneity/endogeneity of uncertainty is not only important to discriminate among two broad classes of theories about the origins of uncertainty, excellently reviewed in Ludvigson *et al.* (2018), but also for its policy implications. Indeed, as suggested by Bloom (2009, pages 626-7), uncertainty shocks may induce a trade-off between policy 'correctness' and 'decisiveness' - it may be better to act decisively (but occasionally incorrectly) than to deliberate on policy, generating policy-induced uncertainty.

We address the exogeneity/endogeneity problem by using a small-scale SVAR model for $Y_t := (a_t, U_{F,t}, U_{M,t})'$ $(n = 3)$, including a measure of real economic activity (a_t), a measure of financial uncertainty $(U_{F,t})$ and a measure of macroeconomic uncertainty $(U_{M,t})$. As explained in Ludvigson *et al.* (2018), the joint use of macroeconomic and financial uncertainty is crucial to disentangle the contributions of two distinct sources of uncertainty and study their pass through to the business cycle. The direction of causality we are concerned with runs from non-uncertainty shocks to financial and macroeconomic uncertainty, not the other way around. As argued in Section 4, one way to achieve this objective is to use a set of external instruments for $\varepsilon_{a,t}$, see the matrices B and $\widetilde{G}$ in eq.s (15)-(17).

In Section 8.1 we summarize the data, in Section 8.2 we present the empirical results obtained with our baseline AC-SVAR model and in Section 8.3 we compare our findings with those obtained by other authors.

8.1 Data

Real economic activity a_t is proxied by the growth rate of the log of the U.S. industrial production index, $a_t := \Delta ip_t$ (source Fred); financial uncertainty $U_{F,t}$ is proxied by a measure of 1-month ahead financial uncertainty taken from Ludvigson *et al.* (2018); macroeconomic uncertainty $U_{M,t}$ is proxied by a measure of 1-month ahead macroeconomic uncertainty taken from Jurado *et al.* (2015).[17] We consider $T = 88$ monthly observations and the same variables as in Ludvigson *et al.* (2018), but differently from these authors, we do not estimate the model on the entire period 1960-2015, but on the subsample 2008M1-2015M4 that we term the 'Great Recession + Slow Recovery' period. Our choice of considering only the period after the Global Financial Crisis is motivated by the empirical results in Angelini *et al.* (2019), who show that the unconditional error covariance matrix of the VAR for $Y_t := (a_t, U_{F,t}, U_{M,t})'$ is affected by at least two major structural breaks in the period 1960-2015. The 'Great Recession + Slow Recovery' period 2008M1-2015M4 is particularly informative to infer whether uncertainty measures respond on-

[17] We consider a version of the index $U_{M,t}$ 'purged' from possible effects of financial variables, see Angelini *et al.* (2019) for details

impact to non-uncertainty shocks as it broadly coincides with the zero lower bound constraint on the short-term nominal interest rate. According to Plante *et al.* (2018)'s argument, uncertainty should be triggered by first moment shocks in this period because of the Fed's inability to offset adverse shocks by conventional policies; see also Basu and Bundick (2015).[18]

8.2 Non-uncertainty shock, empirical results

We are primarily interested in the parameters in the column $b_a := (b_{a,a}, b_{F,a}, b_{M,a})'$ which enters the matrix $\widetilde{G}$ in eq. (17) and capture the instantaneous impact of the shock $\varepsilon_{a,t}$. The specification in eq. (17) pertains to an AC-SVAR model for $W_t := (Y_t', Z_t')'$, where Z_t contains $r = 2$ external instruments for the non-uncertainty shock of the system. In principle, we might include variables in Z_t selected from a set of external instruments correlated with real economic activity, including proxies for the technology shock, the oil shock, investors confidence shocks, loan demand and supply shocks, to give a few examples relating to both aggregate supply and aggregate demand shocks.[19] Unfortunately, given the monthly frequency of our variables and the estimation sample we consider, it is not immediate to find monthly analogs of the series of shocks largely available in the literature at the quarterly frequency, see e.g. Ramsey (2016). However, the flexibility of the AC-SVAR approach allows us to use 'raw' time series in Z_t and employ, under Assumption 2, the reduced form innovations $v_{Z,t} := Z_t - E(Z_t \mid \mathcal{F}_{t-1})$ as external instruments.

We consider the following external instruments for the real economic activity shock $\varepsilon_{a,t}$: (a) innovations obtained from an auxiliary model for $\Delta house_t$, where $house_t$ is the log of new privately owned housing units started (source: Fred); (b) an oil supply shock constructed by following Kilian's (2009) identification strategy, denoted oil_t (see Supplementary Appendix A.12.1 for details); (c) innovations obtained from an auxiliary model for $\Delta hours_t$, where $hours_t$ is the log of hours worked (source: Fred). The baseline AC-SVAR model estimated in this section employes (a,b) as external instruments for $\varepsilon_{a,t}$. Oil shocks might be weak instruments for real economic activity (Stock and Watson, 2012), but this should not in principle affect standard asymptotic inference if the other instrument is strong, see Section 5. However, in the Supplementary Appendix A.12.2 we also estimate an analog of the baseline AC-SVAR model based on

[18] Interestingly, Pellegrino (2017) compares the real effects of a monetary shock in tranquil and turbulent periods by distinguishing the cases of endogenous and exogenous uncertainty. He reports that the responses of real variables to a monetary policy shock gets halved when uncertainty is treated as an endogenous variable.

[19] We prefer not to consider explicitly a monetary policy shock among the list of candidate external instruments for the real economic activity shock. As is known, assessing the impact of unconventional policy (given the sample 2008M1-2015M4) is more challenging than it is for conventional policy, see, among others, Gertler and Karadi (2015) and Roger *et al.* (2018). In the Supplementary Appendix A.12.5 we check to what extent the empirical results obtained in this section are affected by the inclusion of Wu and Xia (2016)'s 'shadow policy rate' (other than the inflation rate) in the system.

the external instruments (a, c), i.e. not including oil shocks.

While oil_t is in shock form, $\Delta house_t$ is a 'raw' variable from which we derive the innovations $v_{Z_1,t} := \Delta house_t - E(\Delta house_t \mid \mathcal{F}_{t-1})$ by estimating an auxiliary dynamic model which is appendend to the original SVAR model. Since $\Delta house_t$ can be considered a predictor of real economic activity, it is reasonable to conjecture that the innovations $v_{Z_1,t} := \Delta house_t - E(\Delta house_t \mid \mathcal{F}_{t-1})$ are not contemporaneosuly correlated with financial and macroeconomic uncertainty shocks.[20]

The baseline AC-SVAR model is given by the VAR for $Y_t := (a_t, U_{F,t}, U_{M,t})'$ along with the auxiliary model for $Z_t := (Z_{1,t}, Z_{2,t})' = (\Delta house_t, \, oil_t)'$ which jointly form the system:

$$Y_t = \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \Pi_3 Y_{t-3} + \Pi_4 Y_{t-4} + \Upsilon_y + u_t \quad (29)$$

$$Z_t = \Theta_1 Z_{t-1} + \Theta_2 Z_{t-2} + \Gamma_1 Y_{t-1} + \Gamma_2 Y_{t-2} + \Upsilon_z + v_{Z,t} \quad (30)$$

where $m = n + r = 5$ and $\ell^{op} = k = 4$. Here the second rows of the matrices Θ_1, Θ_2, Γ_1, Γ_2 and the vector Υ_z (the ones associated with the $Z_{2,t}$-equation) are fixed to zero because $Z_{2,t} \equiv v_{Z_2,t} = oil_t$. The reduced form disturbances $\eta_t := (u_t', v_{Z,t}')'$ of system (29)-(30) are linked to $\xi_t := (\varepsilon_t', \omega_t^{\circ\prime})'$ by the relationship $\eta_t = \widetilde{G}\xi_t$, where the form of the matrix $\widetilde{G}$ is given in eq. (17).

[TABLE 1 HERE]

System (29)-(30) is estimated by setting to zero the autoregressive coefficients in Θ_1, Θ_2, Γ_1 and Γ_2 which are not statistically significant at the 5% significance level. A battery of diagnostic tests on the disturbances of the AC-SVAR model are summarized in Panel A of Table 1, where we report equation-wise: a test of normality; a test for the absence of serial autocorrelation; and a test for the absence of conditional heteroskedasticity. To save space, we have not reported the estimated reduced form parameters, including the covariance matrix Σ_η. Panel A of Table 1 shows that aside from the non-normality of the residuals associated with the a_t-equation, the specified model is neither affected by serial autocorrelation, nor by conditional heteroskedasticity in the residuals. As observed in the previous sections, the absence of conditional heteroskedasticity in the residuals is particularly important for bootstrap inference on the IRFs, see below. The last column in Panel A of Table 1 also summarizes F-type tests for the null hypothesis that (four) lags of Z_t are not statistically significant in the Y_t-equations. These Granger causality tests largely support the triangular structure that characterizes the autoregressive parameters of the AC-SVAR model, see eq.s (8)-(10). This result remarks that the only possible connection between the two external instruments in $v_{Z,t} := (v_{Z_1,t}, v_{Z_2,t})'$ and

[20] It is reasonable to assume that $E(v_{Z_1,t}\varepsilon_{2,t}') = 0_{1\times 2}$, while it is not possible to rule out the condition $E(v_{Z_1,t}\varepsilon_{2,t-i}') \neq 0_{1\times 2}$, $i = 1, 2, ...$ (which however does not violate Assumption 2). The orthogonality condition $E(v_{Z_1,t}\varepsilon_{2,t}') = 0_{1\times 2}$ will be tested empirically.

the instrumented structural shock $\varepsilon_{a,t}$ is by the covariance matrix $\Sigma_{v_Z,u} := E(v_{Z,t}u_t')$ which is a key element of system (29)-(30) and of the proxy-SVAR approach. A Wald test for the null hypothesis $H_0' : vec(\Sigma_{v_Z,u}) = 0_{6\times 1}$ (no correlation between the external instruments and the VAR disturbances) against $H_1' : vec(\Sigma_{v_Z,u}) \neq 0_{6\times 1}$, see Section 5 and the Supplementary Appendix A.9, is equal to W_T^{rel}:=8.55 with a p-value of 0.01, hence the reduced form evidence tends to indirectly reject the hypothesis that the relevance parameters are zero at the 5% level of significance.

Partial shocks identification strategy

In the partial shocks identification strategy, we are solely interested the identification and estimation of the parameters $\widetilde{G}_1 := (b_a', \varphi')' = \vartheta$, where $b_a := (b_{a,a}, b_{F,a}, b_{M,a})'$ and $\phi = vec(\Phi) := (\varphi_{1,a}, \varphi_{2,a})'$. Proposition 1 ensures that the model is identified if at least one among $\varphi_{1,a}$ and $\varphi_{2,a}$ is non-zero in the population. Panel B of Table 1, left-side, summarizes the CMD estimates (with analytic standard errors) obtained with the procedure described in Section 5. The estimated on-impact coefficients $b_{F,a}$ and $b_{M,a}$ are not statistically significant and the overidentification restrictions test $TQ(\hat{\vartheta}_T)_{exog}$ has p-value 0.74 and strongly supports the hypothesis of 'contemporaneous exogeneity' of financial and macroeconomic uncertainty, $b_{F,a} = 0$ and $b_{M,a} = 0$. The immediate interpretation of this result is that financial and macroeconomic uncertainty do not respond contemporaneously to the identified non-uncertainty shock. We inspect the implied IRFs next.

We evaluate the quality of the identification by checking directly whether the (estimated) innovations in new privately owned housing units started ($\hat{v}_{Z_1,t}$) and the oil supply shock ($v_{Z_2,t}$) are relevant for the real economic activity shock. Panel C of Table 1, left-side, reports the estimated correlations between $\hat{v}_{Z_1,t}$ and $\varepsilon_{a,t}$ (given by given by the ratio $\hat{\varphi}_{1,a}/\hat{\sigma}_{v_{Z_1}}$) and between $v_{Z_2,t}$ and $\varepsilon_{a,t}$ (given by the ratio $\hat{\varphi}_{2,a}/\hat{\sigma}_{v_{Z_2}}$). These are 0.30 and 0.25, respectively, and are both statistically significant at the 5% level of significance.

Full shocks identification strategy

If we add the restriction $b_{F,M} = 0$ in the matrix $\widetilde{G}$ in eq. (17) also the financial and macroeconomic uncertainty shocks can be identified from the AC-SVAR model (despite these shocks are not instrumented). The restriction $b_{F,M} = 0$ involves the submatrix B_2 of B which collects the instantaneous impacts of the non-instrumented structural shocks. We borrow the restriction $b_{F,M} = 0$ from Angelini *et al.* (2019) who investigate the endogeneity/exogeneity of uncertainty by exploiting the breaks in unconditional volatility of a VAR for $Y_t := (a_t, U_{F,t}, U_{M,t})'$ across three main macroeconomic regimes of U.S. business cycle. Given the two external instruments $Z_t := (Z_{1,t}, Z_{2,t})' = (\Delta house_t,\ oil_t)'$ and driven by some preliminary evidence, we impose a diagonal structure to the covariance matrix of measurement errors Σ_ω, i.e. we set $\varpi_{2,1} = 0$

in eq. (17). According to Proposition 2(b), with $b_{F,M} = 0$ and $\varpi_{2,1} = 0$ the necessary order condition for identification is satisfied because there are $a_{\widetilde{G}} = 12$ unrestricted (free) elements in and $1/2m(m+1) = 15$ covariance restrictions, hence the system features 3 testable overidentification restrictions if also the rank condition in Proposition 2(a) holds. It is possible to show that also the identification rank condition is satisfied. Actually, it can be easily checked that the AC-SVAR model with the matrix $\widetilde{G}$ in eq. (17) subject to $b_{F,M} = 0$ and $\varpi_{2,1} = 0$ satisfies the sufficient conditions for global identification of Theorem 1 in Rubio-Ramirez *et al.* (2010).

Panel B of Table 1, right-side, summarizes the ML estimates (with analytic standard errors) of the parameters in the matrix $\widetilde{G}$, see Section 6. The table also reports the results of two LR tests: one (LR_T) is a test for the 3 overidentification restrictions featured by the estimated AC-SVAR model and the other (LR_{exog}) is a test for the hypothesis of 'contemporaneous exogeneity' of financial and macroeconomic uncertainty, $b_{F,a} = 0$ and $b_{M,a} = 0$. Both tests provide ample empirical support to the estimated proxy-SVAR model (p-value 0.92) and to the hypothesis of 'contemporaneous exogeneity' (p-value 0.53), confirming the finding already obtained with the partial shocks identification strategy. In this framework we can also evaluate the instantaneous impacts of financial and macroeconomic uncertainty shocks on industrial production growth. The estimated coefficient $b_{a,F}$ is positive (0.049) but is not significant at the 5% level of significance, while the estimated coefficient $b_{a,M}$ is negative (-0.313) and significant, meaning that a one standard deviation macroeconomic uncertainty shock leads to an instantaneous decline in industrial production growth.

Panel C of Table 1, right-side, reports the estimated correlations between $\hat{v}_{Z,t} := (\hat{v}_{Z_1,t}, v_{Z_2,t})'$ and $\varepsilon_t := (\varepsilon_{a,t}, \varepsilon_{F,t}, \varepsilon_{M,t})'$. It is seen that the estimated correlations between $\hat{v}_{Z,t}$ and $\hat{\varepsilon}_{a,t}$ are 0.29 and 0.25, respectively, and are both statistically significant at the 5% level of significance. Instead, the estimated correlations between $\hat{v}_{Z,t}$ and $\hat{\varepsilon}_{2,t} := (\varepsilon_{F,t}, \varepsilon_{M,t})'$ are close to zero and not statistically significant at the 5% level.

Dynamic causal effects

The IRFs generated by the estimated AC-SVAR model in the full shocks identification strategy are plotted in Figure 1 with associated 90%-bootstrap simultaneous confidence bands.[21] The IRFs are estimated by imposing the 'contemporaneous exogeneity' restrictions $b_{F,a} = 0$ and $b_{M,a} = 0$, not rejected by formal testing. Since the tests in Panel A of Table 1 rule out the occurrence of conditional heteroskedasticity in the disturbances, the simultaneous 'sup-t' boot-

Simultaneous confidence bands seem appropriate given our interested in the overall dynamic response (contemporaneous and lagged) of financial and macroeconomic uncertainty measures to the identified real economic activity shock. In the Supplementary Appendix we also plot the same IRFs as in Figure 1 with pointwise 90%-bootstrap confidence bands.

strap confidence bands for the IRFs are computed by combining a nonparametric *iid* resampling scheme for $\hat{\eta}_t := (\hat{u}_t', \hat{v}_{Z,t}')'$ with Algorithm 3 in Montiel Olea and Plagborg-Møller (2019); see the Supplementary Appendix A.10 for details.

[FIGURE 1 HERE]

The first column of Figure 1 plots the responses of the variables in Y_{t+h}:=$(a_{t+h}, U_{F,t+h}, U_{M,t+h})'$ to one standard deviation non-uncertainty shock $\varepsilon_{a,t}$, $h = 0, 1, ...$ Given the estimates in Panel B of Table 1, the IRFs in the first column of Figure 1 are expected to be numerically similar to the ones computed from a partial shocks identification strategy. It is seen that while macroeconomic uncertainty does not respond significantly at any lag to the identified non-uncertainty shock, financial uncertainty responds after one month, but such response is short-lived and lasts one month. The no response of macroeconomic uncertainty and the very short-lived response of financial uncertainty to the identified non-uncertainty shock in Figure 1 are at odds with Plante *et al.* (2018)'s hypothesis of 'endogenous uncertainty'.

In the full shocks identification framework, we can also track the dynamic responses of Y_{t+h}:=$(a_{t+h}, U_{F,t+h}, U_{M,t+h})'$ to the identified uncertainty shocks $\varepsilon_{2,t} := (\varepsilon_{F,t}, \varepsilon_{M,t})'$, $h = 0, 1, ...$ Given our scopes, these responses are not of strict interest but for completeness we plot them in the second and third columns of Figure 1 and postpone their comment in the Supplementary Appendix A.12.6.

Finally, the Supplementary Appendix A.12.2 shows that the main results obtained in the paper are confirmed by changing the oil supply shock with innovations taken from an auxiliary model for changes in hours worked, i.e. the instrument (c) in place of (b). Overall, our empirical analyses support the common practice of ordering (financial and macroeconomic) uncertainty first in SVARs, i.e. as 'the most exogenous' variables of the system.

8.3 Comparison with existing works

The empirical results discussed in the previous section allow us to make direct contact with Angelini *et al.* (2019), Carriero *et al.* (2018) and Ludvigson *et al.* (2018).

Angelini *et al.* (2019) identify a small-scale SVAR for Y_t:=$(a_t, U_{F,t}, U_{M,t})'$ on the period 1960M8-2015M4 by applying a novel 'identification-through-heteroskedasticity' method which exploits the changes in the unconditional volatility of macroeconomic variables across the main U.S. macroeconomic regimes. In line with our results, they find that macroeconomic uncertainty can be approximated as an exogenous driver of real economic activity and that financial uncertainty displays a delayed and short-lived response to industrial production shocks.

Carriero *et al.* (2018) use a novel stochastic volatility approach in SVAR models which include measures of macroeconomic and financial uncertainty (one at a time), along with measures of real economic activity. Their empirical evidence is partly consistent with ours: they document that macroeconomic uncertainty is broadly exogenous to business cycle fluctuations but find that financial uncertainty might, at least in part, arise as an endogenous response to some macroeconomic developments. Carriero *et al.* (2018) do not model financial and macroecononomic uncertainty jointly, and this might explain why their findings are not fully consistent with ours.

Ludvigson *et al.* (2018) employ a SVAR for $Y_t{:=}(a_t, U_{F,t}, U_{M,t})'$ on the period 1960M8-2015M4 and apply a novel set-identification strategy which combines sign-restrictions that are imposed directly on the structural shocks in correspondence of particular events (event constraints) with external instruments (correlation constraints). They instrument the uncertainty shocks by exploiting a measure of the aggregate stock market return and the log difference in the real price of gold, respectively, and this is one key difference with respect to our identification strategy. The specific events constraints they impose to identify the uncertainty shocks pertain mostly to financial uncertainty: the 1987 stock market crash and the 2007-09 financial crisis. They report that while financial uncertainty can be approximated as an exogenous driver of real economic activity, macroeconomic uncertainty is often an endogenous response to output shocks, and this is another major difference with respect to our empirical findings. Ludvigson *et al.* (2018) also find that, while financial uncertainty shocks are contractionary shocks, macro uncertainty shocks have positive effects on real activity, in line with 'growth options' theories. In Ludvigson *et al.* (2018), the main role of the external instruments is to narrow the identification set constructed with the event constraints: the external instruments need not be orthogonal to the non-instrumented structural shocks but the relevance condition boils down to a set of inequality restrictions.

9 Concluding remarks

We have presented a general framework to analyze the identification of proxy-SVARs when $r \geq g$ external instruments are used to identify $1 \leq g < n$ structural shocks of interest. We have discussed 'partial' and 'full' shocks identification strategies and developed novel frequentist estimation methods based on CMD and ML, respectively.

We have applied the suggested proxy-SVAR methodology to analyze whether commonly employed measures of macroeconomic and financial uncertainty respond to non-uncertainty shocks in the U.S., after the Global Financial Crisis. The empirical evidence supports the view that financial and macroeconomic uncertainty can be approximated as exogenous drivers of real economic activity. Our results, however, can not be considered 'final' as they depend on the specific set of external instruments used to identify the non-uncertainty shock.

ACKNOWLEDGMENTS

The authors thank for constructive comments on previous versions of this article the Co-Editor, Michael McCracken, three anonymous referees, Giovanni Caggiano, Andrea Carriero, Toru Kitagawa, Kurt Lunsford, Aaron Mumtaz, Michele Piffer, Barbara Rossi, seminar participants at the University of Melbourne and conference participants at the "12th International Conference of Computational and Financial Econometrics (CFE 2018)" Pisa, December 2018, the "Eighth Italian Congress of Econometrics and Empirical Economics (ICEEE 2019)" Lecce, January 2019, and the "2019 IAAE Annual Conference" Nicosia, June 2019. We are solely responsible for any remaining errors. The second author gratefully acknowledges partial financial support from Almaidea and RFO grants from the University of Bologna.

References

Amisano, G. and Giannini, C. (1997). *Topics in Structural VAR Econometrics*, 2nd edn, Springer, Berlin.

Angelini, G., Bacchiocchi, E., Caggiano, G., and Fanelli, L. (2019), Uncertainty across volatility regimes, *Journal of Applied Econometrics* 34, 437-455.

Arias, J. E., Rubio-Ramirez, J. F. and Waggoner, D. F. (2018a), Inference based on Structural Vector Autoregressions identified with sign and zero restrictions, *Econometrica* 86, 685-720.

Arias, J. E., Rubio-Ramirez, J. F. and Waggoner, D. F. (2018b), Inference in Bayesian SVARs identified with external instruments, *Working Paper,* Federal Reserve, version dated October 2018.

Baker, S.R., Bloom, N. and Davis, S.J. (2016), Measuring economic policy uncertainty, *Quarterly Journal of Economics* 131, 1593–1636.

Basu, S. and Bundik, B. (2015), Endogenous volatility at the Zero Lower Bound: Implications for stabilization policy, *NBER Working Paper* No. 21838.

Bloom, N. (2009), The impact of uncertainty shocks, *Econometrica* 77, 623-685.

Brüggemann, R., Jentsch, C. and Trenkler, C. (2016), Inference in VARs with conditional volatility of unknown form, *Journal of Econometrics* 191, 69-85.

Caggiano, G., E. Castelnuovo, and Pellegrino, G. (2017), Estimating the real effects of uncertainty shocks at the Zero Lower Bound, *European Economic Review* 100, 257-272.

Caldara, D. and Herbst, E. (2019), Monetary policy, real activity, and credit spreads: evidence from Bayesian proxy SVARs, *American Economic Journal: Macroeconomics* 11, 157-192.

Caldara, D. and Kamps, C. (2017), The analytics of SVARs: A unified framework to measure fiscal multipliers, *Review of Economic Studies* 84, 1015-1040.

Carriero, A., Mumatz, H., Theodoritis, K. and Theophilopoulou, A. (2015), The impact of uncertainty shocks under measurement error: A proxy SVAR approach, *Journal of Money, Credit, and Banking* 47, 1223-1238.

Carriero, A., Clark, T. E. and Marcellino, M. (2018), Endogenous uncertainty?, *Federal Reserve Bank of Cleveland, Working Paper* No. 18-05.

Christiano, J.C., Motto, R. and Rostagno, M. (2014), Risk shocks, *American Economic Review* 104, 27–65.

Gertler, M. and Karadi, P. (2015), Monetary policy surprises, credit costs, and economic activity, *American Economic Journal: Macroeconomics* 7, 44-76.

Jentsch, C. and Lunsford, K.C. (2016), Proxy VARs: Asymptotic theory, bootstrap inference and the effects of income tax changes in the united states, *Federal Reserve Bank of Cleveland, Working Paper* No. 16-19.

Jentsch, C. and Lunsford, K. (2019), The dynamic effects of personal and corporate income tax changes in the United States: Comment, *American Economic Review*, forthcoming.

Jurado, K., Ludvigson, S.C. and Ng, S. (2015), Measuring uncertainty, *American Economic Review* 105(3), 1177-1216.

Kilian, L. (2009), Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market, *American Economic Review* 99, 1053-1069.

Kilian, L. and Lütkepohl, H. (2017), *Structural Vector Autoregressive Analysis*, Cambridge University Press.

Ludvigson, S.C., Ma, S. and Ng, S. (2018), Uncertainty and business cycles: exogenous impulse or endogenous response? *Working Paper,* draft dated November 19, 2018.

Lunsford, K. G. (2015), Identifying Structural VARs with a proxy variable and a test for a weak proxy, *Federal Reserve Bank of Cleveland, Working Paper* No. 15-28.

Lütkepohl, H. (2005), *New introduction to multiple time series analysis*, Springer-Verlag, Berlin.

Magnus, J.R. and Neudecker, H. (1999), *Matrix differential calculus with applications in Statistics and Econometrics*, Wiley & Sons, 2nd edition.

Mertens, K. and Montiel Olea, J.L. (2018), Marginal tax rates and income: New time series evidence, *Quarterly Journal of Economics* 133, 1803-1884.

Mertens, K. and Ravn, M. (2013), The dynamic effects of personal and corporate income tax changes in the United States, *American Economic Review* 103, 1212-1247.

Mertens, K. and Ravn, M. (2014), A reconciliation of SVAR and narrative estimates of tax multipliers, *Journal of Monetary Economics* 68, S1-S19.

Mertens, K. and Ravn, M. (2019), The dynamic effects of personal and corporate income tax changes in the United States: Reply to Jentsch and Lunsford, *American Economic Review*, forthcoming.

Montiel Olea, J.L., Plagborg-Møller, M. (2019), Simultaneous confidence bands: Theory, implementation, and an application to SVARs, *Journal of Applied Econometrics* 34, 1-17.

Montiel Olea, J.L., Stock, J.H. and Watson, M.W. (2018), Inference in SVARs identified with an external instrument, *Working Paper,* version dated November 2018.

Newey, W.K and D. McFadden (1994), Large sample estimation and hypothesis testing, *Handbook of Econometrics*, R.F. Engle and D.L. McFadden (eds.), Vol. IV, Chap. 36.

Pellegrino, G. (2017), Uncertainty and monetary policy in the US: A journay into a non-linear territory, Melbourne Institute *Working Paper* No. 6/17.

Plagborg-Møller, M. and Wolf, C.K. (2018), Instrumental variable identification of dynamic variance decompositions, *Working Paper, Princeton University*, version dated November 24, 2018.

Plante, M., Richter, W.A. and Throckmorton, N.A. (2018), The Zero Lower Bound and endogenous uncertainty, *The Economic Journal* 128, 1730-1757.

Ramey, V. (2016), Macroeconomic shocks and their propagation, *Handbook of Macroeconomics* Vol. 2, 71-162.

Rogers, J.H,. Scotti, C. and Wright, J.H. (2018), Unconventional monetary policy and international risk premia, *Journal of Money, Credit and Banking* 50, 1827-1850.

Rothenberg, T.J. (1971), Identication in parametric models, *Econometrica* 39, 577-591.

Rubio-Ramírez, J. F., Waggoner, D.F. and Zha, T. (2010), Structural Vector Autoregressions and algorithms for inference, *Review of Economic Studies* 77, 665-696.

Stock, J.H and Watson, M.W. (2012), Disentangling the channels of the 2007-2009 recession, *Brooking Panel of Economic Activity*, Conference Paper March 22-23.

Stock, J.H., & Watson, M.W. (2018). Identification and estimation of dynamic causal effects in macroeconomics using external instruments. *The Economic Journal* 128, 917-948.

Wu, J. C. and Xia, F.D. (2016), Measuring the macroeconomic impact of monetary policy at the zero lower bound, *Journal of Money, Credit, and Banking* 48, 253-291.

Table 1. Estimated baseline AC-SVAR model

Panel A

	N_{JB}	AR_4	$ARCH_4$	$F_T : Z_t \nrightarrow Y_t$
a_t-eq.	318.21[0.00]	2.31[0.68]	6.06[0.19]	1.39[0.22]
U_{Ft}-eq.	0.85[0.50]	1.35[0.85]	2.45[0.65]	0.24[0.98]
U_{Mt}-eq.	2.22[0.23]	0.20[0.99]	3.98[0.41]	0.39[0.92]
Z_{1t}-eq.	0.89[0.50]	0.47[0.98]	8.47[0.06]	-
Z_{2t}-eq.	1.11[0.50]	3.14[0.54]	0.61[0.96]	-

Panel B

Partial Shocks Identification

$$\widehat{\widetilde{G}}_1 = \begin{pmatrix} \underset{0.1050}{0.5404} \\ \underset{0.0026}{0.0004} \\ \underset{0.0019}{0.0018} \\ \hdashline \underset{0.0085}{0.0251} \\ \underset{0.0583}{0.1368} \end{pmatrix}$$

$$TQ(\vartheta)_{exog} = 0.61[0.74]$$

Full Shocks Identification

$$\widehat{\widetilde{G}} = \left(\begin{array}{ccc:cc} \underset{0.0941}{0.5614} & \underset{0.1502}{0.0493} & \underset{0.1444}{-0.3135} & 0 & 0 \\ \underset{0.0060}{-0.0058} & \underset{0.0022}{0.0215} & 0 & 0 & 0 \\ \underset{0.0027}{0.0008} & \underset{0.0012}{0.0037} & \underset{0.0008}{0.0092} & 0 & 0 \\ \hdashline \underset{0.0089}{0.0245} & 0 & 0 & \underset{0.0061}{0.0811} & 0 \\ \underset{0.0571}{0.1469} & 0 & 0 & 0 & \underset{0.0394}{0.5224} \end{array}\right)$$

$$LR_T = 0.47[0.93] \quad LR_{exog} = 1.27[0.53]$$

Panel C

Correlations (relevance)

	$\hat{\varepsilon}_a$
$\hat{\upsilon}_{Z_1}$	0.30[0.01]
$\hat{\upsilon}_{Z_2}$	0.25[0.02]

Correlations (relevance, orthogonality)

	$\hat{\varepsilon}_a$	$\hat{\varepsilon}_F$	$\hat{\varepsilon}_M$
$\hat{\upsilon}_{Z_1}$	0.29[0.01]	-0.03[0.75]	0.05[0.61]
$\hat{\upsilon}_{Z_2}$	0.25[0.02]	-0.12[0.28]	0.04[0.68]

NOTES: Estimated AC-SVAR model for $Y_t := (a_t, U_{Ft}, U_{Mt})'$ and external instrument $Z_t := (\Delta house_t, oil_t)'$, period 2008:M1-2015:M4 (T=88). Panel A: diagnostic tests. 'N_{JB}' is the Jarque-Bera test for the null of Gaussian disturbances.'AR_4' is the LM-type test for the null of absence of residual autocorrelation against the alternative of autocorrelated disturbances up to 4 lags. '$ARCH_4$' is a test for the null of absence of the ARCH-type conditional heteroskedasticity up to 4 lags. '$F_T : Z_t \nrightarrow Y_t$' is a Granger causality F-type test for the null hypothesis that Z_t do not Granger cause the corresponding equation in Y_t. Numbers in brackets are p-values. Panel B: estimates. Left side, CMD estimates of $\widetilde{G}_1$ with associated standard errors, '$TQ(\vartheta)_{exog}$' is the overidentification restriction test for the null $b_{F,a} = 0$ and $b_{M,a} = 0$. Right side, ML estimates of $\widetilde{G}$ with associated standard errors, 'LR_T' is a test for the 3 overidentification restrictions featured by the estimated model, 'LR_{exog}' is the overidentification test for the null $b_{F,a} = 0$ and $b_{M,a} = 0$. Panel C: ex-post correlations. Left side, ex-post correlations between the structural shock $\hat{\varepsilon}_a$ and the reduced form shocks $\hat{\upsilon}_{Z_1}$ and $\hat{\upsilon}_{Z_2}$ (relevance). Right side, ex-post correlations between the structural shocks $\hat{\varepsilon}_t := (\hat{\varepsilon}_{at}, \hat{\varepsilon}_{Ft}, \hat{\varepsilon}_{Mt})'$ and the reduced form shocks $\hat{\upsilon}_{Z_1}$ and $\hat{\upsilon}_{Z_2}$ (relevance and orthogonality).

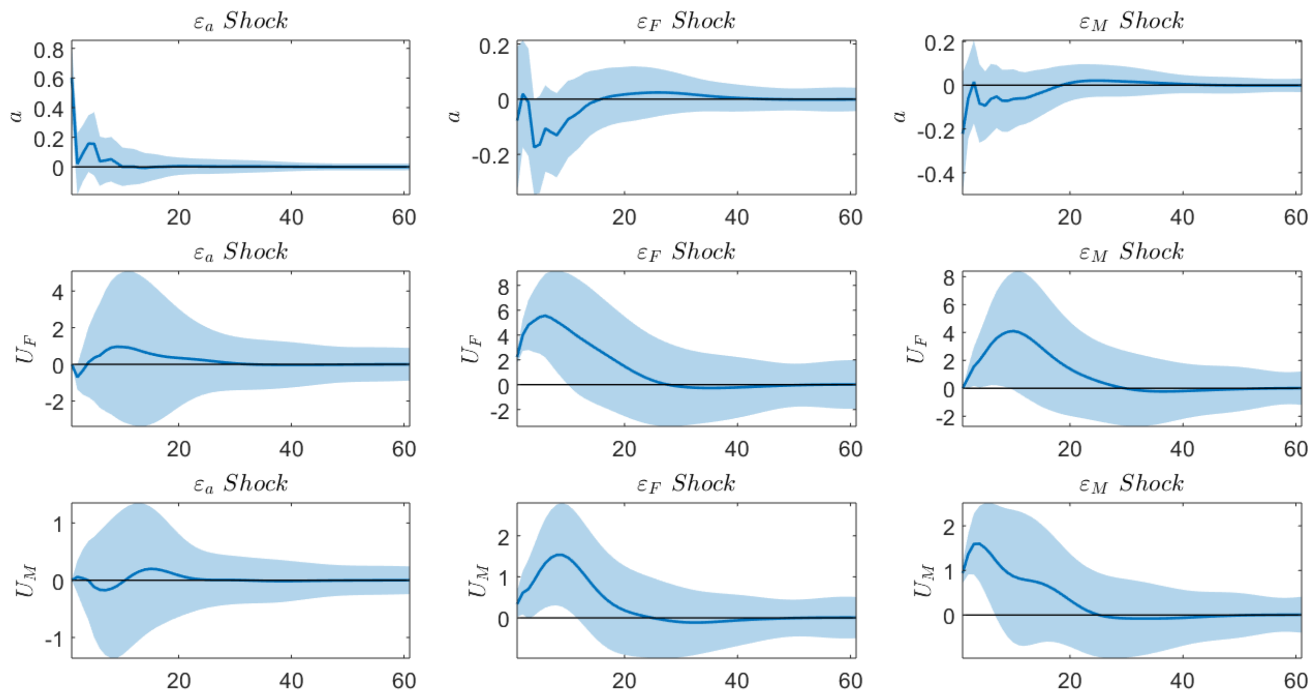

Figure 1: IRFs obtained from the baseline AC-SVAR model for $Y_t := (a_t, U_{Ft}, U_{Mt})'$ and external instrument $Z_t := (\Delta house_t, oil_t)'$, period 2008:M1-2015:M4 (T=88). Blue shaded areas denote 90%-bootstrap simultaneous 'sup-t' confidence bands (Algorithm 3 in Montiel Olea and Plagborg-Møller, 2019). Responses are measured with respect to one standard deviation changes in the structural shocks. The on-impact coefficients are estimated by imposing the null hypothesis $b_{F,a} = 0$ and $b_{M,a} = 0$ of exogenous financial and macro uncertainty.

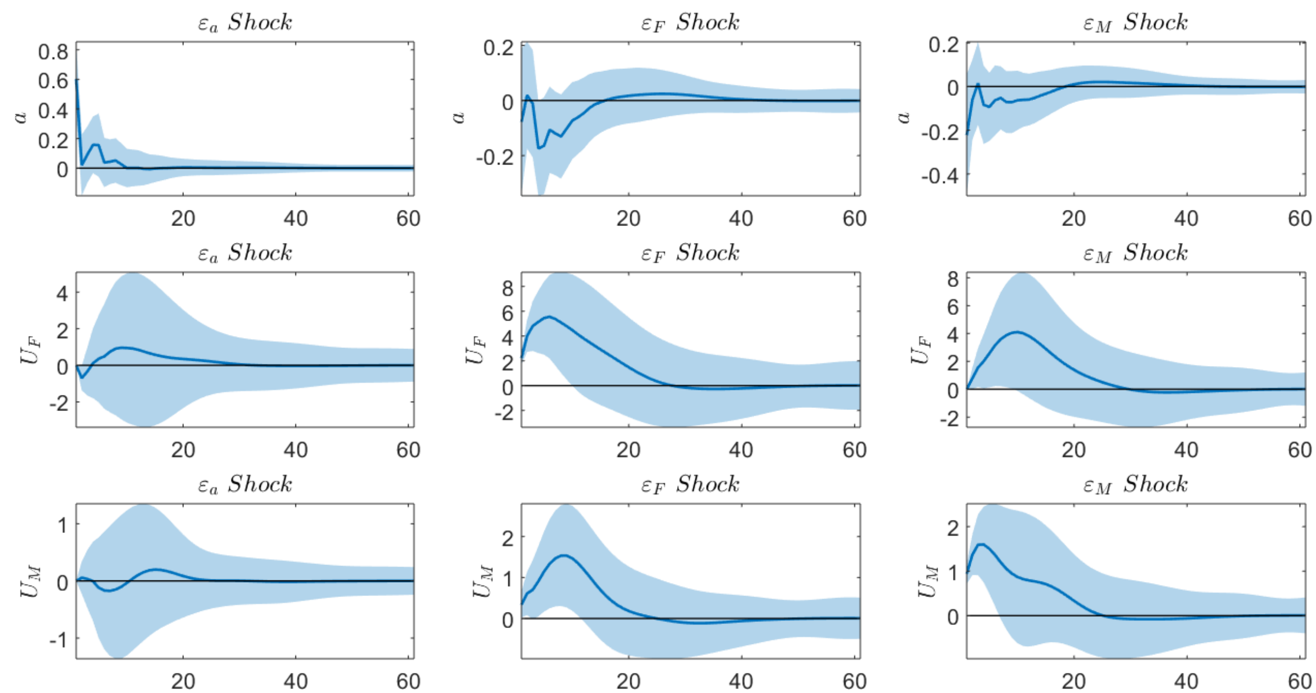

FIGURE 1: *IRFs obtained from the baseline AC-SVAR model for* $Y_t := (a_t, U_{Ft}, U_{Mt})'$ *and external instrument* $Z_t := (\Delta house_t, oil_t)'$, *period 2008:M1-2015:M4 (T=88). Blue shaded areas denote 90%-bootstrap simultaneous 'sup-t' confidence bands (Algorithm 3 in Montiel Olea and Plagborg-Møller, 2019). Responses are measured with respect to one standard deviation changes in the structural shocks. The on-impact coefficients are estimated by imposing the null hypothesis* $b_{F,a} = 0$ *and* $b_{M,a} = 0$ *of exogenous financial and macro uncertainty.*

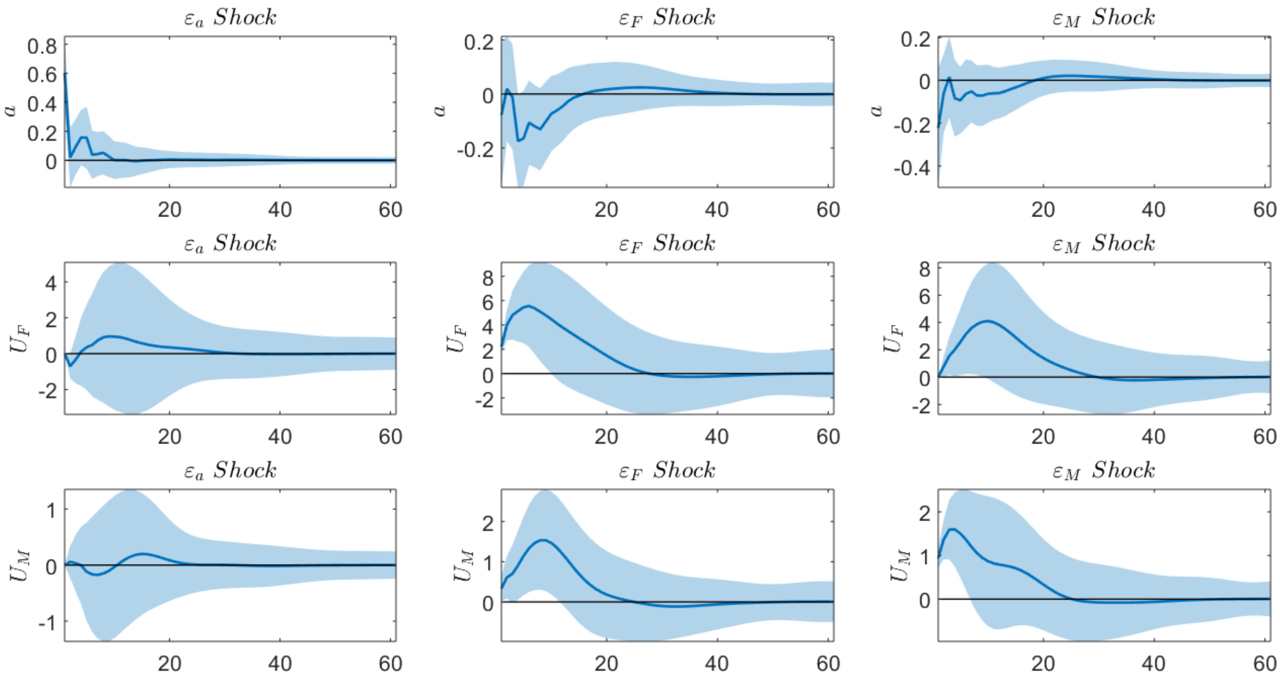

Figure1_IRF_paper.png