AOA estimation with EM lens-embedded massive arrays

Published Version:
Guidi, F. (2018). AOA estimation with EM lens-embedded massive arrays. Piscataway, NJ : IEEE Institute of Electrical and Electronics Engineers [10.1109/VTCSpring.2018.8417728].

Availability:
This version is available at: https://hdl.handle.net/11585/684986 since: 2019-04-16

Published:
DOI: http://doi.org/10.1109/VTCSpring.2018.8417728

AOA Estimation with EM Lens-Embedded Massive Arrays

(Invited Paper)

Francesco Guidi
DEI, University of Bologna, via Venezia 52, I-47521 Cesena (FC), Italy.
e-mail: f.guidi@unibo.it

***Abstract*—Recently, EM lens-embedded massive array antennas have been proposed for next 5G mobile wireless communications, as the adoption of a lens allows to discriminate the AOA of signals in the analog domain, with the possibility to preserve the processing complexity lower with respect to traditional massive arrays. In fact, in such a way, complex ADC chains can be avoided and the number of required antennas can be decreased. By exploiting these advantages, in this paper we study the possibility to use a single EM lens massive array at mm-wave for the AOA estimation of the received signal. In this perspective, ML estimator and practical approaches, tailored for the considered scenario, are derived. Results, obtained for different number of antennas, confirm the possibility to achieve interesting AOA estimation performance with an extremely compact architecture.**

***Index Terms*—AOA estimation, EM lens, massive array, maximum likelihood**

I. Introduction

The joint adoption of millimeter-waves (mm-wave) and massive multiple-input multiple-output (MIMO) technologies has recently shown promising possibilities for next fifth generation (5G) of mobile wireless communications. In fact, thanks to the proposed schemes exploiting a large number of antennas packed into a small area, data rates up to several Gbit per second can be attained [1]. Consequently, new applications can be enhanced from the possibility to integrate multi-antenna systems with laser-like beams in portable devices, thus avoiding expensive dedicated infrastructures [2] . Among all the opportunities, the idea of personal radars has been recently proposed, where mobile users are equipped with mm-wave massive arrays performing beamsteering for the reconstruction of indoor environments [3], [4].

If from one side the densification of antennas opens unexplored ways to conceive the communication, localization and mapping, from the other side an increased architectural complexity is unavoidable at the receiver to allow a proper managing of the data [5]. In fact, even if large antenna arrays have been widely investigated to achieve significant beamforming gains, such systems face several limitations in their practical adoption due to the complex hardware and high power consumption associated with the radio-frequency chains at each antenna element. Several schemes have been already proposed to preserve the complexity as low as possible, as for example sub-arrays architectures or analog and hybrid analog/digital beamforming schemes [6], but currently they are not capable to entirely overcome the problem. To this purpose, one option to put in practice the integration of massive arrays even in portable devices could be the use of electromagnetic (EM) lens-based massive arrays operating at mm-wave. In fact, by making use of a massive array and of a lens to collimate the beams in precise directions, it is possible to spatially discriminate signals in the analog domain [7]–[9]. Consequently, thanks to the lens, there is a unique relation between the incident and the output angles of the impinging and refracted waves, respectively. This operation enhances to reduce the number of antennas with respect to traditional massive arrays, and to move from discrete beamforming architectures towards continuous-aperture-phased arrays. If such a solution has already been proposed and investigated [10], it still needs a dedicated effort to be fully exploited for localization [7]. In fact, despite few works have addressed the direction finding problem [11], [12], several aspects still have to be studied as, for example, the attainable fundamental angle-of-arrival (AOA) estimation performance limits.

In this paper we assume the presence of a reference node equipped with a lens-embedded massive array to detect and localize the surrounding transmitting devices. To this purpose, starting from a tractable signal model taken from the state-of-the-art describing the lens filtering effect, we first derive the maximum likelihood (ML) AOA estimator and, successively, two schemes entailing a lower complexity. By comparing the obtained results with those attainable with classical array schemes, the feasibility of the herein considered EM lens massive array scheme is shown.

The rest of the paper is organized as follows. In Sec. II we report a description of the EM lens-embedded massive array, whereas in Sec. III the considered signal model is detailed. In Sec. IV, the ML approach and the two practical schemes are derived, and the obtained results are reported in Sec. V. Finally, conclusions are drawn in Sec. VI.

II. EM Lens-Embedded Massive Array

A. Traditional Massive Array

Massive arrays have already been widely investigated for communication and localization [5], [6], [13], [14], especially in the perspective of using mm-wave massive arrays for next 5G wireless communication technologies.

Several parameters have been used to describe the characteristics of traditional arrays. Here we consider the array response vector which contains the phase shifting across different antenna elements [15]. In particular, the array response can be defined as

$$\mathbf{b} = [b_1, \ldots, b_m, \ldots, b_{N_\mathrm{A}}] \tag{1}$$

with $b_m = e^{j\,\Psi_m}$, where Ψ_m indicates the phase shift of the mth antenna element with respect to the reference point according to the operational frequency [15], including the information on the AOA of the signal, and N_A being the number of antennas employed.

By exploiting dedicated receiver architectures to re-phase the received signal at each antenna, it has been demonstrated that very precise localization can be achieved with arrays equipped with hundreds of elements, thanks to the possibility to realize accurate beamsteering operations [14]. This capability comes at the prize of ad-hoc phase-shifting networks or analog-to-digital converter (ADC) chains that entail high complexity and cost for the realization of beamsteering operations.

In the following, the spatial filtering effect of the lens, performed in analog, is described.

B. Integration of the EM Lens on Massive Arrays

An EM lens antenna array consists in general of integrating an EM lens with an antenna array, with the intent to exploit the lens capability to precisely collimate the signal in different spatial directions according to the incident angle of the impinging wave on then lens. In this way, antennas are spatially located so that they can gather the different AOA of the signals [10]. Current works in the state-of-the-art have tackled the description of the lens massive array in different, but equivalent, ways. Generally, they consider the EM lens and traditional massive array only separately, by approximating the effect of the lens to that of a spatial discrete Fourier transform filter [8]. Other works instead have studied the lens impact by a power perspective [11], showing how it is distributed on the antennas after the EM lens focusing effect. A detailed analysis has been proposed in [7], where the lens and the massive array are jointly considered and a derivation is proposed to demonstrate the capability to collimate signals in very precise spatial directions. More specifically, it is highlighted that the lens spatial filtering effect can be modelled with a "sinc" function, as it will be accounted for in the rest of this manuscript. Furthermore, the solution in [11] accounts for a uniform linear array, whereas in [7] the array elements are located on the focal arc of the lens, as shown in Fig. 1. In this paper, by following the guidelines given in [7], we consider an array with N_A antennas located on the focal arc of the lens, on the xy-plane, as reported in Fig. 1, with $\theta_m \in [-\pi/2, \pi/2]$ representing the angle of the mth generic antenna element. Then, by assuming that $\tilde{\theta}_m = \sin\theta_m$, antenna elements are equally spaced in the interval $[-1, 1]$ so that

$$\tilde{\theta}_m = \frac{m\,\lambda}{D_y} = \frac{m}{\tilde{D}} \tag{2}$$

where λ is the signal wavelength, and $\tilde{D} = D_y/\lambda$, with D_y being the lens length along the y-axis. Notably, the analysis

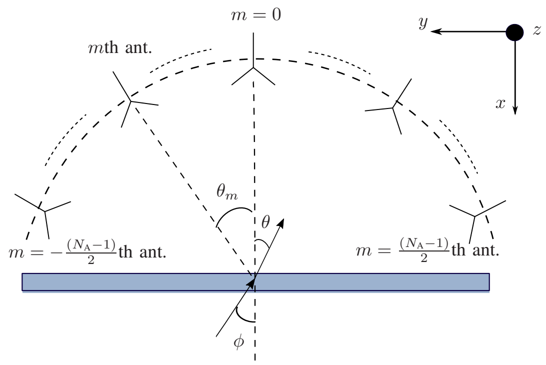

Fig. 1. Top-view of the EM lens massive array architecture.

herein carried out is general and scales according to the operational frequency. Then, according to [7], the relation between $\tilde{D}$ and the required number of antennas N_A is

$$N_\mathrm{A} = 1 + 2\lfloor\tilde{D}\rfloor \tag{3}$$

with $\lfloor\cdot\rfloor$ denoting the largest integer no greater than its argument, so that (3) indicates the need of a higher number of antennas with a larger lens dimension $\tilde{D}$. In addition, according to the considered design, antenna elements are more densely located in the center of the system.

In the following we describe the considered signal model by showing the lens effect.

III. Signal Model

We now consider a generic source in the environment transmitting a signal which is collected by the EM lens-based massive array here considered. Thus, by assuming that the receiver is equipped with N_A antennas, the signal received by the antenna array can be expressed as

$$\mathbf{r}(t) = \mathbf{H}(t) \otimes x(t) + \mathbf{z}(t) \tag{4}$$

where $x(t)$ denotes the transmitted signal, $\mathbf{r}(t) = [r_1(t), \ldots, r_m(t), \ldots, r_{N_\mathrm{A}}(t)]$ the vector containing the received signal at each antenna, $\otimes$ indicates the convolution operation and $\mathbf{z}(t) = [z_1(t), \ldots, z_m(t), \ldots, z_{N_\mathrm{A}}(t)]$, with $z_m(t)$ being the additive white Gaussian noise (AWGN) at the mth antenna element. Then, according to the notation adopted in [7], the impulse response vector comprising both the channel and lens effect can be expressed as

$$\begin{aligned}\mathbf{H}(t) &= [h_1(t), \ldots, h_m(t), \ldots, h_{N_\mathrm{A}}(t)]\\ &= \sum_{l=1}^{L} \alpha_l\, \mathbf{a}(\phi_l)\, \delta(t-\tau_l)\end{aligned} \tag{5}$$

where the generic element $h_m(t)$ represents the channel impulse response at the mth receiving antenna. The multipath components are herein described in the following way: L indicates the number of paths,[1] α_l, τ_l and ϕ_l denote the lth path amplitude, delay and AOA, respectively.

[1]Note that the EM lens massive array should be designed so that $L \ll N_\mathrm{A}$.

The lens effect is included in the vector $\mathbf{a}$ that represents the joint lens-array response at the receiver, with $\mathbf{a}(\phi_l) = [a_1(\phi_l), \ldots, a_m(\phi_l), \ldots, a_{N_\text{A}}(\phi_l)]$. According to [7], the mth element $a_m(\phi_l)$ can be written as[2]

$$a_m(\phi_l) \approx \sqrt{A}\, \mathrm{sinc}(m - \tilde{D}\, \tilde{\phi}_l) \quad (6)$$

where $A = D_y\, D_z/\lambda^2$ is the normalized lens aperture and $\tilde{\phi}_l = \sin(\phi_l)$. Note that (6) shows the capability of the lens to focus the signal in very precise spatial directions. In fact, the power at the output of the lens is spatially distributed as a $[\mathrm{sinc}(\cdot)]^2$ and thus, according to the lens characteristics, only very few antennas collect the received signal for specific AOA. Then, the generic element $h_m(t)$ in (5) can be expressed as

$$h_m(t) = \sum_{l=1}^{L} \alpha_l\, a_m(\phi_l)\, \delta(t - \tau_l)\,. \quad (7)$$

Notably, in a general scenario, the AOA of the impinging signal is usually not aligned with the receiving antenna element, that is

$$\tilde{D}\tilde{\phi}_l = m_l + \epsilon_l \quad (8)$$

where ϵ_l is comprised between 0 (perfect alignment) and 0.5 (signal perfectly centered between two antennas) and m_l indicates the antenna with m_lth index which collects the greatest amount of energy. Thus, in case that the lth received path is perfectly aligned to the m_lth antenna, the AOA estimation problem is simplified to that of picking the index of the antenna that gathers the largest amount of energy. On the other side, as it usually happens in real scenarios, a misalignment is present and needs to be estimated.

To that purpose, in the following we derive the ML approach and two more practical estimators for the direction finding, in order to assess the attainable performance.

IV. AOA Estimation with EM Lens Massive Arrays

The signals received by all the antennas are collected and post-processed together in order to estimate the AOA of the received signal. To take benefit from the spatial filtering allowed by the EM lens, here a ML approach and two simpler but more practical methods are investigated. For the sake of simplicity, we consider only the line-of-sight (LOS) component so that the received signal, at each antenna, can be written in the form

$$r_m(t) = s_m(t) + z_m(t) = \alpha\, a_m(\phi)\, x(t - \tau) + z_m(t) \quad (9)$$

with α and τ indicating the path-gain and the time-of-arrival (TOA), respectively, considered identical at each antenna, and $x(t)$ is the transmitted pulse centered at f_c.

A. Maximum Likelihood

We first investigate the employment of a ML estimator. The likelihood function related to the AOA ϕ can be written as

$$\Lambda(\phi) \propto \prod_{m=1}^{N_\text{A}} \exp\left\{-\frac{1}{N_0}\int_T [r_m(t) - s_m(t)]^2\, dt\right\} \quad (10)$$

[2]Here, it is assumed $a_m(\phi) \in \Re$.

where T is the observation interval, N_0 represents the overall noise power spectral density (PSD) at each antenna. Taking the logarithm and discarding all the terms that do not contribute in maximizing ϕ, the log-likelihood function can be written as

$$\ell(\phi) = \sum_{m=1}^{N_\text{A}} \frac{2}{N_0}\int_T r_m(t) \cdot s_m(t) dt - \sum_{m=1}^{N_\text{A}} \frac{1}{N_0}\int_T s_m^2(t) dt\,. \quad (11)$$

Then, by considering that the last integral in (11) returns the energy of the signal, that is $E_m = \int_T s_m^2(t)dt$, and by accounting for (9) and (11), the log-likelihood function is

$$\ell(\phi) = \frac{1}{N_0}\sum_{m=1}^{N_\text{A}} \left\{2\,\alpha\, a_m(\phi)\, \chi_m - E_m\right\} \quad (12)$$

where $\chi_m = \int_T r_m(t)\, x(t - \tau)dt$.
Finally, the ML estimate of the AOA can be expressed as

$$\hat{\phi} = \arg\max_{\phi}\, [\ell(\phi)] \quad (13)$$

that, in accordance with the previous derivation, yields to

$$\hat{\phi} = \arg\max_{\phi} \sum_{m=1}^{N_\text{A}} \left\{2\,\alpha\, a_m(\phi)\, \chi_m - E_m\right\}. \quad (14)$$

Thus, the AOA estimation process translates into a maximization of a term containing the lens response $a_m(\phi)$.

Analogously to the case with lens, the ML can be easily derived also for the case without lens. To that purpose, an extensively literature concerns the comparison of the two signal models [7], as well as the derivation of the ML for traditional arrays [16], [17].

B. Approach 1: Energy-Based Method

Despite the ML approach represents a benchmark, it usually entails a high complexity for its implementation. The same holds for other schemes, such as the MUSIC or the ESPRIT, when they are exploitable [12]. Thus, a more practical solution is represented by the energy detector, which could be also useful when there is a complete uncertainty on the received waveform shape. In particular, since the relation

$$\hat{\phi} = \mathrm{asin}\left(\frac{\hat{m} + \hat{\epsilon}}{\tilde{D}}\right) \quad (15)$$

holds, it is possible to estimate the AOA of the received signal according to two steps: an initial coarse estimation $\hat{m}$ of m, and a refined estimation $\hat{\epsilon}$ to compensate for the AOA misalignment with respect to the $\hat{m}$th antenna.

For each antenna, the received signal $r_m(t)$ is first passed through an ideal bandpass filter, centered at f_c and with bandwdith W to eliminate the out-of-band noise. Then, energy is evaluated as

$$e_m = \int_0^T [\tilde{r}_m(t)]^2\, dt \simeq \frac{1}{2\,W}\sum_{k=1}^{N} (\tilde{r}_{mk})^2 \quad (16)$$

where $\tilde{\mathbf{r}}_m = [\tilde{r}_{m1}, \ldots, \tilde{r}_{mk}, \ldots, \tilde{r}_{mN}]$ is the vector containing the signal samples, taken at Nyquist rate,[3] of the

[3]In a real scenario, it is more feasible to first down-convert mm-wave signals in the base-band.

filtered version $\tilde{r}_m(t)$ of $r_m(t)$, with $N = 2\,TW$. Then, given $\mathbf{e} = [e_1, \ldots, e_m, \ldots, e_{N_\mathrm{A}}]$, it is possible to estimate m as

$$\hat{m} = \arg\max_{m} \{e_m\} \ . \tag{17}$$

Now define

$$\nu_{\hat{m}} = \max\left(\frac{e_{\mathcal{N}}}{\eta}, e_{\hat{m}} - e_{\mathcal{N}}\right) \tag{18}$$

with $e_{\mathcal{N}}$ being the noise estimate at the output of the receiver and η a normalization coefficient so that $e_{\mathcal{N}}/\eta \ll e_{\mathcal{N}}$. Then, consider

$$\nu_{\hat{m}} \simeq \mathcal{A} \cdot \left[\mathrm{sinc}(\hat{m} - \tilde{D}\tilde{\phi})\right]^2 \tag{19}$$

where $\mathcal{A}$ is constant with respect to m as it contains all the terms that do not depend on the signal spatial distribution, and $\tilde{D}\tilde{\phi}$ is the index related to the true AOA comprising the misalignment ϵ. Since contiguous bins contain the information related to the spatial distribution given by the lens through the "sinc" function, by substituting $\hat{m} - \tilde{D}\tilde{\phi} = \hat{\epsilon}$, it yields to

$$\begin{aligned} \nu_{\hat{m}} &\simeq \mathcal{A} \cdot [\mathrm{sinc}(\hat{\epsilon})]^2 \\ \nu_{(\hat{m}\pm 1)} &\simeq \mathcal{A} \cdot [\mathrm{sinc}(\hat{\epsilon} \pm 1)]^2 \ . \end{aligned} \tag{20}$$

Notably, we have two extreme scenarios: (i) $\epsilon = 0$ (alignment with the $\hat{m}$th antenna) and (ii) $\epsilon = 1/2$ (complete ambiguity between two contiguous indexes). Thus, from (20), we can write

$$\frac{\nu_{\hat{m}}}{\nu_{(\hat{m}\pm 1)}} \simeq \left[\frac{\mathrm{sinc}(\hat{\epsilon})}{\mathrm{sinc}(\hat{\epsilon} \pm 1)}\right]^2 \tag{21}$$

where, according to how the energy is allocated in the $\hat{m}+1$ and $\hat{m}-1$ antenna indexes, it gives

$$\hat{\epsilon} = \begin{cases} \dfrac{1}{1+\sqrt{\frac{\nu_{\hat{m}}}{\nu_{(\hat{m}+1)}}}} & \text{if} \quad \nu_{(\hat{m}+1)} \geq \nu_{(\hat{m}-1)} \\ -\dfrac{1}{1+\sqrt{\frac{\nu_{\hat{m}}}{\nu_{(\hat{m}-1)}}}} & \text{if} \quad \nu_{(\hat{m}+1)} < \nu_{(\hat{m}-1)} \end{cases} \tag{22}$$

Thus, by injecting (17) and (22) in (15), the AOA estimation is performed without requiring an a-priori knowledge of the received signal.

C. Approach 2: Combination of Contiguous Signals

A possible alternative approach, which allows to preserve the complexity affordable, is to exploit the method suggested in [12] where comparable performance to the MUSIC algorithm has been attained. In particular, assuming that N_A signals are collected, and that each received signal is sampled at Nyquist rate as before giving the samples r_{mk}, it is possible to build a received signal matrix $\mathbf{R}$ with size $(N_\mathrm{A} \times N)$. Successively, consider

$$\left\{\hat{m}, \hat{k}\right\} = \arg\max_{m,k} \{r_{mk}\} \tag{23}$$

as the collected peak amplitude, located at the $\hat{m}$th antenna and at the time index $\hat{k}$. Then, we can write

$$\kappa = \frac{r_{\hat{m}\hat{k}} - r_{(\hat{m}\pm 1)\hat{k}}}{r_{\hat{m}\hat{k}} + r_{(\hat{m}\pm 1)\hat{k}}} \tag{24}$$

that, according to (15), yields to write

$$\hat{\phi} = \begin{cases} \mathrm{asin}\left[\left(\frac{\hat{m}+\frac{1}{2}(\kappa-1)}{\tilde{D}}\right)\right] & \text{if} \quad r_{(\hat{m}+1)\hat{k}} \geq r_{(\hat{m}-1)\hat{k}} \\ \mathrm{asin}\left[\left(\frac{\hat{m}-\frac{1}{2}(\kappa-1)}{\tilde{D}}\right)\right] & \text{if} \quad r_{(\hat{m}+1)\hat{k}} < r_{(\hat{m}-1)\hat{k}} \ . \end{cases} \tag{25}$$

From (25) it is evidenced that, as for the Approach 1, this method accounts for a two-step AOA estimation process which allows to refine the coarse estimation given by $\hat{m}$.

V. Numerical Results

We now evaluate the AOA estimation performance of the previously described methods, by considering a source transmitting a pulse with bandwidth $W = 100\,\mathrm{MHz}$ centered at $f_\mathrm{c} = 60\,\mathrm{GHz}$, and with only the LOS component considered, as in the model. At the receiver, we account for a lens-embedded massive array with different N_A (i.e., 15, 25 and 35), which implies a different value of D_y according to (3) (i.e., $3.5\,\mathrm{cm}$, $6\,\mathrm{cm}$ and $8.5\,\mathrm{cm}$, respectively) and thus normalized aperture A. For the sake of simplicity, once D_y is fixed by N_A and f_c, we set $D_z = D_y$.

Results are expressed in terms of the root mean square error (RMSE) of the AOA estimate, which is evaluated as

$$\mathsf{RMSE}\left(\hat{\phi}\right) = \sqrt{\frac{1}{N_\mathrm{c}} \sum_{i=1}^{N_\mathrm{c}} \left[\phi_i - \hat{\phi}_i\right]^2} \tag{26}$$

where N_c is the number of Monte Carlo iterations considered in simulations. For each ith cycle, the AOA ϕ_i is generated according to a uniform distribution in the interval $[-70°, 70°]$ for different signal-to-noise ratio (SNR) per antenna, here defined as

$$\mathsf{SNR}_\mathrm{A} = \frac{E_\mathrm{rx}/T}{N_0\,W} \tag{27}$$

where N_0 is the noise PSD, and E_rx is the energy received by an isotropic antenna as if it would be the receiver antenna. This means that neither the lens effect, nor the overall array gain, are accounted for in SNR_A.

Finally, we assume for simplification that E_m does not depend on the AOA, both in the presence or not of the lens.

A. Results

Fig. 2 reports the obtained results with and without the lens (i.e., traditional arrays), and for different N_A. As expected, the larger is N_A, the better is the AOA estimate. In addition, the use of a lens allows to improve the achievable performance thanks to the increased array aperture. In fact, the same array aperture with traditional massive arrays usually employs a much larger number of antennas [7], thus entailing a higher complexity, especially at 60 GHz.

Fig. 3 shows instead the comparison of the different estimation methods when EM lens-embedded massive arrays are used. Obviously, the ML allows to outperform the other schemes but, due to its complexity, it is hardly exploitable in real scenarios. On the other side, it represents an interesting benchmark for analyzing the performance of the herein considered algorithms. Notably, the method indicated as Approach 2

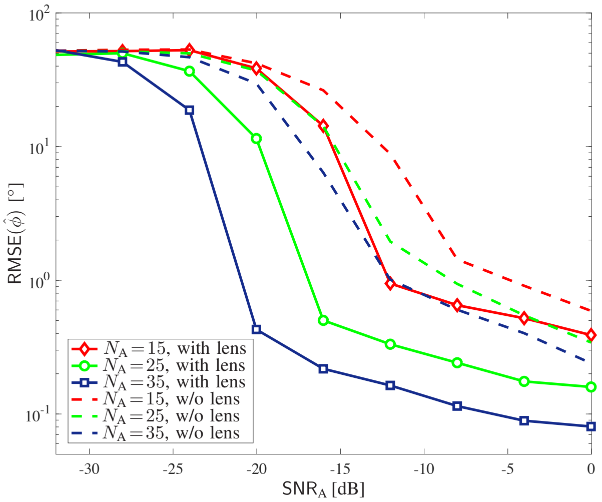

Fig. 2. RMSE as a function of SNR$_A$ with and without the lens.

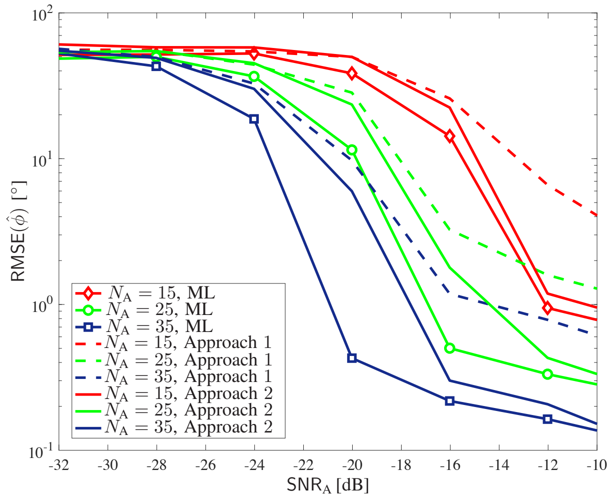

Fig. 3. RMSE as a function of SNR$_A$ for different methods.

in Sec. IV-C allows to attain interesting results with an extremely simple architecture. Furthermore, the energy-based method (here indicated as Approach 1) permits as well to keep the complexity affordable, but with reduced performance. As an example, when adopting the Approach 1 and $N_A = 35$, sub-degree error for $\mathsf{SNR}_A = -10$ dB can be obtained, which is still accurate for several applications [14]. In this perspective, such a choice can represent a good trade-off in terms of achievable AOA estimate, array size and complexity (i.e., affordable number of antennas).

Indeed, according to all the aforementioned considerations, the exploitation of the EM lens-embedded massive arrays with one of the two practical approaches here described, can be a promising solution for all AOA-based localization schemes requiring a low system complexity.

VI. Conclusions

We investigated the AOA estimation performance when a ML approach and practical methods are used for EM lens-embedded massive arrays. After discussing the main advantages offered by the joint adoption of an EM lens and a massive array, we derived different estimators by exploiting an ad-hoc signal model available in the state-of-the-art. Results confirmed that extremely accurate AOA estimate can be achieved by discriminating angles in the analog domain, and thus by preserving the entire processing complexity lower than classical schemes.

Acknowledgments

The author would like to thank A. Guerra and N. Decarli and D. Dardari for the fruitful discussions. This research has been in part funded by the European Space Agency project LOST and by the European Commission through the framework of the H2020 project XCycle under Grant 635975.

References

[1] I. F. Akyildiz *et al.*, "5G roadmap: 10 key enabling technologies," *Computer Networks*, vol. 106, no. Supplement C, pp. 17 – 48, 2016.

[2] L. D. Palma *et al.*, "1-bit reconfigurable unit cell for Ka-band transmitarrays," *IEEE Antennas Wireless Propag. Lett.*, vol. 15, pp. 560–563, 2016.

[3] F. Guidi, A. Guerra, and D. Dardari, "Personal mobile radars with millimeter-wave massive arrays for indoor mapping," *IEEE Trans. Mobile Comp.*, vol. 15, no. 6, pp. 1471–1484, Jun. 2016.

[4] A. Guerra *et al.*, "A millimeter-wave indoor backscattering channel model for environment mapping," *IEEE Trans. Antennas Propag.*, vol. 65, no. 9, pp. 4935–4940, Sep. 2017.

[5] A. F. Molisch *et al.*, "Hybrid beamforming for massive MIMO: A survey," *IEEE Commun. Mag.*, vol. 55, no. 9, 2017.

[6] F. Sohrabi and W. Yu, "Hybrid digital and analog beamforming design for large-scale antenna arrays," *IEEE J. Sel. Topics Signal Processing*, vol. 10, no. 3, pp. 501–513, Apr. 2016.

[7] Y. Zeng and R. Zhang, "Millimeter wave MIMO with lens antenna array: A new path division multiplexing paradigm," *IEEE Trans. Commun.*, vol. 64, no. 4, pp. 1557–1571, Apr. 2016.

[8] J. Brady, N. Behdad, and A. M. Sayeed, "Beamspace MIMO for millimeter-wave communications: System architecture, modeling, analysis, and measurements," *IEEE Trans. Antennas Propag.*, vol. 61, no. 7, pp. 3814–3827, Jul. 2013.

[9] Y. Zeng, R. Zhang, and Z. N. Chen, "Electromagnetic lens-focusing antenna enabled massive mimo: Performance improvement and cost reduction," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1194–1206, 2014.

[10] M. A. Al-Joumayly and N. Behdad, "Wideband planar microwave lenses using sub-wavelength spatial phase shifters," *IEEE Trans. Antennas Propag.*, vol. 59, no. 12, pp. 4542–4552, Dec. 2011.

[11] S. A. Shaikh and A. M. Tonello, "Localization based on angle of arrival in EM lens-focusing massive MIMO," in *Proc. 6th Int. Conf, on Consumer Electronics (ICCE-Berlin)*, Sep. 2016, pp. 124–128.

[12] ——, "Complexity reduced direction finding massive MIMO system using EM lens," in *Workshop on Dependable Wireless Commun. and Localization for the IoT*, Graz, Austria, Sep. 2017.

[13] A. Guerra, F. Guidi, and D. Dardari, "Position and orientation error bound for wideband massive antenna arrays," in *Proc. IEEE Int. Conf. on Commun. (ICC), ANLN Workshop*, Jun. 2015, pp. 853–858.

[14] F. Guidi *et al.*, "Joint energy detection and massive array design for localization and mapping," *IEEE Trans. Wireless Commun.*, vol. 16, no. 3, pp. 1359–1371, Mar. 2017.

[15] C. Balanis, *Antenna Theory*, 3rd ed. Wiley, 2005.

[16] P. Stoica and R. L. Moses, *Spectral analysis of signals*. PRENTICE HALL, 2005, vol. 1.

[17] A. Guerra, F. Guidi, and D. Dardari, "Single anchor localization and orientation performance limits using massive arrays: MIMO vs. beamforming," *arXiv preprint arXiv:1702.01670*, 2017.