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Published Version:
Poaching in media: Harm to subscribers? / Carroni, Elias*. - In: JOURNAL OF ECONOMICS & MANAGEMENT STRATEGY. - ISSN 1058-6407. - STAMPA. - 27:2(2018), pp. 221-236. [10.1111/jems.12238]

Availability:
This version is available at: https://hdl.handle.net/11585/650548 since: 2021-02-28

Published:
DOI: http://doi.org/10.1111/jems.12238

Poaching in media: harm to subscribers?*

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February 8, 2018

Abstract

Two media platforms compete for heterogeneous users bothered by commercials and sell advertisement spaces to firms. In a two–period model, media are allowed to condition subscription prices on the past behavior of users. *Within–group* price discrimination intensifies media competition on the firms' side, as some firms advertise only on one media outlet (single–home), where they can meet early users and switchers. As a consequence, advertising revenues are reduced and this puts an upward pressure on subscription prices. However, price discrimination also induces stronger *within–group* competition to poach the rival's users. Depending on the balance between these two forces, conditioning subscription prices on past behavior might be beneficial or detrimental to users, whereas it is always detrimental to platforms. In relation to *within–group* uniform pricing, total welfare might increase or decrease, as the lower advertising intensity may entail either under–provision or a mitigation of over–provision of advertisements.

JEL codes: L1, D4.

Keywords: Behaviour–Based Price Discrimination, Two–Sided Markets, Media, Platform Competition, Advertisement

*I am grateful to the editor Ramon Casadesus-Masanell and three anonymous referees for helpful reviews on the paper. I also wish to thank Eric Toulemonde, Paul Belleflamme, Marc Bourreau, Lidia Carroni, Marco Delogu, Vincenzo Denicolò, Rosa Branca Esteves, Luca Ferrari, Andrea Mantovani, Gabriella Mezei, Antonio Minniti, Leo Mocciola, Dimitri Paolini, Giuseppe Pignataro and Simone Righi. I am indebted to the participants to the Doctoral Workshop 2012 at UCLouvain, 2013 Ecore Summer School - Governance and Economic Behavior (Leuven), 3rd GAEL Conference - Product differentiation and innovation on related markets (Grenoble). This research was conducted as part of the project Labex MME-DII (ANR11-LBX-0023-01). I acknowledge the "Programma Master & Back - Regione Autonoma della Sardegna" for financial support. All remaining errors are my own.

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1 Introduction

When a firm knows the identity of its customers, it often decides to charge new clients with a lower price in order to capture new demand. There is strong evidence of the fact that this strategy is used in the market of media subscriptions. As pointed out by **?**, price discrimination based on past purchases, called behavior–based price discrimination (BBPD), is very common in subscription markets. In these markets, since transactions are never anonymous, a firm knows the identity of current subscribers and can thus propose discounts to those who did not subscribe in the past. Discounts take different forms such as low introductory prices and free trial memberships. More specific to media, **?** report how a new subscriber for three months to the French newspaper "Le Monde", pays 50 euros whereas a previous customer is charged 131.30 euros. Similar offers can be found in many traditional media (TVs, newspapers and magazines) as well as on online platforms such as Spotify and Deezer, which offer free trial memberships to access their contents.

These strategies have captured the attention of many economists.[1] Their main concern has been the study of the consequences of such practices on firms' profits, consumer surplus and price levels. Roughly speaking, there is a consensus on the conclusion that BBPD reduces firms' profitability, as they compete fiercely to poach the rival's customers, with a consequent benefit for consumers in relation to uniform pricing. This conclusion is quite important from a policy viewpoint. Indeed, the access of firms to data on consumers is something that, from a strictly economic perspective, is ultimately positive for consumers themselves. The idea is that the more information firms possess the more fiercely they compete, and this goes all at the benefit of consumers.

However, media markets have the peculiarity of advertising. All articles studying BBPD neglect the presence of cross–group externalities that typically characterize media markets. It turns out that traditional media as well as new –online– media have the common feature that users (subscribers)[2] are not the only customers, as their profits also come from the advertisers. In economic jargon, these markets are run by two–sided platforms allowing the interaction between different groups of customers linked to each other by cross–group externalities. Namely, the utility that a user enjoys

[1] See **?**, **?** and **?** among others and **?** for an extensive survey on BBPD.
[2] In what follows, the words users and subscribers are used interchangeably.

by subscribing to the service decreases with the number of commercials (nuisance) present in the platform, while advertisers are more satisfied if the number of users increases.

Because of externalities, one of the distinctive features of these markets is the pricing rule, which is different from the general rule that applies in a one–sided framework (i.e., market without externalities). Indeed, the subscription price (advertising intensity) affects not only the demand of subscriptions (advertisement revenues), but also the wellbeing of advertisers (subscribers) who join the platform. Hence, platforms offer a low (often below–cost) price to the group whose participation entails a larger (reduced) marginal benefit to the other group, which becomes the profit–making segment.[3] Hence, media make use of two different kinds of strategies. On the one hand, they sort customers according to their externalities (i.e., *cross–group* price discrimination). On the other hand, once they know the identity and the behavior of the users, they also engage in BBPD within the group of subscribers.

This paper provides a model of two–sided competing media. On one side of the market, heterogeneous subscribers receive a utility coming from contents and suffer the presence of advertisements. On the other side, advertisers want to sell their products and the platforms are a means to reach consumers. In a two–period model, after the first round of subscription decisions, the platforms are allowed to discriminate users on the basis of their past behavior. The aim is to contribute to the literature of two–sided media as well as to the one of pricing under customer recognition. On the one hand, the paper shows that subscribers' switching affects the optimal behaviour of firms and ultimately reduce the provision of advertisements. On the other hand, it demonstrates that, even if platforms are always worse–off in the discriminatory case, subscribers might be worse– or better–off when they can be recognized and discriminated.

The advertisement is interpreted as in the broadcasting models of **?** and **?**. Firms pay broadcasters to advertise their products in order to meet potential consumers among viewers. Advertisements are purely informative and create some social value, given the fact that people exposed to commercials become aware of the existence of products they may like. However, whenever subscribers are repeatedly exposed to the same commercial, there is over–provision of advertisements. Under uniform subscription pricing, there are no movements of subscribers over time. The users' market is a competitive

[3] This is the so–called *Divide & Conquer* firstly proposed by **?**.

bottleneck as in **?**, as firms must advertise their products on a given platform to reach that platform's subscribers. This leads to multi–homing and gives an important power to media outlets, which become monopolistic in the eyes of advertisers. Differently, under BBPD, the switching of subscribers reduces this monopolistic power, as a firm may prefer to advertise its products only on one platform rather than multi–home. Indeed, this choice allows for reaching a sufficient number of subscribers (early subscribers plus switchers) meanwhile saving on the fee paid. This intensifies media competition on the advertisers' side, thus reducing the incentives to make profits on that side. Consequently, the two media set an amount of advertising lower than the one they would have set under uniform pricing.

The mechanisms just discussed allow for distinctive results in terms of subscriber surplus, confirming the fact that the theoretical conclusions and the implications offered by two–sided models can be different from the one that one would find using a one–sided logic.[4] This is because the analysis of any strategy used on one side of the market alone does not take into account what this strategy provokes on the other side. In one–sided oligopolies, price discrimination has often been proven to have a positive impact on consumer surplus. This is due to the fact that firms are very aggressive in order to attract individuals more inclined to buy the rival's product.[5] More specific to the BBPD literature, under repeated purchase, this business–stealing effect often drives towards a consumer–benefiting outcome. In **?**, **?**, **?** and **?**, the negative consequences on firms profits of late business stealing always outweigh the (possible) mitigation of early competition resulting from consumers' anticipation of future discounted prices. Recent articles have demonstrated how BBPD can actually be (partially) detrimental to consumers. In a context where firms have incomplete information about consumers' purchase histories, **?** shows an inverse U–shape relationship between consumer surplus and information accuracy. Moreover, BBPD boosts firms' profits at the detriment of consumers in the presence of a weak over–time correlation between consumers' preferences (**?**) or when consumers are sufficiently myopic in anticipating the future (**?**).

In the present model, pricing subscribers according to their past behavior causes switching which, in turn, changes the optimal decisions of advertisers. This finally leads to a different level of advertising

[4]As pointed out, among others, by **?**, **?** and **?**, given the peculiar features of two–sided markets, the antitrust and regulation policies might suffer a misrepresentation problem if one does not consider the two–sidedness of the market.

[5]See, among others, **?**, who provide a Hotelling model with perfect price discrimination, where the equilibrium prices turn out to be lower–than–uniform as a result of firms being very aggressive in more distant locations.

intensities at equilibrium, which has an impact on the final subscription price. In particular, three forces are at play. First, platforms compete more severely in the second period in order to poach the rival's subscribers. Second, the first–period competition is weakened by the users' anticipation of advantageous offers they will receive in the switching stage. Finally, on top of these two within–group forces, there is also a cross–group force which pushes final subscription prices to rise: the amount of advertising is lowered by switching, and therefore subscribers can be charged a higher price.

Overall, BBPD may boost or reduce subscriber surplus depending on the balance between these three forces, it always reduces platforms' profits, and it enhances advertisers' profits unless the advertising intensity becomes zero. The equilibrium level of advertisements is the key to understand the impact of BBPD on the total welfare, which depends on advertisement provision. Indeed, as in **?** and **?**, the social value of commercials is that they make users aware about products they like and this generates surplus–enhancing transactions between the two groups. Under uniform pricing, the subscribers' market is a competitive bottleneck, as firms need to advertise on a given platform to be known to that platform's subscribers. This brings about an over–provision of advertisements, because users receive more information than needed. Differently, BBPD and subscribers switching reduce the equilibrium amount of advertising, so that the over–provision of advertisements is then mitigated at the benefit of welfare. However, in the limit case in which the nuisance parameter is particularly high, BBDP leads to a full elimination of commercials with a consequent welfare loss due to under–provision of advertisements.

Finally, the paper contributes to the new line of two–sided market literature interested in within–group price discrimination. Proofs of an increasing interest of scholars in within–group price discrimination are the recent empirical investigations of **?** and **?** as well as the theoretical analysis of **?**. Using data on Spanish TVs and French newspapers respectively, **?** and **?** show that competition and advertisement revenues have a negative impact on the likelihood of media to use price discrimination. In a static model of perfect price discrimination with cross–group externalities, **?** show that within-group price discrimination may be positive for platforms at the detriment of the two sides of the market, confirming the fact that the conventional wisdom on one–sided markets can be misleading when analyzing a two–sided market.

The remainder of the paper is as follows. The next section presents the principal elements of the model. After, Section **??** is devoted to the analysis of the two benchmarks of uniform (**??**) and discriminatory pricing (**??**). Then, Section **??** discusses the differences between the two regimes for the advertisers' side, and Section 5 provides a welfare analysis on the effects of BBPD. Finally, Section **??** draws the conclusions.

2 The model

Two competing platforms $i = \{0, 1\}$ aim to offer access to media contents to a population of subscribers (users) in two periods. Platforms offer differentiated contents, represented by a location on a unit line. Namely, platform i offers content $l^i = i$. Ad–spaces are offered to a population of advertisers, who only care about the number of subscribers they can reach in each platform. For the sake of exposition, let us introduce the objectives of the two groups separately.

Subscribers. A mass N of users is interested in platforms' contents, which give an intrinsic utility u. The utility level u is assumed to be sufficiently high for the market to be fully covered. Users preferred content is represented by a location x, they are assumed to be uniformly distributed along the segment, and they face a transportation cost τ per unit of distance from each platform's contents. Moreover, subscribers are bothered by the presence of commercials. This is described by parameter β, which is common to all users. This has to be interpreted as the unitary nuisance cost (negative externality) which each advertiser creates to the users, because the enjoyment of contents is interrupted or disturbed by commercials. The utility of an agent located at x subscribing platform j will be:

$$U^i(x) = u - \beta a^i - p^i - \tau \left| x - l^i \right| \tag{1}$$

where a^i is the amount of advertising on platform i and p^i is the subscription price.

Advertisers. The advertisers' side is modeled as in **?** and **?**, who assume that each platform sets an advertising intensity and heterogeneous advertisers rationally decide whether to join each platform. A mass one of advertisers aim to sell products to the subscribers. Without loss of generality, products are sold at a zero marginal cost. Each producer offers a product of quality α, distributed on the interval $[0, \alpha^{max}]$ according to a p.d.f. G with $G(0) = 0$ and a continuously differentiable density. The lead example is a uniform distribution in the interval $[0, 1]$. Each quality–α advertiser has monopoly power so that, once the advertisement informs each subscriber on product characteristics and related price, the latter is willing to buy a product of quality α at price α. Therefore, each firm advertises price α, as lowering the price does not improve the probability of sale.[6] The access to platform i is needed for each advertiser to pass information on product existence to i–subscribers. All advertisers rationally decide whether to advertise their product in none, one or both platforms. This decision depends on how many subscribers can be met on platform i (i.e., s^i), and also on the amount paid to advertise on platform i (an endogenously determined per–period fee f_i). Accordingly, when s users can be reached by paying a fee f in each time period, the profit of a quality–α advertiser will be $\alpha s - 2f$. Therefore, all firms with selling a product of quality at least equal to $2f/s$ are willing to pay the fee f and, thus, the number of firms willing to advertise is $D_{ad}(f, s) = 1 - G(2f/s)$, which is the demand curve for advertising. The corresponding inverse demand curve is denoted by $f(a, s)$.

Platforms. Each platform i is allowed to decide on the number of advertisements a_i. Profits are made by selling advertising spaces to advertisers and by setting subscription prices. The time profit of platform i is given by:

$$\Pi^i = p^i s^i + f^i a^i \tag{2}$$

and future profits are discounted at a factor δ.

The timing of the model is as follows. At the beginning, platforms simultaneously choose their advertising intensity anticipating the impact on subscription prices and advertising revenues. Then, firms decide on which platform (none, one, or both) to advertise their products. Once an advertiser decides to enter a given platform, it pays a per–period fee f and its commercials are shown until the end of the game. Given their optimal advertisement intensities, platforms compete for users in two periods. In the first one, each platform i optimally sets a price p_1^i. Given the price and advertising

[6]Type α can be also interpreted as follows. Users who are informed on product type α's characteristics have a willingness–to–pay 1 with probability α and 0 with probability $1 - \alpha$.

intensity of each firm, consumers decide which platform to join, while firms decide where to advertise their products. After this first round of purchases, platforms set second–period prices. If previous subscribers can be identified, platform i sets price p_2^{ii} to old subscribers and p_2^{ij} to the new ones. Otherwise, $p_2^{ii} = p_2^{ij} = p_2^i$.[7]

3 Equilibrium analysis

This section is devoted to the equilibrium analysis. Following a backward–induction reasoning, the first focus is on subscription–price competition, with the advertisement intensities of both platforms as given, and then the optimal advertising decisions will be studied.

For the sake of exposition, it is convenient to define the per–user revenue of advertising. For a given inverse demand curve $f(a^i, s^i)$, the per–user market–clearing fee $f(a^i, s^i)/s^i$ can be used to compute the per user–user revenue of advertisements $R(a^i) = f^i a^i / s^i$, which is assumed to be a concave function of a^i. Accordingly, the time profit of platform i can also be expressed as follows:

$$\Pi^i = p^i s^i + R(a^i) s^i \tag{3}$$

The next section is dedicated to the benchmark case of uniform pricing and the subsequent one to the case of pricing based on past subscription behavior.

3.1 Uniform subscription price

Assume there exists a ban on price discrimination or that subscribers cannot be recognized across periods. In this scenario, competing platforms cannot distinguish between old and new subscribers, and $p_1^{ii} = p_2^{ij} = p_1^i = p_u^i$, where u stands for uniform. This would imply that the competition takes the form of a two–period repeated game in which nothing changes from the first to the second period.

[7] Notice that media are free to change price over time but they are assumed to commit earlier to an amount of advertisement. The commitment assumption is made in order to is to capture the fact that changing the amount of advertisement is not an immediate strategic decision to real–world media. A similar assumption is made by **?** in section 7.1, in which they study the impact of switching viewers on advertisement provision by broadcasts.

For this reason, the solution of the repeated game is nothing more than the solution of the per–period game.

For a given amount of advertising and given prices offered by the competing platforms, the number of subscribers attracted by firm i will be:

$$s_u^i = \frac{N}{2} + \frac{p_u^j - p_u^i - \beta(a_u^j - a_u^i)}{2\tau}N, \text{ with } i, j \in \{0, 1\} \text{ and } j \neq i. \tag{4}$$

in both periods. Plugging this demand for subscriptions into the profit function in Eq. (**??**) and differentiating with respect to p^i, the first–order conditions for platform i are:

$$\frac{\partial \Pi_u^i}{\partial p_u^i} = (p_u^i + R(a_u^i))\frac{\partial s^i}{\partial p^i} + s^i = 0 \Leftrightarrow p_u^i(p_u^j) = \frac{t + \beta(a_u^j - a_u^i) - R(a^j) + p_u^j}{2}. \tag{5}$$

Clearly, the best reply of firm i with respect to firm j's price is to take into account differences in advertising intensities and compensate the users for the presence of advertisements by lowering the price by an amount equal to the per–user revenues. Solving the system of best replies, it is straightforward to verify that the sub–game perfect equilibrium prices will be:

$$p_u^i(a_u^i) = \tau + \frac{\beta(a_u^j - a_u^i)}{3} - \frac{2R(a_u^i) + R(a_u^j)}{3} \text{ with } i, j \in \{0, 1\} \text{ and } i \neq j. \tag{6}$$

so that platform i attracts $s_u^i = \frac{N}{2} + \frac{R(a_u^i) - R(a_u^j) + \beta(a_u^j - a_u^i)}{6t}N$ and platform j attracts the remaining $s_u^j = N - s_u^i$.

At the beginning of the game, the platforms simultaneously set the optimal advertisement intensities. The number of advertisements chosen takes into account both the direct effect on the number of subscribers and the indirect effect on the subscription price. Platform i sets a^i to maximize:

$$\Pi_u^i(a^i) = (1 + \delta)[p_u^i(a_u^i) + R(a_u^i)]s_u^i(a_u^i), \tag{7}$$

anticipating the effect that the amount of advertising has on advertising revenues, subscription prices, and subscribing decisions. The equilibrium is summarized in the following lemma:

Lemma 1. *When subscribers cannot be identified, platforms set the equilibrium price* $p_u^* = \tau - R(a_u^*)$

and an advertising intensity $a_u^* = \max\{\hat{a}, 0\}$ *with* $R'(\hat{a}) = \beta$.

Proof. The first–order conditions of this problem are:

$$\frac{\partial \Pi_u^i}{\partial a_u^i} = \left[\frac{\partial p_u^i}{\partial a_u^i} + R'(a_i)\right] s_u^i + \frac{\partial s_u^i}{\partial a_u^i}\left[p_u^i(a_u^i) + R(a_u^i)\right] \leq 0 \tag{8}$$

Given Eq. (**??**) and rearranging terms, an interior solution is such that:

$$\frac{\partial \Pi_u^i}{\partial a_u^i} = (R'(a_i) - \beta)\left[\frac{1}{3} + \frac{R(a_u^i) - R(a_u^j) + \beta(a_u^j - a_u^i)}{9\tau}\right] N = 0 \Leftrightarrow R'(a_u^*) = \beta, \tag{9}$$

with corner solution $a_u^* = 0$ whenever $R'(0) < \beta$. Plugging a_u^* into the prices, we get $p_u^* = \tau - R(a_u^*)$.

■

Lemma **??** delivers two main messages. On the one hand, there is a full pass–through of advertising revenues into lower subscription prices, so that users will enjoy lower prices as long as per–user advertising revenues increase.[8] In other words, users are compensated for the presence of commercials by receiving lower prices. On the other hand, providing a fixed number advertising spaces that equates per–user marginal revenues to the nuisance parameter β is the best solution for the two platforms. Clearly, this amount of advertising is zero whenever β is too high (corner solution). In the opposite limit case in which the advertisement creates no nuisance, each media platform provides the monopoly advertising intensity.

In the present and in the following section, the advertisers' decisions are not explicitly discussed. This choice was made for the sake of expositive clarity, these decisions are analyzed in Section **??**, which aims to underline the differences in equilibrium advertising intensities that emerge going from the uniform to the discriminatory subscription pricing.

3.2 Behavior–based subscription prices

In this section, first–period prices as well as the identity of first–period subscribers are assumed to be observable to both platforms when they choose second–period prices. Following a backward–induction reasoning, the first analysis is on the response of users to second–period prices and then, proceeding

[8] Same full pass–through is present in **?** and in **?**.

backwards step–by–step, the optimal decisions on advertisement intensities will be presented.

Second Period. Once they receive second–period offers, subscribers decide whether to switch or to stay. On the inherited turf of platform i, a subscriber prefers to join again media i rather than switching whenever $u - \beta a^i - p^{ii} - \tau|x - l^i| \geq u - \beta a^j - p^{jj} - \tau|x - l^j|$ which gives the following indifferent location:

$$x_2^i = \frac{1}{2} + \frac{p_2^{ji} - p_2^{ii} + \beta(a^1 - a^0)}{2\tau} \text{ with } i, j \in \{0, 1\} \text{ and } j \neq i. \tag{10}$$

Eq. (**??**) says that, among the agents who subscribed to platform i in period one, all subscribers located between x_i and l^i will remain loyal to platform i, while the remaining ones will switch to the rival. Given a time–1 market–splitting location x_1, the agents located between l^i and x_2^i remain on platform i, so that $s_2^{ii} = |x_2^i - l^i|N$ agents are loyal to i. Conversely, the ones located between x_2^j and x_1 switch from platform j to platform i: this segment is composed of $s_2^{ij} = |x_2^j - x_1|N$ agents.

In what follows, x_1 is assumed to be close enough to $1/2$, so that both platforms succeed to poach some subscribers from the competitor.[9] This allows to conclude the following:

Lemma 2. *When the platforms are allowed to price discriminate between old and new subscribers, for $i, j \in \{0, 1\}$ and $i \neq j$ the second–period equilibrium prices are:*

$$p_2^{ii} = \tfrac{\tau}{3} + \tfrac{2\tau}{3}\tfrac{s_1^i}{N} + \tfrac{\beta(a^j - a^i)}{3} - \tfrac{2R(a^i)+R(a^j)}{3} \quad \textit{and}$$
$$p^{ij} = \tau - \tfrac{4\tau}{3}\tfrac{s_1^i}{N} + \tfrac{\beta(a^j - a^i)}{3} - \tfrac{2R(a^i)+R(a^j)}{3},$$

where $s_1^i/N = |x_1 - l^i|$ is the time–1 market share of platform i on the subscribers' side.

Proof. See Appendix **??**. ■

The own inherited market share affects positively the price to the loyal subscribers, and negatively the one offered to the switchers. This follows directly from the effective power that the size of the first–period market creates on each turf for the "attacking" (else turf) and the "defending" platform (own turf). Clearly, the attack on the rival turf turns out to be more costly as the size of the

[9]This assumption follows **?** and it allows to consider only symmetric switching scenarios. See **?** for an analysis of **?** second period with the past taken as given and **?** for an inter–temporal analysis of asymmetric equilibria.

market already conquered in the first period becomes larger, as the offer to the switchers should be particularly advantageous in order to attract people who would pay a high transportation cost. Moreover, advertisements reduce the subscribers' willingness–to–pay, thus making the price decreasing as a^i gets higher.

First period. Taking into account the possibility of tomorrow's switching, the utility of an agent located at x who joins platform j in the first period and i in the second one is $U^{ij}(x) = u - \beta a^j - p_1^j - \tau \left|x - l^j\right| + \delta(u - \beta a^i - p_2^{ij} - \tau \left|x - l^i\right|)$, with j possibly different from i in case of second-period switching.

The indifferent agent who joins platform j will switch to platform $i \neq j$ in the subsequent period. Therefore the indifferent subscriber locates at:

$$x_1 = \frac{1}{2} + \frac{3\left(p^1 - p^0\right) + 3\beta(1-\delta)\left(a^1 - a^0\right) + \delta(R(a^0) - R(a^1))}{2(3+\delta)\tau} \tag{11}$$

and under full market coverage, it holds that the total number of users joining platform 0 and platform 1 will be $s_1^0 = x_1 N$ and $s_1^1 = (1 - x_1)N$ respectively. It is important to notice that users are less responsive to price compared to the case of uniform pricing (i.e., $-\frac{\partial s^j}{\partial p^j} < -\frac{\partial s_u^j}{\partial p_u^j}$). This is the case because the indifferent user is sensible to the opportunity of switching and enjoying a discounted price tomorrow.[10] Similarly, the indifferent location x_1 takes into account not only the direct impact of the advertising intensities on first–period utility but also on second–period prices.

At time 1, the media platforms correctly anticipate both first–period purchase decisions and the indirect effect on second–period profits, as the market share of period 1 determines the second–period competition for switchers. The inter–temporal equilibrium in the subscription–price competition is described in the following proposition.

Lemma 3. *Let us consider a situation in which advertisement intensities are given. Then the first–period equilibrium prices are equal to:*

[10] If users were completely myopic, they would take into account only the current utility (i.e., $\delta = 0$). In that case the indifferent consumer would respond to price variations as if the price was uniform in both periods.

$$p_1^i = \tau + \frac{\delta\tau}{3} + \frac{\beta(a^j - a^i)}{27 - 11\delta} - \frac{(9 - \delta(8 - 3\delta))R(a^j) + (3\,(6 - \delta(1 + \delta))\,R(a^i)}{27 - 11\delta} \textit{ with } i, j \in \{0, 1\} \textit{ and } i \neq j$$

Proof. See Appendix **??**. ■

When setting its price, platform i takes into account two main aspects. On the one hand, for a given advertising intensity, the first–period demand is less sensitive to price because of the switching offers that are going to be received later on. This is captured by the additional term $\frac{\delta\tau}{3}$, which increases the early price. On the other hand, the advertising intensity will have both the direct effect of reducing the users' willingness–to–pay, and the indirect effect of reducing future prices so as to make switching offers more interesting for users. In a symmetric situation in which $a^i = a^j$, the price boils down to full pass–through of advertising revenues ($p_1^i = \tau + \frac{\delta\tau}{3} - R(a^i)$).

Advertisement intensity. Similarly to the case of uniform subscription prices, let us now analyze the optimal decisions of the two platforms at the beginning of the game, where the optimal advertisement intensities are simultaneously set. Here, the amount of advertising chosen takes into account the direct effect on the first–period competition (prices and number of subscribers) and the indirect effect on the switching phase (prices and number of switchers). The analysis delivers the following equilibrium:

Proposition 1. *When subscribers can be identified, the equilibrium is as follows:*

1. *In the first period, both platforms set the same advertising intensity* $a^* = \max\{\bar{a}, 0\}$ *with* $R'(\bar{a}) = \beta$ *and the same first–period price* $p_1^* = \tau\left(1 + \frac{\delta}{3}\right) - R(a^*)$*, so that they attract half of the subscribers.*

2. *In the second period, both platforms offer a price* $p_2^{ii*} = \frac{2\tau}{3} - R(a^*)$ *to loyal users and a price* $p_2^{ij*} = \frac{\tau}{3} - R(a^*)$ *to switchers.*

3. *In the second period, one third of the agents remain loyal* ($s_2^{ii*} = N/3$) *and one sixth switch to the rival platform* ($s_2^{ij*} = N/3$)*.*

Proof. See Appendix **??**. ■

Similarly to the uniform case, advertising intensity will be such that the marginal revenue equals the nuisance parameter, and when the latter is too high, corner solutions with no advertisement are found. Contextually, also in this case there is full pass–through of advertising revenues into lower subscription prices. Second–period prices are lower than first–period ones as each platform attacks the rival's turf and defends their own turf by charging a lower subscription fee. In the comparison with the uniform pricing, everything depends on the advertising revenues made in each case, which can be different because switchers may create an additional value to advertisers when they decide where to advertise their product. These differences will be explained in the following section.

4 Advertising intensities: the value of switchers

So far, the decisions of the advertisers have been taken as given. In particular, using the per–user advertising revenue, it has been demonstrated that at equilibrium the latter equals the nuisance parameter in both the uniform and the BBPD case, i.e., $R'(a_u^*) = R'(a^*) = \beta$. This section aims to explain, given that $R(a_u^*)$ turns out be different from $R(a^*)$, how the optimal advertising intensity changes across regimes and how this, given the full pass-through, will in turn affect the level of subscription prices. In particular, what follows discusses what each firm prefers to do (no advertisements, single–homing or multi–homing) in the two regimes of uniform subscription price and BBPD.

Before going into detail, it should be noticed that going from a scenario in which subscribers do not switch and to a one in which they do it can make a dramatic difference to advertisers.[11] Intuitively, when users are not mobile between platforms, each media outlet has a monopoly: in order to reach a given subscriber, a firm must advertise on the platform that the subscriber joined. To put it differently, the users' market is a "competitive bottleneck", and the individual decision to show a commercial on platform i is independent of the decision of showing the same commercial on platform j. Differently, when users switch media over time, some of them can be reached through different platforms. This may lead some advertisers to prefer single–homing on one platform (in which first–period subscribers plus second–period switchers can be reached) rather than paying both platforms to reach all users.

[11]See also **?** when they discuss the role of switching viewers in their model of broadcasting.

Let us consider the two cases separately.

Advertisers' decisions: uniform subscription price. A firm selling a product with quality α is willing to enter platform i if it is worth paying the (endogenously determined) fee f_u^i, where the subscript u stands for uniform subscription price. The benefit depends on how many subscribers can be met on platform i. Under uniform pricing, each advertiser anticipates that half of the subscribers can be met on platform i and the other half on platform j. For this reason, a quality–α advertiser is willing to join platform i if $\alpha s_u^i - 2f_u^i \geq 0$ or $\alpha \geq \frac{2f_u^i}{s_u^i} \equiv \bar{\alpha}^i$.

The cutoff $\bar{\alpha}^i$ is the quality of the marginal advertiser that determines the demand curve for advertisement $D_{ad} = 1 - G(\bar{\alpha}^i)$, so that the fee is endogenously determined by $a_u^i = 1 - G\left(\frac{2f_u^i}{s_u^i}\right)$. In a symmetric equilibrium, each platform sets the same subscription price and advertising intensity, so that $s_u^i = 1/2$ in both periods. Therefore, the market–clearing fee will be such that:

$$a_u = 1 - G\left(2\frac{f_u}{s_u}\right) = 1 - G\left(\frac{4f_u}{N}\right) \text{ or, equivalently, } f_u = \frac{G^{-1}(1-a_u)N}{4} \tag{12}$$

Two remarks on Eq. (**??**) are needed. On the one hand, the superscript i can be eliminated. Indeed, since no user switches from one platform to the other, the decision of advertising on platform i is independent of advertising on the other platform, so that in a symmetric equilibrium a firm either multi–homes or does not advertise its product. On the other hand, the choices of each firm are statically based on current subscribers reached only, as no new potential customer is expected to be found in the subsequent period. As a result, the per–user time revenue of advertisement will be $R(a_u) = \frac{a_u f_u}{s_u} = \frac{a_u G^{-1}(1-a_u)}{2}$.

Advertisers' decisions: BBPD. Now, consider the optimal decision of an advertiser when platforms engage in behavior–based price discrimination on the other side of the market. Each advertiser has two options. On the one hand, they may decide to join both platforms (multi–home) and then be sure to reach all users. This will give a profit of αN and will cost $2f^i + 2f^j$. On the other hand, the alternative is to advertise only on platform i (single–home). Compared to multi–homing, this will entail a renounce in terms of potential customers reached but also a saving in terms of fees paid, i.e., the net profit will be $\alpha S^i - 2f^i$, where $S^i = s_1^i + s_2^{ij}$ represents the total number of users reached on platform i (first–period subscribers plus second–period switchers).

Comparing the two options, one can easily notice that there exist two cutoff types $\alpha_{SH}^i \equiv \frac{2f^i}{S^i}$ and $\alpha_{MH}^i \equiv \frac{2f^j}{1-S^i}$, so that all firms with $\alpha \in [\alpha_{SH}^i, \alpha_{MH}^i)$ are willing to single–home on platform i and all firms selling a product of quality above α_{MH}^i will prefer to multi–home. Clearly, high–quality producers face a higher cost of failing to reach all users, so they will always multi–home. Given the symmetry of the problem, Proposition **??** shows that it must be the case that equilibrium advertisement intensities and prices are the same for both platforms. This will lead to a market–clearing fee common to the two platforms and to a total number of subscribers met by single–homing firms equal to $S^i = s_1^i + s_2^{ij} = \left(\frac{1}{2} + \frac{1}{6}\right) N = \frac{2N}{3}$. Hence, a single–homing firm will be indifferent between the two media: to break ties, each joins each of two media with probability 1/2. At the market clearing fee f, the demand for advertising will be equal to the number of advertising spaces offered by each platform, so that the fee will be such that:

$$a = \underbrace{1 - G\left(\frac{2f^j}{1-S^i}\right)}_{\text{multi–homing}} + \underbrace{\frac{1}{2}\left[G\left(\frac{2f^j}{1-S^i}\right) - G\left(\frac{2f^i}{S^i}\right)\right]}_{\text{single–homing}} = 1 - \frac{G\left(\frac{6f}{N}\right) + 2G\left(\frac{3f}{N}\right)}{2} \tag{13}$$

Clearly, the advertising intensity in the two cases will be different because it will result in different market–clearing fees, thus changing the optimal behavior of the two media. In particular, one can verify that:

Lemma 4. *Provided that $G\left(\frac{3\alpha^{max}}{4}\right) > \frac{1}{2}$ (sufficient condition), the advertising intensity will be lower under BBPD.*

Proof. See Appendix **??**. ∎

Lemma **??** highlights a very important result.[12] Under a uniform subscription price, all firms willing to advertise on one platform are also willing to do the same on the rival one, because multi–homing lets them reach all users (half on one platform and half on the other one). Differently, when

[12]The sufficient condition in Lemma **??** is not too stringent and it is clearly verified in the lead example of $\alpha \sim U[0, 1]$. Therefore, it is taken as given in the remainder of the analysis.

media engage in BBPD on the subscribers' side, some firms prefer to advertise only on one platform as this choice lets them meet more than half of the subscribers and lets them save in fees. *Ceteris paribus*, this reduces the advertisement demand of each platform, which will then make less money per user. As a consequence, advertisements become less profitable for the platforms, which opt for a lower number of advertisements. In a sense, the presence of single–homing firms intensifies platform competition on that side of the market, inducing the two media to reduce the amount of advertising.

5 Welfare effects of BBPD

The fact that advertising intensities are lower in the presence of switching subscribers should not surprise. Indeed, switching leads to a situation in which the advertisers' side becomes less attractive (more competition), so the two platforms reduce the amount of advertising meanwhile making subscribers pay a higher price. Indeed, as already discussed in **?**, the full pass–through of advertising revenues into subscription prices implies that the number of commercials does not affect equilibrium profits of the two platforms. In other words, the nuisance parameter β affects the number of commercials as well as the subscription prices, but in terms of profits the two forces go in opposite directions and perfectly offset. This "profit neutrality" on β can be easily proved by looking at the profits of the two media. In the uniform case, each platform sets price $p_u = t - R(a_u^*)$ in both periods. Therefore, the profit will be:

$$\Pi = (1+\delta)\{[p_u^* + R(a_u^*)] \cdot s_1^*\} = \frac{(1+\delta)N\tau}{2}.$$

Under BBPD, given the prices in Proposition **??**, equilibrium profits are:

$$\begin{aligned}
\Pi &= [p_1^* + R(a^*)]s_1^* + \delta[p_2^{ii*}s_2^{ii*} + p_2^{ij*}s_2^{ij*} + R(a^*)(s_2^{ii*} + s_2^{ij*})] \\
&= [\tfrac{\tau N}{2}\left(1+\tfrac{\delta}{3}\right)] + \delta[(\tfrac{2\tau}{3} - R(a^*))\tfrac{N}{3} + (\tfrac{\tau}{3} - R(a^*))\tfrac{N}{6} + R(a^*)\left(\tfrac{N}{3} + \tfrac{N}{6}\right)] \\
&= [\tfrac{\tau}{2}\left(1+\tfrac{\delta}{3}\right)] + \tfrac{5\delta\tau}{18} = \left[\tfrac{(1+\delta)\tau}{2} - \tfrac{\delta\tau}{18}\right]N.
\end{aligned}$$

Comparing the two profits, the following holds.

Proposition 2. *Platforms' profits are lower when users can be identified.*

Hence, when subscribers can be identified, the platforms' profits are reduced because of a fiercer competition in the second period, which prevails on the reduced first–period competition due to the fact that subscribers anticipate the opportunity of switching. This is a standard result of the one–sided market literature of BBPD (e.g., **?**). In the two–sided environment presented here, the nuisance parameter β has no impact on total profits. The market is as if it was one–sided, with the nuisance parameter only determining the surplus distribution across sides. In terms of the impact of BBPD, it means that the main mechanisms (reduced early competition and stronger late competition for switchers) remain unchanged going from a one–sided to a two–sided market. What changes is what happens on each side.

The impact of subscriber poaching on the well–being of each single advertiser will depend on the number of subscribers met and also on the fee. As shown in Lemma **??**, the equilibrium advertising intensity is lower under BBPD. In particular, when advertisers' types are uniformly distributed, it holds that $a^* = \max\left\{\frac{1-3\beta}{2}, 0\right\} < \max\left\{\frac{1-2\beta}{2}, 0\right\} = a^*_u$. Nevertheless, a reduction of the supply of advertising spaces does not result in a higher market–clearing fee, because the demand for advertising is also reduced by the fact that some advertisers single–home. Therefore, the fee is lower under BBPD, i.e., $f^*_u = \frac{(1+2\beta)N}{8} > \frac{(1+3\beta)N}{12} = f^*$.

Clearly, for high values of β, platforms may find it profitable to set the number of commercials to zero. This happens in both regimes $\beta > 1/2$ and in the discriminatory one when $\beta \in \left(\frac{1}{3}, \frac{1}{2}\right)$. Therefore, advertisers are indifferent across regimes in the first case and always prefer the uniform subscription pricing in the second one. When β is low enough to have positive amounts of advertising in both cases, under uniform subscription price, all firms with $\alpha > 2f^*_u/s^*_u = \frac{1}{2}(1+2\beta)$ multi–home on both platforms. Each of them receives a profit equal to $\pi^{ad}_u(\alpha) = \alpha - 4f^*_u = \frac{N(2\alpha-2\beta-1)}{4}$. Hence, in aggregate terms, the total surplus of advertisers will be:

$$\Pi^{ad}_u = \int_{\frac{1}{2}(1+2\beta)}^{1} \pi^{ad}_u(\alpha)d\alpha = \frac{(1-2\beta)^2 N}{16}$$

Differently, when subscribers switch from period one to period two, all firms with $\alpha > \frac{4f^*}{1-S^*} = \frac{1}{2} + \frac{3\beta}{2}$ will multi–home and they receive a profit of $\pi^{ad}_{mh}(\alpha) = \frac{(3\alpha-3\beta-1)N}{3}$. Differently, firms with intermediate $\alpha \in \left(\frac{2f^*}{S^*}, \frac{4f^*}{1-S^*}\right) = \left(\frac{1}{4} + \frac{3\beta}{4}, \frac{1}{2} + \frac{3\beta}{2}\right)$ will single–home in one of the two media and they receive a profit of $\pi^{ad}_{sh}(\alpha) = \frac{N(4\alpha-3\beta-1)}{6}$. Therefore, the aggregate profit of advertisers will be:

$$\Pi^{ad} = \int_{\frac{1}{4}+\frac{3\beta}{4}}^{\frac{1}{2}+\frac{3\beta}{2}} \pi^{ad}_{sh}(\alpha)d\alpha + \int_{\frac{1}{2}+\frac{3\beta}{2}}^{1} \pi^{ad}_{mh}(\alpha)d\alpha = \frac{(1-\beta)(7-15\beta)N}{32}.$$

A simple comparison between the two aggregate profits gives the following result.

Proposition 3. *When $\beta < 1/3$, advertisers are better–off under BBPD. When $\beta \in (1/3, 1/2)$, the uniform pricing is preferred by advertising firms. For higher levels of β, advertising intensity is zero in both regimes.*

Clearly, when $\beta \in (1/3, 1/2)$, the number of advertisements is zero when users switch across periods, so that advertisers cannot make any profit.[13] More interesting is the case in which advertising intensity is positive in both the uniform and the discriminatory regime. In this case, under the discriminatory regime, advertisers face a lower fee in comparison with the uniform subscription pricing. This is because, even if the supply of advertising spaces is contracted, some firms opt for single–homing on a unique platform, thus pushing the fee to decrease. This fee reduction clearly makes multi–homing firms always better–off, as they pay a lower amount to reach the same number of subscribers, i.e., the entire population N. Single–homing firms are also better–off, as they save both on the fee duplication they would face when multi–homing and on the fee reduction. Moreover, even if the advertising spaces are scarcer when subscribers switch media, a higher number of firms advertise on at least one platform. Indeed, the infra–marginal advertiser single–homing in the switching scenario has a lower quality $\left(\alpha = \frac{1}{4} + \frac{3\beta}{4}\right)$ than the infra–marginal advertiser of the uniform case $\left(\alpha = \frac{1}{2} + \frac{3\beta}{2}\right)$.

On the other side of the market, BBPD has two effects. On the one hand, it has the usual *within–group* direct effect: firms compete more fiercely and prices, for given advertising revenues, go down. On the other hand, though, the reduction of advertising revenues associated with BBPD entails a *cross–group* opposite force going towards an increase in the subscription prices. In order to understand which one of the two effects prevails, let us consider a uniform distribution of advertisers types with $\alpha^{max} = 1$. In this case, one can easily find the following result.

[13] See also Figure **??**.

Proposition 4. *When past subscription behavior is observed, then*

1. *first–period subscription prices are higher than uniform;*

2. *in then second–period, prices to new subscribers are higher than uniform if* $\tau < \frac{1}{16}\min\{1 + 6\beta^2, 3 - 12\beta^2\}$*, whereas prices to old subscribers are higher than uniform if* $\tau < \frac{1}{8}\min\{1 + 6\beta^2, 3 - 12\beta^2\}$.

Proof. See Appendix **??** ∎

Point 1 of Proposition **??** is a standard conclusion of the one–sided literature of BBPD and it is simply due to the fact that, when having the opportunity to switch tomorrow, the first–period indifferent subscriber is less responsive to early prices. Differently, points 2 and 3 deserve a more detailed discussion, as they are specific to the two–sidedness of the market. Indeed, for second–period subscription prices, the relationship between transportation cost and nuisance parameter turns out to be crucial. As a matter of fact, these two parameters are simply capturing the *within-side* and *cross–group* effects on prices as discussed above. When τ is sufficiently high, what happens on the side of subscribers is the only thing that matters. As in the traditional one–sided literature of BBPD, knowing the identity of previous subscribers induces platforms to compete more severely, with the consequence of lower subscription prices. In contrast, when β is sufficiently high in relation to τ, what happens on the other side of the market prevails and determines higher discriminatory prices. Indeed, in this case lower advertising revenues correspond to higher subscription prices.

Therefore, compared to a one–sided market, the subscriber surplus is not always higher when the customer's identity is known to the rival platforms. Indeed, discriminating prices can be either higher– or lower–than–uniform and, moreover, also the number of advertisements changes across regimes. When subscription prices are lower–than–uniform, BBPD would surely benefit subscribers, who are offered lower prices and are also bothered by a lower number of commercials. In the opposite case, the augmented subscription prices are accompanied by a reduction of advertising disutility, so that it is not trivial to understand which of the two forces would prevail.

Hereafter, $U^{ij}(x)$ refers to the inter–temporal utility of a user located at x who subscribes to platform j in the first period and platform i in the second one, with possibly $i \neq j$ in case of switching in the second period. This utility will be equal to:

$$U^{ij}(x) = u - \beta a^j - p_1^j - \tau|x - l^j| + \delta(u - \beta a^i - p_2^{ij} - \tau|x - l^i|) \text{ where } i,j \in \{0,1\}.$$

If $i = j$, user x above is loyal to platform j in both periods so that the total transportation cost becomes $2|x - l^j|$, l^j being the location of firm j. On the contrary, when $i \neq j$ the subscriber switches from j to i and faces the transportation cost on the entire Hotelling line. When no subscriber switches in the second period, the subscriber surplus will be:

$$\begin{aligned} S_u &= (1+\delta)\left[\int_0^{1/2}(u - \beta a^* - p_u^* - tx)dx + \int_{1/2}^{1}(u - \beta a^* - p_u^* - t(1-x))dx\right] \\ &= (1+\delta)\left[u + \max\left\{\tfrac{(1-2\beta)^2}{8}, 0\right\} - \tfrac{10\tau}{8}\right]. \end{aligned}$$

In case of switching, the surplus of subscribers is given by:

$$\begin{aligned} S &= \underbrace{\int_0^{1/3} U^{00}(x)dx + \int_{2/3}^{1} U^{11}(x)dx}_{\text{loyal subscribers}} + \underbrace{\int_{1/2}^{2/3} U^{01}(x)dx + \int_{1/3}^{1/2} U^{10}(x)dx}_{\text{switchers}}, \\ &= (1+\delta)\left[u + \max\left\{\tfrac{(1-3\beta)^2}{12}, 0\right\} - \tfrac{(43\delta+45)\tau}{12}\right]. \end{aligned}$$

A comparison between the two subscriber surpluses gives the following proposition.

Proposition 5. *Let*

$$\bar{\tau} \equiv \begin{cases} \frac{3\left(1-6\beta^2\right)(1+\delta)}{4\delta} & \text{if } \beta \leq 1/3 \\ \frac{9\left(4\beta^2-4\beta+1\right)(1+\delta)}{4\delta} & \text{if } \beta \in (1/3, 1/2) \\ 0 & \text{if } \beta > 1/2. \end{cases}$$

If the transportation cost is sufficiently high ($\tau > \bar{\tau}$), then BBPD increases subscriber surplus. Otherwise, BBPD damages subscribers.

Proposition **??** confirms the fact that, depending on a relationship between the transportation cost τ and the nuisance parameter β, subscriber surplus can be reduced when the past subscription behavior is known to the competing media outlets. This is because, even if they compete more fiercely

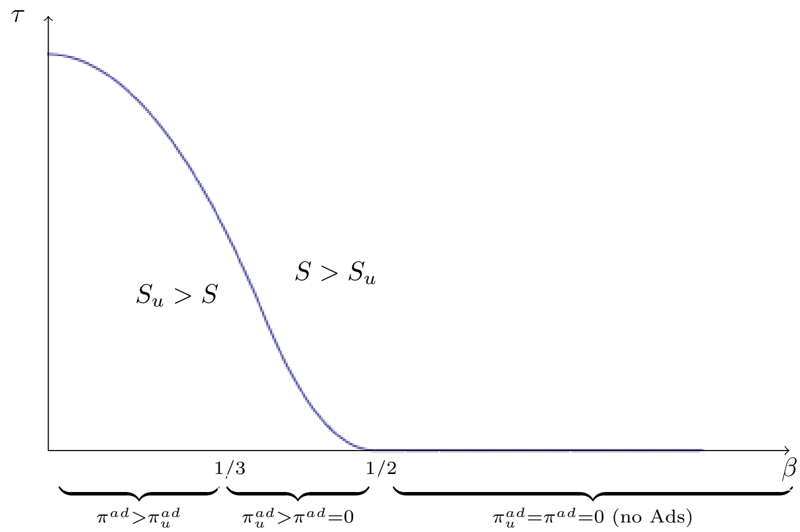

Figure 1: Subscriber surplus and advertisers profits across regimes.

to steal each others' subscribers and the advertisement intensity is lower under the discriminating regime, the profits made on the advertisers' side are reduced and then this is recouped by increasing the subscription price. This leads to the damage of BBPD to subscribers whenever the platforms are not too differentiated (low τ). This is shown in Figure **??**: below the blue curve, subscribers are better–off if prices are uniform, whereas above the curve the opposite is true. Notice that the higher the nuisance parameter, β, the more likely it is that BBPD a benefits subscribers. This is because, the total price (subscription price plus shadow cost of advertisements) reacts more strongly to variations in β in the uniform case. Indeed, one can easily verify that:[14]

$$\frac{\partial(p_u^* + \beta a_u)}{\partial\beta} = \frac{1}{2} - \beta > \frac{1}{2} - \frac{3\beta}{2} = \frac{\partial(p_1^* + \beta a^*)}{\partial\beta}.$$

Hence, given the transportation cost, the total price paid by a subscriber increases faster in the uniform regime, so that the discriminatory pricing becomes more and more attractive as β increases.

In terms of social surplus, comparing $\Pi + \Pi^{ad} + S$ with $\Pi_u + \Pi_u^{ad} + S_u$, one can verify the following Proposition.

[14]The first–period price p_1^* is used in the derivative below, but one can use any price, since the derivative with respect to β would always be the same.

Proposition 6. *If $\beta \geq 1/2$, BBPD and uniform subscription pricing lead to the same aggregate welfare. When $\beta \in (1/3, 1/2)$, the total welfare is reduced by BBPD. Conversely, when $\beta < 1/3$, BBPD increases welfare.*

Only cases with $\beta < 1/2$ are worth being discussed, as for $\beta > 1/2$ advertisement is null in both regimes, and then BBPD brings about a surplus redistribution (from the platforms to the users) without any effect on total surplus. The results of Proposition **??** derive from the fact that the amount of advertising is reduced under BBPD. This may boost or damage total welfare depending on the nuisance. If the nuisance is sufficiently high, the number of advertisements will be zero in the discriminating regime, whereas it is positive under uniform subscription pricing. This leads to an under–provision of advertisements reducing welfare: a positive number of advertisements would be socially beneficial as it would allow some users to be informed and then buy products they would like. This makes uniform pricing preferred because it allows for a positive amount of advertising. However, the advertising intensity chosen by the platforms always lead to over–provision, as users might be shown the same commercial twice. This is wasteful from a social viewpoint since, once informed, a subscriber will simply buy the advertised product without needing to be informed the second time. Therefore, whenever BBPD allows for a positive amount of advertising ($\beta < 1/3$), it will be welfare–enhancing with respect to uniform pricing. Indeed, the severity of the problem of over–provision of commercials is mitigated by the fact that some advertisers are able to reach users just being present in only one platform. This reduces the total number of advertisements, which will be closer to its social optimum level.

6 Conclusion

The main message of the literature of pricing under customer recognition is that sellers face a problem similar to a prisoner's dilemma. Once they have information on consumers' past purchase behavior, each firm has incentives to attack the rival's turf. This intensifies competition and makes discriminatory pricing beneficial to consumers. The present paper sheds light on the consequences of BBPD by looking at media markets. In the traditional media markets as well as in the emerging markets of online content provision, platforms engage in BBPD by offering discounted prices to subscribers and, meanwhile, they sell advertisement spaces to firms interested in reaching new demand. The paper proves that cross–group externalities do involve some distinctive additional effects that enrich the set of possible outcomes, both in terms of surplus distribution and in terms of total welfare.

In a two–sided environment, subscription prices take into account both a direct within–side effect and an indirect one related to the other side of the market. On the one hand, when competing media offer differentiated contents, the level of differentiation (measured by the transportation cost) entails some market power which pushes subscription prices to rise. On the other hand, since the commercials create nuisance to subscribers (measured by β), there is a full pass–through of advertising revenues into lower subscription prices.

In relation to within–group uniform pricing, price discrimination based on the past subscription behavior of users changes the two forces above in opposite ways. On the one hand, the platforms tend to directly compete more severely to poach the rival's inherited subscribers. This somehow reduces the extent to which platforms exploit content differentiation and it is directly related to the subscribers' response to prices. On the other hand, the switching of subscribers resulting from BBPD leads some advertisers to prefer single– to multi–homing, with a reduction in the demand of advertising spaces. The consequence is that platforms make less money on commercials and then the subscription price is pushed to increase. The balance between these two opposite forces depends on the relative size of the differentiation and the nuisance parameter. Second–period subscription prices are higher than uniform (lower–than–uniform) when β (τ) is sufficiently high, and first–period subscription prices are always higher than uniform.

Overall, BBPD always reduces the platforms' profits and increases those of the advertisers (unless the advertising intensity becomes zero), whereas the subscriber surplus might increase or decrease depending on τ and β. In sum, the equilibrium level of commercials is crucial to understand the impact of BBPD, as total welfare comparisons essentially depend on advertisement provision. Indeed, commercials have a social value for their informative role: users become aware of products they like, and this generates surplus–enhancing transactions between the two groups. Under uniform pricing, the two media outlets are monopolistic and the subscribers' market is a competitive bottleneck. As a consequence, the demand for advertising spaces is maximal and this results in an over–provision of advertisements, because users repeatedly receive information on the same products. Differently, the switching of subscribers reduces the equilibrium amount of advertising because some firms prefer to advertise only on one platform rather than multi–homing. Indeed, this choice allows for reaching a sufficient number of subscribers (early subscribers plus switchers) meanwhile saving on the fee paid. This intensifies competition on the advertisers' side, thus reducing the incentives to make profits on that side. The over–provision of advertisements present in the uniform case is then mitigated at the benefit of welfare. However, in the limit case in which the nuisance parameter is particularly high, BBDP leads to a full elimination of commercials with a consequent welfare loss due to the under–provision of advertisements.

Appendix A

A.1 Proof of Lemma ??.

Platform i maximizes the following profit:

$$\Pi_2^i = p_2^{ii} s_2^{ii} + p_2^{ij} s_2^{ij} + R(a^i)[s_2^{ii} + s_2^{ij}]$$

Differentiating with respect to prices, one gets:

$$\frac{\partial \Pi_2^i}{\partial p_2^{ii}} = [p_2^{ii} + R(a^i)]\frac{\partial s_2^{ii}}{\partial p_2^{ii}} + s_2^{ii} = 0$$

$$\frac{\partial \Pi_2^i}{\partial p_2^{ij}} = [p_2^{ij} + R(a^i)]\frac{\partial s_2^{ij}}{\partial p_2^{ij}} + s_2^{ij} = 0$$

Using the first FOC, one gets that the best–response price charged to old subscribers by platform i will be:

$$p_2^{ii}(p_2^{ji}) = \frac{t + \beta(a^j - a^i) - R(a^i) + p_2^{ji}}{2}.$$

Using the second FOC, one gets that the price charged to switchers by platform j will be:

$$p_2^{ji}(p_2^{ii}) = \frac{t(1-2|x_1 - l^j|) + \beta(a^i - a^j) + p_2^{ii} - R(a^j)}{2}.$$

Solving the system of best replies and considering that $|x_1 - l^j|$ is simply the first–period market share of platform j, the prices in the proposition are found.

A.2 Proof of Lemma ??.

Platform i maximizes

$$\pi_1^i + \delta\pi_2^i = (p_1^i + R(a^i))s_1^i + \delta\pi_2^i,$$

where $s_1^i = N|x_1 - l^i| = \frac{N}{2} + \frac{3\left(p_1^j - p_1^i\right) + 3\beta(1-\delta)\left(a^j - a^i\right) + \delta(R(a^i) - R(a^j))}{2(3+\delta)\tau}N$ and $\pi_2^i = \frac{5t}{9}\left(1 - 2s_1^i(1-s_1^i)\right) + \frac{2}{9}(2 - s_1^i)\left(\beta(a^j - a^i) + R(a^i) - R(a^j)\right) + \frac{[\beta(a^j - a^i + R(a_i) - R(a^j))]^2 N}{9t}$ is simply found by plugging second–period prices in Proposition **??** into the second–period profit function.

The first–order conditions are given by:

$$\frac{\partial\pi^i}{\partial p_1^i} = [p_1^i + R(a^i)]\frac{\partial s_1^i}{\partial p_1^i} + s_1^i + \delta\frac{\partial\pi_2^i}{\partial s_1^i}\frac{\partial s_1^i}{\partial p_1^i} = 0$$

$$p_1^i(p_1^j) = \frac{(9-7\delta)}{18-4\delta}p_1^j + \frac{\beta\left(9+3\delta^2-8\delta\right)(a^j - a^i) + \delta(3\delta(R(a^i) - R(a^j))) - (9+4\delta)R(a^i) + \delta R(a^j) + \left(\delta^2+6\delta+9\right)t}{18-4\delta}$$

Solving the system of best replies $p_1^i(p_1^j)$ and $p_1^i(p_1^j)$, the price stated in proposition in found.

A.3 Proof of Proposition ??

Plugging p_1^i found in Lemma **??** into the profit, the profit becomes:

$$\Pi^i(a^i) = N\left(\frac{t + (9+8\delta)}{18} + A[\beta(a^j - a^i) + R(a^i) - R(a^j)] + B[\beta(a^j - a^i) + R(a^i) - R(a^j)]^2\right)$$

where $A = \frac{27+\delta(9-10\delta)}{3(27-11\delta)}$ and $B = \frac{81-\delta(18+\delta(55-16\delta))}{2t(27-11\delta)}$

Differentiating $\Pi^i(a^i)$ with respect to a^i, one gets:

$$\frac{\partial \Pi^i}{\partial a^i} = [A + 2B(\beta(a^j - a^i) + R(a^i) - R(a^j))](R'(a^i) - \beta)$$

In a symmetric equilibrium, an interior solution will be such that $R'(a^*) = \beta$, with corner solution $a^* = 0$ whenever $R'(0) < \beta$. Plugging a^* into the the first–period prices, we get a common first–period price $p_1^* = \tau\left(1 + \frac{\delta}{3}\right) - R(a^*)$, so that each platform attracts half of the subscribers ($x_1 = 1/2$). Plugging $x_1 = 1/2$ into the second–period prices, both platforms offer a price $p_2^{ii*} = \frac{2\tau}{3} - R(a^*)$ to loyal users and a price $p_2^{ij*} = \frac{\tau}{3} - R(a^*)$ to switchers. Consequently, $s_2^{ii*} = N/3$ and $s_2^{ij*} = N/3$.

A.4 Proof of Lemma ??.

To see why, consider the case of BBPD and assume the fee is f_u. Clearly, the fee f_u does not clear the market. Indeed, plugging f_u of Eq. (**??**) into Eq. (**??**), it can be proved that:

$$\phi(a) = \underbrace{1 - \frac{G\left(3\frac{G^{-1}(1-a)}{2}\right) + 2G\left(3\frac{G^{-1}(1-a)}{4}\right)}{2}}_{\text{demand of advertisement at fee } f_u} - \underbrace{a}_{supply} < 0 \tag{14}$$

Let us prove it. Defining $x = 3\frac{G^{-1}(1-a)}{4}$ and differentiating $\phi(a)$ we get:

$$\frac{\partial \phi(a)}{\partial a} = \frac{3}{4}g(2x) + \frac{3}{4}g(x) - 1 < 0$$

Hence, since the LHS of Eq. (**??**) is decreasing in a, it is sufficient to demonstrate that the inequality is satisfied when $a = 0$. Eq. (**??**) becomes:

$$\phi(a=0) = 1 - \frac{G\left(3\frac{G^{-1}(1)}{2}\right) + 2G\left(3\frac{G^{-1}(1)}{4}\right)}{2} < 0$$

$$\Leftrightarrow$$

$$1 - \frac{G\left(3\frac{\alpha^{max}}{2}\right) + 2G\left(\frac{3\alpha^{max}}{4}\right)}{2} < 0$$

Since $G\left(3\frac{\alpha^{max}}{2}\right) = G\left(\frac{\alpha^{max}}{2}\right) = 1$, it is sufficient that $G\left(\frac{3\alpha^{max}}{4}\right) > \frac{1}{2}$. When this condition is satisfied, demand for advertisement is lower than the supply, so that, for any given advertising intensity, it holds that the market–clearing fee is higher in the uniform case ($f < f_u$).

In order to understand how this result affects equilibrium advertising intensities, one can verify from Eqs. (**??**) and (**??**) that the fee in both cases is a monotonically decreasing function of a which takes value 0 when $a = 1$. Moreover, if $G\left(\frac{3\alpha^{max}}{4}\right) > \frac{1}{2}$ then it must also hold that $\lim_{a_u \to 0} f_u > \lim_{a \to 0} f$, since the market–clearing fee is always lower under BBPD in the subscribers' side. As a consequence, being the fee always monotonically decreasing in a, one can also find that $\frac{\partial f}{\partial a} > \frac{\partial f_u}{\partial a_u}$, because the fee f_u must decrease faster than f to reach 0 when $a = 1$.

Since at equilibrium the per–user revenue equates the nuisance parameter in both cases:

$$R'(a) = \frac{2}{N}\left[\frac{\partial f}{\partial a}a + f\right] = \beta = \frac{2}{N}\left[\frac{\partial f_u}{\partial a_u}a_u + f_u\right] = R'(a_u) \tag{15}$$

Hence, $\frac{\partial f}{\partial a}a + f = \frac{\partial f_u}{\partial a_u}a_u + f_u$ together with $f_u > f$ and $\frac{\partial f}{\partial a} > \frac{\partial f_u}{\partial a_u}$ imply that $a^* < a_u^*$.

A.5 Proof of Proposition ??

Eq. (**??**) gives an advertising revenue equal to:

$$R(a_u) = \frac{(1-a_u)a_u}{2}$$

when the subscription prices are uniform. Therefore, the equilibrium advertising intensity will be $a_u^* = \max\left\{\frac{1-2\beta}{2}, 0\right\}$ and consequently $R(a_u^*) = \max\left\{\frac{1-4\beta^2}{8}, 0\right\}$. Differently, under BBPD and subscribers switching, Eq. (**??**) gives an advertising revenue equal to:

$$R(a) = \frac{(1-a)a}{3}$$

Hence, the equilibrium advertising intensity will be $a^* = \max\left\{\frac{1-3\beta}{2}, 0\right\}$, so that $R(a^*) = \max\left\{\frac{1-9\beta^2}{12}, 0\right\}$. Plugging $R(a_u^*)$ and $R(a^*)$ into the corresponding equilibrium prices, the uniform equilibrium price will be $p_u^* = \tau - \max\left\{\frac{1-4\beta^2}{8}, 0\right\}$, whereas, under BBPD, users receive a first–period price equal to $p_1^* = \frac{\tau(3+\delta)}{3} - \max\left\{\frac{1-9\beta^2}{12}, 0\right\}$. Then, in the second period, the price to old subscribers will be

$p_2^{ii*} = \frac{2\tau}{3} - \max\left\{\frac{1-9\beta^2}{12}, 0\right\}$ and switchers are offered $p_2^{ij*} = \frac{\tau}{3} - \max\left\{\frac{1-9\beta^2}{12}, 0\right\}$. Simple comparison of prices gives the result in the proposition.