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Published:
DOI: http://doi.org/10.1016/j.ijheatmasstransfer.2018.09.088

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International Journal of Heat and Mass Transfer,Volume 129,2019, p. 427-431

The final published version is available online at:
https://doi.org/10.1016/j.ijheatmasstransfer.2018.09.088

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ABSTRACT

This paper presents data reduction of average friction factor of gas flow through adiabatic micro-channels. In the case of micro-channel gas flow at high speed, the large expansion occurs near the outlet and the pressure gradient along the length is not constant with a significant increase near the outlet. This results in flow acceleration and a decease in gas temperature. Therefore the friction factor of micro-channel gas flow should be obtained with measuring both the pressure and temperature. The data reductions on friction factors were carried out under the assumption of isothermal flow for numerous experimental and numerical studies since temperature measurement *of micro-channel gas flow at high speed is quite difficult due to the measurement limitations. In the previous study, it was found that the gas temperature can be determined by the pressure under the assumption of one dimensional flow in an adiabatic channel (Fanno flow). Therefore in the present study data reduction to estimate friction factors between two relatively distant points considering the effect of a decrease in temperature is introduced with the temperature determined by the measured pressure at a specific location. The Friction factors obtained by using the present data reduction are examined with the available literature and the results are compared with empirical correlations on Moody chart.*

Key words : average friction factor, turbulent, gas flow, micro-tube, flow choking

INTRODUCTION

The pressure loss (drop or difference) determined by a friction factor between two points is one of the significant factors to design micro piping lines for MEMS (Micro Electro Mechanical System). Since the experimental work by Wu and Little [1] who obtained the average friction factors using pressure difference between the inlet and the outlet of gases in micro-channels, many experimental and numerical studies on friction factor in a micro-channel have been undertaken. The friction factor for gases with large variations in the physical properties flowing through channels was obtained under the assumption of isothermal flow by most of the researchers, e.g. Turner et al. [2], Asako et al. [3], Tang et al. [4], Lorenzini et al. [5], Yang et al. [6] and Hong et al. [7] due to the measurement limitation of gas temperature flowing through a channel. The literature is surveyed in the article by Kawashima and Asako [8].

However, in the case of micro-channel gas flow at high speed, a large expansion occurs near the outlet and the pressure gradient along the length is not constant with a significant increase near the outlet. This results in flow acceleration and a decease in gas temperature. Therefore the friction factor of micro-channel gas flow should be obtained with measuring both the pressure and temperature. In actual situation, micro-channel gas flow does not stay isothermal and shows a strong decrease in temperature near the outlet for adiabatic walls. Recently, Kawashima and Asako [8] found that the gas temperature can be determined by the pressure under the assumption of one dimensional flow in an adiabatic channel (Fanno flow) to obtain the friction factor considering the effect of a decrease in gas temperature.

This is the motivation of the present study. In this paper an equation is proposed to calculate the average friction factors between relatively distant points considering the effect of a decrease in gas temperature.

DERIVATION OF AVERAGE FANNING FRICTION FACTOR FOR ADIABATIC WALL

The four multiples of the Fanning friction factor for an adiabatic wall (Fanno flow) is defined by [8]

$$f_{\mathrm{f}} = \frac{4\tau_{\mathrm{w}}}{\frac{1}{2}\rho u^2} = \frac{2D}{p}\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right) - \frac{2Dp}{\rho^2 u^2 RT}\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right) - \frac{2D}{T}\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right) \tag{1}$$

The pressures at the ports that are located adjacently are measured to obtain a semi-local friction factor. In such a case the change in temperature between two ports is small. Therefore, the temperature which appears in the second term in the right hand side of Eq. (1) can be treated as constant. Integrating Eq. (1) with the average of the temperatures at two pressure ports, $T = \frac{T_1 + T_2}{2}$ where T_1 and T_2 are the temperatures at the pressure ports 1 and 2, the following equation is obtained.

$$f_{\mathrm{f}}^* = \frac{1}{x_2 - x_1}\int_{x_1}^{x_2} f_{\mathrm{f}}\mathrm{d}x = \frac{D}{x_2 - x_1}\left(-2\ln\frac{p_1}{p_2} + \frac{2\left(p_1^2 - p_2^2\right)}{R\left(T_1 + T_2\right)\dot{G}^2} + 2\ln\frac{T_1}{T_2}\right) \tag{2}$$

The temperature in Eq. (2) can be determined solving the following quadratic equation obtained by total temperature balance between given two points (inlet and x) [8].

$$\alpha\frac{\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R^2}{2c_{\mathrm{p}} p^2}T^2 + T - \left(T_{\mathrm{in}} + \frac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}}\right) = 0 \tag{3}$$

where p is the pressure at the pressure port. The temperature is determined by

$$T = \frac{-1 + \sqrt{1 + 4 \times \alpha \dfrac{\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R^2}{2c_{\mathrm{p}} p^2} \times \left(T_{\mathrm{in}} + \dfrac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}} \right)}}{2 \times \alpha \dfrac{\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R^2}{2c_{\mathrm{p}} p^2}} \tag{4}$$

where the inlet values of velocity, density and temperature are obtained with isentropic process between the inlet and the stagnation under the assumption of ideal gas.

Kawashima and Asako [9] measured three local pressures near the outlet of a PEEK (polyether ether ketone) micro-tube with D = 514.4 μm and L = 50 mm and obtained semi-local friction factors between two pressure ports that are located adjacently using Eqs. (2) and (4). Hong et al. [10] also measured three local pressures near the outlet of a glass micro-tube with D = 397 μm and L = 120 mm to obtain semi-local friction factors between two adjacently located pressure ports. The effect of a decrease in gas temperature between two pressure ports is relatively small since the distances between two pressure ports are less than 0.1L.

However in the case that two locations are not close such as the inlet and the outlet, T in the second term of Eq. (1) can not be considered as a constant. It should be integrated between x_1 and x_2. Therefore, in the present study, the focus is to obtain the friction factor between two relatively distant points. The above T (Eq. (4)) is a function of p. The average Fanning friction factor integrating Eq. (1) between x_1 and x_2 is

$$\begin{aligned} f_{\mathrm{f,ave}} &= \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} f_{\mathrm{f}} \, \mathrm{d}x = \frac{D}{x_2 - x_1} \left\{ \int_{x_1}^{x_2} \left(\frac{2}{p} \mathrm{d}p \right) - \int_{x_1}^{x_2} \left(\frac{2p}{\rho^2 u^2 RT} \mathrm{d}p \right) - \int_{x_1}^{x_2} \left(\frac{2}{T} \mathrm{d}T \right) \right\} \\ &= \frac{D}{x_2 - x_1} \left[\begin{array}{l} -2 \ln \dfrac{p_1}{p_2} + 2 \ln \dfrac{T_1}{T_2} - \dfrac{1}{\left(\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R \times \left(T_{\mathrm{in}} + \dfrac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}} \right) \right)} \\ \times \left\{ \dfrac{p_2^2 - p_1^2}{2} + \dfrac{B^2}{2} \ln \dfrac{p_2 + \sqrt{p_2^2 + B^2}}{p_1 + \sqrt{p_1^2 + B^2}} + \dfrac{1}{2} \left(p_2 \sqrt{p_2^2 + B^2} - p_1 \sqrt{p_1^2 + B^2} \right) \right\} \end{array} \right] \end{aligned} \tag{5}$$

where

$$B^2 = 4 \times \alpha \frac{\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R^2}{2c_{\mathrm{p}}} \times \left(T_{\mathrm{in}} + \frac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}} \right) \tag{6}$$

Therefore, the average Fanning friction factor between x_1 and x_2 can be obtained from Eq. (5). Eq. (5) will also help piping design where pressure loss determination between relatively distant points is done using average friction factors.

EXPERIMENTAL AND NUMERICAL ESTIMATIONS OF AVERAGE FANNING FRICTION FACTOR

Table 1 Experimental results on average friction factors for turbulent gas flow in micro-tubes

	D (μm)	Gas	Re	Micro-tube Outer surface	Average friction factor
Choi et al. (1991) [11]	3.0 ~ 81.2	N_2	20 ~ 18000	Silica tube Not clear	Isothermal flow lower than Blasisu eq.
Yu et al. (1995) [12]	52 ~ 191	N_2	250 ~ 20000	Silica tube Nearly adiabatic	Isothermal flow lower than Blasisu eq.
Tang et al. (2007) [4]	55 ~ 201	N_2	28 ~ 6200	Fused silica tube Not covered	Isothermal flow lower than Blasisu eq.
Yang et al. (2012) [6]	86, 308, 920	Air	150 ~18800	Stainless steel tube Inserted in a vacuum chamber	Isothermal flow higher than *Blasius* eq.

There are a few literature which investigate friction factors of turbulent gas flow through micro-tubes as tabulated in Table 1. The most average friction factors between the micro-tube inlet and outlet are obtained under the assumption of isothermal flow and the obtained average friction factors are lower than the *Blasius* equation. Only the exception is friction factors obtained by Yang et al. [6].

At least the inlet temperature, the inlet pressure, the mass flow rate and the outlet pressure are required to obtain the average friction factor from Eq. (2) or Eq. (5). These raw data are not shown in the literature. We asked Professor Tang [4] and Professor Yang [6] to show us the raw data but they failed to find the data since they were collected long time ago. Then, the experiments and numerical calculations for nitrogen gas flowing through a stainless steel micro-tube (D = 867 μm, L = 200mm and Ra = 0.48 μm) whose inner surface is relatively smooth were carried out for data reduction. The average Fanning friction factors between the inlet and the outlet, $f_{f,\ ave}$ were obtained under the assumption of Fanno flow from Eq. (5) with both the experimentally measured and numerically calculated data. For both of them, the micro-tube outlet pressure is assumed to be at atmospheric pressure for the micro-tube gas flow discharged into the atmosphere. Detailed description of the experimental setup and the numerical methodology are documented in the previous studies [13 and 14] and will not be given.

The experimentally and numerically obtained $f_{\mathrm{f,\ ave}}$ is plotted in Fig. 1 as a function of Reynolds number. The f_{f}^{*} obtained from Eq. (2) is also plotted in the figure. The viscosity, which is evaluated at the stagnation temperature by using REFPROP [15] is used to calculate the Reynolds number. The solid line and dotted line in the figures represent the values obtained by the theoretical formula (f = 64/Re) and $f = 0.3164/Re^{0.25}$ (*Blasius* equation) for incompressible flow theory, respectively. Since the effect of inner surface roughness on micro-tube flows is relatively large compared with conventional tube flows, the inner surface roughness of micro-tubes used for the experiment were measured. In order to measure the roughness of the inner surface of the tube, a part of the micro-tube is cut. The arithmetic mean heights of the surface of the micro-tubes were measured with a 3D laser scanning confocal microscope for profilometry (Keyence, VK-X260). The arithmetic mean height of the stainless steel micro-tube used in this study was 0.448 μm. The value of the inner relative surface roughness of the micro-tubes was 5.17×10^{-4}. Therefore the inner surface of the micro-tube used in this study is considered to be smooth. For reference, the following *Colebrook-White* equation (Eq. 7) [16] calculating the friction factor of turbulent flow in a rough pipe is plotted in the figure with the red dotted line.

$$\frac{1}{\sqrt{f_{\mathrm{f}}}} = -2\log\left(\frac{2.51}{Re\sqrt{f_{\mathrm{f}}}} + \frac{k_{\mathrm{s}}}{3.72D}\right) \tag{7}$$

where k_{s} is equivalent sand grain surface roughness. In order to employ the above *Colebrook-White* equation obtained with k_{s}, of the stainless steel micro-tube, the k_{s} is assumed to be $1\times Ra$. The values obtained by the *Colebrook-White* equation nearly coincide with *Blasius* equation within Re = 20000. And in the range of Re > 20000, it deviates from *Blasius* equation.

Mach numbers obtained using the measured local pressures at three locations near the micro-tube outlet are plotted in Fig. 2 as a function of Reynolds number. The values of Mach number increase with increasing Reynolds number and level off in the range Re > 23000 since the flow is choked.

Due to the steep decrease in gas static temperature near the outlet from gas expansion, T in the second term of Eq. (1) can not be treated as a constant. Therefore, the values of f_f^* obtained from Eq. (2) deviate from those of $f_{f,\,ave}$ obtained from Eq. (5) at Re > 10000 for Fig. 2 as the gas velocity and bulk temperature differences between the inlet and the outlet are large.

In the range of Re > 23000, they deviate greatly from those of $f_{f,\,ave}$ obtained from Eq. (5) since the assumption of $p_{out} = p_{atm}$ is not valid with flow choking. When the flow is choked, the gas velocity and bulk temperature at a specific cross section inside a tube remain unchanged, and the outlet pressure is higher than the back pressure (atmospheric pressure) with an increase in Reynolds number. However, the outlet bulk temperature obtained under the assumption of $p_{out} = p_{atm}$ does not remain unchanged and rather decreases. Therefore the arithmetic average bulk temperature decreases and f_f^* increases at Re > 23000.

Both experimental and numerical values of $f_{f,\,ave}$ obtained from Eq. (5) are in excellent agreement with *Blasius* equation even though the outlet flow is under-expanded with flow choking and the outlet pressure is higher than the atmospheric pressure in the range of Re > 23000. The measure of under-expansion at the outlet is relatively small in this tube. And the relatively larger pressure difference between the inlet and the outlet is not significantly affected by under-expansion at the outlet.

In the case of the high speed gas flow in an adiabatic tube, the gas temperature decreases in the tube. This results in viscosity change. In order to account for the variation of the viscosity, the viscosity which is evaluated at the average of the inlet and the outlet temperatures, is used to calculate the Reynolds number. The average friction factor $f_{f,\,ave}$ obtained from Eq. (5) vs the Reynolds number based on the average temperature is also plotted Fig. 1. Then the obtained uncertainty of the friction factor is represented in the figure. The uncertainty of the pressure measured by the pressure transducer (Krone KDM30, 0~1 MPa) is ±2.5 kPa (±0.25 % F.S.). In the lower Re range, the uncertainty of the friction factor is slightly higher compared to the uncertainty of the friction factor in the higher Re range. The corresponding uncertainty of the friction factor obtained in the range of $2590 \leq Re \leq 43204$ is less than ±0.0095 and more than ±0.0002. In the same Reynolds number, the uncertainty of the friction factor obtained by Eq. (2) and Eq. (5) is also the same. The average friction factor $f_{f,\,ave}$ vs the Reynolds number based on the average temperature is slightly higher than the average friction factor $f_{f,\,ave}$ vs the Reynolds number based on the stagnation temperature, since the Reynolds number based on the average temperature is larger than the Reynolds number based on the stagnation temperature. As a result of that, the average friction factor $f_{f,\ ave}$ vs the Reynolds number based on the average temperature is in very close agreement with *Blasius* equation.

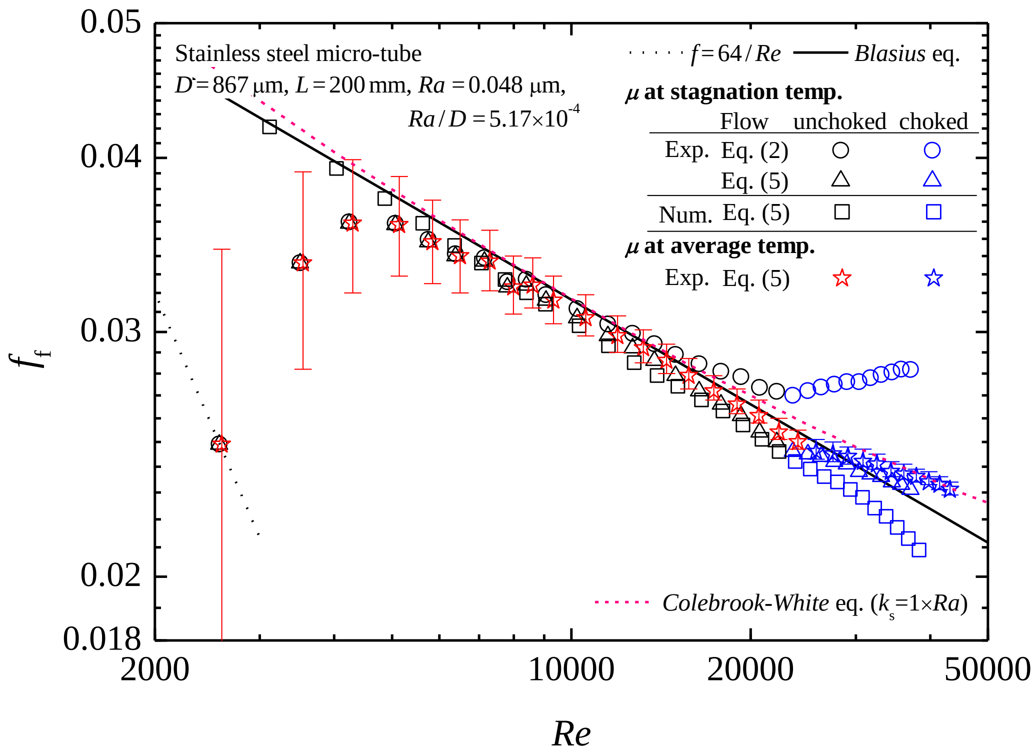

Fig.1 Fanning friction factor as a function of *Re*

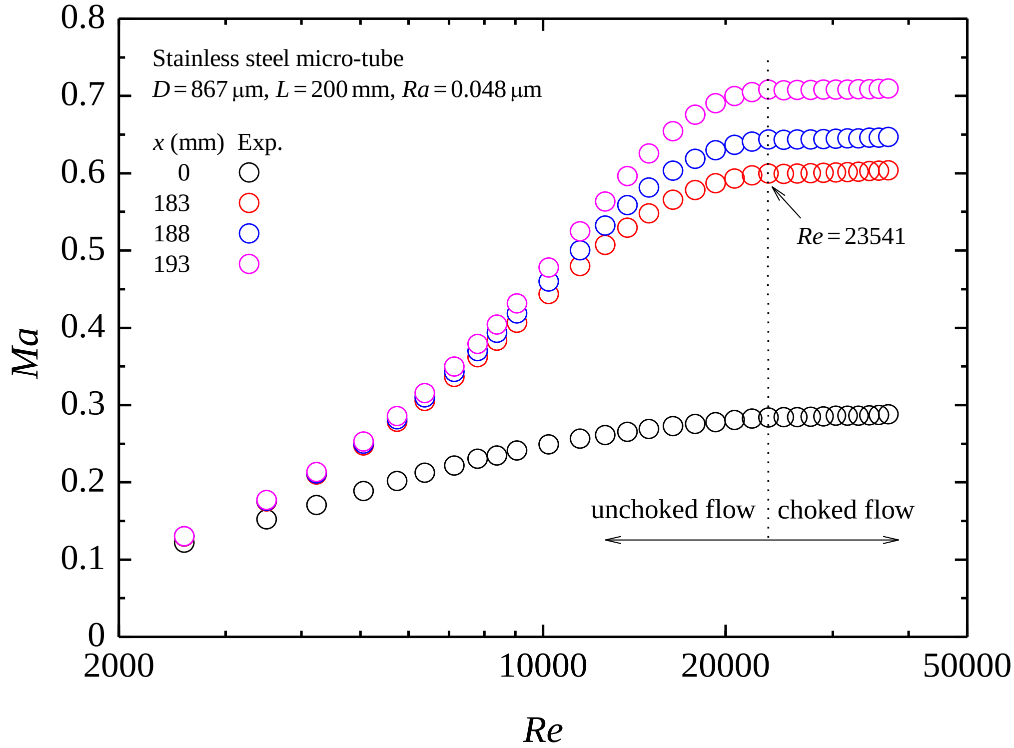

Fig. 2 Mach number as a function of *Re*

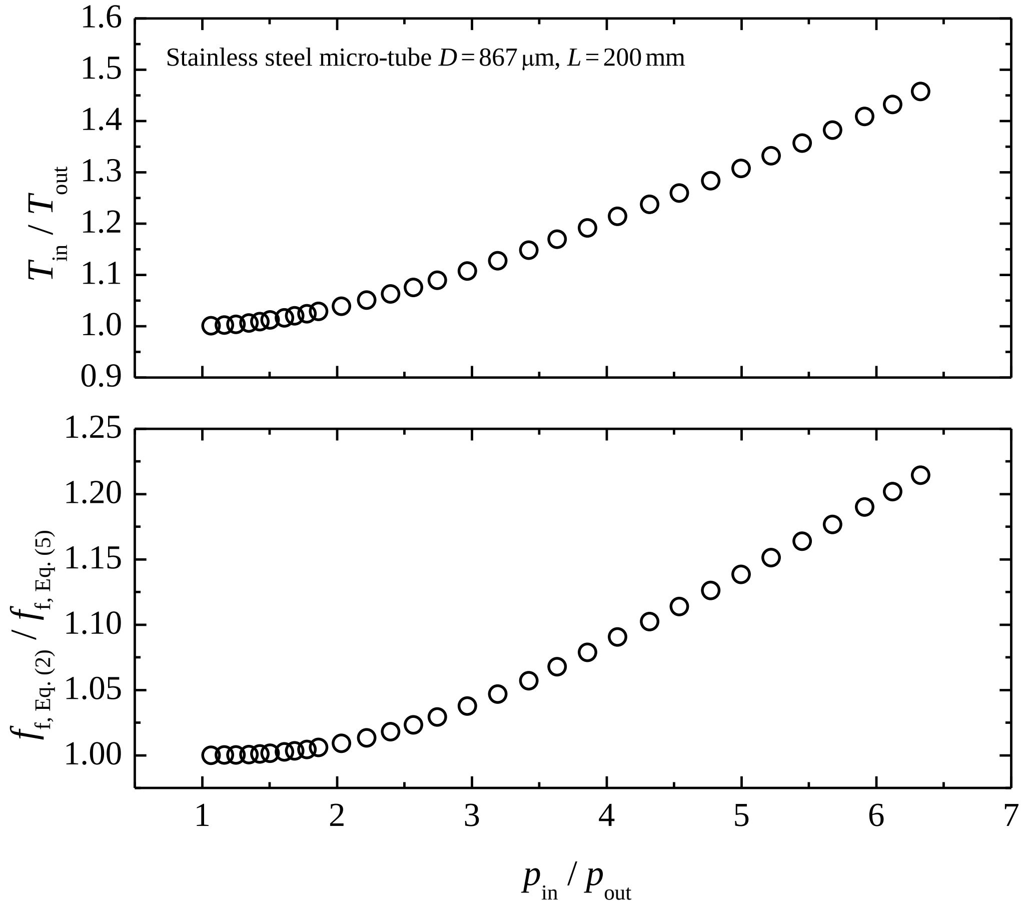

Fig. 3 T_{in} / T_{out} and $f_{f,\ Eq.\ (2)} / f_{f.\ Eq.\ (5)}$ as a function of the pressure ratio

In the present experiment, the pressure and temperature in the chamber at the upstream section of the micro-tube were measured. The measured pressure and temperature can be considered as the stagnation values, since the gas velocity in the chamber is very low. The temperature and pressure at the inlet of the micro-tube are obtained with an isentropic process between the inlet and the stagnation under the assumption of ideal gas. Also the pressure loss at the tube entrance is considered. The square edged type is assumed for the entrance configuration. The outlet pressure is assumed to be at atmospheric pressure since the gas at the outlet is discharged into the atmosphere. The outlet temperature of the thermally insulated micro-tube can be obtained by solving Eq. (4) since the outlet pressure is assumed to be at atmospheric pressure. As a result of that, the correlation between the temperature ratios at two points (the inlet and the outlet) and their corresponding pressure ratios were obtained. The temperature ratio,

T_1/T_2 is plotted in Fig. 3 as a function of its pressure ratio. The ratio of the friction factors obtained by Eq. (2) and Eq. (5), $f_{f,\text{ Eq. (2)}} / f_{f.\text{ Eq. (5)}}$ is also plotted in the figure. T_1/T_2 and $f_{f,\text{ Eq. (2)}} / f_{f.\text{ Eq. (5)}}$ increase with an increase in the pressure ratio. T_1/T_2 is 10 % higher than isothermal value at p_{in}/p_{out}=3 and $f_{f,\text{ Eq. (2)}}$ is 5 % higher than $f_{f.\text{ Eq. (5)}}$ at p_{in}/p_{out}=3. Therefore, Eq. (5) should be used instead of Eq. (2) when the pressure ratio p_{in}/p_{out} is greater than 3 if the 5% difference is allowed.

CONCLUSION

The average Fanning friction factor between two relatively distant points is derived with the temperature determined by the pressure measured at a location to estimate friction factor between the inlet and outlet for gas flow through adiabatic micro-channels. The following conclusions were reached.

(1) The experimental and numerical Fanning friction factors obtained from Eq (5) in the turbulent flow regime are in excellent agreement

(2) Both experimental and numerical average friction factor $f_{f,\text{ ave}}$ obtained from Eq. (5) nearly coincide with *Blasius* equation even though the outlet flow is under-expanded with flow choking.

(3) The average friction factor $f_{f,\text{ ave}}$ vs the Reynolds number based on the average temperature is in very close agreement with *Blasius* equation.

NOMENCLATURE

B^2	calculated value from Eq. (6)	
c_p	specific heat at constant pressure	J/(kg K)
D	micro-tube diameter	m
f_f	Fanning friction factor	-
f_f^*	semi-local Fanning friction factor	-
$f_{f,\text{ ave}}$	average Fanning friction factor	-
$\dot{G}$	mass flow rate per unit area	kg/(s m^2)
k_s	equivalent sand grain surface roughness	m
L	micro-tube length	m
Ma	Mach number	-
n	pressure port number	
p	static pressure	Pa

x	coordinates	m
R	gas constant	J/(kg·K)
Ra	arithmetic mean roughness	m
Re	Reynolds number	-
T	static temperature	K
u	velocity component	m/s
α	kinetic energy correction factor	-
γ	specific heat ratio	-
μ	viscosity	Pa·s
ρ	density	kg/m^3
τ_w	shear stress on wall	Pa

subscript

ave	cross sectional average value
in	inlet

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