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Innovation, specialization and growth in a model of structural change

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Abstract:
In this paper we investigate the process of creation and destruction of industries as it stems from productivity increasing innovations and from the induced changes of consumption patterns. In our model industries whose demand increases experience an expansion of the number of intermediate goods and hence of their research effort, while those whose demand declines undergo a cost-cutting restructuring with a corresponding reduction of the number of intermediates. We show that if aggregate consumption is concentrated on high (low) priority goods in the early (later) stages of the economy's development and spread out more evenly in an intermediate stage, then the diversification of the economy over the development path is inversely U-shaped: a result that is consistent with the empirical evidence in Imbs and Wacziarg 2003 "Stages of Diversification." *American Economic Review* 93: 63–86.

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1 Introduction

The process of economic development is unequivocally one of structural change. The pattern of demand, and accordingly supply, is subject to shifts related to increasing income per head. Studies undertaken by a number of economists, see for instance the early work by Chenery (1960), later by Chenery, Robinson, and Syrquin (1986) more recently followed by Matsuyama (2002 and 2008), have indeed upheld the view that the root of such shifts lies with income-per-head increasing productivity growth. Structural change, however, is clearly not confined to demand shifts. It is a well recorded fact that the progress of technology applied to industry has brought about an extraordinary expansion of implements assisting labor of ever more sophisticated design and specialized function (David 1976; Rosenberg 1983; Basalla 1988). This fact suggests that in some industries productivity increases are also due to an increase in the number of production inputs and specialized tasks, resulting in an increase in the degree of specialization within the industry. Yet, whilst some industries witness an increase in specialized inputs, it is likewise well documented that others experienced a reduction in their number, undergoing a rationalization process aimed at reducing production costs.[1]

The aim of this paper is to investigate the dynamic interplay between structural change, the degree of specialization of each industry and the diversification of the economy, the former relating to the degree to which the production process of a given final good is assisted by specific intermediate inputs produced by specialized sectors and the latter characterizing the range of final goods. More specifically, the purpose of this paper is to characterize the dynamic feedbacks between the degree of specialization of inputs within industries and economic growth along the development path. Imbs and Wacziarg (2003), using shares in total employment and in value added at various levels of disaggregation, show that countries diversify over most of their development path but that in later stages of their development, sectoral diversification decreases, and hence that the overall pattern of diversification is inverse-U shaped. In particular, these authors show that after an initial stage in which specialization is concentrated in few industries, economies start to diversify. As a consequence, some industries undergo a process of rationalization decreasing specialization while others experience an increase. They further show that at a later stage, specialization starts to concentrate again on fewer industries. We propose a model where innovation, growth and structural change stem from R&D efforts, and that is consistent with this empirical evidence.

We consider an economy with many industries, each supplying a final good. In each industry there are sectors producing differentiated industry-specific intermediate goods. Their number is taken to be a proxy for

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the degree of specialization of the industry: the more intermediate goods are used, the greater the degree of specialization. In our model the economy is more concentrated (less diversified) if there are only few highly specialized industries, while it is less concentrated (more diversified) the more evenly is the distribution of the degree of specialization of industries. On the demand side we assume that consumers have non homothetic preferences and consume goods according to their income, prices, and preferences following a hierarchy of needs. In particular, we consider consumers facing a variety decision, that is, what type of goods to consume, and a quantity decision, that is, how much of each good to consume. As prices of high priority goods fall due to productivity enhancing innovations, consumers reduce their consumption in favor of lower priority ones.

In each industry three types of innovation events may occur: a vertical innovation, a specialization event and one due to rationalization. While the former type is standard in endogenous growth models, the latter two are novel and, in our model, drive the economy's degree of diversification. In each intermediate good producing sector of a given industry a technology follower invests in R&D to dethrone the technology leader (Aghion and Howitt 1992). If the follower is successful then intermediate good producers' and industry productivities increase, leading, because of competition, to a decline in the price of the final good. A decline in the price of goods, by increasing the real income of consumers, may lead them to spend less on high priority goods in favor of less priority ones. This change in aggregate expenditure results in a change in firm profits. Consequently, profits in industries producing high priority goods decline while profits in those producing low priority ones increase. In the former industries, the decrease in demand and profits leads to a rationalization effort aimed at cutting costs by reducing the number of intermediate goods used. In the latter, the increase in demand leads to an expansion of intermediate goods, that is, to a deepening of specialization. These events, by raising productivity and per capita income, set in motion another demand share change feeding back onto the vertical innovation race.[2] The industry structure is, therefore, endogenously determined (Romer 1987; Ciccone 2002).

The next step is to study the implications for the economy's growth rate. It will be shown that if this distribution is more even, then the expected growth rate is lower. This is due to the fact that the contribution to growth is higher where arrivals of innovations are higher as a consequence of each intermediate good producer's R&D effort. Economies in which there is a greater number of intermediate good producers grow faster. Moreover, the more specialization is concentrated in fewer industries, the higher is the average growth rate. On the contrary, the more evenly distributed is the degree of specialization, the lower is the economy's growth rate. This phenomenon is consistent with the theory of conditional convergence which claims that lagging behind economies feature higher per capita growth rates. For empirical evidence and discussion, see Jones (2016).

We further show that if aggregate consumption is concentrated on high (low) priority goods in the early (later) stages of the economy's development and spread out more evenly in an intermediate stage, then the diversification of the economy over the development path is inversely U-shaped: a result that is consistent with the empirical evidence in Imbs and Wacziarg (2003). In an economy poised at the initial stage of a development path, where the aggregate consumption pattern is limited to few essential goods, our model predicts a high specialization in a few industries and implying a low degree of diversification of the economy. As prices decline due to vertical innovations, consumption of low priority goods increases and the degree of specialization of existing industries becomes more evenly distributed. The degree of diversification of the economy accordingly increases. Further price decreases lead to a further increase in the consumption of low priority goods enhancing the degree of specialization in these industries at the expense of high priority ones; consequently, the economy becomes less diversified. Moreover, our model predicts that the economy's growth rate is also U-shaped over the development path, being higher at early and lower in intermediate stages, which is in keeping with the empirical evidence on conditional convergence (see Jones 2016), and increases again in later stages.

The paper is structured as follows. Section 2 discusses the relevant literature. Section 3 describes the demand side of the economy, Section 4 sets out the production structure while Section 5 describes the innovative process that takes place within the sphere of intermediate goods producers as well as the specialization and rationalization that there occur. In Section 6 we characterize the economy's equilibrium, its growth rate and its pattern of diversification over the development path. Section 7 contains some discussion and Section 8 draws the paper to a close. Proofs are placed in the Appendix.

2 Literature review

There is a growing literature concerning demand- and supply-induced structural change and economic growth (see Acemoglu 2009; Matsuyama 2008, Chapter 20; Herrendorf, Rogerson, and Valentinyi 2014). On the demand side, Pasinetti (1981) puts forth a theory of consumers' demand to account for its evolution centered on an order of consumption expenditure determined by hierarchical needs and investigates its consequences for structural change. Kongsamut, Rebelo, and Xie (2001) study demand driven structural change in a model with exogenous technological (labor augmenting) progress where consumers have different income elasticities of demand for the different goods. Matsuyama (2002) studies the mechanics of the development process by investigating the relationship between income distribution, expanding markets and productivity increases due to learning by doing. Individuals have non-homothetic preferences with a well-defined priority over the space of consumer goods and differ only in their heterogeneous incomes. The author studies the interplay between a trickle down effect in which lower prices due to high-income individuals' consumption make goods more affordable for those earning low incomes and a trickle up effect in which lower prices due to low-income individuals' consumption allow those with high-incomes to spend more on lower priority goods. The implication of the resulting joint impact for the development process is then analyzed. In a similar vein, Foellmi and Zweimüller (2006 and 2008) study the growth and development features of an economy in a model with demand-induced structural change in which R&D investment is aimed at introducing new products. Foellmi, Wuergler, and Zweimüller (2014) study a model where consumers have non-homothetic preferences and where process and product innovation incentives depend on income distribution.[3] Supply-induced structural change has been pioneered by Baumol (1967) and formally investigated in Ngai and Pissarides (2007). This latter paper aims at explaining the implications of different sectoral total factor productivity growth rates for structural change and at reconciling these features with an aggregate balanced growth path. Similarly, Acemoglu and Guerrieri (2008) consider a multisector economy with capital deepening and factor proportion differences across sectors featuring a non-balanced growth path.

Recent literature studies the interaction between supply and demand induced structural change. Buera and Kaboski (2009) integrate the two strands of this literature by considering a model with sector-biased productivity growth and non-homothetic preferences. Another paper by Guilló, Papageorgiou, and Perez-Sebastian (2011) likewise considers a two-sector overlapping generation model that features endogenous technological progress and encompasses both theories of structural change. These authors investigate biased TFP growth as an endogenous response to non-homothetic preferences and in a calibrated version of their model aim to identify the characteristics of technical change that reproduce the observed evolution of sectoral TFP growth. Our paper is most closely aligned with this more recent literature since it studies the interaction between supply and demand inducing structural change given that specialization is a form of directed technological change (see Gancia and Zilibotti 2005 for a survey).[4] In our model, changes in the demand pattern induce changes in firm profits, which lead to changes in industry specialization and, as a consequence, to differential productivity growth across industries.[5] The focus of our paper lies, in fact, in the dynamic interplay between structural change and the diversification pattern of an economy along its development path. Nevertheless, we differ from the current literature in modeling choices. On the demand side, the mentioned papers either consider a single representative consumer with non-homothetic preferences (Laitner 2000; Kongsamut, Rebelo, and Xie 2001; Foellmi and Zweimüller 2008; Buera and Kaboski 2009; Guilló, Papageorgiou, and Perez-Sebastian 2011), or derive aggregate demand functions from heterogeneous consumers with a hierarchy of needs and exogenous saturation levels (see Matsuyama 2002 and Foellmi and Zweimüller 2006). The latter approach, where individuals consume as much as possible of as many goods, is closest to our approach but we consider consumers solving a variety and quantity problem: they choose, therefore, how many goods and how much of each good to consume. Furthermore, our paper differs from the extant literature in respect to the innovation process. In the current literature the main innovation process concerns product innovation (Foellmi and Zweimüller 2006; 2008), while productivity increases are either due to learning by doing (Matsuyama 2002) or are a side effect of product innovation (Foellmi and Zweimüller 2006).[6] In Guilló, Papageorgiou, and Perez-Sebastian (2011) technological progress is due to an expanding variety of inputs while the paper we present focuses instead on the effects of productivity changes that are due to vertical innovations, specialization and rationalization. In our paper, the overall number of intermediate goods remains constant over time, while the number of intermediate goods in each industry depends on the evolution of aggregate demand. The productivity of each single intermediate good increases over time because of an R&D race between an incumbent and an entrant. Quite importantly and contrary to Guilló, Papageorgiou, and Perez-Sebastian (2011), we do not assume increasing returns to specialization (that is, efficiency increasing as the number of inputs increases) as would result from a production function such as in Romer (1990) and Jones (1995) but obtain increasing returns at the aggregate level as a result of specialization within the various industries.

3 Consumption pattern

Our aim is to characterize the aggregate consumption pattern in terms of the proportion of aggregate income spent on goods produced in an economy and how this proportion changes as the economy evolves over time.

For this purpose we first analyze the consumer's problem of how many and how much of each good to consume and then characterize the aggregate consumption pattern.

Consider an economy with $1, \ldots, j, \ldots, J$ differentiated goods with prices $p_{1,t}, \ldots, p_{j,t}, \ldots, p_{J,t}$ at time t. For simplicity's sake we abstract from the individuals' saving decision and consider an economy populated by non-overlapping generations of $\mathscr{L}$ identical individuals who are non-altruistic and live for a small time period dt. What we are trying to model is the way higher priority goods are substituted with lower priority ones as the price of high priority goods declines. Accounting explicitly for the individuals' saving decision in the framework of an intertemporal optimization problem would greatly complicate our analysis. An endogenous saving decision might affect the pace at which this substitution occurs but it would neither change its qualitative result nor provide qualitative insights into the dynamic interplay between structural change, the degree of specialization of each industry and the diversification of the economy, which is the focus of our paper. Individuals inelastically supply homogenous labor, own the firms and consume. Time is continuous.[7]

For given prices and income R, consumers have to decide how many goods, and how much of each, to consume: a variety and quantity choice problem. The instantaneous utility function of the representative individual born at time t consuming $y_{1,t}, \ldots, y_{j,t}$ quantities of the first j goods is[8]

$$u(j,t) = Log\left(C_j\right) + \alpha_j \sum_{h=1}^{j} d_h Log\left(y_{h,t}\right) \tag{1}$$

where d_h, for $h = 1, \ldots, J$, weighs goods in utility terms, C_j accounts for the individual's eagerness to consume good j and α_j affects the concavity of the utility function as the variety consumed changes and thus affects the preference ordering of goods consumed. Note that C_j and α_j change as the variety consumed changes (as j changes), while d_h weighs the single good consumed in a given basket. Hence, as shown and discussed below, the former parameters affect the variety choice problem (that is, how many and which goods to consume) while the latter affects the quantity choice problem (that is, how much of each good to consume). (1) is a Cobb-Douglas utility function where the consumer's utility depends on the quantity as well as on the variety of goods consumed. Formally, for given prices $p_{1,t}, \ldots, p_{J,t}$, and income R_t, each individual solves a static optimization problem

$$\begin{aligned} U(j,t) &= \max_{y_{1,t}, \ldots, y_{j,t}} u(j,t) \\ &\text{s.t.} \sum_{h=1}^{j} p_{h,t} y_{h,t} \leq R_t \end{aligned} \tag{2}$$

and

$$max_{j \in \{1,\ldots,J\}} U(j,t) \tag{3}$$

where (2) is the individual's quantity and (3) her variety choice problem.

We make the following Assumption.

Assumption 1

1. *The difference* $\alpha_{j+1} - \alpha_j$ *is positive and non-decreasing in* j *;*
2. *constants* C_j *for* $j = 1, \ldots, J$ *satisfy the condition that* $\chi(j) \equiv \frac{C_{j+1}}{C_j} \frac{\left(\frac{R}{j+1}\right)^{(j+1)\alpha_{j+1}}}{\left(\frac{R}{j}\right)^{j\alpha_j}}$ *is non-increasing in j;*
3. d_h *is non-decreasing while* $\frac{d_h}{d_1 + d_2 + \ldots + d_h}$ *is non-increasing in h, for* $h = 1, \ldots, J$.

Assumption 1.1 requires that the degree of concavity of the individual's utility function does not increase as variety j is increased. Assumption 1.2 requires that C_j is such that the rate of change of the indirect utility obtainable by equally distributing income on the number of goods in a given basket, hence independently of prices, does not increase in j.[9] This property generates a hierarchy such that high priority goods weigh more in determining indirect utility than those of lesser priority. d_h affects, for a given variety choice, the quantity of each single good h consumed by an individual. Assumption 1.3 implies that this weight is non-decreasing in h but does not increasingly do so. From (1) it is clear that C_j has no impact on the quantity consumed for a given variety choice, but it compares the utility deriving from different bundles of goods and thus affects the consumer's variety choice.

For the sake of concreteness, we explicitly calculate the level of income that allows an offspring of an individual who consumed only one good to increase the number of consumed goods to two, as a function of their respective prices. More formally, omitting for simplicity's sake the time index t, the consumer chooses to consume only one good if [see (2) and (3)]

$$\begin{array}{ll} max_{x_1} C_1 x_1^{d_1\alpha_1} > & max_{x_1,x_2} C_2\left(x_1^{d_1} x_2^{d_2}\right)^{\alpha_2} \\ \text{s.t. } p_1 x_1 = R & \text{s.t. } p_1 x_1 + p_2 x_2 = R \end{array}$$

The inequality holds if

$$R < \left(\frac{C_1}{C_2}\left(\frac{d_1^{d_1} d_2^{d_2}}{(d_1 + d_2)^{(d_1+d_2)}}\right)^{-\alpha_2} p_1^{d_1(\alpha_2-\alpha_1)} p_2^{d_2\alpha_2}\right)^{\frac{1}{(d_1+d_2)\alpha_2 - d_1\alpha_1}}$$

Clearly, if the price of the first good declines sufficiently, then the offspring consumes also the second good whatever its price, this event happening earlier the larger the impatience to consume the second good, i.e. the larger the constant C_2.[10] We finally remark that the condition that the difference $\alpha_2 - \alpha_1$ be positive assures that individuals reach a satiety threshold as the price of the first good declines, whatever are the other quantities.

4 The production structure

Each final good is produced by a technologically vertically integrated industry. In particular, each industry j consists of a perfectly competitive final good producer and a continuum of measure k_j of monopolistic, industry specific intermediate goods producers. In each period of time in each of the intermediate good producers, an outsider firm invests in R&D to dethrone the incumbent monopolist, owner of the best practice technology.

To keep the model as simple as possible, we assume that all intermediate goods are strictly complementary[11] and thus the production function of final good j is

$$y_{j,t} = \min_{k\in[0,k_{j,t}]} \left\{a_{j,t}(k)\, x_{j,t}(k)\,, b_{j,t} l_{j,t}^{y}\right\} \tag{4}$$

where $a_{j,t}(k)$ is the productivity of input k and $x_{j,t}(k)$ its quantity while $b_{j,t}$ is specific labor productivity and $l_{j,t}^{y}$ employment. We assume that $a_{j,0}(k) = a_0$, $b_{j,0} = b_0$ for each k and j. The advantage of production function (4) is that it does not assume upfront increasing returns to specialization and thus, if increasing returns to specialization occur, they are obtained as a result rather than as an assumption. $k_{j,t}$ indicates the degree of specialization in industry j: the larger $k_{j,t}$ the more specialized inputs are. $k_{j,t}$ will be endogenously determined in a following section.

Efficient utilization of all inputs implies that

$$y_{j,t}^{s} = a_{j,t}(k)\, x_{j,t}(k) = b_{j,t} l_{j,t}^{y} \text{ for each } k \in \left[0, k_{j,t}\right] \tag{5}$$

The production function of intermediate good $k \in [0, k_{j,t}]$ belonging to industry j is

$$x_{j,t}(k) = \eta l_{j,t}(k)\,, \tag{6}$$

where $l_{j,t}(k)$ is the amount of labor employed and η is the constant labor productivity. Let $l_{j,t}^{x} = \int_0^{k_{j,t}} l_{j,t}(k)\, dk$ be the overall employment by intermediate good producers in industry j at time t.

The economy's nominal wage rate is kept constant throughout and hence it is the economy's numéraire[12] and normalized to 1.

5 Innovations

Consider first vertical innovations. Innovations are idiosyncratic, intermediate good producer specific productivity- enhancing events that are the result of a technology race between an outsider firm investing in

R&D and an insider who, because of Arrow's replacement effect, does not. We assume that these events have an industry-wide impact.[13] This statement holds either because some innovations simply raise the overall industry productivity without altering input requirements[14] or because they create technological imbalances. Historians of technology and factual observation indicate that an innovative event that upsets a technological equilibrium becomes a *focusing device* that prompts adjustments wherever, within the set of complementary inputs, frictions and mishaps happen in consequence (Rosenberg 1983; Mokyr 1990).[15] More formally, we assume that a vertical innovation occurring at time t in intermediate good producer k of any industry j raises the productivity of all inputs $b_{j,t}$ and $a_{j,t}(k)$, $\forall k$ by a factor λ

$$a_{j,t}(k) = a_{j,t-}(k)\, e^{\lambda} \text{ for } k \in \left[0, k_{j,t}\right] \text{ and } b_{j,t} = b_{j,t-}e^{\lambda}$$

The innovator can push the price of the intermediate good down to a level that ousts the current incumbent, capturing all the monopoly rents. To see this, let $u_{j,t}(k)$ be the price of intermediate good k in industry j at time t , then because of perfect competition between final good producers, the price of the final good is

$$p_{j,t} = \int_0^{k_{j,t}} u_{j,t}(k) \frac{1}{a_{j,t}(k)} dk + \frac{1}{b_{j,t}}$$

Using (5) and assuming that all intermediate good producers set the same price u, price $p_{j,t}$ can be written as

$$p_{j,t} = \frac{1}{b_{j,t}} \left(1 + u\eta k_{j,t}\delta\right) \tag{7}$$

where $\delta = \frac{b_0}{\eta a_0}$.

In the production of intermediate goods, as a consequence of (5), $l_{j,t}(k) = \delta l^{y}_{j,t}$, and thus profits for intermediate good producing firms are

$$\pi_{j,t}(k) = (\eta u - 1)\, \delta l^{y}_{j,t}$$

Given an innovation at time t, the lowest price that allows an incumbent to realize non-negative profits is $u' = \frac{1}{\eta}$ and the corresponding final good price is

$$p'_{j,t} = \frac{1}{b_{j,t-}} \left(1 + \delta k_{j,t}\right) \tag{8}$$

If innovators charge u, then the price of the final good is (7) and the incumbent is dethroned if u is such that $p'_{j,t} = p_{j,t}$. Since $b_{j,t-} = e^{-\lambda} b_{j,t}$, the value of u such that $p'_{j,t} = p_{j,t}$ is

$$u = \frac{e^{\lambda}\left(1 + \delta k_{j,t}\right) - 1}{k_{j,t}\delta} \frac{1}{\eta}$$

and the corresponding profits of innovators are

$$\pi_{j,t}(k) = \left(e^{\lambda} - 1\right)\left(1 + \frac{1}{\delta k_{j,t}}\right) \delta l^{y}_{j,t}$$

Given h workers employed in the R&D activity, vertical innovations occur according to a Poisson process with arrival rate h and R&D costs over the small time period dt are $C(h)dt$. For simplicity's sake we assume that $C(h) = \frac{a}{2}h^2 + \frac{F}{2}$, where the first term accounts for variable costs and the latter for fixed production costs.

Specialization and rationalization concern the change in the number of inputs used in the production of a final good. In particular, we assume that the deepening of specialization at time t consists of the appearance of a new intermediate good that brings about an overall productivity increase equal to λ, while rationalization consists in the disappearance of one intermediate.

6 Innovation, growth and structural change

In this section we characterize the economy's equilibrium, its growth rate and its diversification pattern over the development path. In our model, the degree of diversification of an economy is higher the more evenly the number of intermediate goods is distributed among the industries. On the contrary, the degree of diversification is lower the more intermediate goods are concentrated in fewer industries. Following Imbs and Wacziarg (2003), let the Gini index over $\{l_{j,t}\}_{j=1,\dots,J}$, where $l_{j,t} = l^y_{j,t} + l^x_{j,t}$, and over $\{p_{j,t}y_{j,t}\}_{j=1,\dots,J}$ be measures of the degree of diversification of an economy. It follows that the degree of diversification is low (high) if the Gini index is high (low).

Consider first the solution to the consumers' problem. In the appendix we show that Assumption 1 states sufficient conditions such that a unique maximum j^*_t exists and that the individual demand function is:

$$y_{j,t} = \begin{cases} \frac{d_j}{\sum_{h=1}^{j^*_t} d_h} \frac{R_t}{p_{j,t}} & \text{for each } j \le j^*_t \\ 0 & \text{for each } j > j^*_t \end{cases} \tag{9}$$

Note that, as a consequence of Assumption 1.3, the fraction of income spent by an individual on a high priority good is not larger than that on a low priority one and that this fraction is non-increasing in the number of goods consumed.

Consider an individual born at time t consuming goods $1, \dots, j^*_t$. Suppose that from time t to time $t + dt$ the price of good j^*_t declines. As shown in the Appendix, the individual's descendant revises the quantity and possibly also the variety choice. If the variety consumed remains unchanged, the descendant's consumption of good j simply increases compared with her ascendant's choice. The price decrease may also trigger an increase in the variety of the consumption bundle. In this case, compared with the ascendant's consumption pattern, the descendant decreases the consumption of some or all the high priority goods to accommodate the consumption of an additional variety of lower priority.[16]

Consumers are by assumption identical. Let Y_t be the economy's aggregate income, where $Y_t = \mathscr{L}R_t = \mathscr{L}\sum_{j=1}^{J} y_{j,t}p_{j,t}$, then we can write the aggregate demand for good j as

$$y^d_{j,t} = \beta_{j,t}\frac{Y_t}{p_{j,t}} \tag{10}$$

with $\beta_{j,t} = I_{j\le j^*_t}\frac{d_j}{\sum_{h=1}^{j^*_t} d_h}$, where $I_{j\le j^*_t}$ is an indicator function, taking 1 if $j \le j^*_t$ and 0 otherwise. $\beta_{j,t}$'s are expenditure shares that indicate the fraction of aggregate income consumers spend on a given good j at time t. Price changes may trigger changes in the aggregate consumption pattern. For instance, a fall in price may lead to an increase in the number of goods j^*_t and thus to lower shares for some products and higher for others. As real income rises as a consequence of productivity gains, the share of the goods that are less essential increases whilst that of those that are more so concomitantly decreases; hence, goods placed in the higher part of the product sequence ordering become progressively weightier.[17]

Our economy is in equilibrium if the consumption goods market is in equilibrium:

$$y^s_{j,t} = y^d_{j,t} = \beta_{j,t}\frac{Y_t}{p_{j,t}}. \tag{11}$$

If this condition is realized then, using (5) and (8), we obtain

$$\pi_{j,t}(k) = \left(e^{\lambda} - 1\right)\frac{1}{k_{j,t}}\beta_{j,t}Y_t$$

Full employment and constant aggregate profits lead to constant nominal income $Y_t = Y$ (see the Proof "Derivation of equilibrium income" in the Appendix). Assuming no-arbitrage among industries we obtain, at the equilibrium, $\pi_{j,t}(k) = \pi$

$$\pi_{j,t}(k) = \left(e^{\lambda} - 1\right)\frac{1}{k_{j,t}}\beta_{j,t}Y = \pi$$

from which $\frac{\beta_{j,t}}{k_{j,t}} = \gamma$. Thus, an increase (decrease) in $\beta_{j,t}$ increases (decreases) $k_{j,t}$, triggering a specialization (rationalization) event. We assume that λ is sufficiently large such that the increase in costs due to the lengthening of the production process is outweighed by the productivity gain brought by the specialization event and thus the new technology has lower production costs than the old one.

Specialization (rationalization) is the consequence of an increase (a decrease) in demand and hence profits. We assume that producers whose intermediate good has been made redundant by a rationalization process switch their innovative effort to the newly created intermediate good in the expanding industry. In this way, the expected present value of an innovator is independent of the industry she is operating in and can be recursively written as

$$rV = \pi - h_f V \tag{12}$$

where r is the discount factor and h_f is the outsider's R&D investment.

The firm's innovative effort is the solution of the following maximization problem

$$W = \max_h \left[hV - \frac{a}{2}h^2 - \frac{F}{2} \right] \tag{13}$$

Since we are assuming that there is no entry barrier, specialization occurs as long as the net expected present value of a vertical innovation remains positive, which endogenously determines $k_{j,t}$ and thus γ^*. More formally, the free-entry condition $W = 0$ yields

$$k_{j,t} = \frac{1}{\sqrt{aFr + F}} \left(e^\lambda - 1\right) \beta_{j,t} Y \tag{14}$$

and thus

$$\bar{k} = \sum_{j=1}^{J} k_j = \frac{1}{\sqrt{aFr + F}} \left(e^\lambda - 1\right) Y \tag{15}$$

In the following we are going to characterize the output level as well as the economic growth rate and introduce the simplifying assumption $r = 0$.

The size of the available labor force, $\mathscr{L}$, is allocated as follows $\mathscr{L} = L_t^y + L_t^x + H_t$, and $L_t^y = \sum_{j=1}^{J} l_{j,t}^y$, $L_t^x = \sum_{j=1}^{J} \int_0^{k_{j,t}} l_{j,t}(k)\, dk$ and $H_t = \sum_{j=1}^{J} k_{j,t} h_{j,t}$ and thus aggregate output is (see the Proof "Derivation of equilibrium income" in the Appendix) :

$$Y = \frac{\mathscr{L}}{\frac{1}{e^\lambda} + \sqrt{\frac{1}{Fa}}\left(1 - e^{-\lambda}\right)} \tag{16}$$

Given that $\mathscr{L}$ is constant, an increase in the costs of R&D, that is, an increase in either F or a, implies a drop in the number of employees devoted to this task and therefore determines a shift in employment from R&D to final production. This shift increases total output and income Y. There is a consequence in terms of the total number of specialized intermediate goods $\bar{k}$ and industry specialization k_j. An increase in variable R&D costs a, by increasing aggregate income Y, increases aggregate profits, an effect that sustains a greater number of specialized intermediate good producers and therefore more specialization in each industry j. An increase in the fixed costs of R&D, besides having the indirect effect of increasing the total number of specialized intermediate goods through higher total profits, has a direct negative effect on the entrance of new specialized firms. The former does not counterbalance the latter and hence an increase in F decreases $\bar{k}$ and k_j.[18]

Note furthermore that Y remains constant over time and that it is an increasing function of $\mathscr{L}$. Real income increases because of price declines. Furthermore, let us define $f(\mathscr{L}) = \left(1 - e^{-\lambda}\right) \frac{Y}{F}$, then $k_j = \beta_j f(\mathscr{L})$ while $\bar{k} = f(\mathscr{L})$ remains constant over time, $f(\mathscr{L})$ increasing in $\mathscr{L}$ and decreasing in F.

We next characterize the economy's growth rate.

Proposition 1

The economy's growth rate is

$$g_{Y_R} = \sqrt{\frac{F}{a}} \sum_{j=1}^{J} \beta_j \left(e^{\lambda \beta_j f(\mathcal{L})} - 1 \right) \tag{17}$$

Proof.

In the Appendix. ■

This result lends itself to some interesting interpretations. The first observation is that g_{Y_R} is the weighted average of the single industries' productivity growth rates, the latter depending on the dimension of their markets. In other words, the greater is the market of an industry, the greater its specialization, and hence the industry's productivity growth rate contributing to the aggregate in accordance to its specific weight. As in other models of firm and industry specific knowledge accumulation, ours predicts scale effects at the industry level, those at the aggregate one becoming vanishing small as the economy becomes very large.[19]

The second observation is that the growth rate depends on a specific distribution of demand shares; thus, a stage of development denoted by the corresponding demand shares is accordingly identified. The distribution of demand shares corresponds to a distribution of the intermediate good producers (that is, a distribution of specialization) amongst the various industries. It is indeed straightforward to see that the aggregate growth factor is an average of the various industries' own factor weighed by aggregate demand shares, the latter having been shaped by the development process. As this process unfolds thanks to real income growth assigning greater weights to goods that are less essential, the economy grows through two stages of diversification. At an initial stage, the impact of innovations is spread over a narrow number of primary industries and hence the degree of diversification is subject to increase. In a later stage, the degree of diversification may decline if consumers spend relatively more of their income on lower priority goods than on higher priority ones. This fact has important consequences for countries at different stages of this process. Consider, as an example, two countries with the same general features: the less developed having a demand pattern and production structure concentrated in the smaller range of high priority goods and industries; the more developed one having a production and demand pattern more evenly spread over a larger number. According to the above stated argument, the first grows faster than the second but as the aggregate demand of the more developed country gets again more concentrated, entering the second stage, its growth rate increases rendering the catching up process of the less developed one more difficult.[20]

A third observation follows immediately from the previous two. An economy that manages to concentrate its aggregate output on fewer industries, other things being equal, achieves higher aggregate growth for the simple reason that specialization is also more concentrated: the same $\bar{k}$ distributed on fewer $j's$.[21] The reason for this result is that the greater the degree of specialization of an industry, the greater the number of intermediate good producers that are affected by an innovation in the industry. In particular, because of our assumptions, as a single intermediate good producer becomes more productive, it raises the productivity of all inputs. As a consequence, the aggregate effect of a single innovation is therefore stronger the more specialization is concentrated. This configuration may occur in economies that, in spite of possessing a high real income per capita, have specialized through foreign trade in the production of a relatively small number of goods that they export while importing many more scoring comparative advantage and higher growth. This view finds support in Steingress (2015) where, in cross-country analysis comprising a panel of 130 countries, it is shown that exports are more concentrated than imports[22] and in Sakyi, Villaverde, and Maza (2015) where it is shown that trade openness increases with income. Furthermore, it is a well established result of relevant theory that international trade by increasing the demand for the goods subject to relative specialization enlarges the size of the market of the industry where it occurs, leading to a yet higher productivity growth in the industry and, depending on its weight in the aggregate economy, to a higher aggregate growth rate (for empirical evidence see Frankel and Romer 1999, and Alcalá and Ciccone 2004). It must, however, be stressed that graduating from a stage of development to the next depends crucially on the research and development process that finally yields innovations, productivity growth and ultimately the increase of income per head that reshapes the distribution of demand shares. It is on this logical sequence of events that the development process hinges upon, the sooner the innovation-led virtuous circle of productivity growth is ignited, the faster will growth be.

A final result concerns the degree of diversification over the development path.

Proposition 2

If aggregate consumption is concentrated on high (low) priority goods in the early (later) stages of the economy's development and spread out more evenly in an intermediate stage, then the diversification of the economy over the development path is inversely U-shaped

The proof of this result is as follows. Since Y is constant over time, from (11) it follows that $p_{j,t}y_{j,t}$ is proportional to $\beta_{j,t}$. Moreover, it is easy to see that overall industry employment $l_{j,t}$ is proportional to $k_{j,t}$.[23] We argued that because of a no arbitrage condition, $k_{j,t}$ is proportional to $\beta_{j,t}$, i.e. $k_{j,t} = \frac{\beta_{j,t}}{\gamma}$. Changes in $\beta_{j,t}$ are triggered by vertical innovations that lead, because of competition between an insider and an outsider, to price decreases reshaping the industry structure. More specifically, an increase (decrease) in $\beta_{j,t}$ leads to an increase (decrease) in $k_{j,t}$. Given the consumers' problem, individuals increase the consumption of low priority goods at the expense of high priority ones along the economy's development path. As a consequence, the Gini index of the distribution of $\{\beta_{j,t}\}_{j=1,\dots,J}$ is high in the early and later stages of the economy's development path and low in an intermediate stage. This fact implies a low degree of diversification in early and later stages and a high one in an intermediate stage. A result which is consistent with the empirical evidence in Imbs and Wacziarg (2003). Moreover our model predicts that the economy's growth rate is also U-shaped over the development path, being higher at early and later stages, and lower at intermediate ones.

7 Discussion

From the growth rate equation (17) it might be inferred that our model exhibits strong scale effects as the size of the population of the economy increases.[24] This result, however, has to be interpreted in the light of our assumption of a finite, exogenously given number of final good industries. Akin to other models with firm and industry-specific knowledge accumulation, our model exhibits strong industry level scale effects while aggregate ones decrease as the number of goods increases. Laincz and Peretto (2006) point out that in models with firm-specific knowledge production, what matters for economic growth is the scale of the firm or industry as opposed to the scale of the economy. In particular, by resorting to a very general multi-industry growth model with industry-specific knowledge production and variety expanding R&D, these authors show that the number of products in equilibrium is proportional to the population size and that in consequence aggregate scale effects are absent while those pertaining to industries remain strong. In a similar vein, if one assumes in our model that the number of final goods produced is proportional to the population size and that as the population size increases the number of goods also increases, then aggregate scale effects are completely eliminated. Nevertheless, scale effects at the industry level remain strong, a fact in the Smithian tradition that is also consistent with the empirical evidence documented in Backus, Kehoe, and Kehoe (1992).

A final issue concerns cross-industry spill-overs and the diffusion of innovations. We do not incorporate the latter in our model but recognize that they may play an important role in reality. If one assumes an R&D activity as in Andergassen, Nardini, and Ricottilli (2009) where innovations are the result both of in-house R&D and, from a technological viewpoint, of a local search for information, then structural change would be driven by local and global firms' interaction. In such a model the growth rate of the economy would depend on the extent of productivity spill-overs but less on the degree of the economy diversification maintaining strong industry scale effects, aggregate ones becoming vanishing small as the number of intermediate good producers in the industry becomes large.

8 Conclusions

This paper argues that productivity increases driven by firms' innovative efforts lead to income per head increases and to a reshaping of the aggregate consumption pattern such that shares of low priority goods increase at the expense of high priority ones. As a consequence and over time, intermediate inputs that are initially concentrated in few industries producing high priority goods become more evenly distributed across industries, while their distribution gets more concentrated in later stages of development on lower priority goods, a result which is in keeping with the empirical findings in Imbs and Wacziarg (2003).

The change in aggregate demand pattern induced by vertical innovations along the development path gives rise to a process of creation and destruction of intermediate good producers. Industries whose demand increases experience an expansion of the number of intermediate goods and hence of their research effort, while those whose demand declines undergo a cost-cutting restructuring with a corresponding reduction of the number of intermediates. Since the growth rate depends on the innovation contribution of each intermediate good producer and since the productivity growth rate in an industry is greater the greater is demand, we find that the more even is the distribution of the latter the slower is the economy's growth rate.

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Appendix

The consumption function. The solution to the consumer's problem is as follows. Let j_t^* be the optimal variety at time t, that is $U\left(j_t^*, t\right) > U\left(j, t\right)$ for each j other than j_t^*, then (9) follows and from which the indirect utility can be obtained. Assumption 1 states sufficient conditions for the existence of j_t^*. More particularly, it guarantees that the utility is monotonically increasing in j for the consumption bundles with variety lower than j_t^* and monotonically decreasing for baskets with variety larger than j_t^*, that is,

$$U\left(j, t\right) < U\left(j+1, t\right) \text{ for each } j = 1, ..., j_t^* - 1 \tag{18}$$

and

$$U\left(j+1, t\right) < U\left(j, t\right) \text{ for each } j = j_t^*, ..., J - 1 \tag{19}$$

To see this, taking into account (9), write $U\left(j, t\right) \lesseqgtr U\left(j+1, t\right)$, i.e. the inequality of indirect utilities, as $\Omega\left(j, t\right) \lesseqgtr \chi\left(j, t\right)$, where $\Omega\left(j, t\right) \equiv \left(\frac{\sum_{h=1}^{j} d_h}{d_j} p_{j+1,t}\right)^{\alpha_{j+1}} \Pi_{h=1}^{j} \left(\frac{\sum_{h=1}^{j} d_h}{d_j} p_{h,t}\right)^{\alpha_{j+1}-\alpha_j}$. Assumptions made on the production and demand side of the economy guarantee that if at time $t = 0$ prices of higher priority goods (in reference to j_t^*) are lower than those of lower priority goods then this is true also for $t > 0$. As a consequence, Assumption Assumption 1 (1) and (3) guarantee that $\Omega(j,t)$ is increasing in j. Conditions (18) and (19) can be written as follows

$$\Omega\left(j, t\right) < \chi\left(j, t\right) \text{ for each } j = 1, ..., j_t^* - 1$$

and

$$\Omega\left(j, t\right) > \chi\left(j, t\right) \text{ for each } j = j_t^*, ..., J - 1$$

Since at each time period t, $\Omega(j,t)$ and $\chi(j,t)$ cross at most once, an unique j_t^* exists.

Changes in prices or income lead to a revision of the individual's quantity choice and eventually also of the variety one. Suppose that at time t price $p_{j,t}$, $j \leqslant j_t^*$, declines. This event decreases $\Omega\left(j_t^*, t\right)$ and may trigger an increase in the variety of the consumption bundle. In this case, a re-balancing of the individual's consumption pattern occurs, decreasing the consumption of some or all high priority goods to accommodate the consumption of an additional variety of lower priority. If the variety consumed remains unchanged, the individual's consumption of good j simply increases. ■

Derivation of equilibrium income. Given the assumption $r = 0$ and since in equilibrium $h_f = h$, (12) becomes $V = \frac{\pi}{h}$; consequently the first order conditions for problem (13) yield

$$\begin{aligned} h_{j,t}^k = \quad & h_{j,t} = \frac{V_{j,t}}{a}, \\ \pi_{j,t} = \quad & F,. \\ \text{and } h_{j,t} = \quad & \sqrt{\frac{F}{a}} \end{aligned} \tag{20}$$

Employment in both final and intermediate good producer can be characterized in terms of aggregate output. Since $l_{j,t}^y = \beta_{j,t} \frac{1}{b_{j,t}} \frac{Y_t}{p_{j,t}}$ and $x_{j,t}\left(k\right) = \beta_{j,t} \frac{1}{a_{j,t}(k)} \frac{Y_t}{p_{j,t}}$, the number of workers engaged in producing a final good j is

$$l_{j,t}^y = \beta_{j,t} \frac{Y_t}{e^{\lambda}\left(1 + \delta k_{j,t}\right)} \tag{21}$$

From (21) and (5) it follows that employment by intermediate good producers in each industry is

$$l^x_{j,t} = \int_0^{k_{j,t}} l_{j,t}(k)\,dk = \beta_{j,t}\int_0^{k_{j,t}} \frac{b_{j,t}}{\eta a_{j,t}(k)}\frac{Y_t}{e^{\lambda}\left(1+\delta k_{j,t}\right)}dk = \beta_{j,t}Y_t\frac{\delta k_{j,t}}{e^{\lambda}\left(1+\delta k_{j,t}\right)} \tag{22}$$

Summing over j in the above formula and in (21), total manufacturing employment turns out to be:

$$L_t = \sum_{j=1}^{J}\left(l^y_{j,t}+l^x_{j,t}\right) = \frac{Y_t}{e^{\lambda}}. \tag{23}$$

The third of (20) and (15) allow us to calculate the employment H_t that followers hire to conjure up the next round of innovations.

Considering (23) and having in mind that the assumption that the overall employment is constant $\mathscr{L} = L^y_t + L^x_t + H_t$, H_t is, then, equal to

$$\mathscr{L} - \frac{Y_t}{e^{\lambda_v}} = H_t \tag{24}$$

Finally from (24), (15) and the third of (20),

$$\mathscr{L} = Y\left\{\frac{1}{e^{\lambda}} + \sqrt{\frac{1}{Fa}}\left(1-e^{-\lambda}\right)\right\}. \tag{25}$$

aggregate final good output (16) follows. ■

Proof of Proposition 1.

Using the accounting definition of aggregate income and taking the time derivative, aggregate growth,[25] is:

$$\frac{\dot{Y}_t}{Y_t} = \sum_{j=1}^{J}\beta_{j,t}\left(\frac{\dot{p}_{j,t}}{p_{j,t}}+\frac{\dot{y}_{j,t}}{y_{j,t}}\right).$$

In this model, however, the nominal wage rate is kept constant and productivity gains translate into proportionally lower prices such that the real wage rate and likewise real monopolists' profits grow in step with productivity. The long-run real growth $Y_{R,t}$ turns out to be:

$$\frac{\dot{Y}_{R,t}}{Y_{R,t}} = \sum_{j=1}^{J}\beta_j\frac{\dot{y}_{j,t}}{y_{j,t}},$$

from which we can characterize the long-run growth rate, recalling that the Poisson arrival rate is $h_{j,t} = \sqrt{\frac{F}{a}}$, as:

$$g_{Y_R} = \sqrt{\frac{F}{a}}\sum_{j=1}^{J}\beta_j\left(e^{k_j\lambda}-1\right).$$

Since $k_j = \beta_j f(\mathscr{L})$, the expression in (17) follows. ■

Notes

1 See cases of technological innovated products discussed by Mokyr (1990), Chapter 6. As an example, consider the case of telegraphy. Growth of demand for this kind of communication led to the introduction of specialized inputs stemming from several separate technological innovations. The recent decline of demand has brought about radical simplification until commercial telegraphy has all but disappeared. Another case in point concerns the radio. Following an increase in construction complexity associated with increasing demand, transistors and integrated circuits radically rationalized its production process in the context of ever more sluggish demand.

2 One could also obtain the same inverted-U shaped pattern of diversification along the development path in an exogenous growth framework. Nevertheless, we adopt an endogenous growth framework since we wish to stress that R&D is enhanced in those sectors in which demand expands as a consequence of an endogenous process rather than an exogenous one. We would like to thank our referee for pointing this out.
3 Buera and Kaboski (2012a, 2012b)) analyze the consequences of domestic production on structural change in the manufacturing and service sector.
4 Compared with the related literature on directed technological change in the present model it is the demand induced process of creation and destruction of intermediate good sectors that directs the pace of technological change.
5 Boppart (2014) studies supply and demand induced structural change introducing a sub-class of price-independent-generalized linear preferences assuming exogenously given TFP growth rates. Boppart and Weiss (2015) study the dynamic interplay between directed technical change and non-homothetic preferences. Both papers, however, do not address the issue of diversification of an economy along its development path.
6 In Laitner (2000), Kongsamut, Rebelo, and Xie (2001), Foellmi and Zweimüller (2008), and Buera and Kaboski (2009) productivity growth is exogenous.
7 In this paper we consider J as exogenously defined. In Section 7 we briefly discuss an extension of our model where J is endogenously determined.
8 One could also allow for preference heterogeneity, where individuals' preferences are the same with respect to the ordering of goods but differ with respect to the rate at which they substitute a higher priority good with a lower priority one. Differences in this rate of substitution may be due to an impatience effect or because of status seeking and demonstration effects as described in Banerjee and Duflo (2011).
9 To better grasp this point, consider that should a decrease of indirect utility occur by adding one more good in the equally distributed basket, a further increase of such goods would yield an even larger drop in indirect utility.
10 In the case of preference heterogeneity where some individuals already consume the second good, a similar effect applies when the price of commodity two declines.
11 What is needed for the model to work is the imperfect substitutability between differentiated inputs, since this gives each intermediate good producing firm sufficient market power to introduce innovations and new inputs. For simplicity's sake we have taken this to an extreme where all inputs are perfect complements but the qualitative results hold also in a more general case.
12 Note that since labor in our model is homogeneous because of arbitrage in the equilibrium all workers earn the same wage.
13 Note that to simplify exposition we assume that direct and indirect (intermediate good) labor productivity increases at the same rate. Since, as explained below, productivity increases lead to price declines, relaxing this assumption would make the latter to be given by the weighted average of the different productivity growth rates a fact that would not alter our qualitative results. Moreover, we abstract from spill-overs between different industries. See Section 7 for a discussion.
14 A first example of specialization is the introduction of the separate condenser in the Newcomen atmospheric engine by James Watt. Without basically modifying the boiler piston cylinder and pump, Watt managed to obtain a lower cost per kWh thanks to an astonishing 75% cut of fuel consumption. A further example of specialization is the introduction of a regenerative heat exchanger (the so called Cowper stove) to the blast furnace, which highly increased the productivity without changing the furnace itself. A more recent example is the substitution of welding and riveting in the machine industry with new generation glues: this innovation has substantially decreased the time of execution and improved the stress distribution over the manufactured piece. Examples of vertical innovations are Trevithick's fire-tube boiler and, later, the water-tube boiler by Babcock and Wilcox that led to a sensible fall of the cost per kWh without changing other engine components. A similar case is the shift from low rotation speed side-valve car engines to high rotation speed overhead-valve engines. In the construction industry special alloy micropiles by underpinning buildings have substantially improved their foundation without altering other components.
15 The nature of focusing devises is such that they guide innovations where technological imbalances arise. Examples of this fact abound. Classical ones mentioned by Nathan Rosenberg are the coupling of improved engines entailing faster speeds in automobiles made feasible by better breaking systems and lathes turning at higher speeds made implementable by high-speed steel. i.e. steel alloy combining tungsten, vanadium and chromium, that raised the hardness of cutting tools (Rosenberg 1976, chapter 6). An other well known example is the introduction of Watt's rotative steam engine in the textile industry: despite the high cost of this new production factor the strong increases in the productivity of all other factors resulted in a much more profitable production process. In this case the considerable increase of the processing speed required many adjustments of the existent looms and water-powered spinning frame.
16 If individuals' preferences are heterogeneous, some individuals being more impatient than others, their solution to the variety choice problem may be different, leading to $j^*_{i,t}$ for $i = 1, \ldots, \mathscr{L}$. In this case, consumption choices of individual i depend not only on prices of goods of priority higher than or equal to $j^*_{i,t}$, but also on prices of goods of lower priority. In particular, individual i might increase the variety consumed if the price decline of good $j^*_{i,t} + 1$ is sufficiently strong (see also footnote 10).
17 Note that, since the numéraire is the nominal wage, as the economy evolves the level of expenditure on high-priority goods in nominal terms declines, but in real terms it may well increase.
18 From the point of view of aggregate income, while income from wages remains constant, income from production of the fixed cost, which we have not explicitly modeled, changes as the costs of R&D vary. In particular, income from the production of fixed costs is $\bar{k}F$, which, using (15) and (16), can be written as $\bar{k}F = \left(1 - e^{-\lambda}\right) \frac{\mathscr{L}}{\frac{1}{e^{\lambda}} + \sqrt{\frac{1}{Fa}}\left(1 - e^{-\lambda}\right)}$ and is thus increasing in both a and F. The intuition for this result is the following. An increase in F produces two effects. On the one hand, an increase in F, for a given $\bar{k}$, increases income while, on the other hand, it decreases $\bar{k}$. The former effect dominates the latter. An increase in a increases $\bar{k}$ and therefore increases aggregate income.
19 For a more detailed discussion on scale effects see Section 7.
20 This result is obtained without taking cross-industry spill-overs into account. See Section 7 for a discussion.
21 A greater concentration on fewer final products may also lead to a higher aggregate risk and a more volatile growth and with a corresponding negative feedback on innovation incentives in the spirit of Acemoglu and Zilibotti (1997); this may be a promising future research avenue.
22 Country-specific studies provide some further support. For instance, Uchida and Cook (2005) have found evidence, in their case for East Asian economies, of a strong relationship between trade and diversification, the latter being ascribed to structural reasons relating to technological advancement. Rammer and Schubert (2016) find that Germany is in the process of becoming less and less diversified since the cutting edge of innovation is becoming increasingly concentrated in fewer but larger firms to the exclusion of small and medium size enterprises.
23 Considering $l^y_{j,t}$ in (21), $l^x_{j,t}$ in (22), and income share $\beta_{j,t}$ (14) overall employment in industry j is given by

$$l^y_{j,t} + l^x_{j,t} = \frac{\sqrt{aF}r + F}{e^{\lambda}\left(e^{\lambda} - 1\right)} k_{j,t}.$$

24 For a discussion of scale effects see Jones (2005) and Laincz and Peretto (2006).
25 By equilibrium between demand and supply $y^s_{j,t} = y^d_{j,t} = \beta_{j,t}\frac{Y_t}{p_{j,t}}$ this means that $\frac{p_{j,t}y_{j,t}}{Y_t} = \beta_{j,t}$.

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