The impact of consumer multi-homing on advertising markets and media competition

The Impact of Consumer Multi-homing on Advertising Markets and Media Competition

by

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This Version: October 14, 2016

We develop a model of advertising markets in an environment where consumers may switch (or "multi-home") across publishers. Consumer switching generates inefficiency in the process of matching advertisers to consumers, because advertisers may not reach some consumers and may impress others too many times. We find that when advertisers are heterogeneous in their valuations for reaching consumers, the switching-induced inefficiency leads lower-value advertisers to advertise on a limited set of publishers, reducing the effective demand for advertising and thus depressing prices. As the share of switching consumers expands (e.g., when consumers adopt the internet for news or increase their use of aggregators), ad prices fall. We demonstrate that increased switching creates an incentive for publishers to invest in quality as well as extend the number of unique users, because larger publishers are favored by advertisers seeking broader "reach" (more unique users) while avoiding inefficient duplication. *JEL Classification Numbers*: L11, L82

Keywords. advertising, media, newspapers, matching, switching, multi-homing, single-homing, tracking, two-sided markets, platforms

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This paper was previously circulated with the title "Will the Internet Destroy the News Media?" and "The Impact of the Internet on Advertising Markets for News Media." We thank participants at the ZEW Platforms Conference (Mannheim), Toulouse Network Conference (Seattle), 2010 Media Economics Workshop (CUNY), Annual IO Theory Workshop (Duke), Conference on Internet and Software (Toulouse), 2010 Economics of Information and Communication Technologies (Paris), 2011 International Industrial Organization Conference (Boston), 2011 CRES Conference (St Louis), 2011 NBER Summer Institute (Digitization), 2011 FTC Annual Microeconomics Conference, 2014 IO Workshop (Manchester), 2015 Summer Institute on Competitive Strategy (Berkeley), the 2015 International Conference in Game Theory (Stonybrook), the 2016 WZB Lecture in Social Sciences (Berlin), the 2016 Provost Lecture (Carnegie Mellon), and at seminars at Monash, Dartmouth, Harvard, Yale, Virginia, Toronto, Stanford, Chicago, Columbia, George Mason, London School of Economics, Boston College, Google, Microsoft Research, Harvard Business School, CSEF in Naples, Autonoma, Bologna, Ca Foscari, MIT, Berkeley, Northwestern, Queensland, and Melbourne for helpful comments. Special thanks to Michael Riordan, Hanna Halaburda, Francisco Ruiz-Aliseda, Simon Anderson, Sergio Parreiras and Glen Weyl for helpful discussions. We thank 3 anonymous referees and the co-editor for useful comments and Jing Xia, Carlo Didonna and Gleb Romanyuk for outstanding research assistance. Financial support from the Toulouse Network on Information Technology and SSHRC is gratefully acknowledged. Responsibility for all views expressed lies with the authors and not with any affiliated organization.

1 Introduction

This paper studies advertising markets in settings where consumers allocate their attention across multiple publishers. We examine how an increase in consumers' propensity to switch across publishers affects advertising prices, publisher profits, and publisher content strategy. We are motivated by the observation that frequent consumer switching is an essential distinguishing feature of online news consumption relative to physical newspapers (Gentzkow and Shapiro, 2011; Varian, 2010). In the older print world, consumers' attention was concentrated on a single publisher. Web browsers, search engines, aggregators and social network make it easy for consumers to move between publishers and increase consumer switching among publishers (Athey and Mobius, 2012), while free access removes other barriers. Indeed, consumer behavior in news in the post-internet world looks qualitatively more similar to consumer behavior in media such as radio and television: the same user consumes many different shows. The Internet simply accelerates the time scale. We demonstrate here that switching creates significant complications for how advertisers choose their strategies and, in turn, leads to lower ad prices and publisher profits. [1] Further, there are changes in publisher incentives to invest in attracting different types of audience: the returns to attracting more unique users increase.

When the attention of a consumer is concentrated (as was arguably the case in the world of print-only newspapers), advertisers, who want to reach all news consumers a given number of times, can advertise on all publishers and achieve their objectives. Although they have to contend with market power by the publishers, the process of using advertising to reach consumers is efficient: there is limited waste in terms of reaching the same consumer multiple times. Further, by advertising in many newspapers, they can

[1] The shift of news distribution from print to the Internet has created many changes for the news industry, including a dramatic decline in advertising revenue. A recent report of the Federal Communication Commission (Waldman et.al., 2011) found that U.S. newspaper advertising revenues dropped 47% from 2005 to 2009. The ad revenue decline is pronounced even when controlling for factors such as circulation, decline in revenues from classified ads, and the business cycle. According to the Newspaper Association of America (www.naa.org), since 2000, total advertising revenue earned by its member U.S. newspapers declined by 57% in real terms to be around $27 billion in 2009. Much of this decline was in revenue from classifieds, but total display advertising revenue fell around 40%. In contrast, circulation over the same period declined by 18%. Ad revenue as a share of GDP also declined by 60%.

avoid missing consumers. When consumers switch between publishers in unpredictable ways, this changes the decision facing advertisers as to how they distribute their ads across publishers, as the movement of consumers may mean they show too many ads to some consumers and not enough to others.

In an environment of consumer switching, a primary concern for advertisers planning an advertising campaign has been how to most effectively meet "reach" and "frequency" objectives (Boyd and Leckenby, 1985). Reach refers to the number of unique users who see a campaign, while frequency refers to the number of times that an advertiser impresses each consumer. Standard media planning attempts to allocate a budget to maximize the number of consumers who see a desired frequency of advertisements, where too few or too many ads are viewed as wasteful (Cannon and Riordan, 1994). In television or radio, an advertiser uses ratings data and information about audiences to select a set of programs on which to advertise, paying attention to audience overlap. Adding a program to the mix with high overlap will bring relatively few *new* consumers relative to the size of the audience. In the early days of the internet, advertisers and analysts alike recognized that achieving reach and frequency goals could prove challenging, making internet advertising inefficient (Dreze and Zufryden, 1998); perhaps surprisingly, despite advances in technology such as tracking cookies, the challenges of achieving reach and frequency goals in cross-publisher campaigns remain substantial, as consumers use multiple devices, browsers, and apps to consume media.

In the context of these challenges, a stylized fact in advertising is that media content with greater reach—that is, more unique users—generally sells for a higher price per user.[2] Although there are a variety of potential explanations for this discrepancy, industry participants recognize that one important advantage of large audiences is that they provide a one-stop shop for wide reach without inefficient duplication (Greene, 2013). This paper provides a theoretical model that captures the benefits to publishers of achieving high reach, while simultaneously allowing us to answer a variety of questions about competition and media strategy.

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[2] In display advertising, see Lella and Lipsman (2015). Recently, this has been referred to as the "ITV Premium Puzzle" (Competition Commission, 2003). However, the relationship has been noted previously by Fisher, McGowan and Evans (1980) and Chwe (1988). See also Crampes, Haritchabalet and Jullien (2009). Goettler (2012) provides recent empirical verification of such advantages.

Our model is tailored to analyze the implications of *changes* in consumer loyalty (or, equivalently, in "switching"), motivated by the experience of newspapers through the move to online consumption and the increased use of aggregators, intermediaries, and social networks to access news. We first assess the impact of switching on fundamentals, that is, market prices and profits. We then show how switching affects publisher strategy in terms of quantity and quality of content. Switching leads advertisers to be more selective in their advertising behavior. In particular, we find that some moderate-value advertisers focus their advertising on a single publisher, so that they can ensure that each advertisement they purchase creates value by reaching a new consumer. If they expanded their scope to additional publishers, each impression would be less valuable on average, because of duplication, and these advertisers would find the value of advertising less than the market price. On the other hand, higher-value advertisers find the value of advertising across publishers to be greater than the market price, and, thus, they choose to frequent multiple publishers. This endogenously determined "mixed-homing" behavior is a consequence of consumer switching, and the choice of advertisers to single-home reduces the effective advertising demand, depressing prices.[3]

Our model differs from traditional economic analyses of competition in media markets in several important ways. Most studies start from the perspective that consumers allocate their attention to a single publisher (this is termed "single-homing" in the multi-sided platform literature).[4] Given this, if there are advertisers that want to place ads in front of all consumers, those advertisers will be forced to advertise on all publishers (i.e., "multi-home"). This eliminates competition among publishers for those advertisers (Anderson and Coate, 2005; Armstrong and Wright, 2007). Subsequent empirical work has challenged this prediction with evidence that (as described above) larger publishers command a premium in terms of ad revenue per consumer; and that,

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[3] A countervailing effect outside our model is that with more data about consumers, publishers can sell more targeted advertising. See, e.g., Iyer, Soberman and Vilas-Boas (2005), who analyze advertiser strategy when advertisements can be targeted; Athey and Gans (2010) for an analysis of the impact of targeting technology on ad prices; and Bergmann and Bonatti (2011) for an analysis of the interaction between online and offline media competition in an environment with targeted advertising.

[4] Haigu and Lee's (2011) analysis of two-sided platforms that connect together content (e.g. games, applications) and users is one of the few studies of multi-sided markets where the focus is on the endogenous choice of multi-homing on one side of the market (content providers). The driving force behind multi-homing in their paper is exclusive contracts between platforms and one side of the market.

competition exists among publishers on the advertising-side (Brown and Williams (2002); Brown and Alexander (2005)). [5]

In order to make predictions about the impact of switching on overall market prices, our model keeps the supply of user attention fixed while varying the degree of user switching, allowing us to isolate the effect of switching. This assumption contrasts somewhat with the existing theoretical literature, which often assumes that if a consumer visits an additional publisher, their supply of attention increases proportionally. Some observers have noted that "supply of advertising space is infinite on the internet," (Rice, 2010) implying that the increase in the number of web pages available is primarily responsible for falling advertising prices for newspapers. However, user attention is still scarce, even on the internet, and a distinguishing feature of our model is the focus on the change in the market structure and consumer behavior rather than the increase in supply.

The paper proceeds as follows. Section 2 sets up the model and defines a market equilibrium and efficiency benchmarks. Section 3 then examines the impact of consumer switching on advertising markets and publisher profits. Our baseline model assumes price-taking behavior by participants. We then extend the model to allow publishers to endogenously choose their advertising capacities, so that switching also affects publisher ability to exercise market power over advertisers. Section 4 tackles questions about publisher content strategy. These include the returns to readership (where we show that switching implies that publishers with larger readerships command advertising premia), the returns to depth (where we show that the returns to keeping users on a publisher for multiple attention periods diminish as switching increases) and the returns to reaching many unique users (which increase with switching). Although most previous media models ignored the role reach and frequency considerations play in advertiser demand, in practice, these considerations are central to how publishers position themselves to their advertising customers. For example, an advantage of Facebook today is its ability to offer advertisers the ability to reach a large fraction of the population a specified number of

[5] Anderson, Foros and Kind (2016) and Ambrus, Calvano and Reisinger (2016) also attempted to reconcile the facts using models that relax single-homing of consumers. The mechanism that leads to lower equilibrium prices and profits in these papers is quite distinct from ours. In these models, prices decrease due to publishers competing down the price charged to advertisers for access to multi-homers. So switching causes a reallocation of the advertising surplus while leaving the total surplus *unchanged*. In our setting, switching has real effects in that it *degrades* the value of the inventory as formally argued in Section 3.

times, even as users access the internet through multiple devices.[6] Our model sheds new light on these facts and empirical findings. Section 5 summarizes our findings as well as additional managerial implications, while Appendix Section 6 provides supporting technical analysis and proofs.

2 The Model

Our goal is to build a simple and tractable model of the allocation of user attention to ads on different publishers, incorporating the crucial issues of duplication (due to advertisers endogenously purchasing multiple ads on multiple publishers) and the possibility of missing consumers. We begin by taking publisher characteristics as exogenous. In later sections, we endogenize publisher choices such as the amount of advertising sold and investments in quality and content.

2.1 Consumer attention and advertising inventory

There is a continuum (unit mass) of consumers and two media publishers (*i* and *j*). Consumers are endowed with a limited amount of attention – here assumed to be two units – which they allocate across publishers. The set of consumers is (exogenously) partitioned into three subsets: (i) those who devote all of their attention to publisher *i* (*i*'s "loyals"); (ii) those who devote all of their attention to publisher *j* (*j*'s "loyals"); and (iii) those who devote 1 unit to each ("switchers"). Let D_i^l , D_j^l and D^s with $D_i^l + D_j^l + D^s = 1,$ denote the measures of these subsets.[7]

For each unit of user attention, publisher *i* can potentially display to the user a_i units of advertising (advertising "impressions" in industry parlance). We refer to a_i (< ½) as

[6] For example, Facebook includes a reach and frequency-planning tool for advertisers (Osborne, 2014). Facebook's marketing materials assert: "because Facebook targeting is based on real people and not cookies (or other identity proxies), we can more accurately control the reach and frequency across devices to better help advertisers achieve their business goals" (Facebook for Business, 2014).

[7] Note that our model keeps the total amount of consumer attention fixed, while the parameter describing the extent of switching may vary. As described in the introduction, this assumption allows us to isolate the role of switching. What is important for our qualitative results is that as consumer switching increases, the amount of attention per consumer does not grow linearly with the number of outlets visited.

the capacity of publisher i, which we take to be exogenous initially and endogenize later.[8] The total supply of impressions by publisher i is equal to $(2D_i^l + D^s)a_i$. Note that loyal consumers can be matched to twice as many advertisers than switchers on a given publisher. For part of the analysis, we assume symmetric consumer demand and hence drop subscripts: $D_1^l = D_2^l = D^l$.

2.2 Advertiser preferences and advertising products

There is a unit mass of advertisers. Advertisers do not directly value ad impressions. In practice, most advertisers have a target range for the number of impressions per consumer (e.g., between 3 and 8 is often considered ideal for online display advertising (Yuan, Wang and Zhao, 2013)). In our model, we capture the idea of diminishing returns in the number of impressions per consumer by simply assuming that each advertiser values only the first impression on each consumer and all additional impressions are wasted. This is a simplifying assumption; what matters for our analysis is that there are diminishing returns to duplicate ad impressions on the same consumer.

We further assume that advertiser preferences are additive in the number of consumers who view their ads, and that all users are equally valuable to a given advertiser.[9] In addition, advertisers are heterogeneous in their value per reached consumer. This value is denoted v and is distributed uniformly on $[0,1]$.

Since advertisers have preferences over the number of unique consumers they impress, it is useful to develop notation for the number of consumers they expect to impress as a function of their advertising purchases. This, in turn, requires us to specify how the number of impressions purchased by an advertiser on a publisher translates into unique users. Let $\Phi(n_1, n_2)$ denote the number of unique users an advertiser expects to impress when it purchases n_1 units of advertising on publisher 1 and n_2 units of advertising on publisher 2. The main property we rely on for Φ in the results that follow

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[8] As the mass of consumers is normalized to one, we restrict ad capacity to be less than ½. This assumption allows for positive excess demand of i's impressions when its price is close to zero.

[9] An alternative specification might have advertisers aim to reach a specific number of consumers (Athey and Gans, 2010) or a specific consumer type (Athey and Gans, 2010; Bergemann and Bonatti, 2011).

is a subadditivity condition, $\Phi(n_1, 0) + \Phi(0, n_2) > \Phi(n_1, n_2)$.[10] That is, when an advertiser multi-homes, the number of unique consumers impressed is less than the sum of the number of unique consumers that would be impressed by single-homing on each publisher. This feature arises in any setting where there is some switching of users across publishers, but the publishers are not able to perfectly track which consumers have already seen an advertisement on another publisher. Even with advances in online advertising technology, given that consumers often access media through multiple devices (e.g. mobile phones and computers), imperfect tracking is an accepted feature of advertising.

To keep our analysis tractable, we assume a particular functional form (that meets the subadditivity condition):

$$\Phi(n_1, n_2) = n_1 + n_2 - \frac{n_1 n_2}{4 \cdot \left(D_1^l + \frac{1}{2} D_s\right)\left(D_2^l + \frac{1}{2} D_s\right)} D_s. \tag{1}$$

This expression says that expected reach is equal to the total number of impressions purchased less those expected to be wasted. To interpret how (1) corresponds to an advertising product offered by a publisher, suppose publishers associate advertisements with particular pieces of content (e.g. web pages within their site, physical pages within printed media, or time blocks in audio or video programming). Consumers do not consume the same piece of content twice, and, thus, never see the same advertisement twice on a particular publisher (for simplicity, we do not consider the case where advertisers purchase advertising in more than one piece of content on the same publisher).[11] Suppose that there are two pieces of content per publisher. One piece of content requires one unit of attention, so loyal consumers consume both pieces from their preferred publisher, while switchers choose a random piece from each publisher (these choices being independent across publishers). The number of people reading a piece of

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[10] In Athey, Calvano and Gans (2012) we study the problem of a sender who wishes to inform as many receivers as possible. We link a number of properties of the Φ function (or communication technology) to communication strategies that maximize reach. (when to concentrate, when to spread messages, etc.).

[11] Our working paper (Athey, Calvano and Gans 2013) establishes that our basic results hold in that case, and, in addition, it is possible that a set of very high-value advertisers does indeed choose to advertise on both. In Athey, Calvano and Gans (2013), we also explored publishers offering a guaranteed impression contract (see Fisher, 2014) alongside the one studied here. This latter option increases the efficiency of the market but it is not prevalent in the real world. Because of imperfect tracking, there is no guarantee of a certain number of impressions on unique visitors to a publisher.

content is $D_i^l + \frac{1}{2}D^s$. Thus, when an advertiser purchases n_i impressions on a particular piece of content on publisher i, a fraction $\frac{n_i}{D_i^l+\frac{1}{2}D^s}$ of the readers of that content see the advertisement.[12] Suppose an advertiser purchases (n_1, n_2) impressions on publisher 1 and 2, respectively, with the advertising associated with a specific piece of content on each publisher. A switching consumer views both of these pieces of content with probability $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$, and is impressed by the advertiser on publisher 1 with probability = $n_1 / (D_1^l + \frac{1}{2}D^s)$ and on publisher 2 with (probability = $n_2 / (D_2^l + \frac{1}{2}D^s)$. Thus, the number of switching consumers who see twice the same ad is given by the final term in (1).

To understand the implications of the expression, note that (i) if all consumers are exclusive ($D^s = 0$) then the total reach is $n_1 + n_2$ and there is no duplication; (ii) if $n_i = 0$ then the total reach is equal to n_j and there is no duplication; (iii) if all consumers visit both publishers ($D^s = 1$and $D_i^l = D_j^l = 0$) then the total reach is equal to 1 minus the probability that a given consumer is not impressed, that is, $1-(1-n_1)(1-n_2)$.[13]

Given the additivity of advertiser preferences for reaching users, if offered advertising at a constant price per impression, an advertiser will either choose to purchase all available ad impressions on a given piece of content at a publisher, or none. All advertisers can, in principle, purchase advertising at neither, one, or both publishers. In line with the literature, in what follows, we refer to these actions as "not purchasing," "single-homing on *i*," and "multi-homing," respectively. The payoffs associated with the different choices are then:

i. Single-home on 1 $\quad (v - p_1)(D_1^l + \frac{1}{2}D^s)$

ii. Single-home on 2 $\quad (v - p_2)(D_2^l + \frac{1}{2}D^s)$

iii. Multi-home $\quad v\left(D_1^l + D_2^l + \frac{3}{4}D^s\right) - p_1 D_1^l - p_2 D_2^l - (p_1 + p_2)\frac{1}{2}D^s$

A key observation is that switching is a *source* of diminishing returns from multi-homing; that is, from purchasing additional impressions on a *different* publisher. To build

[12] For specific media, purchasing n_i ads means, say, "showing the ad to the first n_i consumers who request a particular URL" or "printing the ad on a subset n_i of the newspapers produced."

[13] In contrast, Butters (1977) approximates the outcome under no tracking with $\Phi(n_1, n_2) = 1 - e^{-n_1-n_2}$.

intuition, suppose that publishers are symmetric in their readership ($D_1^l = D_2^l = D^l$) and ad capacities ($a \equiv a_1 = a_2$), and assume equal prices $p \equiv p_1 = p_2$ (indeed, this will be the case in equilibrium when publishers are symmetric). By assumption, $2D^l + D^s = 1$. By single-homing on either publisher, the expected return is equal to the value of informing half the population, $\frac{1}{2}v$, while the expected expenditure is $\frac{1}{2}p$. If the advertiser were to multi-home instead, it would purchase an additional $\frac{1}{2}$ impressions at a cost of $\frac{1}{2}p$. However, the return to that would be less than the value of informing the other half of the population $(D^l + \frac{1}{4}D^s)v \leq \frac{1}{2}v$ due to duplicated impressions. Returns are constant only if $D^s = 0$.

Let $\tilde{n}_i(\mathbb{I}_i) = (D_i^l + \frac{1}{2}D^s)\mathbb{I}_i$ denote the number of impressions an advertiser consumes on publisher i given D_i^l, D^s, and the advertiser's choice $\mathbb{I}_i \in \{0,1\}$ of whether to advertise on publisher i. Given unit price p_i, publisher i's profits are

$$\pi_i = p_i\left(2D_i^l + D^s\right)a_i. \tag{2}$$

2.3 Market Equilibrium

A *market equilibrium* is a tuple $\left(\left(\tilde{\mathbb{I}}_1(v), \tilde{\mathbb{I}}_2(v)\right)_v, \hat{p}_1, \hat{p}_2\right)$ where advertisers choose optimally among single and multi-homing, and where advertising supply equals demand: (i) For each advertiser v,

$$\left(\tilde{\mathbb{I}}_1(v), \tilde{\mathbb{I}}_2(v)\right) \in \arg \max_{\mathbb{I}_1, \mathbb{I}_2} v\Phi\left(\tilde{n}_1(\mathbb{I}_1), \tilde{n}_2(\mathbb{I}_2)\right) - \tilde{n}_1(\mathbb{I}_1)\hat{p}_1 - \tilde{n}_2(\mathbb{I}_2)\hat{p}_2,$$

and (ii) For each publisher i, $\int_0^1 \tilde{\mathbb{I}}_i(v)dv = 2a_i$.[14]

In what follows, we refer to $(\hat{p}_1, \hat{p}_2)$ as the prices that clear the market for impressions.

Diminishing returns from multi-homing implies that advertisers *sort*, in equilibrium, with relatively higher types patronizing more publishers overall. Figure 1 illustrates the sorting that arises in the symmetric case. Type v_m is indifferent between

[14] That is, each publisher can serve up to a_i advertisers per unit of content. There are two units of content. So total supply is $2a_i$ advertisers.

single-homing on either publisher or multi-homing; that is, $v_m = p\frac{D^l+\frac{1}{2}D^s}{D^l+\frac{1}{4}D^s}$. Type v_s is, instead, indifferent between single-homing and nothing; that is, $v_s = p$. In what follows, we refer to advertisers in $(v_m, 1]$ and $[v_s, v_m]$ as "high" and "low" types, respectively.

To solve for the market equilibrium, the publishers' respective aggregate demands have to equal their supply. Note that, with equal prices, all low types are simultaneously indifferent between single-homing on either publisher. In the symmetric publisher case, per impression prices must equalize across publishers and single-homing advertisers must split equally across publishers. Since both thresholds v_m and v_s are monotone decreasing in p, the aggregate demand of each publisher, $1 - v_m + \frac{1}{2}(v_m - v_s)$, decreases in p, and a market equilibrium necessarily exists. In Appendix 6.1, we demonstrate this formally for the more general asymmetric publisher case. We show that for each (a_1, a_2), there is a unique pair of market clearing prices, and we derive the corresponding inverse demand.

2.4 Benchmark: Efficient Matching

It is useful to describe a benchmark in this environment. The first-best allocation of consumers' attention to advertisers is such that the highest value advertisers are allocated with priority to scarce advertising inventory, and there is no duplication. Let v_i denote the marginal advertiser allocated to consumers loyal to publisher i, and let v_s denote the marginal advertiser allocated to consumers who switch. An efficient allocation of advertisers to consumers involves allocating all advertisers with $v \geq v_i$ to publisher i's loyal consumers and those with v_{sw} to those who switch. Thus, the values of the marginal advertisers are the unique solutions to $2a_1 = 1 - v_1$, $2a_2 = 1 - v_2$, and $a_1 + a_2 = 1 - v_{sw}$.[15]

Could such an allocation be implemented in practice? It might require both publishers to make use of a common ad platform, or publishers might share user data; in both cases, the publishers would need to separately sell impressions for loyals and

[15] To see how this first best might be implemented in practice, consider a scenario where there exists a public record that keeps track of all consumer/ad matches. More realistically, suppose that both publishers outsource their advertising to a third party, labeled "ad-platform." The platform acquires the publishers' entire advertising inventory and can keep track, say by planting "cookies" on the consumers' web browsers, of all previous consumer/ad matches.

switchers. The prices that equate demand and supply for each type of consumer are equal to $\hat{p}_i = v_i$ and $\hat{p}_{sw} = v_{sw}$, respectively. Prices for loyals and switchers will be the same in the efficient matching case if publishers are symmetric (if $a_i = a_j$ then $\hat{p}_i = \hat{p}_j = \hat{p}_{sw}$), but if publishers are asymmetric ($a_i > a_j$), then $\hat{p}_i < \hat{p}_{sw} < \hat{p}_j$, since publisher j has a lower supply of ad space.

3 Switching and Market Outcomes

How does consumer switching shape equilibrium advertiser behavior and publisher profits? With fixed supply, the market equilibrium properties are basically inherited from the properties of the aggregate demand for impressions. In particular, a reduction in the aggregate demand for one publisher implies lower prices and profits for that publisher. Throughout Section 3, we restrict attention to the case where publisher readership is symmetric.

3.1 Impact on prices and profits

In equilibrium, the aggregate demand of publisher i equals:

$$1 - v_m + \tfrac{1}{2}(v_m - v_s) = 1 - \tfrac{1}{2}\left(p\tfrac{1}{1-\frac{1}{2}D^s} + p\right) \tag{3}$$

Thus, aggregate demand is falling in D^s. The arrow in Figure 1 depicts the effect of a marginal increase in D^s. As switching increases, the high types scale back on advertising and become low types due to the increased duplication on switchers. The following result directly follows from the observation that demand is monotone decreasing in D^s.

Proposition 1. *Under symmetry, equilibrium ad prices and publisher profits are decreasing in* D^s.

Note that setting demand equal to supply, $2a$, gives $\hat{p} = \frac{2(2-D^s)}{4-D^s}(1-2a)$ with profit $\hat{\pi} = \hat{p}a$. What can be said about the equilibrium allocation? Duplication reduces demand and, therefore, causes prices to drop below the level that would arise without switching. This means that some low-value advertisers that would not have had access to consumer attention in an efficient matching benchmark gains access in this case. So two kinds of

"mismatches" occur in equilibrium. The combination of inefficient waste (due to duplication) and inefficient use of attention (due to ad/consumer mismatches) is necessarily suboptimal from the perspective of the total advertiser and publisher welfare (and would also be suboptimal from a consumer perspective, had we spelled out a richer model where consumers receive surplus from matching with advertisers, or are annoyed by repetitious ads).

Switching is critical: when $D^s = 0$ equilibrium publisher profits are the same as those that would arise under efficient matching. In this regard, our model delivers at least two distinct predictions. First, *as the share of consumers who are switchers increases, the share of advertisers that single-home in equilibrium increases*. Second, it predicts that *those advertisers that place the lowest value on reaching consumers will be the single-homers*. Advertisers for which the value of reaching the marginal consumer is high will spread their advertising across many publishers.

In our setting, switching has real effects in that it *degrades* the value of the inventory. One way to understand this property of our model is to consider the advertisers' dual problem of minimizing the expenditure needed to guarantee a certain reach

$$\min_{n_1,n_2}(n_1+n_2)p \quad s.t. \quad \Phi(n_1,n_2) \geq \phi$$

where $\phi > \frac{1}{2}$. Switching degrades the inventory in the sense that a higher D^s raises the minimum number of impressions $n_1 + n_2$ required to get at least a reach of ϕ. This reduces the market price of ad impressions.

3.2 Endogenous advertising capacity

So far, we held advertising capacity fixed for each publisher. In this section, we relax this assumption by allowing publishers to choose their respective ad capacities, which is equivalent to allowing them to choose ad prices. We show that our main comparative statics generalize to this case. In addition, we can characterize the extent to which the payoff externalities due to switching reduce advertising prices beyond the effects already highlighted. However, we continue to abstract from another force

considered in the literature, which is that users find advertising annoying, so that competition for users may mitigate incentives to increase ad capacity.[16]

Formally, we study the strategic game in which the publishers choose simultaneously their capacity levels. Different publishers provide advertisers substitutable means to reach switching customers. We show that in equilibrium, substitutability creates upward pressure on advertising quantities (and corresponding downward pressure on prices) much as it does in ordinary oligopoly settings. The analysis is quite similar to that of a standard Cournot game, with the complication that publisher objective functions are non-concave, leading to best response functions that are non-monotone and discontinuous.[17]

Formally, we define firm i's best reply correspondence as $a_i^*(a_j) = \arg\max_{a_i} 2a_i\tilde{p}_i(a_i, a_j)(D^l + \frac{1}{2}D^s)$, where $\tilde{p}_i$ denotes the market clearing price as a function of the capacity choices.[18] Consider first the benchmark case $D^s = 0$. As each consumer is exclusive to one publisher, the problem reduces to a textbook monopoly problem. In the absence of switching, there are no externalities across publishers. Advertisers purchase i's impressions if and only if $v \geq p_i$. One can easily derive that $\tilde{p}_i(a_i, a_j) = 1 - 2a_i$. Then $a_i^*(a_{-i}) = \frac{1}{4}$, and the corresponding market clearing price is the monopoly price of $\frac{1}{2}$.

Suppose now that $D^s > 0$. Figure 2 depicts the qualitative shape of the best reply correspondence of publisher 1. The four regions labeled A to D correspond to the different competitive regimes that arise as a function of publisher 2's capacity. The non-monotonicity of the best reply correspondence shows that capacities are at first strategic complements and then strategic substitutes. Figure 3 illustrates how changing capacity affects advertiser sorting for different regions of (a_1, a_2). Consider first the case where a_2

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[16] This has been a particular focus of models of media economics. Anderson and Coate (2005) modeled viewership levels as endogenous to the level of advertising on a publisher as those ads created a nuisance for consumers. They demonstrate that when the number of publishers increase (i.e., there was more publisher competition), publishers would, in fact, reduce annoying advertising levels as a result of strengthened competition for consumers.

[17] A related paper in the two-sided matching literature is Azevedo (2014) that explores a Cournot like game when firms are being matched with workers.

[18] In Appendix Section 6.1, we derive the expression of $\tilde{p}_i$ for the more general asymmetric case.

is close to zero. Then publisher 1 can sell all of its impressions at the monopoly price as if publisher 2 were not active. Publisher 2's price is high at that small output, and it serves a small set of high-value, multi-homing advertisers. In the second case, suppose that a_2 is arbitrarily high. Then publisher 2 sets a very low price, serving all of the single-homing advertisers, and publisher 1's best response is then to select a low quantity, essentially the monopoly output for those advertisers who are already advertising on publisher 2. Notice that these multi-homing advertisers account for the inefficiency when choosing whether to also advertise on publisher 1. So the unit price is a fraction of the monopoly one: $\tilde{p}_1(a_1, a_2) = (1-2a_1)(1-\frac{1}{2}D^s)$. In both cases, the best reply quantity for firm 1 equals the monopoly level of ¼. In regions B and C, publisher 2, by increasing a_2, exerts a price externality on publisher 1. In region B, when publisher 2's capacity is moderately low, publisher 1's best response is to supply the lowest capacity that keeps the rival out of the market for single-homers (and, thus, advertiser behavior is as shown in Figure 3 region B). It basically behaves as a constrained monopolist. Eventually, as publisher 2's capacity increases (region C), publisher 1 "accommodates entry" in terms of competition for single-homers, and firms compete head-on for single-homers. We show that the unique symmetric Pure Strategy Nash Equilibrium (PSNE) lies in this region and is given by $a_1^* = a_2^* = \frac{1}{3}$. However, $D^s < \frac{4}{9}$, there is no PSNE.

Proposition 2: *In the game where both publishers simultaneously set their capacities and publishers are symmetric, a Pure Strategy Nash Equilibria exists and it is unique if and only if* $D^s \geq \frac{4}{9}$ *with* $a_1^* = a_2^* = \frac{1}{3}$. *In addition, equilibrium ad prices and publisher profits are decreasing with switching* (D^s).

The slope of the best reply function in the neighborhood of the PSNE reflects the basic Cournot intuition: ad capacities are strategic substitutes, and further ad capacity is over-provided relative to the monopoly level due to the price externality. The symmetric equilibrium exists so long as neither publisher has an incentive to deviate to a lower quantity and corner the market for multi-homers. The profitability of such a deviation rises with the willingness to pay of those multi-homing advertisers, which, in turn, increases as D^s (and thus inefficiency) decreases.

Since payoffs are jointly continuous in capacities, at least one symmetric mixed strategy equilibrium exists when $D^s < \frac{4}{9}$. In Appendix Section 6.4, we characterize a simple mixed strategy equilibrium in which both firms randomize over a support of cardinality 2. We show that such an equilibrium exists for $D^s \geq \frac{1}{9}$. Combining our findings about this mixed strategy equilibrium and our result about the unique PSNE, Figure 4 illustrates the equilibrium average capacity, price and profit as a function of D^s. In all of the equilibria shown, the average capacity is strictly larger than the monopoly one of ¼, and prices and profits are also strictly lower as well as decreasing in D^s. We leave a further analysis of more complex mixed strategy equilibria for future work.

In Appendix 6.3, we also consider PSNE for publishers with asymmetric readership. We find that the publisher with higher readership (and, thus, higher share of loyals) chooses lower advertising capacity and receives higher prices. Intuitively, the larger publisher internalizes to a larger extent the effect of increasing ad capacity on prices.

4 How Consumer Switching Affects Content Strategy

We now turn to consider the impact of consumer switching on other aspects of the news media. In this section, we examine the impact on content, looking first at "vertical" differentiation between publishers in terms of quality and then turning to the amount and type of content a publisher might choose to provide. To keep our analysis tractable, we maintain the assumption that advertising capacity is fixed throughout this section.

4.1 Investing in Readership

To shed light on how switching affects the returns to acquiring consumers and, therefore, the incentive to invest in "quality," suppose that one publisher is able to

generate a higher loyal readership share than the other[19] ($D_1^l < D_2^l$), but the publishers are otherwise symmetric.[20]

To begin, we argue that with asymmetric readership, prices must be different for the two publishers. Figure 5 illustrates how sorting would work with equal prices. Since publisher 2 is larger, if prices are equal across publishers, single-homing advertisers strictly prefer publisher 2, since it allows advertisers to impress (and thus gain ($v - p$) for) more unique consumers. Note that this argument relies on there being single-homing advertisers, which in turn requires $D^s > 0$. Formally:

Proposition 3. *Assume that outlet 1 has fewer loyal consumers than outlet 2* ($D_1^l < D_2^l$) *and ad capacities are equal* ($a_1 = a_2$). *Then equilibrium ad prices are lower for outlet 1 than for outlet 2* ($\hat{p}_1 < \hat{p}_2$) *if and only if there is a positive share of switchers* ($D^s > 0$). *If there are no switchers* ($D^s = 0$), *ad prices are the same across outlets* ($\hat{p}_1 = \hat{p}_2$).

Figure 6 shows the allocation of advertisers in equilibrium.

The fact that "more popular" publishers, in terms of readership share, command a premium for their ad space is a known puzzle in traditional media economics (Goettler, 2012). In a canonical model, consumers are equally valuable regardless of the publisher they are on, yet, in practice, advertising rates are typically higher on larger publishers. Here, because duplicated ad impressions are avoided within a publisher, placing ads on only the larger publisher involves less expected waste than when you place ads on the smaller publisher or spread them across publishers. So, the larger publisher can command a premium and this fact can, arguably, account for some of the observed wedge in impression prices between large and small publishers. It is useful to note that the price premium here is commanded over *all* advertising impressions.

This finding sheds light on how switching affects the payoff from using various strategies that may affect publisher size. Not only does attracting consumers increase the base upon which publishers can earn advertising revenue, it also increases their priority in

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[19] We use "quality" in the economic sense that it drives more consumer demand as opposed to some other criteria that might be applied to news content. It is well-established that changes in content strategy do affect the shares of newspaper readers (George and Waldfogel, 2006).

[20] If our model included more than two publishers or periods, we could model asymmetric shares of switchers, but with only two publishers and periods, switchers must divide their time evenly.

the advertising market. Thus, imperfect tracking makes competitive pressures for consumers somewhat more intense than traditional media economics would imply.[21]

Similarly, there has been much discussion regarding paywalls and how these interact with advertising businesses. Common wisdom is that paywalls are a substitute for advertising that might otherwise cause consumer annoyance (see Anderson and Coate, 2005; Casadesus-Masanell and Zhu, 2010). Here, however, a paywall would have an indirect effect through the advertising market. Regardless of how a paywall is actually structured,[22] we speculate that the effect would be to reduce the *relative* size of the publisher establishing it. Thus, in addition to losing consumers (and their associated advertising revenue), the paywall also reduces the publisher's priority in the advertising market.

4.2 Investing in Depth

Proposition 3 suggests that investing for large readership might be a desirable strategy. But what about the level of engagement for each reader? In this section, we build on this by assessing the returns to investing in sufficiently "deep" content to keep readers engaged for more time. For example, how do content strategies that capture the attention of a large audience for small amounts of time per user (such as BuzzFeed) compare to those that lead to loyal, deeply engaged readers (generally regarded as the strategies of established newspapers)? Is it worth investing for full attention?

Our baseline model assumes that both publishers have deep enough content to fully capture an individual's attention for 2 periods (that is, two pieces of content in our simple model). Here we consider what happens if depth (or *how much* content to provide) is endogenous. We assume that prior to consumers and advertisers making any choices, the

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[21] In contrast, Anderson et al. (2016) and Ambrus et al. (2016) build models where platforms cannot appropriate the rent associated with switching consumers. As a result, larger publishers command a per viewer premium, as they have more exclusive viewers. In our model, a bigger publisher (i.e., with more consumers) is valuable to advertisers regardless of the ratio of switchers to exclusive viewers. Moreover, in these alternative models, since switching consumers are both marginal to publishers and also do not attract advertising revenue, switching *decreases* the incentives to invest in quality and a larger readership.

[22] In our working paper (Athey, Calvano and Gans, 2013), we demonstrated this for *micropayments* (which reduced the overall number of consumers visiting a publisher), *subscriptions* (which reduced the incentive for would-be switchers to allocate partial attention to a publisher and so become exclusive to the other publisher) and a *limited paywall* (that only charged if a consumer wanted to allocate more than one unit of attention to a publisher and, therefore, induced them to become a switcher).

publishers simultaneously choose whether to have full (*f*) or shallow (*s*) content. On the supply side, choosing *s* over *f* entails an additional payoff of *c* interpreted as cost savings. However, on the demand side, a "shallow content publisher" can be allocated at most one unit of attention and, therefore, all their consumers are switchers.

Let d^l be a publisher's loyals and d^s be the number of switchers if both publishers provide full content. If a publisher should opt instead to provide only shallow content, they may not retain all of their original loyal consumers as some (captured here by a parameter $\gamma \in [0,1]$) may prefer depth over other attributes. In this case, the sole full publisher would have $d^l(1+\gamma)$ loyals while the shallow publisher would secure $d^l(1-\gamma)+d^s$ switchers over the two periods. Table 1 summarizes the share allocations for various content depth choices.

Table 1: Market Shares for full (*f*) and shallow (*s*) content

	D_1^l	D_2^l	D^s
(f,f)	d^l	d^l	d^s
(f,s)	$d^l(1+\gamma)$	0	$d^l(1-\gamma) + d^s$
(s,f)	0	$d^l(1+\gamma)$	$d^l(1-\gamma) + d^s$
(s,s)	0	0	1

Our goal is to characterize the PSNE of the game and study how outcomes are affected by *c*. Notice that all subgames stemming from the publishers' content are special instances of the asymmetric readership model developed above and in the Appendix. Profits are still given by (2), so that $\pi_i = p_i(2D_i^l + D^s)a_i$, where D_i^l, D^s and p_i are replaced by the entries in Table 1, and the market clearing prices correspond to the subgame considered.

Let $\hat{\pi}(\cdot,\cdot)$ and $\hat{p}(\cdot,\cdot)$ denote the equilibrium profits and prices given the own investment strategy (first argument) and the rival's (second argument). A key driver of the equilibrium outcome is a fundamental property of the game that we call (strong) strategic substitutability:

$$\hat{\pi}(f,s) - \hat{\pi}(s,s) > \hat{\pi}(f,f) - \hat{\pi}(s,f) \qquad \text{(SS)}$$

When (SS) is *not* satisfied, the returns from "catching up" with the rival's content exceed those of "moving ahead." This immediately implies that for all c, no equilibrium exists with asymmetric content strategies. In contrast, when (SS) is satisfied, there exist values of c where the equilibrium is asymmetric. To build intuition, we rewrite both sides of the (SS) condition as follows using Table 1:

$$d^l(1+\gamma)\hat{p}(f,s) + \hat{p}(f,s) - \hat{p}(s,s) > d^l(1+\gamma)\hat{p}(f,f) + (\hat{p}(f,f) - \hat{p}(s,f))(1 - d^l(1+\gamma))$$

Notice that providing a second unit of content pays a double dividend regardless of the rival's choice. First, the market price goes up due to the lower switching. Indeed Proposition 2 implies that $\hat{p}(f,\cdot) - \hat{p}(s,\cdot) > 0$. Second, the quantity of impressions served trivially goes up with the extra attention supplied by the acquired loyals. A key observation is that the first publisher to supply full content taps into the rival customer base, attracting all "would-be loyal" consumers, while the second (or follower) publisher investing merely retakes some of those consumers back. So the larger γ or the lower d^l, the more likely that the (SS) condition is satisfied. Indeed, it is possible to show that (SS) is satisfied if and only if $d^l > \frac{1-3\gamma}{2+2\gamma}$ (see the Appendix Section 6.5 for a proof).

The following result characterizes the equilibrium of the investment game:

Proposition 4. *(i) Suppose that (SS) holds. For all $d^s < 1$ there exists $\underline{c} < \overline{c}$ such that in equilibrium: For $c < \underline{c}$, both publishers choose to provide full content; for $c \in (\underline{c}, \overline{c})$ one publisher chooses full content while the other chooses shallow content and for $c > \overline{c}$ neither publisher chooses full content. Finally $\underline{c} \to \overline{c} \to 0$ as $d^s \to 1$.*

The final result in this proposition shows that increasing the share of switchers decreases the returns for one and eventually both publishers to provide full content. This happens regardless of whether (SS) holds or not. Intuitively, full content is most valuable when providing it builds up a loyal customer base. So with switching, publishers' premium for full content evaporates. Proposition 4 provides an interesting perspective on the way news and other content might be organized on the Internet. Because of the duplication issues associated with imperfect tracking, the relative returns to full versus shallow content are skewed towards shallow content.[23]

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[23] It is possible to consider more elaborate contracts that may allow advertisers to single-home on one publisher, when all consumers are switching, and still impress all consumers. In this situation, there will be

4.3 Competition Between a High-Reach Outlet and Multiple Publishers

Propositions 3 and 4 focus on the impact of consumer switching on content strategy for publishers. Here we consider the distinct question of the role of types of publishers whose main intention is to reach a large share of consumers for at least one attention period. We have in mind here firms such as Facebook that attract large reach without necessarily being a consumer's only news source (indeed, consumers often click on links and visit other news sites to read the stories). Does the alignment of such a publisher's content strategy with the interests of advertisers (who also want to reach consumers rather than engage them) give it a superior position in advertising markets?

To consider this, we add a third outlet to our model and suppose that a share, d^r (where r denotes "reach") devote one unit of their attention to a high-reach site (referred to as HR). We assume that $d^r < 1$, which rules out the possibility that high value advertisers can impress all consumers by simply advertising on HR, but that it is significant in that $d^r > \frac{1}{2}$. As above, we assume d^l is the share of consumers on a full content publisher who allocated both units to that publisher; that is, in the notation of Table 1, $D^l = (1-d^r)d^l$ while $(1-d^r)d^s$ switch between publishers 1 and 2 and $\frac{1}{2}d^r$ switch between any publisher and HR.

Table 2 shows the reach and number of impressions that advertisers gain using various allocation strategies.

Table 2: Advertiser Reach and Impressions with a High Reach Outlet

	Reach	Number of Impressions
Single-Home on HR	d^r	d^r
Multi-Home on 1 and 2	$d^r + (1-d^r)(2d^l + \frac{3}{4}d^s)$	1
Multi-Home on *i* and HR	$d^r + (1-d^r)(d^l + \frac{1}{2}d^s)$	$\frac{1}{2}(d^r + 1)$
Single-Home on *i*	$\frac{1}{2}d^r + (1-d^r)(d^l + \frac{1}{2}d^s)$	$\frac{1}{2}$

Importantly, single-homing on HR means that there are no wasted impressions, but $1-d^r$ consumers are missed. In addition, multi-homing on i and HR is dominated by

pressures towards unbundling of news content across publishers, with consumers choosing their own aggregate bundles and seeing distinct ads on each one.

multi-homing on both 1 and 2 when $d^r > \frac{1}{2}$. This is because, by multi-homing on both 1 and 2, the advertiser impresses all HR consumers in addition to those who do not visit HR. This means that switchers between HR and either publisher 1 or 2 do not affect advertiser demand (i.e., do not cause waste) and thus $\hat{p}_1 = \hat{p}_2 = \frac{2(2-d^s)}{4-d^s}(1-2a)$, as in our baseline model. As HR only attracts single-homing advertisers, $\hat{p}_r = 1-2a > \hat{p}_1 = \hat{p}_2$ so long as $d^s > 0$. This means that HR earns a higher price per impression than other publishers, and further that return is increasing in d^s. Thus, what gives rise to a distinct return to a "high reach specialist" is that consumers switch between regular publishers. A natural prediction from our model is that switching can increase the profitability of outlets, such as Facebook, who deliver large reach.

It is important to note, however, that HR profitability relies on it being of sufficient size. If $d^r < \frac{1}{2}$, multi-homing on HR and another publisher is no longer dominated and, for single homers, higher valued advertisers may select a publisher over HR. In this situation, we cannot demonstrate that HR commands a premium in advertising markets.

5 Conclusion / Managerial Implications

This paper analyzes the impact of the increased consumer switching (alongside imperfect tracking of these switchers) on advertising market prices as well as strategies for publishers. Summarizing, our model generates a number of empirical predictions:

1. Consumer switching results in an increased share of advertisers single-homing on individual publishers.
2. Consumer switching is associated with a fall in publisher advertising prices and profits, as well as decreased efficiency in allocating advertisements to users.
3. Consumer switching increases the competition among publishers in the advertising market, increasing advertising levels of publishers and (further) decreasing prices.
4. Publishers with higher readership shares attract higher per-consumer revenues.
5. Consumer switching increases a publisher's incentives to invest in quality content that attracts a greater share of consumers.
6. Consumer switching reduces a publisher's incentives to provide full content that can serve each consumer's full attention relative to offering shallower content that

engages users for short time periods; that is, reach becomes relatively more valuable than depth of readership.

7. "High-reach" publishers that engage a large share of the market for a short period of time gain priority over traditional publishers in the advertising market, and an increase in consumer switching among traditional publishers increases advertising prices on the high-reach publisher.

The model also helps interpret the recent history of advertising prices for news, and has further predictions about how news publishers might optimally respond to further changes, as consumers increase their use of social media and other intermediaries that promote switching across news publishers. Our model predicts that increased consumer switching is associated with inefficiency in advertising markets and a corresponding fall in advertising revenues. This is, indeed, something that we have seen with regard to news media.[24,25] To respond to these trends, our model suggests that media mergers may help publishers counter-act the effects of increased switching. If merged entities can pool information about users across websites, they may be able to gain priority in the advertising market and attain higher prices per user, while simultaneously improving efficiency in the allocation of advertisers to users.

More broadly, these results imply that strategic opportunities for publishers will depend on the factors that affect the costs of consumer switching, including the nature of tracking technologies (and regulations surrounding them), advertising platforms, and information sharing across publishers. Beyond mergers, publishers may be able to influence these factors, engage in information sharing, or increase their use of advertising platforms that help manage reach and frequency.

Our findings about publisher content strategy outlined in the previous section also have direct managerial implications: in a nutshell, consumer switching should lead

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[24] The decline in advertising revenue has been almost unanimously attributed to the rise of the Internet. However, many forces influencing supply and demand appear to be favorable for the industry. Online advertising created products and services that might be *more* valuable to advertisers (e.g., enhanced ads, targeting capabilities, improved measurement, search ads; see Evans (2008, 2009)). Chandra and Kaiser (2014) demonstrate that magazines that tailor content to specific consumer groups continue to command a premium in ad rates; the premium is positively associated with higher internet use of the consumer base.

[25] Another theme is that online or digital ads are less effective than ads that are on paper, but a variety of evidence is inconsistent with that hypothesis (see Dreze and Hussherr, 2003; Lewis and Reiley, 2014; Goldfarb and Tucker, 2011a, 2011b).

publishers to prioritize investing in reach, while publishers remain vulnerable to losing priority in the advertising market to "high-reach" publishers such as Facebook.

Our results have further implications for publisher strategy in response to competition from media that does not offer advertising (for instance, publicly-funded media or smaller non-profit publishers or blogs). Numerous studies have documented that for-profit media publishers object to public publishers being able to sell advertising.[26] This stance is seen as a puzzle in traditional media economics, since advertising usually annoys consumers and, hence, public publishers using advertising would allow traditional publishers to attract users away from public publishers. Suppose that, in our baseline model, one publisher eliminates advertising. This has two effects. First, while that publisher still captures (scarce) consumer attention, it decreases the effective supply of advertising capacity in the market. Second, unlike switchers between mainstream publishers, switchers between non-advertising and advertising publishers do not contribute to the wasted impressions problem. Consequently, the switch to eliminating advertising reduces duplication. This increases the demand for advertisements.[27] These two effects – a decrease in supply and an increase in demand – combine to raise equilibrium impression prices and profits for the advertising publisher. Thus, our model suggests that for-profit news publishers would indeed suffer in advertising markets from public news publishers using advertising.

Our modeling approach can be potentially enriched in a number of directions to offer insights into other aspects of online advertising markets. For example, throughout this paper, we assume that advertisements are equally effective on both publishers. We did not consider strategies such as tailored content that attracts a large share of a narrow audience that is attractive only to a subset of advertisers (e.g. local news versus national news, as in Athey and Gans (2010) and Bergemann and Bonatti (2011)). Combining considerations of tailoring, consumer switching, and advertising markets presents a fruitful area for future research.

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[26] For example, see Filistrucchi et al. (2011) for an empirical analysis of the French advertising ban on prime-time state television.

[27] A recent paper by de Corniere and Taylor (2014) develops this point. They show that if Google were to "divert" attention to websites with little or no ads, it could achieve higher ad-prices (due to scarcer supply) trading off search traffic (due to lower attractiveness).

6 Appendix

6.1 Existence and Uniqueness of Market Equilibrium

Proposition A1. *For all* $a_1, a_2 > 0$ *and* $D_1^l, D_2^l, D^s \geq 0$, *a market equilibrium exists.*

PROOF: Let $g(\mathbb{I}_1, \mathbb{I}_2; v, p_1, p_2) = v\Phi\left(\tilde{n}_1(\mathbb{I}_1), \tilde{n}_2(\mathbb{I}_2)\right) - \tilde{n}_1(\mathbb{I}_1)p_1 - \tilde{n}_2(\mathbb{I}_2)p_2$. The choice set is finite so the set of maximizers of g is non-empty. Define $Q_i(p_i, p_{-i})$ as the correspondence mapping price vectors to the aggregate demand of publisher i. $Q_i(p_i, p_{-i}) = \{q: \int_0^1 \tilde{\mathbb{I}}_i(v, p_i, p_{-i})dv = q$ for some selection $\tilde{\mathbb{I}}_i(v, p_1, p_2)$ from the set of maximizers of $g\}$. Note that there is an arbitrarily high real number, denoted $\bar{p}$, such that $Q_i(p_i, p_{-i}) = 0$ for all $p_i, p_{-i} \geq \bar{p}$. The following properties follow immediately from the properties of the solution of the advertiser program:

Q-1. Q_1 and Q_2 are well-defined and upper hemi-continuous correspondences for all positive p_1, p_2.

Q-2. (Monotonicity in own price.) If $p_i' > p_i$ and $Q_i(p_i, p_{-i}) \neq \{0\}$, then $q > q'\ \forall q \in Q_i(p_i, p_{-i}), q' \in Q_i(p'_i, p_{-i})$, i=1,2.

Q-3. (Weak monotonicity in rival's price.) If $p_{-i}' > p_{-i}$ then $q \geq q'\ \forall q \in Q_i(p_i, p_{-i}), q' \in Q_i(p'_i, p_{-i})$, i=1,2.

Q-4. (Monotonicity in both prices.) If $p_i' > p_i$ and $p_{-i}' > p_{-i}$ then the aggregate demand for at least one publisher should be strictly lower with (p'_i, p'_{-i}) than (p_i, p_{-i}).

Define the following auxiliary mapping $p_i^*: [0, \bar{p}] \times [0, \bar{a} = \frac{1}{2}] \to [0, \bar{p}]$ where $p_i^*(p_{-i}, a_i) = \{p_i: 2a_i \in Q_i(p_i, p_{-i})\}$. Note that this mapping is single-valued by Q-2 and weakly increasing by Q-3. By definition, a pair of equilibrium prices is equivalent to any point in the square $[0, \bar{p}] \times [0, \bar{p}]$ at which the two auxiliary functions intersect. Existence of such an intersection follows by applying Tarski's fixed point theorem to the auxiliary mapping (p_i^*, p_{-i}^*) from the compact set $[0, \bar{p}] \times [0, \bar{p}]$ into itself. In summary, for all $a_1, a_2 > 0$ there is a selection of the set of maximizers of g, $\tilde{\mathbb{I}}_i(v, p_i, p_{-i})$, and a pair of prices such that $\int_0^1 \tilde{\mathbb{I}}_i(v, p_i, p_{-i})dv = 2a_i$.

Proposition A2. *Suppose that* $\left(\left(\tilde{\mathbb{I}}_1(v), \tilde{\mathbb{I}}_2(v)\right)_v, \hat{p}_1, \hat{p}_2\right)$ is a market equilibria and $D_1^l \leq D_2^l$. Then

$$(\hat{p}_1, \hat{p}_2) = \begin{cases} \left((1-2a_1)\dfrac{D_1^l + \frac{1}{4}D^s}{D_1^l + \frac{1}{2}D^s}, 1-2a_2\right) & a_1 < a_2 - \dfrac{D^s(1-2a_2)}{8D_1^l + 2D^s} \\ \left(\dfrac{2(1-a_1-a_2)(4D_1^l + D^s)}{8D_1^l + 3D^s}, x\right) & a_2 - \dfrac{D^s(1-2a_2)}{8D_1^l + 2D^s} \leq a_1 \leq a_2 + \dfrac{D^s(1-2a_2)}{8D_1^l + 4D^s} \\ \left((1-2a_1), (1-2a_2)\dfrac{D_2^l + \frac{1}{4}D^s}{D_2^l + \frac{1}{2}D^s}\right) & a_1 > a_2 + \dfrac{D^s(1-2a_2)}{8D_1^l + 4D^s} \end{cases}$$

where x is a placeholder for $\frac{16D_1^l(a_2(D_1^l - 2D_2^l) + D_2^l) + 6(D_1^l + D_2^l - 2a_2D_2^l)D^s - 2(-1+a_2)(D^s)^2 - 2a_1(2D_1^l + D^s)(4D_1^l + D^s)}{(2D_2^l + D^s)(8D_1^l + 3D^s)}$.

PROOF: We start deriving basic properties of the equilibrium strategies $\left(\tilde{\mathbb{I}}_1(v), \tilde{\mathbb{I}}_2(v)\right)_v$:[28]

Property 1: *If* $\left(\tilde{\mathbb{I}}_1(v''), \tilde{\mathbb{I}}_2(v'')\right) = (0,0)$ *then for all* $v' < v''$, $\left(\tilde{\mathbb{I}}_1(v'), \tilde{\mathbb{I}}_2(v')\right) = (0,0)$

Proof (by contrapositive): If for some v' *we have* $\left(\tilde{\mathbb{I}}_1(v'), \tilde{\mathbb{I}}_2(v')\right) \neq (0,0)$ *then the corresponding payoff* $v' * \Phi - \hat{p}_1\tilde{n}_1 - \hat{p}_2\tilde{n}_2$ *must be weakly larger than 0. As the payoff function is linear in* v*, it follows that for all* $v'' > v'$*, the same strategy would lead to* $v'' * \Phi - \hat{p}_1\tilde{n}_1 - \hat{p}_2\tilde{n}_2 > 0$ *and so* $\left(\tilde{\mathbb{I}}_1(v''), \tilde{\mathbb{I}}_2(v'')\right) = (0,0)$ *would be suboptimal.*

Property 2: *If* $\left(\tilde{\mathbb{I}}_1(v'), \tilde{\mathbb{I}}_2(v')\right) = (1,1)$ *then for all* $v'' > v'$, $\left(\tilde{\mathbb{I}}_1(v''), \tilde{\mathbb{I}}_2(v'')\right) = (1,1)$

Proof: The premise of the statement says that:

$$v'\Phi(\tilde{n}_1(1), \tilde{n}_2(1)) - p_1\tilde{n}_1(1) - p_2\tilde{n}_2(1) \geq \max_{\mathbb{I}_1,\mathbb{I}_2} v'\Phi\left(\tilde{n}_1(\mathbb{I}_1), \tilde{n}_2(\mathbb{I}_2)\right) - \tilde{n}_1(\mathbb{I}_1)\hat{p}_1 - \tilde{n}_2(\mathbb{I}_2)\hat{p}_2.$$

Next observe that $\Phi(\tilde{n}_1(1), \tilde{n}_2(1)) > \Phi\left(\tilde{n}_1(\mathbb{I}_1), \tilde{n}_2(\mathbb{I}_2)\right)$ *for all strategies* $(\mathbb{I}_1, \mathbb{I}_2) \neq (1,1)$. *It follows that for all* $v'' > v'$,

$$v''\Phi\left(\tilde{n}_1(1), \tilde{n}_2(1)\right) - p_1\tilde{n}_1(1) - p_2\tilde{n}_2(1) > \max_{\mathbb{I}_1,\mathbb{I}_2} v''\Phi\left(\tilde{n}_1(\mathbb{I}_1), \tilde{n}_2(\mathbb{I}_2)\right) - \tilde{n}_1(\mathbb{I}_1)\hat{p}_1 - \tilde{n}_2(\mathbb{I}_2)\hat{p}_2.$$

Property 3: *If* $D_1^l < D_2^l$ *and* $\tilde{\mathbb{I}}_2(v') = 1$ *then for all* $v'' > v', \tilde{\mathbb{I}}_2(v'') = 1$.

Proof: Suppose $\tilde{\mathbb{I}}_2(v') = 1$ and *consider the equilibrium strategy at some* $v'' > v'$ which necessarily belongs to: $\left(\tilde{\mathbb{I}}_1(v''), \tilde{\mathbb{I}}_2(v'')\right) \in \{(0,0), (1,0), (0,1), (1,1)\}$. Property 1 implies $\left(\tilde{\mathbb{I}}_1(v''), \tilde{\mathbb{I}}_2(v'')\right) \neq (0,0)$. *We are left to show that* $\left(\tilde{\mathbb{I}}_1(v''), \tilde{\mathbb{I}}_2(v'')\right) \neq (1,0)$. *If* $\tilde{\mathbb{I}}_1(v') = 1$ *then* $\left(\tilde{\mathbb{I}}_1(v''), \tilde{\mathbb{I}}_2(v'')\right) = (1,1)$ *by property 2. If* $\tilde{\mathbb{I}}_1(v') = 0$ *then* $(v' - \hat{p}_2) * (D_2^l + 1/2D^s) \geq (v' - \hat{p}_1) * (D_1^l + 1/2D^s)$ *which together with the assumption* $D_1^l < D_2^l$ *implies that* $(v' - \hat{p}_2) * (D_2^l + 1/2D^s) > (v' - \hat{p}_1) * (D_1^l + 1/2D^s)$ *and hence that that* $\left(\mathbb{I}_1(v''), \mathbb{I}_2(v'')\right) = (1,0)$ *cannot be optimal.*

Let $v_{k,l}$ denote the advertiser indifferent between action k and action l, where the set of possible actions is denoted {12,1,2,0} (multi-homing, single-homing on publisher 1, single-homing on publisher 2, or doing nothing). Property 1, 2 and 3 jointly imply that advertisers' can sort according to their strategies in three possible ways:

- Case 1: types in $[v_{12,2}, 1]$ multi-home, types in $[v_{2,0}, v_{12,2})$ single home on 2, and all lower types do nothing.
- Case 2: types in $[v_{12,1}, 1]$ multi-home, types in $[v_{1,0}, v_{12,1})$ single home on 1 and all lower types do nothing.
- Case 3: types in $[v_{12,2}, 1]$ multi-home, types in $[v_{1,2}, v_{12,2})$ single home on 2, types in $[v_{0,1}, v_{1,2})$ single-home on 1 and all lower types do nothing.

Consider case 1. With these strategies, the aggregate demand for 1 and 2, respectively, under the uniform distribution assumption, are single-valued: $Q_1(p_1, p_2) = 1 - v_{12,2}$ and $Q_2(p_1, p_2) = 1 - v_{2,0}$. Imposing $Q_i(p_i, p_{-i}) = 2a_i$ for each i yields unique market-clearing prices $\hat{p}_1 = (1-2a_1)\frac{D_1^l + \frac{1}{4}D^s}{D_1^l + \frac{1}{2}D^s}$ and $\hat{p}_2 = 1 - 2a_2$.

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[28] We assume that in the case of indifference between advertising and not advertising agents break ties in favor of advertising. We also assume that if a set of measure zero agents are indifferent between only advertising in media 1 and only advertising in media 2, they break ties in favor of only advertising in 2. This assumption allows us to refer to $\left(\tilde{\mathbb{I}}_1(v), \tilde{\mathbb{I}}_2(v)\right)_v$ as functions (instead of correspondences) when $D_1^l > D_2^l$ and has no effect on the equilibrium because ties are a measure zero event.

Finally, these strategies are indeed optimal only if single-homing on 1 is dominated, that is if $v_{1,2} \leq v_{2,0}$ which yields:

$$a_1 \leq a_2 - \frac{D^s(1-2a_2)}{8D_1^l+2D^s}.$$

So as long as this inequality is satisfied this is an equilibrium. Next, consider case 2. Then the aggregate demands are single valued and given by $Q_1(p_1,p_2) = 1 - v_{0,1}$ and $Q_2(p_1,p_2) = 1 - v_{1,12}$. Imposing $Q_i(p_i,p_{-i}) = 2a_i$ for each i yields market-clearing prices $\hat{p}_1 = 1 - 2a_1$ and $\hat{p}_2 = (1-2a_2)\frac{D_2^l+\frac{1}{4}D^s}{D_2^l+\frac{1}{2}D^s}$. These strategies are indeed optimal if single-homing on 1 is preferred to single-homing on 2 by all types $v < v_{12,1}$ which leads to the condition $v_{12,1} \leq v_{1,2}$ which is equivalent to $a_1 \geq a_2 + \frac{D^s(1-2a_2)}{8D_1^l+4D^s}$.

Finally consider case 3. The aggregate demands are $Q_1(p_1,p_2) = 1 - v_{12,2} + v_{1,2} - v_{0,1}$ and $Q_2(p_1,p_2) = 1 - v_{1,2}$. Imposing $Q_i(p_i,p_{-i}) = 2a_i$ for each i yields market-clearing prices

$$\hat{p}_1 = \frac{2(1-a_1-a_2)(4D_1^l+D^s)}{8D_1^l+3D^s} \text{ and}$$

$$\hat{p}_2 = \frac{16D_1^l(a_2(D_1^l-2D_2^l)+D_2^l)+6(D_1^l+D_2^l-2a_2D_2^l)D^s-2(-1+a_2)(D^s)^2-2a_1(2D_1^l+D^s)(4D_1^l+D^s)}{(2D_2^l+D^s)(8D_1^l+3D^s)} \quad \text{(A2)}$$

These strategies are indeed optimal if $v_{2,0} \leq v_{1,2}$ (which guarantees that types in $[v_{0,1}, v_{1,2})$ prefer single-homing on 1 to single-homing on 2) and if $v_{1,2} \leq v_{12,1}$ (which guarantees that types in $[v_{0,1}, v_{1,2})$ prefer single-homing on 1 to multi-homing. Notice that $v_{1,2} \leq v_{12,2}$ is always verified so types in $(v_{1,2}, v_{12,2})$ strictly prefer single-homing on 2. Plugging the above prices into $v_{2,0} \leq v_{1,2} \leq v_{12,1}$ yields:

$$a_2 - \frac{D^s(1-2a_2)}{8D_1^l+2D^s} \leq a_1 \leq a_2 + \frac{D^s(1-2a_2)}{8D_1^l+4D^s} \qquad \text{(A3)}$$

So as long as this inequality is satisfied this is an equilibrium. Note that to each pair of quantities corresponds a unique pair of market clearing prices so the inverse demand is well defined if $D_1^l < D_2^l$.

Suppose $D_1^l = D_2^l = D^l$. Then all types who choose to single home in equilibrium either strictly prefer single-homing on 1, strictly prefer single-homing on 2 or are indifferent. This, together with properties 1 and 2 allows to restrict attention to the following candidate strategies. Case 1: types in $[v_{12,2}, 1]$ multi-home, types in $[v_{2,0}, v_{12,2})$ single home on 2, and all lower types do nothing. Case 2: types in $[v_{12,1}, 1]$ multi-home, types in $[v_{1,0}, v_{12,1})$ single home on 1 and all lower types do nothing. Case 3: types in $[v_{12,2} = v_{12,1}, 1]$ multi-home, types in $[v_{1,0} = v_{2,0}, v_{12,2})$ single home on some outlet and all lower types do nothing. Cases 1 and 2 play out exactly as above where $D_1^l < D_2^l$ and therefore lead to the same conditions. For case 3, let y denote the measure of advertisers with intermediate valuation who choose to single home on outlet 1. The aggregate demands are $Q_1(p_1,p_2) = 1 - v_{12,2} + y(v_{12,2} - v_{0,1})$ and $Q_2(p_1,p_2) = 1 - v_{12,2} + (1-y)(v_{12,2} - v_{0,1})$. Imposing $Q_i(p_i,p_{-i}) = 2a_i$ for each i, $0 \leq y \leq 1$ and $v_{0,1} = v_{0,2}$ gives prices $\hat{p}_1 = \hat{p}_2 = \frac{2(1-a_1-a_2)}{3+2D_1^l}(1+2D_1^l)$ and unique thresholds as in (A3). These strategies are indeed optimal.

6.2 Proof of Proposition 2

Suppose $D^s > 0$ and a PSNE (a^*, a^*) exists. By symmetry in a neighborhood of the equilibrium the inverse demand in Proposition A2 reduces to: $\tilde{p}_i(a_i, a^*) = 2(1-a_i-a^*)(2-D^s)(4-D^s)^{-1}$. The unique stationary point of profits $a_i\tilde{p}_i(a_i, a^*)$ in this neighborhood is $(1-a^*)/2$. By symmetry $(1-a^*)/2 = a^*$ which implies $a^* = \frac{1}{3}$. In summary, if a PSNE exists then $a_1^* = a_2^* = \frac{1}{3}$., with profits $\frac{2}{9}\frac{2-D^s}{4-D^s}$. It follows that an equilibrium exists if and only if $\frac{1}{3} \in \arg\max_{a_i} \tilde{p}_i\left(a_i, \frac{1}{3}\right) a_i$. So suppose that $a_{-i} = \frac{1}{3}$, and

consider i's best reply. All actions in $[a_{iL}, a_{iH}] \cap \frac{1}{3}$ (as defined in (A3) and (A1)) are strictly dominated by $\frac{1}{3}$, which is a feasible interior local maximum of the profit function over the interval $[a_{iL}, a_{iH}]$.

Now consider actions falling outside $[a_{iL}, a_{iH}]$. First, note that profits for $a_i \geq a_{iH}$ are always less than at $a_i = \frac{1}{3}$. In this region profits are $a_i(1 - 2a_i)$, which is decreasing. So the highest possible value takes place at a_{iH}, and it is easy to check that for $a_{iH}(1 - 2a_{iH}) \leq \frac{2}{9}\frac{2-D^s}{4-D^s}$.

For $a_i \leq a_{iL}$, we distinguish 2 cases. If $D^s \geq \frac{2}{3}$, then $a_{iL} \leq \frac{1}{4} < a_{iH}$. In this lower region profits are $a_i(1 - 2a_i)(1 - \frac{1}{2}D^s)$, which is maximized at $a_i = \frac{1}{4}$ at a value of $\frac{2-D^s}{16}$, which is less than $\frac{2}{9}\frac{2-D^s}{4-D^s}$ for $D^s \geq \frac{2}{3}$. Thus, all actions lower than $\frac{1}{3}$ are strictly dominated. If $D^s < \frac{2}{3}$, then $\frac{1}{4} < a_{iL}$. Since $a_i = \frac{1}{4}$ is a local maximum, then a necessary and sufficient condition for a PSNE to exist is that profits evaluated at $(a_i = \frac{1}{4}, a_{-i} = \frac{1}{3})$ do not exceed profits evaluated at $a_i = a_{-i} = \frac{1}{3}$. That is: $\frac{1}{2}(1-\frac{1}{2})\frac{D^l + \frac{1}{4}D^s}{D^l + \frac{1}{2}D^s} \leq \frac{4\,(2 - D^s)}{9\,(4 - D^s)}$ which is equivalent to $D^s \geq \frac{4}{9}$.

6.3 Endogenous capacity: PSNE with Asymmetric Publishers

Proposition A3. *In the game where both publishers simultaneously set their capacities, if a Pure Strategy Nash Equilibrium exists, then it is unique with:*

$$a_1^* = \frac{16(D_1^l)^2 + 3D_1^l(D^s - 8D_2^l) - D^s(9D_2^l + D^s)}{40(D_1^l)^2 + D_1^l(6D^s - 64D_2^l) - 3D^s(8D_2^l + D^s)} \quad \textit{and} \quad a_2^* = \frac{8(D_1^l)^2 - 16D_1^lD_2^l - D^s(6D_2^l + D^s)}{40(D_1^l)^2 + D_1^l(6D^s - 64D_2^l) - 3D^s(8D_2^l + D^s)}.$$

For the special case in which $D_1^l = D_2^l$, a Pure Strategy Equilibria exists if and only if $D^s \geq \frac{4}{9}$ and the expressions simplify to $a_1^* = a_2^* = \frac{1}{3}$. In addition equilibrium prices and profits decrease with D^s.

PROOF: Recall that (a_1^*, a_2^*) denotes a PSNE of the game. Best response behavior by publisher 1 requires that $a_1^* = \arg\max_{a_1} \hat{p}_1(a_1, a_2^*)2a_1$. Given the expression for the inverse demand derived in Appendix Section 6.1, publisher 1's profit as a function of a_1 is defined by three different sub-functions, each sub-function applying to three different subdomains, depending on whether $a_1 < a_{1L}(a_2^*)$, $a_{1L}(a_2^*) \leq a_1 \leq a_{1H}(a_2^*)$, or $a_1 > a_{1H}(a_2^*)$. It can be verified by differentiating each sub-function that each sub-function is strictly concave in a_i for all admissible values of D_1^l, D_2^l and D^s. Further, the objective function is continuous and piecewise differentiable for all admissible values of D_1^l, D_2^l and D^s. In the first part of the proof we show that $a_1^* \in (a_{1L}(a_2^*), a_{1H}(a_2^*))$ (and symmetric arguments imply that $a_2^* \in (a_{2L}(a_1^*), a_{2H}(a_1^*))$). This immediately implies that the global maximum of the profit function must be a solution to the usual system of first order conditions corresponding to the sub-function where $a_i \in [a_{iL}(a_j^*), a_{iH}(a_j^*)]$.

To show that $a_1^* \in (a_{1L}(a_2^*), a_{1H}(a_2^*))$, we proceed by contradiction. Suppose that $a_1^* > a_{1H}(a_2^*)$, which implies $a_2^* < a_{1L}(a_1^*)$. If $D^s > 0$, by definition of a_{1H} we get $a_1^* > a_2^*$. However, the first-order conditions of the sub-function that apply to publishers 1 and 2 when $a_1^* > a_{1H}(a_2^*)$ and $a_2^* < a_{1L}(a_1^*)$ respectively are necessary for an interior solution, and they imply that $a_1^* = \frac{1}{4}$ and $a_2^* = \frac{1}{4}$, a contradiction. The same contradiction arises if we posit $0 < a_1^* < a_{1L}(a_2^*)$. Suppose now that $a_1^* = a_{1L}$. To see that this can't be optimal for publisher 1, we use Lemma 1:

Lemma 1. *Let $f: A \to \mathbb{R}$ denote a real valued continuous function characterized by strictly concave, continuous and differentiable sub-functions $g: A \to \mathbb{R}$ and $h: A \to \mathbb{R}$ in the subdomains $x \leq \tilde{x}$ and $x > \tilde{x}$*

respectively. Let x^f, x^h and x^g denote the unique maximizers of f, h, and g, respectively, over A and suppose they all belong to the interior of A. If $x^h > x^g$ then $x^f \neq \tilde{x}$.

PROOF: Suppose that $x^f = \tilde{x} = x^g$. Then the left derivative of f at $\tilde{x}$ must be equal to zero and since $x^h > x^g$ by assumption then the right derivative must be strictly positive, a contradiction. Suppose now that $x^f = \tilde{x} = x^h$, then the right derivative of f at must be equal to zero while the left derivative must be positive, also a contradiction. Finally, suppose $x^f = \tilde{x}$ and $x^g, x^h \neq x^f$. Then the left and right derivative of f around $\tilde{x}$ must be strictly negative. This contradicts $x^h > x^g$. To see this, note that a negative left (right) derivative implies $x^g > \tilde{x}$ ($x^h < \tilde{x}$).

To verify the premises of Lemma 1, we shall show that the maximum over $(0,\frac{1}{2})$ of the two different sub-functions that apply when $a_1 < a_{1L}(a_2^*)$ and $a_{1L}(a_2^*) \leq a_1 \leq a_{1H}(a_2^*)$ are ordered. By solving the corresponding first order conditions, one finds that the stationary points are $\frac{1}{4}$ and $\frac{1-a_2^*}{2}$ respectively. Since $a_2^* < \frac{1}{2}$ we have $\frac{1}{4} < \frac{1-a_2^*}{2}$. So Lemma 1 applies and $a_1^* \neq a_{1L}$, a contradiction. Finally, suppose that $a_1^* = a_{1H}(a_2^*)$ which is equivalent to $a_2^* = a_{2L}(a_1)$. Publisher 2's stationary point for the sub-function that applies when $a_{2L}(a_1^*) \leq a_1 \leq a_{2H}(a_1^*)$ reaches a minimum over the simplex $\{D_1^l, D_2^l, D^s : D_1^l + D_2^l + D^s = 1\}$ when D^s is equal to $\frac{1-a_1^*}{2}$ for $D^s = 0$. Applying Lemma 1 to publisher 2's decision problem leads to a $a_2^* \neq a_{2L}$, a contradiction. We conclude that if a SPNE exists then $a_1^* \in (a_{1L}(a_2^*), a_{1H}(a_2^*))$, and (a_1^*, a_2^*) are the solution to the following system of equations:

$$\frac{\partial}{\partial a_1}(\hat{p}_1(a_1, a_2) \cdot 2a_1) = 0; \ \frac{\partial}{\partial a_2}(\hat{p}_2(a_1, a_2) \cdot 2a_2) = 0,$$

where $\hat{p}_1$, $\hat{p}_2$ in the above equation are given by the relevant subfunction (defined in equation (A2)). Solving the above system yields the closed form expressions given in the statement of the proposition.

6.4 Mixed strategy Equilibrium

We look for a symmetric Mixed Strategy Nash Equilibrium in which firms play only two actions with positive probability. Let $\overline{a}_i$ and $\underline{a}_i$ denote these actions, and let α and 1- α denote the probabilities with which they are used. Consider the following set of conditions, which (given that this is a two player game with continuous payoffs) are necessary for the actions and mixing probabilities to constitute a Mixed Strategy Nash Equilibria (MSNE): (i) $\overline{a}_i$ and $\underline{a}_i$ yield the same profit to publisher *i*, and (ii) $\overline{a}_i$ and $\underline{a}_i$ are each global optimizers of publisher *i*'s objective. If, in addition, we knew that any optimizer must be interior and in addition publisher *i*'s objective was everywhere differentiable, then we could replace condition (ii) with first-order necessary conditions for optimality. Instead, we proceed by characterizing the solution to first-order conditions for optimality as well as condition (i), and then we verify that the solution satisfies (ii). This approach yields a system of three polynomial equations in three unknowns:

$$\begin{cases} 2\overline{a}_i\left(\alpha\tilde{p}_i(\overline{a}_i,\overline{a}_j)+(1-\alpha)\tilde{p}_i(\overline{a}_i,\underline{a}_j)\right) = 2\underline{a}_i\left(\alpha\tilde{p}_i(\underline{a}_i,\overline{a}_j)+(1-\alpha)\tilde{p}_i(\underline{a}_i,\underline{a}_j)\right) \\ \dfrac{\partial 2a_i\left(\alpha\tilde{p}_i(a_i,\overline{a}_j)+(1-\alpha)\tilde{p}_i(a_i,\underline{a}_j)\right)}{\partial a_i} = 0 \ \text{ if } a_i \in \{\overline{a}_i, \underline{a}_i\} \\ \overline{a}_i = \overline{a}_j = \overline{a} \text{ and } \underline{a}_i = \underline{a}_j = \underline{a} \end{cases}$$

This system admits one solution that can be recovered in closed form:

$$\overline{a}^c = \frac{4 - D^s + D^s\alpha}{4(4 - D^s - \alpha + D^s\alpha)} \text{ and } \underline{a}^c = \frac{4 - 2D^s + D^s\alpha}{2(6 - 3D^s + 2\alpha + D^s\alpha)}$$

and α^c is the unique real solution to the polynomial equation $a + bx + cx^2 + dx^3 + ex^4 + fx^5 = 0$, where:

$$a = -512 + 2176D^s - 3104D^{s2} + 2104D^{s3} - 740D^{s4} + 130D^{s5} - 9D^{s6}$$
$$b = 1280 - 2688D^s + 3920D^{s2} - 3552D^{s3} + 1688D^{s4} - 384D^{s5} + 33D^{s6}$$
$$c = -768 + 2176D^s - 2112D^{s2} + 1872D^{s3} - 1256D^{s4} + 404D^{s5} - 46D^{s6}$$
$$d = 512 - 1280D^s + 1248D^{s2} - 544D^{s3} + 336D^{s4} - 176D^{s5} + 30D^{s6}$$
$$e = 256D^s - 400D^{s2} + 208D^{s3} - 36D^{s4} + 26D^{s5} - 9D^{s6}$$
$$f = 32D^{s2} - 32D^{s3} + 8D^{s4} + D^{s6}.$$

Now, we check whether there are other candidate optimizers of the publishers' objective functions that do not satisfy first-order conditions. Inspecting the objective function it is apparent that there is at least one other critical point that coincides with the monopoly solution of $a_i = \frac{1}{4}$. This observation allows us to derive a necessary condition on D^s for the candidate solution to be an equilibrium. The condition is that the objective function at $a_i = \frac{1}{4}$ is not larger than the value it achieves when computed at the candidate actions:

$$\tfrac{1}{2}\left(\alpha p_i(\tfrac{1}{4}, \overline{a}_j) + (1-\alpha)p_i(\tfrac{1}{4}, \underline{a}_j)\right) \le 2\underline{a}_i\left(\alpha p_i(\underline{a}_i, \overline{a}_j) + (1-\alpha)p_i(\underline{a}_i, \underline{a}_j)\right).$$

This inequality reduces to D^s being larger than the only real solution of the following polynomial equation:

$$-65536 + 622592x - 270336x^2 + 1871872x^3 - 6939904x^4 + 12041088x^5$$
$$-12957824x^6 + 9740800x^7 - 5412664x^8 + 2286836x^9 - 742206x^{10}$$
$$+184185x^{11} - 34184x^{12} + 4520x^{13} - 384x^{14} + 16x^{15} = 0$$

The above numerically approximates to 0.117796. When D^s is greater than this value, then the solution to the first-order conditions stated above yields higher profit to the publishers than the alternative strategy of ¼.

More generally, since the payoff function of publisher i is not smooth in own capacity, we cannot exclude the possibility that there are additional local maxima that may yield higher profits than our candidate strategies. To confirm that the candidate equilibrium is indeed an equilibrium we numerically calculated profits for a fine grid of values of the switching parameter D^s between the values of $\frac{1}{9} \approx 0.117796$ and $\frac{4}{9}$ and for a fine grid of advertising capacities. The numerical calculations confirm that for all values of D^s in this range the candidate solution is a mixed strategy equilibrium.

6.5 Proof of Proposition 4

We first show that (SS) holds if and only if $d^l > \frac{1-3\gamma}{2+2\gamma}$. Plugging the equilibrium profits and rearranging, the (SS) condition is equivalent to:

$$d^l(1+\gamma)\hat{p}(f,s) + \hat{p}(f,s) - \hat{p}(s,s) > d^l(1+\gamma)\hat{p}(f,f) + (\hat{p}(f,f) - \hat{p}(s,f))(1 - d^l(1+\gamma))$$

The previously derived closed form expressions for the market clearing prices, (proposition A2) allow us to recover the price function:

$$\hat{p}(f,s) = \tfrac{2(1+2d^l(1+\gamma))}{3(1+d^l(1+\gamma))}(1-2a),\ \hat{p}(f,f) = \tfrac{2(1+2d^l)}{3+2d^l}(1-2a),\ \hat{p}(s,f) = \hat{p}(s,s) = \tfrac{2}{3}(1-2a).$$

Plugging these into the (SS) condition and re-arranging yields

$$2(1-2a)d^l(3\gamma+2(\gamma+1)d^l-1)(9+6D^l)^{-1}>0,$$

Given that $a<\frac{1}{2}$ by assumption and since $0\le d^l\le\frac{1}{2}$ by definition, this in turn is equivalent to the following condition: $3\gamma+2(\gamma+1)d^l-1>0 \Leftrightarrow d^l>\frac{1-3\gamma}{2\gamma+2}$.

Let $\underline{c}=\{c\ge 0:\pi(f,f)-\pi(s,f)=c\}$ and $\overline{c}=\{c\ge 0:\pi(f,s)-\pi(s,s)=c\}$. If (SS) holds, then $\underline{c}<\overline{c}$. If $c>\overline{c}$, it is a dominant strategy not to invest, and the unique Nash equilibrium is (*s*, *s*). If $c<\underline{c}$, then it is a dominant strategy to invest and (f,f) is the unique equilibrium outcome. If $\underline{c}<c<\overline{c}$, only a pair of asymmetric pure strategy equilibria exists. That is, an equilibrium exists and is unique up to permutations of the indexes. When c is equal to one of the two thresholds, then, the symmetric and asymmetric equilibria coexist.

The last statement of the proposition follows from observing that with $d^s=1$, providing shallow content has no effect on prices. That is $\hat{p}(f,s)=\hat{p}(s,s)=\hat{p}(s,f)=\frac{2}{3}(1-2a)$ and, hence, $\hat{\pi}(f,s)-\hat{\pi}(s,s)=\hat{\pi}(f,f)-\hat{\pi}(s,f)=0$, which implies $\underline{c}=\overline{c}=0$.

Figure 1: Sorting with Symmetric Publishers

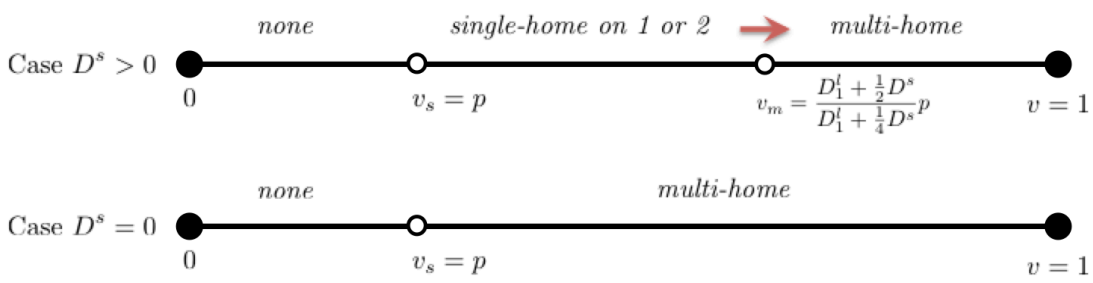


Figure 2: Best Reply of Publisher 1

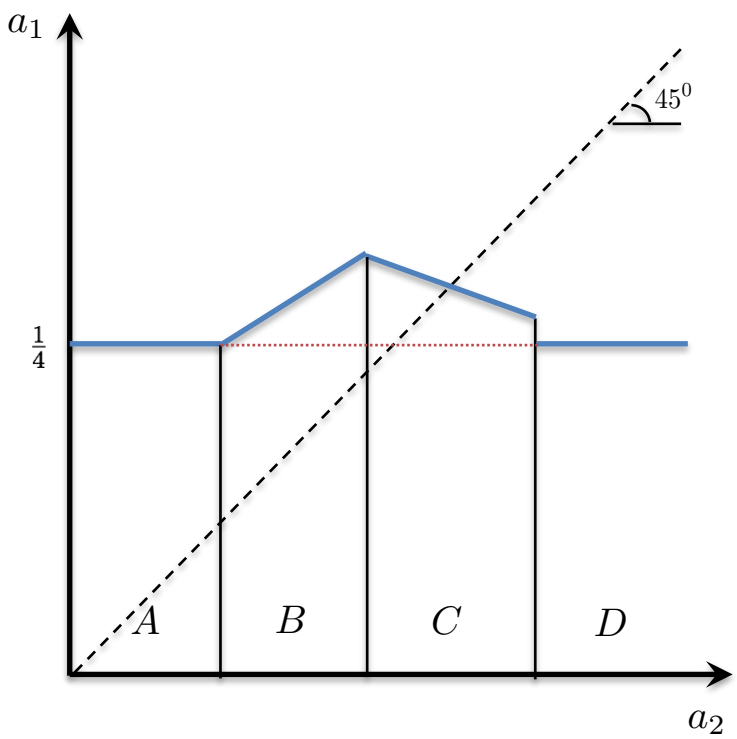


Figure 3: Sorting with Endogenous Capacity

$v_{k,l}$ denotes the advertiser indifferent between action k and action l where $k,l \in \{12,1,2,0\}$.

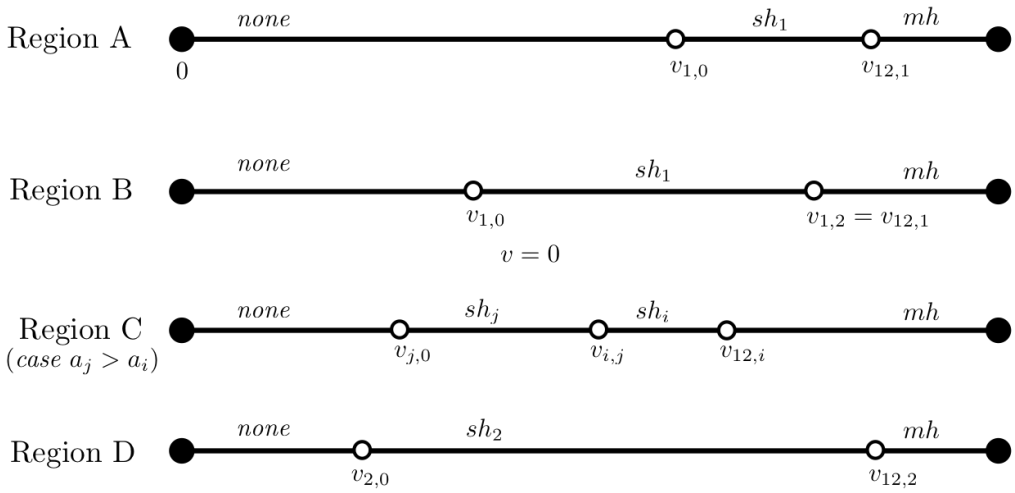


Figure 4: Impact of Switching with Endogenous Capacity

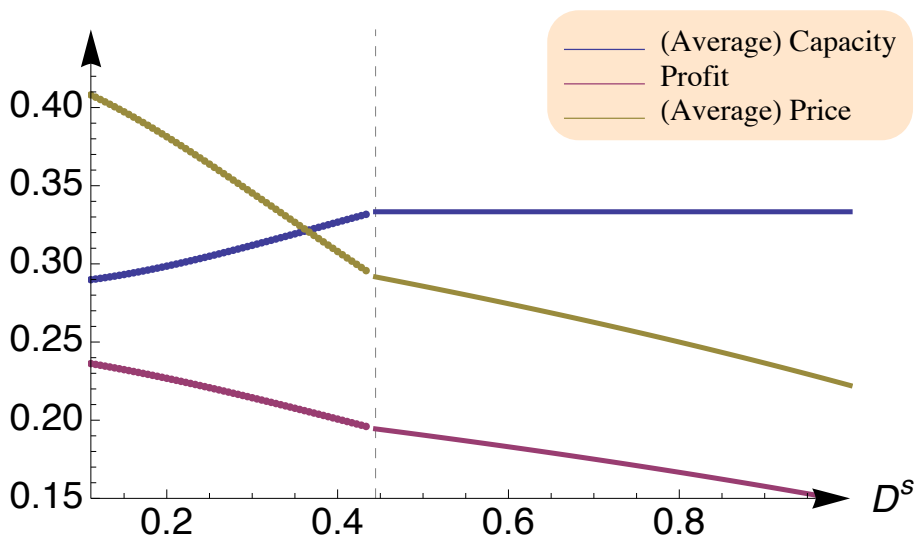


Figure 5: Sorting with Asymmetric Readership and Single Price

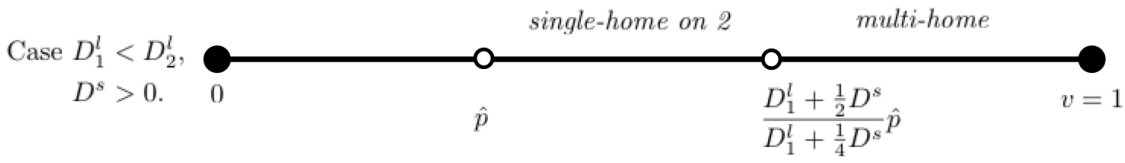
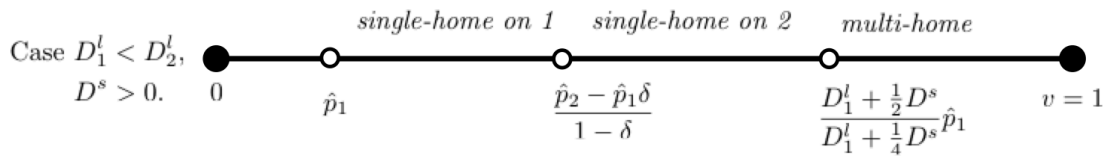


Figure 6: Sorting with Asymmetric Readership and Differential Prices



$\delta := \frac{D_1^l + \frac{1}{2}D^s}{D_2^l + \frac{1}{2}D^s}$; measure of relative size

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