Bootstrap lag selection in DSGE models with expectations correction

Published Version:
Angelini, G. (2020). Bootstrap lag selection in DSGE models with expectations correction. ECONOMETRICS AND STATISTICS, 14, 38-48 [10.1016/j.ecosta.2017.09.002].

Availability:
This version is available at: https://hdl.handle.net/11585/631687 since: 2023-03-10

Published:
DOI: http://doi.org/10.1016/j.ecosta.2017.09.002

Angelini, G. (2020). Bootstrap lag selection in DSGE models with expectations correction. Econometrics and Statistics, 14, 38-48.

The final published version is available online at:

https://doi.org/10.1016/j.ecosta.2017.09.002

Bootstrap Lag Selection in DSGE Models with Expectations Correction

Giovanni Angelini*

School of Economics Management and Statistics, University of Bologna, Italy

Abstract

A well known feature of DSGE models is that their dynamic structure is generally not consistent with agents' forecasts when the latter are computed from 'unrestricted' models. The expectations correction approach tries to combine the structural form of DSGE models with the best fitting statistical model for the data, taken the lag structure from dynamically more involved state space models. In doing so, the selection of the lag structure of the state space specification is of key importance in this framework. The problem of lag selection in state space models is quite an open issue and bootstrap techniques are shown to be very useful in small samples. To evaluate the empirical performances of our approach, a Monte Carlo simulation study and an empirical illustration based on U.S. quarterly data are provided.

Keywords: Dynamic stochastic general equilibrium model, Expectations correction, Dynamic misspecification, Bootstrap, Model selection

*Corresponding author.

E-mail address: giovanni.angelini3@unibo.it (G. Angelini).

1. Introduction

In recent years, there has been a growing interest on the part of central banks and academia in the use of Dynamic Stochastic General Equilibrium (DSGE) models in policy analysis and forecasting. Henry and Pagan (2004) and An and Schorfheide (2007), among many others, remark that these models have a limited time series performance and are able to capture only stylized aspects of the business cycle. As is known log-linearized DSGE models can be represented in the ABCD form:

$$x_t = A(\theta) x_{t-1} + B(\theta) \varepsilon_t, \quad (1)$$
$$y_t = C(\theta) x_{t-1} + D(\theta) \varepsilon_t, \quad (2)$$

where x_t is the vector of state variables, y_t is the vector of observable variables and ε_t is the vector of structural shocks. The matrices $A(\theta), B(\theta), C(\theta)$ and $D(\theta)$ are of comparable dimensions and are functions of the structural parameters collected in the vector θ. In general, the state space model in (1)-(2) will give rise to VARMA-type representations of y_t, see, e.g. Hannan and Deistler (1988) and more recently Morris (2016,2017). Instead, for the analysis of the cases in which y_t can be given a fundamental and finite-order VAR representation when the $D(\theta)$ matrix in (2) is square, we refer to Fernández-Villaverde et al. (2007), Ravenna (2007), Franchi and Vidotto (2013) and Franchi and Paruolo (2015). The approach we propose in this paper does not need a finite-order VAR representation and it can be applied also to the DSGE models with a VARMA-type representation. More generally, From the ABCD representation in (1)-(2), the maximum likelihood estimation of θ is based on the innovation form:

$$x_{t+1|t} = A(\theta) x_{t|t-1} + K_t(\theta) u_t,$$
$$y_t = H x_{t|t-1} + u_t,$$

where $H(\theta) = C(\theta) A(\theta)^{-1}$, $K_t(\theta)$ is the Kalman gain and the innovations u_t are assumed to be Gaussian. What typically happens in practice is that, given the maximum likelihood Kalman filter based estimate of θ, $\hat{\theta}$, the innovation residuals $\hat{u}_t = y_t - H(\hat{\theta})\hat{x}_{t|t-1}$ are found to be autocorrelated. As an example, Figure 1 reports the autocorrelation functions of the innovation terms $\hat{u}_t$ obtained from the estimation of the DSGE model of An and Schofrfheide (2007). The significant autocorrelations one finds in the innovation residuals imply that the estimated DSGE model is not able to fully capture the actual dynamic structure present in the data. This issue of omitted dynamics can be a possible cause of rejection of the theoretical model when it is compared with an 'unrestricted' state space model. DSGE models imply a set of nonlinear restrictions on the state space representation they generate, denoted Cross Equation Restrictions (CER), which are the natural metric through which these models should be empirically evaluated (Hansen, 2004; Hansen and Sargent, 1980; 1981). The CER can be divided into two different groups: (i) nonlinear cross restrictions, which map the structural parameters into the reduced form parameters; (ii) restrictions on the lag structure of the variables.

This problem is well known in the literature and different solutions have been recently proposed. Lubik and Schorfheide (2004) suggest specifying a theoretically micro-founded model with less restrictive dynamic, which possibly captures all frictions. This adjustment, however, is not always possible. Curdia and Reis (2010) and Smets and Wouters (2007) suggest using a richer dynamic specification, like ARMA-type models, for the disturbances of the DSGE model. Another solution is the DSGE-VAR approach introduced by Del Negro et al. (2007), which attempts to fill the gap between theory and the data by specifying a Bayesian Vector AutoRegression (BVAR) model whose priors are centred on the estimated DSGE model. The idea of the authors is to evaluate whether the posterior estimates of the BVAR are far from the priors. Consolo, Favero and Paccagnini (2009) extend the idea of Del Negro et al. (2007) by introducing factor analysis and proposing the DSGE-FAVAR model. Another solution recently proposed in the literature is the Expectations Correction approach [DSGE_{ExC}, hereafter] proposed by Angelini and Fanelli (2016). The aim of DSGE_{ExC} is to connect the theoretical model with the information provided by the data. In the DSGE_{ExC}, the dynamics of the best fitting statistical model for the data, which is a state space model, is used to define a 'pseudo structural' form which has the same dynamic structure as the agents' forecasting model. The main idea of the DSGE_{ExC} is to derive a data-driven process to determine the lag structure of the DSGE model without assuming an ARMA-type structure for the disturbances.

In this paper we propose an exhaustive analysis of the empirical performance of the DSGE_{ExC} approach by focusing our attention on the model selection issue when the reduced form solution of the DSGE is a state space model which involves more lags than suggested by the theoretical model. The model selection issue has been much analysed in the literature and many information criteria have been proposed over the last years, after the ground-breaking works of Akaike (1973; 1974). The Akaike Information Criterion (AIC) is probably the most widely used tool for time series model selection, even in the context of state space models and even though many competitors are nowadays available. Among these criteria we have the Hannan and Quinn Information Criterion (HQC, Hannan and Quinn, 1979) and the Bayesian Information Criterion (BIC, Schwarz, 1978). Exploiting the work of Stoffer and Wall (1991), who first applied bootstrap techniques to state space models, a couple of papers have addressed the issue of model selection in state space models using bootstrap techniques. Among these papers we underline the work of Cavanaugh and Shumway (1997), who derive a completely new bootstrap-based AIC criterion called AICb.

This paper is organized as follows. Section 2.1 introduces the main ideas by describing the Expectations Correction approach for a univariate example. Section 2.2 generalises this idea to the case of DSGE models. Section 3 proposes the bootstrap algorithm to specify the DSGE_{ExC} model. Section 4 proposes a Monte Carlo simulation study based on an univariate case and Section 5 illustrates how our approach works for the DSGE model of An and Schorfheide (2007) based on U.S. quarterly data. Section 6 concludes the paper.

2. Dynamic Misspecification and Expectations Correction

2.1. Univariate example

In this section we discuss a simple example which introduces the concept of expectations correction. Assuming that the economy is described by the univariate structural model:

$$z_t = \gamma_f E_t z_{t+1} + \gamma_b z_{t-1} + \omega_t, \quad \omega_t \sim WN(0,1), \tag{3}$$

where γ_f and γ_b are the structural parameters, z_t is a scalar unobservable variable, $E_t z_{t+1} = E(z_{t+1} \mid \mathcal{F}_t)$ is the expectation operator conditional on the information set $\mathcal{F}_t$ and ω_t is a white noise process with variance 1. Given the condition $\gamma_f + \gamma_b < 1$, the unique stable rational expectations solution of the structural model in (3) is given by:

$$z_t = \tilde{a} z_{t-1} + \tilde{g} \omega_t, \tag{4}$$

where $\tilde{a} = a(\theta)$ and $\tilde{g} = g(\theta)$ are the reduced form parameters which depend nonlinearly on the vector of structural parameter $\theta = (\gamma_f, \gamma_b)'$. In this setup, the mapping between the parameters of model (4) and the structural parameters is given by:

$$\begin{aligned} \gamma_f \tilde{a}^2 - \tilde{a} + \gamma_b &= 0, \\ (1 - \gamma_f \tilde{a})^{-1} &= \tilde{g}, \end{aligned} \tag{5}$$

where $\tilde{a}$ is the real stable solution (i.e. $\tilde{a} \in (0, 1)$) of (5). To make this simple example similar to a real case, it is assumed that z_t is not directly observable, so we need the following measurement equation:

$$y_t = z_t + \upsilon_t, \quad \upsilon_t \sim WN(0, 1). \tag{6}$$

where y_t is an observable scalar variable and the measurement error υ_t is a white noise process with variance 1 and independent of ω_t. The system given by (4) and (6) is a state space model which can be easily cast into ABCD form as in (1)-(2) with $x_t = z_t$, $A(\theta) = \tilde{a}$, $B(\theta) = (\tilde{g}, 0)$, $C(\theta) = \tilde{a}$, $D(\theta) = (\tilde{g}, 1)$ and $\varepsilon_t = (\omega_t, \upsilon_t)'$. The problem of dynamic misspecification occurs when the actual autocorrelation structure of $y_1, y_2, ..., y_T$ is not consistent with that implied in system (4) and (6), i.e. when the number of lags in the state equation (4) is insufficient for capturing the whole dynamics present in the data. Assume that the autocorrelation structure of the data is well described by the following state space model:

$$z_t = a_1 z_{t-1} + a_2 z_{t-1} + \epsilon_t, \quad \epsilon_t \sim WN(0, \sigma_\epsilon^2), \tag{7}$$

$$y_t = z_t + \upsilon_t, \quad \upsilon_t \sim WN(0, 1), \tag{8}$$

where the coefficient related to the second lag $a_2 \neq 0$ and ϵ_t is a white noise process with variance σ_ϵ^2. The state space model in (7)-(8) can be cast into ABCD form as in (1)-(2) by putting:

$$x_t = \begin{pmatrix} z_t \\ z_{t-1} \end{pmatrix}, \quad A = \begin{pmatrix} a_1 & a_2 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

$$\varepsilon_t = \begin{pmatrix} \epsilon_t \\ \upsilon_t \end{pmatrix}, \quad C = \begin{pmatrix} a_1 & a_2 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 1 \end{pmatrix}.$$

Compared to the reduced form in (4) and (6), the statistical model in (7)-(8) involves an additional lag of the state variable z_t. The state space model in (7)-(8) can be considered as the reduced form of the structural model in (3) if the following restrictions holds:

$$\begin{aligned} a_1 &= \tilde{a}, \qquad (9) \\ \sigma_\epsilon^2 &= \tilde{g}^2, \\ a_2 &= 0. \qquad (10) \end{aligned}$$

It is now clear that the restriction in (10) conflicts with the autocorrelation structure of the data captured by the system (7)-(8) based on $a_2 \neq 0$. In a situation like this, if the data generating process belongs to the class of models in (7)-(8), and the econometrician estimates the structural model in (3), the estimator of the structural parameters θ is distorted because of the omission of an important regressor (the second lag of the state variable z_t). The zero restrictions in (10) are implicit, and very often, practitioners are not aware of their role and importance in the empirical performance of NK-DSGE models. Indeed the zero restrictions in (10) can be a possible cause of the statistical rejection of the DSGE model. With our approach, relaxing these restrictions, it is possible to reduce the empirical rejection of the theoretical model. Testing the validity of an NK-DSGE when the restrictions in (10) conflict with the actual autocorrelation structure of the data might distort the overall evaluation process.

The main method adopted in the literature to fix this problem is to consider a different process for the error term ω_t in (3), with a richer dynamic structure. For instance, if we assume that:

$$\omega_t = \rho\omega_{t-1} + \zeta_t, \quad \zeta_t \sim WN(0,1),$$

we implicitly augment the dynamics of the reduced form solution (Crudia and Reis, 2010; Smets and Wouters, 2007). Alternatively, Zanetti (2008) suggests adding measurement errors to (6) in order to capture all the comovements in the data. The expectations correction approach starts from a different point of view. We assume that the state space model in (7)-(8) corresponds to the agents' forecast model. Starting from this assumption, from the original structural model in (3), we define a 'pseudo structural' form which amends the theoretical model with the 'correct' number of lags, according to the agents' forecast model. This leads to the following 'pseudo structural' model:

$$z_t = \gamma_f E_t z_{t+1} + \gamma_b z_{t-1} + \gamma_{bb} z_{t-2} + \omega_t^*, \quad \omega_t^* \sim WN(0,1), \qquad (11)$$

where γ_{bb} is an auxiliary parameter whose role is to reconcile the agents' expectations with the theoretical model. The additional term $\gamma_{bb}z_{t-2}$ can be interpreted as the expectations correction term and its role is to guarantee that the reduced form solution associated with the 'pseudo structural' model in (11) has the same representation as the agents' forecast model in (7)-(8). Indeed, the reduced form solution of the 'pseudo structural' model in (11) is given by the state space model:

$$z_t = \tilde{a}_1 z_{t-1} + \tilde{a}_2 z_{t-2} + \epsilon_t^*, \quad \epsilon_t^* \sim WN(0, \sigma^2_{\epsilon_t^*}), \tag{12}$$

$$y_t = z_t + \upsilon_t, \quad \upsilon_t \sim WN(0, 1), \tag{13}$$

where the reduced form parameters are connected to $\theta^* = (\gamma_g, \gamma_b, \gamma_{bb})'$ through the following set of restrictions:

$$(1 - \gamma_f \tilde{a}_1)\tilde{a}_1 = (\gamma_f \tilde{a}_2 + \gamma_b), \tag{14}$$

$$(1 - \gamma_f \tilde{a}_1)\tilde{a}_2 = \gamma_{bb},$$

$$(1 - \gamma_f \tilde{a}_1)^{-2} = \sigma^2_{\epsilon_t^*}. \tag{15}$$

Comparing this set of restrictions to that in (9)-(10), we can observe that no zero restrictions that reduce the lag order arise. In this way we have amended the dynamic structure of the system without changing the autocorrelation structure of the disturbances.

2.2. Expectations Correction in DSGE models

In this section we generalize the expectations correction idea to the case of DSGE models. Let $Z_t = (Z_{1,t}, Z_{2,t}, \cdots, Z_{n,t})'$ be an $n_z \times 1$ vector of endogenous variables. Assume the economy is described by the following structural model:

$$\Gamma_0 Z_t = \Gamma_f E_t Z_{t+1} + \Gamma_b Z_{t-1} + \Pi\omega_t, \tag{16}$$

where $\Gamma_i = \Gamma_i(\theta)$, $i \in \{0, b, f\}$ are $n_z \times n_z$ and Π is a $n_z \times n_\omega$, whose elements depend on the $n_\theta \times 1$ vector of structural parameters θ, ω_t is a $n_\omega \times 1$ vector of disturbances. The matrix Γ_0 is assumed to be non-singular, while Γ_f and Γ_b can be singular and Γ_b possibly zero. As common in the literature, we complete the structural system specifying an autoregressive process for the disturbances ω_t in (16):

$$\omega_t = R\omega_{t-1} + \epsilon_t, \tag{17}$$

where R is a $n_\omega \times n_\omega$ diagonal stable matrix (i.e. with its eigenvalues inside the unit disk), and ϵ_t is a fundamental white noise term with covariance matrix Σ_ϵ.

Assuming that there exist an unique stable solution of the system, one way to express the reduced form solution associated with the system (16)-(17) is:

$$\begin{pmatrix} Z_t \\ Z_{t-1} \end{pmatrix} = \begin{pmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 \\ I_{n_z} & 0_{n_z \times n_z} \end{pmatrix} \begin{pmatrix} Z_{t-1} \\ Z_{t-2} \end{pmatrix} + \begin{pmatrix} \tilde{\Psi} \\ 0_{n_z \times n_z} \end{pmatrix} \epsilon_t, \tag{18}$$

where $\tilde{\Phi}_1 = \Phi_1(\theta)$, $\tilde{\Phi}_2 = \Phi_2(\theta)$ and $\tilde{\Psi} = \Psi(\theta)$ depend nonlinearly on θ through the CER:

$$\begin{aligned} \left(\Gamma_0 + \check{\Pi}\Gamma_f\right)\Phi_1 - \Gamma_f\left(\tilde{\Phi}_1^2 + \tilde{\Phi}_2\right) + \check{\Gamma}_b &= 0_{n_z \times n_z}, \\ \left(\Gamma_0 + \check{\Pi}\Gamma_f\right)\tilde{\Phi}_2 - \Gamma_f\left(\tilde{\Phi}_1\tilde{\Phi}_2\right) - \check{\Pi}\Gamma_b &= 0_{n_z \times n_z}, \\ \left(\check{\Pi}^{-1}\left(\Gamma_0 - \Gamma_f\tilde{\Phi}_1\right)\right)^{-1} &= \tilde{\Psi}, \end{aligned}$$

where $\check{\Pi} = \Pi^{-1}R\Pi$ and $\check{\Gamma}_b = \Gamma_b + \check{\Pi}\Gamma_0$. It can be proved (Castelnuovo and Fanelli, 2015) that the stability of the matrix $W(\theta) = (\Gamma_0 + \check{\Pi}\Gamma_f - \Gamma_f\tilde{\Theta}_1)^{-1}\Gamma_f$ is sufficient for uniqueness (determinacy). The solution is not unique (i.e. there are multiple stable solutions) if $A(\theta)$ has eigenvalues inside the unit disk but the matrix $W(\theta)$ has eigenvalues outside the unit disk, see Binder and Pesaran (1995), section 2.3. For more details about the derivation of the CER, see Bårdsen and Fanelli (2015) and Castelnuovo and Fanelli (2015). Assuming that the vector of endogenous variables Z_t is not completely observable, the measurement system which connects Z_t to the observable variables $y_t = (y_{1,t}, y_{2,t}, ..., y_{n_y,t})'$, can be written as:

$$y_t = Hx_t + V\upsilon_t, \tag{19}$$

where the $n_y \times 2n_z$ matrix H and the $n_y \times n_\upsilon$ matrix V are selection matrices and the $n_\upsilon \times 1$ vector υ_t, $n_\upsilon \leq n_y$, is the measurement error with covariance matrix Σ_υ. The link between (18)-(19) and the state space model in (1)-(2)

is straightforward:

$$x_t = \begin{pmatrix} Z_t \\ Z_{t-1} \end{pmatrix}, \quad A(\theta) = \begin{pmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 \\ I_{n_z} & 0_{n_z \times n_z} \end{pmatrix}, \tag{20}$$

$$B(\theta) = \begin{pmatrix} \tilde{\Psi} & 0_{n_z \times n_v} \\ 0_{n_z \times n_z} & 0_{n_z \times n_v} \end{pmatrix}, \quad C(\theta) = HA(\theta),$$

$$\varepsilon_t = \begin{pmatrix} \epsilon_t \\ \upsilon_t \end{pmatrix}, \quad D(\theta) = \begin{pmatrix} HB(\theta) & V \end{pmatrix}. \tag{21}$$

The ABCD solution described by the matrices in (20)-(21) is based on two lags of the endogenous variables. This can be inconsistent with the dynamics that characterise the observable data. As we have already stressed in the univariate example, the problem of omitted dynamics can affect the inference. Assume that the number of lags necessary to catch the actual dynamics in the observable data is $k^{op} > 2$, which means that the agents' forecast model belongs to the class of state space models with k^{op} lags. In this case the original structural model in (16)-(17) is too simple to describe the dynamics of the economy. Hence, to fully capture the actual dynamics, we construct a 'pseudo structural' model defined by:

$$\Gamma_0 Z_t = \Gamma_f E_t Z_{t+1} + (\Gamma_b + \dot{\Gamma}_b) Z_{t-1} + \sum_{j=2}^{k^{op}-1} \Upsilon_j Z_{t-j} \mathbb{I}_{\{k^{op} \geq 3\}} + \Pi \epsilon_t, \tag{22}$$

$$\epsilon_t = R\epsilon_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN\left(0, \Sigma_\varepsilon\right), \tag{23}$$

where $\mathbb{I}_{\{.\}}$ is the indicator function and the $n_z \times n_z$ matrices $\Upsilon j, j = 2, ..., k^{op} - 1$ contain the expectations correction terms. Note that the $n_z \times n_z$ matrix $\dot{\Gamma}_b$ is another expectations correction matrix which contains parameters associated with the first lag of Z_t which are missing in the original structural model. The vector of the parameters associated to the 'pseudo structural' model in (22)-(23) is $\theta^* = (\theta', \zeta')'$, where ζ' is the vector containing the expectations correction parameters present in the matrices $\Upsilon j, j = 2, ..., k^{op} - 1$ and $\dot{\Gamma}_b$. Together with the measurement error in (19), the solution of the 'pseudo structural' model in (22)-(23) is a state space, which can be cast into ABCD form:

$$x_t = \begin{pmatrix} Z_t \\ Z_{t-1} \\ \vdots \\ Z_{t-k^{op}+1} \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} \epsilon_t \\ \upsilon_t \end{pmatrix},$$

$$A(\theta) = \begin{pmatrix} \tilde{\Phi}_1 & \dots & \tilde{\Phi}_{k^{op}-1} & \tilde{\Phi}_{k^{op}} \\ I_{n_z} & \dots & 0_{n_z \times n_z} & 0_{n_z \times n_z} \\ \vdots & \ddots & \vdots & \vdots \\ 0_{n_z \times n_z} & \dots & I_{n_z} & 0_{n_z \times n_z} \end{pmatrix}, \quad B(\theta) = \begin{pmatrix} \tilde{\Psi} & 0_{n_z \times n_\upsilon} \\ 0_{n_z \times n_z} & 0_{n_z \times n_\upsilon} \\ \vdots & \vdots \\ 0_{n_z \times n_z} & 0_{n_z \times n_\upsilon} \end{pmatrix},$$

$$C(\theta) = HA(\theta), \quad D(\theta) = \begin{pmatrix} HB(\theta) & V \end{pmatrix},$$

where H is an $n_y \times k^{op} n_z$ selection matrix and $\tilde{\Phi}_1 = \Phi_1(\theta), \tilde{\Phi}_2 = \Phi_2(\theta), ..., \tilde{\Phi}_{k^{op}} = \Phi_{k^{op}}(\theta)$ and $\tilde{\Psi} = \Psi(\theta)$ depend non-linearly on θ^* through the restrictions:

$$\begin{aligned} \left(\Gamma_0 + \check{\Pi}\Gamma_f\right)\tilde{\Phi}_1 - \Gamma_f\left(\tilde{\Phi}_1^2 + \tilde{\Phi}_2\right) + \check{\Gamma}_b &= 0_{n_z \times n_z}, \\ \left(\Gamma_0 + \check{\Pi}\Gamma_f\right)\tilde{\Phi}_2 - \Gamma_f\left(\tilde{\Phi}_1\tilde{\Phi}_2 + \tilde{\Phi}_3\right) + \Upsilon_2 - \check{\Pi}\Gamma_{bb} &= 0_{n_z \times n_z}, \\ \left(\Gamma_0 + \check{\Pi}\Gamma_f\right)\tilde{\Phi}_3 - \Gamma_f\left(\tilde{\Phi}_1\tilde{\Phi}_3 + \tilde{\Phi}_4\right) + \Upsilon_3 - \check{\Pi}\Upsilon_2 &= 0_{n_z \times n_z}, \\ &\vdots \\ \left(\Gamma_0 + \check{\Pi}\Gamma_f\right)\tilde{\Phi}_{k^{op}} - \Gamma_f\left(\tilde{\Phi}_1\tilde{\Phi}_{k^{op}}\right) - \check{\Pi}\Upsilon_{k^{op}-1} &= 0_{n_z \times n_z}, \\ \left(\Pi^{-1}\left(\Gamma_0 - \Gamma_f\tilde{\Phi}_1\right)\right)^{-1} &= \tilde{\Psi}, \end{aligned}$$

where $\check{\Pi} = \Pi^{-1}R\Pi, \check{\Gamma}_b = \Gamma_{bb} + \check{\Pi}\Gamma_0$ and $\Gamma_{bb} = \Gamma_b + \dot{\Gamma}_b$. For a complete description of the derivation of the restrictions for the expectations correction approach, see the supplementary material of Angelini and Fanelli (2016). In this way, if the agents' forecast model belongs to the class of state space models with k^{op} lags, the reduced form solution of the 'pseudo structural' model in (22)-(23) is consistent with the dynamics of the data.

3. Optimal lag selection

A key role in this exercise is the determination of $\hat{k}^{op}$, i.e. the optimal number of lags to capture the dynamics in the data. In this section we propose the complete algorithm used for specifying the 'pseudo structural' model in (22)-(23). Assuming that the solution of the structural model belongs to class of state space models:

$$
\begin{aligned}
Z_t &= A_1 Z_{t-1} + \ldots + A_k Z_{t-k} + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \Sigma_\varepsilon\right), & (24)\\
y_t &= H x_t + V \upsilon_t, & (25)
\end{aligned}
$$

we want to determine the value of $k^{op} \in \{1, 2, \ldots, k^{max}\}$ which best fits the data. In doing so, we consider different information criteria: the Akaike Information Criterion (AIC, Akaike, 1973,1974), the Hannan and Quinn Information Criterion (HQC, Hannan and Quinn, 1979), the Bayesian Information Criterion (BIC, Schwarz, 1978), and the bootstrap-based AIC criterion (AICb, Cavanaugh and Shumway, 1997). Once k^{op} has been determined, we can amend the dynamic structure of the DSGE model with the expectations correction terms. Following the non-parametric bootstrap proposed by Stoffer and Wall (1991), our approach can be summarized in the following steps:

1. Estimate the model in (24)-(25) with $k = 1$ by using a standard maximum likelihood Kalman filter approach.
2. Compute the AIC, BIC and HQC criteria

$$
\begin{aligned}
\text{AIC}_k &= -2\ell_k\left(\hat{\tau}|y_t\right) + 2q,\\
\text{HQC}_k &= -2\ell_k\left(\hat{\tau}|y_t\right) + 2q\log\log(T),\\
\text{BIC}_k &= -2\ell_k\left(\hat{\tau}|y_t\right) + q\log(T),
\end{aligned}
$$

   where $\ell_k\left(\hat{\tau}|y_t\right)$ is the maximum value of the log-likelihood function associated with the system (24)-(25) where the number of lags is k, $\hat{\tau}$ is the $q \times 1$ vector of the parameters, and T is the sample size.
3. Compute the innovation form of the estimated model:

$$
\begin{aligned}
\hat{x}_{t+1|t} &= \hat{A}\hat{x}_{t|t-1} + \hat{K}_t\hat{\xi}_t, & (26)\\
y_t &= H\hat{x}_{t|t-1} + \hat{\xi}_t, & (27)
\end{aligned}
$$

   where

$$
x_t = \begin{pmatrix} \hat{Z}_{t+1|t} \\ \vdots \\ \hat{Z}_{t-k+2|t-k+1} \end{pmatrix}, \quad \hat{A}\begin{pmatrix} \hat{A}_1 & \hat{A}_2 & \ldots & \hat{A}_k \\ I_{n_z} & 0_{n_z\times n_z} & \ldots & 0_{n_z\times n_z} \\ \vdots & \ddots & \ldots & \vdots \\ 0_{n_z\times n_z} & 0_{n_z\times n_z} & I_{n_z} & 0_{n_z\times n_z} \end{pmatrix},
$$

where $\hat{\xi}_t$ is the vector of innovation residuals with estimated covariance matrix $\Sigma_{\xi,t}$ and $\hat{K}_t$ is the Kalman gain.

4. Given the innovation residuals $\hat{\xi}_t = y_t - H\hat{x}_{t|t-1}$, derive the corresponding standardized innovations:
$$\bar{\xi}_t = \Sigma_{\xi,t}^{-1/2}\hat{\xi}_t,$$
5. Sample with replacement, T times, from $\bar{\xi}_1, \bar{\xi}_2, \ldots, \bar{\xi}_T$ to obtain the bootstrap sample of standardized innovations $\bar{\xi}_1^*, \bar{\xi}_2^*, \ldots, \bar{\xi}_T^*$.
6. Generate the bootstrap sample $y_1^*, y_2^*, \ldots, y_T^*$ by solving, recursively, for $t = 1, \ldots, T$, the innovation form in (26)-(27):
$$\begin{aligned} \hat{x}_{t+1|t}^* &= \hat{A}\hat{x}_{t|t-1}^* + \hat{K}_t\Sigma_{\xi,t}^{-1/2}\bar{\xi}_t^*, \\ \hat{y}_t^* &= H\hat{x}_{t|t-1}^* + \Sigma_{\xi,t}^{-1/2}\bar{\xi}_t^*, \end{aligned}$$
with the initial condition $\hat{x}_{1|0}^* = \hat{x}_{1|0}$ fixed as in the data.
7. Estimate the model in (24)-(25) using y_t^* instead of y_t.
8. Repeat steps 5-7, B times, obtaining a sequence of bootstrap estimators of τ, $\hat{\tau}^{1,*}, \hat{\tau}^{2,*}, \ldots, \hat{\tau}^{B,*}$.
9. Compute the AICb criterion of Cavanaugh and Shumway (1997):
$$\text{AICb}_k = -2\ell_k\left(\hat{\tau}|y_t\right) + 2\left\{\frac{1}{B}\sum_{b=1}^{B} -2\left(\ell_k^b\left(\hat{\tau}^{b,*}|y_t\right) - \ell_k\left(\hat{\tau}|y_t\right)\right)\right\},$$
where $\ell_k^b\left(\hat{\tau}^*|y_t\right)$ is the value of the log-likelihood function computed using the maximum likelihood estimator of τ, $\hat{\tau}^{b,*}$, related to the b^{th} bootstrap iteration and the original sample y_t.
10. Repeat steps 1-9 for each $k = 1, \ldots, k^{max}$, obtaining a sequence of information criteria, $\text{AIC}_1, \ldots, \text{AIC}_{k^{max}}$, $\text{HQC}_1, \ldots, \text{HQC}_{k^{max}}$, $\text{BIC}_1, \ldots, \text{BIC}_{k^{max}}$, and $\text{AICb}_1, \ldots, \text{AICb}_{k^{max}}$.
11. Select the value of k^{op} in accordance with one of the information criteria considered:
$$\begin{aligned} k_{AIC}^{op} &= \arg\min_k \text{AIC}, \\ k_{HQC}^{op} &= \arg\min_k \text{HQC}, \\ k_{BIC}^{op} &= \arg\min_k \text{BIC}, \\ k_{AICb}^{op} &= \arg\min_k \text{AICb}. \end{aligned}$$

12. Specify and estimate the 'pseudo structural' model in (22)-(23) setting the value of k^{op} following one of the criteria considered above and using a Kalman filter maximum likelihood approach for the estimation.

4. Monte Carlo study

In this section we present the results of a Monte Carlo simulation study based on the univariate example of Section 2.1 in order to evaluate which of the proposed criteria perform better. The Data Generating Process (DGP) is given by (11) with $k^{op} = 2$, $\gamma_f = 0.4$,$\gamma_b = 0.3$, and $\gamma_{bb} = 0.1$. For this simulation we employed 1000 simulations, 100 bootstrap replications, and 3 different sample sizes, $T = 50, 100$ and 500. The results are reported in Table 1.

TABLE 1. Monte Carlo study.

	Model selection											
	T=50				T=100				T=500			
Lag	AIC	HQC	BIC	AICb	AIC	HQC	BIC	AICb	AIC	HQC	BIC	AICb
1	0.167	0.265	0.186	0.191	0.145	0.198	0.168	0.091	0.178	0.212	0.117	0.071
2	**0.272**	**0.416**	**0.524**	**0.567**	**0.394**	**0.576**	**0.729**	**0.827**	**0.448**	**0.697**	**0.850**	**0.893**
3	0.143	0.173	0.150	0.166	0.267	0.178	0.087	0.067	0.234	0.082	0.023	0.032
4	0.114	0.098	0.097	0.055	0.149	0.039	0.011	0.012	0.118	0.008	0.01	0.004
5	0.093	0.045	0.043	0.021	0.021	0.009	0.005	0.003	0.016	0.001	0	0
6	0.085	0.003	0	0	0.019	0	0	0	0.006	0	0	0
7	0.074	0	0	0	0.005	0	0	0	0	0	0	0
8	0.052	0	0	0	0	0	0	0	0	0	0	0

	Estimation											
	T=50				T=100				T=500			
	Pseudo structural		Structural		Pseudo structural		Structural		Pseudo structural		Structural	
Param.	$\hat{\theta}_T$	s.e.	$\hat{\theta}_T$	s.e.	$\hat{\theta}_T$	s.e.	$\hat{\theta}_T$	s.e.	$\hat{\theta}_T$	s.e.	$\hat{\theta}_T$	s.e.
$\gamma_f = 0.4$	0.471	0.196	0.398	0.312	0.396	0.169	0.395	0.290	0.402	0.077	0.322	0.063
$\gamma_b = 0.3$	0.198	0.105	0.396	0.126	0.289	0.098	0.428	0.084	0.298	0.056	0.424	0.038
$\gamma_{bb} = 0.1$	0.164	0.080	-	-	0.124	0.063	-	-	0.099	0.030	-	-

Table 1: Model selection (top panel) and estimation (bottom panel) related to the Monte Carlo simulation study presented in Section 4. In the top panel are reported the selection frequencies associated to each criterion considered for $k = 1, ..., 8$. In bold the selection frequencies associated to the true number of lags, i.e. $k = 2$. 'Pseudo structural' in the bottom panel represents the model with the auxiliary expectations correction term and 'structural' indicates the model with no additional lags ($\gamma_{bb} = 0$). The Monte Carlo experiment is based on 1000 simulations, 100 bootstrap iterations, and three different sample sizes, T=50,100 and 500.

In the top panel are the results regarding the model selection and in the bottom panel the results for what concerns the estimation of the structural parameters. Looking at the results in the top panel we obtain very useful information about the 'pseudo structural' specification. First, it is possible to observe that the AIC criterion has the worst ability in selecting the true number of lags, indeed the frequencies of selecting the true model are 27.2%, 39.4%, and 44.8% for $T = 50, 100$ and 500, respectively. Second, the AICb criterion produces the best results, having a percentage of selecting the true model of 56.7%, 82.7%, and 89.3% for $T = 50, 100$ and 500, respectively. Regarding the other criteria, the percentage of selecting the true number of lags for the HQC criterion is 41.6%, 57.6% and 69.7% for $T = 50, 100$ and $T = 500$, respectively, while the BIC criterion selects the true value of k for 52.4%, 72.9%, and 85.0% of the cases for $T = 50, 100$ and $T = 500$, respectively. From the bottom panel of Table 1 we can see how the omission of the second lag in the model specification leads to a wrong inference. Indeed, the estimation of the structural parameters γ_f and γ_b is distorted if we do not take into account that $\gamma_{bb} \neq 0$. In particular, we can observe that on increasing the sample size T, the values of the estimated parameters associated to the 'pseudo structural' model tend to become close to the true values, while the estimation of the parameters associated to the 'structural' form (in which $\gamma_{bb} = 0$), diverges from the true values. In particular, in the latter case, the estimate of γ_f is 0.398 if we consider $T = 50$ and became 0.322 if we increase the sample size to $T = 500$, in relation to a population value equal to 0.4. Looking at the results for γ_b, the estimates are stabler even though far from the true value 0.3. Indeed, the estimates of γ_b are 0.396, 0.428 and 0.424 for $T = 50, 100$ and $T = 500$, respectively.

5. Empirical illustration

The empirical analysis presented in this section is based on the DSGE model of An and Schorfheide (2007), which is estimated on U.S. quarterly data for the 'Great Moderation' period (1984Q2-2008Q3, $T = 98$). The model is described by the following equations:

$$\begin{aligned}
\tilde{x}_t &= E_t\tilde{x}_{t+1} + g_t - E_t g_{t+1} - \frac{1}{\tau}(r_t - E_t\pi_{t+1} - E_t z_{t+1}), && (28)\\
\pi_t &= \beta E_t\pi_{t+1} + \kappa(\tilde{x}_t - g_t), && (29)\\
r_t &= \rho_r r_{t-1} + (1-\rho_r)\psi_1\pi_t + (1-\rho_r)\psi_2(\tilde{x}_t - g_t) + \varepsilon_{r,t}, && (30)\\
g_t &= \rho_g g_{t-1} + \varepsilon_{g,t}, && (31)\\
z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}, && (32)
\end{aligned}$$

where $\varepsilon_{r,t} \sim WN(0,\sigma_i^2), i = r,g,z,$ (28) is a forward-looking output-gap equation where $\tilde{x}_t = x_t - x_t^p$ is the unobserved output gap (x_t is the output and x_t^p is the potential output), (29) is a forward-looking New-Keynesian Phillips Curve (NKPC) with slope κ and inflation rate π_t, (30) is the monetary policy rule with policy rate r_t, while (31)-(32) define two autoregressive processes of order one for the aggregate supply (g_t) and demand (z_t) disturbances (see An and Schorfheide (2007) for a derivation and discussion of the system in (28)-(32)). In the notation of (16)-(17), we have

$$Z_t = \begin{pmatrix} \tilde{x}_t \\ \pi_t \\ r_t \end{pmatrix}, \quad \epsilon_t = \begin{pmatrix} z_t \\ g_t \\ \varepsilon_{r,t} \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{g,t} \\ \varepsilon_{r,t} \end{pmatrix},$$

$$\Gamma_0 = \begin{pmatrix} 1 & 0 & \tau^{-1} \\ -\kappa & 1 & 0 \\ -(1-\rho_r)\psi_2 & -(1-\rho_r)\psi_1 & 1 \end{pmatrix}, \quad \Gamma_f = \begin{pmatrix} 1 & \tau^{-1} & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Pi = \begin{pmatrix} \tau^{-1}\rho_z & (1-\rho_g) & 0 \\ 0 & -\kappa & 0 \\ 0 & -(1-\rho_r)\psi_2 & 1 \end{pmatrix}, \quad \Gamma_b = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \rho_r \end{pmatrix},$$

$$\Sigma_\varepsilon = \begin{pmatrix} \sigma_z^2 & 0 & 0 \\ 0 & \sigma_g^2 & 0 \\ 0 & 0 & \sigma_r^2 \end{pmatrix}, \quad R = \begin{pmatrix} \rho_z & 0 & 0 \\ 0 & \rho_g & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The output gap $\tilde{x}_t$ is treated as unobservable, so we need a measurement equation which connects the observable variables $y_t = (\Delta x_t, \pi_t r_t)'$ to the state variables Z_t. The measurement system has the same structure as in

(19):

$$x_t = \begin{pmatrix} \tilde{x}_t \\ \pi_t \\ r_t \\ \tilde{x}_{t-1} \\ \pi_{t-1} \\ r_{t-1}) \end{pmatrix}, \quad H = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad (33)$$

and υ_t is a scalar white noise process with variance σ_υ^2. The reduced form solution of the system in (28)-(33) belongs to the class of state space models with representation as (1)-(2). Following the results in the top panel of Table 2, the AICb criterion of Cavanaugh and Shumway (1997) selects $k^{op} = 3$, so the 'pseudo structural' model is specified as follows:

$$\begin{aligned}
\tilde{x}_t &= E_t\tilde{x}_{t+1} + g_t - E_t g_{t+1} - \frac{1}{\tau}(r_t - E_t\pi_{t+1} - E_t z_{t+1}) + \zeta_1^{\tilde{x}}\tilde{x}_{t-1} + \zeta_2^{\tilde{x}}\tilde{x}_{t-2}, \\
\pi_t &= \beta E_t\pi_{t+1} + \kappa(\tilde{x}_t - g_t) + \zeta_1^{\pi}\pi_{t-1} + \zeta_2^{\pi}\pi_{t-2}, \\
r_t &= \rho_r r_{t-1} + (1-\rho_r)\psi_1\pi_t + (1-\rho_r)\psi_2(x_t - g_t) + \zeta_2^{r} r_{t-2} + \varepsilon_{r,t}, \\
g_t &= \rho_g g_{t-1} + \varepsilon_{g,t}, \\
z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}. \qquad (34)
\end{aligned}$$

The results of the estimations are presented in the bottom panel of Table 2.

As pointed out in Angelini et al. (2016), the DSGE model of An and Schorfheide (2007) is strongly identified if the vector of structural parameters is restricted to $\theta = \left(\kappa, \rho_r, \sigma_g^2, \sigma_\upsilon^2\right)$ while the other parameters are fixed to the values suggested by Komunjer and Ng (2011). The vector of the structural parameters related to the 'pseudo structural' form is $\theta^* = \left(\kappa, \rho_r, \sigma_g^2, \sigma_\upsilon^2, \zeta_1^{\tilde{x}}, \zeta_2^{\tilde{x}}, \zeta_1^{\pi}, \zeta_2^{\pi}, \zeta_2^{r}\right)$. The inclusion in the system of additional parameters could change the identifiability of the 'pseudo structural' model with respect to the standard one of An and Schorfheide (2007). For this reason, for each estimated model, we check whether the minimality (controllability and observability) and local identification conditions discussed in Komunjer and Ng (2011) are satisfied. From the results in the bottom panel of Table 2 we notice that the majority (three out of five) of the expectations correction parameters $\zeta = \left(\zeta_1^{\tilde{x}}, \zeta_1^{\pi}, \zeta_2^{\tilde{x}}, \zeta_2^{\pi}, \zeta_2^{r}\right)$ are significant at the 5% level, which underlines the mismatch between the dynamics of the agents expectations model and the dynamics of the structural model in (28)-(32). Another important result

TABLE 2. Estimation of the structural parameters and model selection.

Model selection				
Lags	AIC	HQC	BIC	AICb
1	283.469	299.152	368.800	284.556
2	239.968	265.062	**358.936**	242.305
3	222.606	**257.110**	361.302	**221.907**
4	**214.982**	258.896	370.892	224.114
5	220.932	274.256	408.959	245.525
6	235.700	298.434	468.490	275.943
7	242.722	314.866	504.581	320.521
8	247.966	329.520	533.870	346.744

Estimation				
	Pseudo structural		Structural	
Param.	$\hat{\theta}_T$	s.e.(Hess.)	$\hat{\theta}_T$	s.e.(Hess.)
κ	0.110	0.043	0.317	0.017
ρ_r	0.945	0.033	0.803	0.019
$\zeta_1^{\tilde{x}}$	-0.900	0.838	-	-
ζ_1^{π}	-0.552	0.165	-	-
$\zeta_2^{\tilde{x}}$	-0.899	0.323	-	-
ζ_2^{π}	-0.573	0.257	-	-
ζ_2^{r}	-0.045	0.058	-	-
σ_g^2	0.112	0.098	0.132	0.206
σ_v^2	0.057	0.011	0.101	0.215

Table 2: In the top panel the values of the criteria used for the model selection. In bold the selected number of lags for the criteria considered. In the bottom panel the estimation of the parameters associated to the 'Pseudo structural' model in (34) and the estimation of the parameter related to the 'Structural' system in (28)-(32). The rest of the parameters are calibrated to the values in Komunjer and Ng (2010).

taken from Table 2 is how the omission of relevant lags in the specification leads to a distorted estimation of the structural parameters. Indeed, looking at the slope of the New-Keynesian Phillips Curve, κ, it decreases from 0.317 to 0.110 if we consider the 'pseudo structural' model, indicating less impact of the output-gap, $\tilde{x}_t$, on the inflation rate π_t if the true dynamics are taken into account. In contrast, in what concerns the policy rule, we notice that the autoregressive parameter ρ_r significantly increases (0.945 as opposed to

0.803) if we estimate the ‘pseudo structural’ model. Finally, looking at the innovation autocorrelations for the ‘structural’ and for the ‘pseudo structural’ models reported in Figure 1 and Figure 2, respectively, we can observe that the significant autocorrelations found in the baseline theoretical model are not present in the innovations estimated in the ‘pseudo structural’ model.

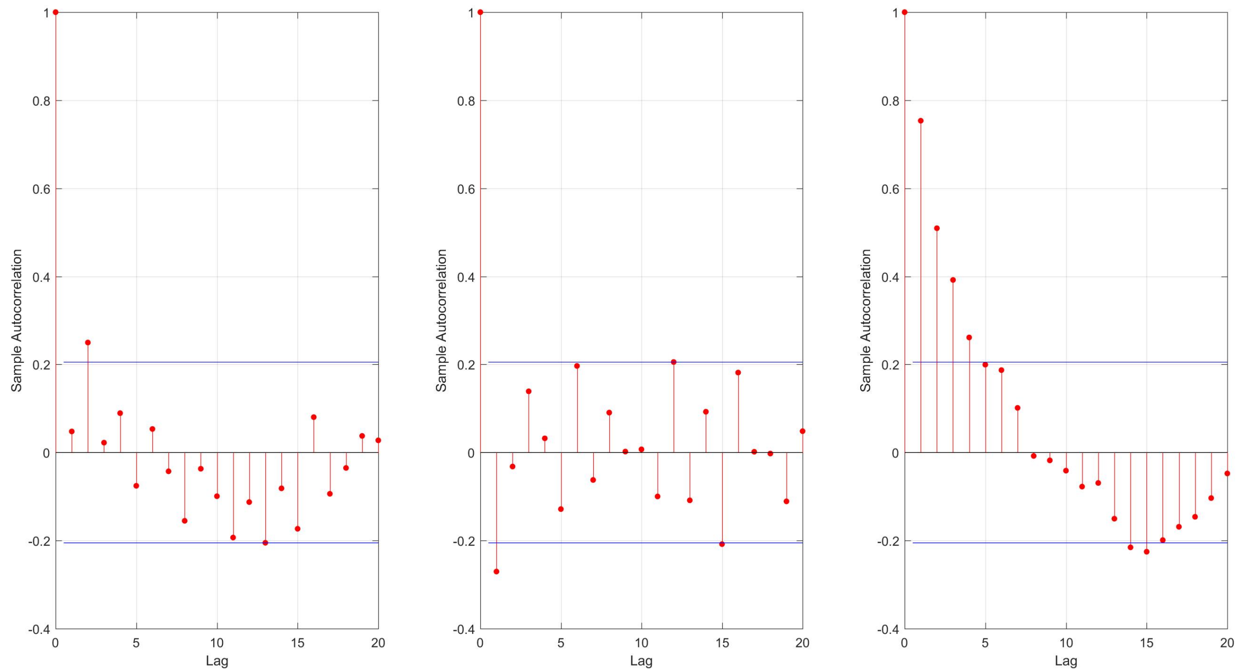

Figure 1: Sample autocorrelation functions of the innovation residuals based on Kalman filter maximum likelihood estimation of the An and Schorfheide's (2007) DSGE model estimated on U.S. quarterly data in the 'Great Moderation' (1984Q2-2008Q3, T=98). Blue lines represent the 95% confidence bands.

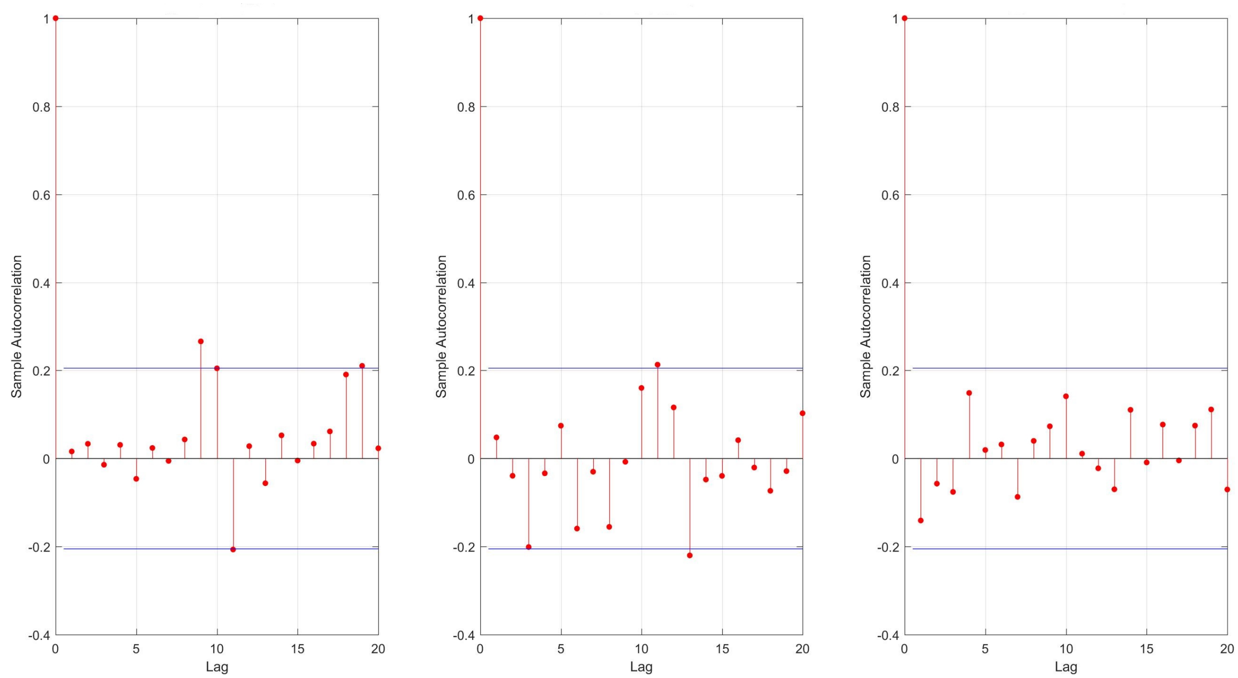

Figure 2: Sample autocorrelation functions of the innovation residuals based on Kalman filter maximum likelihood estimation of the 'pseudo Structural' approach applied on the An and Schorfheide's (2007) DSGE model estimated on U.S. quarterly data in the 'Great Moderation' (1984Q2-2008Q3, T=98). Blue lines represent the 95% confidence bands.

6. Conclusions

The expectations correction approach is a way to deal with the dynamic misspecifications which characterize DSGE models, using the information present in the data directly in the model specification. In this paper we have analysed the importance of model selection in the specification of a 'pseudo structural' DSGE model that is able to capture the actual dynamics present in the data. The results suggest that bootstrap techniques can be very useful for model selection, indeed, the bootstrap AICb criterion proposed by Cavanaugh and Shumway (1997) has the best empirical performance. Our approach is illustrated through a Monte Carlo simulation study and an empirical illustration based on the DSGE model of An and Schorfheide (2007), estimated on U.S. quarterly data.

Acknowledgments

The author is grateful to the Editor and two anonymous referees for their detailed comments on the original version of the paper.

References

Akaike, H. (1973), Information theory and an extension of the maximum likelihood principle. In 2nd International Symposium on Information Theory (Edited by B. N. Petrov and F. Csáki), 267-281. Akadémia Kiadó, Budapest.

Akaike, H. (1974), A new look at the statistical model identification. *IEEE Trans. Automat. Control* 19, 716-723.

An, S., Schorfheide, F. (2007), Bayesian analysis of DSGE models, *Econometric Reviews* 26, 113-172.

Angelini, G., Fanelli, L. (2016), Misspecification and expectations correction in New-Keynesian DSG. models, *Oxford Bulletin of Economics and Statistics* 78(5), 623-649.

Angelini, G., Cavaliere, G., Fanelli, L. (2016), Bootstrapping DSGE models, Dipartimento di Scienze Statistiche "Paolo Fortunati", Alma Mater Studiorum Università di Bologna, Quaderni di Dipartimento Serie Ricerche 3/2016.

Bårdsen, G., Fanelli, L. (2015), Frequentist evaluation of small DSGE models, *Journal of Business and Economic Statistics* 33, 307-322.

Binder, M., Pesaran, M. H. (1995), Multivariate rational expectations models and macroeconomic modelling: a review and some new results. In M. H. Pesaran and M. Wickens (eds.), *Handbook of Applied Econometrics*, pp. 139-187 (Ch. 3). Oxford: Blackwell.

Castelnuovo, E., Fanelli, L. (2015), Monetary policy indeterminacy and identification failures in the U.S.: Results from a robust test, *Journal of Applied Econometrics* 30, 857-1010.

Cavanaugh, J. E., Shumway, R. H. (1997), A bootstrap variant of AIC for state-space model selection, *Statistica Sinica* 7, 473-496

Consolo, A., Favero, C.A., Paccagnini, A. (2009), On the statistical identification of DSGE models, *Journal of Econometrics* 150, 99-115.

Cúrdia,V., Reis, R. (2010), Correlated disturbances and U.S. business cycle, *NBER* Working Paper 15774.

Del Negro, M., Schorfheide, F., Smets, F., Wouters, R. (2007), On the fit of New Keynesian models, *Journal of Business and Economic Statistics* 25, 123-143.

Fernández-Villaverde, J., Rubio-Ramírez, J., Sargent, T., Watson, M. (2007), ABCs (and Ds) of understanding VARs, *American Economic Review* 97, 1021-1026.

Franchi, M., Vidotto, A. (2013), A check for finite order VAR representations of DSGE models, *Economic Letters* 120, 100-103.

Franchi, M., Paruolo, P. (2015), Minimality of state space solutions of DSGE models and existence conditions for their VAR representation, *Computational Economics* 46, 613-626.

Hannan, E.J., Deistler, M. (1988), *The Statistical Theory of Linear Systems*, SIAM ed. Wiley, New York.

Hannan, E. J., Quinn, B. G. (1979), The determination of the order of an autoregression, *Journal of the Royal Statistical Society Series B* 41, 190-195.

Hansen, L.P. (2014), Nobel Lecture: Uncertainty outside and inside economic models, *Journal of Political Economy* 122, 945-987.

Hansen, L. P., Sargent, T. J. (1980), Formulating and estimating dynamic linear rational expectations models, *Journal of Economic Dynamics and Control* 2, 7-46.

Hansen, L. P., Sargent, T. J. (1981), Linear rational expectations models for dynamically interrelated variables, in: R. E. Lucas Jr. and T. J. Sargent, (Eds.), *Rational Expectations and Econometric Practice.* University of Minnesota Press, Minneapolis, 127–156.

Henry, S.G.B., Pagan, A.R. (2004), The econometrics of the New Keynesian policy model. Introduction, *Oxford Bulletin of Economics and Statistics* 66, Supplement, 581–607.

Komunjer, I., Ng, S. (2011), Dynamic identification of dynamic stochastic general equilibrium models, *Econometrica* 79, 1995-2032.

Lubik, T. A., Schorfheide, F. (2004), Testing for indeterminacy: an application to U.S. monetary policy, *American Economic Review* 94, 190-217.

Morris, S.D. (2016), VARMA representation of DSGE models, *Economic Letters* 138, 30-33.

Morris, S.D. (2017), DSGE pileups, *Journal of Economic Dynamics and Control* 74, 56-86.

Ravenna, F. (2007). 'Vector autoregressions and reduced form representations of DSGE models', *Journal of Monetary Economics*, Vol. 54, pp. 2048-2064.

Schwarz, G. (1978), Estimating the dimension of a model, *Annanls of Statistics* 6, 461-464.

Smets, F., Wouters, R. (2007), Shocks and frictions in U.S. business cycles, *American economic Review* 97, 586-606.

Stoffer, D.S., Wall, K. (1991), Bootstrapping state-space models: Gaussian maximum likelihood estimation and the Kalman filter, *Journal of the American Statistical Association* 86, 1024-1033.

Zanetti, F. (2008), Labor and investment frictions in a real business cycle model, *Journal of Economic Dynamics and Control* 32, 3294-3314.