

# Temporalised Belief Revision in the Law

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**Abstract.** This paper presents a belief revision operator for legal systems that considers time intervals. This model relates techniques about belief revision formalisms and time intervals with temporalised rules for legal systems. Our goal is to formalise a temporalised belief base and corresponding timed derivation, together with a proper revision operator. This operator may remove rules when needed or adapt intervals of time when contradictory norms are added in the system.

**Keywords.** Norm Change, Belief Revision, Temporal Reasoning

## 1. Introduction and Motivation

One peculiar feature of the law is that it necessarily takes the form of a dynamic normative system [22,21]. Despite the importance of norm-change mechanisms, the logical investigation of legal dynamics was for long time underdeveloped. However, research is rapidly evolving and recent contributions exist.

In the eighties a pioneering research effort was devoted by Alchourrón, Gärdenfors and Makinson [4] to develop a logical model (AGM) for also modeling norm change. As is well-known, the AGM framework distinguishes three types of change operation over theories. Contraction is an operation that removes a specified sentence  $\phi$  from a given theory  $\Gamma$  (a logically closed set of sentences) in such a way that  $\Gamma$  is set aside in favor of another theory  $\Gamma_{\phi}^{-}$  which is a subset of  $\Gamma$  not containing  $\phi$ . Expansion operation adds a given sentence  $\phi$  to  $\Gamma$  so that the resulting theory  $\Gamma_{\phi}^{+}$  is the smallest logically closed set that contains both  $\Gamma$  and  $\phi$ . Revision operation adds  $\phi$  to  $\Gamma$  but it is ensured that the resulting theory  $\Gamma_{\phi}^{*}$  be consistent. Alchourrón, Gärdenfors and Makinson argued that, when  $\Gamma$  is a code of legal norms, contraction corresponds to norm derogation (norm removal) and revision to norm amendment. AGM framework has the advantage of being very abstract, as it works with theories consisting of simple logical assertions. For this reason, it can capture basic aspects of the dynamics of legal systems, such as the change obligations and permissions [7,14].

Some research has been carried out to reframe AGM ideas within richer rule-based logical systems [24,23]. However, also these attempts suffer from some drawbacks of

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standard AGM, among them the fact that the proposed frameworks fail to handle the temporal aspects of norm change: indeed, legal norms are qualified by temporal properties, such as the time when the norm comes into existence and belongs to the legal system, the time when the norm is in force, the time when the norm produces legal effects, and the time when the normative effects hold. Since all these properties can be relevant when legal systems change, [14] argues that failing to consider the temporal aspects of legal dynamics poses a serious limit to correctly model norm change in the law.

Unlike rich but complex frameworks such as the one of [14], this paper claims that belief revision techniques—which are based on an abstract and elegant machinery—can be reconciled with need to consider several temporal patterns of legal reasoning. In this work we are thus interested in the formalisation of a belief revision operator applied to an epistemic model that considers rules and time. We enrich a simple logic language with an interval-based model of time, to represent validity and effectiveness of a norm. The revision operator may remove rules when needed or adapt intervals of time when newer, contradictory norms are introduced in the system.

The layout of the paper is as follows. Section 2 shows an example to motivate the main ideas of our proposal. Section 3 proposes the notions of temporalised belief base and temporalised derivation. Section 4 presents a norm revision operator based on temporalised belief base. Section 5 reports on related work. Some conclusions end the paper.

## 2. Motivating Example

Let us first of all present a concrete example that will serve to motivate the main ideas of our proposal.

**EXAMPLE 1.** *Consider the following pieces of information regarding a legislative attempt to ease tax pressure for people that have been unemployed.*

- *A citizen was unemployed from 1980 to 1985.*
- *If unemployed from 1980 to 1983, then a tax exemption applies from 1984 to 1986, in order to increase individual savings.*
- *New authorities in government revoke tax exemption for years 1985 and 1986.*
- *Tax exemption reinstated for the year 1985 due to agreements with labor unions.*

*However, later on the legislators approved a new normative establishing that finally there is no tax-exemption for all citizens for the years 1985 and 1986.*

The previous situation seems to establish that, at the end, a tax exemption applies only for year 1985 for a while, before being revoked.

## 3. Legal System as Temporalised Belief Base

The problem of representing temporal knowledge and temporal reasoning arises in many disciplines, including Artificial Intelligence. A usual way to do this is to determine a *primitive* to represent time, and its corresponding *metric relations*. There are in the literature two traditional approaches to reasoning with and about time: a point based approach, as in [14], and an interval based approach as in [5,9]. In the first case, the emphasis is put on *instants* of time (e.g., timestamps) and a relation of precedence among them. In

the second case, time is represented as continuous sets of instants in which something relevant occurs. These intervals are identified by the starting and ending instants of time.

In this work, time intervals (like in [6,9]) will be considered. This design decision has been taken because it simplifies the construction of an revision operator which will be introduced below. That is, following the semantics of the temporalised rules proposed in [14] and explained in Section 3 (an adapted version), the revision operator in many cases only consists in modifying the intervals to maintain the consistency.

Besides, different temporal dimensions will be taken into account. That is, as it is mentioned in [14], in a normative system, norms have different temporal dimensions: the time of **validity** of a norm (when the norm enters in the normative system) and the time of **effectiveness** (when the norm can produce legal effects). Thus, if one wants to model norm modifications, then normative systems must be modelled by more complicated structures. In particular, a normative system is not just the set of norms valid in it, but it should also consider the normative systems where the norms are effective.

### 3.1. Preliminaries and Notation

We will adopt a propositional language  $\mathbb{L}$  with a complete set of boolean connectives:  $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ . Each formula in  $\mathbb{L}$  will be denoted by lowercase Greek characters:  $\alpha, \beta, \delta, \dots, \omega$ . We will say that  $\alpha$  is the complement of  $\neg\alpha$  and vice versa. The characters  $\sigma$  will be reserved to represent cut function for a change operator. We also use a consequence operator, denoted  $Cn(\cdot)$ , that takes sets of sentences in  $\mathbb{L}$  and produces new sets of sentences. This operator  $Cn(\cdot)$  satisfies *inclusion* ( $A \subseteq Cn(A)$ ), *idempotence* ( $Cn(A) = Cn(Cn(A))$ ), and *monotony* (if  $A \subseteq B$  then  $Cn(A) \subseteq Cn(B)$ ). We will assume that the consequence operator includes classical consequences and verifies the standard properties of *supraclassicality* (if  $\alpha$  can be derived from  $A$  by deduction in classical logic, then  $\alpha \in Cn(A)$ ), *deduction* ( $\beta \in Cn(A \cup \{\alpha\})$  if and only if  $(\alpha \rightarrow \beta) \in Cn(A)$ ) and *compactness* (if  $\alpha \in Cn(A)$  then  $\alpha \in Cn(A')$  for some finite subset  $A'$  of  $A$ ). In general, we will write  $\alpha \in Cn(A)$  as  $A \vdash \alpha$ . Note that the AGM model [4] represents epistemic states by means of belief sets, that is, sets of sentences closed under logical consequence. Other models use belief bases; i.e., arbitrary sets of sentences [10,18]. Our epistemic model is based on an adapted version of belief bases which have additional information (time intervals). The use of belief bases makes the representation of the legal system state more natural and computationally tractable. That is, following [20] (page 24), we consider that legal systems sentences could be represented by a limited number of sentences that correspond to the explicit beliefs of the legal system.

### 3.2. Time Interval

We will consider a universal finite *set of time labels*  $\mathbb{T} = \{t_1, \dots, t_n\}$  strictly ordered; each time label will represent an unique time instant. Simplifying the notation, we assume that  $t_i - 1$  is the immediately previous instant to the instant  $t_i$  and  $t_i + 1$  is the immediately posterior instant to the instant  $t_i$ .

Like in [16] we propose temporalised literals, however, we use intervals. We will consider an interval like finite ordered sequence of time labels  $t_i, \dots, t_j$  where  $i, j$  are natural numbers ( $i \leq j$ ) and  $t_i, \dots, t_j \in \mathbb{T}$  denoting instances of time or *timepoints*. Thus we have expressions of the type  $\alpha^{interval}$ , where *interval* can be as follow:

- $[t_i, t_i]$ : meaning that  $\alpha$  holds at time  $t_i$ . Following [14]  $\alpha$  is transient (holding at precisely one instant of time). For simplicity  $[t_i, t_i] = [t_i]$ .
- $[t_i, \infty]$ : meaning that  $\alpha$  holds from  $t_i$ . Following [14]  $\alpha$  is persistent from  $t_i$ .
- $[t_i, t_j]$ : meaning that  $\alpha$  holds from time  $t_i$  to  $t_j$  with  $t_i < t_j$ .

Then we will consider a set of time intervals  $\mathbb{I}$  which contains intervals as those described previously. Thus, for simplicity, we can have expressions like  $\alpha^J$  where  $J \in \mathbb{I}$ . Intervals in  $\mathbb{I}$  will be denoted by uppercase Latin characters:  $A, B, C, \dots, Z$ . Two intervals may not be disjoint, as defined next.

**Definition 1.** *Contained interval.* Let  $R, S \in \mathbb{I}$  be two intervals. We say that  $R$  is contained in  $S$ , denoted  $R \subseteq S$  if and only if for all  $t_i \in R$  it holds that  $t_i \in S$ .

**Definition 2.** *Overlapped interval.* Let  $R, S \in \mathbb{I}$  be two intervals. We say that  $R$  and  $S$  are overlapped, denoted  $R \approx S$  if and only if there exists  $t_i \in R$  such that  $t_i \in S$ .

**EXAMPLE 2.** Let  $R, S, V \in \mathbb{I}$  where  $R = [t_3, t_7]$ ,  $S = [t_4, t_6]$  and  $V = [t_5, t_9]$  with  $t_3, t_4, t_5, t_6, t_7, t_9 \in \mathbb{T}$ . Then  $S \subseteq R$ ,  $R \approx V$  and  $S \approx V$ .

### 3.3. Temporalised Belief Base

As rules are part of the knowledge, they are subject of temporal validity too: the *time of force* of a rule, i.e., the time when a rule can be used to derive a conclusion given a set of premises. In this perspective we can have expressions like

$$(\alpha^{[t_a, t_b]} \rightarrow \beta^{[t_c, t_d]})^{[t_e, t_f]}$$

meaning that the rule is in force from timepoint  $t_e$  to  $t_f$ , or in other words, we can use the rule to derive the conclusion at time from time  $t_e$  to  $t_f$ . The full semantics of this expression is that from time  $t_e$  to  $t_f$  we can derive that  $\beta$  holds from time  $t_c$  to  $t_d$  if we can prove that  $\alpha$  holds from time  $t_a$  to  $t_b$ . But now we are doing a derivation from time  $t_e$  to  $t_f$ , so the conclusion  $\beta^{[t_c, t_d]}$  is derived from time  $t_e$  to  $t_f$  and the premise  $\alpha^{[t_a, t_b]}$  must be derived from time  $t_e$  to  $t_f$  as well. In the same way a conclusion can persist, this applies as well to rules and then to derivations.

Thus, it is possible to define *temporalised belief base* which will contain temporalised literal and temporalised rules (see Example 3). This base represents a legal system in which each temporalised sentence defines a norm whose time interval determines the validity and effectiveness time.

**EXAMPLE 3.** A legal system can be represented by the temporalised belief base  $\mathbb{K} = \{\alpha^{[t_1, t_3]}, \alpha^{[t_4]}, (\alpha^{[t_1, t_4]} \rightarrow \beta^{[t_4, t_6]})^{[t_4, t_6]}, \beta^{[t_5, t_6]}, \beta^{[t_6, t_8]}, \beta^{[t_{10}]}, \delta^{[t_{11}]}, (\delta^{[t_{11}]} \rightarrow \beta^{[t_{15}, t_{20}]})^{[t_5, t_6]}, \omega^{[t_2, t_8]}, (\omega^{[t_4]} \rightarrow \beta^{[t_6, \infty]})^{[t_{21}, t_{22}]}, \varepsilon^{[t_1, \infty]}\}$ .

This type of belief base representation implies that a sentence can appear more than once in a temporalised belief base; but from the point of view of the temporalised sentences stored in the temporalised belief base there is no redundancy because each temporalised sentence has different time interval. For instance, consider Example 3,  $\alpha$  appears two times, but with different intervals. In this case, we will say that  $\alpha$  is **intermittent** and it means that  $\alpha$  is held from  $t_1$  to  $t_3$  and it is held in the instant  $t_4$ . Besides, if the intervals of a sentence are overlapped ( $\beta^{[t_5, t_6]}$ ,  $\beta^{[t_6, t_8]}$  in Example 3), despite that the time interval of the sentence intuitively be only one ( $[t_5, t_8]$ ), we decided to maintain all versions because will be more suitable when we will model the dynamics of the legal system.

### 3.4. Temporalised Derivation

Note that a norm can explicitly be in a temporalised belief base,  $\alpha^{[t_5]} \in \mathbb{K}$  in Example 3. However, a norm can implicitly be represented in a temporal belief base if some conditions are held. For instance, in Example 3,  $\beta$  is implicitly represented with  $\omega^{[t_2, t_8]}$ ,  $(\omega^{[t_4]} \rightarrow \beta^{[t_6, \infty]})^{[t_{21}, t_{22}]}$  due to the antecedent of the rule is held in  $t_4$  by the temporalised sentence  $\omega^{[t_2, t_8]}$ . Next, temporalised derivation for a sentence are defined to capture this notion. To do this, first we define a temporalised derivation in a time instant and then we give a definition of temporalised derivation in time interval. The last mentioned is based on the previous.

**Definition 3.** *Temporalised derivation in a time instant.* Let  $\mathbb{K}$  be a set of temporalised sentences and  $\alpha^{[t_i]}$  be a temporalised sentence. We say that  $\alpha^{[t_i]}$  is derived from  $\mathbb{K}$  (denoted  $\mathbb{K} \vdash^t \alpha^{[t_i]}$ ) if and only if:

- $\alpha^J \in \mathbb{K}$  and  $t_i \in J$ , or
- $(\beta^H \rightarrow \alpha^P)^Q \in \mathbb{K}$  and  $t_i \in P$  and  $\mathbb{K} \vdash^t \beta^{[t_j]}$  for all  $t_j \in H$ .

**Definition 4.** *Temporalised derivation in a time interval.* Let  $\mathbb{K}$  be a set of temporalised sentences and  $\alpha^{[t_i, t_j]}$  be a temporalised sentence. We say that  $\alpha^{[t_i, t_j]}$  is derived from  $\mathbb{K}$  (denoted  $\mathbb{K} \vdash^t \alpha^{[t_i, t_j]}$ ) if and only if  $\mathbb{K} \vdash^t \alpha^{[t_p]}$  for all  $t_p \in [t_i, t_j]$ .

To compute the temporalised derivation of a sentence checking each instant of the intervals benefits us in special cases where implicit sentences need temporalised sentences with overlapped intervals as antecedents. To determine the time interval of the implicitly derived temporal sentence, the temporal consequence will be defined below.

**Definition 5.** *Temporalised consequence.* Let  $\mathbb{K}$  be a set of temporalised sentences and  $\alpha^{[t_i, t_j]}$  be a temporalised sentence. We say that  $\alpha^{[t_i, t_j]}$  is a temporalised consequence of  $\mathbb{K}$  ( $\alpha^{[t_i, t_j]} \in \text{Cn}^t(\mathbb{K})$ ) if and only if  $\mathbb{K} \vdash^t \alpha^{[t_i, t_j]}$ .

**EXAMPLE 4.** Consider again the temporalised belief base of Example 3. Then,  $\mathbb{K} \vdash^t \beta^{[t_4, \infty]}$ , that is,  $\beta^{[t_4, \infty]} \in \text{Cn}^t(\mathbb{K})$ ; and  $\mathbb{K} \vdash^t \alpha^{[t_1, t_4]}$ , that is,  $\alpha^{[t_1, t_4]} \in \text{Cn}^t(\mathbb{K})$ .

Note that the underlying semantics of this type of derivation (legal system) differs from that in propositional logic when we want to represent the knowledge [9]. Note that, following Definition 4, the **interval of an implicitly derived sentence** will be the interval of the consequent of the rule that derives the conclusion of the proof. For instance, suppose that  $\mathbb{K} = \{\gamma^{[t_2, t_5]}, (\gamma^{[t_3, t_4]} \rightarrow \varepsilon^{[t_6, t_9]})^{[t_1, \infty]}\}$  then the time interval of  $\varepsilon$  is  $[t_6, t_9]$ .

In this proposal, a **contradiction** arise when two complementary sentences can be derived with time intervals overlapped. For instance, suppose  $\mathbb{K} = \{\alpha^{[t_2, t_9]}, \neg\alpha^{[t_1, t_3]}\}$ , in this case there exist a contradiction. However, consider  $\mathbb{K} = \{\alpha^{[t_5]}, \neg\alpha^{[t_1, t_3]}\}$ , in this case, we will say that  $\mathbb{K}$  does not have contradictions. Moreover, we will say that a temporalised belief base is **temporally consistent** if the base does not have contradictions. The temporalised belief base of Example 3 is temporally consistent.

## 4. Legal Belief Revision

A legal system should be temporally consistent, i.e., it cannot contain contradictory norms at any time. Hence, we propose a **norm prioritised revision operator** that allows to consistently add a temporalised sentence  $\alpha^{[t_i, t_j]}$  to a consistent legal system  $\mathbb{K}$ .

This special revision operator is inspired in the rule semantics explained above in Section 3 (an adapted version from that proposed in [14]). Thus, following the concept of consistency proposed in Section 3, the revision operator may remove temporalised sentences or, in some cases, may only modify the intervals to maintain consistency.

To incorporate a norm  $\neg\beta^J$  into a legal system, it is necessary to consider all possible contradictions that may arise if the norm is added without checking for consistency. For this, it is necessary to compute all proofs of  $\beta$  considering only those temporalised sentences  $\beta^P$  whose effectiveness time is overlapped with the time interval  $J$ , that is,  $J \approx P$ . Note that, computing all minimal proofs of a temporal sentence considering only those which time interval is overlapped with the time interval of the input sentence, is an optimized version. Next, a set of minimal proof for a sentence will be defined.

**Definition 6.** Let  $\mathbb{K}$  be a temporalised belief base and  $\alpha^J$  a temporalised sentence. Then,  $\mathbb{H}$  is a minimal proof of  $\alpha^J$  if and only if

1.  $\mathbb{H} \subseteq \mathbb{K}$ ,
2.  $\alpha^P \in \text{Cn}^t(\mathbb{H})$  with  $J \approx P$ , and
3. if  $\mathbb{H}' \subset \mathbb{H}$ , then  $\alpha^P \notin \text{Cn}^t(\mathbb{H}')$  with  $J \approx P$ .

Given a temporalised sentence  $\alpha^J$ , the function  $\Pi(\alpha^J, \mathbb{K})$  returns the set of all the minimal proofs for  $\alpha^J$  from  $\mathbb{K}$ .

**REMARK 1.** Each set of  $\Pi(\alpha^J, \mathbb{K})$  derives  $\alpha$  in at least one time instant of  $J$ .

**EXAMPLE 5.** Consider the temporalised belief base of Example 3. Then  $\Pi(\beta^{[t_5, t_6]}, \mathbb{K}) = \{\mathbb{H}_1, \mathbb{H}_2, \mathbb{H}_3, \mathbb{H}_4\}$  where:

- $\mathbb{H}_1 = \{\alpha^{[t_1, t_3]}, \alpha^{[t_4]}, (\alpha^{[t_1, t_4]} \rightarrow \beta^{[t_4, t_6]})^{[t_4, t_6]}\}$ ,
- $\mathbb{H}_2 = \{\beta^{[t_5, t_6]}\}$ ,
- $\mathbb{H}_3 = \{\beta^{[t_6, t_8]}\}$ ,
- $\mathbb{H}_4 = \{\omega^{[t_2, t_8]}, (\omega^{[t_4]} \rightarrow \beta^{[t_6, \infty]})^{[t_{21}, t_{22}]}\}$

Note that  $\mathbb{H}_1$  is minimal due to  $\alpha$  should be derived from  $t_1$  to  $t_4$  to use the rule  $(\alpha^{[t_1, t_4]} \rightarrow \beta^{[t_4, t_6]})^{[t_4, t_6]}$  hence,  $\alpha^{[t_1, t_3]}$  and  $\alpha^{[t_4]}$  should be in  $\mathbb{H}_1$ .

Our operator is based on a selection of sentences in the knowledge base that are relevant to derive the sentence to be retracted or modified. In order to perform a revision, following kernel contractions [19], this approach uses *incision functions*, which select from the minimal subsets entailing the piece of information to be revoked or modified. We adapt this notion of incision function proposed in [19] to our epistemic model. An incision function only selects sentences that can be relevant for  $\alpha$  and at least one element from each  $\Pi(\alpha^J, \mathbb{K})$ , as follows.

**Definition 7. Incision function.** Let  $\mathbb{K}$  be a temporalised belief base, an incision function  $\sigma$  for  $\mathbb{K}$  is a function such that for all  $\alpha^J \in \text{Cn}^t(\mathbb{K})$ :

- $\sigma(\Pi(\alpha^J, \mathbb{K})) \subseteq \bigcup(\Pi(\alpha^J, \mathbb{K}))$ .
- For each  $\mathbb{H} \in \Pi(\alpha^J, \mathbb{K})$ ,  $\mathbb{H} \cap \sigma(\Pi(\alpha^J, \mathbb{K})) \neq \emptyset$ .

In Hansson's work it is not specified how the incision function selects the sentences that will be discarded of each minimal proof. In our approach, this will be solved by considering those sentences that can produce legal effects in favour of a possible contradiction with the new norm. Thus, if the new norm is  $\neg\beta^J$  then the incision function will select the temporalised sentences  $\beta^P$  or  $(\alpha^Q \rightarrow \beta^F)^H$  of each  $\Pi(\beta^J, \mathbb{K})$ .

**Definition 8.** *Search consequence function.*  $Sc: \mathbb{L} \times \mathbb{K} \mapsto \mathbb{K}$ , is a function such that for a given sentence  $\alpha$  and a given temporalised base  $\mathbb{K}$  with  $\mathbb{H} \subseteq \mathbb{K}$ ,

$$Sc(\alpha, \mathbb{H}) = \{\alpha^J : \alpha^J \in \mathbb{H}\} \cup \{(\beta^P \rightarrow \alpha^Q)^R : (\beta^P \rightarrow \alpha^Q)^R \in \mathbb{H} \text{ and } \beta \in \mathbb{L}\}$$

**Definition 9.** *Consequence incision function.* Given a set of minimal proofs  $\Pi(\alpha^J, \mathbb{K})$ ,  $\sigma^c$  is a consequence incision function if it is a incision function for  $\mathbb{K}$  such that

$$\sigma^c(\alpha^J, \mathbb{K}) = \bigcup_{\mathbb{H} \in \Pi(\alpha^J, \mathbb{K})} Sc(\alpha, \mathbb{H})$$

**EXAMPLE 6.** Consider Examples 3 and 5. Then,  $Sc(\beta, \mathbb{H}_1) = \{(\alpha^{[t_1, t_4]} \rightarrow \beta^{[t_4, t_6]})^{[t_4, t_6]}\}$ ,  $Sc(\beta, \mathbb{H}_2) = \{\beta^{[t_5, t_6]}\}$ ,  $Sc(\beta, \mathbb{H}_3) = \{\beta^{[t_6, t_8]}\}$ , and  $Sc(\beta, \mathbb{H}_4) = \{(\omega^{[t_4]} \rightarrow \beta^{[t_6, \infty]})^{[t_21, t_22]}\}$ . Thus,  $\sigma^c(\beta^{[t_5, t_6]}, \mathbb{K}) = \bigcup_{\mathbb{H} \in \Pi(\beta^{[t_5, t_6]}, \mathbb{K})} Sc(\beta, \mathbb{H}) = \{(\alpha^{[t_1, t_4]} \rightarrow \beta^{[t_4, t_6]})^{[t_4, t_6]}, \beta^{[t_5, t_6]}, \beta^{[t_6, t_8]}, (\omega^{[t_4]} \rightarrow \beta^{[t_6, \infty]})^{[t_21, t_22]}\}$

As mentioned before, the revision operator may remove temporalised sentences or, in some cases, may modify the intervals to maintain consistency. Next, a temporal projection will be defined based on a given time interval. The idea here is, given a temporalised belief base  $\mathbb{K}$  and given a time interval  $[t_i, t_j]$ , to return a temporalised belief base  $\mathbb{K}'$  containing those sentences from  $\mathbb{K}$  whose time intervals be out of  $[t_i, t_j]$ .

**Definition 10.** *Excluding temporal projection.* Let  $\mathbb{K}$  be a temporalised belief base and let  $[t_i, t_j]$  be a time interval where  $t_i, t_j \in \mathbb{T}$ . A excluding temporal projection of  $\mathbb{K}$  from  $t_i$  to  $t_j$ , denoted  $\mathbb{K}_{i,j}^{out}$ , is a subset of  $\mathbb{K}$  where for all  $\alpha^{[t_p, t_q]} \in \mathbb{K}$ ,  $\mathbb{K}_{i,j}^{out}$  will contain:

- $\alpha^{[t_p, t_i-1]}$  if  $t_p < t_i$ ,  $t_q \geq t_i$  and  $t_q \leq t_j$ .
- $\alpha^{[t_j+1, t_q]}$  if  $t_p \geq t_i$ ,  $t_q > t_j$  and  $t_p \leq t_j$ .
- $\alpha^{[t_p, t_i-1]}$  and  $\alpha^{[t_j+1, t_q]}$  if  $t_p < t_i$ ,  $t_q > t_j$ .
- $\alpha^{[t_p, t_q]}$  if  $t_q < t_i$  or  $t_p > t_j$ .

**REMARK 2.** Note that the case in which  $t_p \geq t_i$  and  $t_q \leq t_j$  the temporal sentence it is not considered. In this case, this sentence is erased.

**EXAMPLE 7.** Consider Example 6 and suppose that  $S$  is a temporalised belief base and  $S = \sigma^c(\beta^{[t_5, t_6]}, \mathbb{K})$ . Then,  $S_{i_6}^{out} = \{(\alpha^{[t_1, t_4]} \rightarrow \beta^{[t_4, t_6]})^{[t_4, t_6]}, \beta^{[t_7, t_8]}, (\omega^{[t_4]} \rightarrow \beta^{[t_7, \infty]})^{[t_21, t_22]}\}$ .

Following the notion of excluding temporal projection (Definition 10) a norm prioritized revision operator can be defined. That is, an operator that allows to consistently add temporalised sentences in a temporalised belief base. If a contradiction arises, then the revision operator may remove temporalised sentences or modify the corresponding intervals in order to maintain consistency.

**Definition 11.** Let  $\mathbb{K}$  be a temporalised belief base and  $\alpha^{[t_i, t_j]}$  be a temporalised sentence. The operator “ $\otimes$ ”, called prioritized revision operator, is defined as follow:

$$\mathbb{K} \otimes \alpha^{[t_i, t_j]} = (\mathbb{K} \setminus S) \cup \overset{out}{S}_i^{t_j} \cup \{\alpha^{[t_i, t_j]}\}$$

where  $S = \sigma^c(-\alpha^{[t_i, t_j]}, \mathbb{K})$

**EXAMPLE 8.** Consider Example 3 and suppose that a new norm  $\neg\beta^{[t_5, t_6]}$  it is wished to add. To do this, it is necessary to do  $\mathbb{K} \otimes \neg\beta^{[t_5, t_6]}$ . Consider Examples 5 and 6. Then,  $\mathbb{K} \otimes \neg\beta^{[t_5, t_6]} = \{\alpha^{[t_1, t_3]}, \alpha^{[t_4]}, (\alpha^{[t_1, t_4]} \rightarrow \beta^{[t_4]})^{[t_4, t_6]}, \beta^{[t_7, t_8]}, \beta^{[t_{10}]}, \delta^{[t_{11}]}, (\delta^{[t_{11}]} \rightarrow \beta^{[t_{15}, t_{20}]})^{[t_5, t_6]}, \omega^{[t_2, t_8]}, (\omega^{[t_4]} \rightarrow \beta^{[t_7, \infty]})^{[t_{21}, t_{22}]}, \varepsilon^{[t_1, \infty]}, \neg\beta^{[t_5, t_6]}\}$ . Note that, this new temporalised base is temporally consistent.

The following example shows how our operator works in a particular situation when a legal system undergoes many changes and has rules that complement each other.

**EXAMPLE 9.** Consider following temporalised belief base  $\mathbb{K} = \{\beta^{[t_1, t_{10}]}, (\beta^{[t_1, t_5]} \rightarrow \alpha^{[t_1, t_5]})^{[t_1, \infty]}, (\beta^{[t_6, t_{10}]} \rightarrow \alpha^{[t_6, t_{10}]} )^{[t_1, \infty]}, \delta^{[t_4]}\}$ . Note that,  $\mathbb{K} \vdash \alpha^{[t_1, t_{10}]}$  because  $\mathbb{K} \vdash \alpha^{[t_i]}$  for all  $t_i \in [t_1, t_{10}]$ . Suppose that it is necessary to adopt  $\neg\alpha^{[t_1, t_{10}]}$ . To do this, it is necessary to compute all the minimal proofs of  $\alpha^{[t_1, t_{10}]}$  in  $\mathbb{K}$ . In this case,  $\Pi(\alpha^{[t_1, t_{10}]}, \mathbb{K}) = \{ \{\beta^{[t_1, t_{10}]}, (\beta^{[t_1, t_5]} \rightarrow \alpha^{[t_1, t_5]})^{[t_1, \infty]}, (\beta^{[t_6, t_{10}]} \rightarrow \alpha^{[t_6, t_{10}]} )^{[t_1, \infty]}\} \}$ . Then,  $S = \sigma^c(\alpha^{[t_1, t_{10}]}, \mathbb{K}) = \{ (\beta^{[t_1, t_5]} \rightarrow \alpha^{[t_1, t_5]})^{[t_1, \infty]}, (\beta^{[t_6, t_{10}]} \rightarrow \alpha^{[t_6, t_{10}]} )^{[t_1, \infty]} \}$ . Thus,  $\overset{out}{S}_{t_{10}}^{t_1} = \emptyset$ . Therefore,  $\mathbb{K} \otimes \neg\alpha^{[t_1, t_{10}]} = \{ \beta^{[t_1, t_{10}]}, \delta^{[t_4]}, \neg\alpha^{[t_1, t_{10}]} \}$ .

## 5. Related work

Alchourrón and Makinson were the first to logically study the changes of a legal code [2,3,1]. The addition of a new norm  $n$  causes an enlargement of the code, consisting of the new norm plus all the regulations that can be derived from  $n$ . Alchourrón and Makinson distinguish two other types of change. When the new norm is incoherent with the existing ones, we have an *amendment* of the code: in order to coherently add the new regulation, we need to reject those norms that conflict with  $n$ . Finally, *derogation* is the elimination of a norm  $n$  together with whatever part of the legal code that implies  $n$ .

[4] inspired by the works above proposed the so called AGM framework for belief revision. This area proved to a very fertile one and the phenomenon of revision of logical theories has been thoroughly investigated. It is then natural to ask if belief revision offers a satisfactory framework for the problem of norm revision. Some of the AGM axioms seem to be rational requirements in a legal context, whereas they have been criticised when imposed on belief change operators. An example is the *success* postulate, requiring that a new input must always be accepted in the belief set. It is reasonable to impose such a requirement when we wish to enforce a new norm or obligation. However, it gives rise to irrational behaviors when imposed to a belief set, as observed in [11].

The AGM operation of contraction is perhaps the most controversial one, due to some postulates such as recovery [14,25], and to elusive nature of legal changes such as derogations and repeals, which are all meant to contract legal effects but in remarkably different ways [14]. Standard AGM framework is of little help here: it has the advantage



of being very abstract—it works with theories consisting of simple logical assertions—but precisely for this reason it is more suitable to capture the dynamics of obligations and permissions than the one of legal norms. In fact, it is hard in AGM to represent how the same set of legal effects can be contracted in many different ways, depending on how norms are changed. For this reason, previous works [12,13,14] proposed to combine a rule-based system with some forms of temporal reasoning.

Difficulties behind standard AGM have been considered and some research has been carried out to reframe AGM ideas within reasonably richer rule-based logical systems, combining AGM ideas with Defeasible Logic [23,15] or Input/Output Logic [7,24]. [25] suggested a different route, i.e., employing in the law existing techniques—such as iterated belief change, two-dimensional belief change, belief bases, and weakened contraction—that can obviate problems identified in [14] for standard AGM.

In this paper we showed to extend base revision with temporal reasoning, and, in particular, with time intervals. Our approach, like in [14], is able to deal with constituents holding in an interval of time, thus an expression  $\implies a^{[t_1, t_2]}$  meaning that  $a$  holds between  $t_1$  and  $t_2$  can be seen as a shorthand of the pair of rules from [14] (defeasible and defeater)  $\implies a^{[t_1, pers]}$  and  $\rightsquigarrow \neg a$ . We have taken this design decision because it simplifies the construction of the revision operator: following the semantics of the temporalised rule proposed in [14] and explained in Section 3 (an adapted version), the revision operator in many cases only consists in modifying the intervals to maintain the consistency.

Interval and duration based temporal defeasible logic have been developed [6,17]. [17] focuses on duration and periodicity and relationships with various forms of causality. [6] proposed a sophisticated interaction of defeasible reasoning and standard temporal reasoning (i.e., mutual relationships of intervals and constraints on the combination of intervals). In both cases it is not clear whether the techniques employed there are relevant to the application to norm modifications, and such works consider only a single temporal dimension.

In [8], belief revision in a temporal logic context is also addressed. However, they use modal operators over possible worlds to model belief changes. Here we are focused in a propositional language with time intervals following kernel contraction construction proposed in [19].

## 6. Conclusions and Future Work

In this work we have introduced a time-based belief revision operator for legal systems. A temporalised belief base and a temporalised belief derivation was defined, following the formalisation of temporal rules, suitable to model examples in the legal area. In this special belief base each piece of information is decorated with a time interval. In this scenario our novel belief revision operator allows the consistent addition of temporalised sentences in a temporalised belief base. If a contradiction arises, then the revision operator may either remove conflictive temporalised sentences or modify the intervals.

Change operators are presented following the AGM model [4] where the operators are defined through constructions and representation theorems. In this paper, the operator was defined through construction. As future work, rationality postulates will be given and its corresponding Representation Theorem for this new revision operator. This theorem proves the correspondence between the set of postulates and the construction.

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